DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125



# COURNOT OLIGOPOLY WITH INFORMATION SHARING

Lode Li



SOCIAL SCIENCE WORKING PAPER 561

January 1985

ABSTRACT

This paper studies the incentives for information sharing among firms in a Cournot oligopoly facing a linear uncertain demand and an affine conditional expectation information structure. No information sharing is found to be the unique equilibrium in two cases in which the signals with equal precision are assumed indivisible and infinitely divisible. However, the nonpooling equilibrium converges to the situation where the pooling strategies are adopted as the amount of information increases. Hence, the efficiency is achieved in the competitive equilibrium as the number of the firm become large.

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## 1. INTRODUCTION

This paper studies the incentives for information sharing among firms in an oligopolistic industry in which there is some uncertainty in the demand function. We characterize equilibrium behavior in a model where firms may observe private signals about the true state of the demand, each firm first chooses a level of information that it commits to share with others and then chooses a level of production based on the information both from private sources and the "common pool."

The model is a two-stage game. In the first stage, firms select levels of information to release which can be non-, partial, or full. Then private signals are generated and an "outside agency" conducts the transmission of the private information according to the firms' commitments. In the second stage, each firm observes its private signal, the levels of information-sharing selected by other firms and the publicized signals. The firms then determine their output level based on the information available. The equilibrium notion we use is that of a subgame-perfect Nash equilibrium. We proceed by solving the second stage first and the first stage is then solved by assuming that payoffs from the first stage are determined by the equilibrium behavior in the second-stage subgame. We derive pure strategy Nash equilibria that are symmetric and subgame perfect under a symmetric information structure where firms receive private signals with equal precision. No information sharing is found to be the unique dominant equilibrium. However the ex post behavior of the nonpooling equilibrium converges almost surely to that of the information pooling situation when the total amount of information in the industry becomes large. Consequently the competitive limit will be reached when the number of firms increases.

Several recent papers (Clarke [1982], Gal-Or [1984], Novshek and Sonnenschein [1982], etc.) have addressed the same issue we discuss here. Two generalizations are made in this paper. First, in contrast to Clarke and Gal-Or where the signals are assumed to be normally distributed, our assumption, that the expectation of the true state conditional on the signals is linear in the signals, is general to include many interesting distributions which are especially appropriate here because they may obey the nonnegativity constraints of the inverse demand. Secondly, the results in this paper are derived for Cournot oligopoly with n firms and then the assymptotic properties of the equilibrium can be studied. The result, that no information-sharing is the unique equilibrium even when the signals are correlated, is consistent with the result of a duopoly in Clarke and Gal-Or. Our limiting result, that firms are indifferent between no pooling and pooling when the total amount of information is large,



coincides with that of Novshek and Sonnenschein because their model is an approximation of ours when the signals are sufficiently accurate.

The next section lays out the general model. In section 3, a unique Bayesian Nash equilibrium is derived for the second-stage game. The characterization of the information-sharing game and the asymptotic properties of the equilibrium are presented in section 4.

## 2. THE MODEL

Consider an oligopoly with n firms producing a product at no cost. The inverse demand is given by

$$p = a + \theta - bQ, \tag{2.1}$$

where a, b > 0, and $\theta$ is the true state of the world which is generated according to a distribution $g(\theta)$ with zero mean. Before deciding its output quantity, each firm observes a signal for $\theta$. The signal observed by firm i is $y_i$. Then $y_i$ is generated according to $h(y_i|\theta)$. Both these distributions are assumed to have finite variance. We define

$$t_i = \frac{1}{E[\mathrm{Var}(y_i|\theta)]} \tag{2.2}$$

as the measure of the amount of data firm i is to receive, which is the expected conditional precision of $y_i$. And let $R = \frac{1}{\mathrm{Var}\theta}$ be the precision of the prior. The distributions g, h and $t_i$ are common knowledge.

Before learning their signals, firms are required to commit themselves to release a fixed amount of information to a common pool to be made "available" to all firms by an "outside agency." Assume signal $y_i$ can be divided linearly into two parts: the amount of information revealed, $\hat{y}_i$, and the amount concealed, $\bar{y}_i$. And $\hat{y}_i$ has the expected conditional precision $\tau_i$ ( $\le t_i$) where

$$\tau_i = \frac{1}{E[Var(\hat{y}_i | \theta)]} \tag{2.3}$$

One may view $y_i$ as the sample of the observations generated by the true state of the world and $\hat{y}_i$ is the sample of a subset. Also note that $t_i$ and $\tau_i$ are directly proportional to the sample sizes. Therefore $\tau_i$ is a measure of the amount of information revealed by firm i; namely, if $\tau_i = 0$, there is no information sharing; if $\tau_i = t_i$, there is complete information sharing; and if $0 < \tau_i < t_i$, there is partial information sharing. The value of $\tau_i$ is chosen prior to and independent of the actual realization of $y_i$.

The "agency" reports to each firm the messages $(\tau_1,\ldots,\tau_n)$ and $(\hat{y}_1,\ldots,\hat{y}_n)$ after they are selected. Therefore the information that firm i can use for an output decision consists of its private signal $y_i$ or $(\hat{y}_i,\bar{y}_i)$ and the reported information $(\hat{y}_j,\ j \ne i)$. Denote $(y_i,\hat{y}_j,\ j \ne i)$ by $X_i$.

The further assumptions on the information structure are as follows:



<u>Assumption 1</u>.

$$E[y_i|\theta] = E[\hat{y}_i|\theta] = E[\bar{y}_i|\theta] = \theta.$$

Hence, the firms' private signals and transmitted signals are all unbiased estimators of $\theta$.

<u>Assumption 2</u>.

$$E[\theta|X_i] = a_0 + \bar{a}_i \cdot X_i \text{ and } \bar{a}_i \text{ are n-tuples of constants.}$$

That is, each firm's expectation of the uncertainty is affined in the available signals.

<u>Assumption 3</u>.

$$\hat{y}_i,\bar{y}_i,\ i=1,2,\ldots n \text{ are independent, conditional on } \theta.$$

As pointed out by Li, McKelvey and Page [1985], the above assumptions are general enough to include a variety of interesting prior-posterior distribution pairs for different modeling purposes. For example, the Gamma-Poisson and the Beta-Binomial are reasonable here since we wish to impose the nonnegativity constraints on the intercept of the demand function.

<u>Lemma 1</u>. Suppose random variables $\theta$ and $Z = (z_1,z_2,\ldots,z_n)$ have the following properties: $E[z_i|\theta] = \theta$, for all i; $E[\theta|Z] = c_0 + \bar{c} \cdot Z$, $\bar{c} = (c_1,c_2,\ldots,c_n)$; and $z_i$ are independent conditional on $\theta$. Then

(i) $E[\theta|z_i] = E[z_j|z_i] = \frac{\rho_i}{\rho_i + R} z_i + \frac{R}{\rho_i + R} E[\theta]$, for all i, $j \ne i$, where $\rho_i = \frac{1}{E[Var(z_i|\theta)]}$ and $R = \frac{1}{Var(\theta)}$.

(ii) $z = \sum_{i=1}^{n} d_i z_i$ is unbiased and is sufficient in the estimation of prior mean,

where $d_i = \frac{\rho_i}{\sum_{j=1}^{n} \rho_j}$.

Proof: By conditional independence,

$$E[z_j|z_i] = E[E(z_j|\theta,z_i)|z_i] = E[E(z_j|\theta)|z_i] = E[\theta|z_i]. \tag{2.4}$$

But

$$E[\theta|z_i] = E[E(\theta|Z)|z_i]$$

$$= E\left[c_0 + \sum_{j=1}^{n} c_j z_j | z_i\right]$$

$$= c_0 + c_i z_i + \sum_{j\neq i} c_j \, E[\theta|z_i]. \tag{2.5}$$

Hence,

$$E[\theta|z_i] = \frac{c_0 + c_i z_i}{1 - \sum_{j\neq i} c_j} \tag{2.6}$$

is linear in $z_i$. Using a result from Ericson [1968], we have

$$E[\theta|z_i] = \frac{\rho_i}{\rho_i + R} z_i + \frac{R}{\rho_i + R} E[\theta]. \tag{2.7}$$



It follows, from equations (2.6) and (2.7), that

$$\frac{\rho_i}{\rho_i + R} = \frac{c_i}{1 - \sum_{j\neq i} c_j} \text{ and } \frac{RE[\theta]}{\rho_i + R} = \frac{c_0}{1 - \sum_{j\neq i} c_j}. \tag{2.8}$$

Then,

$$c_i = \frac{\rho_i}{R + \sum_{j=1}^{n}\rho_j}, \quad i > 0, \text{ and } c_0 = \frac{RE[\theta]}{R + \sum_{j=1}^{n}\rho_j}. \tag{2.9}$$

Clearly, $z = \sum_{i=1}^{n} d_i z_i = \frac{R + \sum_j \rho_j}{\sum_j \rho_j} \cdot \sum_{i=1}^{n} c_i z_i$ is unbiased and $E[\theta|Z] = c_0 + \frac{\sum_j \rho_j}{R + \sum_j \rho_j} z$.

Q.E.D.

In view of the above proof, the assumption that $z_i$ are conditionally independent may be replaced by $E[z_j|z_i]$ are linear in $z_i$ for $j \neq i$. By carefully defining the correlation between $z_i$ and $z_j$, the results in the paper will still be valid.

Applying Lemma 1, we can obtain the following results. First,

$$y_i = \frac{\tau_i}{t_i}\hat{y}_i + \frac{\beta_i}{t_i}\bar{y}_i, \text{ and } \beta_i = t_i - \tau_i = \frac{1}{E[Var(\bar{y}_i|\theta)]}. \tag{2.10}$$

Secondly, define

$$x_i \equiv \frac{t_i}{\alpha_i - R} y_i + \sum_{j\neq i} \frac{\tau_j}{\alpha_i - R} \hat{y}_j, \tag{2.11}$$

where

$$\alpha_i = t_i + \sum_{j \neq i} \tau_j + R. \tag{2.12}$$

It then follows that $x_i$ is unbiased and

$$E[\theta | X_i] = \frac{\alpha_i - R}{\alpha_i} x_i. \tag{2.13}$$

Finally,

$$E\left[\hat{y}_i \ \hat{y}_j\right] = E\left[\hat{y}_j \ \bar{y}_i\right] = E\left[\hat{y}_i \ \bar{y}_i\right] = \frac{1}{R}, \ i \neq j,$$

$$\mathrm{Var}\left[\hat{y}_i\right] = \frac{1}{\tau_i} + \frac{1}{R} \text{ and } \mathrm{Var}\left[\bar{y}_i\right] = \frac{1}{\beta_i} + \frac{1}{R}. \tag{2.14}$$

## 3. MARKET EQUILIBRIUM

In this section, we fix $(\tau_1, \tau_2, \ldots, \tau_n)$ and derive the Bayesian equilibrium strategy functions $q_i^* = q_i^*(X_i)$ for the second-stage subgame. The market equilibrium is found to be unique. The following lemma is crucial in the proof of the uniqueness of the equilibrium and we proceed with it first.

<u>Lemma</u> <u>2</u>. Suppose the vectors of random variables $X_i$ satisfy the following equations

$$g_i(X_i) = -\sum_{j=1}^{n} E[g_j(X_j) | X_i], \qquad \text{for all } i. \tag{3.1}$$

Then,



$$g_i(X_i) = 0 \qquad \text{a.s. for all } i. \tag{3.2}$$

<u>Proof</u>. Taking the expectations of both sides of (3.1) conditional on $g_i(X_i)$, we have

$$Z_i = -\sum_{j=1}^{n} E[Z_j | Z_i] \tag{3.3}$$

where $Z_i = g_i(X_i)$. Multiplying (3.3) by $Z_i$, taking the expectations and then summing both sides of (3.3) over all i, we get

$$\sum_{i=1}^{n} \sigma_{ii} = \sum_{i=1}^{n} E[Z_i Z_i] = -\sum_{i=1}^{n} \sum_{j=1}^{n} E\left[Z_i E[Z_j | Z_i]\right]$$

$$= -\sum_{i=1}^{n} \sum_{j=1}^{n} E[Z_i Z_j] = -\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} \tag{3.4}$$

Note that $\sigma_{ii} \geq 0$ for all i and $\sum_{i} \sum_{j} \sigma_{ij} \geq 0$ since $(\sigma_{ij})$ is semi-positively definite. So (3.4) implies $E\left[(g_i(X_i))^2\right] = 0$, for all i. That is, $g_i(X_i) = 0$ almost surely for all i.

Q.E.D.

<u>Proposition</u> <u>1</u>. For any fixed $(\tau_1, \ldots, \tau_n)$, there is a unique Bayesian equilibrium to the second-stage game. The equilibrium strategy for each firm is linear (affine) in its information from the private source as well as the "common pool."

<u>Proof</u>: The expected profit for firm i given its information $X_i$ is

$$E[\pi_i(q, \tau, \theta) | X_i] = q_i(a - bq_i - b\sum_{j \neq i} E[q_j | X_i] + E[\theta | X_i]). \tag{3.5}$$

The first order conditions yield

$$2q_i^* = \frac{a}{b} + \frac{1}{b}\left[\frac{t_i}{\alpha_i} y_i + \sum_{j\neq i} \frac{\tau_j}{\alpha_i} \hat{y}_j\right] - \sum_{j\neq i} E[q_j^* | X_i]. \qquad (3.6)$$

Define the candidate linear strategies as

$$q_i \equiv A_0^i + \sum_{j=1}^{n} A_j^i \hat{y}_j + A_{n+1}^i y_i \qquad (3.7)$$

and subtract $2q_i$ from both sides of equation (3.6). We have

$$2\left[q_i^* - (A_0^i + \sum_{j=1}^{n} A_j^i \hat{y}_j + A_{n+1}^i y_i)\right]$$

$$= \left[\frac{a}{b} - 2A_0^i\right] + \sum_{j\neq i}\left[\frac{\tau_j}{\alpha_i b} - 2A_j^i\right]\hat{y}_j - 2A_j^i \hat{y}_i + \left[\frac{t_i}{\alpha_i b} - 2A_{n+1}^i\right] y_i - \sum_{j\neq i} E[q_j^* | X_i]$$

$$= \sum_{j\neq i} A_0^j + \sum_{j\neq i}\left[\frac{\tau_j}{t_j} A_{n+1}^j + \sum_{k\neq i} A_j^k + \frac{\tau_j}{\alpha_i} \sum_{k\neq i} \frac{\beta_k}{t_k} A_{n+1}^k\right]\hat{y}_j$$

$$+ \sum_{j\neq i} A_i^j \hat{y}_i + \frac{t_i}{\alpha_i}\left[\sum_{j\neq i} \frac{\beta_j}{t_j} A_{n+1}^j\right] y_i - \sum_{j\neq i} E\left[q_j^* | X_i\right]$$

$$= - \sum_{j\neq i} E\left[q_j^* - (A_0^j + \sum_{k=1}^{n} A_k^j \hat{y}_k + A_{n+1}^j y_j) | X_i\right]. \qquad (3.8)$$

The third equation in (3.8) can be verified as follows by using the results (2.10)-(2.13). Note that

$$E\left[A_0^j + \sum_k A_k^j \hat{y}_k + A_{n+1}^j y_j \middle| X_i\right]$$



$$= A_0^j + \sum_k A_k^j \hat{y}_k + A_{n+1}^j E\left[\frac{\tau_j}{t_j} \hat{y}_j + \frac{\beta_j}{t_j} y_j \middle| X_i\right]$$

$$= A_0^j + \sum_k A_k^j \hat{y}_k + \frac{\tau_j}{t_j} A_{n+1}^j \hat{y}_j + \frac{\beta_j}{t_j} \frac{t_i}{\alpha_i} A_{n+1}^j y_i$$

$$+ \frac{\beta_j}{t_j} A_{n+1}^j \sum_{k\neq i} \frac{\tau_k}{\alpha_i} \hat{y}_k. \qquad (3.9)$$

It then follows,

$$\sum_{j\neq i} E\left[A_0^j + \sum_k A_k^j \hat{y}_k + A_{n+1}^j y_j \middle| X_i\right]$$

$$= \sum_{j\neq i} A_0^j + \sum_{j\neq i} \sum_{k\neq i} A_k^j \hat{y}_k + \sum_{j\neq i} A_i^j \hat{y}_i + \sum_{j\neq i} \frac{\tau_j}{t_j} A_{n+1}^j \hat{y}_j$$

$$+ \frac{t_i}{\alpha_i}\left[\sum_{j\neq i} \frac{\beta_j}{t_j} A_{n+1}^j\right] y_i + \sum_{j\neq i} \sum_{k\neq i} \left[\frac{\tau_k}{\alpha_i} \frac{\beta_j}{t_j} A_{n+1}^j \hat{y}_k\right]$$

$$= \sum_{j\neq i} A_0^j + \sum_{j\neq i}\left[\frac{\tau_j}{t_j} A_{n+1}^j + \sum_{k\neq i} A_j^k + \frac{\tau_j}{\alpha_i} \sum_{k\neq i} \frac{\beta_k}{t_k} A_{n+1}^k\right]\hat{y}_j$$

$$+ \sum_{j\neq i} A_i^j \hat{y}_i + \frac{t_i}{\alpha_i}\left[\sum_{j\neq i} \frac{\beta_j}{t_j} A_{n+1}^j\right] y_i. \qquad (3.10)$$

And the second equation in (3.8) holds if $A_j^i$, $i=1,\ldots,n$, $j=0,1,\ldots,n+1$ satisfy the following $n(n + 2)$ linear equations:

$$\frac{a}{b} - 2A_0^i = \sum_{j\neq i} A_0^j , \qquad i=1,\ldots,n, \qquad (3.11)$$

$$\frac{\tau_i}{\alpha_i b} - 2A_j^i = \frac{\tau_i}{t_j} A_{n+1}^j + \sum_{k\neq i} A_j^k + \frac{\tau_i}{\alpha_i} \sum_{k\neq i} \frac{\beta_k}{t_k} A_{n+1}^k, \quad i,j=1,\ldots,n, i\neq j. \quad (3.12)$$

$$-2A_i^i = \sum_{j\neq i} A_i^j, \quad i=1,\ldots,n, \text{ and} \quad (3.13)$$

$$\frac{t_i}{\alpha_i b} - 2A_{n+1}^i = \frac{t_i}{\alpha_i} \sum_{j\neq i} \frac{\beta_j}{t_j} A_{n+1}^j, \quad i=1,\ldots,n. \quad (3.14)$$

It is tedious but, fortunately, not very difficult to solve this system of equations. Obviously $A_0^i$ and $A_{n+1}^i$ can be solved independently in the systems of equations (3.11) and (3.14) respectively. Equation (3.14) can help to reduce (3.13) to be n equations for $A_i^i$ and then $A_j^i$ follows directly. The solution is given as follows (see Appendix A for details):

$$A_0^i = \frac{a}{(n+1)b}, \quad (3.15)$$

$$A_i^i = \frac{\tau_i((n+1)\delta_i - \sum_k 2\delta_k)}{b(n+1)(1 + \sum_k \beta_k \delta_k)}, \quad (3.16)$$

$$A_j^i = \frac{2\tau_j[(n+1)\delta_i - \sum_k \delta_k]}{b(n+1)(1 + \sum_k \beta_k \delta_k)}, \quad j \neq i, \quad (3.17)$$

$$A_{n+1}^i = \frac{t_i \delta_i}{b(1 + \sum_k \beta_k \delta_k)}, \quad (3.18)$$

where



$$\delta_i = \frac{1}{2\alpha_i - \beta_i}. \quad (3.19)$$

Now writing

$$g_j(X_j) = q_j^*(X_j) - (A_0^j + \sum_{k=1}^n A_k^j \hat{y} + A_{n+1}^j y_j), \quad (3.20)$$

it follows from (3.8) that each i's Bayesian strategy $q_i^*$ must satisfy

$$g_i(X_i) = -\sum_{j=i}^n E\left[g_j(X_j) | X_i\right] \quad \text{for any } x_i. \quad (3.21)$$

By Lemma 2, $g_i(X_i) = 0$ almost surely, and hence

$$q_i^* = q_i = A_0^i + \sum_{k=1}^n A_k^i \hat{y}_k + A_{n+1}^i y_i, \quad \text{a.s., } i=1,\ldots,n. \quad (3.22)$$

Q.E.D.

The expected profit of firm i in this subgame can be easily expressed as a function of its strategy choice, that is

$$E\left[\pi_i | X_i\right] = b\left[q_i^*(X_i)\right]^2. \quad (3.23)$$

## 4. INFORMATION SHARING

The payoff function that starts at the first stage can be derived by using equation (3.23). Denote the payoff for i by

$$\Pi_i(\tau_1,\ldots,\tau_n) = E\left[E(\pi_i | X_i)\right] = bE\left[q_i^2\right]. \quad (4.1)$$

But

$$E\left[q_i^2\right] = E\left[(A_0^i + \sum_{j=1}^n A_j^i \hat{y}_j + A_{n+1}^i y_i)^2\right]$$

$$= \frac{a^2}{(n+1)^2 b^2} + \frac{1}{(n+1)^2 b^2 R} D_i \tag{4.2}$$

where

$$D_i(\tau_1,\ldots,\tau_n) = \left[(\sum_{j=1}^{n} \tau_j) B_1^i + \beta_i B_2^i\right]^2 + R\left[(\sum_j \tau_j)(B_1^i)^2 + \beta_i (B_2^i)^2\right], \tag{4.3}$$

$$B_1^i = \frac{2((n+1)\delta_i - \sum_k \delta_k)}{1 + \sum_k \beta_k \delta_k}, \text{ and} \tag{4.4}$$

$$B_2^i = \frac{(n+1)\delta_i}{1 + \sum_k \beta_k \delta_k}. \tag{4.5}$$

The second equation in (4.2) follows from (2.14) and is shown as follows:

$$E\left[(A_0^i + \sum_j A_j^i \hat{y}_j + A_{n+1}^i \, y_i)^2\right]$$

$$= A_0^i + E\left[(\sum_{j\neq i} A_j^i \hat{y}_j + (A_i^i + \frac{\tau_i}{t_i} A_{n+1}^i)\hat{y}_i + \frac{\beta_i}{t_i} A_{n+1}^i \, \bar{y}_i)^2\right]$$

$$= \frac{a^2}{(n+1)^2 b^2} + \frac{1}{(n+1)^2 b^2} E\left[(\sum_j \tau_j \, B_1^i \, \hat{y}_j + \beta_i \, B_2^i \, \bar{y}_i)^2\right],$$

where $B_1^i$ and $B_2^i$ are defined as above. By (2.14), we have

$$E\left[(\sum_j \tau_j \, B_1^i \, \hat{y}_j + \beta_i \, B_2^i \, \bar{y}_i)^2\right] = \frac{1}{R}(\sum_j \tau_j \, B_1^i + \beta_i \, B_2^i)^2 + \left[\sum_j \tau_j (B_1^i)^2 + \beta_i (B_2^i)^2\right].$$

Then



$$\Pi_i(\tau_1,\ldots,\tau_n) = \frac{a^2}{(n+1)^2 b} + \frac{1}{(n+1)^2 bR} D_i(\tau_1,\ldots,\tau_n). \tag{4.6}$$

In fact, these explicitly calculated payoff functions enable us to investigate the equilibria of the games with asymmetric information, i.e. $t_i \neq t_j$ for some i,j. But for the purpose of simplicity and illustration, we assume $t_i = t$ for all i in the rest of the paper.

<u>Proposition</u> <u>2</u>. Complete information sharing is dominated by no information sharing when the information is symmetric.

<u>Proof</u>. Calculate

$$\Delta_i \equiv \Pi_i(0,\ldots,0) - \Pi_i(t,\ldots,t)$$

$$= \frac{1}{(n+1)^2 bR} (D_i(0,\ldots,0) - D_i(t,\ldots,t))$$

$$= \frac{1}{(n+1)^2 bR} \left[\frac{(n+1)^2 t(t+R)}{((n+1)t + 2R)^2} - \frac{nt}{nt+R}\right]$$

$$= \frac{(n-1)^2 t(t+R)}{(n+1)^2 b(nt+R)((n+1)t+2R)^2} > 0 \quad \text{for } n \geq 2. \tag{4.7}$$

Q.E.D.

Note that $\Delta_i$ diminishes as n or t goes to infinity. That means the net gains of no pooling and full pooling become close when the total amount of information is large in the industry or the signals the firms receive are sufficiently accurate. The first result follows from the fact that the price in the oligopoly with privately held information converges almost surely to the price in the pooled information situation as long as the information is not costly (see Li, McKelvey and Page [1985]; Palfrey [1985]). Whereas the second result is consistant with Novshek and Sonnenschein's finding for a duopoly case since their model is a good approximation only when t is sufficiently large.

Until now, we have not specified the constraints on the strategy space of the game. The question depends on the structure of the information. A natural choice for the strategy space is $[0,t] \subset R_+$ if the signal is infinitely divisible. But this is not true in many other situations. For instance, the precision $\tau_i$ might be a function of the signal only through the number of observations. So we have to consider two cases: the discrete and the continuous strategy spaces. In the discrete case we only investigate an extreme case, i.e. where a firm chooses to either not reveal any of its private information, or chooses to reveal all of it. And then the symmetric equilibrium for the continuous game is examined.

Proposition 3. Suppose $\tau_i \varepsilon \{0,t\}$, i=1,...,n. Then for $n \geq 2$ and $t > 0$, $\tau_1 = \tau_2 = \cdots = \tau_n = 0$ is the unique Nash equilibrium.

Proof. Since the game is symmetric, we assume, without loss of generality, that $\tau_i = t$, i=1,...,k − 1 and $\tau_i = 0$, i=k + 1,...,n, and denote the payoff of player k if $\tau_k = 0$ by $\Pi_k(0)$ and the payoff if $\tau_k = t$ by $\Pi_k(t)$. It is sufficient to show $\Pi_k(0) - \Pi_k(t) > 0$ for k=1,...,n because that means any player will be worse off by revealing



its signal in any case, and hence no pooling is the unique equilibrium.

Clearly, it is equivalent to show $D_k(0) - D_k(t) > 0$ for all k. By (4.3)-(4.5),

$$\begin{aligned} D_k(0) - D_k(t) &= \left[(k-1)t\, B_1^k(0) + t\, B_2^k(0)\right]^2 + R\left[(k-1)t[B_1^k(0)]^2 + t[B_2^k(0)]^2\right] \\ &\quad - [kt\, B_1^k(t)]^2 - Rkt[B_1^k(t)]^2 \\ &= \left[(k-1)tB_1^k(0) + tB_2^k(0) - ktB_1^k(t)\right]\left[(k-1)tB_1^k(0) \right. \\ &\quad \left. + tB_2^k(0) + ktB_1^k(t)\right] \\ &\quad + (k-1)Rt\left[B_1^k(0) - B_1^k(t)\right]\left[B_1^k(0) + B_1^k(t)\right] \\ &\quad + Rt\left[B_2^k(0) - B_1^k(t)\right]\left[B_2^k(0) + B_1^k(t)\right], \end{aligned} \tag{4.8}$$

$$B_1^k(0) = \frac{(k-1)t + 2R}{[(k-1)t + R][(n+k)t + 2R]}, \tag{4.9}$$

$$B_2^k(0) = \frac{n+1}{(n+k)t + 2R}, \text{ and} \tag{4.10}$$

$$B_1^k(t) = \frac{(n+k+1)t + 2R}{(kt + R)[(n+k+1)t + 2R)}. \tag{4.11}$$

Direct calculation shows that

$$G(k) \equiv (k-1)tB_1^k(0) + tB_2^k(0) - ktB_1^k(t)$$

$$= -R[B_1^k(0) - B_1^k(t)]$$

$$= \frac{Rt\left[n(k-1)(n+k+1)t^2 + [(n-1)(n+3k-1) + 2(k-1)]tR + 2(n-1)R^2\right]}{[(k-1)t + R](kt+R)[(n+k)t + 2R][(n+k+1)t + 2R]}$$

$$> 0, \quad \text{and} \tag{4.12}$$

$$Rt[B_2^k(0) - B_1^k(t)] = [(k-1)t + R]G(k) > 0, \qquad \text{for } n \geq 2. \tag{4.13}$$

Therefore,

$$D_k(0) - D_k(t) = (kt + R)G(k)[B_2^k(0) + B_1^k(t)] > 0,$$
$$\text{for } k=1,2,\ldots,n \text{ and } n \geq 2. \tag{4.14}$$

Q.E.D.

<u>Proposition</u> 4. Suppose $\tau_i \in [0,t]$, $i=1,\ldots,n$. For any given $n \geq 2$ and $t > 0$, $\tau_1 = \tau_2 = \cdots = \tau_n = 0$ is the unique symmetric equilibrium.

<u>Proof</u>: Note that $\tau_i = 0$ for all i is the symmetric equilibrium, then $\frac{\partial \Pi_i}{\partial \tau_i}\Big|_{\tau_1=\tau_2=\ldots=0} \leq 0$ for all i and $\tau_i = \tau$, $0 < \tau \leq t$ is the symmetric equilibria, then $\frac{\partial \Pi}{\partial \tau_i}\Big|_{\tau_1=\tau_2=\ldots=\tau} \geq 0$ for all i. Using the fact that

$$\frac{\partial \Pi_i}{\partial \tau_i} = \frac{1}{(n+1)^2 bR}\frac{\partial D_i}{\partial \tau_i}, \tag{4.15}$$

$$\frac{\partial D_i}{\partial \tau_i} = 2\left[\sum_j \tau_j B_1^i + \beta_i B_2^i\right]\left[B_1^i - B_2^i + \sum_j \tau_j \frac{\partial B_1^i}{\partial \tau_i} + \beta_i \frac{\partial B_2^i}{\partial \tau_i}\right]$$



$$+ R\left[(B_1^i)^2 - (B_2^i)^2 + 2\sum_j \tau_j B_1^i \frac{\partial B_1^i}{\partial \tau_i} + 2\beta_i B_2^i \frac{\partial B_2^i}{\partial \tau_i}\right], \tag{4.16}$$

$$B_1^i\Big|_\tau = \frac{2}{2\alpha + (n-1)\beta}, \tag{4.17}$$

$$B_2^i\Big|_\tau = \frac{n+1}{2\alpha + (n-1)\beta}, \tag{4.18}$$

$$\frac{\partial B_1^i}{\partial \tau_i}\Bigg|_\tau = \frac{2(n-1)(\alpha + n\beta)}{(2\alpha - \beta)(2\alpha + (n-1)\beta)}, \quad \text{and} \tag{4.19}$$

$$\frac{\partial B_2^i}{\partial \tau_i}\Bigg|_\tau = \frac{(n+1)(n-1)\beta}{(2\alpha - \beta)(2\alpha + (n-1)\beta)}, \tag{4.20}$$

where

$$\alpha = t + (n-1)\tau + R, \qquad \beta = t - \tau, \tag{4.21}$$

we can calculate

$$\frac{\partial \Pi_i}{\partial \tau_i}\Big|_\tau = \frac{1}{(n+1)^2 bR}\left[-\frac{(n-1)(n+3)R}{(2\alpha + (n-1)\beta)^2} - \frac{2\beta(n-1)(n+1)(4\alpha + (n-1)\beta)R}{(2\alpha - \beta)(2\alpha + (n-1)\beta)^3}\right]$$

$$= -\frac{n-1}{(n+1)^2 b}\left[\frac{n+3}{((n+1)t + (n-1)\tau + 2R)^2} + \frac{2(t-\tau)(n+1)((n+3)t + 3(n-1)\tau + 4R)}{(t + (2n-1)\tau + 2R)((n+1)t + (n-1)\tau + 2R)^3}\right]$$

$$< 0 \quad \text{for } n \geq 2,\ 0 < t,\ R < \infty. \qquad (4.22)$$

Therefore, $\tau_i = \tau$, $0 < \tau \leq t$, for all i are not equilibria. We then verify that $\left.\frac{\partial \Pi_i}{\partial \tau_i}\right|_{\tau_j=0,\, j\neq i} < 0$ for $n \geq 2$ and $0 \leq \tau_i \leq t$ (see Appendix B), and conclude the proof.

Q.E.D.

Propositions 2-4 show that no pooling is the unique symmetric equilibrium which always dominates full information pooling. Our results are solved for an oligopoly with n firms, and hence the asymptotic properties of the equilibrium can be examined when the market becomes large. For example, in the continuous game, it is easy to see by equation (4.22) $\left.\frac{\partial \Pi_i}{\partial \tau_i}\right|_{\tau} \to 0$ as $n \to \infty$, for any $0 \leq \tau \leq t$. That is, any amount of communication among firms is consistent with an equilibrium as long as the market is sufficiently large. On the other hand, letting t go to infinity, we also have $\left.\frac{\partial \Pi_i}{\partial \tau_i}\right|_{\tau} \to 0$ for any $n \geq 2$, $0 \leq \tau \leq t$. To summarize the two limiting effects, denote by $T = nt$ the total amount of information ex ante and $y = \frac{1}{n}\sum_{i=1}^{n} y_i$ the realization ex post. Then the equilibrium output of the industry is

$$Q^* = \sum_{i=1}^{n} q_i^* = \frac{n}{(n+1)b}\left[a + \frac{(n+1)Ty}{((n+1)T + 2Rn)}\right].$$

Consider a situation in which the pooling strategies are adopted. The total output then is a trivial standard oligopoly solution, i.e.



$$Q = \sum_{i=1}^{n} q_i = \frac{n}{(n+1)b}\left[a + \frac{T}{T+R}\, y\right].$$

<u>Proposition 5</u>. $Q^* - Q$ converges to zero almost surely as $T \to \infty$.

<u>Proof</u>: Simply notice that as $T \to \infty$, the difference

$$Q^* - Q = -\frac{n(n-1)TR}{(n+1)(T+R)((n+1)T + 2Rn)}\, y$$

converges to zero almost surely for $n \geq 2$.

Q.E.D.

Since demand is linear, convergence of $Q^*$ to Q implies the convergence of the equilibrium price (with privately held information) to the price in the pooled information situation. Consequently, the ex ante expectations such as profits and total social welfare also converge correspondingly in the normal sense. Therefore, in an industry with a sufficient amount of information, the oligopolists behave as if the information is pooled. The competitive price will certainly be efficient when the number of firms becomes large.

We conclude the paper with some more remarks. First, we show that there are no asymmetric equilibria only for the case in which partial revelation of the information is not allowed. But the class of symmetric equilibria is natural to examine first since firms are assumed to have access to equally accurate information. However our analysis has provided a basis (the explicitly calculated payoffs) for the investigation of the asymmetric equilibria in a symmetric information setting (our conjecture is that no pooling is the only equilibrium there) and the equilibrium behavior under asymmetric information structure as well. Secondly, in proposition 4, we assume the strategies which firms employ are continuous in the first-stage game. In many cases such as when $\tau_i$ are scaled sample sizes, it is not true. But the equilibrium characterization is still a good approximation when the strategy space is discrete. Finally, the results in this paper provide a support of Li, McKelvey and Page [1985] where we investigate the equilibrium behavior of a Cournot oligopoly with endogenous information acquisition under the assumption that firms will hold the information privately after the acquisition. A unique symmetric equilibrium is found there. What we show here is that this equilibrium is sustainable because any sharing agreement is not an equilibrium.



## APPENDIX A

Solution to the system of equations (3.11)-(3.14).

Equation (3.5) is easy to solve. Now rewrite (3.14) to be

$$A^i_{n+1} = \frac{t_i}{2\alpha_i - \beta_i}\left[\frac{1}{b} - \sum_j \frac{\beta_j}{t_j} A^j_{n+1}\right] \qquad \text{(A1)}$$

and then

$$A^j_{n+1} = \frac{t_j(2\alpha_i - \beta_i)}{t_i(2\alpha_j - \beta_j)} A^i_{n+1} \qquad \text{(A2)}$$

Substituting (A2) into the right side of (A1) and collecting the terms, we have

$$A^i_{n+1} = \frac{\dfrac{t_i}{2\alpha_i - \beta_i}}{b\left[1 + \sum_j \dfrac{\beta_j}{2\alpha_j - \beta_j}\right]} . \qquad \text{(A3)}$$

By (3.13),

$$\sum_{k\neq i} A^k_j = -A^j_j - A^i_j \qquad \text{for } i \neq j. \qquad \text{(A4)}$$

Using (A3) and (A4), we can derive from (3.12) that

$$A^i_j - A^j_j = \frac{\tau_i}{\alpha_i b} - \frac{\tau_i}{t_j} A^j_{n+1} - \frac{\tau_i}{\alpha_i} \sum_{k\neq i} \frac{\beta_k}{t_k} A^k_{n+1}$$

$$= \frac{\tau_j\left[\frac{2}{2\alpha_i - \beta_i} - \frac{1}{2\alpha_j - \beta_j}\right]}{b\left[1 + \sum_k \frac{\beta_k}{2\alpha_k - \beta_k}\right]}, \; j \neq i. \tag{A5}$$

Summing both sides of (A5) over i (i ≠ j) and using (3.13) again, we get

$$-(n + 1)A_j^j = \frac{\tau_j\left[\sum_{k\neq j}\frac{2}{2\alpha_k - \beta_k} - \frac{n-1}{2\alpha_j - \beta_j}\right]}{b\left[1 + \sum_k \frac{\beta_k}{2\alpha_k - \beta_k}\right]}, \text{ or}$$

$$A_j^j = \frac{\tau_j\left[\frac{n+1}{2\alpha_j - \beta_j} - \sum_k \frac{2}{2\alpha_k - \beta_k}\right]}{b(n+1)\left[1 + \sum_k \frac{\beta_k}{2\alpha_k - \beta_k}\right]}. \tag{A6}$$

It directly follows from (A5) and (A6) that

$$A_j^i = \frac{2\tau_j\left[\frac{n+1}{2\alpha_i - \beta_i} - \sum_k \frac{1}{2\alpha_k - \beta_k}\right]}{b(n+1)\left[1 + \sum_k \frac{\beta_k}{2\alpha_k - \beta_k}\right]}, \; i \neq j. \tag{A7}$$

Q.E.D.



## APPENDIX B

Letting $\tau_j = 0$, $j \neq i$, we have

$$B_1^i = \frac{2}{h_i}\left[(n+1)\tau_i + t + 2R\right], \tag{B1}$$

$$B_2^i = \frac{n+1}{h_i}(2\tau_i + t + 2R), \tag{B2}$$

$$\frac{\partial B_1^i}{\partial \tau_i} = \frac{2}{h_i^2}(n-1)(t+2R)[(n+2)t + 2R], \tag{B3}$$

$$\frac{\partial B_2^i}{\partial \tau_i} = \frac{n+1}{h_i^2}(n-1)t(t+2R), \tag{B4}$$

where

$$h_i = [n+3)t + 4R]\tau_i + [(n+1)t + 2R](t+2R). \tag{B5}$$

Note that

$$B_1^i - B_2^i = -\frac{1}{h_i}(n-1)(t+2R) < 0, \text{ for } n \geq 2, \tag{B6}$$

and equation (4.16). It follows

$$\frac{\partial D_i}{\partial \tau_i} < 2\left[\tau_i B_1^i + (t - \tau_i)B_2^i\right]\left[B_1^i - B_2^i + \tau_i \frac{\partial B_1^i}{\partial \tau_i} + (t - \tau_i)\frac{\partial B_2^i}{\partial \tau_i}\right]$$

$$+ 2R\left[\tau_i B_1^i \frac{\partial B_1^i}{\partial \tau_i} + (t - \tau_i) B_2^i \frac{\partial B_2^i}{\partial \tau_i}\right]$$

$$= - \frac{2R}{h_i^3}(n - 1)(n + 1)(t - \tau_i)(t + 2R)(2\tau_i + t + 2R)[(n + 3)t + 4R]$$

$$\leq 0, \text{ for } 0 \leq \tau_i \leq t \text{ and } n \geq 2. \qquad (B7)$$

Q.E.D.



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