DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

AN EFFICIENT MARKET MECHANISM WITH ENDOGENOUS INFORMATION ACQUISITION: COURNOT OLIGOPOLY CASE

Lode Li

SOCIAL SCIENCE WORKING PAPER 571

May 1985

ABSTRACT

The question of how an efficient competitive equilibrium could be reached through a pricing mechanism in which the information acquisition is endogenously determined is addressed. The traditional oligopoly market is extended to include an ex ante information market when there is uncertainty either in the cost function or the demand function. Equilibrium behavior is characterized in a two-stage noncooperative game involving n production firms and m research firms in the industry. As the environment becomes more competitive, meaning, both the information market and the tangible good market become large, the equilibrium random price of the product converges almost surely to its competitive price level with certainty and consequently the total social welfare (consumer plus producer surplus) is maximized.

AN EFFICIENT MARKET MECHANISM WITH ENDOGENOUS INFORMATION ACQUISITION: COURNOT OLIGOPOLY CASE*

Lode Li

California Institute of Technology

1. INTRODUCTION

Two lines of research in the literature motivate this study. First, Wilson (1977), later extended by Milgrom (1979), shows that under fairly general conditions the winning bid converges to the true value of the object at auction as the number of bidders becomes large and indicates that the bidding can serve as a basis of competitive price formation. As Matthews (1984) points out, however, that endogeneity of costly information acquisition can alter the convergence property of auction mechanism. Secondly, the question whether diverse imperfect information may be processed via other market pricing mechanism as perfectly as it is processed in an auction is studied by Li (1985) and Palfrey (1985) in a Cournot-Bayesian oligopoly setting, and it is shown that the random equilibrium price converges almost surely to the perfectly competitive price as the number of firms becomes arbitrarily large. But again, the amount of

* I gratefully acknowledge the insightful suggestion of my former teacher Morton Kamien. The paper is a continuation work on the basis of "Optimal Research for Cournot Oligopolists," a joint work with my colleagues Richard D. McKelvey and Talbot Page. I have also benefited from my discussion with Leonid Hurwicz and Quang H. Vuong.

information each firm acquires is not only exogenously determined but costless as well. Li, McKelvey and Page (1985) show in great detail, the sources of the inefficiency are inadequate amount of information acquired as well as incomplete information pooling when the amount of information obtained is costly. Therefore the convergence properties described above fail if endogeneity of information acquisition is introduced by simply associating a positive cost to obtaining the relevant information.

In this paper the market mechanism studied in the previous work is augmented by introducing an extra market for ex ante information trading and studies the equilibrium value of the information. We show that this augmentation may eliminate the weakness suffered from the endogenous information acquisition in the previous models and that the convergence theorem prevails as the environment (both the information market and the tangible good market) becomes more competitive.

We study an oligopolistic industry in which there is uncertainty either in the cost function or the demand function. In addition, there is a group of firms who undertake research aiming at resolving the uncertainty. By subcontracting research to these firms, each production firm can acquire exclusive information which may help it resolve the uncertainty and thereby make a more informed decision on the level of output. It is noteworthy that the industries which specialize in information acquisition and processing have been booming for years. For instance, an enormous number of marketing research or

consulting firms are in service for a variety of industries. Regarding the cost information research, the chemical industry has a long tradition of subcontracting the preliminary toxicological tests of new products (which are critical in estimating the liabilities that may have to be paid if the products are eventually found to be toxic) to independent laboratories and research institutes. Therefore, developing an equilibrium model to study the behavior of production firms versus research firms is not only of theoretic interest but also of practical importance.

Equilibrium behavior is characterized in a noncooperative game involving n + m players - the n production firms and m research firms in the industry. Prior to observing the information, each research firm chooses a research level to sell whereas each production firm chooses a level of information to buy in a simultaneous move. Then, the research firms undertake research and producers observe the exclusive signals from the contracted research results and simultaneously choose levels of production based on the data. This is a two-stage game. The equilibrium definition we use is that of a subgame perfect Nash equilibrium. In the first stage, the ex ante information that we refer to as research is traded in a measurement of the expected precision of the signal. It is assumed that the research contract is sold at a single price per unit of research. The payoff to each production firm can be determined as the expected net profit assuming the Cournot-Bayesian equilibrium behavior follows in the second stage. The payoff to each research firm is the total amount it extracts from selling the research data. The Nash equilibrium of this game uniquely determines the value of research as well as the total amount of research traded. In the second stage game, the unique Cournot-Bayesian equilibrium determines the ex post price of the tangible good. This two-stage game parallels the two-stage game introduced by Kamien and Tauman (1984a, 1984b) in which a patent holder licenses cost reducing invention to the producers in a n-firm oligopoly by means of a fee. What this two-stage game amounts to, in their analysis, as here, is first finding the demand function for research and then maximizing against it. The demand function for research is derived from the Bayesian solution of the second stage of the game and then using it in the first stage of the game.

This paper is arranged in five sections, including the present one. Section 2 is devoted to stating the assumptions, developing the notations, and presenting the specifications of the game. A unique subgame perfect Nash equilibrium is derived in Section 3. A convergence theorem is proved and the efficiency implication is discussed in Section 4. Some concluding remarks follow.

2. THE MODEL

Consider an industry with n identical firms producing a homogenous output. The production firms face a stochastic inverse demand function given by $D^{-1}(Q,\theta) = a + \theta - bQ$, where $Q = \sum_{i=1}^{n} q_i$ is the total quantity produced, and θ is the true state of the world which is unknown when the output decisions are made. Producers have identical

constant unit cost of production, c, such that $0 < c \leq a$. While we assume the uncertainty arises in the intercept of the demand function, the analysis is identical if the uncertainty arises from the coefficient c in the cost function. There are also m identical research firms conducting research to acquire information concerning the true state of the world θ.

Without loss of generality, we assume that θ is generated according to a probability distribution $g(\theta)$ with <u>zero</u> mean. Denote by y_{ij} the signal that producer i is to receive from researcher j. Assume y_{ij} is generated according to the conditional distribution $h(y_{ij}|\theta,t_{ij})$ where

$$t_{ij} \equiv \frac{1}{E[\mathrm{Var}[y_{ij}|\theta]]} . \tag{1}$$

The measurement t_{ij} is used as the level of research that producer i contracts from researcher j prior to the observation of the signal y_{ij}. It is a measure of the expected precision of the state-relevant research data in trade. By using this measurement, we may treat the ex ante research contracts as ordinary goods. This can be justified as follows. First, the higher the quantity t_{ij}, the lower the expected variance and therefore the better the quality of the resulting signal. Second, note that t_{ij} is directly proportional to the number of observations if the data is obtained from independent sampling. In the example of the toxicological tests of chemicals, t_{ij} is directly proportional to, say, the number of animals used in the test, other things being equal. Third, we shall see shortly that only

t_{ij} affect the payoffs to producers prior to observing the signals y_{ij} and the expected profit of producer i strictly increases as the amount of information to receive, t_i, increases, where $t_i = \sum_{j=1}^{n} t_{ij}$. Therefore, the value of t_{ij} plays a similar role as the quantity measurement of other inputs in the production process.

Further assumptions on the information structure are as follows:

Assumption A1 (independence) y_{ij}, $i = 1,\ldots,n$, $j = 1,\ldots,m$, are independent conditional on θ.

Assumption A2 (unbiasedness) $E[y_{ij}|\theta] = \theta$ for all i, j.

Assumption A3 (linearity) $E[\theta|Y] = \gamma + \delta \cdot Y$ where γ is a constant, δ is a vector of n X m constants, $Y = (Y_1,\ldots,Y_n)$ is the signal generated by researchers for producers, and $Y_i = (y_{i1},\ldots,y_{im})$ is exclusively for producer i.

Assumption A1 ensures the noncooperative nature of the game. That is, each research firm conducts its research independently and each production firm receives exclusive signals. Assumption A2 implies that the research data is "good" information and then all it matters is its precision. Assumption A3 restricts the assumed probability distributions to a special class which is wide enough to include many interesting prior-posterior distribution pairs such as normal-normal, beta-binomial and gamma-Poisson (see Degroot (1970) and Ericson (1969)). It not only enables us to get the explicit form and

the uniqueness of the equilibrium, but also facilitates the information trading. Let $X_j = (y_{1j},\ldots,y_{nj})$ be all the signals provided by research firm j. We have the following lemma (See Li (1985) for the proof).

Lemma 1 Suppose A1, A2, and A3 hold. Then

$$E[\theta|y_{ij}] = \frac{t_{ij}}{t_{ij} + R} y_{ij}, i = 1,\ldots,n, j = 1,\ldots m, \tag{2}$$

$$E[\theta|Y_i] = \frac{t_i}{t_i + R} y_i, \ i = 1,\ldots,n, \text{ and} \tag{3}$$

$$E[\theta|X_j] = \frac{\tau_j}{\tau_j + R} x_j, \ j = 1,\ldots,m, \text{ where } R \equiv \frac{1}{Var[\theta]}, \tag{4}$$

$$y_i \equiv \frac{1}{t_i}\sum_{j=1}^{m} t_{ij} y_{ij}, \ t_i \equiv \frac{1}{E[Var[y_i|\theta]]} = \sum_{j=1}^{m} t_{ij}, \tag{5}$$

$$x_j \equiv \frac{1}{\tau_j}\sum_{i=1}^{n} t_{ij} y_{ij}, \ \tau_j \equiv \frac{1}{E[Var[x_j|\theta]]} = \sum_{i=1}^{n} t_{ij} \ . \tag{6}$$

The above lemma addresses the equivalence between receiving one signal y_i and m signals $y_{i1},\ldots,y_{im}$ for producer i, and also the equivalence between generating one signal x_j and n signals $y_{1j},\ldots,y_{nj}$ for researcher j. More explicitly, note that $t_{ij} = 0$ means researcher j is not to provide any information for producer i. So, production firm i is indifferent in contracting research from any specific researchers in any amount as long as the total amount of contracted research $t_i = \sum_j t_{ij}$ remains the same since signals $y_{i1}, \ldots, y_{im}$ are used in an aggregated way as if one signal y_i with expected precision t_i is received. Similarly, researcher j only cares the total level of research sold, τ_j. Therefore the research contracts can be additively aggregated or disaggregated as if they were ordinary commodities. Finally, we assume that the underlying probability distributions are common knowledge. Hence, $g(\theta)$ is the common prior distribution that firms have for θ. Each production firm can compute a posterior $f(\theta|y_i, t_i)$ for θ on the basis of a chosen level of contracted research t_i and the resulting information y_i.

The model is a two stage game. In the first stage, the true value of θ is generated. Production firm i contracts a research level t_{ij} with research firm j. Or, in other words, each research firm chooses a level of research τ_j to sell and each production firm chooses a level of research t_i to buy. In the second state, conditional on the contracted research levels t_{ij}, the signals $Y_1, \ldots, Y_n$ are generated. Producer i observes the research levels contracted by other producers t_j, $j \neq i$, but only observes his <u>own</u> private signal Y_i. Production firms then determine their output levels $(q_1, \ldots, q_n)$.

Next we define the payoff function of each player in the game. Given the choices $((t_1, q_1), \ldots, (t_n, q_n))$ by the n production firms and the price of a unit of research v, the profit of producer i is then of the form,

$$\pi_i((t_1, q_1), \ldots, (t_n, q_n), v, \theta) = q_i(a - c + \theta - b\sum_{j=1}^{n} q_j) - vt_i. \quad (7)$$

The payoff to producer i in the second stage game is the expected profit given signal Y_i and the choice levels $(t_1, \ldots, t_n)$, i.e., $E[\pi_i|Y_i, (t_1, \ldots, t_n)]$, and the payoff to producer i in the first stage then is the profit which is to obtain in the second stage with strategy choices $(q_1, \ldots, q_n)$ expected over possible value of Y_i,

$$\Pi_i(t_1, \ldots, t_n, v) = E[E[\pi_i|Y_i, (t_1, \ldots, t_n)]]. \quad (8)$$

The payoff to researcher j is simply its profit

$$\Pi_j^r(\tau_1, \ldots, \tau_n, v) = \tau_j v \quad (9)$$

The equilibrium notion we adopt is that of a subgame perfect Nash equilibrium. Formally, a (n + m)-tuple of strategies $(\tau_1^*, \ldots, \tau_m^*, (t_1^*, q_1^*), \ldots, (t_n^*, q_n^*))$ is an equilibrium if (i) for each i, $q_i^*(Y_i, (t_1 \ldots, t_n))$ is the best response of producer i given other firms' choices $q_j^*(Y_j, (t_1, \ldots, t_n))$, $j \neq i$, for each $(t_1, \ldots, t_n)$ and almost every Y_i in the second stage game, (ii) for each i, $t_i^*(v)$ is the best response of producer i given other firms' choices $t_j^*(v)$, $j \neq i$, and the second stage Bayesian strategies $(q_1^*, \ldots, q_n^*)$, for each $v \in \mathbb{R}_+$ in the first stage, (iii) $\sum_{j=1}^{m} \tau_j^* = \sum_{i=1}^{n} t_i^*$, and (iv) for each j, τ_j^* is the best responses of researcher j given τ_k^*, $k \neq j$ and $((t_1^*, q_1^*), \ldots, (t_n^*, q_n^*))$.

Basically, we are looking for an market-clearing price v^*, under which the payoff to each player is maximized. Since production firms are assumed to be price-takers in the first stage, the market-clearing condition (iii) determines a demand function for research provided that conditions (i) and (ii) are satisfied. In fact, the research firms in this game play the role of the Stackelberg leaders in the sense that they determine the producers' reaction function or demand function for the ex ante information as a function of the price v, and then strategically exploit the demand curve. The research firms can carry out this kind of inference if we assume that the linear demand and cost function of production, and their coefficients a, b, and c, are common knowledge. In fact, as we will show later, the knowledge of the linear demand and linear cost plus coefficient b is sufficient for researchers to infer the reaction function of producers.

3. EQUILIBRIUM

In this section, we derive the subgame perfect Nash equilibrium for the game, which is unique if the game is continuous. We proceed by solving the second stage game first.

<u>Proposition 1</u> For any fixed $(t_1,\ldots,t_n)$, there is a unique Bayesian-Cournot equilibrium to the second stage game for almost every $(Y_1,\ldots,Y_n)$. The equilibrium strategy for each producer i is of the form

$$q_i(y_i) = \frac{a - c}{(n+1)b} + \frac{\dfrac{t_i}{t_i + 2R}}{b\left(1 + \sum_{j=1}^{n} \dfrac{t_j}{t_j + 2R}\right)} y_i, \tag{1}$$

and the payoff to each producer i with the Bayesian strategy is

$$E[\pi_i | y_i] = bq_i^2 - vt_i. \tag{2}$$

The rigorous proof of the theorem has been given by Li, McKelvey and Page (1985). The uniqueness of the equilibrium and the explicit forms enable us to determine the payoffs to producers for the first stage game in terms of their choices $(t_1,\ldots,t_n)$ and the price of research v, i.e.,

$$\Pi_i(t_1,\ldots,t_n,v) = \frac{1}{b(n+1)^2}(a-c)^2 + \frac{t_i(t_i + R)}{bR(t_i + 2R)^2\left(1 + \sum_{j=1}^{n} \dfrac{t_j}{t_j + 2R}\right)^2} - vt_i. \tag{3}$$

Thus, Π_i is the payoff function of producer i in the first stage given Bayesian strategies follows in the second stage, and the first stage game is then reduced to a game with complete information. We now specify the strategy space for players in the first stage game. We choose a simplest and most natural one which is to let each player's strategy space be the non-negative reals, $\mathbb{R}_+$. Note that this choice is doable when $g(\theta)$ is normal and the relevant signals are also normally distributed with mean θ. Other information structures may restrict the strategic choice of the precision of the signal, t_i or τ_j, as the expected precision may be a function of the signal only through the number of observations. In this case, the appropriate strategy space is the scaled non-negative integers. Nevertheless, the equilibrium of the game with continuous strategy space is a good approximation to the symmetric equilibrium in mixed strategy of the discrete game given the same parameters (see Li, McKelvey and Page (1985)). The continuity assumption of the strategy space also facilitates the direct application of calculus technique as we commonly do with ordinary commodities. A choice of $t_i \in \mathbb{R}_+$ for producer i simply selects a signal of a given expected precision t_i. We impose a upper bound $\bar{\tau}$ (> 0) for each research firm's choice τ_i to indicate the technical difficulties of exhausting the uncertainty completely, i.e., $\tau_j \in [0,\bar{\tau}]$ for j = 1,...,m.

<u>Proposition 2</u> For any given value v

(i) There is a unique Nash equilibrium $(t_1(v),\ldots,t_n(v))$ for production firms which is symmetric, i.e., $t_1(v) = \cdots = t_n(v)$.

(ii) The equilibrium t_i is a strictly decreasing and convex function of v.

The proof can also be found in Li, McKelvey and Page (1985). In fact, letting $t(v) = t_i(v)$ for all i, we have the following equation (first order condition) which determines the functional relation between the equilibrium t and the value v,

$$v = \frac{(3n-1)t^2 + 2R(n+2)t + 4R^2}{b(t+2R)((n+1)t+2R)^3}. \tag{4}$$

Denote the total demand for research from the production firms by $T \equiv \sum_{i=1}^{n} t_i = nt$. Equation (4) can be rewritten as

$$v(T) = \frac{n^2[(3n-1)T^2 + 2n(n+2)RT + 4n^2R^2]}{b(T+2Rn)((n+1)T+2Rn)^3}. \tag{5}$$

Equation (5) provides a downward-sloping and convex inverse demand function for research (Fig. 1). Clearly, it is not necessary for one to know parameters a and c in infering this demand curve. Notice coefficient b is also irrelevant in the decision making by researchers (see Proposition 3). So we can weaken the common knowledge assumption in the preceeding section by assuming the linear demand and linear cost structure is common knowledge for all players while the coefficients a, b, and c are common knowledge only among producers.

Now, the equilibrium value of research can be obtained by examining the strategic behavior of the research firms subject to the inverse demand function (5). By the market clearing condition, the payoff to research firm j is

$$\Pi_j^r(\tau_1,\ldots,\tau_m) = \tau_j v(T) = \tau_j v\left(\sum_{k=1}^{m} \tau_k\right). \tag{6}$$

Note that the profit function of each research firm is always positive except $\Pi_j^r(0,\tau_{-j}) = 0$ and $\Pi_j^r(\infty,\tau_{-j}) = 0$ where $\tau_{-j} = (\tau_1,\ldots,\tau_{j-1},\tau_{j+1},\ldots\tau_m)$. Though $\Pi_j^r(\cdot,\tau_{-j})$ is not concave, we still have the following nice result.

<u>Proposition 3</u> For fixed m and n, there is a unique Nash equilibrium $(\tau_1^*,\ldots\tau_m^*)$ and $\tau_1^* = \ldots = \tau_m^* = \bar{\tau}$ if $m \geq 2$.

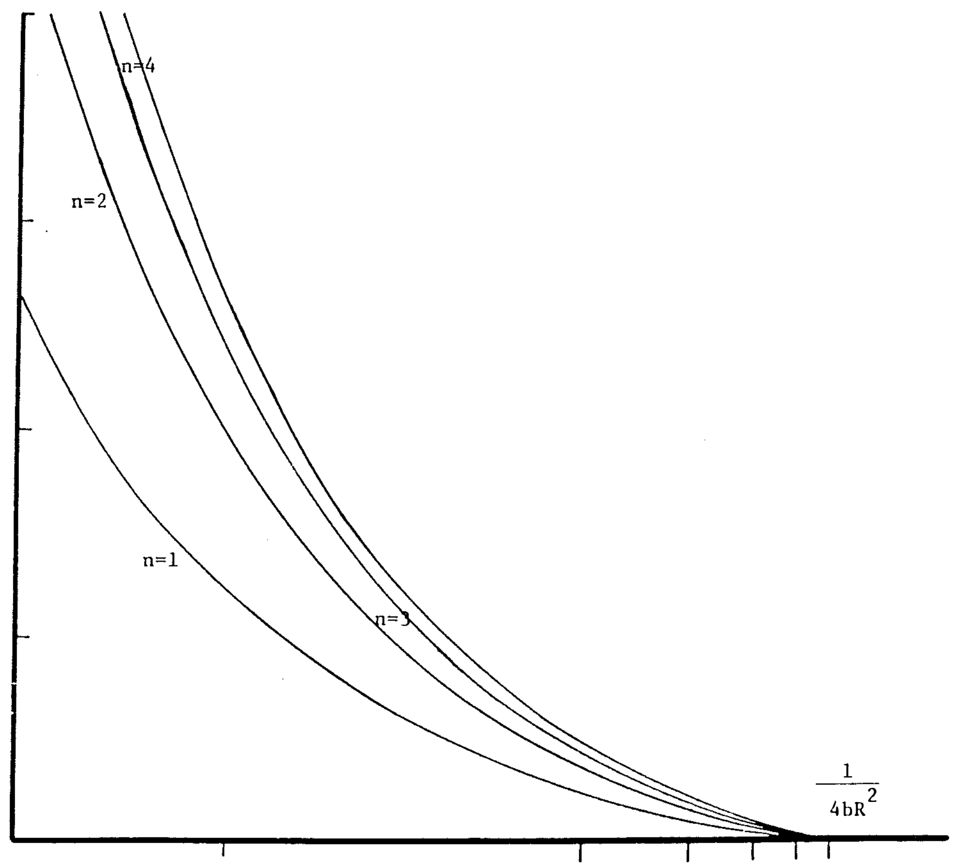
<u>Proof</u>: For m = 1,

$$\Pi_1^r = \tau_1 v(\tau_1) = \frac{n^2\tau_1[(3n-1)\tau_1^2 + 2n(n+2)\tau_1 R + 4n^2R^2]}{b(\tau_1+2Rn)((n+1)\tau_1+2Rn)^3} \tag{7}$$

AMOUNT OF RESEARCH DEMANDED BY PRODUCERS

AS A FUNCTION OF PRICE v

TOTAL AMOUNT OF RESEARCH (T)



PRICE PER UNIT OF RESEARCH (v)

FIGURE 1

Direct calculation shows that

$$\frac{\partial \Pi_1^r}{\partial \tau_1} = \frac{n^2 h(\tau_1)}{b(\tau_1 + 2Rn)^2((n+1)\tau_1 + 2Rn)^4}, \tag{8}$$

where

$$h(\tau_1) = -(n+1)(3n-1)\tau_1^4 - 4n(n^2+3)R\tau_1^3 - 4n^2(n^2-4n+6)R^2\tau_1^2 + 16n^3R^3\tau_1 + 16n^4R^4 . \tag{9}$$

Polynomial h has the following properties: $h(0) > 0$, $h(\infty) < 0$ and the sequence of its coefficients switch only once in sign. Hence h has only one positive root τ_1^0, and this implies $\frac{\partial \Pi_1^r}{\partial \tau_1} > 0$ for $\tau_1 < \tau_1^0$, $\frac{\partial \Pi_1^r}{\partial \tau_1} < 0$ for $\tau_1 > \tau_1^0$, and $\frac{\partial \Pi_1^r}{\partial \tau_1}(\tau_1^0) = 0$. We then conclude $\tau_1^* = \min(\tau_1^0, \bar{\tau})$ is the unique maximum point.

For $m \geq 2$,

$$\frac{\partial \Pi_j^n}{\partial \tau_j} = v(T) + \tau_j v'(T), \tag{10}$$

where $T = \sum_{j=1}^{m} \tau_j$.

Note that

$$v'(T) = -\frac{2n^2}{6(T+2Rn)^2((n+1)T+2Rn)^4} [(n+1)(3n-1)T^3 + 2Rn(3n^2+4n+3)T^2$$

$$+ 4R^2n^2(n^2+2n+5)T + 8R^3n^3(n+1)] < 0 \qquad (11)$$

and

$$v(T) + \frac{T}{2}v'(T) = \frac{n^2}{6(T+2Rn)^2((n+1)T+2Rn)^4} [2nR(n^2+4n-3)T^3+4R^2n^2(6n-1)T^2$$

$$+ 8R^3n^3(n+3)T + 16R^4n^4] > 0. \qquad (12)$$

So, for $\tau_j \leq \frac{T}{2}$ or $\tau_j \leq \sum_{k \neq j} \tau_k$,

$$\frac{\partial \prod_j^r}{\partial \tau_j} = v(T) + \tau_j v'(T) \geq v(T) + \frac{T}{2}v'(T) > 0. \qquad (13)$$

Suppose $(\tau_1^*, \ldots, \tau_m^*)$ with some j such that $\tau_j^* < \bar{\tau}$ is an equilibrium. Let $J \subset \{1,\ldots,m\}$, and $J = \{j \mid \tau_j^* < \bar{\tau}\}$. Then observation (13) implies $\tau_j^* > \sum_{k \neq j} \tau_k^*$ for $j \in J$. This implies $(|J| - 2)\sum_{j \in J} \tau_j^* + (m - |J|)\bar{\tau} < 0$ for $m \geq 2$, a contradiction. Hence $\tau_j^* = \bar{\tau}$, $j = 1,\ldots,m$ is the unique equilibrium.

Q.E.D.

It is worth mentioning that the research firms will sell as much information as they can for $m \geq 2$ is not true in general. It depends on the specific form of the reaction function resulting from the special assumptions of the probability distributions and the linear demand function for tangible goods, and the assumption that the information is produced at no cost. However, the qualitative description of the equilibrium behavior will not be altered in more general settings.

Let us summarize the equilibrium behavior of the players in this game. As Stackelberg leaders, the equilibrium strategy τ_j^* for each research firm determined in Proposition 3 is his best choice given other researchers' choices τ_k^*, $k \neq j$ and the strategic reactions of production firms. The total research in trade is $T^* = \sum_{j=1}^m \tau_j^*$ and the equilibrium value (price) of the ex ante information is $v^* = v(T^*)$. Given v^*, the equilibrium strategy $t_i(v^*)$ for each production firm determined in Proposition 2 is his best response conditioning on other producers' strategies $t_k(v^*)$, $k \neq i$ and the ex post Bayesian equilibrium output choices of the oligopoly. Finally, the Bayesian strategy $q_i(y_i,(t_1^*,\ldots,t_n^*))$ determined in Proposition 1 gives producer i's best choice based on his private signal y_i generated according to $h(\cdot|\theta,t_i^*)$.

4. CONVERGENCE AND EFFICIENCY

In this section, we show that the convergence property is regained with the augmented market mechanism described above, and the competitive equilibrium is efficient as the number of producers as well as the number of researchers become large.

The equilibrium amount of research producer i buys t_i^* depends on m and n. We denote t_i^* by t_i^{mn}, and y_i^{mn} is the signal generated according to $h(\cdot|\theta,t_i^{mn})$. Also let $Y_{mn} = \frac{1}{n}\sum_{i=1}^n y_i^{mn}$. Then conditional on θ, Y_{mn} is a sum of n independent and identically distributed random variables which depend on n and m through t^{mn}. It is easy to see the total output ex post is

$$Q_{mn} = \frac{n}{(n+1)b}[a - c + \frac{(n+1)T_{mn}}{(n+1)T_{mn} + 2nR} Y_{mn}]. \quad (1)$$

And $\bar{Q} = \frac{a-c}{b}$ is the competitive total output if the true state θ is known and takes its mean value zero. We are seeking the condition under which Q_{mn} converges to $\bar{Q}$ conditional on $\theta = 0$.

Obviously,

$$\lim_{\substack{m\to\infty \\ n\to\infty}} \frac{(n+1)T_{mn}}{(n+1)T_{mn} + 2nR} = 1, \text{ and} \quad (2)$$

$$\lim_{n\to\infty} \frac{n(a-c)}{(n+1)b} = \frac{a-c}{b} . \quad (3)$$

Notice that for $m \geq 2$, $T_{mn} = m\bar{\tau} = nt^{mn}$ or $t^{mn} = \frac{m}{n} \cdot \bar{\tau}$. Suppose we let $m = kn$, $k > 0$. Then y_i^{mn} do not depend on m and n and have the same distribution. Therefore the fact that

$$Y_{mn} \overset{a.s.}{\to} 0, \text{ as } m, n \to \infty \quad (4)$$

follows directly from the strong law of large numbers. Combining (2), (3), and (4), we have

$$Q_{mn} \overset{a.s.}{\to} \bar{Q}. \quad (5)$$

However, we can obtain a slightly stronger result (Proposition 4) by using the following uniform strong law of large numbers given by

Mickey (1963, p.40).

Lemma 2 (the uniform strong law of large numbers)

Let g be a function on Z X D where Z is a Euclidean space and D is a compact subset of a Euclidean space. Let g(z,d) be a continuous function of d for each z and a measurable function of z for each d. Assume also that $|g(z,d)| \leq h(z)$ for all z and d, where h is integrable with respect to a probability distribution function F on Z. If $z_1, z_2, \cdots$ is a random sample from F then for almost every sequence $\{z_i\}$

$$\frac{1}{n}\sum_{i=1}^{n} g(z_i, d) \to \int g(z,d)dF(z) \quad (6)$$

Uniformly for all d in D.

Applying Lemma 2, we have

Proposition 4 Given $\theta = 0$, Q_{mn} converges almost surely to $\bar{Q}$ as $n \to \infty$ and $\frac{n}{m} \leq B$ where B is a positive constant.

Proof: Let $z_i = \sqrt{t_i^{mn}}\, y_i^{mn}$. Then

$$E[z_i | \theta = 0] = 0 \quad (7)$$

$$E[Var[z_i | \theta]] = t_i^{mn} E[Var[y_i^{mn} | \theta]] = 1. \quad (8)$$

We assume conditional on $\theta = 0$, z_i are i.i.d. random variables. In fact a sequence of i.i.d. random variables $\{z_i\}$ can be obtained in this manner in many examples such as t_i^{mn} varies only via the number of observations under general distribution assumptions and t_i^{mn} varies continuously under normality assumption. Let

$$d_{mn} = \sqrt{\frac{1}{t^{mn}}} = \sqrt{\frac{n}{m\bar{\tau}}}. \tag{9}$$

It follows that

$$Y_{mn} = \frac{1}{n}\sum_{i=1}^{n} d_{mn} z_i. \tag{10}$$

Since $\frac{n}{m} \leq B$,

$$0 \leq d_{mn} \leq \sqrt{B\bar{\tau}^{-1}} \equiv \bar{B}. \tag{11}$$

Let $g(z,d) = dz$, $D = [0,\bar{B}]$, $h(z) = \bar{B}z$ and F be the conditional distribution function of z_i given $\theta = 0$. Then all the conditions for Lemma 2 are satisfied and we have

$$\frac{1}{n}\sum_{i=1}^{n} dz_i \overset{a.s.}{\rightarrow} 0 \tag{12}$$

uniformly for all d in D. Whence for any convergence sequence $\{d_{mn}\}$, $d_{mn} \in D$,

$$\frac{1}{n}\sum_{i=1}^{n} d_{mn} z_i \overset{a.s.}{\rightarrow} 0, \tag{13}$$

which is equivalent to

$$Y_{mn} \overset{a.s.}{\rightarrow} 0, \text{ as } n \rightarrow \infty, \frac{n}{m} \leq B. \tag{14}$$

Q.E.D.

The purpose of proving the above slightly stronger result is to illustrate where the convergence theorem might break down in the previous models with endogenous information acquisition. Condition $\frac{n}{m} \leq B$ is critical for the proof. Suppose m stays unchanged. Then this condition is violated when n goes to infinity and the proof collapses. This clearly indicates that competition among the information providers plays an essential role in reaching an efficient competitive equilibrium when the endogeneity of costly information acquisition is introduced. Since demand is linear, convergence of Q_{mn} to $\bar{Q}$ implies the convergence of the equilibrium random price to the perfectly competitive price which is equal to the marginal cost of production c.

As one can expect, the ex ante total social welfare from the information Cournot equilibrium converges to an efficient one as the market becomes more competitive in the sense that it maximizes the total social welfare (consumer plus producer surplus). Suppose that for any given level of research T (expected precision), a social planner selects a level of output Q based on a observed signal $\bar{y}$ generated according to $h(\cdot|\theta,T)$ to maximize the conditional total social welfare defined as $E[W|\bar{y}]$, where

$$W = \int_0^Q D^{-1}(Q,\theta)dQ - cQ - vT$$

$$= (a - c + \theta)Q - \frac{b}{2}Q^2 - vT. \tag{16}$$

The maximizer $Q_e(\bar{y})$ is found to be

$$Q_e(\bar{y}) = \frac{1}{b}[a - c + \frac{T}{T + R}\bar{y}], \qquad (17)$$

and the expected total social welfare following the choice $Q_e(\bar{y})$ is

$$W_e(T,v) = \frac{1}{2b}(a - c)^2 + \frac{1}{2bR}\frac{T}{T + R} - vT. \qquad (18)$$

On the other hand, we can also calculate the expectation of the total social welfare resulting from the equilibrium behavior of the oligopolists, i.e.,

$$W_n(T^*) = \frac{n(n + 2)}{2(n + 1)^2 b}(a - c)^2$$

$$+ \frac{1}{2bR} \cdot \frac{nT^*((n + 2)T^* + 3nR)}{((n + 1)T^* + 2nR)^2} - v^*T^*. \qquad (19)$$

<u>Proposition 5</u>

(i) $W_n(T^*) < W_e(T^*,v^*)$ for any m, $n \geq 1$,

(ii) If $\frac{n}{m} \leq B$, then

$$\lim_{n \to \infty} W_n(T^*) = \lim_{n \to \infty} W_e(T^*,v^*) = \sup_{T,v} W_e(T,v).$$

That is, the competitive limit maximizes the expected total social welfare.

<u>Proof</u>:

(i) It can be calculated

$$W_e(T^*,v^*) - W_n(T^*) =$$

$$\frac{(a - c)^2}{2b(n + 1)^2} + \frac{1}{2bR} \cdot \frac{T^*(T^* + nR)^2}{(T^* + R)((n + 1)T^* + 2nR)^2} > 0$$

for m, $n \geq 1$. (20)

(ii) Note that $\frac{n}{m} \leq B$ implies $T^* = m\bar{\tau} \geq nB^{-1}\bar{\tau}$. Therefore it is easy to see that as $n \to \infty$,

$$v(T^*)T^* = \frac{n^2T^*[(3n - 1)T^{*2} + 2n(n + 2)T^*R + 4n^2R^2]}{b(T^* + 2nR)((n + 1)T^* + 2Rn)^3} \to 0, \qquad (21)$$

$$W_e(T^*,v^*) - W_n(T^*) \to 0, \text{ and} \qquad (22)$$

$$W_e(T^*,v^*) \to \frac{1}{2b}(a - c)^2 + \frac{1}{2bR} = \sup_{T,v} W_e(T,v), \qquad (23)$$

by equation (3.5), (18) and (20).

Q.E.D.

5. CONCLUSION

This paper was motivated by the idea how an efficient competitive equilibrium could be obtained in a setting where diverse partial information is acquired by individuals endogenously. We show that the remedy is to introduce an ex ante information market. Though the problem is solved for a special case, namely a Cournot oligopoly with linear demand and linear conditional expectation information structure, the treatment of an information equilibrium can be applied elsewhere. For instance, Milgrom (1981) considers a special example of a Vickrey auction with information acquisition. The two stage game does provide a demand curve for any given price c of the signal, and the treatment in our paper is applicable. Certainly, one should admit the possibility that the outcome of trading may depend critically on the nature of the trading process, and that the variety of possible outcomes may not be representable by any single model. However, the notion of the equilibrium value of information studied in this paper will be useful in the discussions of the value of information in other trading processes. Our treatment of information ex ante as if it is an ordinary commodity by defining its proper measurement may also be enlightening.

REFERENCES

Blume, L. and Easley, D. "On the Game Theoretic Foundations of Market Equilibrium with Asymmetric Information," working paper, 1984.

Clarke, R. N. "Collusion and the Incentives for Information Sharing." Bell Journal of Economics 15 (1984):383-394.

Crawford, V. and Sobel, J. "Strategic Information Transmission." Econometrica 50 (1982):1431-1451.

DeGroot, M. Optimal Statistical Decisions. New York: McGraw Hill, 1970.

Ericson, W. A. "A Note on the Posterior Mean of a Population Mean." Journal of the Royal Statistical Society 31 (1969):332-334.

Gal-Or, E. "Information Sharing in Oligopoly." Econometrica, forthcoming.

Grossman, S. and Stiglitz, J. "On the Impossibility of Informationally Efficient Markets." American Economic Review 70 (June 1980):393-409.

Harsanyi, J. "Games with Incomplete Information Played by Bayesian Players." Management Science 14 (1967-1968):159-182, 320-334, 486-502.

Kamien, M. I. and Tauman, Y. "The Private Value of a Patent: A Game Theoretic Analysis." Journal of Economics 4(1984):93-118.

Kamien, M. I. and Tauman, Y. "Fees Versus Royalties and the Private Value of a Patent." Northwestern University Discussion Paper No. 583, 1984.

Lee, T. "Resource Information Policy and Federal Resource Leasing." Discussion Paper no. 81-11. San Diego: University of California, Department of Economics, 1981.

Li, L. "Cournot Oligopoly with Information Sharing." Social Science Working Paper no. 561. Pasadena: California Institute of Technology, 1985.

Li, L., McKelvey, R. D., and Page, T. "Optimal Research for Cournot Oligopolists." Social Science Working Paper no. 563. Pasadena: California Institute of Technology, 1985.

Matthews, S. "Information Acquisition in Discriminatory Auctions," in

Bayesian Models in Economic Theory, eds. M. Boyer and R. E. Kihlstrom. Elsevier Science Publishers B.V., (1984), p. 181-207.

Mickey, M. R., Mundle, P. M., Walker, D. N. and Glinski, A. M. "Test Criteria for Pearson Type III Distributions." Aeronautical Research Laboratories Office of Aerospace Research, Wright Patterson Air Force Base ARL63-100, Dayton, Ohio, 1963.

Milgrom, P. "A Convergence Theorem for Competitive Bidding with Differential Information." Econometrica 47 (1979):679-688.

Milgrom, P. "Rational Expectation, Information Acquisition, and Competitive Bidding." Econometrica 49 (1981):921-943.

Milgrom, P. and Weber, R. "A Theory of Auctions and Competitive Bidding." Econometrica 50 (1982):1089-1122.

Novshek, W. and Sonnenschein, H. "Fulfilled Expectations Cournot Duopoly with Information Acquisition and Release." Bell Journal of Economics 13 (1982):214-218.

Palfrey, T. "Uncertainty Resolution, Private Information Aggregation, and the Cournot Competitive Limit." Review of Economic Studies (1985), forthcoming.

Vives, X. "Duopoly Information Equilibrium: Cournot and Bertrand." Journal of Economic Theory 34 (1984):71-94.

Wilson, R. B. "A Bidding Model of Perfect Competition." Review of Economic Studies 44 (1977):511-518.