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FAIRNESS, SELF-INTEREST, AND THE POLITICS OF THE PROGRESSIVE INCOME TAX

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ABSTRACT

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All advanced democracies have adopted income taxes with considerable progression in marginal tax rates. To explain this we examine the nature of individual and collective preferences over alternative tax schedules, in the context of a simple two-sector model. We first consider the case of altruistic or "sociotropic" citizens who view the income tax as a means of achieving a fairer or more egalitarian distribution of income. We show that greater marginal-rate progressivity may well be less fair; that a "fairest" tax, however defined, is always a linear or "flat-rate" schedule in which all incomes are taxed at the same marginal rate; and that with a purely sociotropic electorate there exists a flat-rate schedule which is a majority equilibrium. We then show that with "self-interested" voters who seek to minimize their own tax burdens, greater marginal-rate progression may well be preferred by middle- and upper-income voters; that for middle-income citizens the optimal schedule is a sharply progressive one; and that within the set of individually optimal schedules there exists a majority equilibrium, which is a progressive schedule which minimizes the burden on median-income or middle class citizens, at the expense of lower- and upper-income taxpayers.

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1. INTRODUCTION

Distributional issues are the most divisive and potentially destabilizing a democratic society can face, and clearly the taxation of incomes is the most direct and transparent, and hence potentially most explosive, redistributive mechanism.

Yet in practice the potential instabilities do not seem to arise often. A highly stylized but broadly accurate summary of the salient facts might run as follows: All advanced industrial democracies impose direct taxes on incomes, and the income tax typically serves redistributive as well as revenue-raising ends. The incidence of the tax burden seems quite stable over time; while income tax schedules are revised from time to time, the changes are typically rather modest and incremental (e.g. readjusting brackets for inflation), and hardly of the chaotic and large-scale kind we might expect if the political process underlying the changes were driven by majority coalitions of "have-nots" getting together to tax away the incomes of the minority of "haves." The effective (as distinct from statutory) incidence across income classes is even more stable. Income tax paid as a fraction of before-tax income typically increases somewhat with income. The average effective tax rate is thus mildly progressive, and achieves some redistribution of incomes. The degree of redistribution is quite modest, however, and falls far short of complete equalization of after-tax income.

At the same time, however, statutory marginal rates increase more rapidly with income, in many cases dramatically: in Australia, Belgium and France, for example, the marginal tax rate on taxable (declared) income varies from zero in the lowest tax bracket to 60% or more in the highest bracket.* Statutory marginal rates show considerable progression in all the advanced industrial democracies, with maximum rates ranging from 39.6% in Denmark to a high of 83% in Great Britain. The extreme rates are to some degree misleading, of course, since often they apply to only a minuscule portion of the taxpaying population. A better idea of the effective progression of rates can be obtained from Table 1, which shows the statutory marginal

INSERT TABLE 1 ABOUT HERE

rates for taxpayers at the tenth, fiftieth and ninetieth percentiles of the before-tax income distribution, for various countries. The countries are grouped, somewhat subjectively, according to the degree

* Data from OECD (1981), for central government income tax schedules in effect during the mid-seventies. Inter-country comparisons are subject to the usual caveats concerning differing definitions of taxable income, joint versus individual taxation, the role of social security contributions and local income taxes, and so forth.

and nature of marginal-rate progressivity. In Australia and Great Britain the marginal rate is constant across all incomes in this range; despite the nominal progressivity of their tax schedules (from 0 to 61.5% in Australia, 34% to 83% in Great Britain), the income tax in these countries is, effectively, a linear or flat-rate tax for most of the population. In Austria, Germany, and Denmark the marginal rate is constant across low and middle incomes, but then shows some progression in the upper-income range. In the next group of countries there is some modest increase in rates over the lower half of the distribution, but most of the progression again occurs at higher income levels. The progression in rates extends down through all income levels in Norway and Sweden, and in the last two countries, Ireland and Belgium, the increase is actually greater over the lower half of the income distribution. The tax schedules chosen by different countries thus vary considerably. All, however, seem committed to some degree of progression; with only one apparently inadvertant exception,* marginal rates always increase, and never decrease, with income.

* In Belgium, because of a ceiling on the income tax surcharge the effective marginal rate decreases slightly in the highest bracket.

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It seems natural enough to suppose that in a democracy the choice of a tax schedule should reflect the preferences of the citizenry, and should be compatible with majority rule. But how well can theory account for these empirical facts, and explain the observed democratic preference for progressive taxation?

There have been a few relevant studies. A common framework is as follows: an income tax schedule must be chosen by majority voting over some set of admissable schedules. If the ith individual's before-tax income is Y_i, his tax burden under the schedule T is $T(Y_i)$, and his after-tax consumption or disposable income is $y_i = Y_i - T(Y_i)$. His average tax rate is thus $T(Y_i)/Y_i$, while his marginal rate is $t(Y_i) \equiv \frac{dT(Y_i)}{dY}$. An admissable schedule must raise enough revenue to meet a fixed, exogenously given revenue target, $R = \sum_i T(Y_i)$, and must typically satisfy certain "fairness" constraints as well (e.g. $T(Y) \leq Y$, $0 \leq t(Y) \leq 1$, perhaps $\frac{d(t)}{dY} \geq 0$).

In Foley's (1967) analysis, before-tax incomes are taken as exogenous and independent of the tax schedule chosen, and citizens are assumed to be egoistically motivated and strive to maximize their own disposable incomes in choosing among schedules. In Foley's main result the admissable class is further restricted to the class of linear or "flat-rate" schedules of the form $T(Y) = \alpha + \beta Y$, where β is the constant marginal tax rate, and α is a lump-sum tax payment (if $\alpha > 0$) or credit (if $\alpha < 0$). (In the latter, "negative income tax"

case the average rate $T(Y)/Y$ increases in Y, so the schedule would be progressive in the average (but not marginal) sense.) Foley shows that under these assumptions majority voting yields a transitive ordering of the set of admissable schedules, so an equilibrium outcome exists. If incomes are distributed in the usual left-skewed fashion, however, the equilibrium is at $\beta = 1$, and thus results in complete equalization of after-tax incomes — an empirically implausible result.

One unrealistic assumption underlying this result is that incomes are exogenous, so that even extensive redistributions do not affect the size of the pie. Romer (1975) has analyzed a more realistic model in which citizens can respond to high tax rates by substituting untaxable leisure for taxable work effort. Again, the admissable schedules are assumed to be linear, and voting is "egoistic," with each voter trying to maximize a Cobb-Douglas utility function which depends solely on "own" after-tax income, and leisure. Romer shows that when the range of the admissable tax rates is restricted to those at which all citizens would continue to work (rather than choose to be voluntarily unemployed and live off their tax credit), voter preferences over tax rates are single-peaked, so majority rule is transitive and there again exists an equilibrium. Unlike Foley's this is typically an interior equilibrium which lies between the extremes of complete redistribution or none at all, a more plausible result. However when the restrictions (which relate to the underlying parameters of the model) are relaxed, an equilibrium need not exist. Moreover even when there is an equilibrium, it may (again depending on the underlying parameters) be at $\alpha > 0$, and thus be a regressive one. Since the results are sensitive to the precise specification and parameter values of the model, it would presumably require careful econometric investigation to see how well this model can account for our various stylized facts.

It seems quite clear, however, that neither it nor the Foley model can explain the prevalence of increasing-rate schedules. As Foley shows, once nonlinear tax schedules which allow for the possibility of varying marginal rates are introduced into the feasible set, voting cycles become inevitable, and no majority equilibrium exists; the same will clearly also be true in the Romer structure. Since nonlinear schedules with increasing marginal rates are clearly admissable members of the political agendas of all advanced industrial democracies, self-interested voting would lead to gross instability and cycling over tax structures, with new majority coalitions perpetually emerging and overturning the existing tax code in favor of a new one which favors them. This picture of perpetual chaos is hardly plausible empirically.

An alternative possibility is that citizens view the income distribution as a pure public good (Thurow (1971)), and that their preferences over redistributive tax schedules are primarily reflections of their views on fairness and social justice. The implications of this approach for majority voting have been explored by Hamada (1973). Again, before-tax incomes are taken to be exogenous and independent of taxes. The voter in question, j, assumes that all

citizens share a common utility or welfare function, U^j, strictly concave in income, but differ in their incomes. Thus if citizen i has an after-tax income of y_i, his welfare (in j's view) is $U^j(y_i)$. Each citizen j has an essentially Benthamite social welfare function, in which social welfare is simply the sum of individual welfare levels; thus citizen j, when judging two tax schedules T, T' which yield post-tax income distributions y and y' respectively, will prefer T to T' if and only if

$$\sum_i U^j(y_i) > \sum_i U^j(y_i').$$

Since different citizens may have different social welfare functions it is not self-evident as to whether majority voting over some set of schedules is well-behaved.

It turns out there is a majority equilibrium under this structure; it is, however, identical to Foley's, and results in complete equalization of after-tax incomes. In one respect the public-goods equilibrium is more robust, since Hamada places no restrictions on the form of admissable schedules (the Foley result depends heavily on their linearity). On the other hand Hamada also shows, in his major result, that in general there will be majority cycles over the non-equilibrium schedules. Thus the equilibrium is not a stable one, and there is no guarantee that successive majority votes over arbitrary changes in the tax code would necessarily converge on it. The more fundamental problem, however, is that the public goods equilibrium is implausibly egalitarian.

The incorporation of incentive effects could possibly lead to more plausible results. We know of no attempt to do this directly; some results in the optimal taxation literature, however, are at least suggestive, since the optimal taxation problem is closely akin in structure to the choice problem confronting a representative "Benthamite" voter in the Hamada framework. Shesenski's (1972) analysis of optimal linear taxation suggests that within the class of linear schedules, our representative (or median) voter would prefer a progressive schedule with $\alpha > 0$. Romer (1976) has pointed out, however, that this result rests on some rather special assumptions; moreover, because of pervasive nonconvexities which arise from the structure of the problem, it appears most unlikely that single-peakedness would hold except in special and atypical cases, so even within the class of linear tax schedules, the existence of a voting equilibrium (progressive or not), is problematical in the Hamada framework, once incentive effects are allowed for.

With nonlinear, increasing-rate schedules equilibrium is of course even less likely. Moreover other results in the optimal taxation literature suggest that our Benthamite, social-welfare-oriented voter, given a free choice on the form of the tax schedule, might well prefer one which is approximately linear (e.g., section 9 of Mirrlees (1971)), or even one in which the marginal tax rate diminishes with income (Theorem 4 of Sadka (1976)). The apparently stable democratic preference for increasing-rate schedules therefore remains rather mysterious, in light of these results.

* * *

In this paper we explore the issues of individual and collective choice of an income tax schedule in the context of a simple model which incorporates incentive effects which are similar in spirit, but different in detail, from those of the Romer and optimal taxation variety. We assume, conventionally, that individuals vary in their potential income-earning abilities. However in our model an individual faced with a high tax rate on his labor income responds, not by substituting untaxable leisure for taxable work effort, but rather by working in an untaxed "underground" economy, at a lower (but tax-free) wage rate. Individual welfare is measured by total (taxable and underground) after-tax income; the "fairness" of any tax thus depends directly on the after-tax income distribution it induces (rather than of the essentially unidentifiable cardinal utilities which play a central role in the optimal taxation literature).

In part 2 of the paper we describe the model and develop some necessary preliminary machinery and results; the reader uninterested in techinical details may wish to concentrate on sections 2.1 and 2.4, which contain the essential results for the subsequent argument, and skim sections 2.2 and 2.3. In part 3 we examine the relationship between fairness and the form of the tax schedule. We first show (Proposition 3.4) that for any progressive schedule T we can always find a <u>less</u> progressive schedule T′ which is fairer according to <u>any</u> fairness criteria; thus the relationship between fairness and marginal-rate progressivity is not necessarily positive, and may well be a perverse one. We then consider the choice of a fair tax schedule from the point of an individual decision-maker (a benevolent despot, an elected representative, or a "sociotropic" citizen-voter who views distributional issues in public-goods terms). In Proposition 3.6 we show that any such individual will always choose a linear or "flat-rate" schedule, no matter what particular fairness criterion he employs (though of course the parameters of the optimal linear schedule will depend on the criterion); a progressive schedule is never optimal (or "fairest") under any measure of fairness. We then turn to the issue of collective choice of a tax schedule by majority rule. In Proposition 3.8 we show that with sociotropic voters who judge tax proposals according to their fairness (variously measured), there necessarily exists a majority equilibrium. The equilibrium schedule is, once again, a linear one.

The fact that there exists a stable or equilibrium tax schedule is a step in the right direction; however the fact that the equilibrium is not progressive leaves unexplained the pervasive and apparently stable democratic preference for increasing-rate schedules. We thus turn elsewhere for an explanation, and in Part 4 examine the choice of a tax schedule from the viewpoint of a selfish or "egoistic" citizen-taxpayer, who is interested only in maximizing his welfare, rather than promoting social justice or distributional fairness. In Proposition 4.1 we show that a majority of such citizen-taxpayers, consisting of those in the middle- and upper-income ranges, may indeed prefer more progressivity in income taxation. This preference has

nothing to do with fairness — the more-progressive tax schedule they prefer over the status quo is unambiguously <u>less</u> fair — but rather arises from the fact that greater progression in marginal rates can actually reduce the tax burden on middle- and upper-income taxpayers, at the expense of the poor. In Proposition 4.2, we show that a "selfish" citizen-taxpayer interested in minimizing his own tax burden, and given a free choice on the form of the schedule, would choose a sharply progressive one, which imposes a low (in fact, zero) marginal rate on lower incomes, and high rates on large incomes. Individual preferences over different schedules of this form of course vary, since the parameters of the individually optimal schedule depend on the individual's own position in the income (or ability) distribution; in Proposition 4.3, however, we show that there nevertheless exists a majority equilibrium within the set of such schedules. The equilibrium is the most-preferred schedule of the median-income — i.e., typical middle-class — taxpayer.

Our results thus suggests that the observed stability and progressivity of income taxation in democratic societies has little to do with fairness or equity considerations, but rather arises from the success of the middle class in minimizing its own tax burden, at the expense of upper- and low-income taxpayers. This conclusion is, of course, quite reminiscent of Director's Law of Income Redistribution (Stigler (1970)).

2. PRELIMINARY RESULTS

2.1 The Model

We assume that individuals vary in their ability to earn incomes and that this ability, a composite of intrinsic intellectual and physical capacity, work ethic and energy, education, skills and training, and the like, is indexed by a single number n. The distribution of ability in the population is described by a distribution F, which we assume to have a continuous density $f \equiv \frac{dF}{dn}$. The ability index is normalized to lie between zero and one for all individuals in the population, and we assume there are at least a few individuals of every ability level in this range (i.e. $f(n) > 0$ for all $n \varepsilon (0,1)$). The total (or mean) ability level is $\bar{n} \equiv \int_0^1 n \cdot dF(n)$.

Individuals are worker-consumers in a simple one-good economy with two production sectors, a "legal", taxable sector, and an "underground", untaxable sector. We can think of an individual of ability n as possessing n units of standardized labor, which he can allocate as he chooses between the two sectors. We again normalize so that the wage rate in the taxable sector is always unity. Thus, if an individual of ability n chooses to work entirely in the taxable sector he can earn a pretax, taxable income of $X = n$. Conversely, if he works entirely in the untaxable sector his (untaxable) income is $z = w \cdot n$, where w is the prevailing wage rate in the underground economy; and if he works ℓ_n units in the latter and $n - \ell_n$ units in the former, his total pretax income is $Y = X + z = (n - \ell_n) + w \cdot \ell_n$. If he is

taxed T(X) on his taxable (declared) income, his after-tax taxable income is $x = X - T(X)$, while his total after-tax income is $y = x + z$.

The wage rate in the untaxed sector in general depends on the total labor supply L to that sector. We assume the wage schedule w(L) (or inverse labor demand function) is a strictly decreasing, twice-continuously differentiable function, with $w(0) \leq 1$ (otherwise individuals would work in the untaxed sector irrespective of tax considerations), and $w(\bar{n}) > 0$.

A <u>tax schedule</u> is a function T which specifies for any level of (taxable) income X the amount of tax T(X) to be paid. Taxable incomes necessarily lie between zero and one, and for the schedules of interest below we can without loss of generality suppose that tax liabilities or credits also lie in this interval; hence a tax schedule is a function defined on this domain and range, i.e. $T: [0,1] \rightarrow [-1,+1]$. We shall confine attention to schedules which are continuous, and continuously differentiable except possibly at some finite number of income levels which define different "tax brackets." Thus for any T the <u>marginal-rate</u> schedule $t \equiv \frac{dT}{dX}$ is defined except possibly at some finite number of points (and continuous wherever defined). An <u>admissable</u> schedule is one which is an increasing function of income, and whose marginal rate does not exceed 1 (i.e. $t(X) \varepsilon [0,1]$ for all X at which t(X) is defined), and which is not regressive in the sense that the marginal rate is also an increasing function of income (i.e. $t(X') \geq t(X)$ if $X' > X$). (It would also be natural to require $T(X) \leq x$ i.e. that tax liabilities not exceed taxable income. As shown in Section 2.4 below, however, this constraint is automatically satisfied by feasible schedules, so we do not impose it explicitly.)

Some descriptive terms for different kinds of schedules will be useful. A linear or flat-rate schedule is one of the form $T(X) = \alpha + \beta X$, whose marginal rate $t(X) = \beta$ is constant for all incomes. A progressive schedule T is one whose marginal-rate schedule t is strictly increasing over some range of incomes, or equivalently $t(0) < t(1)$. One schedule T is more progressive than another, T' (or equivalently, T' is less progressive than T) if the marginal-rate schedule t crosses t' from below; i.e. if there is some income level X^* such that $t(X) \leq t'(X)$ for $X < X^*$ and $t(X) \geq t'(X)$ for $X > X^*$, with strict inequalities holding on $I - \{X^*\}$ for some open interval I which contains X^*.

A feasible schedule is one which raises just enough revenue to meet an exogenously given revenue target. Since we are interested in redistributional issues, we shall suppose that the target is zero, so that the income taxation is purely redistributional. Thus, if we denote by R_T the total tax collected under the admissable schedule T, then T is feasible if and only if $R_T = 0$.

We now examine some consequences of these definitions and assumptions.

2.2 Individual Labor Supply

Let the tax schedule T and wage rate w be fixed. If an individual of type n works ℓ_n units in the untaxed sector and $(n - \ell_n)$

in the taxable sector (where $0 \leq \ell_n \leq n$), his after-tax consumption is $y = X - T(X) + z = (n - \ell_n) - T(n - \ell_n) + w\ell_n$. His problem is to maximize this quantity with respect to ℓ_n.

Let us suppose, initially, that the schedule T is continuously differentiable and that the marginal rate t is strictly increasing on [0,1]. Then, differentiating with respect to ℓ_n, we have $\frac{dy_n}{d\ell_n} = w - [1 - t(n - \ell_n)]$, and since t is increasing the maximizing value $\hat{\ell}_n$ will be as follows:

(2.1a) If $w > 1 - t(0)$ then $\hat{\ell}_n = n$ for all n.

(2.1b) If $w < 1 - t(1)$ then $\hat{\ell}_n = 0$ for all n.

(2.1c) If $w \in [1 - t(1), 1 - t(0)]$ and $w < 1 - t(n)$, then $\hat{\ell}_n = 0$.

(2.1d) If $w \in [1 - t(1), 1 - t(0)]$ and $w \geq 1 - t(n)$ then $\hat{\ell}_n$ must satisfy $w = 1 - t(n - \hat{\ell}_n)$.

Note that, since t is strictly increasing, there will be a unique $\hat{\ell}_n$ satisfying (2.1d). Hence all individuals of type n choose the same $\hat{\ell}_n$, which can be expressed as a labor supply function $\hat{\ell}_T(n;w)$. For $w \in [1 - t(1), 1 - t(0)]$ define $n_T(w)$ as the unique (since $[1 - t(\cdot)]$ is continuous and strictly decreasing on this interval) ability level for which $1 - t(n_T(w)) = w$, and let $n_T(w) = 1$ for $w < 1 - t(1)$ and $n_T(w) = 0$ for $w > 1 - t(0)$. Evidently $n_T(\cdot)$ is a continuous, decreasing function of w, and is strictly decreasing on $(1-t(1), 1-t(0))$. The individual labor supply function can then be explicitly characterized as follows:

$$\hat{\ell}_T(n;w) = \begin{cases} 0 & \text{if } n \leq n_T(w) \\ n - n_T(w) & \text{if } n \geq n_T(w) \end{cases} \tag{2.2}$$

using (2.1) above.

The relationship between these various quantities can be seen on Figure 2.1 below. An individual with taxable income equal to

FIGURE 2.1 ABOUT HERE

$n_T(w)$ faces a marginal rate $t(n_T(w))$ equal to $1 - w$. Any individual with $n < n_T(w)$ faces a lower marginal rate even if he works entirely in the taxable sector, so $\hat{\ell}_n = 0$, and his untaxable income $z_T(n)$ is zero, while his taxable income $X_T(n)$ is $n - \hat{\ell}_n = n$. An individual for whom $n \geq n_T(w)$ will work only $n_T(w)$ hours in the untaxable sector. Hence taxable income $X_T(n) = \ell_T(n) = n_T(w)$ is constant for all $n \geq n_T$, while untaxable income $z_T(n) = w \cdot \hat{\ell}_n = w(n - n_T(w))$ increases with n, at rate w. Total pretax income $Y_T(n) = X_T(n) + z_T(n)$ thus increases with n, with slope 1 for $n \leq n_T(w)$, and with slope w thereafter.

2.3 Equilibrium

The individual labor supply function $\hat{\ell}_T$ yields a total labor supply of $\hat{L}_T(w) \equiv \int_0^1 \hat{\ell}_T(n;w)dF(n)$ to the untaxed sector. A wage rate w_0 is an equilibrium if it clears the labor market, i.e. if $w(\hat{L}_T(w_0)) = w_0$. We now show that such an equilibrium wage necessarily exists, and is unique.

As noted earlier, the wage in the untaxed sector is a strictly decreasing function of the aggregate labor supplied to that sector. Since $1 - t$ is also decreasing on $[0,1]$, $w(0) \leq 1 - t(1)$ implies $w(L) < 1 - t(0)$ for any L, so from (2.1b) everyone will work only in the taxable sector, and $w_0 = w(0)$ at the unique equilibrium in this case.

Similarly, from (2.1a), $w(\bar{n}) \geq 1 - t(0)$ implies $L_0 = 1$ and hence that $w_0 = w(\bar{n})$ is the unique equilibrium.

The remaining possibility is $w(0) > 1 - t(1)$, $w(\bar{n}) < 1 - t(0)$. Let us suppose, initially, that T is continuously differentiable and t is strictly increasing everywhere. Define the function $N: [0,1] \to [0,\bar{n}]$ by $N(n') = \int_{n'}^1 (n - n')dF(n)$. Then $\frac{dN}{dn'} = -\int_{n'}^1 dF(n) < 0 = -[1 - F(n')]$, for $n' \in [0,1]$. Evidently N is a continuous, strictly decreasing function on $[0,1]$ with $N(0) = \bar{n}$, $N(\bar{n}) = 0$. From (2.2), the aggregate labor supply to the untaxed sector is $\hat{L}_T(w) = N(n_T(w))$, so in view of the properties of n_T (namely, that it is strictly decreasing on $(1 - t(1), 1 - t(0))$) and N, $\hat{L}_T(w)$ is continuous and strictly increasing on $(1 - t(1), 1 - t(0))$, with $\hat{L}_T(w) = \bar{n}$ for $w \geq 1 - t(0)$, $\hat{L}_T(w) = 0$ for $w \leq 1 - t(1)$. Moreover the labor demand function w^{-1} is continuous and strictly decreasing on $[w(0), w(\bar{n})]$ with $w^{-1}(w) = \bar{n}$ for $w \leq w(\bar{n})$, $w^{-1}(0) = 0$ for $w \geq w(0)$. Since, by hypothesis $w(0) > 1 - t(1)$ and $w(\bar{n}) < 1 - t(0)$, the excess demand function $w^{-1} - \hat{L}_T$ has a value of zero at some unique point w_0 in the interval, so w_0 is the unique equilibrium (see Figure 2.2).

FIGURE 2.2 ABOUT HERE

Let us denote by w_T^* the unique equilibrium under T. If we define $n_T^* \equiv n_T(w_T^*)$, then $1 - t(n_T^*) = w_T^*$, and the individual labor supply function $\hat{\ell}_T(n;w_T^*)$ given by 2.2 is optimal for all n, given T and w_T^*, and supports the equilibrium in the sense that $\int \hat{\ell}_T(n;w_T^*)dF(n) = \hat{L}_T(w_T^*) = w^{-1}(w_T^*)$.

These arguments are readily extended to nondifferentiable schedules. Since any admissable schedule has at most a finite number of points at which t is discontinuous, the right-hand and left-hand derivatives, t^+ and t^-, exist everywhere. The conditions (2.1a,b) remain valid as stated, as is (2.1c) when the inequality $w < 1 - t(n)$ is replaced by the inequality $w < 1 - t^+(n)$. In (2.1d) the equality $w = 1 - t(n - \hat{\ell}_n)$ is replaced by $w \in [1 - t^+(n - \hat{\ell}_n),\ 1 - t^-(n - \hat{\ell}_n)]$. For any $w \in [1 - t(1), 1 - t(0)]$ there is still a unique ability level $n_T(w)$ such that $w \in [1 - t^+(n_T(w)), 1 - t^-(n_T(w))]$, and n_T is still a continuous, increasing function. (If x_0 is a point of nondifferentiability evidently $n_T(w) = x_0$ for all $w \in [1 - t^+(x_0), 1 - t^-(x_0)]$, so n_T is no longer strictly increasing on this interval however.) Hence the individual labor supply is still given by (2.2) and the argument proceeds as before. The resulting equilibrium w_T^* and threshhold ability level n_T^* defining the individual labor supply function are still unique, though now satisfying $1 - w_T^* \in [t^-(n_T^*), t^+(n_T^*)]$.

Things become slightly more complicated when the marginal-rate schedule t is not strictly increasing, i.e. is constant over some

interval (or "bracket") $[\underline{X}, \bar{X}]$, with

$$t(X) \;\begin{cases} < k & \text{for } X < \underline{X} \\ = k & \text{for } x \in [\underline{X}, \bar{X}] \\ > k & \text{for } X > \bar{X} \end{cases}$$

Without loss of generality we can suppose there is only one such interval, and that $w(0) > k > w(\bar{n})$. When the wage rate takes on the value $w = 1 - k$, some of the individual labor supply decisions become ambiguous. In particular, if $n > \underline{X}$ the first-order condition (2.1d) holds for all ℓ_n such that $X_n = n - \ell_n \in [\underline{X}, \bar{X}]$, since $t(X_n) = k = 1 - w$ for all such ℓ_n. Let $\bar{\ell}_T(n;w)$ and $\underline{\ell}_T(n;w)$ be functions which specify the largest and smallest such ℓ_n for each n, i.e.

$$\bar{\ell}_T(n;1 - k) = \underline{\ell}_T(n;1 - k) = 0 \quad \text{for } n < \underline{X},$$

$$\bar{\ell}_T(n;1 - k) = n - \underline{X} \quad \text{for } n \geq \underline{X}, \text{ and}$$

$$\underline{\ell}_T(n;1 - k) = \begin{cases} 0 & \text{for } n \in [\underline{X}, \bar{X}] \\ n - \bar{X} & \text{for } n > \bar{X} \end{cases}$$

For $w \neq 1 - k$ the first-order condition (2.1) again holds at a unique $\hat{\ell}_n$ for each n, so $n_T(w)$ can be defined as before, and

$$\bar{\ell}_T(n;w) = \underline{\ell}_T(n;w) = \begin{cases} 0 & \text{for } n < n_T(w) \\ n - n_T(w) & \text{for } n \geq n_T(w) \end{cases}$$

Any $\hat{\ell}_T(n;w) \in [\underline{\ell}_T(n;w), \bar{\ell}_T(n;w)]$ is a possible individual labor supply function, so the aggregate labor supply $L_T^* = \int \hat{\ell}_T(n;w)dF(n)$ can lie anywhere in th e interval $[\underline{L}_T(w),\overline{L}_T(w)]$, where $\underline{L}_T(w) = \int \underline{\ell}_T(n;w)dF(n)$ and $\overline{L}_T(w) = \int \overline{\ell}_T(n;w)dF(n)$. Aggregate labor supply is thus described by a correspondence $L_T^*(w) = [\underline{L}_T(w),\overline{L}_T(w)]$, which is interval-valued at $w = 1 - k$, and single-valued elsewhere. It is readily verified that L_T^* is upper hemi-continuous, so existence of an equilibrium w_T^* follows from a straightforward fixed-point argument. To show uniqueness, note that the endpoint functions $\underline{L}_T$, $\overline{L}_T$ are increasing on $[1 - t(1), 1 - t(0)]$, $\underline{L}_T(w) = \overline{L}_T(w)$ at all $w \neq 1 - k$, and that

$$\lim_{w \overset{-}{\to} 1-k} \underline{L}_T(w) = \underline{L}_T(1 - k) = N(\overline{X})$$

$$\lim_{w \overset{+}{\to} 1-k} \overline{L}_T(w) = \overline{L}_T(1 - k) = N(\underline{X})$$

Hence since $w^{-1}(w_T^*) \in [\underline{L}_T(w_T^*),\overline{L}_T(w_T^*)]$, $\underline{L}_T(w) \geq \overline{L}_T(w_T^*) \geq w^{-1}(w_T^*)$ for $w > w_T^*$, so since $w^{-1}(w)$ is strictly decreasing, clearly $w^{-1}(w) < \underline{L}_T(w)$, i.e. $w^{-1}(w) \notin [\underline{L}_T(w),\overline{L}_T(w)]$ for any $w > w_T^*$. Similiarly, $w^{-1}(w) \notin [\underline{L}_T(w),\overline{L}_T(w)]$ for all $w < w_T^*$.

If the unique equilibrium w_T^* does not coincide with a flat, (2.2) still holds and all individual labor supplies, incomes and taxes paid are uniquely determined. If $w_T^* = 1 - k$, however, $\hat{\ell}_n$ is ambiguous for $n > \underline{X}$, as noted earlier. From individual maximization $y_T(n)$ must be constant for all optimal $\hat{\ell}_n$, so individual and aggregate post-tax incomes are uniquely determined. Moreover, since the aggregate labor

supply is fixed at $L_T^* = w^{-1}(w_T^*) = \int \hat{\ell}_n dF(n)$, evidently $\int \hat{z}_T(n)dF(n) = \int w_T^*\hat{\ell}_n dF(n) = w_T^* L_T^*$, while $\int \hat{X}_T(n)dF(n)* = \int(n - \hat{\ell}_n)dF(n) = 1 - L_T^*$, so aggregate untaxable and taxable incomes, and hence total taxes collected, are also uniquely determined. However taxes paid or pretax incomes cannot be unambiguously determined for those individuals with $n > \underline{X}$.

Since the aggregate labor supply L_T^* must lie in the interval $[\underline{L}_T(1 - k),\overline{L}_T(1 - k)] = [N(\overline{X}),N(\underline{X})]$ there exists a unique $n_T^* \in [\underline{X},\overline{X}]$ such that $N(n_T^*) = L_T^*$, and it is readily verified that the individual labor supply function

$$\hat{\ell}(n;1 - k) = \begin{cases} n & \text{for } n < n_T^* \\ n - n_T^* & \text{for } n \geq n_T^* \end{cases}$$

is optimal for all n, and supports the equilibrium (since $\int \hat{\ell}(n;1 - k)dF(n) = N(n_T^*) = L_T^*$).

Henceforth we shall assume all individuals act according to this particular labor supply function; with this convention pre-tax incomes and taxes can be uniquely determined for all n, even for schedules whose marginal rate is not strictly increasing.

2.4 Further Results

To summarize the previous section, for any admissable T there exists a unique equilibrium w_T^*, and a unique threshold ability level n_T^* such that $t(n_T^*) = 1 - w_T^*$, and

$$\hat{\ell}_T(n;w_T^*) = \begin{cases} n & \text{for } n < n_T^* \\ n - n_T^* & \text{for } n \geq n_T^* \end{cases}$$

is an optimal labor supply for all n, and supports the equilibrium. If $w(0) \leq 1 - t(1)$ the equilibrium is $w_T^* = w(0)$ and $n_T^* = 1$; alternatively if $w(\bar{n}) \geq 1 - t(0)$ the equilibrium is at $w_T^* = w(\bar{n})$, and $n_T^* = 0$. Otherwise the equilibrium is an interior one, $w(\bar{n}) < w_T^* < w(0)$, with $0 < n_T^* < 1$. If $X_T(n)$, $z_T(n)$ and $Y_T(n)$ denote the pre-tax taxable, untaxable and total income of a person of type n, evidently

$$X_T(n) = \begin{cases} n & \text{for } n \leq n_T^* \\ n_T^* & \text{for } n > n_T^* \end{cases}$$

$$z_T(n) = \begin{cases} 0 & \text{for } n \leq n_T^* \\ n - n_T^* & \text{for } n > n_T^* \end{cases}$$

$$Y_T(n) = \begin{cases} n & \text{for } n \leq n_T^* \\ n_T^* + w_T^*(n - n_T^*) & \text{for } n > n_T^* \end{cases}$$

using the labor-supply function (2.2) at $w = w_T^*$. All incomes increase with n, and total income $Y_T(n)$ increases strictly with n. Similarly if we denote by $C_T(n) = T(X_T(n))$ the tax actually collected from n, and by $x_T(n) = X_T(n) - T(X_T(n))$ and $y_T(n) = x_T(n) + z_T(n) = Y_T(n) - C_T(n)$ his post-tax declared (taxable) and total income, then evidently

$$C_T(n) = \begin{cases} T(n) & \text{for } n \leq n_T^* \\ T(n_T^*) & \text{for } n > n_T^* \end{cases}$$

and

$$y_T(n) = \begin{cases} n - T(n) & \text{for } n \leq n_T^* \\ n - T(n_T^*) + w_T^*(n - n_T^*) & \text{for } n > n_T^* \end{cases}$$

Note that $y_T(n)$ is a strictly increasing function of n, and that C_T is an increasing function whose marginal rate of increase (right-hand derivative) c_T is

$$c_T(n) = \begin{cases} t^+(n) \in [0,1] & \text{for } n < n_T^* \\ 0 & \text{for } n \geq n_T^* \end{cases}$$

Notice also that for any tax T the effective schedule C_T depends only on the shape of T(X) for $X \leq n_T^*$. Thus, if $T(X) = T'(X)$ for all $X \leq n_T^*$, then $C_T(n) = C_{T'}(n)$ for all n; ie. the effective schedules C_T and $C_{T'}$ are identical. Moreoever after-tax incomes y_T, $y_{T'}$ are also identical for all n. Hence we shall say the schedules T and T' are <u>equivalent</u>. The relationship between T, $Y_T(n)$, $C_T(n)$ and $y_T(n)$ is shown in Figure 2.3 below.

FIGURE 2.3 ABOUT HERE

The total revenue collected under the schedule T is

$$R_T = \int_0^1 C_T(n)dF(n).$$

Note that the labor-supply correspondence $\hat{L}_T^*$ depends only on the marginal-rate schedule (t, or t^-, t^+ for non-differentiable T).

Thus if T and T′ are two tax schedules whose marginal schedules are the same—or equivalently, if $T' = T + k$ is a vertical translation of T—then $L_T^* = L_{T'}^*$, so they must yield the same equilibrium wage $w_T = w_{T'}$, and threshold ability levels $n_T^* = n_{T'}^*$. Hence individual labor supplies and pre-tax incomes are the same under either schedule. Taxes collected and post-tax incomes are not, however, since $C_T(n) = C_{T'}(n) + k$, $y_{T'}(n) = y_T(n) - k$, while the total revenues collected are $R_{T'} = R_T + k$. Hence, for any T, there exists a unique feasible schedule $T' = T + k$ which raises precisely zero net revenue.

Note that $C_T(0) > 0$ would imply $C_T(n) > 0$ for all n, since C_T is an increasing function, which in turn implies $R_T > 0$ and hence that the schedule T is not feasible. On the other hand $C_T(0) \leq 0$ implies $T(0) \leq 0$ and hence, (since $t \leq 1$ on $[0,1]$) that $T(X) \leq X$ for all X. Thus, for feasible schedules, taxes imposed never exceed pretax income.

Let T be a differentiable schedule such that $w_T^* \varepsilon (w(\bar{n}),w(0))$, whence $n_T^* \varepsilon (0,1)$, and let T′ be another differentiable schedule such that $t'(n_T^*) = t(n_T^*)$. From (2.1c), $1 - t(n_T^*) = w_T^*$. If we consider individual labor supply under the schedule T′ and with the wage rate w_T^*, then (2.1c,d) apply (since $w_T^* = 1 - t'(n_T^*) \varepsilon [1 - t'(1),1 - t'(0)]$). Evidently $n < n_T^*$ implies $1 - t'(n) \geq w_T^* = 1 - t(n_T^*)$, since t is increasing. If the latter is strict then $\hat{\ell}_n = 0$, from (2.1c). If it holds with equality then $\hat{\ell}_n = 0$ is still optimal, since $1 - t'(n - \hat{\ell}_n) = 1 - t'(n) = w_T^*$ and (2.1d) holds. On the other hand $\hat{\ell}_n = n - n_T^*$ is optimal for $n \geq n_T^*$,

since $1 - t'(n - \hat{\ell}_n) = 1 - t'(n_T^*) = w_T^*$. so (2.1d) again holds. Hence there exists an individually optimal labor supply function $\hat{\ell}_{T'}(n;w_T^*)$ which yields an aggregate labor supply $\hat{L}_{T'}(w_T^*) = \int \hat{\ell}_{T'}(n;w_T^*)$ such that $\hat{L}_{T'}(w_T^*) = N(n_T^*) = L_T(w_T^*)$. Since w_T^* is the equilibrium under T, $w_T^* = w(\hat{L}_T(w_T^*)) = w(\hat{L}_{T'}(w_T^*))$ so w_T^* is also an equilibrium (which must be unique) under T′, and $n_T^* = n_{T'}^*$. It is easily seen that these conclusions also hold for nondifferentiable schedules T, T′ if $1 - w_T^* \varepsilon [t'^-(n_T^*),t'^+(n_T^*)]$, and the extension to the cases $n_T^* = 0$ and $n_T^* = 1$ is obvious. Thus we have:

<u>Proposition 2.1</u> If T is a feasible schedule for which $w(\bar{n}) < w_T^* < w(0)$, and if T′ is another feasible schedule such that $1 - w_T^* \varepsilon [t'^-(n_T^*),t'^+(n_T^*)]$ (or equivalently $t(n_T^*) = t'(n_T^*)$ if both schedules are differentiable at n_T^*) then both schedules induce the same equilibrium, i.e. $w_T^* = w_{T'}^*$, with $n_T^* = n_{T'}^*$. Hence pretax incomes are the same, i.e. $X_T(n) = X_{T'}(n)$, $z_T(n) = z_{T'}(n)$ and $Y_T(n) = Y_{T'}(n)$, for all n.

The same conclusions also hold for T′ such that $t'(n_T^*) \leq 1 - w_T^*$ if $w_T^* = w(0)$ and $n_T^* = 1$, or for T′ such that $t'(n_T^*) \geq 1 - w_T^*$ if $w_T^* = w(\bar{n})$ and $n_T^* = 0$.

3. FAIRNESS AND PROGRESSIVITY

3.1 <u>Fairness</u>

We assume that individual welfare depends directly on after-tax income or consumption. Hence the fairness of any tax schedule T can be judged solely in terms of the fairness of the after-tax income distribution it induces, with more egalitarian distributions being fairer than less egalitarian ones, ceteris paribus. Rather than work with a specific index of income inequality, we instead assume that the fairness of any income distribution can be assessed in terms of a social welfare function of the form $S_W(T) = \int W(y_T(n))dF(n)$; as Atkinson (1970) has pointed out, the usual inequality measures can all be rationalized by social welfare functions of this kind. We shall refer to W as the evaluation function: we assume it to be twice continuously differentiable, strictly increasing, and strictly concave on [0,1]. (The second property implies that the welfare of every individual, no matter how wealthy, is given some weight, and the third implies that income equalization is positively valued by the social welfare function.) If $S_W(T') = \int W(y_{T'}(n))dF(n) > \int W(y_T(n))dF(n) = S_W(T)$, the tax schedule T' is conditionally fairer than T, conditional upon the particular evaluation function W (a different function W' might order them differently). Any admissable W induces a (transitive, complete) ordering of the admissable schedules. If no admissable T' is conditionally fairer than T (and if T itself is admissable), then the schedule T is optimal for W; as it turns out, in the structure we consider here there exists an optimal schedule for each W, and with some weak restrictions on the wage function w, all schedules which are optimal for W are equivalent (as defined on p. 12). If T' is conditionally fairer than T according to every evaluation function W, then we shall say T' is unconditionally or unambiguously fairer than T. Evidently the "unambiguously fairer" relation is a partial (transitive) ordering of the admissable schedules, and is equal to the intersection (over all W) of the "conditionally fairer" relations.

The conditional relationship is the one which would guide a single decision maker, such as a benevolent despot or elected representative interested in promoting social justice, or a voter who judges tax proposals from a social or "sociotropic" point of view. We examine the relationship between progressivity and conditional fairness in Section 3.3 below. Proposition 3.6 shows that any such decision maker, given a free choice of tax schedules, will always select a linear or flat-rate schedule, rather than a progressive one. Alternatively, in a democracy citizens may have their own views on what constitutes fairness, and may vote accordingly (for specific tax reform proposals, or for candidates who advocate such proposals). Since different citizens may employ different criteria W, their individual orderings of the admissable tax schedules in general will differ, so majority rule may well not yield a consistent social ordering of tax schedules. In Section 3.4 we show that (again, under the weak restrictions on the wage function w) there nevertheless exists a majority equilibrium $\hat{T}$, and this equilibrium schedule is linear over $[0,n_{\hat{T}}]$. This equilibrium, it should be noted, assumes that voters view tax changes in a purely disinterested and altruistic fashion, and judge them solely in terms of whether they lead to a fairer post-tax income distribution. They thus view the income distribution as a pure public good, in Thurow's (1980) sense, and are not influenced by considerations of their own tax burden. (The other extreme, that citizens vote in a self-interested fashion to minimize their own tax burden, is considered in part 4 below.)

It will be useful to first note some useful facts about the "unambiguously fairer" relationship. The first two are well known.

Proposition 3.1 (Pareto). If T' and T are two schedules such that $y_{T'}(n) \geq y_T(n)$ for all n, with strict inequality for some (nonnull, measurable set of) n, then T' is unambiguously fairer than T.

Proposition 3.1 is an immediate consequence of the fact that W is a strictly increasing function.

The second proposition, from Atkinson (1970, pp. 245–248), says that one income distribution is unambiguously fairer than another if it can be obtained from the latter by redistributing income from the richer to the poorer. In the present context this assertion can be more precisely stated as follows:

Proposition 3.2 If T' and T are two schedules such that $\int y_{T'}(n)dF(n) = \int y_T(n)dF(n)$, and if there exists an ability level n' such that $y_{T'}(n) \geq y_T(n)$ for $n < n'$ and $y_{T'}(n) \leq y_T(n)$ for $n > n'$, with strict inequality holding for some (nonnull measurable set of) n, then T' is unambiguously fairer than T.

The third proposition says that a tax schedule T' which yields lower total income than a schedule T cannot be unambiguously fairer than T.

Proposition 3.3 If $\int y_T(n)dF(n) > \int y_{T'}(n)dF(n)$ then T' is not unambiguously fairer than T.

Proof Let $\gamma = \int y_T(n)dF(n) - \int y_{T'}(n)dF(n) = \int [y_T(n) - y_{T'}(n)]dF(n) > 0$. Consider the weighting function W_δ defined by $W_\delta(y) = y - \delta y^2$. For any $\delta \in (0,1)$, W_δ is increasing and strictly concave on [0,1]. Now, $S_{W_\delta}(T) - S_{W_\delta}(T') = \int W_\delta(y_T(n))dF(n) - \int W_\delta(y_{T'}(n))dF(n) = \int [y_T(n) - y_{T'}(n)]dF(n) + \delta \cdot \int [y^2_{T'}(n) - y^2_T(n)]dF(n) = \gamma + \delta\tau$, where $\tau = \int [y^2_{T'}(n) - y^2_T(n)]dF(n) \in [-1,1]$. If $\tau \geq 0$ then clearly $S_{W_\delta}(T) - S_{W_\delta}(T') \geq 0$. Otherwise, if $\tau < 0$, choose $\delta \in (0,1)$ such that $\delta < -(\gamma/\tau)$; then $S_{W_\delta}(T) - S_{W_\delta}(T') > \gamma - (\gamma/\tau)\tau = 0$. In either case we have $S_{W_\delta}(T) > S_{W_\delta}(T')$, so T is conditionally fairer than T' for W_δ and thus T' is not unambiguously fairer than T.

QED

Finally, we should note that if T is equivalent to T' then $y_T(n) = y_{T'}(n)$ for all n so $S_W(T) = S_W(T')$ for all W. Thus anyone ranking tax schedules by any social welfare function S_W will be indifferent between equivalent schedules.

3.2 Fairness and the Degree of Progressivity

As noted earlier, a schedule T is less progressive than T', if there exists an income level X^* such that $t'(X) \leq t(X)$ (respectively $\geq$) for $X < X^*$ (respectively $>$), with strict inequality (except at X^*)

on some open interval containing X^*. Proposition 3.4 below expresses one relationship between fairness and degree of progressivity in this sense; it shows that the relationship is a perverse one:

Proposition 3.4. If T is a feasible, differentiable schedule which is progressive over the interval $[0,n_T^*]$ (ie. $t(0) < t(n_T^*)$), with $0 < n_T^* < 1$ and $0 < t(n_T^*) < 1$, then there exists a less progressive schedule T′ which is unambiguously fairer than T.

Proof Let T be a schedule satisfying the above conditions, and let T′ be another differentiable schedule such that $t'(n_T^*) = t(n_T^*)$, $t'(X) > t(X)$ for $X < n_T^*$, and $t'(X) < t(X)$ for $X > n_T^*$, as shown on Figure 3.1 below. Evidently we can always find such a schedule,

FIGURE 3.1 ABOUT HERE

and clearly T′ is less progressive than T. From Proposition 2.1, both schedules induce the same equilibrium, with $w_T^* = w_{T'}^*$, and $n_T^* = n_{T'}^*$. If it were true that $T'(n_T^*) \leq T(n_T^*)$ the effective tax rate would satisfy $C_{T'}(n) = T'(n_{T'}^*) \leq T(n_T^*) = C_T(n_T^*)$ for $n \geq n_T^* = n_T^*$, and $C_{T'}(n) = T'(n) < T(n) = C_T(n)$ for $n < n_T^*$ (since $t'(n) > t(n)$ over this range), implying $\int C_{T'}(n)dF(n) < \int C_T(n)dF(n) = 0$ (since $n_T^* > 0$) and hence that the total tax collected under T′ does not meet the revenue target, i.e. T′ is not feasible. Thus if T′ is feasible (clearly we can ensure this by taking a vertical translation of the original schedule) it must be true that $T'(n_T^*) > T(n_T^*)$, implying $C_{T'}(n) > C_T(n)$ for all $n > n_T^*$. Moreover, the fact that $t'(n) > t(n)$ for $n < n_T^*$ and

that $\int (C_{T'}(n) - C_T(n))dF(n) = 0$ (from feasibility) imply that there must exist some $n' \varepsilon (0,n_T^*)$ such that $T'(n) = C_{T'}(n) > C_T(n) = T(n)$ for $n > n'$, $T'(n) = C_{T'}(n) < C_{T'}(n) = T(n)$ for $n < n'$, with equality at $n = n'$. All before-tax incomes are the same under T and T′ (from Proposition 2.1), so after-tax incomes under T will be greater for $n < n'$, and lower for $n > n'$. Moreover since total after-tax incomes are the same (since total taxes collected are the same, from feasibility), Proposition 3.2 applies, and implies that T′ is unambiguously fairer than T, as asserted.

QED

The restriction to differentiable schedules is clearly not essential, and the inequalities on n_T^* and $t(n_T^*)$ are needed only to ensure that T′ be less progressive than T in the precise sense of the earlier definition: without these conditions, we could still find an unambiguously fairer schedule T′ such that $t'(n) \geq t^+(n)$ for $n < n_T^*$ and $t'(n_T^*) \leq t^-(n)$ for $n > n_T^*$, though unless both equalities were strict T′ would not be less progressive than T in the sense defined earlier. In fact, the following variant of Proposition 3.4 is readily established:

Proposition 3.5 For any feasible schedule T which is progressive over $[0,n_T^*]$, there exists a feasible flat-rate schedule T′ which is unambiguously fairer than T.

Proof To show this first suppose $n_T^* = 0$. Then everyone works only in the underground economy, and $y_T(n) = w(\bar{n}) \cdot n$ for all n. The flat-rate schedule $T'(X) = 0$ for all X is also feasible, but evidently everyone works entirely in the taxable sector under it, so $y_{T'}(n) = n > w(\bar{n}) \cdot n = y_T(n)$ for all n. Hence, from Proposition 3.1, T' is unambiguously fairer than T.

Otherwise, if $n_T^* > 0$, let $T'(X) = \beta X + \alpha$ with $\beta = 1 - w_T^* \in [t^-(n_T^*), t^+(n_T^*)]$. Since t^- and t^+ are increasing, with $t^+(X) \le t^-(n_T^*)$ if $X < n_T^*$, it follows that $t^+(X) \le \beta$ for $X < n_T^*$. Moreover the inequality must be strict for some nonnull set of X, else we would have $t(0) = \beta = t(n_T^*)$, contrary to the hypothesis that T is progressive over $[0, n_T^*]$. Hence, from the same reasoning as used to establish Proposition 3.4, there must exist $n' \in (0, n_T^*)$ such that $y_{T'}(n) > y_T(n)$ for $n < n'$, $y_{T'}(n) < y_T(n)$ for $n > n'$, so T' is unambiguously fairer than T.

QED

3.3 Individual Choice of a Fair Tax Schedule

Proposition 3.5 implies, in particular, that no schedule T which is progressive over $[0, n_T^*]$ can be optimal for any evaluation function W, so that under any such function W the optimal schedule T_W (if one exists) is necessarily a flat-rate schedule over this range. It is straightforward to show that such an optimum does in fact exist.

Proposition 3.6 For any evaluation function W there exists a conditionally optimal feasible schedule T_W which is linear. The tax rate β_W for any such schedule satisfies $1 - \beta_W \in [w(\bar{n}), w(0)]$.

Proof By Proposition 3.5, if a conditionally optimal tax exists for W then it must be linear on $[0, n_T^*]$. Consider any linear (on [0,1]) schedule T defined by $T(X) = \beta X + \alpha$. From Section 2.2 above there is a unique equilibrium wage w_T^* in the untaxed sector, and the average pretax income in the taxable sector is then $\bar{X}_T = \bar{n} - \hat{L}_T(w_T^*)$. Since T is feasible, government net revenues are zero, and we have $\int T(X_T(n))dF(n) = \beta \bar{X}_T + \alpha = 0$, or $\alpha = -\beta \bar{X}_T$. Thus α is determined uniquely as a function of β, $\tilde{\alpha}(\beta) = -\beta \bar{X}$, so there is a one-to-one correspondence between feasible linear tax schedules and tax rate parameters. For T defined by $T(X) = \beta X + \tilde{\alpha}(\beta)$, let $\tilde{y}(\beta, n) = y_T(n) = (1 - \beta)X_T(n) + z_T(n) - \tilde{\alpha}(\beta)$. Then define $\tilde{S}_W$: $[0,1] \to \mathbb{R}$ by $\tilde{S}_W(\beta) = \int_0^1 W(\tilde{y}(\beta, n)dF(n) = \int_0^1 W(y_T(n)dF(n) = S_W(T)$. $\tilde{S}_W$ then is a simple function of one real variable, rather than a functional on tax schedules like S_W. Also, $\tilde{S}_W(\beta) \ge \tilde{S}_W(\beta')$ if and only if $S_W(T) \ge S_W(T')$, where T and T' are feasible linear tax schedules with tax rates β and β' respectively. Notice that $\tilde{S}_W$ is useful only for ranking feasible linear schedules. Since F has a continuous density, and W and $\tilde{y}$ are integrable functions, continuous in their arguments, $\tilde{S}_W$ is continuous [by Apostol p. 281, Theorem 10.38]. The interval [0,1] is compact, so S_W attains its maximum for some $\beta_W \in [0,1]$. Then the tax schedule T_W defined by $T_W(X) = \beta_W X + \tilde{\alpha}(\beta_W)$ is a conditionally optimal schedule for W.

Now we show that $1 - \beta_W \in [w(\bar{n}), w(0)]$.

Suppose $1 - \beta_W > w(0)$. Then, by Section 2.3 the equilibrium wage rate is $w(0)$, and $n^*_{T_W} = 1$. Then $\tilde{y}(\beta_W, n) = (1 - \beta_W)n + \beta_W\bar{n}$, $\forall n$. Consider the tax T' defined by $T'(X) = \beta' X + \tilde{\alpha}(\beta')$, with $\beta' = 1 - w(0) > \beta_W$. The equilibrium wage is again $w(0)$, and $n^*_{T'} = 1$, so $\tilde{y}(\beta', n) = (1 - \beta')n + \beta'\bar{n}$ $\forall n$. Then we have $\tilde{y}(\beta', n) - \tilde{y}(\beta_W, n) = (\beta' - \beta_W)(\bar{n} - n)$, so $\tilde{y}(\beta, \bar{n}) = \tilde{y}(\beta_W, \bar{n})$, $\tilde{y}(\beta', n) > \tilde{y}(\beta_W, n)$ for all $n < \bar{n}$, and $\tilde{y}(\beta', n) < \tilde{y}(\beta_W, n)$ for all $n > \bar{n}$. Thus, by Proposition 3.2, T' is unambiguously fairer than T_W so T_W could not be conditionally optimal for W.

Next, suppose $1 - \beta_W < w(\bar{n})$. Then, again looking at Section 2.3, we see that the equilibrium wage rate is $w(\bar{n})$, and $n^*_{T_W} = 0$. So, $\tilde{y}(\beta_W, n) = w(\bar{n})n$ $\forall n$. But, if we consider T' defined by $T'(X) = 0$ $\forall X$, then the equilibrium wage rate is $w(0)$ and $n^*_{T'} = 1$, so $\tilde{y}(\beta', n) = n$ $\forall n$. Then $\tilde{y}(\beta', n) > \tilde{y}(\beta_W, n)$ $\forall n$, so by Proposition 3.1 T' is unambiguously fairer than T_W, and thus T_W could not be conditionally optimal for W. Hence $1 - \beta_W \in [w(\bar{n}), w(0)]$.

QED

Of course, because any schedule T' equivalent to T_W is also optimal for W, the set of optimal taxes is infinite. However, the following proposition estableishes that under some weak conditions on the wage function w, the optimal linear schedule is unique. Thus all

optimal schedules are equivalent, so they all induce the same effective schedule.

<u>Proposition 3.7</u> If the wage function w satisfies $\frac{d^2w(L)}{dL^2} > \frac{-2\left[\frac{dw(L)}{dL}\right]^2}{1 - w(L)}$ for every $L \in [0, \bar{n}]$ then for any evaluation function W the function $\tilde{S}_W$ is strictly concave. Hence the optimal linear schedule T_W is unique, and any (possibly nonlinear) optimal schedule T'_W is equivalent to T_W. Moreover, voter preferences over the set of linear schedules are single-peaked, if every voter i judges schedules according to some social welfare function S_{W_i}.

<u>Proof</u> Recall that $\tilde{S}_W$, introduced in the previous proposition, is defined by $\tilde{S}_W(\beta) = \int W(\tilde{y}(n,\beta))dF(n)$. Consider all linear taxes with tax rate β satisfying $1 - \beta \in [w(\bar{n}, w(0)]$. If $\tilde{S}_W$ is defined by $\tilde{S}_W(\beta) = \int_0^1 W(\tilde{y}(\beta, n))dF(n)$ is concave over this set of β, then $S_W(T)$ has a unique maximum T_W among such taxes, and hence, by Propositions 3.5 and 3.6, any optimal schedule must coincide with T_W on $[0, n^*_{T_W}]$ and thus is equivalent to T_W. Single-peakedness follows directly if $\tilde{S}_W$ is concave for every W—simply order the linear taxes on the real line by their tax rate parameter.

Since W is concave, $\tilde{S}_W$ will be concave if $\tilde{y}$ is concave in β for each n. We now show this.

It is clear from Section 2.3 that for T defined by $T(X) = \beta X - \alpha(\beta)$, the equilibrium wage rate in the untaxed sector must be $w_T = 1 - \beta$. Then $\tilde{y}(\beta,n) = (1-\beta)n + \beta\bar{X}_T = (1-\beta)n + \beta(\bar{n} - w^{-1}(1-\beta))$. Since w^{-1} is twice differentiable, $\tilde{y}$ is also, and we have

$$\frac{\partial\tilde{y}}{\partial\beta} = -n + (\bar{n} - w^{-1}(1-\beta)) + \beta\frac{dw^{-1}(1-\beta)}{dw}, \text{ and}$$

$$\frac{\partial^2\tilde{y}}{\partial\beta^2} = \frac{dw^{-1}(1-\beta)}{dw} - \beta\frac{d^2w^{-1}(1-\beta)}{dw^2}.$$

By definition, $w(w^{-1}(1-\beta)) = 1-\beta$, so

$$-\frac{dw}{dL}\cdot\frac{dw^{-1}}{dw} = -1, \quad \text{or} \quad \frac{dw^{-1}}{dw} = \left[\frac{dw}{dL}\right]^{-1};$$

hence
$$-\frac{d^2w}{dL^2}\frac{dw^{-1}}{dw} = \left[\frac{dw^{-1}}{dw}\right]^2\cdot\frac{d^2w^{-1}}{dw^2}, \quad \text{or} \quad \frac{d^2w^{-1}}{dw^2} = -\frac{d^2w}{dL^2}\cdot\left[\frac{dw}{dL}\right]^{-3},$$

so
$$\frac{\partial^2\tilde{y}}{\partial\beta^2} = 2\left[\frac{dw(w^{-1}(1-\beta))}{dL}\right]^{-2} + \beta\frac{d^2w(w^{-1}(1-\beta))}{dL^2}\left[\frac{dw(w^{-1}(1-\beta))}{dL}\right]^3.$$

This is negative at all β if

$$\frac{d^2w(w^{-1}(1-\beta))}{dL^2} > \frac{2\left[\frac{dw(w^{-1}(1-\beta))}{dL}\right]^2}{\beta}. \quad \text{for all } \beta \in [1-w(0), 1-w(\bar{n})].$$

Letting $L = w^{-1}(1-\beta)$, this condition can be written

$$\frac{d^2w(L)}{dL^2} > \frac{2\frac{dw(L)}{dL}}{1-w(L)} \quad \text{for all } L \in [0,\bar{n}].$$

QED

3.4 Fair Taxation Under Majority Rule

In view of Propositions 3.5, 3.6 and 3.7 above, we can easily extend the analysis from the individual case to group decision-making under majority rule. Define a majority equilibrium $\hat{T}$ as a tax schedule which no other schedule can defeat in a pairwise majority vote. If voters have different weighting functions W_i their preferences over tax schedules will be different, so the possibility of voting cycles arises and a majority equilibrium may not exist. Proposition 3.7, along with the well known result on single-peakedness and majority rule (Black (1958)), implies that majority votes over linear schedules will be consistent (i.e. different pairwise votes will be transitive), and that at least one such schedule $\hat{T}$ will satisfy the median voter condition and be able to defeat any other such schedule in a pairwise vote. However $\hat{T}$ is only a restricted equilibrium, within the set of linear schedules; Proposition 3.6 does not preclude the possibility that some progressive schedule T′ could defeat $\hat{T}$, and hence that there is no majority equilibrium within the set of admissable schedules. However Proposition 3.5, in conjunction with 3.6, does preclude this possibility, and implies that $\hat{T}$ is a general equilibrium:

Proposition 3.8 Under the condition of Proposition 3.7, there exists a majority equilibrium, i.e. a feasible schedule $\hat{T}$ which cannot be defeated by any other feasible schedule in a pairwise majority vote. Moreover every such equilibrium is linear over the interval $[0,n^*_{\hat{T}}]$.

<u>Proof</u> We must prove that no non-linear (i.e. progressive) schedule can defeat the majority-preferred flat-rate schedule $\hat{T}$. If there were such a schedule T′ which defeats $\hat{T}$, then the set of voters C who prefer T′ to T constitute a majority. From Proposition 3.5, there exists a flat-rate schedule T′′ which is unambiguously fairer than T′, so every $i \varepsilon C$ prefers T′′ to T′, and hence (from the transitivity of individual preference) also prefers T′′ to $\hat{T}$. But this means that the flat-rate schedule T′′ defeats $\hat{T}$ in a pairwise vote, which is impossible. Hence no such T′ can defeat $\hat{T}$, i.e. $\hat{T}$ is an equilibrium against all admissable, feasible schedules. The fact that every such $\hat{T}$ must be linear over $[0,n^*_{\hat{T}}]$ follows directly from Proposition 3.5.

QED

As a final observation, we note that all majority equilibria are equivalent if the distribution of voters has strictly positive density at the median.

4. SELF-INTEREST AND PROGRESSIVITY

4.1 <u>A Preliminary Result</u>

We now consider the problem from the viewpoint of citizen-taxpayers interested in promoting their own welfare, narrowly interpreted, rather than pursuing broader social ends of fairness or

economic equality. Individual welfare, in this context, is again measured by after-tax consumption; thus an individual of ability n will prefer one schedule T to another, T′, if and only if $y_T(n) > y_{T'}(n)$. In general citizens at differing earning abilities will have differing preferences over such schedules, and in particular may view more and less progressive schedules differently. A preliminary result which gives some insight into this relationship is as follows (n_m denotes the median ability level; the differentiability assumption is unnecessary, and the result could easily be strengthened in various ways):

<u>Proposition 4.1</u> If T is a differentiable schedule such that $0 < t(n^*_T) < 1$, then there exists a more progressive schedule T′ which is favored by upper-income and opposed by lower-income taxpayers. If $0 < t(n_m) < 1$, we can find such a schedule T′ which is preferred by a majority, consisting of middle- and upper-income citizens.

<u>Proof</u> Let $(\underline{X},\overline{X})$ be the interval over which $t(X) \varepsilon (0,1)$ for all $X \varepsilon (\underline{X},\overline{X})$; then by hypothesis n_m, $n^*_T \varepsilon (\underline{X},\overline{X})$ so there exist points $n' \varepsilon (\underline{X},n_m) \cap (\underline{X},n^*_T)$. Let T′ be a differentiable schedule which crosses T at some such n′, i.e. $T'(n') = T(n')$, with $T'(X) > T(X)$ for $X < n'$, $T'(X) < T(X)$ for $X > n'$, and whose marginal-rate schedule t satisfies $t'(n^*_T) = t(n^*_T)$, $t'(X) < t(X)$ for $X \varepsilon (\underline{x},n^*_T)$, and $t'(X) > t(X)$ for $X \varepsilon (n^*_T,\overline{x})$, with $t'(X) = t(X)$ elsewhere. Such a schedule is shown in Figure 3.1. Evidently we can always find such a schedule, and can make it lie as close to T on the interval $[n',n^*_T]$ as

desired. Clearly T' is more progressive than T (relative to the point n_T^* and interval $(\underline{X},\overline{X})$). Since $t'(n_T^*) = t(n_T^*)$, both schedules have the same equilibrium and induce the same before-tax incomes, i.e. $Y_{T'}(n) = Y_T(n)$ for all n, from Proposition 2.1. Evidently $C_{T'}(n) = T'(n) > T(n) = C_T(n)$ for $n < n'$, while $C_{T'}(n) < C_T(n)$ for $n > n'$, so by choosing T' to lie sufficiently close to T over the interval (n', n_T^*) we can ensure that

$$\int_0^{n'} [C_{T'}(n) - C_T(n)]dF(n) + \int_{n'}^1 [C_{T'}(n) - C_T(n)]dF(n) =$$

$$\int_0^1 [C_{T'}(n) - C_T(n)]dF(n) = 0,$$ and hence that T' is feasible. Since before-tax incomes are the same under either schedule, clearly taxpayers with $n < n'$ will prefer the less progressive schedule T, while those with $n > n'$ prefer T'; moreover since $n' < n_m$ this latter set constitutues a majority, which consists of upper- and middle-income (or -ability) taxpayers.

QED

Thus an increase in progressivity may well redistribute incomes upwards, and hence (from Proposition 3.2) lead to an unambiguously <u>less</u> fair income distribution; nevertheless, if the more progressive schedule T' is properly chosen, it may be preferred by a majority of taxpayers, and thus prevail over the fairer schedule T. (This does not imply that every more-progressive schedule redistributes incomes in this way, of course, or that every less-fair schedule is necessarily more progresive.)

4.2 Individual Choice of a Tax Schedule

Let us now consider the choice of a tax schedule from the viewpoint of a simple taxpayer interested in minimizing his own tax burden, or more accurately, maximizing his own after-tax income. We shall say a feasible schedule $\hat{T}$ is <u>optimal</u> for n_0 if it maximizes the after-tax income $y_T(n_0)$ of an individual of ability n_0 over the set of feasible schedules T, i.e. if $y_{\hat{T}}(n_0) \geq y_T(n_0)$ for all such T. Heuristically, an optimal schedule is one which shifts as much as possible of the tax burden to other taxpayers; since (from feasibility) the total revenue is constant, this will minimize his own burden, and <u>ceteris paribus</u> maximize his after-tax income. A schedule such as T in Figure 4.1

FIGURE 4.1 ABOUT HERE

is clearly not optimal, since T' collects more revenue from $\{n: n < n_0\}$ and $\{n: n > n_0\}$; hence (assuming T feasible) there will be a downward translation of T' which is also feasible, and which reduces n_0's tax and hence increases his after-tax income. A schedule like T' (if feasible) might or might not be optimal for n_0: since any admissable schedule must be an increasing function, clearly no such schedule could impose still higher taxes on lower incomes without at the same time increasing n_0's own tax. On the other hand it might or might not be possible to increase the burden on $\{n: n > n_0\}$. A higher tax on such incomes will increase the taxes collected from $\{n: n \in (n_0, n_T^*)\}$, but will also shift n_T^* to the left and may therefore collect more or less from upper-income taxpayers; if T' is optimal, evidently it must maximize the revenue collected from $\{n: n \geq n_0\}$. A formal characterization of the optimal schedules is as follows:

<u>Proposition 4.2</u> For any n_0, there exists an optimal schedule of the form

$$T^0(X) = \begin{cases} \alpha_0 & \text{for } X \leq n_0 \\ \alpha_0 + \beta_0(x - n_0) & \text{for } X > n_0 \end{cases}$$

where β_0 maximizes the revenue collected from $\{n: n \geq n_0\}$ over all schedules of this form, and where α_0 is chosen to ensure feasibility. Moreover any optimal schedule $\hat{T}$ must be equivalent to such a T^0.

<u>Proof</u> We first show that a feasible schedule of the form T^0 exists. Let α and n_0 be fixed, and for any $\beta \varepsilon [0,1]$ denote by T_β the schedule of the form

$$T_\beta(X) = \begin{cases} \alpha & \text{for } n \leq n_0 \\ \alpha + \beta(X - n_0) & \text{for } n > n_0 \end{cases}$$

Let $n^*(\beta) \equiv n^*_{T_\beta}$. If $\beta \leq 1 - w(0)$ everyone works completely in the taxable sector, so $n^*(\beta) = 1$. Alternatively, no matter how large β becomes, all $n \leq n_0$ will continue to work in the taxable sector, though if $\beta \geq 1 - w(N(n_0))$ individuals with $n > n_0$ will work only n_0 units in the taxable sector, and $(n - n_0)$ in the untaxed sector, so the total labor supply to the untaxed sector will be $N(n_0)$; hence $n^*(\beta) = n_0$ for $\beta \geq 1 - w(N(n_0))$. For $\beta \varepsilon (1 - w(0), 1 - w(N(n_0)))$ evidently $n^*(\beta) \varepsilon (n_0, 1)$, where from equilibrium $1 - \beta = w(N(n^*(\beta)))$.

Since w is continuous and strictly decreasing on $[0,\bar{n}]$, and N is continuous and strictly decreasing on [0,1], it follows that $n^*(\beta) = N^{-1}(w^{-1}(1 - \beta))$ is a continuous (in fact differentiable) strictly decreasing function on [0,1].

The tax collected from an individual of type n is

$$C_{T_\beta}(n) = \begin{cases} \alpha & \text{for } n \leq n_0 \\ \alpha + \beta(n - n_0) & \text{for } n \varepsilon (n_0, n^*(\beta)) \\ \alpha + \beta(n^*(\beta) - n_0) & \text{for } n \geq n^*(\beta) \end{cases}$$

so the total revenue collected is

$$R_{T_\beta} \equiv \int_0^1 C_{T_\beta}(n)dF(n)$$

$$= \int_0^1 \alpha dF(n) + \int_{n_0}^{n^*(\beta)} \beta(n - n_0)dF(n) + \int_{n^*(\beta)}^1 \beta(n^*(\beta) - n_0)dF(n)$$

$$= \alpha + \int_{n_0}^1 \beta(n - n_0)dF(n) - \int_{n^*(\beta)}^1 \beta(n - n^*(\beta))dF(n)$$

$$= \alpha + \beta[N(n_0) - N(n^*(\beta))],$$

and maximizing the revenue raised from $n \geq n_0$ is (for fixed α) equivalent to maximizing the quantity $\beta[N(n_0) - N(n^*(\beta))]$. Let g be defined by $g(\beta) = \beta[N(n_0) - N(n^*(\beta))]$. Evidently g is a continuous bounded function on [0,1], so it has a maximum, which is clearly strictly positive. For $\beta \geq 1 - w(N(n_0))$, $n^*(\beta) = n_0$, so $g(\beta) = 0$, and thus the maximizing value β_0 cannot lie in this range. Moreover for $\beta \leq 1 - w(0)$, $n^*(\beta) = 1$ so $g(\beta) = \beta \cdot N(n_0)$ which is strictly increasing in β, while if $\beta > 1 - w(0)$, $n^*(\beta) < 1$ and $g(\beta) > [1 - w(0)]N(n_0) = f(1 - w(0))$. Hence the maximum must lie in the interval $(1 - w(0), 1 - w(N(n_0)))$, and $n^*(\beta_0) \varepsilon (n_0, 1)$. Clearly we can choose α_0 to ensure feasibility.

Hence a feasible schedule T^0 of the indicated form exists. It will be optimal for n_0 if and only if there is no other feasible schedule T' such that $y_{T'}(n_0) < y_{T^0}(n_0)$. Since $Y_{T^0}(n_0) = n_0 \geq Y_{T'}(n_0)$ for any such T', clearly the above inequality can hold only if $C_{T'}(n_0) < C_{T^0}(n_0)$.

Let T' be any feasible schedule such that $C_{T'}(n_0) \leq C_{T^0}(n_0) = \alpha_0$. We will show that the inequality cannot be strict, so T^0 is optimal, and that T' coincides with a piecewise linear optimal schedule T'' (possibly distinct from T^0) of the indicated form over the interval $[0, n^*_{T^0}]$, which (since T' is optimal) proves the result.

Since $C_{T'}(n)$ is nondecreasing for $n < n^*_{T'}$, and is constant for $n \geq n^*_{T'}$, $n^*_{T'} \leq n_0$ would imply $C_{T'}(n) \leq \alpha_0$ for all n. But since $C_{T^0}(n) = \alpha_0$ for $n \leq n_0$ and $C_{T^0}(n) > \alpha_0$ for $n > n_0$, this would imply $\int C_{T'}(n)dF(n) < \int C_{T^0}(n)dF(n) = 0$, so T' would not be feasible, contrary to hypothesis. Hence it must be true that $n^*_{T'} > n_0$. Also, since $C_{T'}$ is nondecreasing, $C_{T'}(n) \leq C_{T'}(n_0) \leq \alpha_0 = C_{T^0}(n)$ for all $n \leq n_0$, so $\int_0^{n_0} C_{T'}(n)dF(n) \leq \int_0^{n_0} C_{T^0}(n)dF(n)$. Moreover this inequality

would be strict if $C_{T'}(n_0) < \alpha_0$, i.e. if T^0 is not optimal.

Now consider the portion of T' over $[n_0, 1]$. Define T'' as the piecewise linear schedule

$$T''(x) = \begin{cases} T'(n_0) & \text{for } X \leq n_0 \\ T'(n_0) + \beta''(X - n_0) & \text{for } X > n_0 \end{cases}$$

where $\beta'' = \dfrac{T'(n^*_{T'}) - T'(n_0)}{n^*_{T'} - n_0}$, implying $T''(n_0) = T'(n_0)$, and $T''(n^*_{T'}) = T'(n^*_{T'})$. Since t' is defined and continuous almost everywhere on $[0,1]$, $T'(X) = T'(n_0) + \int_{n_0}^{X} t'(z)dz$ for any $X > n_0$, and thus $T'(X) - T''(X) = \int_{n_0}^{X} [t'(z) - t''(z)]dz = \int_{n_0}^{X} [t'(z) - \beta'']dz$. If $t'^+(n_0) > \beta''$ then since $t'(n) > t'^+(n_0)$ for all $n > n_0$ (such that $t'(n)$ exists) we would have $T'(n^*_{T'}) > T''(n^*_{T'})$, which is impossible. Similarly $t'^+(n_0) < \beta''$ would imply $t'^-(n^*_{T'}) > \beta''$ and $T'(X) < T''(X)$ for all $X < n^*_{T'}$. In this case, since $1 - \beta'' > 1 - t'^-(n^*_{T'}) \geq w(N(n^*_{T'}))$, evidently $n^*_{T''} > n^*_{T'}$, so $C_{T'}(n) < C_{T''}(n)$ for all $n > n_0$, and thus $\int_{n_0}^{1} C_{T''}(n)dF(n) > \int_{n_0}^{1} C_{T'}(n)dF(n) \geq \int_{n_0}^{1} C_{T^0}(n)dF(n)$. But T'' is a vertical translation downward (since $T'(n_0) \leq \alpha_0$) of some schedule which is of the same form as Tsup0. By construction T^0 maximizes total taxes collected from the set of individuals with $n \geq n_0$ among taxes of this form, so $\int_{n_0}^{1} C_{T^0}(n)dF(n) \geq \int_{n_0}^{1} C_{T''}(n)dF(n)$, so the inequality above cannot hold, a contradiction. Hence $t'^+(n_0) = \beta''$, from which it follows that $t'(X) = \beta''$ and hence $T'(X) = T''(X)$ for all $X \varepsilon [n_0, n^*_T]$. This implies that $n^*_{T''} = n^*_{T'}$, and hence that $C_{T''}(n) = C_{T'}(n)$ for all $n \geq n_0$, whence $\int_{n_0}^{1} C_{T'}(n)dF(n) = \int_{n_0}^{1} C_{T''}(n)dF(n) \leq \int_{n_0}^{1} C_{T^0}(n)dF(n)$.

This and the fact that $\int_{1}^{n_0} C_{T'}(n)dF(n) \leq \int_{1}^{n_0} C_{T^0}(n)dF(n)$ (established in the previous paragraph) imply, from feasibility, that both inequalities must be equalities, i.e. $\int_{n_0}^{1} C_{T^0}(n)dF(n) = \int_{n_0}^{1} C_{T''}(n)dF(n) = \int_{n_0}^{1} C_{T'}(n)dF(n)$ and $\int_{0}^{n_0} C_{T^0}(n)dF(n) = \int_{0}^{n_0} C_{T'}(n)dF(n)$. As noted earlier if $C_{T'}(n_0) < \alpha_0$ the second equality could not hold; hence $C_{T'}(n_0) = \alpha_0$ for any such T', so T^0 is optimal for n_0.

This last equality implies T' is also optimal. Since $C_{T'}(n) = T'(n)$ is increasing for $n \leq n_0$ $\int_{0}^{n_0} C_{T^0}(n)dF(n) = \int_{0}^{n_0} C_{T'}(n)dF(n)$ implies $T'(n) = C_{T'}(n) = \alpha_0$ for $n \leq n_0$. Hence T' coincides with T'' over $[0, n^*_{T''}]$. Since $C_{T''} = \alpha^0 = C_{T^0}$ for $n \leq n_0$ and $\int_{n_0}^{1} C_{T''}(n)dF(n) = \int_{n_0}^{1} C_{T^0}(n)dF(n)$ it follows that T'' is a feasible, optimal schedule of the indicated form, which proves the result.

QED

The individually optimal schedule is thus a sharply progressive one, which imposes a marginal tax rate of zero on incomes below n_0, and a positive and sizeable rate β_0 on incomes greater than n_0. Since $C_{T^0}(n)$ is unaffected by the t(X) for $X \geq n^*_{T^0}$, the shape of the tax schedule for high incomes is irrelevant for n_0 (or $\{n: n \geq n^*_{T^0}\}$); for appearance's sake all might well agree to impose sharply increasing marginal rates on such income levels, but these rates would never become effective. Increasing marginal rates over the range $(n_0, n^*_{T^0})$ would reduce the tax collected from upper-income taxpayers, however, so from n_0's point of view the optimal schedule should be linear over this range.

Individuals of a given (potential) income level n_0 will prefer a schedule whose kink is located at this income level. Thus if lower-income taxpayers can control the choice, the kink will be located far to the left, and the schedule will resemble a linear or flat-rate schedule over most of its range. Conversely, upper-income taxpayers would choose a schedule which is essentially a constant or per-capita tax for most of the population. If the political process is controlled by middle-income citizens, however, the resulting schedule will be a sharply progressive one, in which major segments of the population confront quite different marginal rates. Progressive income taxation of this kind is an effective means for the middle class to minimize their own tax burden, at the expense of lower- and upper-class taxpayers. Proposition 4.1 and the results of the previous section strongly suggest that the observed social preference for progressive income taxation has much more to do with individual self-interest and the desire to maximize personal welfare, rather than any attempt to promote social justice.

4.3 Self-Interested Voting

We now consider the question of majority voting over alternative tax schedules. The following preliminary result will be useful.

Comment 4.1 Under the condition of Proposition 3.6, there exists a unique T^0 for each n_0. The parameters α_0, β_0 are continuously differentiable functions of n_0, with $\frac{d\alpha_0}{dn_0} > 0$ and $\frac{d\beta_0}{dn_0} < 0$.

Proof As noted in the proof of Proposition 4.2, β^0 is a maximum of $g(\beta) = \beta[N(n_0) - N(n^*(\beta)]$, and lies in the interval $(1 - w(0), 1 - w(N(n_0)))$ in which g is continuously differentiable. Differentiating we get $g'(\beta) = [N(n_0) - N(n^*(\beta))] - \beta N'(n^*(\beta))n^{*\prime}(\beta) = [N(n_0) - N(n^*(\beta))] + \beta/w'(N(n^*(\beta)))$, where the fact that $N'(n^*(\beta))n^{*\prime}(\beta) = -1/w'(N(n^*(\beta)))$ follows from differentiating the equilibrium condition $w(N(n^*(\beta))) - 1 + \beta = 0$. Differentiating again we get

$$\begin{aligned} g''(\beta) &= -N'(n^*(\beta))n^{*\prime}(\beta) + [w'(N(n^*(\beta)))]^{-1} \\ &\quad -\beta[w'(N(n^*(\beta)))]^{-2}w''(N(n^*(\beta)))N'(n^*(\beta)n^{*\prime}(\beta) \\ &= 2(w')^{-1} + \beta w''/(w')^3 \\ &= [2w'^2 + \beta w'']/(w')^3. \end{aligned}$$

The condition of Proposition 3.7 implies that the quantity in square brackets is positive, and since w' is negative it follows that $g''(\beta) < 0$ for all $\beta \varepsilon [1 - w(0), 1 - w(N(n_0))]$, and hence that there exists a unique maximum β_0 in this interval. From the first-order condition $g'(\beta) = 0$ it follows that

$$\beta_0 = -w'(N(n^*(\beta_0))]N(n_0) - N(n^*(\beta_0))].$$

Hence, differentiating with respect to n_0, we have

$$\begin{aligned} \frac{d\beta_0}{dn_0} &= \frac{d}{dn_0}[-w'(N(n^*(\beta_0)))[N(n_0) - N(n^*(\beta_0))]] \\ &= -w'\left[N'(n_0) - N'(n^*(\beta_0))n^{*\prime}(\beta_0)\frac{d\beta_0}{dn_0}\right] \\ &\quad -[N(n_0) - N(n^*(\beta_0))]w''N'(n^*(\beta_0)n^{*\prime}(\beta_0)\frac{d\beta_0}{dn_0} \\ &= -w'N'(n_0) - w'\cdot\frac{1}{w'}\frac{d\beta_0}{dn_0} + [N(n_0) - N(n^*(\beta_0))]\frac{w''}{w'}\frac{d\beta_0}{dn_0} \\ &= -w'N'(n_0) - \left[1 + \frac{\beta_0 w''}{(w')^2}\right]\frac{d\beta_0}{dn_0}, \end{aligned}$$

whence

$$\frac{d\beta_0}{dn_0} = \frac{-w'^3N'(n_0)}{[2w'^2 + \beta_0 w'']} < 0,$$

since w' and N' are negative, and since the condition of Proposition 3.7 implies that $[2w'^2 + \beta_0 w'']$ is positive. Since the total tax collected under any schedule T^0 is $R_{T^0} = \alpha_0 + \beta_0[N(n_0) - N(n^*(\beta))]$, feasibility requires $\alpha_0 = -\beta_0[N(n_0) - N(n^*(\beta))]$, so

$$\frac{d\alpha_0}{dn_0} = -\beta_0\left[N'(n_0) - N'(n^*(\beta_0))n^{*\prime}(\beta_0)\frac{d\beta_0}{dn_0}\right]$$

$$-[N(n_0) - N(n^*(\beta_0))]\frac{d\beta_0}{dn_0}$$

$$= -\beta_0 N'(n_0) - \beta_0(1/w')\frac{d\beta_0}{dn_0} + \left[\frac{\beta_0}{w'}\right]\frac{d\beta_0}{dn_0}$$

$$= -\beta_0 N'(n_0) > 0,$$

since $\beta_0 > 0$ and $N'(n_0) < 0$.

QED

Denote by Θ the set of individually optimal schedules T^0 for some n_0, and by T_m the optimal schedule of the median (ability level) voter. Then:

<u>Proposition 4.3</u> Under the condition of Proposition 3.7, all taxpayer's preferences are single-peaked on Θ, and the median schedule T_m is a majority equilibrium within this set.

<u>Proof</u> Denote by $\hat{y}_n(n_0)$ the after-tax income of an individual with ability n, under the schedule T^0 which is optimal for n_0. We shall show that

$$\frac{d\hat{y}_n(n_0)}{dn_0} \begin{cases} < 0 & \text{for } n < n_0 \\ > 0 & \text{for } n > n_0, \end{cases}$$

which implies n's preferences are single-peaked over Θ, with (from Propositions 4.2 and 4.3) his most-preferred schedule at $n_0 = n$.

For $n \leq n_0$, under the schedule T_0 evidently $Y_n = n$, $C_{T^0}(n) = \alpha_0$ and $y_n = n - \alpha_0$, so $\frac{dy_n}{dn_0} = -\frac{d\alpha_0}{dn_0} > 0$, from Comment 4.1.

For $n \varepsilon (n_0, n^*_{T^0}]$, $Y_n = n > n_0$, so $C_{T^0}(n) = \alpha_0 + \beta_0(n - n_0)$ and $y_n = n - \alpha_0 - \beta_0(n - n_0)$, while if $n > n^*_{T^0}$ then $Y_n = n^*_{T^0} + w^*_{T^0}(n - n^*_{T^0}) = n^*_{T^0} + (1 - \beta_0)(n - n^*_{T^0}) = n - \beta_0(n - n^*_{T^0})$, $C_{T^0}(n) = \alpha_0 + \beta_0(n^*_{T^0} - n_0)$, so $y_n = n - \alpha_0 - \beta_0(n - n_0)$, so this relation holds for all $n > n_0$. Differentiating with respect to n_0 we have

$$\frac{dy_n}{dn_0} = -\frac{d\alpha_0}{dn_0} + \beta_0 - (n - n_0)\frac{d\beta_0}{dn_0}$$

$$= \beta_0 N'(n_0) + \beta_0 - (n - n_0)\frac{d\beta_0}{dn_0}$$

$$= [1 + N'(n_0)]\beta_0 - (n - n_0)\frac{d\beta_0}{dn_0}$$

Since $\frac{d\beta_0}{dn_0} < 0$ from Comment 4.1, and $\beta_0 > 0$ and $n > n_0$, the expression will be positive if $[1 + N'(n_0)] \geq 0$. Recall that $N'(n_0) = -[1 - F(n_0)]$, so $1 + N'(n_0) = F(n_0) > 0$ for all $n_0 > 0$.

QED

The schedule T_m is only an equilibrium within the restricted subset of Θ of schedules. Unlike Proposition 3.7, the above result cannot be extended to show that T_m is a majority equilibrium within the entire set of admissable schedules; in general no such equilibrium will exist in the purely self-interested, redistributive situation being considered here.

Proposition 4.3 is nevertheless suggestive as to the likely outcome under pure self-interested voting over the set of individually optimal schedules; the equilibrium schedule is the one which is optimal for the median-ability, i.e. middle-income voter, and is a sharply progressive one, as noted earlier. The result can also be given an alternative interpretation in a delegation or representative democracy framework: in most democracies citizens normally cannot vote directly on alternative tax schedules; rather, they vote for representatives, and delegate the various decisions of government, including those on taxation, to these elected delegates. Representatives of varying backgrounds will have differing attitudes toward taxation and redistribution, and their views may well reflect the preferences or interests of their own "class" (i.e. in the context of our simple model, those of similar ability levels). To the extent this is so, and that such distributional issues are important electoral considerations, the equilibrium outcome will be election of a median-ability representative, whose own inclinations on taxation issues will tend toward the kind of progressive taxation which favors the middle class.

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	Marginal Tax Rate for Taxpayers at		
	Tenth Percentile:	Median:	Ninetieth Percentile:
U.K.	.34	.34	.34
Australia	.335	.335	.335
Austria	.28	.28	.33
Germany	.22*	.22	.38
Denmark	.144	.144	.288
Italy	.10	.13	.22
Canada	.18	.21	.28
Netherlands	.20	.25	.39
U.S.	.16	.22	.36
France	.15	.20	.40
Norway	.06	.16	.33
Sweden	.02	.19	.31
Ireland	.20	.35	.45
Belgium	.21	.37	.44

Table 1. Marginal Tax Rates Applicable to Taxpayers at Selected Positions in the Before-Tax Income Distribution, Various Countries

Source: Income Tax Schedules. Distribution of Taxpayers and Revenues. OECD, Paris. 1981. Data from Table 2, p. 19, except as noted.

* Taken from country graph on page 28.

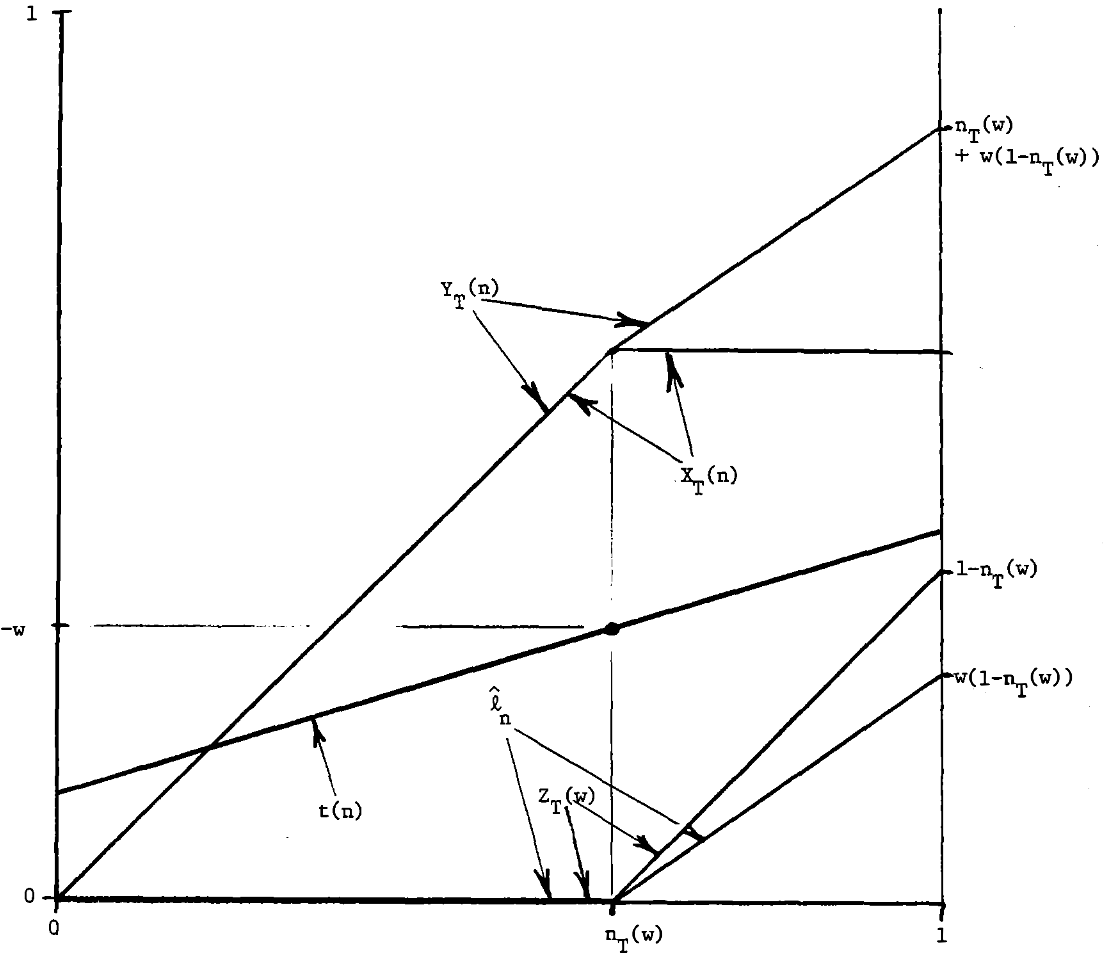

FIGURE 2.1

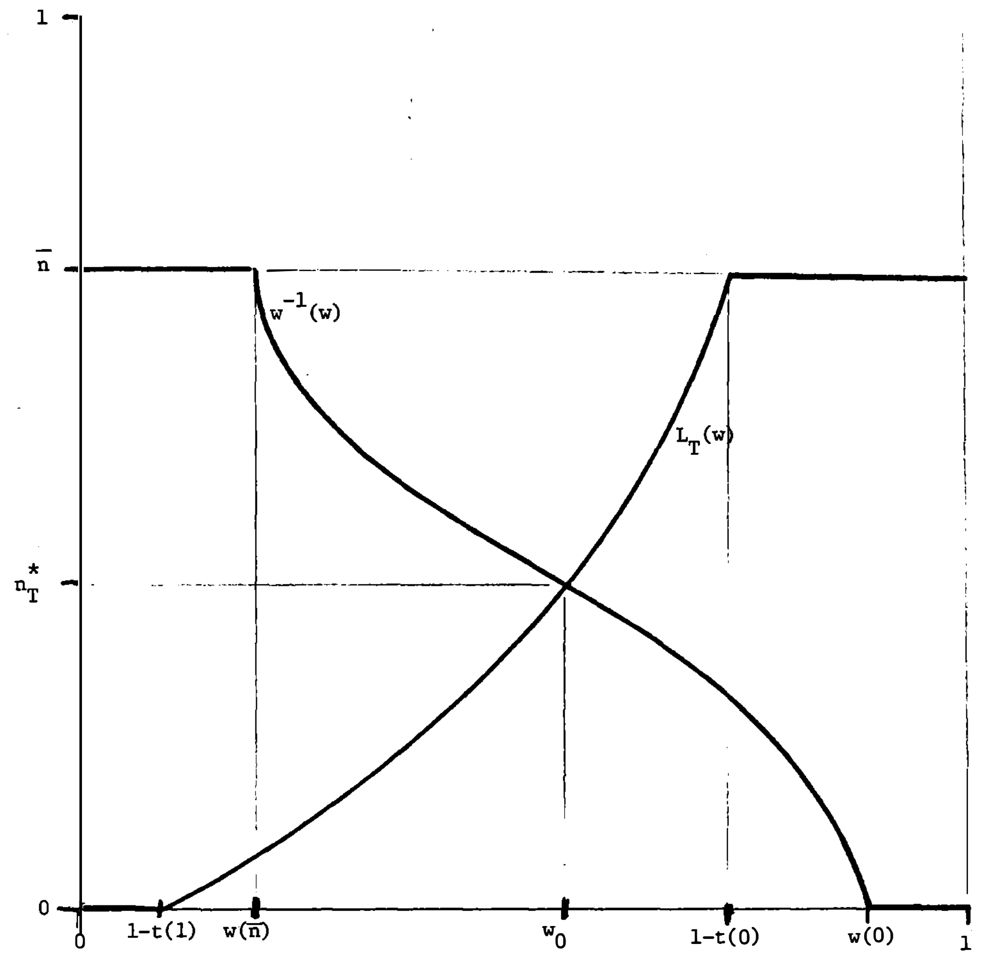

FIGURE 2.2

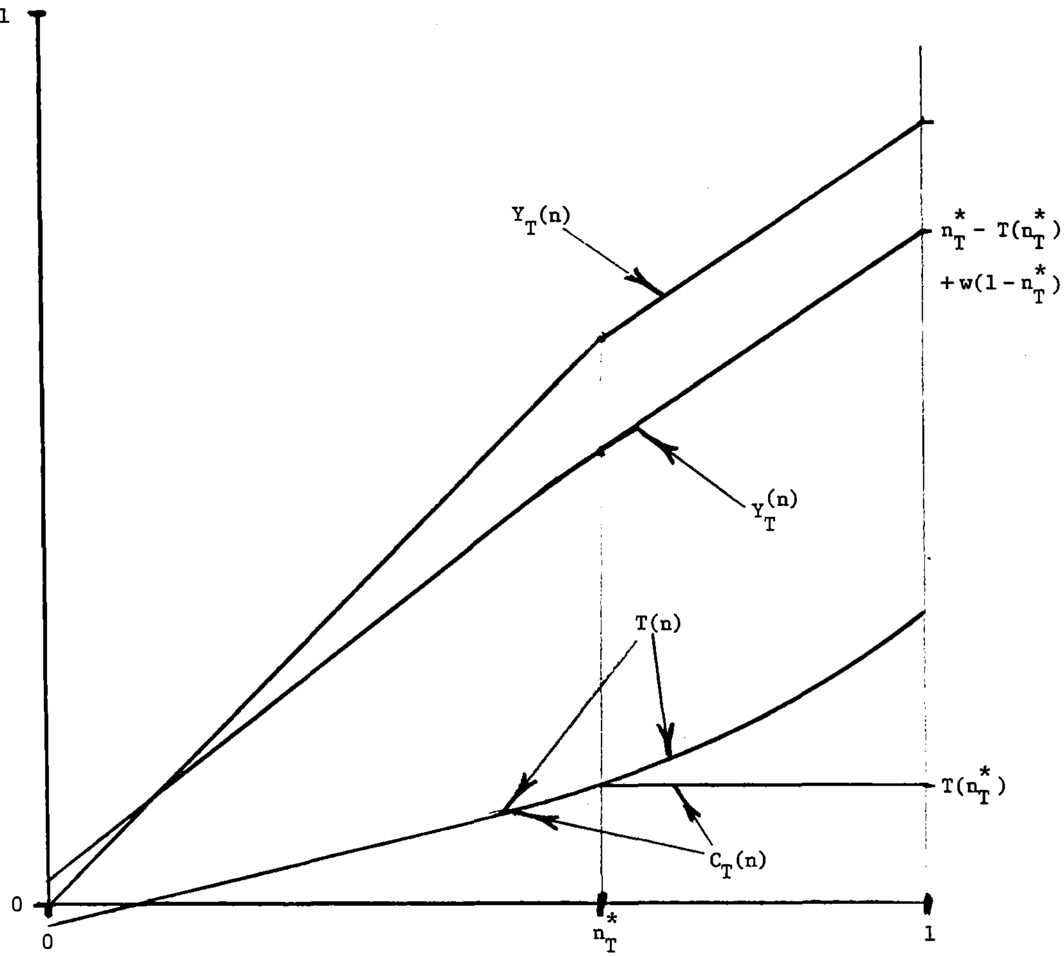

FIGURE 2.3

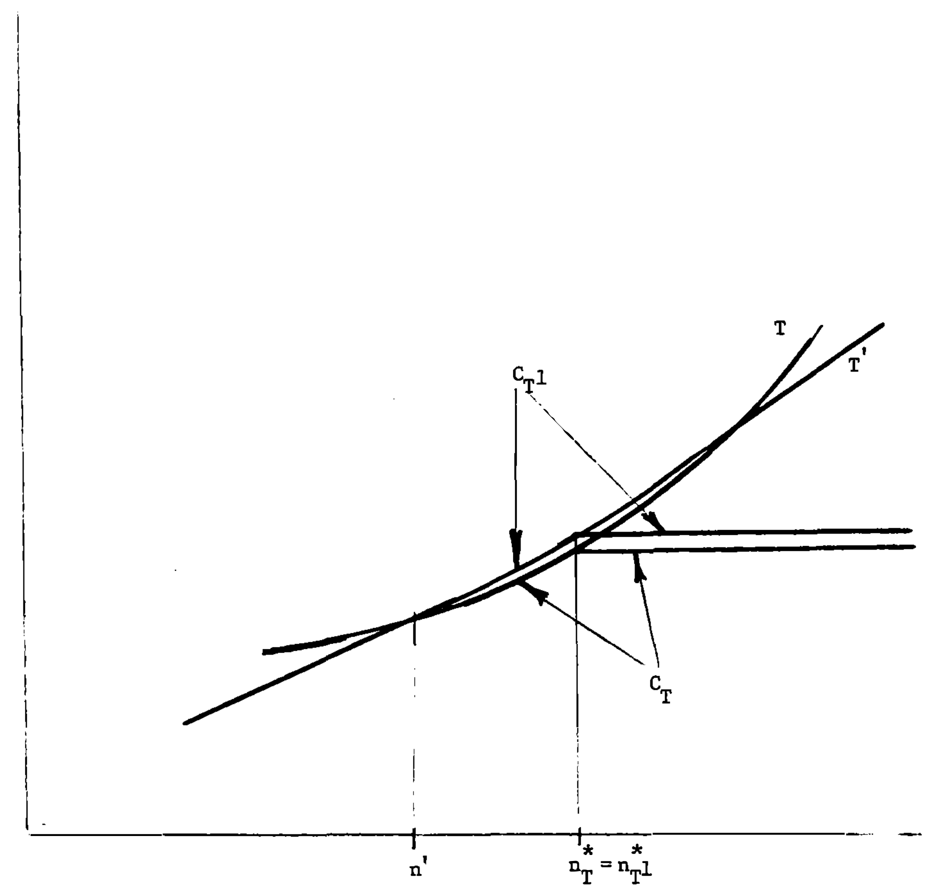

FIGURE 3.1

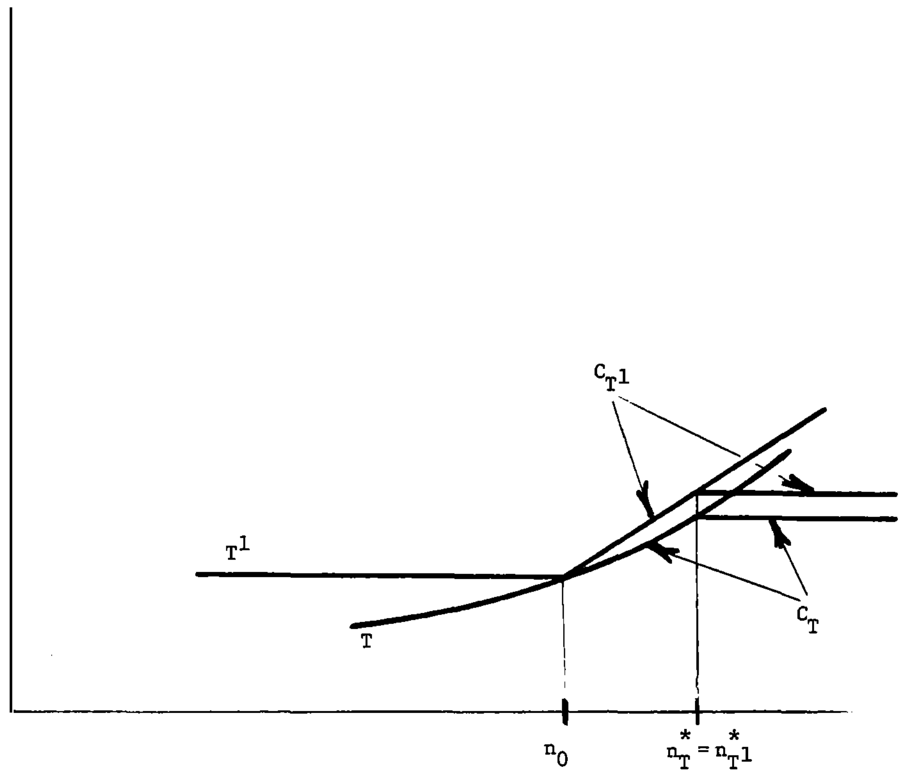

FIGURE 4.1