DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

SPECULATION AND PRICE STABILITY UNDER UNCERTAINTY: A GENERALIZATION

Da-Hsiang Donald Lien

SOCIAL SCIENCE WORKING PAPER 536

July 1984

Speculation and Price Stability Under Uncertainty: A Generalization

Da-Hsiang Donald Lien

Abstract

Since Friedman maintained that profitable speculation necessarily stabilizes prices, the necessary and sufficient conditions for his conjecture to hold have been derived following ex post analyses. However, within these frameworks, no uncertainty is involved.

In this paper we assume the nonspeculative excess demand functions are always linear but with random slopes and intercepts (i.i.d. across time). Employing dynamic programming approaches, the optimal complete speculation sequence for a monopolistic speculator (which maximizes his long-run expected profits) can be characterized. Furthermore, Friedman's conjecture holds under this sequence.

As for competitive speculation cases, we consider three variants arising from deviations of the monopolistic case. Of these, two models establish the property that Friedman's conjecture holds for optimal speculation sequences. However, since this conjecture might be falsified for the other model, a necessary condition is derived. Also, an example is given which shows that, if uncertainties are involved, a destabilizing optimal speculation sequence exists even with linear nonspeculative excess demand functions.

Speculation and Price Stability Under Uncertainty: A Generalization*

Da-Hsiang Donald Lien

I. Introduction

In arguing the case for flexible versus fixed exchange values, it was maintained by Friedman [2, p. 175] that profitable speculation necessarily stabilizes prices. Thereafter, several studies tried to verify Friedman's conjecture. The most general results were derived by Farrell [1] and Schimmler [5]. Specifically, Farrell showed that, (i) for a two-period model, any continuous, negatively sloped nonspeculative excess demand function would validate Friedman's conjecture if there is no lag structure; and (ii) for a T-period model with $T \geq 3$, negatively sloped linear nonspeculative excess demand is necessary and sufficient for Friedman's conjecture to be true if there is no lag structure. Schimmler generalized Farrell's results to the case of lag-responsive excess demand, deriving similar results.

In Lien [4], a basic error underlying the proofs by Farrell and Schimmler was identified and corrected. With this correction, the Farrell results are valid.

All these results hold under a deterministic framework using an ex post viewpoint. In a recent paper, Jesse and Radcliffe [3] examined Friedman's conjecture in an environment with uncertainties in future nonspeculative excess demand functions. Their conclusions show that, assuming the future (nonspeculative) excess demand functions are linear with constant slope and random intercepts (i.i.d. across time), expected profit maximizing speculation will stabilize market prices, thus the Jesse-Radcliffe results extend Telser's original ex post finding [6] to an ex ante setting.[1]

Motivated by their results, in this paper, we assume both intercepts and slopes are random variables (with independent, identical joint distribution across time), which is a generalization of their model. The generalized model is presented in Section II. Under this framework, an optimal speculation sequence can be characterized in terms of a dynamic programming approach. The results are derived in Section III. Section IV considers the effects of profitable speculation on price stability. Section V extends the analysis to the case of competitive speculation. Finally, some concluding remarks are given in Section VI.

II. The Model

Consider a discrete time abstract market model where the associated commodity is storable. Let t = 1, 2, ..., T, ... denote each period. Within each period all transactions are assumed to take place at the same price.

There are two types of agents in the market: speculators and nonspeculators. It is assumed that speculators face a linear nonspeculative excess demand function in each period with the intercepts and slopes varying from period to period. Specifically, the nonspeculative excess demand in period t, N_t (defined as the difference between nonspeculative demand and supply at every given price) takes the following form:

$$D_t: \quad N_t = a_t - b_t P_t \tag{1}$$

where (a_t, b_t) is a nonnegative random vector, identically and independently distributed over time.

In each period t, the speculators observe the nonspeculative excess demand function D_t (but face uncertainty about future demands), and then choose a speculative transaction S_t to maximize their long-run expected profits, where $S_t > 0$ denotes speculative sales and $S_t < 0$ denotes speculative purchases.[2]

To make clear the effects of speculators on market prices, we consider only complete speculative sequence.[3] I.e. a speculation sequence $\{S_1, S_2, \ldots\}$ is complete if and only if it satisfies

$$\sum_{t=1}^{\infty} S_t \equiv 0 \tag{2}$$

Consider the case of a monopolistic speculator. At time t = 1, the problem for the speculator can be written as:

$$\text{(P1)} \qquad \max_{\{S_t\}} E\left[\sum_{t=1}^{\infty} \pi_t\right]$$

$$\text{subject to:} \quad \sum_{t=1}^{\infty} S_t \equiv 0 \tag{3}$$

$$\text{with} \qquad \pi_t = P_t S_t = (a_t - S_t)\frac{S_t}{b_t}$$

where $P_t = \dfrac{(a_t - S_t)}{b_t}$ is the market price in period t when the associated speculative transaction is S_t. $q_t = \dfrac{a_t}{b_t}$ is the market price when there are no speculators in the market in period t.

III. Optimal Speculation Sequence

Before proceeding further, note that for $E\left[\sum_{t=1}^{\infty}\pi_t\right]$ to be positive, $\{S_t\}$ must be a random vector. Otherwise,

$$E\left[\sum_{t=1}^{\infty}\pi_t\right] = E\left[\sum_{t=1}^{\infty}(a_t - S_t)\frac{S_t}{b_t}\right]$$

$$= \sum_{t=1}^{\infty}E\left[(a_t - S_t)\frac{S_t}{b_t}\right] = E(\frac{a_t}{b_t})\sum_{t=1}^{\infty}S_t - E(\frac{1}{b_t})\sum_{t=1}^{\infty}S_t^2 \qquad (4)$$

$$= -E(\frac{1}{b_t})\sum_{t=1}^{\infty}S_t^2 \le 0$$

where we used equation (2) to derive the last equality. Therefore, we know if there is any speculative sequence which generates positive expected long-run profits, it must be a random vector (i.e. S_t must depend on observed a_i, b_i for $i \le t$). All the other speculation sequences are irrelevant in examining Friedman's conjecture.

Now we apply the dynamic programming approach to solve (P1). First, define $E_tV_r(k)$ as the maximum of $E_t\left[\sum_{i=r}^{\infty}\pi_i\right]$ subject to $\sum_{i=r}^{\infty}S_i = k$, where the expectation is taken at time t. Then, using the principle of optimality, we have

$$E_tV_t(k) = \max_{S_t}\{(a_t - S_t)\frac{S_t}{b_t} + E_tV_{t+1}(k - S_t)\} \qquad (5)$$

To satisfy equation (5),

$$\frac{\partial E_tV_{t+1}(k - S_t)}{\partial S_t} + \frac{a_t}{b_t} - 2\frac{S_t}{b_t} = 0 \qquad (6)$$

Theorem 1

$$\frac{\partial E_tV_{t+1}(k - S_t)}{\partial S_t} = -E(\frac{a_t}{b_t}),\ \forall\ S_t \qquad (7)$$

[Proof]

Let δ be an arbitrary positive number. For every S_t, assume $\{x_{t+1}, x_{t+2}, \cdots\}$ is the optimal speculation sequence which achieves $E_tV_{t+1}(k - S_t)$. Now, insert $(-\delta)$ into the sequence, and define

$y_{t+1} = -\delta$, and $y_i = x_{i-1}$, $\forall\ i \ge t+2$

such that $\sum_{i=t+1}^{\infty}y_i = \sum_{i=t+1}^{\infty}x_i - \delta = k - S_t - \delta$.

Therefore, replacing S_t by $S_t + \delta$, we have

$$E_tV_{t+1}(k - S_t - \delta) - E_tV_{t+1}(k - S_t)$$

$$\ge E\left[-(a_t + \delta)\frac{\delta}{b_t}\right] = -E(\frac{a_t}{b_t})\delta - E(\frac{1}{b_t})\delta^2$$

which implies

$$\frac{E_tV_{t+1}(k - S_t - \delta) - E_tV_{t+1}(k - S_t)}{\delta} \ge -E(\frac{a_t}{b_t}) - E(\frac{1}{b_t})\delta$$

$$\Rightarrow \frac{\partial E_tV_{t+1}(k - S_t)}{\partial S_t} \ge -E(\frac{a_t}{b_t}) \qquad (8)$$

by letting $\delta \to 0$.

Similarly,

$$E_tV_{t+1}(k - S_t) - E_tV_{t+1}(k - S_t - \delta)$$

$$\ge E\left[(a_t - \delta)\frac{\delta}{b_t}\right] = E(\frac{a_t}{b_t})\delta - E(\frac{1}{b_t})\delta^2$$

which implies

$$\frac{E_tV_{t+1}(k - S_t - \delta) - E_tV_{t+1}(k - S_t)}{\delta} \lesssim -E(\frac{a_t}{b_t}) + E(\frac{1}{b_t})\delta$$

$$\Rightarrow \frac{\partial E_tV_{t+1}(k - S_t)}{\partial S_t} \lesssim -E(\frac{a_t}{b_t}) \tag{9}$$

again, by letting $\delta \to 0$.

Combining equations (8) and (9), the proof is completed.

QED

<u>Theorem 2</u>

The optimal speculation sequence $\{S_t^*\}$ can be characterized by:

$$S_t^* = \frac{a_t}{2} - \frac{b_t}{2}E(\frac{a_t}{b_t}), \forall\ t. \tag{10}$$

if it generates positive long-run expected profits.

[Proof]

Inserting equation (7) into equation (6),

$$\frac{a_t}{b_t} - E(\frac{a_t}{b_t}) - \frac{2S_t}{b_t} = 0$$

$$\Rightarrow S_t = \frac{a_t}{2} - \frac{b_t}{2}E(\frac{a_t}{b_t}).$$

Note that equations (6) and (7) hold for every t, then the proof is completed.

QED

<u>Corollary 1</u>

In an optimal speculation sequence, the speculators will sell (i.e. $S_t^* > 0$) if $\frac{a_t}{b_t} > E(\frac{a_t}{b_t})$ and buy (i.e. $S_t^* < 0$) if $\frac{a_t}{b_t} < E(\frac{a_t}{b_t})$.

<u>Corollary 2</u>

(i) When b_t is a constant,

$$S_t^* = \frac{1}{2}(a_t - Ea_t), \forall\ t, \tag{11}$$

hence $S_t^* > 0$ if and only if $a_t > Ea_t$.

(ii) When a_t is a constant,

$$S_t^* = \frac{a}{2}(1 - b_tE(\frac{1}{b_t})), \forall\ t, \tag{12}$$

hence $S_t^* > 0$ if and only if $\frac{1}{b_t} > E(\frac{1}{b_t})$.

Corollary 1 shows that the speculator's activity depends on the difference between $\frac{a_t}{b_t}$ and its expected value $E(\frac{a_t}{b_t})$ in every period t. Noting that $\frac{a_t}{b_t}$ is the market equilibrium price in period t where there is no speculation, this result can be used to provide some intuition as to the interpretation of the speculator's behavior. In period t, the speculator knows the nonspeculative excess demand D_t. Using this information, he can calculate the price which will prevail in the market in the absence of speculation (i.e. $\frac{a_t}{b_t}$). If this price is higher than average price, then he should sell in the market; if it is lower, then he should buy.

Corollary 2 considers two special cases: First, when b_t is a constant we have Jesse-Radcliffe's result which states that speculators should sell if nonspeculative excess demand is above average and buy if it is below average. Secondly, when a_t is a constant, then speculators should sell if nonspeculative excess demand is less responsive than average and buy if it is more responsive than average,[4] where responsiveness is measured by b_t.

IV. Profitability and Stability

Given equation (10), the market price can be solved by letting $S_t^* = N_t$, i.e.,

$$S_t^* = \frac{a_t}{2} - \frac{b_t}{2}E(\frac{a_t}{b_t}) = a_t - b_tP_t$$

$$\Rightarrow \frac{a_t}{2} + \frac{b_t}{2}E(\frac{a_t}{b_t}) = b_tP_t \tag{13}$$

$$\Rightarrow P_t = \frac{1}{2}(\frac{a_t}{b_t}) + \frac{1}{2}E(\frac{a_t}{b_t})$$

Therefore, if q_t (the market price without speculation) is greater than the average price $E(q_t)$, then $P_t < q_t$. On the other hand, if $q_t < E(q_t)$, then $P_t > q_t$,[5] where $q_t = \frac{a_t}{b_t}$. Obviously, the introduction of speculators into the market will enhance price stability (measured by the variance of price). Mathematically,

$$\text{Var } P_t = \frac{1}{4}\text{ Var}(\frac{a_t}{b_t}) < \text{var}(\frac{a_t}{b_t}) = \text{Var } q_t \tag{14}$$

This result shows that, if the optimal speculation sequence is $\{S_t^*\}$,

then Friedman's conjecture is justified, i.e. the conclusion holds if $\{\ {}^*_t\}$ generates positive profits since $S_t \equiv 0$, $\forall$ t is a feasible policy and the best over nonrandom policy space.

As a special case, assume $b_t \equiv b$, $\forall$ t where b is a constant, then $E(S_t^*) = 0$ and $E(\pi_t) = \frac{\text{Var}(a_t)}{4b} > 0$. Therefore, we know $\{S_t^*\}$ as derived from the first order conditions is actually the optimal speculation sequence since $E(\pi_t) > 0$. On the other hand, if $a_t \equiv a$, $\forall$ t, then

$$E(S_t^*) = \frac{a}{2}[1 - E(b_t)E(\frac{1}{b_t})] < 0 \text{ and}$$

$$E(\pi_t) = \frac{a^2}{4}E(\frac{1}{b_t})[1 - E(b_t)E(\frac{1}{b_t})] < 0.$$[6]

Therefore, in this case, $\{S_t^*\}$ derived above is not the optimal speculation sequence. However, in the general case,

$$E(S_t^*) = \frac{1}{2}[E(a_t) - E(b_t)E(\frac{a_t}{b_t})], \tag{15}$$

$$\text{and} \quad E(\pi_t) = E(p_tS_t^*) = \frac{1}{4}[E(\frac{a_t^2}{b_t}) - E(b_t)(E(\frac{a_t}{b_t}))^2] \tag{16}$$

Hence, for $E(\pi_t) > 0$, we must require $E(\frac{a_t^2}{b_t}) > E(b_t)[E(\frac{a_t}{b_t})]^2$.

Since the sign of $E(\pi_t)$ is ambiguous, to carry further, we assume $a_t = b_tC_t$ where b_t and C_t are independent.[7] In this case,

$$E(S_t^*) = \frac{1}{2}[E(a_t) - E(b_t)E(C_t)] = 0, \text{ and}$$

$$\begin{aligned} E(\pi_t) &= \frac{1}{4}[E(b_t C_t^2) - E(b_t)(E(C_t))^2] \\ &= \frac{1}{4}[E(b_t)E(C_t^2) - E(b_t)(E(C_t))^2] \\ &= \frac{1}{4}E(b_t)[E(C_t^2) - (E(C_t))^2] > 0. \end{aligned}$$

Therefore, $\{S_t^*\}$ as derived from the first order conditions is actually the optimal speculation sequence.

V. Competitive Speculation

In the above section, a monopolistic speculator is assumed to take into account the effects of his actions on market prices when solving (P1). An alternative is the case of competitive speculation. In this section, we consider three variants associated with this idea (the case of the monopolistic speculator is named Model 1).

Model 2: $S_t^* = \frac{b_t}{2}[C_t - EC_t] + e_t$, where e_t is independent of (b_t, C_t) and $\{e_t\}$ is a sequence of i.i.d. random variables with $E(e_t) = 0$, $Var(e_t) > 0 \ \forall\ t$.

Model 3: $S_t^* = \frac{b_t}{2}[C_t - EC_t] + Z_t b_t$, where Z_t is independent of (b_t, C_t) and $\{Z_t\}$ is a sequence of i.i.d. random variables with $E(Z_t) = 0$, $Var(Z_t) > 0 \ \forall\ t$.

Model 4: $S_t^* = \frac{W_t b_t}{2}[C_t - EC_t]$, where W_t is independent of (b_t, C_t) and $\{W_t\}$ is a sequence of i.i.d. random variables with $E(W_t) = 1$, $Var(W_t) > 0 \ \forall\ t$.

where we have already imposed the assumption: $a_t = b_t C_t, \forall\ t$. These models are the natural generalizations of the competitive speculation models considered by Jesse and Radcliffe [3].

Under Model 2,

$P_t = \frac{1}{2}[C_t - EC_t] - \frac{e_t}{b_t} \Rightarrow VarP_t = \frac{1}{4} Var\ C_t + E(e_t^2)E(\frac{1}{b_t^2})$. On the other hand,

$$E\pi_t = E(P_t S_t^*) = \frac{E(b_t)}{4} Var\ C_t - E(e_t^2)E(\frac{1}{b_t}) \qquad (17)$$

Therefore, $E\pi_t > 0$ implies $E(b_t)Var\ C_t > 4E(e_t^2)E(\frac{1}{b_t})$. Nonetheless, for Friedman's conjecture to hold,[8] we need $E(e_t^2)E(1/b_t^2) \le \frac{3}{4}Var\ C_t$.

Theorem 3

Under Model 2, a necessary condition for Friedman's conjecture to be falsified[9] is,

$$E(b_t)E(1/b_t^2) > 3E(\frac{1}{b_t}). \qquad (18)$$

[Proof]

If Friedman's conjecture does not hold, then we have

$$E(e_t^2)E(\frac{1}{b_t^2}) > \frac{3}{4}Var\ C_t \text{ and } E(b_t)Var\ C_t > 4E(e_t^2)E(\frac{1}{b_t})$$

$$\Rightarrow E(e_t^2)E(b_t)E(\frac{1}{b_t^2}) > \frac{3}{4}Var\ C_t E(b_t), \text{ since } b_t > 0$$

$$\Rightarrow E(e_t^2)E(b_t)E(\frac{1}{b_t^2}) > 3E(e_t^2)E(\frac{1}{b_t})$$

$$\Rightarrow E(b_t)E(\frac{1}{b_t^2}) > 3E(\frac{1}{b_t}), \text{ since } E(e_t^2) > 0.$$

QED

Hence, when equation (18) holds, there are some cases where Friedman's conjecture does not hold. As an example where equation (18) holds, let $b_t = 1$ with probability $\frac{1}{2}$; 9 with probability $\frac{1}{2}$. Then,

$E(b_t) = 5$; $E(\frac{1}{b_t^2}) = \frac{41}{81}$; $E(\frac{1}{b_t}) = \frac{5}{9}$ which implies

$E(b_t)E(\frac{1}{b_t^2}) = \frac{205}{81} > 3E(\frac{1}{b_t}) = \frac{5}{3}$. Furthermore, let $E(e_t^2) = 2$ and $Var(C_t) = 1$, then

$$E(\pi_t) = \frac{5}{4} - \frac{5}{9} \cdot 2 = \frac{5}{36} > 0 \text{ and}$$

$$E(e_t^2)E(\frac{1}{b_t^2}) = \frac{82}{81} > \frac{3}{4}Var(C_t) = \frac{3}{4}.$$

Therefore, we have an optimal speculation sequence such that Friedman's conjecture is falsified. However, if we assume $b_t = 1$ with probability $\frac{1}{2}$; 2 with probability $\frac{1}{2}$, then equation (18) does not hold.

Theorem 4

Under Model 3, Friedman's conjecture always holds.

[Proof]

In this case, $S_t^* = \frac{b_t}{2}[C_t - EC_t] + Z_t b_t$

$\Rightarrow P_t = \frac{1}{2}C_t + \frac{1}{2}EC_t - Z_t$.

Hence, $E\pi_t = E(P_t S_t^*) = \frac{E(b_t)}{4}Var\ C_t - E(b_t)Var\ Z_t$ and $Var\ P_t = \frac{1}{4}Var\ C_t + Var\ Z_t$. Now, if $E\pi_t > 0$, then $Var\ C_t > 4Var\ Z_t \Rightarrow Var\ P_t < \frac{1}{2}Var\ C_t < Var\ C_t$, then Friedman's conjecture holds.

QED

Theorem 5

Under Model 4, Friedman's conjecture always holds.

[Proof]

In this case, $S_t^* = \frac{W_t b_t}{2}(C_t - EC_t)$. By market equilibrium conditions,

$$b_t C_t - b_t P_t = \frac{W_t b_t}{2}(C_t - EC_t)$$

$$\Rightarrow P_t = C_t - \frac{W_t}{2}(C_t - EC_t).$$

Hence, $Var\ P_t = Var(C_t) + \frac{1}{4}Var(W_t)\ Var(C_t) - E(W_t)Var(C_t) = \frac{1}{4}Var(W_t)Var(C_t)$, since $E(W_t) = 1$. On the other hand,

$$E\pi_t = E(P_t S_t^*) = E\left[\frac{W_t b_t}{2}C_t(C_t - EC_t) - \frac{W_t^2 b_t}{4}(C_t - EC_t)^2\right]$$

$$= \frac{1}{4}E(b_t)Var(C_t) - \frac{1}{4}E(b_t)Var(W_t)Var(C_t).$$

Therefore, if $E\pi_t > 0$, then $Var(C_t) > Var(W_t)Var(C_t)$ $\Rightarrow Var\ P_t < \frac{1}{4}Var(C_t) < Var(C_t)$, and hence Friedman's conjecture is satisfied.

QED

VI. Conclusions

In this paper, we have examined Friedman's conjecture in an uncertainty framework. Specifically, we assumed speculators only know that the future nonspeculative excess demand functions are linear, but with random intercepts and random slopes. Assuming some conventional properties (i.e. the coefficients are i.i.d. across time and $a_t = b_t C_t$ where b_t and C_t are stochastically independent), we have characterized the optimal speculation sequence in the case of a monopolistic speculator as equation (10). In this case, Friedman's conjecture holds, assuming that the speculation sequence is optimal (i.e. expected profit maximizing).

However, in the case of competitive speculation (i.e. Model 2), there are some situations where Friedman's conjecture does not hold (an example is also given in Section V) even with a linear nonspeculative excess demand function. The key factor is the probability distribution function of the slope b_t. For the other competitive speculation models (Models 3 and 4), Friedman's conjecture always holds even in this uncertainty framework.

NOTES

* I am indebted to James Quirk for helpful comments and editings, also to Richard McKelvey for helpful comments. In preparing Section IV, I benefited from useful discussions with David Grether and Quang Vuong. All errors, of course, remain mine.

1. Of minor interests there is a typing error in their paper [3, p. 130]. I.e. instead of x_n, the sum of speculative transactions prior to period n should be expressed by $-x_n$. Otherwise, we will have

$$x_n = \sum_{t=n+1}^{N} S_t = \sum_{t=n}^{N} S_t - S_n = x_{n-1} - S_n$$

$$\Rightarrow \; x_{n-1} = x_n + S_n,$$

rather than $x_{n-1} = x_n - S_n$. Also, one major difference between their analysis versus Farrell's is that Jesse and Radcliffe adopt ex ante analyses while Farrell's approach is ex post.

2. Actually, in period t, speculators observe $\{D_i\}$ with $i \leq t$. However, since $\{a_i, b_i\}$ are i.i.d. and there is no lagged term in D_t, then $\{D_i\}$ with $i < t$ provides no information.

3. This idea was originated by Telser [6].

4. Since $N_t = a_t - b_t P_t$, hence $\frac{\partial N_t}{\partial P_t} = -b_t$. Therefore, if and only if $b_1 > b_2$, then $\frac{1}{b_1} < \frac{1}{b_2}$ and the nonspeculative excess demand function is more responsive under b_1 than under b_2 when $\frac{1}{b_1} < \frac{1}{b_2}$.

5. Note that $E(P_t) = E(\frac{a_t}{b_t}) = E(q_t)$, hence introduction of speculators into the market does not change the expected market prices.

6. To show this, note that $cov(b_t,\frac{1}{b_t}) = 1 - E(b_t)E(\frac{1}{b_t})$. Hence we only have to show $cov(b_t,\frac{1}{b_t}) < 0$. Intuitively, since $f(x) = \frac{1}{x}$ has negative slope everywhere when $x > 0$, and $cov(Y,\frac{1}{Y})$ measures the linear dependence between Y and $\frac{1}{Y}$ (where Y is a random variable), hence $cov(Y,\frac{1}{Y}) \leq 0$. Furthermore, $cov(Y,\frac{1}{Y}) = 0$ only when Y is degenerate. Mathematically, since $f(x) = \frac{1}{x}$ is a convex function over $(0,\infty)$, hence, by Jensen's inequality, $E[f(y)] \geq f[E(Y)]$. I.e. $E(\frac{1}{Y}) \geq \frac{1}{E(Y)} \Rightarrow 1 \leq E(\frac{1}{Y})E(Y) \Rightarrow cov(Y,\frac{1}{Y}) \leq 0$. Moreover the equality holds only when $Y = E(Y)$ almost surely. Nonetheless when Y can take on negative values, this result does not hold due to the concavity of $f(x)$ over $(-\infty,0)$. As an example, let $Y = -2$ with probability $\frac{8}{70}$; $-\frac{1}{2}$ with probability $\frac{8}{70}$; $\frac{1}{3}$ with probability $\frac{27}{70}$; 3 with probability $\frac{27}{70}$. Then, $EY = \frac{16}{70} - \frac{4}{70} + \frac{9}{70} + \frac{81}{70} = 1$. Also, $E(\frac{1}{Y}) = 1 \Rightarrow E(Y)E(\frac{1}{Y}) = 1 \Rightarrow cov(Y,\frac{1}{Y}) = 0$. Yet, Y is a nondegenerate random variable. We owe our thanks to David Grether for providing this example, and to Quang Vuong for helpful discussions.

7. Note that this assumption implies: when a_t is a constant, b_t must also be constant. Otherwise, $cov(b_t,\frac{a_t}{b_t}) = a_t cov(b_t,\frac{1}{b_t}) < 0$, and therefore b_t and $C_t = \frac{a_t}{b_t}$ are not stochastically independent.

Also, instead of this assumption, let a_t and b_t be stochastically independent. Then

$$E(\pi_t) = \frac{1}{4}[E(\frac{a_t^2}{b_t}) - E(b_t)(E(\frac{a_t}{b_t}))^2]$$

$$= \frac{1}{4}[E(a_t^2)E(\frac{1}{b_t}) - E(b_t)(E(a_t))^2(E(\frac{1}{b_t}))^2]$$

$$= \frac{1}{4}E(\frac{1}{b_t})[E(a_t^2) - E(b_t)E(\frac{1}{b_t})(E(a_t))^2]$$

Although $E(a_t^2) > (E(a_t))^2$, $E(b_t)E(\frac{1}{b_t}) > 1$, hence the sign of $E(\pi_t)$ is indetermined.

Actually Friedman's conjecture is concerned with all profitable speculations and here we only considered optimal speculations. Therefore if Friedman's conjecture is falsified in the optimal speculation case, then it is already invalid. Nonetheless, if it holds under optimal speculation, whether it will hold for all other non-optimal profitable speculations remains an open question.

Obviously, when $b_t \equiv b$ is a constant, $E(b_t)E(\frac{1}{b_t^2}) = \frac{1}{b} = E(\frac{1}{b_t})$. Hence equation (18) does not hold and Friedman's conjecture is justified. This result also showed up on [3].

REFERENCES

[1] Farrell, M. J. "Profitable Speculation." Economica 33 (May 1966):183-93.

[2] Friedman, Milton. Essays in Positive Economics. Chicago: University of Chicago Press, 1953.

[3] Jesse, R. R. and Radcliffe, R. C. "On Speculation and Price Stability Under Uncertainty." Review of Economics and Statistics 63 (February 1981):129-32.

[4] Lien, Da-Hsiang D. "Profitable Speculation and Linear Excess Demand." Social Science Working Paper 521. Pasadena, CA: California Institute of Technology, March 1984.

[5] Schimmler, Jörg. "Speculation, Profitability, and Price Stability--A Formal Approach." Review of Economics and Statistics (1973):110-14.

[6] Telser, Lester G. "A Theory of Speculation Relating Profitability and Stability." Review of Economics and Statistics 41 (August 1959):295-301.