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A MODEL OF TAX COMPLIANCE WITH BUDGET-CONSTRAINED AUDITORS

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ABSTRACT

In the midst of various taxpayer "revolts" and federal budget deficits of unprecedented magnitude, noncompliance with federal and state income tax laws has become an issue of significant policy concern. If the IRS' budget is limited, the probability that any individual taxpayer will be audited depends on the behavior of other taxpayers. Thus the problem of compliance involves a "congestion" effect, which generates strategic interaction among taxpayers as well as between taxpayers and the IRS. This paper reflects an initial attempt to explore how the combination of a strategic IRS and asymmetric information affects the traditional theoretical results on tax compliance behavior.

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1. INTRODUCTION

In the midst of various taxpayer "revolts" and federal budget deficits of unprecedented magnitude, noncompliance with U.S. federal and state income tax laws has become an issue of significant policy concern. An American Bar Association Conference held in 1983 devoted to the topic suggested that the "compliance gap" is running at 10 to 15 percent of total taxable income, resulting in uncollected taxes totalling about \$100 billion. More recently, the President's Private Sector Survey on Cost Control reported that IRS tax examinations have decreased from 2.4 percent of all returns in 1977 to 1.7 percent in 1983.

Despite the shocking nature of noncompliance estimates, our understanding of the relationship between compliance with the tax laws and the structure of sanctions and efficacy of law enforcement strategies is in an infant stage. The economics-based literature on crime and punishment has made little genuine progress since the basic theoretical structure was established fifteen years ago by Gary Becker (1968). Much work has been done by economists using this general model since then, including additional theoretical work as well as extensive empirical work in the context of noneconomic crimes such as murder and noneconomic penalties such as death. The empirical work to date remains highly controversial (e.g., Ehrlich, 1975 and 1977) and the improvements in theory have failed to yield additional practical, policy-relevant insights (e.g., Polinsky and Shavell, 1979; Landes,

1983). In particular, there has been little effort to develop and test theoretical models designed to be useful to federal, state or local law enforcement agencies in establishing and implementing effective programs.

The model we present in this paper differs substantially from those found in the extant economics literature on tax evasion. Much of that literature treats noncompliance as a problem in decisionmaking under uncertainty where the taxpayer faces given probabilities of detection and conviction, and given tax and penalty functions (e.g., Allingham and Sandmo, 1972, Srinivasan, 1973; Yitzhaki, 1974). The problem then is to characterize compliance incentives given these exogenous parameters. Extensions of this basic approach often have focused on standard labor supply decisions (e.g., Andersen, 1977; Baldry, 1979; Pencavel, 1979) or on trade-offs between labor supply in primary markets where the wage is observable and secondary markets where it is not (e.g., Isachsen and Strom, 1980; Sandmo, 1981). In some cases attention has been given to the optimal choice of probabilities of detection for proscribed behavior, usually giving the government free rein to set a number of instruments in order to maximize a measure of social welfare (e.g., Fishburn, 1979; Kemp and Ng, 1979; Polinsky and Shavell, 1979; Sandmo, 1981; Landsberger and Meilijson, 1982). While Landsberger and Meilijson (1982) and Greenberg (1984) do condition the probability of audit in a given period on whether the taxpayer was caught under-reporting in some prior period, in none of these models is the likelihood of audit in a given period ever made dependent on currently observed behavior, such as a level of income reported to the IRS. It is clear, however, that actual IRS audits are not entirely random, even within classes of taxpayers, and it is important to introduce this fact into economic models. It is also important, then, to ask how the probability of audit, contingent on reported income, is determined by the IRS.

There are two basic ways to approach the problem. The first is via the principal-agent framework, viewing the IRS as principal and the taxpayer as agent. A recent example of this kind of model can be found in Reinganum and Wilde (forthcoming). A second way to approach the problem is via game theory using, in particular, techniques from the literature on games of incomplete information. The difference between the two approaches is that the latter presumes no precommitment on the part of any actor, be it the IRS or taxpayers. Taking the structure of taxes and fines as given, the IRS picks an audit strategy and the taxpayers pick reporting strategies, each of which must be optimal against the other.

While it seems natural to apply game theory to the problem of tax compliance, there are surprisingly few models which do so. The two leading examples are due to Landsberger and Meilijson (1982) and Greenberg (1984). However, these authors focus on repeated games, and, as noted above, only tie audit probabilities to whether a taxpayer was caught under-reporting in the past -- they do not incorporate the crucial role of self-reporting in the tax-compliance game. This paper, along with a companion piece (Graetz, Reinganum and Wilde, 1983), reflects an initial attempt to explore how the combination of a strategic IRS and asymmetric information affects the traditional theoretical results on tax compliance behavior.

One of the most prevalent features of actual law enforcement systems is the presence of budget constraints--one seldom hears police chiefs, judges or wardens complaining about too many officers, too few cases on the docket, or excess prison space. There is evidence that the IRS is also constrained to audit fewer returns than is optimal. Vitez (1983) states that in 1979 the IRS information matching program generated 9.4 million cases of under-reporting, but only 3.6 million of these were investigated. These investigations

resulted in an average ex post revenue/cost ratio of 10:1. While the marginal ex ante revenue/cost ratio was undoubtedly lower, there is no reason to believe that it was unity. Clearly, information on the actual value of the latter would need to be obtained in order to properly evaluate the efficacy of expanding the IRS budget in generating increased compliance. In any event, we take as our starting point in this paper the presence of a budget constraint, although we do not necessarily assume it is binding. If the budget constraint is binding, however, the probability that any individual taxpayer will be audited depends on the behavior of other taxpayers. This "congestion" feature of the compliance problem has been largely ignored in the crime and punishment literature generally (except see Nagin (1978) who points out that the observed negative relationship between index crime rates and sanction levels may be explained by a constraint on total prison populations), and appears nowhere in the literature on tax compliance. Some authors do consider the effects of budget constraints (e.g., Greenberg, 1984), but none explicitly model the strategic interaction between taxpayers so the congestion effect never arises. There are also surveys which report that noncompliance is more likely as a taxpayer knows more other taxpayers who have failed to comply (e.g., Spicer and Lundstedt, 1976; Song and Yarbrough, 1978). But the explanations offered for this phenomenon usually are psychological or sociological, not economic.

In Section 2 we present a model in which two taxpayers confront an IRS able to audit at most one of them. The IRS is permitted to use a sequential auditing strategy, conditioning its decision to audit on the number of unaudited low reports remaining. We show that whenever a Nash equilibrium exists with a sequential auditing strategy, an alternative equilibrium exists in which the IRS employs a nonsequential strategy, and equilibrium taxpayer compliance is the same. In Section 3 this model is generalized to an

arbitrarily large number of taxpayers confronting an IRS that can audit at most some fraction of them. Considering the taxpayers as a group, taking the IRS audit strategy as given, we find that multiple equilibrium reporting strategies are possible if the budget constraint is neither too low nor too high. However, once the IRS is added as a strategic actor, we find that the equilibrium is often unique, and that a binding budget constraint generally leads to total noncompliance. If the budget constraint is not binding, both audits and noncompliance are probabilistic, the unique equilibrium being similar to the one analyzed in Graetz, Reinganum and Wilde (1983). In Section 4 we discuss these results in more detail, and conclude in Section 5 with some general remarks on future work.

2. THE BASIC MODEL: TWO TAXPAYERS, ONE AUDIT

Consider a situation in which two identical taxpayers confront an IRS which can afford to audit at most one of them. Income is uncertain and takes one of two values, I_L or I_H , where $I_H > I_L > 0$. The probability of high income is q . Taxpayers observe their own income and then make a report to the IRS, which only observes the taxpayer's true income if an audit is performed at cost c , where $c > 0$. The tax levels are T_L and T_H for low and high income respectively, where $T_H > T_L \geq 0$. If a taxpayer is audited and found to be under-reporting, a fine of F is collected in addition to the unpaid taxes. We assume $T_i + F \leq I_i$ for $i = L, H$ and $T_H \leq I_L$, so it is possible, in principle, for a low income taxpayer to report high income. Tax rates and fines are taken as given by all parties.

Taxpayers and the IRS are assumed to be risk-neutral, but the results are not substantially affected if the taxpayers are risk averse. Taxpayers are assumed to minimize expected net tax system costs (taxes plus fines) and

the IRS maximizes expected net revenue (taxes plus fines minus audit costs). In principle, taxpayers with low income could report I_H . But it would never pay to do this since $T_L < T_H$. Thus we need only consider whether a taxpayer with high income reports honestly.

$$\alpha_i \equiv \text{Prob}\{\text{taxpayer } i \text{ reports } I_L \mid \text{taxpayer } i \text{ has income } I_H\}.$$

The IRS observes the total number of low income reports and high income reports. It then decides on (1) an ordering of taxpayers and (2) a set of associated audit probabilities which are exercised sequentially until either all taxpayers or the budget is exhausted. As with high income reports by taxpayers with low income, it never pays the IRS to audit those who report high income since doing so increases audit costs without increasing expected tax revenues. Thus we need only condition audit probabilities on the number of taxpayers who have reported low income and have not yet faced an audit lottery. Furthermore, since these taxpayers are indistinguishable to the IRS, the order in which they face the sequence of audit lotteries is random. Define, for $i = 1$ and 2 ,

$$\beta_i = \text{Prob}\{\text{IRS audits the next (randomly chosen) taxpayer who reported } I_L \text{ when } i \text{ such taxpayers remain}\}.$$

We have elsewhere (1983) used this model to analyze the effect of "honest" taxpayers (who always report truthfully) on the compliance behavior of "strategic" taxpayers (who select a reporting strategy so as to maximize expected net income). In that model no budget constraint is imposed on the IRS so there is no direct interaction between the reporting strategies of the taxpayers -- a Nash Equilibrium for the game can be characterized simply by considering the interaction between the IRS and a representative taxpayer.

However, once a budget constraint is imposed on the IRS, the likelihood of audit facing one taxpayer depends on the reporting strategy of the other taxpayers. Thus it becomes useful to consider first the interaction between the taxpayers in selecting their reporting strategies, taking the IRS audit strategy as given.

2a. Taxpayer Equilibria

In this model, the likelihood of audit depends on two factors, the probability of "exposure" to an "audit lottery" and the probability of audit given exposure. For example, suppose one taxpayer always reports low income and suppose $\beta_1 = 1 = \beta_2$, so that the IRS would always audit any taxpayer who reports low income if it had the resources to do so. If the second taxpayer reports low income, he or she will be indistinguishable from the first taxpayer and will thus face a probability of exposure of one-half (since in this model there are only two taxpayers and at most one can be audited). However, given exposure, the probability of audit is one. Of course, it need not be; β_1 and β_2 will ultimately be determined in equilibrium. But for now we take them as given, and analyze the taxpayers' reporting strategies, α_1 and α_2 .

Suppose taxpayer i observes high income. If he or she reports low income then there is a chance of being audited. If the other taxpayer, say taxpayer j , reports high income, which happens with probability $(1 - \alpha_j)q$, then the probability of audit is just β_1 . If the other taxpayer reports low income, which happens with probability $1 - q + \alpha_j q$, then the probability of audit depends on whether the other taxpayer is audited first, since only one can be audited. But taxpayers are indistinguishable to the IRS except for their reported incomes, so the probability of audit is

$(1/2)\beta_2 + (1/2)(1 - \beta_2)\beta_1$. Thus we can write the expected cost of reporting I_L with probability α_i when income is actually I_H (given that the other taxpayer uses the strategy α_j and the IRS uses the strategy (β_1, β_2)) as

$$\begin{aligned} C(\alpha_i; \alpha_j, \beta_1, \beta_2) \equiv & \alpha_i \left[(1 - \alpha_j)q[\beta_1(T_H + F) + (1 - \beta_1)T_L] \right. \\ & + (1 - q + \alpha_j q) \{ [(1/2)\beta_2 + (1/2)(1 - \beta_2)\beta_1](T_H + F) \\ & \left. + [1 - (1/2)\beta_2 - (1/2)(1 - \beta_2)\beta_1]T_L \right] \\ & + (1 - \alpha_i)T_H. \end{aligned} \quad (1)$$

A best response for agent i is a strategy, $\hat{\alpha}_i(\alpha_j; \beta_1, \beta_2)$ such that $C(\hat{\alpha}_i(\alpha_j; \beta_1, \beta_2); \alpha_j, \beta_1, \beta_2) \leq C(\alpha; \alpha_j, \beta_1, \beta_2)$ for all other strategies α . A taxpayer equilibrium is a pair of strategies, $\alpha_1^*(\beta_1, \beta_2)$ and $\alpha_2^*(\beta_1, \beta_2)$, such that $\hat{\alpha}_i(\alpha_j^*; \beta_1, \beta_2) = \alpha_i^*(\beta_1, \beta_2)$ for $i = 1, 2$; that is, $\alpha_1^*(\beta_1, \beta_2)$ and $\alpha_2^*(\beta_1, \beta_2)$ are best responses to each other, given β_1 and β_2 .

For notational convenience, define

$$\begin{aligned} k &= (T_H - T_L)/(T_H - T_L + F), \\ k_1 &= \beta_2 + (1 - \beta_2)\beta_1, \end{aligned}$$

and

$$k_2 = 2\beta_1 q + [\beta_2 + (1 - \beta_2)\beta_1](1 - q).$$

Proposition 1: A taxpayer equilibrium (TE) always exists and is symmetric, but may not be unique. Let $\alpha^*(\beta_1, \beta_2)$ denote a symmetric TE. Two cases are possible.

- (A) $\beta_2 - (1 + \beta_2)\beta_1 > 0$: In this case $k_2 < k_1$. Furthermore,
- (i) If $k \leq k_2/2$, then $\alpha^*(\beta_1, \beta_2) = 0$ is the unique TE;
 - (ii) If $k_2/2 < k < k_1/2$, then the unique TE is given by

$$\alpha^*(\beta_1, \beta_2) = \alpha^0(\beta_1, \beta_2) \equiv (2k - k_2)/q[\beta_2 - (1 + \beta_2)\beta_1]. \quad (4)$$

(iii) If $k_1/2 \leq k$, then $\alpha^*(\beta_1, \beta_2) = 1$ is the unique TE;

(B) $\beta_2 - (1 + \beta_2)\beta_1 \leq 0$: In this case $k_1 \leq k_2$. Furthermore,

(i) If $k < k_1/2$, then $\alpha^*(\beta_1, \beta_2) = 0$ is the unique TE;

(ii) If $k_1/2 \leq k \leq k_2/2$, three TE exist, both pure strategy TE and the mixed strategy TE $\alpha^0(\beta_1, \beta_2)$ given in (4) above;

(iii) If $k_2/2 \leq k$, then $\alpha^*(\beta_1, \beta_2) = 1$ is the unique TE.

The proof of this result and those that follow can be found in the appendix to this paper. Cases (A) and (B) of the proposition are illustrated in Figures 1(A) and 1(B), respectively. In Figure 1(B), k_2 is presumed to be less than 1 although this need not be so. It is always the case that $k_1 < 1$. By definition, $k = 1$ when $F = 0$ and k goes to zero as F gets large. Thus, if the fine is low, no compliance is always a TE, and if it is high enough, full compliance is always a TE. If it is of intermediate size (relative to β_1 and β_2), then the mixed strategy TE also emerges. In this situation it is case (B) that is of particular interest since it illustrates the congestion effect mentioned in the introduction. Moreover, this case arises whenever $\beta_1 = \beta_2$ or whenever $\beta_1 = 1$, neither of which is unnatural.

2b. The Full Compliance Game

Since all taxpayer equilibria are symmetric, the IRS can treat taxpayers identically, and needs only consider the total number of unaudited low income reports (relative to its remaining budget) in order to determine the optimal audit probability with which to confront the next randomly selected taxpayer. Since in this model the budget constraint allows only one audit and there are only two taxpayers, the problem is reduced to selecting β_1 and β_2 , where β_1 applies when one unaudited low income report remains and β_2

applies when both taxpayers have reported low income. Define the posterior probability that a taxpayer who reports low income actually has high income to be μ . Then

$$\mu = \alpha q / [1 - q + \alpha q] \quad (5)$$

where α is some symmetric taxpayer reporting strategy. Expected revenue when only one unaudited low income report remains is

$$R^1(\beta_1; \alpha) = \beta_1 [\mu(T_H + F) + (1 - \mu)T_L - c] + (1 - \beta_1)T_L \quad (6)$$

and when two unaudited low income reports remain is

$$R^2(\beta_1, \beta_2; \alpha) = \beta_2 [\mu(T_H + F) + (1 - \mu)T_L - c] + (1 - \beta_2)R^1(\beta_1; \alpha) \quad (7)$$

Equation (7) includes only expected income from the audited taxpayer since it applies only when both taxpayers report low income -- only one can be audited and the taxpayer who doesn't get audited will pay T_L whomever he or she turns out to be. This constant T_L is irrelevant to the decision problem of the IRS.

A best response for the IRS is a pair of strategies $\hat{\beta}_1(\alpha)$ and $\hat{\beta}_2(\alpha)$ such that $R^1(\hat{\beta}_1(\alpha); \alpha) \geq R^1(\beta_1; \alpha)$ and $R^2(\hat{\beta}_1(\alpha), \hat{\beta}_2(\alpha); \alpha) \geq R^2(\beta_1, \beta_2; \alpha)$ for any other strategies β_1 and β_2 . A Nash Equilibrium is a set of strategies $(\bar{\alpha}, \bar{\beta}_1, \bar{\beta}_2)$ such that (i) $\bar{\alpha}$ is a taxpayer equilibrium given $\bar{\beta}_1$ and $\bar{\beta}_2$, and (ii) $\bar{\beta}_i = \hat{\beta}_i(\bar{\alpha})$ for $i = 1, 2$; that is, $\bar{\beta}_1$ and $\bar{\beta}_2$ are best responses to $\bar{\alpha}$.

Lemma 1: The best response functions for the IRS are:

$$\hat{\beta}_1(\alpha) = \begin{cases} 0 & \text{if } \alpha \leq a \\ [0, 1] & \text{if } \alpha = a \\ 1 & \text{if } \alpha > a \end{cases}$$

and

$$\hat{\beta}_2(\alpha) = \begin{cases} 0 & \text{if } \alpha < a \\ [0,1] & \text{if } \alpha \geq a \end{cases}$$

where

$$a = (1 - q)c/q(T_H + F - T_L - c). \quad (8)$$

Proposition 2: A Nash Equilibrium (NE) always exists, but it may not be unique. Four cases are possible.

(A) $c > q(T_H + F - T_L)$: Here $(\bar{\alpha}, \bar{\beta}_1, \bar{\beta}_2) = (1, 0, 0)$ is the unique NE

(B) $c \leq q(T_H + F - T_L)$: Three subcases are possible.

(i) $k > (1 + q)/2$: Here $(\bar{\alpha}, \bar{\beta}_1, \bar{\beta}_2) = (1, 1, \beta_2)$ is a NE for any

$$\beta_2 \in [0, 1].$$

(ii) $(1 + q)/2 \geq k \geq 1/2$: Here $(\bar{\alpha}, \bar{\beta}_1, \bar{\beta}_2) = (1, 1, \beta_2)$ is a NE for any

$$\beta_2 \in [0, 1]. \text{ If}$$

$$\alpha^0(1, \beta_2) = (1 + q - 2k)/q > a, \quad (9)$$

where α^0 is defined in (4) and a is defined in (8), then two other NE exist, one with $(\bar{\alpha}, \bar{\beta}_1, \bar{\beta}_2) = (\alpha^0(1, \beta_2), 1, \beta_2)$ for any $\beta_2 \in [0, 1]$, and the other with $(\bar{\alpha}, \bar{\beta}_1, \bar{\beta}_2) = (a, b_1, b_2)$, where b_1 and b_2 are any $b_1 \in [0, 1]$ and $b_2 \in [0, 1]$ such that

$$b_1 = \frac{b_2[aq + (1 - q)] - 2k}{(a - 1)q(1 + b_2) - (1 - b_2)}; \quad (10)$$

(iii) $k \leq 1/2$: Here $(\bar{\alpha}, \bar{\beta}_1, \bar{\beta}_2) = (a, b_1, b_2)$ is a NE for any $b_1 \in [0, 1]$

and $b_2 \in [0, 1]$ such that (10) holds.

To interpret Proposition 2, recall that $k = (T_H - T_L)/(T_H - T_L + F)$, so that it is inversely related to the level of the fine. If $c > q(T_H + F - T_L)$ then it never pays to audit anyone and no one complies.

Otherwise there is always some auditing. How much depends on the size of the fine relative to the other parameters. When the fine is small enough -- case B(i) -- no one ever complies, but as many low income reports as possible are audited. Since $\bar{\beta}_1 = 1$, $\bar{\beta}_2$ is irrelevant and if both taxpayers report low income they face equal chances of being audited. If the fine is somewhat higher -- case B(ii) -- multiple equilibria are possible, some in which $\bar{\beta}_1 = 1$ and others in which both $\bar{\beta}_1$ and $\bar{\beta}_2$ are between zero and one. In these equilibria there is a unique compliance strategy given by $\bar{\alpha} = a$, but $\bar{\beta}_1$ and $\bar{\beta}_2$ can trade off against each other in a variety of ways. If the fine is high enough -- case B(iii) -- these latter equilibria are the only ones that exist.

The following Corollary is of interest in its own right, and proves useful for subsequent analysis.

Corollary 1: Suppose $c \leq q(T_H + F - T_L - c)$ and $1 + q - 2k > qa$; that is, condition (9) holds. Then there exists $b \in (0, 1)$ such that (10) holds with $b_1 = b_2 = b$.

In other words, if there exists a pair of audit probabilities, b_1 and b_2 , such that $(\bar{\alpha}, \bar{\beta}_1, \bar{\beta}_2) = (a, b_1, b_2)$ is a NE, then there is a nonsequential strategy which is also a NE with $\bar{\alpha} = a$. Therefore, the IRS can forego the more complicated sequential strategies in which $\bar{\beta}_1$ and $\bar{\beta}_2$ differ, in favor of the simpler strategy of exposing all taxpayers to the same audit probability until its budget is exhausted. We will make use of this fact in the next section.

3. THE MODEL WITH MANY TAXPAYERS

Several variations of the basic model analyzed in Section 2 are possible, but we will only consider one in this paper. It is of some policy

importance to understand the trade-off between fines and the likelihood of audit in generating increases in compliance. To do that in this model we let the number of taxpayers get arbitrarily large and assume that only some fraction of them can be audited by the IRS. Otherwise we retain the assumptions of the basic model. However, in light of Corollary 1, we will focus on nonsequential IRS strategies in which the probability of audit given exposure is constant.

The analysis proceeds much as in Section 2. In particular, it will still be the case that taxpayers with low income will never report high income, and the IRS will never audit high income reports. Thus, we need only consider the taxpayers' decision whether to report low income when true income is high and the IRS's decision whether to audit when low income reports are observed. We again proceed with taxpayer equilibria first and then add the IRS to the problem.

3a. Taxpayer Equilibria

Assume there exists an arbitrarily large number of taxpayers but the IRS can only afford to audit λ percent of them, where $0 < \lambda < 1$. As before, whether a taxpayer gets audited depends on two factors, the probability of exposure and the probability of audit given exposure. The latter is denoted by β . If $\beta \leq \lambda$, then by the law of large numbers, the probability of exposure is one, since β percent of those exposed get audited, and λ percent of the population can be audited in principle. If $\beta > \lambda$, then the probability of exposure depends on the reporting strategies of the other taxpayers. In particular,

$$\text{Prob}(\text{exposure to an audit} | I_L \text{ is reported}) = \min\left(1, \frac{\lambda/\beta}{1 - q + \alpha q}\right), \quad (11)$$

where α is the symmetric reporting strategy used by all taxpayers. The logic behind (11) is that λ/β is the expected limit on the percentage of taxpayers who can be audited. But $1 - q + \alpha q$ is the expected percentage who report I_L . Thus $(\lambda/\beta)/(1 - q + \alpha q)$ is the expected proportion of those who report I_L who actually face the audit lottery β . It turns out that $(\lambda/\beta)/(1 - q + \alpha q) < 1$ in equilibrium if and only if $\beta < \lambda/(1 - q)$. Hence if $\beta \geq \lambda/(1 - q)$, we can write the expected tax-system costs for a taxpayer using reporting strategy α when all other taxpayers use γ as

$$C(\alpha; \gamma, \beta) = \alpha[\beta(T_H + F) + (1 - \beta)T_L] + (1 - \alpha)T_H, \quad (12)$$

and if $\beta < \lambda/(1 - q)$

$$C(\alpha; \gamma, \beta) = \alpha\left\{\left[\frac{\lambda/\beta}{1 - q + \gamma q}\right][\beta(T_H + F) + (1 - \beta)T_L] + \left[1 - \frac{\lambda/\beta}{1 - q + \gamma q}\right]T_L\right\} + (1 - \alpha)T_H. \quad (13)$$

Best responses and taxpayer equilibria are defined analogously to those in Section 2.

Proposition 3: When the number of taxpayers is arbitrarily large and audits are limited to λ percent, a taxpayer equilibrium (TE) always exists and is symmetric, but may not be unique. Three cases are possible, depending on λ and k .

- (A) $\lambda < k(1 - q)$: Here $\alpha^*(\beta) = 1$ is the unique TE.
 (B) $k(1 - q) \leq \lambda \leq k$: Here $\alpha^*(\beta) = 1$ is always a TE. Furthermore, if $\beta \geq k$, two other TE exist, $\alpha^*(\beta) = 0$, and

$$\alpha^*(\beta) = \alpha^0 \equiv \frac{\lambda - (1 - q)k}{qk}. \quad (14)$$

- (C) $k < \lambda$: Here $\alpha^*(\beta) = 1$ is a TE if $\beta \leq k$ and $\alpha^*(\beta) = 0$ is a TE if $\beta \geq k$.

Figures 2(A), 2(B), and 2(C) illustrate these three cases. Notice that $\alpha^*(\beta) = a^0$, the mixed strategy TE given in (14), is independent of β , but is increasing in F and λ . Thus equilibrium noncompliance increases with the penalty F and with the IRS' audit capability. These peculiar features will be discussed further in Section 4.

3b. The Full Compliance Game

The problem facing the IRS, given a symmetric taxpayer equilibrium α , is exactly the same as in Section 2. Thus the next proposition is immediate, the proof following exactly as that of Proposition 2. The definition of Nash Equilibrium for this case also mimics that of Section 2.

Proposition 4: When the number of taxpayers is arbitrarily large and audits are limited to λ percent, a Nash Equilibrium (NE) always exists, but it may not be unique. Four cases are possible.

- (A) $c > q(T_H + F - T_L)$: Here $(\bar{\alpha}, \bar{\beta}) = (1, 0)$ is the unique NE.
- (B) $c \leq q(T_H + F - T_L)$: Three subcases are possible.
- (i) $\lambda < k(1 - q)$: Here $(\bar{\alpha}, \bar{\beta}) = (1, 1)$ is the unique NE.
 - (ii) $k(1 - q) \leq \lambda \leq k$: Here $(\bar{\alpha}, \bar{\beta}) = (1, 1)$ is always a NE. If $a^0 = a$, where a is defined in (8), and a^0 is defined in (14) then $(\bar{\alpha}, \bar{\beta}) = (a, \beta)$ is a NE for all $\beta \geq k$. If $a^0 > a$, then $(\bar{\alpha}, \bar{\beta}) = (a^0, 1)$ is a NE.
 - (iii) $\lambda > k$: Here $(\bar{\alpha}, \bar{\beta}) = (a, k)$ is the unique NE.

The three subcases (in which $\bar{\beta} > 0$) are illustrated in Figures 2(A), 2(B) and 2(C), along with the various taxpayer equilibria for the limiting case.

4. DISCUSSION

In a sense, the limiting case of Section 3 is the more interesting of the two models. The assumption of an arbitrarily large number of taxpayers eliminates certain second-order effects and thus greatly simplifies the results. In particular, the mixed strategy taxpayer equilibria are independent of β so comparative statics are easier. For example, when $\lambda < k(1 - q)$, $(\bar{\alpha}, \bar{\beta}) = (1, 1)$ is the unique NE, so the marginal effects of changes in the underlying parameters are nil; when $k(1 - q) \leq \lambda \leq k$, two NE are possible (ignoring the knife-edge case), $(\bar{\alpha}, \bar{\beta}) = (1, 1)$ and $(\bar{\alpha}, \bar{\beta}) = (a^0, 1)$; and when $\lambda > k$, the unique NE is $(\bar{\alpha}, \bar{\beta}) = (a, k)$. The effects of changes in T_L , T_H , F , λ , 1 and c on a^0 , a , and k are given in Table 1.

The anomalous equilibrium in this model is the mixed strategy equilibrium when $k(1 - q) \leq \lambda \leq k$: $(\bar{\alpha}, \bar{\beta}) = (a_0, 1)$. Here $\lambda/\bar{\beta} < 1 - q + \bar{\alpha}q$ so that the budget constraint is binding. But relaxing the budget constraint induces an increase in noncompliance since the IRS ability to audit more taxpayers must be compensated by an increase in noncompliance to keep individual taxpayers indifferent between complying and not complying. Similar logic explains the fact that a^0 is increasing in q , F and T_H and decreasing in T_L .

The more natural equilibrium in which the taxpayers play mixed strategies (as well as the IRS in this case) occurs when $\lambda > k$. In this case, however, the budget constraint is not binding and the comparative statics are similar to those found in Graetz, Reinganum and Wilde (1983). Increases in the tax on high income or decreases in the tax on low income increase both compliance and auditing. Increases in the fine increase compliance and decrease auditing. Increases in the proportion of taxpayers who can be audited have no effect since the budget constraint is not binding. Increases

in the probability of high income or decreases in the cost of auditing have no effect on auditing but both increase compliance.

The conclusion here is that when the budget constraint is binding, one equilibrium is for taxpayers never to comply and the IRS to audit as many as possible, each with probability one. Of course, the effective probability of audit is then λ . If λ is small enough this is the only equilibrium. If it is slightly larger, another equilibrium in which some compliance occurs can exist. However, this equilibrium is dominated in expected cost terms, as viewed by the taxpayers, by the "never comply" alternative. If taxpayers are able to coordinate on the "never comply" taxpayer equilibrium (e.g., organize a "tax revolt"), then we would expect to observe the pure strategy equilibrium $(\bar{\alpha}, \bar{\beta}) = (1, 1)$ rather than any mixed strategy equilibrium. Thus, generally speaking, a binding budget constraint yields pure strategy outcomes. If λ is high enough, a more sensible mixed strategy equilibrium occurs (and is unique). This equilibrium involves both probabilistic audits and probabilistic compliance.

A natural question which this raises is when do the various cases occur? To get a feel for this consider the following example using fines proportional to evaded taxes; i.e., $F = \pi(T_H - T_L)$. Here $k = 1/(1 + \pi)$ so $(\bar{\alpha}, \bar{\beta}) = (1, 1)$ is an equilibrium outcome unless $\lambda > 1/(1 + \pi)$. If π were on the order of .5 to 1.0, we need λ greater than .67 in the former case and .5 in the latter. Overall, the IRS currently audits about 2 percent of all returns, although perhaps as many as 5 percent are considered for possible auditing. Even if $\pi = 3$ (which is highly unlikely in practice), it would require $\lambda > .25$ to yield any compliance at all.

A final point concerns the IRS objective function. Suppose compliance is important to the IRS, in addition to expected net revenue. If the maximum

proportion of taxpayers the IRS can audit is small -- $\lambda < k(1 - q)$ -- which is likely to be the case in practice, then nothing the IRS can do will yield an equilibrium in which everyone complies. However, if the maximum proportion of taxpayers the IRS can audit is somewhat larger -- $k \geq \lambda \geq k(1 - q)$ -- then if the IRS audits as many taxpayers as it can afford -- $\bar{\beta} = 1$ -- it is possible that taxpayers will always comply. Of course, the IRS would maximize expected revenue by setting $\bar{\beta} = 0$ if taxpayers set $\bar{\alpha} = 0$, but if it left $\bar{\beta} = 1$, then it would suffer identical audit costs and only forego the lost fine revenue. But this is only a possibility since $\bar{\alpha} = 0$ is not the unique taxpayer equilibrium in this case and, in fact, it is dominated in expected cost terms, as viewed by the taxpayers, by $\bar{\alpha} = 1$. In the third case, $\lambda > k$, the IRS is in a better position: a marginal increase in the probability of audit above the equilibrium value of k will guarantee full compliance at a trivial increase in audit costs. It would, however, still entail a substantial decrease in fine revenue.

5. CONCLUSION

The primary results of this paper are two-fold. First, a budget constraint on IRS auditing introduces a nontrivial interdependence between taxpayers' reporting strategies. Second, in the simple models used here, an equilibrium in which the budget constraint is binding almost always has high-income taxpayers under-reporting with probability one and the IRS always auditing as many taxpayers who report low income as possible. Thus, in the limiting case, for example, each taxpayer faces an effective audit probability of λ . When the budget constraint is not binding the equilibrium audit probability is analogous to that found in Graetz, Reinganum and Wilde (1983) (it differs slightly due to the assumption of risk neutrality and the absence

of habitually "honest" taxpayers).

The most obvious and desirable extension of this model is to consider a continuous distribution of income defined on an interval $[I_1, I_2]$ where $0 \leq I_1 < I_2$. This introduces a number of technical difficulties. In particular, the choice of audit probability must be conditioned on reported income and can take virtually any form. Nevertheless, the restriction to two income levels is clearly strong and needs to be relaxed. The more general problem outlined above is addressed in Reinganum and Wilde (1984) under the assumption that the IRS faces no fixed budget, but rather maximizes its tax revenue net of auditing costs.

APPENDIX

Proof of Proposition 1: Taking the derivative of equation (1) with respect to α_1 , substituting $\alpha^* = \alpha_j = \alpha_i$ for symmetry, and setting the result equal to zero gives

$$(1 - \alpha^*)q[\beta_1(T_H + F - T_L) + T_L] + (1 - q + \alpha^*q)\{(k_1/2)(T_H + F - T_L) + T_L\} - T_H = 0, \quad (A1)$$

where $k_1 = \beta_2 + (1 - \beta_2)\beta_1$. Solving for α^* and substituting $k = (T_H - T_L)/(T_H - T_L + F)$, and $k_2 = 2\beta_1q + [\beta_2 + (1 - \beta_2)\beta_1](1 - q)$ gives $\alpha^*(\beta_1, \beta_2)$ as defined in (4):

$$\alpha^*(\beta_1, \beta_2) = \alpha^0(\beta_1, \beta_2) \equiv (2k - k_2)/q[\beta_2 - (1 + \beta_2)\beta_1]. \quad (A2)$$

If the denominator is positive, then a mixed strategy TE exists and is unique so long as $0 < 2k - k_2 < q[\beta_2 - (1 + \beta_2)\beta_1]$. The right hand side of this inequality reduces to $k < k_1/2$. If these inequalities fail, pure strategy equilibria exist and are unique (that cases A(i), A(ii), and A(iii) are mutually exclusive is easy to show -- see below).

If $\beta_2 - (1 + \beta_2)\beta_1 \leq 0$, then the mixed strategy equilibrium α^0 still exists, but now when $k_1/2 \leq k \leq k_2/2$. Since in this case $k_1 < k_2$, the pure strategy equilibria exist simultaneously with the mixed strategy equilibrium on the range $[k_1/2, k_2/2]$. To see this, note that if one taxpayer always complies, then the cost of noncompliance to the other taxpayer is

$$C(1, 0; \beta_1, \beta_2) = q[\beta_1(T_H + F) + (1 - \beta_1)T_L] + (1 - q)\{(k_1/2)(T_H + F - T_L) - T_L\}.$$

This is greater than or equal to T_H (the cost of complying) if and only if $k \leq k_2/2$. Similarly, if one taxpayer never complies, then the other taxpayer will never comply if and only if $k \geq k_1/2$.

Q.E.D.

Proof of Lemma 1: This lemma follows trivially from differentiating equations (6) and (7).

Q.E.D.

Proof of Proposition 2: If $a > 1$, then $a \geq a$ is impossible so the only equilibrium has $\bar{\beta}_1 = 0 = \bar{\beta}_2$ in which case $\bar{a} = 1$. From (8), $a > 1$ is equivalent to $c > q(T_H + F - T_L)$ -- case (A).

If $a < 1$, several possibilities exist. First, suppose $a > a$. Then $\hat{\beta}_1(a) = 1$ and $\hat{\beta}_2(a)$ is irrelevant. Furthermore, $\beta_2 - (1 + \beta_2)\beta_1 = -1 < 0$ so case (B) of Proposition 1 applies. With $\beta_1 = 1$, $k_1 = 1$ and $k_2 = (1 + q)$. Since we are assuming $a > a$, the pure strategy equilibrium in which $a^* = 0$ is impossible. Thus we need only check the mixed strategy taxpayer equilibrium a^0 and the pure-strategy $a^*(1, \beta_2) = 1$. The latter is an equilibrium so long as $k \geq k_1/2$, or in this case, $k \geq 1/2$. The mixed strategy taxpayer equilibrium is part of a full equilibrium so long as $k \leq k_2/2$, or in this case $k \leq (1 + q)/2$ (given $a^0(1, \beta_2) > a$).

Next, suppose $a^0(\beta_1, \beta_2) = a$. Using the definition of a^0 , this is equivalent to

$$\beta_1 = \frac{\beta_2[aq + (1 - q)] - 2k}{(a - 1)q(1 + \beta_2) - (1 - \beta_2)} \equiv f(\beta_2). \quad (A3)$$

Clearly $f(0) > 0$, but $f(1) < 1$ if and only if $aq < 1 + q - 2k$. Furthermore, $\text{sgn } f'(\beta_2) = \text{sgn}[aq - q - 1 + 2k]$. Hence a solution to (A3) such that

$\beta_1 \in (0,1)$ and $\beta_2 \in (0,1)$ exists if and only if condition (9) holds. In this case $f(0) < 1$ and $f'(\beta_2) < 0$ so in fact a whole range of such solutions exist. Condition (9) is implied by $k \leq 1/2$ but not by $k \leq (1 + q)/2$. Hence the difference between subcases (ii) and (iii). Finally, $a < a$ can never be part of a full equilibrium since in this case $\hat{\beta}_1(a) \equiv 0 = \hat{\beta}_2(a)$ which implies $k_1 = 0 = k_2$, so that case B(iii) of Proposition 1 applies and $a^*(0,0) = 1$ is the taxpayers' best response. This contradicts $a < a < 1$.

Q.E.D.

Proof of Corollary 1: Recall the definition of $f(\beta_2)$ from the proof of Proposition 2. Since $f(0) > 0$ and $f(0) < 1$ under condition (9), and since $f(\cdot)$ is differentiable with $f'(\cdot) < 0$, it follows that $f(1) < f(0) < 1$. By the Intermediate Value Theorem, there exists $b \in (0,1)$ such that $f(b) = b$.

Q.E.D.

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TABLE 1

COMPARATIVE STATICS IN THE LIMITING CASE

	a^0	a	k
T_L	++	+	-
T_H	++	-	+
F	++	-	-
λ	+	0	0
q	++	-	0
c	0	+	0

* Given $k(1 - q) \leq \lambda \leq k$, the relevant case for a^0 to be part of a NE.

FIGURE 1(A)

TAXPAYER EQUILIBRIUM WHEN $\beta_2 - (1 + \beta_2)\beta_1 > 0$

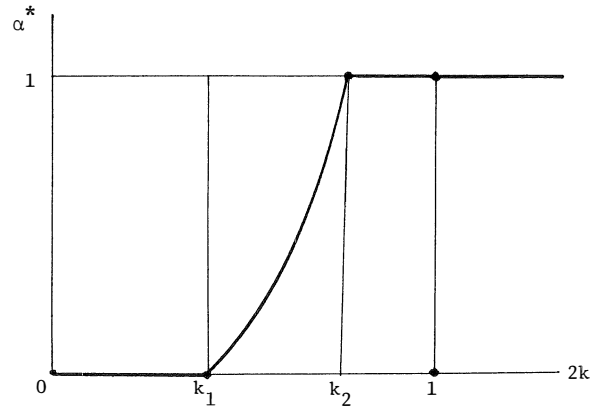


FIGURE 1(B)

TAXPAYER EQUILIBRIUM WHEN $\beta_2 - (1 + \beta_2)\beta_1 \leq 0$

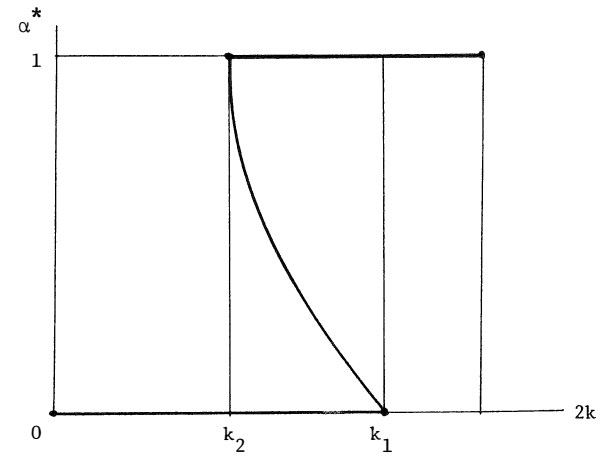


FIGURE 2(A)

LIMITING EQUILIBRIUM WHEN $\lambda < k(1 - q)$

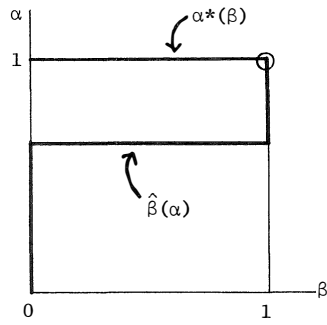


FIGURE 2(B)

LIMITING EQUILIBRIUM WHEN $k(1 - q) \leq \lambda \leq k$

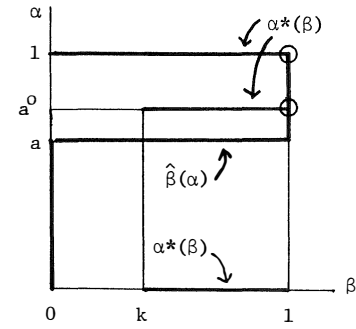


FIGURE 2(C)

LIMITING EQUILIBRIUM WHEN $K < \lambda$

