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ABSTRACT

This paper provides a model of both R&D and production in procurement processes where firms invest in R&D and compete for a government procurement contract. The optimal incentive procurement contract is characterized to maximize the government's expected welfare. Explicit consideration of the R&D process changes the standard results in several ways. If the traditional Baron-Myerson (1982) type contract is used where there is costly R&D, the government buys too little from the contractor and pays too little. Raising the price paid encourages private R&D and raises the government's welfare. The form of the optimal procurement contract depends on the number of firms. With R&D and optimal procurement the government prefers more than one firm to invest in R&D and to bid for the production contract. But too much competition may discourage private R&D investment and leave the government worse off. Other features of optimal procurement and R&D expenditures are also discussed.

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I. INTRODUCTION

It is commonly believed that there is a serious cost overrun problem with defense procurements in the United States when a sole supplier is chosen. It is also commonly believed that the introduction of competition among suppliers could generate substantial price reductions. For most defense procurements, a costly research and development (R&D) effort is required of the competitors before production of the product. After a competition, winning contractors may include their R&D costs in the price of the product, but losing contractors must absorb their R&D costs. Thus, requiring competition to reduce costs may actually discourage R&D and lead instead to higher costs. How do we deal with this problem? This paper is concerned with the design of the optimal incentive procurement contract when R&D is an important prior condition to production.

Recent advances in the economic theory of optimal procurement mainly deal with only two issues. The first concerns the selection of the contractors to produce the item being procured at the lowest cost. The item could be manufacturing facilities, new weapons systems or electricity generation plants. The second concerns attempts to induce the chosen contractor to produce the item at the lowest cost. Asymmetry of information between the buyer (the government) and the potential firms is often emphasized. It is usually assumed that potential firms have some private information about production costs at the time they are chosen and that many of decisions made at the production stage by the chosen contractor are too costly for the buyer to observe or audit. These decisions affect the final cost of production and hence influence both the buyer's and the contractor's welfare. Potential contractors are allowed to bid for an incentive contract or bid from a menu of incentive contracts. Since the nature of the item is known by both the buyer and the contractors in advance, incentive procurement contracts are usually designed either to minimize the buyer's total cost for a fixed level of output or to maximize the buyer's surplus when the purchase quantity is variable. In

both cases the contracts also give the firms incentives to reveal their private information and/or make desirable decisions at the production stage. The earlier literature on this topic can be found in Demsetz (1968) and Loeb and Magat (1979). It was extended later on by Baron and Myerson (1982), Laffont and Tirole (1986), McAfee and McMillan (1986, 1987), and others. In particular, Riordan and Sappington (1987) and Dasgupta and Spulber (1987) have argued that because of competition, more efficient outcomes are possible than if there is only a single supplier. Research and development issues have not been analyzed in this literature on procurement. Besen and Terasawa (1987) have recently provided a selective survey of the literature.[1]

At the beginning of many procurements, neither the buyer nor the potential contractors have much information about either the technology or the item itself. Therefore, the buyer must find this information by himself or ask the contractors. For example, United States defense acquisitions are usually characterized by a three-stage process: concept design, development, and production. Most of the time, what the government is buying is not only the item itself, but also information about the item and the technology. Procurements generate information for the government about improving the technical performance and lowering the manufacturing cost. There is usually a tradeoff between encouraging production efficiency and encouraging R&D. These features of procurements make it very difficult to apply the current theory, which only considers the production stage, to many actual cases of defense procurements. We need to link the different stages of the procurement process and to investigate R&D behavior in procurements.

There exists a large literature on R&D races for a patent with a fixed rent in private markets. Among others are Loury (1979), Lee and Wilde (1980), Dasgupta and Stiglitz (1980),[2] and recently Sah and Stiglitz (1987). Using stochastic racing models and dynamic game theoretic models, they emphasize the effects of market structure on private marginal returns to firms from innovations, and investigate the relationship between marginal private returns and social returns from innovations. In particular, Sah and Stiglitz (1987) have provided a set of conditions under which the total expenditure and the pace of innovation in an industry are invariant to the number of firms. These approaches cannot be applied to procurement cases directly because, in procurements, the government is able to manage and control the supplier's R&D behavior indirectly through the choice of the prize for innovation.[3] Traditional R&D models treat the prize as exogenous, but the government can offer a production contract as the prize for which the firms compete. The marginal private returns and social returns from R&D depend on the quantity to be procured which is specified in the production contract. The results of R&D races in private markets may be changed in the procurement case.

We will link the R&D stage and the procurement stage and concentrate on the use of R&D to reduce production costs. The outcome of such R&D is information about the production technology. This information is stochastically related to the R&D effort. We first consider a principal-agent model of the procurement process in Part II. The production stage is characterized by a linear-cost technology with an unknown marginal cost. The R&D outcome is the potential marginal cost which the firm can affect by exerting some effort. The procurement process is modeled as follows: first, the government announces a menu of production contracts and commits itself to offer the same contracts at the latter stage. Second, after observing the general contracts, the firm invests in R&D. Third, the firm observes the marginal cost outcome and selects a contract it

prefers from the contract menu. Finally, the firm produces and gets paid according to the contract it has accepted. We have assumed that the government makes a full commitment before R&D. If the government can not make a credible commitment, both the firm and the government might have incentives to renegotiate after the R&D is done and change to different contracts. We do not consider the renegotiation issue in detail in this paper.[4]

As a benchmark, the first-best solution is briefly discussed in Section II.2. In section II.3, we discuss the nonobservability of the R&D investments and the R&D outcome by the government and characterize the incentive compatible procurement contracts. We show that under any incentive compatible contract, the larger the quantity procured, the more effort the firm exerts in its R&D activity. Thus, increasing the procured amount is one way to encourage R&D activities. Section II.4 characterizes the optimal incentive procurement contract which maximizes the government's expected welfare. Moral hazard exists in this situation due to nonobservability of R&D investment by the government. R&D has an effect on the optimal production level opposite to the adverse selection effect. With costly R&D, the government offers a higher and steeper payment schedule, compared to the traditional Baron-Myerson (1982) contract where R&D is costless. Also, the optimal contract generates positive expected profits for the firm. It is such positive expected profits that encourage the firm to do R&D.

Part III extends the analysis to the case where there are many identical firms competing for procurement contracts. The potential R&D outcomes by different firms are assumed to be independent. We first discuss Nash equilibrium behavior with respect to R&D expenditures given an arbitrary incentive production contract. When the R&D technology exhibits constant marginal returns to expenditures, we find an invariance result similar to Sah and Stiglitz (1987). But when the R&D technology exhibits diminishing marginal returns to expenditures, we find that Sah and Stiglitz's invariance result does not hold and that more competition could reduce the potential marginal production cost to its lower bound. The specific nature of the R&D technology plays an important role in determining the relationship between R&D and the structure of the industry.

We then characterize the optimal incentive production contract and the optimal level of R&D investments for the competition case in Section III.3. When the R&D technology exhibits constant marginal returns to expenditures, we show that the optimum procurement quantity schedule is dependent on the number of competing firms. This contrasts with Dasgupta and Spulber (1987) and Riordan and Sappington (1987) who find that the optimal quantity schedule does not depend on the number of the firms. No R&D behavior was considered in their models. We also find that total expenditure on R&D and the pace of innovation depend on the number of firms because of the buyer's control of the prize for innovation. Thus, Sah and Stiglitz's invariance result does not hold when the prize is endogenous. In other word, the number of firms really matters to the optimal procurement. We also find that the government prefers more than one firm to participate in private R&D activity and to bid for the production contract.

In summary, we have provided a model of both R&D and production in procurement processes where one or more firms invest in private R&D and compete for a government procurement contract. The optimal incentive procurement contract has been characterized to maximize the government's expected welfare. We have found the following interesting results: 1) If the traditional Baron-Myerson (1982) contract is used where there is costly R&D, the government

buys too little from the contractor and pays too little. Raising the price paid encourages private R&D investment and raises the government's expected welfare. 2) The contractor earns positive expected profits under the optimal incentive contract. It is such positive profits that encourage the firm to invest in private R&D. 3) Unlike the invariance results found by Riordan and Sappington (1987) and Sah and Stiglitz (1987), the optimal incentive production contract, the total equilibrium expenditure on R&D, and the pace of innovation in our model depend on the number of competing firms in the industry. 4) The government prefers more than one firm to invest in private R&D and to bid for the production contract. But too much competition may discourage private R&D investment and lead to a reduction in government welfare.

Before proceeding to our formal analysis, we would like to discuss several recent papers which have tried to link R&D and production in procurements. Rob (1986) has included the R&D process in his model of procurement contracts by viewing R&D activity as searching behavior. The optimal stopping rule allows him to derive the average actual production cost which depends on the unit searching cost and the cutoff level. The government prespecifies a quantity and price for the project to minimize its outlay on the project. The chosen contractor agrees to disclose the technological information that results from its R&D effort, which is the key assumption in his model. As a result, it is optimal for the government to award only a fraction of the project to a single supplier while the remainder is competitively purchased. This is so called "educational" or "learning" buy. We will see in Section III.1 that Rob's model of R&D behavior as a search process can be viewed as a special case of our model. The R&D technology in his paper exhibits constant marginal returns to expenditures.

Besen and Terasawa (1988) have also developed a simple and different model of research and development and production which captures some features of defense acquisitions. In their model, the level of technical performance and the amount of hardware to be procured are fixed. In addition, the target cost in the production contract will be the maximum level acceptable to the government. The contracts in both stages are linear functions of target cost and actual cost, and are exogenously given. They find that production will not be carried out efficiently, that cost overruns will be commonplace, and that contractors can be expected to incur losses during research and development. Our paper differs from theirs in that we will design the optimal incentive procurement contract instead of assuming a linear contract exogenously.

Dasgupta (1987) also considers a two-period procurement model with one buyer and n identical firms. Given a second-period sealed-bid auction, each firm chooses an investment level and the buyer chooses the reserve price simultaneously. At the Nash equilibrium, there is underinvestment relative to the social optimum (cooperative solution) because of "opportunistic" behavior. But for most defense procurements, the government and suppliers need not move simultaneously. In our model, we consider a Stackelberg game in which the government is the leader. In this case the government has indirect control of the supplier's R&D decisions through the optimal choice of production contracts.

II. A PRINCIPAL-AGENT MODEL

II.1. The Model

We model (defense) procurement as a two-stage process. In the first stage, research and development (R&D) is conducted by a firm, but no goods are procured or acquired. The outcome of this stage is better knowledge about the cost of producing the item. In the second stage, the item is produced by the firm. The output of the item can be observed by both the firm and government. We assume that there is no uncertainty in the production stage.[5] We will refer to the government as the principal and the firm as the agent in this Part of the paper.

We first consider a single agent. The agent produces a good at a constant marginal cost which is unknown to both the agent and the principal in the first stage. But the agent can take an action x in the R&D process and find out what the marginal cost is, where x can be either money spent or a level of effort such as assigning the best engineers in the firm to this R&D project. Let $x \in [0, \bar{x}]$, where $\bar{x} > 0$ is the budget. The R&D outcome (marginal cost) is uncertain, so we represent it by a random variable Y. We assume that Y is generated according to the following production function:

$$Y = f(x, \Omega),$$

where Ω is a random variable with known support, $Pr(\Omega \leq \omega) = G(\omega)$, $f_1 \leq 0$, $f_{12} \neq 0$, $f_{11} \geq 0$. The realized R&D output is $y = f(x, \omega)$. The more effort the agent spends, the lower the marginal cost it may find.

Individual actions directed towards innovation cannot be observed by the principal and hence cannot be contracted upon. The agent may not have an incentive to invest much in the R&D stage. Thus, there may exist a moral hazard problem due to the incongruity of the incentives of the agent and the principal.

Both the principal and the agent are assumed to be risk-neutral. Thus the problem of risk-sharing is not an issue in this paper. The utility for the principal from procuring an amount Q and paying P will be $W = B(Q) - P$, where P and Q are nonnegative real variables and $B(Q)$ is the benefit function, $B'(Q) > 0$, $B''(Q) < 0$. The utility U for the agent is a function of the income R (or the profit $R = P - yQ$) and the level of effort x, which is assumed to be additively separable. That is, $U = R - cx$, where cx is the R&D cost,[6] $c \geq 0$ is a known constant. Everything except ω and x is common knowledge.

The timing of the game is the following. First, the principal announces a menu of general contracts and commits itself to offer the same contract at the latter stage. Secondly, after observing the general contracts, the agent invests in R&D. Third, the agent observes the R&D outcomes and selects a contract it prefers from the contract menu (announces its type). Finally, the agent produces and gets paid according to the contract it has accepted (see Figure 1).

(Figure 1 here)

Since it is too costly for the principal to observe or audit the R&D output, the agent may not want to submit true information about the R&D output. This asymmetry of information results in another incentive problem. By the revelation principle (see the Appendix), we can concentrate on

the mechanisms for which the principal asks the agent to report its R&D result and which lead the agent to report its private information truthfully. Based on this marginal cost information, the principal will offer the agent a procurement contract which specifies an amount $Q(y)$ to be procured and a total payment $P(y)$ to the agent.

To proceed formally, we introduce the following notation. Since Y is a random variable, let

$$H(y \mid x) \equiv Pr(Y \leq y) = \int_{f(x, \omega) \leq y} dG(\omega)$$

be the cumulative distribution of Y and and $h(y \mid x) \equiv H_y(y \mid x)$ be the associated density with the support $[\underline{y}, \bar{y}]$, $0 \leq \underline{y} \leq \bar{y}$. We assume the support does not move with x. We also assume that for every $x \in (0, \bar{x})$, $H_x(y \mid x) > 0$ for all $y \in (\underline{y}, \bar{y})$, so that a change in x has a nontrivial effect on the distribution of y. Specially, it will shift the distribution of y to the left in the sense of first-order stochastic dominance (see Figure 2). The more effort the agent spends, the higher the probability that the agent will find a marginal cost less than y. It is also assumed that $H_{xx} < 0$ for all $x \in (0, \bar{x})$ and $y \in (\underline{y}, \bar{y})$, and that $H(\bar{y} \mid x) = 1$ for any x, so $\bar{y}$ represents the current common knowledge of the marginal cost. That is, both the agent and principal know that the good can be produced at the marginal cost $\bar{y}$. We also assume that $H(y \mid 0) = 0$ for all $y < \bar{y}$. If the agent does not spend any effort on R&D, the current marginal cost $\bar{y}$ won't be reduced. The principal can always procure $\bar{Q} \geq 0$ at the marginal cost $\bar{y}$ such that $B'(\bar{Q}) = \bar{y}$. The principal gets the total surplus $\bar{S} = B(\bar{Q}) - \bar{y} \, \bar{Q}$ while the agent earns zero. It is easy to see that $\bar{S} \geq 0$ since $B(Q)$ is concave function. We can assume $\bar{Q} > 0$ for simplicity.

(Figure 2 here)

An example of a distribution that satisfies the assumptions is $H(y \mid x) = 1 - (1 - y)^x$, $y \in [0, 1]$, $x \in [0, \bar{x}]$. Let x be the integer number of times the agent draws its cost from a uniform distribution on $[0, 1]$, then $H(y \mid x)$ represents the distribution of the lowest-order statistic. That is, the agent chooses the lowest cost from the x drawings. In this case, the R&D process is like searching behavior and thus the optimal investment level x is the optimal stopping level of the drawing. This is similar to the optimal stopping approach of Rob (1986). Another example of a distribution that satisfies our assumptions is $H(y \mid x) = 1 - e^{-\alpha(x)y}$, $y \in [0, \infty)$, and $x \geq 0$, where $\alpha(x)$ is a positive, increasing, and concave function. Here, the R&D outcome is subject to an exponential distribution and the R&D technology exhibits decreasing marginal returns to expenditures. Since $E(Y \mid x) = 1 / \alpha(x)$, the more effort the agent exerts, the lower the expected marginal cost the agent observes. This is the distribution of innovation that appears in the literature on stochastic R&D races (see Loury 1979, Reinganum 1988, and others). The difference is that in their models y represents the uncertain date at which the R&D project will be successfully completed.

II.2. The First-Best Solution

Before analyzing incentive procurement contracts, we look at the first-best solution. Suppose that the effort x is observable to both the principal and the agent, and the R&D outcome y can also be observed by both parties at the end of the R&D stage. A Pareto-optimum $[Q^*(y), x^*]$

can be computed by maximizing the principal's expected welfare given that the agent's expected profit is no less than a certain level $\bar{\pi}$. We assume $\bar{\pi} = 0$ without loss of generality. The first order conditions give the following equations:

$$B'(Q^*(y)) = y \tag{1}$$

$$\int_{\underline{y}}^{y} Q^*(y) H_x(y \mid x^*) dy - c = 0 \tag{2}$$

Since the R&D output is observable, the principal can procure efficiently. That is, the procurement amount Q^* is chosen such that the marginal benefit equals the marginal cost. For this optimal quantity $Q^*(y)$, the marginal social benefit of investment x, which is $\int^y Q^*(y) H_x(y \mid x^*) dy$, equals the marginal cost of the investment x. Therefore, we have both production efficiency and investment efficiency when investment and R&D outcomes are observable to both parties. The optimal payment transfer $P^*(y)$ to the agent is determined such that the agent earns exactly zero profit. The principal gets the total surplus $\bar{S} + S^*$, where $S^* = \int_{\underline{y}}^{y} Q^*(y) H(y \mid x^*) dy - cx^*$ is the surplus from the R&D under the efficient arrangement of production. We assume that S^* is positive, that is to say, the R&D is meaningful.

II.3. Unobservable R&D Investment and R&D Outcomes

It is a common phenomenon that R&D outcomes cannot be observed or verified by the principal directly. It is also costly for the principal to audit the agent to get this information. Thus, each agent has private information about the R&D outcome and reports this information strategically (e.g. the agent may not tell the truth). This results in an adverse selection problem. Also, the principal can not observe the agent's investment level. This results in a moral hazard problem.

If the principal only cares about efficiency, then he can delegate the production decision and R&D decision to the agent and allow the agent to keep the entire social surplus. Loeb and Magat (1979) have discussed this situation in detail without R&D. The result is also true in the case of costly R&D. The agent will choose quantity and investment level for R&D to maximize the sum of its expected profits and the principal's expected welfare. Since both the agent and the principal are risk neutral, this full delegation results in the first-best solution. The principal does not get any benefit. This solution is not employed in practice for obvious political reasons.

If the principal's objective is to maximize its own expected welfare, the full delegation of decisions is not optimal. The principal should behave monoposonistically. An incentive procurement contract is needed to maximize the principal's expected welfare such that the agent wants to reveal its private information and to invest in R&D. By the revelation principle,[7] we need only to consider incentive compatible direct revelation mechanisms. Given a revelation contract $[Q(y), P(y)]$, the agent chooses effort x and gets a realized R&D output $y = f(x, \omega)$. Then the agent reports y' which depends on y: we can denote $y' = \phi(y)$. The expected utility for the agent if it chooses $\phi(y)$ and x is

$$EU(\phi, y; x) = \int_{\underline{y}}^{\bar{y}} \left[P(\phi(y)) - yQ(\phi(y)) \right] h(y \mid x) dy - cx,$$

where $R(y, y') = P(y') - yQ(y')$ is the profit for the agent from reporting y' given that y is the true R&D output. The agent will choose its R&D strategy x and its reporting strategy $\phi(y)$ to maximize its expected utility.

We can consider the agent's optimal choice of the reporting strategy $\phi(y)$ first. Given any x, the agent will choose ϕ to maximize the above expected utility subject to $\phi(y) \in [\underline{y}, \bar{y}]$ for any $y \in [\underline{y}, \bar{y}]$. We are interested in incentive compatible contracts $[Q(y), P(y)]$ which give the agent an incentive to report the true R&D outcome. That is, we want $\phi(y) = y$ for all y to be the agent's optimal strategy under this contract $[Q(y), P(y)]$. Solving this simple optimal control problem, we obtain the following

Lemma 1: A contract $[Q(y), P(y)]$ is incentive compatible if and only if $Q'(y) \leq 0$ and $R'(y) = -Q(y)$ for all $y \in [\underline{y}, \bar{y}]$, where $R(y) = P(y) - yQ(y)$.

The proof is standard and is given in the Appendix. Lemma 1 offers a necessary and sufficient condition for a production contract $[Q(y), P(y)]$ to be incentive compatible. We can see that incentive compatibility requires the monotonicity of $Q(y)$ and $P(y)$. The lower marginal cost the agent finds, the bigger the procurement amount it will produce, and the higher the payment it gets. We also have a decreasing, convex compensation rule $R(y)$. Reducing the marginal cost results in a greater profit share at an increasing rate. This gives the agent an incentive both to reduce the marginal cost and to report the true R&D outcome. From Lemma 1, we can also calculate the compensation rule $R(y) = R(\bar{y}) + \int_y^{\bar{y}} Q(\tilde{y}) d\tilde{y}$ for any y. If $Q(y) \geq \tilde{Q}(y)$ for all y and $R(\bar{y}) \geq \tilde{R}(\bar{y})$, then $R(y) \geq \tilde{R}(y)$ for any $y < \bar{y}$. Thus, under an incentive contract, a higher procurement amount generally gives the agent a higher profit share.

We now consider the agent's optimal R&D strategy. Let $EU(x)$ be the agent's expected utility under the truthful reporting of the R&D output given the effort level x. That is,

$$EU(x) = \int_{\underline{y}}^{\bar{y}} R(y) h(y \mid x) dy - cx.$$

Integrating the right hand side by parts and using Lemma 1, we get

$$EU(x) = R(\bar{y}) + \int_{\underline{y}}^{\bar{y}} Q(y) H(y \mid x) dy - cx.$$

At the beginning of the R&D stage, the agent chooses an investment level $\tilde{x} \in [0, \bar{x}]$ to maximize ex ante profit $EU(\tilde{x})$. That is, the optimal investment strategy x satisfies

$$x \in \underset{\tilde{x} \in [0, \bar{x}]}{\text{argmax}} \quad R(\bar{y}) + \int_{\underline{y}}^{\bar{y}} Q(y) H(y \mid \tilde{x}) dy - c\tilde{x}. \tag{3}$$

Assume that Inada's 'derivative conditions' are satisfied; that is, $\lim_{x \to 0} H_x(y \mid x) = +\infty$ and $\lim_{x \to \bar{x}} H_x(y \mid x) = 0$ for all $y \in (\underline{y}, \bar{y})$. Then for any $c > 0$ and $Q(y)$ such that $Q(y) \geq 0$ for all y and $Q(y) > 0$ for a nonzero measure subset of $[\underline{y}, \bar{y}]$, the solutions for (3) are interior solutions. The Inada conditions mean that there is a great potential to increase the probability of finding the marginal production cost less than y in the initial investment, and the increasing rate of that probability diminishes when the investment level reaches a certain upper bound $\bar{x}$. Thus, a certain level of investment between zero and the upper bound $\bar{x}$ is optimal for the agent.

For these interior solutions of (3), the first order condition gives

$$\int_{\underline{y}}^{\bar{y}} Q(y) H_x(y \mid x) dy - c = 0, \tag{4}$$

$x \in (0, \bar{x})$. Since we assume that $H_{xx} < 0$ for $y \in (\underline{y}, \bar{y})$, $EU(x)$ is strictly concave in x if $Q(y) \geq 0$ and $Q(y) \neq 0$ on a nonzero measure subset of $[\underline{y}, \bar{y}]$. The second order condition for (3) is satisfied and the interior solution of (4) is unique. Therefore, we can use the first order approach and substitute (4) for (3). In the extreme case where $c = 0$, the solution for (3) is the boundary $\bar{x}$. Remember $\bar{x}$ is known by the principal.

Before considering the optimal procurement contract, we discuss the effect of the quantity $Q(y)$ on the R&D strategy. Given $Q(y) \geq 0$ and $Q(y) \neq 0$, let $x^*(Q)$ be the solution for (4) and $EU = EU(x^*(Q))$ be the agent's expected utility under $[Q(y), x^*(Q)]$. Then we can show the following

Lemma 2: Given any two incentive compatible contracts $[Q(y), P(y)]$ and $[\tilde{Q}(y), \tilde{P}(y)]$, suppose $Q(y) \geq \tilde{Q}(y) \geq 0$ for all $y \in [\underline{y}, \bar{y}]$. Then i) $x^*(Q) \geq x^*(\tilde{Q})$, and ii) if $R(\bar{y}) = \tilde{R}(\bar{y})$ then $EU \geq E\tilde{U}$.

The proof is by contradiction and is given in the Appendix. Lemma 2 implies that a bigger procurement project, in the sense of higher Q at every y, makes the agent invest more in the R&D process and earn a higher expected profit. The agent always prefers a larger sized project. Therefore, a simple way to encourage R&D is to increase the size of the project or to procure more from the same agent. Another obvious implication of Lemma 2 is that, relative to the first-best solution, underprocurement results in underinvestment in R&D.

II.4. Optimal Incentive Procurement Contracts

Now, we are ready to look at the principal's optimization problem and characterize optimal incentive procurement contracts when the principal's objective is to maximize its own expected welfare. The principal will choose $Q(y)$, $P(y)$, and x to maximize its expected welfare subject to the incentive compatibility constraint, the ex ante and interim individual rationality constraints, and the agent's R&D decision constraint. We will initially ignore the global incentive compatibility constraint $Q'(y) \leq 0$. If the final solution does not satisfy this global incentive constraint, we need to use Guesnerie and Laffont's (1984) technique to get the optimal incentive contract. We do not repeat their arguments here. Under the local incentive compatibility constraint, the payment $P(y)$

can be solved in terms of $Q(y)$ by using $P(y) = yQ(y) + R(y)$ and $R(y) = R(\bar{y}) + \int_y^{\bar{y}} Q(\tilde{y})d\tilde{y}$. Thus, the optimal incentive contract is determined by solving the following optimization problem:

$$(P) \quad \text{Max} \quad \int_{\underline{y}}^{\bar{y}} \left[B(Q(y)) - Q(y)\left[y + \frac{H(y|x)}{h(y|x)} \right] \right] h(y|x)dy - R(\bar{y})$$

$$Q(y), R(\bar{y}), x$$

$$s.t. \quad (IIR) \quad R(\bar{y}) + \int_{\underline{y}}^{\bar{y}} Q(\tilde{y})d\tilde{y} \geq 0$$

$$(R\&D) \quad \int_{\underline{y}}^{\bar{y}} Q(y)H_x(y|x)dy - c = 0 \tag{4}$$

$$(EIR) \quad \int_{\underline{y}}^{\bar{y}} Q(y)H(y|x)dy + R(\bar{y}) - cx \geq 0 \tag{5}$$

The principal will choose $R(\bar{y})$ as low as possible under the interim individual rationality constraint (IIR). In other word, $R(\bar{y}) = 0$. The agent earns zero profit if its marginal production cost is the publicly known $\bar{y}$. The principal offers a contract $[\bar{P}, \bar{Q}]$ and gets the social surplus $\bar{S} = B(\bar{Q}) - \bar{y}\,\bar{Q}$, where $\bar{P} = \bar{y}\,\bar{Q}$ and $\bar{Q}$ is determined by $B'(\bar{Q}) = \bar{y}$.

Since the objective function EW is concave and differentiable in $Q(y)$ and x, and since EU is linear in $Q(y)$, concave and differentiable in x, the sufficient conditions for (P) are satisfied (see Theorem 8.C.5 in Takayama 1985). Let $\hat{Q}(y)$, $R(\bar{y})$, and $\hat{x}$ be the solution to the optimization problem (P) and $E\hat{U} = \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)H(y|\hat{x})dy + R(\bar{y}) - c\hat{x}$ be the agent's expected profit under the optimum contract. Then, because of the (IIR) constraint, $\hat{R}(\bar{y}) = 0$. We can show the following:

Proposition 1: Suppose $c > 0$, then $0 < \hat{x} < \bar{x}$ and $E\hat{U} > 0$.

Proof: First, we want to show $\hat{x} > 0$. Suppose $\hat{x} = 0$. Then $E\bar{W} = \bar{S}$. We know $Q^*(y) > 0$ for all y and $Q^*(y)$ is decreasing in y. There exists at least one $\tilde{Q}(y) > 0$ such that $\tilde{Q}(y)$ is decreasing in y and a little lower than $Q^*(y)$ for $y < \bar{y}$ and the same as $Q^*(y)$ at $y = \bar{y}$. Then $B'(\tilde{Q}(y)) > B'(Q^*(y)) = y$ because $B(Q$ is strictly concave. Let $\tilde{x}$ satisfy $\int_{\underline{y}}^{\bar{y}} \tilde{Q}(y)H_x(y|\tilde{x})dy = c0$ and $\int_{\underline{y}}^{\bar{y}} \tilde{Q}(y)H(y|\tilde{x})dy - c\tilde{x} \geq 0$. Then $\tilde{x} > 0$ since $\lim_{x\to 0} H_x(y|x) = +\infty$. Thus,

$$E\bar{W} = \bar{S} - \int_{\underline{y}}^{\bar{y}} \left[B'(\tilde{Q}(y)) - y \right] \tilde{Q}'(y)H(y|\tilde{x})dy > \bar{S} = E\hat{W}.$$

That is, the principal could choose $[\tilde{Q}(y), \tilde{x}]$ which gives him higher welfare than $\bar{S}$. Thus, $\hat{x} = 0$ cannot be the optimal solution to (P). Therefore, $\hat{x} > 0$.

Second, we claim $\hat{Q}(y)$ is positive at least on a non-zero measure subset of $[\underline{y}, \bar{y}]$. Otherwise, we will have $\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)H_x(y|\hat{x})dy = 0$. Then from the $(R\&D)$ constraint, $c\hat{x} = 0$ and hence $c = 0$. This contradicts the assumption $c > 0$. Thus the claim holds. Furthermore, using Inada's

'derivative conditions' and the assumption $c > 0$, we know $x < \bar{x}$.

Finally, we can show $E\hat{U} > 0$. From the (EIR) constraint, $E\hat{U} \geq 0$. We only need to show $E\hat{U} \neq 0$. Suppose $E\hat{U} = 0$. Let $\delta(x) = \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)H(y \mid x)dy - cx$, then $\delta(x)$ is continuous over $[0, \bar{x}]$, $\delta(\hat{x}) = 0$, and $\delta'(\hat{x}) = 0$. Since $\hat{Q}(y) \geq 0$ for all y and $\hat{Q}(y) > 0$ on a nonzero measure subset, and since $H_{xx} < 0$ for all $x \in (0, \bar{x})$ and $y \in (\underline{y}, \bar{y})$, $\delta(x)$ is strictly concave in x. Thus, $\hat{x}$ is a maximum point of $\delta(x)$. For $x \in (0, \hat{x}]$, $\delta(x) \leq \delta(\hat{x}) = 0$, and $\delta(0) = 0$. In summary, we obtain $\delta(\hat{x}) = 0$, $\delta(0) = 0$, and $\delta(x) \leq 0$ for any $x \in (0, \hat{x})$. These together contradict the continuity and strict concavity of $\delta(x)$. Therefore, $E\hat{U} = \delta(\hat{x}) > 0$.

Q.E.D.

The principal offers a production contract which allows the agent to invest a positive amount in R&D and to earn positive expected profits . This is a nice way to reward the agent for innovation since the R&D outcome cannot be observed or verified directly by the principal. It is easy to show that the result in Proposition 1 will not be true when either the R&D investment or the R&D outcome are observable to the principal. The principal will be able to extract the full surplus from the agent in these cases. Therefore, it is the interaction of the non-observability of the R&D investment and non-observability of the R&D outcome by the principal that allows the risk-neutral agent to earn positive profits.

The interim individual rationality (IIR) constraint also played a key role in Proposition 1. If the (IIR) constraint is relaxed, $R(\bar{y})$ can be negative. Since the choice of $R(\bar{y})$ does not affect the solution of $Q(y)$ and x, the principal could choose $R(\bar{y}) = \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)H(y \mid \hat{x})dy - c\hat{x}$. The agent earns a zero expected profit. The principal extracts the whole surplus. The agent with a higher cost observation will end up with a negative ex post profit if it accepts the contract, and hence will drop out before production unless the principal can force the agent to produce.

Let λ and μ be the multipliers associated with the constraints (EIR) and $(R\&D)$, respectively. At the optimum $[\hat{Q}(y), \hat{R}(\bar{y}), \hat{x}]$, $\lambda E\hat{U} = 0$. From Proposition 1, $E\hat{U} > 0$. Thus, $\lambda = 0$. Then the optimal quantity $\hat{Q}(y)$, optimal investment level $\hat{x}$, and $\hat{\mu}$ are simultaneously determined by the following equations:

$$\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)H_x(y \mid \hat{x})dy - c = 0, \tag{4}$$

$$B'(\hat{Q}(y)) - y = \frac{H(y \mid \hat{x})}{h(y \mid \hat{x})} - \hat{\mu}\frac{H_x(y \mid \hat{x})}{h(y \mid \hat{x})}, \tag{6}$$

$$\int_{\underline{y}}^{\bar{y}} \left[B'(\hat{Q}(y)) - y \right] \hat{Q}'(y)H_x(y \mid \hat{x})dy = \hat{\mu}\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)H_{xx}(y \mid \hat{x})dy. \tag{7}$$

Consider the extreme case when $c = 0$. In this case, R&D is costless. The more the agent invests, the higher profit it earns because the marginal benefit of investment is positive. Thus, the agent will invest the upper bound level $\bar{x}$ in R&D, which is known by the principal. The $(R\&D)$

constraint in (P) is then not binding. The agent observes y from the distribution $H(y \mid \bar{x})$ without any cost. Then, the optimal procurement amount $\hat{Q}_0(y)$ is determined by

$$B'(\hat{Q}_0(y)) = y + \frac{H(y \mid \bar{x})}{h(y \mid \bar{x})}. \tag{8}$$

This is exactly the Baron and Myerson (1982) solution. Because of information asymmetry, there is an information cost $H(y \mid \bar{x}) / h(y \mid \bar{x})$ paid by the principal under the optimal incentive contract in order to induce the agent to reveal its private information y. The principal chooses the quantity $\hat{Q}_0(y)$ such that the marginal benefit of the quantity equals the marginal production cost plus the marginal information cost. There is an adverse selection effect (also see Baron and Myerson 1982 in the regulation context). This effect results in underprocurement as compared to the case where y is observable to the principal.

Suppose $c > 0$; that is, R&D is costly. The agent invests in R&D to balance the benefits and costs. The $(R\&D)$ constraint is binding in this case. Formally, we have

Lemma 3: Suppose $H_x(y \mid x) > 0$, $1 + \frac{\partial(H/h)}{\partial y} \geq 0$, and $\frac{\partial(H_x/h)}{\partial y} \geq 0$ for all $x \in (0, \bar{x})$ and $y \in (\underline{y}, \bar{y})$. Then $\hat{\mu} > 0$.

Proof: The proof is by contradiction. If $\hat{\mu} \leq 0$, then from (6), we obtain $B'(Q(y)) - y > 0$, and

$$B''(\hat{Q}(y))\hat{Q}'(y) = 1 + \frac{\partial(H/h)}{\partial y} - \hat{\mu}\frac{\partial(H_x/h)}{\partial y}$$

which implies that $\hat{Q}'(y) < 0$ by the assumptions. Given $\hat{Q}(y)$, taking the derivative of EW with respect to x and integrating by parts, we get

$$\frac{\partial EW}{\partial x}\Big|_{\hat{x}} = -\int_{\underline{y}}^{\bar{y}} \left[B'(\hat{Q}(y)) - y \right] \hat{Q}'(y) H_x(y \mid \hat{x}) dy.$$

It is easy to see this term is positive because $H_x > 0$ and $\hat{Q}'(y) < 0$. Since $H_{xx} < 0$, then equation (7) implies that $\hat{\mu}$ should be positive. This contradicts the previous hypothesis. Thus $\hat{\mu} > 0$ and $\frac{\partial EW}{\partial x}\Big|_{\hat{x}} > 0$ by (7).

Q.E.D.

The principal wants more R&D but can not control x. The preferences for investment are not consistent between the agent and the principal. Moral hazard exists because the principal can not observe the agent's investment decision. Since $H_x(y \mid \hat{x}) > 0$ for any $y \in (\underline{y}, \bar{y})$, then $-\hat{\mu}\frac{H_x(y \mid \hat{x})}{h(y \mid \hat{x})} < 0$. This negative term (moral hazard) has an opposite effect on $\hat{Q}(y)$ to the adverse selection effect

represented by the information cost term $\dfrac{H(y\,|\,\hat{x})}{h(y\,|\,\hat{x})}$. From (6), we can see that the combination of the two opposite effects determines the extent to which production is carried out inefficiently (see Figure 3).

(Figure 3 here)

Since $\hat{\mu}$ is the multiplier for the $(R\&D)$ constraint, it can be easily shown that $\dfrac{\partial E\hat{W}(c)}{\partial c} = -\hat{\mu}$ for $c > 0$, where $E\hat{W}(c)$ is the principal's expected welfare at the optimal solution $[\hat{Q}(y), R\,(\bar{y}), \hat{x}]$. Under the conditions in Lemma 3, $\hat{\mu} > 0$ and hence $\dfrac{\partial E\hat{W}(c)}{\partial c} < 0$. Using the optimal contract, the more costly the R&D is, the worse-off the principal is. Thus, the principal actually pays indirectly for part of the R&D cost.

Now, we compare these two cases: $c = 0$ and $c > 0$. In the following, we let $\hat{Q}(y)$ and $\hat{x}$ be the solution to (P) when $c > 0$. We have

Proposition 2: Suppose $\dfrac{\partial(H/h)}{\partial x} \geq 0$ for any $x \in (0, \bar{x})$ and $y \in [\underline{y}, \bar{y}]$, and the assumptions in Lemma 3 hold. Then i) $\hat{Q}(y) > \hat{Q}_0(y)$ for any $y \in (\underline{y}, \bar{y})$, $\hat{Q}(\underline{y}) = \hat{Q}_0(\underline{y})$, and $\hat{Q}(\bar{y}) \geq \hat{Q}_0(\bar{y})$; ii) $\hat{P}(y) > \hat{P}_0(y)$ for any $y \in [\underline{y}, \bar{y})$ and $\hat{P}(\bar{y}) \geq \hat{P}_0(\bar{y})$.

Proof: From Proposition 1, $0 < \hat{x} < \bar{x}$. Since $\dfrac{\partial(H/h)}{\partial x} \geq 0$ by the assumption, then

$$\frac{H(y\,|\,\bar{x})}{h(y\,|\,\bar{x})} \geq \frac{H(y\,|\,\hat{x})}{h(y\,|\,\hat{x})} > \frac{H(y\,|\,\hat{x})}{h(y\,|\,\hat{x})} - \hat{\mu}\frac{H_x(y\,|\,\hat{x})}{h(y\,|\,\hat{x})}$$

for all $y \in (\underline{y}, \bar{y})$. Comparing (6) with (8), we know $B'(\hat{Q}(y)) < B'(\hat{Q}_0(y))$ and hence $\hat{Q}(y) > \hat{Q}_0(y)$ for all $y \in (\underline{y}, \bar{y})$ since $B(Q)$ is strictly concave. When $y = \underline{y}$, $B'(\hat{Q}(\underline{y})) = B'(\hat{Q}_0(\underline{y}))$ from (6) and (8) and thus $\hat{Q}(\underline{y}) = \hat{Q}_0(\underline{y})$. When $y = \bar{y}$, $H(\bar{y}\,|\,\bar{x}) / h(\bar{y}\,|\,\bar{x}) \geq H(\bar{y}\,|\,\hat{x}) / h(\bar{y}\,|\,\hat{x})$ form the assumption. Comparing (6) with (8), we obtain $\hat{Q}(\bar{y}) \geq \hat{Q}_0(\bar{y})$.

Since $\hat{P}(y) = y\hat{Q}(y) + \int_y^{\bar{y}} \hat{Q}(\tilde{y})d\tilde{y}$ and $\hat{P}_0(y) = y\hat{Q}_0(y) + \int_y^{\bar{y}} \hat{Q}_0(\tilde{y})d\tilde{y}$, then it is easy to see $\hat{P}(y) > \hat{P}_0(y)$ for any $y \in [\underline{y}, \bar{y})$ and $\hat{P}(\bar{y}) \geq \hat{P}_0(\bar{y})$.

Q.E.D.

The assumption $\dfrac{\partial(H/h)}{\partial x} \geq 0$ in Proposition 2 means that the hazard rate H/h faced by the principal due to the nonobservability of R&D investment increases when the agent increases the level of investment. In this case, if R&D is costly, the principal offers a higher quantity schedule and a higher payment schedule as well. The principal prefers a relatively bigger production project if R&D is costly.

When $\hat{Q}'(y) < 0$ for every $y \in (\underline{y}, \bar{y})$, there exists an inverse function $y = \hat{Q}^{-1}(Q)$. Then the payment $\hat{P}(y)$ can be written as $P = \hat{P}(\hat{Q}^{-1}(Q)) \equiv P(Q)$ given any procurement amount Q. The agent

will reveal its cost information y by choosing the quantity Q. A separating incentive procurement contract $[\hat{Q}(y), \hat{P}(y)]$ can be implemented by a simple nonlinear payment schedule $P = P(Q)$ under which the agent chooses the quantity to produce. When $\hat{Q}'(y) = 0$ over a subinterval of $[\underline{y}, \bar{y}]$, the optimal contract specifies pooling. The principal can not distinguish the different types and offers the same production contract.

Compare two seperating contracts $[\hat{Q}(y), \hat{P}(y)]$ and $[\hat{Q}_0(y), \hat{P}_0(y)]$. Let $P(Q) \equiv \hat{P}(\hat{Q}^{-1}(Q))$, $P_0(Q) \equiv \hat{P}_0(\hat{Q}_0^{-1}(Q))$, $Q_h \equiv \hat{Q}(\underline{y})$, and $Q_l \equiv \hat{Q}(\bar{y})$, then

Proposition 3: Suppose the assumptions in Proposition 2 hold, then $P'(Q) > P'_0(Q)$ for any $Q \in (Q_l, Q_h)$ and $P(Q) > P_0(Q)$ for any $Q \in [Q_l, Q_h]$.

Proof: First, we show $P'(Q) > P'_0(Q) > 0$ for any $Q \in (Q_l, Q_h)$. By the definition of $P(Q)$ and the self-selection property (or incentive compatibility), we obtain

$$P'(Q) = \frac{\hat{P}'(\hat{Q}^{-1}(Q))}{\hat{Q}'(\hat{Q}^{-1}(Q))} = \hat{Q}^{-1}(Q).$$

Similarly, $P'_0(Q) = \hat{Q}_0^{-1}(Q) > 0$. From Proposition 2, $\hat{Q}^{-1}(Q) > \hat{Q}_0^{-1}(Q)$ for any $Q \in (Q_l, Q_h)$. Thus, $P'(Q) > P'_0(Q) > 0$ for any $Q \in (Q_l, Q_h)$.

Second, we prove $P(Q) > P_0(Q)$ for any $Q \in [Q_l, Q_h]$. Since both $P(Q)$ and $P_0(Q)$ are continuous and increasing in Q, and since $P(Q)$ is steeper than $P_0(Q)$, we only need to show $P(Q_h) > P_0(Q_h)$ and $P(Q_l) > P_0(Q_l)$. In fact, since $\hat{Q}^{-1}(Q_h) = \underline{y}$, we have $P(Q_h) = \hat{P}(\hat{Q}^{-1}(Q_h)) = \hat{P}(\underline{y})$ and $P_0(Q_h) = \hat{P}_0(\hat{Q}_0^{-1}(\bar{Q})) = \hat{P}_0(\underline{y})$. By Proposition 2, $\hat{P}(\underline{y}) > \hat{P}_0(\underline{y})$ and thus $P(Q_h) > P_0(Q_h)$. Since $Q_l = \hat{Q}(\bar{y})$, then $\bar{\bar{y}} \equiv \hat{Q}_0^{-1}(Q_l) < \bar{y}$ by Proposition 2. Then $P(Q_l) = \hat{P}(\bar{y}) = \bar{y}\hat{Q}(\bar{y}) = \bar{y}Q_l$ and

$$P_0(Q_l) = \hat{P}_0(\bar{\bar{y}}) = \bar{\bar{y}}\hat{Q}_0(\bar{\bar{y}}) + \int_{\underline{y}}^{\bar{\bar{y}}} \hat{Q}_0(y)dy.$$

$$< \bar{y}\hat{Q}_0(\bar{\bar{y}}) + (\bar{y} - \bar{\bar{y}})\hat{Q}_0(\bar{\bar{y}}) = \bar{y}\hat{Q}_0(\bar{\bar{y}}) = \bar{y}Q_l$$

Thus, $P(Q_l) > P_0(Q_l)$.

<div align="right">Q.E.D.</div>

Proposition 3 implies that with more costly R&D the principal should offer a higher and steeper payment schedule (see Figure 4). For any given quantity Q, the principal pays more in the case of costly R&D than that in the case of costless R&D. In other words, if the Baron-Myerson-type contract were used when R&D is costly, the principal would buy too little from the agent and pay too little to the agent. Raising the price paid raises the principal's welfare. Therefore, whether R&D is costly or not certainly affects the principal's decision and the principal's welfare as well. When designing an incentive procurement contract, the principal cannot ignore the agent's private R&D investment behavior.

As we discussed before, because of an adverse selection effect, the asymmetry of information results in an underprocurement relative to the first-best solution. R&D (or moral hazard) has an opposite effect on the determination of procurement quantity to the adverse selection effect. When R&D is costly, the optimal quantity schedule $\hat{Q}(y)$ may still be different from the first-best quantity schedule $Q^*(y)$. To end this section, we will consider a special class of R&D technologies and illustrate how $\hat{Q}(y)$ may be different from the first-best level $Q^*(y)$:

Proposition 4: Suppose $H(y \mid x) = 1 - [1 - F(y)]^x$, where $x \geq 0$, $F(y)$ is an arbitrary cumulative distribution with the support $[\underline{y}, \bar{y}]$ and the density function $f(y)$, and $f(y)/[1 - F(y)]$ is nonincreasing over $(\underline{y}, \bar{y})$. Then

    i) $\hat{\mu} > 0$;

    ii) if $\hat{x} \geq \hat{\mu}$, then $\hat{Q}(y) < Q^*(y)$ for all $y \in (\underline{y}, \bar{y})$;

    iii) if $\hat{x} < \hat{\mu}$, then there exists $y_0 \in (\underline{y}, \bar{y})$ such that $\hat{Q}(y_0) = Q^*(y_0)$, $\hat{Q}(y) > Q^*(y)$
        for $y \in (\underline{y}, y_0)$, and $\hat{Q}(y) < Q^*(y)$ for $y \in (y_0, \bar{y})$.

See the Appendix for the proof. The conditions in ii) and iii) are not primitive conditions. They depend on the structure of the benefit function $B(Q)$ and distribution function $F(y)$. Because of the interaction of the adverse selection effect and the moral hazard effect, there is a possibility that the optimal quantity $\hat{Q}(y)$ is higher than the first-best quantity $Q^*(y)$ for lower marginal costs. But for higher marginal costs, the optimal quantity $\hat{Q}(y)$ is lower than the first-best quantity. That is, there is underprocurement relative to the first-best when the marginal cost is relatively high.

III. COMPETITION FOR PROCUREMENT CONTRACTS

Some defense procurements are organized so that many suppliers compete for the right to be the sole contractor on the production of the good to be procured. In order to get the procurement contract, potential firms will invest in R&D activities and become informed about the potential product and technologies before the competitive bidding starts. How does the procurement contract affect R&D behavior? Does the optimal quantity to be procured depend on the number of bidders? Does the total expenditure on R&D and the pace of innovation depend on the structure of the industry? Does competition improve production efficiency and R&D efficiency? In this part of the paper, we extend the analysis in Part II and discuss the effects of competition on R&D expenditures and procurement contracts. We will first describe a basic model which is an extension of the principal-agent model in Part II, and study the Nash equilibrium behavior of R&D expenditures for an arbitrary incentive procurement contract. Then we characterize the optimal incentive contract and discuss properties of this contract.

III.1. The Model

Suppose that there is one buyer and n firms, where n is exogenous.[8] The benefit function for the buyer $B = B(Q)$ is the same as before. The production cost function for firm i is $C_i(Q) = y_i Q$, where y_i is constant marginal cost and is unknown to the buyer and all firms before R&D. But each firm can observe its own marginal cost by investing in R&D. Suppose that firm i invests (capital) x_i and observes y_i which is drawn from a cumulative distribution $H(y_i | x_i)$ with the density function $h(y_i | x_i)$ and the support $[\underline{y}, \bar{y}]$. The R&D outputs among different firms are independently and identically distributed.[9] The R&D cost for firm i is cx_i, where $c > 0$ is a known constant. Each firm's R&D output is only observed by itself, but not observed by the buyer and other firms. Therefore, each firm has private information about its R&D output and will use this information strategically if it is optimal to do so.

At the beginning of the R&D stage, the buyer announces a payment schedule $Q = Q(b)$ and promises that the firm with the lowest bid $\bar{b}$ by an exogenously given date will get the contract $[Q(\bar{b}), Q(\bar{b})\bar{b}]$.[10] We want to determine the optimal payment schedule $Q = \hat{Q}(b)$ for the buyer. Under this contract, firm i's strategy will be $b_i = b_i(y_i)$, $i = 1, \ldots, n$. Since firms are symmetric and the contract is also symmetric, it is reasonable to consider symmetric equilibrium $b_i = b(y_i)$ only. If as is usually the case, the bidding function $b(y_i)$ is increasing function, then the firm with the lowest bid $\bar{b}$ is the firm with the lowest marginal cost $\bar{y}$. In these cases, we only have to consider incentive compatible direct revelation contracts $[Q(y), P(y)]$, where $P(y) = Q(y)b(y)$ and y is the lowest marginal cost. The timing of the game is then as follows: First, the buyer announces production contracts $[Q(y), P(y)]$ and promises that the firm with the lowest (reported) marginal cost $\bar{y}$ by an exogenously given date will be awarded a contract $[Q(\bar{y}), P(\bar{y})]$. Second, each firm invests and observes its marginal cost y_i at the given date. Then the firm with the lowest marginal cost $\bar{y}$ gets the contract. Finally, the winning firm produces $Q(\bar{y})$ and gets paid by $P(\bar{y})$.

We want truthful reporting to be a noncooperative Nash equilibrium. Given that other firms report true information, firm i reports y_i' which depends on its true information y_i. We denote its strategy as $y_i' = \phi_i(y_i)$. Then the probability of firm i winning is

$$K(y_i' \mid x_{-i}) \equiv Pr(y_i' \leq Y_j, j \neq i, j = 1, \cdots, n)$$

$$= \Pi_{j \neq i} \left[1 - H(y_i' \mid x_j) \right],$$

where Y_j is a random variable and represents firm j's R&D outcome (marginal cost). The ex ante profits for firm i from using strategy $\phi_i(y_i)$ and x_i given that the other firms use x_{-i} and y_{-i} is

$$EU_i(\phi_i(y) \mid x_1, \ldots, x_n) = \int_{\underline{y}}^{\bar{y}} \left[P(\phi_i(y)) - yQ(\phi_i(y)) \right] K(\phi_i(y) \mid x_{-i}) h(y \mid x_i) dy - cx_i.$$

Similar to Lemma 1 in Section II.3, the necessary and sufficient conditions for firm i to tell the truth ($\phi_i(y) = y$ for all y) are that $Q'(y) / Q(y) \leq \sum_{j \neq i} h_j / (1 - H_j)$ and that the payment $P(y)$ is

$$P(y) = yQ(y) + \int_{\underline{y}}^{\bar{y}} Q(\tilde{y}) K(\tilde{y} \mid x_{-i}) d\tilde{y} / K(y \mid x_{-i}). \tag{9}$$

For the truth-telling Nash equilibrium, we can simply write firm i's ex ante profits $EU_i(x_1, \ldots, x_n)$ as

$$EU_i(x_1, \ldots, x_n) = \int_{\underline{y}}^{\bar{y}} Q(y) K(y \mid x_{-i}) H(y \mid x_i) dy - cx_i. \tag{10}$$

We will look at noncooperative Nash equilibrium behavior in R&D expenditures for both an arbitrary incentive contract and the optimal incentive contracts in the next sections. Since most of results will depend on the structure of the R&D technology, we first classify R&D technology:

Condition (A): For any $y \in (\underline{y}, \bar{y})$, $-\log[1 - H(y \mid x)]$ is linear in $x (x \geq 0)$.

Condition (B): For any $y \in (\underline{y}, \bar{y})$, $-\log[1 - H(y \mid x)]$ is strictly concave in $x (x \geq 0)$.

Lemma 4: Condition (A) is equivalent to $H(y \mid x) = 1 - [1 - F(y)]^x$, where $x \geq 0$ and $F(y)$ is an arbitrary cumulative distribution function over $[\underline{y}, \bar{y}]$.

The proof is quite straightforward and is omitted here. Suppose that a firm has a prior distribution $F(y)$ about its marginal production cost y. The firm can do an experiment with a fixed cost c and observe marginal cost y^1 which is drawn from the distribution $F(y)$. If this experiment can be repeated k times independently, then the firm observes a sequence of the marginal cost realizations $(y^1, \ldots, y^k)$. The firm will choose the minimum marginal cost y^m. From statistical theory, y^m is a realization of a random variable (the minimum-order statistic) with the distribution $H(y \mid k) = 1 - [1 - F(y)]^k$. Thus, we have an independent search process. Rob (1986) used a similar

optimal search model of R&D behavior in procurements. In general, we can allow $k = x$ to be an continuous variable and to represent the expenditure on R&D. This R&D process of marginal cost reduction is just an independent search process. The first x dollars have the same effect on the minimum marginal cost as the last x dollars. To some extent, this process is subject to constant marginal returns in the number of experiments or expenditures. Similarly, condition (B) represents R&D processes which exhibit diminishing marginal returns to expenditures. Diminishing marginal returns to scale may be a good description of most R&D processes in reality. A relatively simple form that satisfies (B) is

Condition (B_1): $H(y \mid x) = 1 - [1 - F(y)]^{\alpha(x)}$, where $x \geq 0$, $F(y)$ is an arbitrary cumulative distribution function over $[\underline{y}, \bar{y}]$, $\alpha'(x) > 0$, $\alpha''(x) < 0$, $\alpha(0) = 0$, and $\alpha'(0) = +\infty$.

It is easy to check that (B_1) satisfies condition (B), so a (B_1) technology exhibits diminishing marginal returns to expenditures on R&D. In this case, with the belief that y is drawn from a distribution $F(y)$, the firm could not make an independent experiment and then simply take the lowest marginal cost observation. The later experiments are not as productive as the earlier experiments. Let $F(y) = 1 - e^{-y}$ over $[0, \infty)$, then $H(y \mid x) = 1 - e^{-\alpha(x)y}$ which is also an exponential distribution. This distribution is adopted in the literature on stochastic R&D races (see Reinganum 1988 for a survey). Diminishing marginal returns to scale is also assumed in this literature.

III.2. Arbitrary Incentive Contract

Consider an arbitrary direct revelation (possibly non-optimal) contract $[Q(y), P(y)]$, which is assumed to be independent of n in this section. Under this contract, if truth-telling is a Nash equilibrium, then each firm's expected profits can be written as (10). At the beginning of the R&D stage, each firm chooses an investment level to maximize its expected profits. If Inada's 'derivative conditions' are satisfied, the Nash equilibrium $(x_1, \ldots, x_n)$ satisfies the first order condition:

$$\int_{\underline{y}}^{\bar{y}} Q(y)K(y \mid x_{-i})H_{x_i}(y \mid x_i)dy - c = 0 \tag{11}$$

for $i = 1, 2, \ldots, n$, where H_{x_i} is the derivative of $H(y_i \mid x_i)$ with respect to x_i. Since EU_i is concave in x_i, the second order conditions are satisfied.

Under condition (A), the equilibrium condition (11) becomes

$$-\int_{\underline{y}}^{\bar{y}} Q(y)[1 - F(y)]^x \log[1 - F(y)]dy = c \tag{12}$$

which is the same for all firms, where $x = \sum_{i=1}^{n} x_i$ is the total expenditure on R&D. Given $Q(y)$, the equilibrium condition (12) determines x. Both symmetric and asymmetric Nash equilibria on R&D expenditures exist and the total expenditure $x = \sum_{i=1}^{n} x_i$ determined by (12) is independent of n, the

number of firms. The total expenditure is the same under different equilibria. The expected minimum marginal cost is

$$E(y^m \mid x) = -\int_{\underline{y}}^{\bar{y}} y \, d \prod_{i=1}^{n} [1 - H(y \mid x_i)]$$

$$= \underline{y} + \int_{\underline{y}}^{\bar{y}} [1 - F(y)]^x \, dy$$

It is also independent of the number of firms. Therefore, we get the same invariance results as Sah and Stiglitz (1987) do.

     In our model, firms compete for a profitable production contract instead of a prize with a fixed rent as in Sah and Stiglitz and other models in the R&D races literature. An incentive production contract $[Q(y), P(y)]$ can generate a profit $R(y) = P(y) - yQ(y)$ which decreases with the R&D outcome y (the marginal cost). The winning firm is not just awarded a prize, it will get a better prize if its R&D outcome is of higher quality. This can be viewed as being similar to a variable patent system in private markets. The reason we have the invariance result here is quite intuitive. Remember that each firm has the same R&D technology and does R&D independently. The R&D technology exhibits constant marginal returns to expenditures and it is just like an independent search process. Therefore, when one firm does k experiments, it has the same effect on the observed minimum marginal production cost as if k firms each did one experiment. When the prize is predetermined and independent of the number of firms, the total number of experiments or total R&D expenditure for all firms is independent of the number of firms. Under this specified environment, one firm will do what n firms will do. Therefore, it is not surprising to get the invariance result under the particular R&D technology. Later, we will show that if the R&D technology exhibits diminishing marginal returns to expenditures or if the buyer chooses the incentive contract optimally, the above invariance result does not hold. We would conjecture that if the R&D technologies among firms are dependent, the invariance result does not hold either.

     If the R&D process exhibits diminishing marginal returns to expenditures on R&D, Nash equilibrium behavior on R&D expenditure will be different from above. We first show that each firm will invest the same amount in equilibrium.

Proposition 5: Under condition (B), i) only a symmetric Nash equilibrium on R&D expenditures exists; ii) the individual expenditure $x(n)$ decreases with n; and iii) $\lim_{n \to \infty} x(n) = 0$.

Proof: We first show that any Nash equilibrium $(\hat{x}_1, \ldots, \hat{x}_n)$ at the R&D stage is symmetric. If not, there exists $i \neq j$ such that $\hat{x}_i \neq \hat{x}_j, \hat{x}_i > 0$ and $\hat{x}_j > 0$. Since $\hat{x}_i$ and $\hat{x}_j$ satisfy (11), we get

$$\int_{\underline{y}}^{\bar{y}} Q(y) \prod_{k=1}^{n} [1 - H(y \mid \hat{x}_k)] \left[\frac{H_x(y \mid \hat{x}_i)}{1 - H(y \mid \hat{x}_i)} - \frac{H_x(y \mid \hat{x}_j)}{1 - H(y \mid \hat{x}_j)} \right] dy = 0.$$

Condition (B) implies that $H_x(y \mid x) / [1 - H(y \mid x)]$ is decreasing in x for all $x > 0$ and $y \in (\underline{y}, \bar{y})$. Then the above equation cannot be true. This contradiction implies that any Nash equilibrium is symmetric.

Let $x(n)$ be the the individual R&D expenditure, then the equilibrium condition (11) becomes

$$\int_{\underline{y}}^{\bar{y}} Q(y)[1 - H(y \mid x(n))]^{n-1} H_x(y \mid x(n)) dy = c. \tag{11'}$$

Considering n as a real variable and taking the derivatives of both sides of (11') with respect to n, we obtain

$$\frac{\partial x(n)}{\partial n} \int_{\underline{y}}^{\bar{y}} Q(y)[1 - H(y \mid x(n))]^{n-2} \Big[[1 - H(y \mid x(n))] H_{xx}(y \mid x(n)) + (1-n) H_x^2(y \mid x(n)) \Big] dy$$

$$+ \int_{\underline{y}}^{\bar{y}} Q(y)[1 - H(y \mid x(n))]^{n-1} H_x(y \mid x(n)) \log[1 - H(y \mid x(n))] dy = 0.$$

Condition (B) implies that $(1-H) H_{xx} + H_x^2 < 0$ for all $x > 0$ and $y \in (\underline{y}, \bar{y})$. Thus, $\dfrac{\partial x(n)}{\partial n} \le 0$; that is, $x(n)$ decreases with n.

Furthermore, $\{x(n)\}$ is a monotonic decreasing sequence with the lower bound 0. Then there exists a limit $x_0 \ge 0$ of the sequence when n approaches infinity. We will show that $x_0 = 0$. If not, then $H(y \mid x_0) > 0$ for $y > \underline{y}$ by the assumption and hence $[1 - H(y \mid x(n))]^{n-1} \to 0$ for $y > \underline{y}$ when $n \to \infty$. Let $n \to \infty$ in equation (11'), we get $c = 0$ which contradicts to $c > 0$ by the assumption. Therefore, $x(n) \to x_0 = 0$.

Q.E.D.

When more firms enter the R&D race game, each existing firm will invest less in R&D. In the limit, the individual expenditure approaches zero.

It is not clear how the total expenditure and the pace of innovation depend on the number of firms under diminishing marginal returns to expenditure. In a special case of (B), we find the following dependence result which differs from Sah and Stiglitz (1987):

Proposition 6: Under (B_1), i) $E(y^m \mid x(n))$ decreases with n; ii) $\lim\limits_{n \to \infty} E(y^m \mid x(n)) = \underline{y}$, where $E(y^m \mid x(n)) = \underline{y} + \int_{\underline{y}}^{\bar{y}} [1 - F(y)]^{n\alpha(x(n))} dy$ is the expected minimum marginal cost.

Proof: Under condition (B_1), the Nash equilibrium on R&D expenditures is symmetric by Proposition 5. Let $x(n)$ be the individual equilibrium expenditure, then the equilibrium condition (11) can be written as

$$\int_{\underline{y}}^{\bar{y}} Q(y)[1 - F(y)]^{n\alpha(x(n))} \log[1 - F(y)] dy = -c/\alpha'(x(n)). \tag{11''}$$

By Proposition 5, $x(n)$ decreases with n and $x(n) \to 0$ when $n \to \infty$. Since $\alpha(x)$ is strictly concave function, (11″) implies that $n\alpha(x(n))$ increases with n. It is easy to calculate the expected minimum marginal cost as the following:

$$E(y^m \mid x(n)) = -\int_{\underline{y}}^{\bar{y}} y \, d[1 - F(y)]^{n\alpha(x(n))}$$

$$= \underline{y} + \int_{\underline{y}}^{\bar{y}} [1 - F(y)]^{n\alpha(x(n))} dy$$

which decreases with n. Since $\alpha'(0) = +\infty$ from the assumption, then $\alpha'(x(n)) \to +\infty$. From (11″), it must be $n\alpha(x(n)) \to +\infty$. Thus, $E(y^m \mid x(n)) \to \underline{y}$.

Q.E.D.

Because of diminishing marginal returns to expenditures the cost reduction by each firm is limited. But different firms can do R&D independently. When more and more firms invest in R&D, minimum marginal costs are expected to be reduced. In the limit, the expected minimum marginal cost could reach the lower bound $\underline{y}$. But at the same time, the total expenditures on R&D may increase with the number of firms. As a particular example, let $\alpha(x) = x^\alpha$, $0 < \alpha < 1$, and let $Q(y) = -f(y)/\log[1 - F(y)]$. Assume that $f'(y) < 0$ for $y \in (\underline{y}, \bar{y})$, then it is easy to check that $Q'(y) \le 0$. The global incentive condition is satisfied. In this case, the equilibrium condition (11) becomes

$$nx(n) + x(n)^{1-\alpha} = \alpha/c.$$

When $\alpha = 1$, the total expenditure is $nx(n) = 1/c$, which is independent of n as we showed before. But for $0 < \alpha < 1$, $nx(n)$ varies with n. From Proposition 5, we know that $x(n)$ decreases with n and $x(n) \to 0$ when $n \to +\infty$. From the above equation, we obtain that $nx(n)$ increases with n and $nx(n) \to \alpha/c$ when $n \to +\infty$. Thus, the total expenditure $nx(n)$ increases with the number of firms and has a finite limit in this example. Therefore, when the R&D technology exhibits diminishing marginal returns to expenditures, more competition with a finite amount of total expenditures on R&D could reduce the marginal production cost to the lower bound.

III.3. Optimal Incentive Contract

Under the incentive condition (9), we can easily calculate the buyer's expected utility $EW(Q(y), x_1, \ldots, x_n)$. The buyer will choose quantity $Q(y)$ and R&D expenditures $(x_1, \ldots, x_n)$ to maximize its expected utility EW subject to the firms' individual rationality constraints, self-selection constraints, and the Nash equilibrium conditions on R&D expenditures. An optimal incentive contract will be determined by the following optimization problem:

$$(P_n) \ \text{Max} \int_{\underline{y}}^{\bar{y}} \left[\left[B(Q(y)) - yQ(y) \right] \sum_{i=1}^{n} K(y \mid x_{-i}) h(y \mid x_i) - Q(y) \sum_{i=1}^{n} K(y \mid x_{-i}) H(y \mid x_{-i}) \right] dy$$

$Q(y), x_1, \ldots, x_n$

$$s.t. \quad \int_{\underline{y}}^{\bar{y}} Q(y)K(y \mid x_{-i})H_{x_i}(y \mid x_i)dy - c = 0, \quad i = 1, \ldots, n \tag{11}$$

$$\int_{\underline{y}}^{\bar{y}} Q(y)K(y \mid x_{-i})H(y \mid x_i)dy - cx_i \geq 0, \quad i = 1, \ldots, n \tag{13}$$

where the global incentive constraint is ignored. Guesnerie and Laffont's (1984) argument can be used if the global incentive constraint does not hold for the solution to (P_n). We can see that when $n = 1$ the optimization problem (P_n) is just (P) which we discussed in Section II.4. EU_i is strictly concave in x_i, $EU_i = 0$ when $x_i = 0$, and $EU_i > 0$ for some $x_i > 0$ and for any nontrivial distribution $H(y \mid x)$. Thus, for an interior solution $x_i > 0$ of (P_n), the individual rationality constraint (13) must be nonbinding. In order to encourage each firm to do R&D, positive ex ante profits are required.

Let μ_i and λ_i be the multipliers for (11) and (13), respectively, then $\lambda_i = 0$ for all i because of the nonbinding constraints (13). The necessary conditions for the optimal procurement quantity $Q(y)$ and R&D expenditures $(x_1, \ldots, x_n)$ are

$$B'(Q(y)) = y + \frac{\sum\limits_{i=1}^{n} K(y \mid x_{-i})H(y \mid x_i)}{\sum\limits_{i=1}^{n} K(y \mid x_{-i})h(y \mid x_i)} - \frac{\sum\limits_{i=1}^{n} \mu_i K(y \mid x_{-i})H_{x_i}(y \mid x_i)}{\sum\limits_{i=1}^{n} K(y \mid x_{-i})h(y \mid x_i)} \tag{14}$$

$$\int_{\underline{y}}^{\bar{y}} \left[B'(Q(y)) - y \right] Q'(y)K(y \mid x_{-i})H_{x_i}(y \mid x_i)dy = \mu_i \int_{\underline{y}}^{\bar{y}} Q(y)K(y \mid x_{-i})H_{x_i x_i}(y \mid x_i)dy \tag{15}$$

Thus, equation systems (9), (11), (14), and (15) simultaneously determine the optimal contract $[Q(y), P(y)]$, the optimal R&D expenditures $(x_1, \ldots, x_n)$, and the multipliers $(\mu_1, \ldots, \mu_n)$.

Lemma 5: If $1 + \dfrac{\partial}{\partial y}\left[\dfrac{\sum\limits_i K(y \mid x_{-i})H(y \mid x_i)}{\sum\limits_i K(y \mid x_{-i})h(y \mid x_i)} \right] > 0$ and $\dfrac{\partial}{\partial y}\left[\dfrac{K(y \mid x_{-i})H_{x_i}(y \mid x_i)}{\sum\limits_i K(y \mid x_{-i})h(y \mid x_i)} \right] \geq 0$ for all

$y, x_1, \ldots, x_n$ and all i, then there exists at least one j such that $\mu_j > 0$.

Proof: The proof is by contradiction and similar to the proof of Lemma 3. If not, then $\mu_j \leq 0$ for all j. From (14), we obtain $B'(Q(y)) - y > 0$ and

$$B''(Q(y)) = 1 + \frac{\partial}{\partial y}\left[\frac{\sum\limits_i K(y \mid x_{-i})H(y \mid x_i)}{\sum\limits_i K(y \mid x_{-i})h(y \mid x_i)} \right] - \sum_i \left[\mu_i \frac{\partial}{\partial y}\left[\frac{K(y \mid x_{-i})H_{x_i}(y \mid x_i)}{\sum\limits_i K(y \mid x_{-i})h(y \mid x_i)} \right] \right]$$

By the assumption, the above equation implies $Q'(y) < 0$. Thus,

$$\frac{\partial EW}{\partial x_i} = -\int_{\underline{y}}^{\bar{y}} \left[B'(Q(y)) - y \right] Q'(y) K(y \mid x_{-i}) H_{x_i}(y \mid x_i) dy > 0$$

Since $H_{x_i x_i} < 0$, then (15) implies $\mu_i > 0$. This is a contradiction. Therefore, there exists at least one j such that $\mu_j > 0$.

Q.E.D.

It may not be easy to interpret the assumptions in Lemma 5. But if we restrict our attention to technologies (A) or (B_1) we can provide a standard condition. Under technology (A) or (B_1), from (15) it can be shown that $\mu_1 = \cdots = \mu_n \equiv \mu$. Then the assumptions in Lemma 5 become much simpler:

Lemma 6: Under R&D technology (A) or (B_1), if $f(y)/[1 - F(y)]$ is nonincreasing in y, then $\mu > 0$.

The proof is similar to the proof of Lemma 5 and is omitted here. Since $h(y \mid x)/[1 - H(y \mid x)] = \alpha(x) f(y)/[1 - F(y)]$ under technology (A) or (B_1), the condition in Lemma 6 means that for any investment level x the hazard rate $h(y \mid x)/[1 - H(y \mid x)]$ is nonincreasing in y, which is the standard regularity condition. Under condition (A), (14) and (15) become

$$B'(Q(y)) = y + \frac{1 - F(y)}{f(y)x} \left[\sum_{i=1}^{n} [1 - F(y)]^{-x_i} - n + n\mu \log[1 - F(y)] \right] \tag{14'}$$

$$\int_{\underline{y}}^{\bar{y}} \left[B'(Q(y)) - y \right] Q'(y)[1 - F(y)]^x \log[1 - F(y)] dy$$

$$= \mu \int_{\underline{y}}^{\bar{y}} Q(y)[1 - F(y)]^x \log^2[1 - F(y)] dy \tag{15'}$$

Thus, the optimal procurement quantity, equilibrium R&D expenditures, and μ are simultaneously determined by (12), (14'), and (15'). We have the following results:

Proposition 7: Under condition (A), if $f(y)/[1 - F(y)]$ is nonincreasing in y, then the optimal procurement quantity depends on the number of potential firms, as do the total expenditure and the pace of innovation.

Therefore, in the presence of costly R&D investments, the optimal procurement quantity is dependent on the number of potential firms . This contrasts with Riordan and Sappington (1987), and Dasgupta and Spulber (1987) who find that the optimal quantity schedule does not depend on the number of the firms in awarding monopoly franchises and in normal procurements, respectively.

But no R&D behavior was considered in their models. In the presence of R&D behavior, the design of incentive contracts is based upon the buyer's belief about the firms' private information (the R&D outcome). Thus, investment behavior actually has some influence on procurement contracts. The number of potential firms plays an important role in determining the optimal procurement quantity and the investment level. But how the optimal quantity and the R&D expenditure depend on the number of firms in this case is not clear.

Let $EW(n)$ be the principal's expected welfare under the optimal incentive contract, then we know

Proposition 8: Under conditions in Lemma 6, $\dfrac{dEW(n)}{dn}\Big|_{n=1} > 0$.

In other word, the principal prefers that more than one firm participate in R&D and bid for the procurement contract. It is not clear whether the government is always better-off when more firms participate in R&D and compete for the contract. What we know is the following:

Proposition 9: Suppose that the conditions in Proposition 7 hold, and that the optimal solution $Q_n(y)$ and $\sum_{i=1}^{n} x_i$ have limits $Q_0(y) \geq 0$ and $x_0 \geq 0$ when $n \to \infty$, respectively, and $Q_n(y)$ decreases with y. Then $B'(Q_0(y)) = y$ and $\lim_{n \to \infty} E(y^m \mid x(n)) = y_0 > \underline{y}$.

We have ex post production efficiency in the limit, that is, marginal benefit equals marginal cost given the technology, but the expected minimum marginal cost might increase with the number of firms because $Q_n(y)$ and hence $nx(n)$ might decrease with n. The expected minimum marginal cost may not reach the lower bound $\underline{y}$ in the limit. Thus, in the case of costly R&D, more competition improves production efficiency, but may discourage R&D. Production may be socially more efficient but at a relatively higher R&D cost.[11]

IV. CONCLUDING REMARKS

In this paper, we have created a basic model of the R&D and production process. We have examined what should happen if the government manages and controls the R&D stage indirectly by awarding an appropriate production contract. The optimal production contract is characterized to maximize the government's expected welfare given that each firm has incentives to invest in R&D and to report its true private information. There exists inefficiency in procurements because of unobservable R&D outcomes and moral hazard in R&D activities. Costly R&D has an opposite effect on the production decision to the adverse selection effect (Baron and Myerson 1982). If the Baron-Myerson type contract is used in this case, the government buys too little and pays too little as compared with its optimum. Therefore, the government prefers to take into account the pre-contract R&D behavior when offering a procurement contract.

The analysis of the competition case shows that the number of competing firms matters. The optimal incentive production contract offered by the government depends on the number of firms. This is in sharp contrast to Riordan and Sappington (1987) and Dasgupta and Spulber (1987) where no R&D behavior has been considered. The total expenditures on R&D and the pace of innovation in the industry are also dependent of the number of firms. This is different from the invariance result of Sah and Stiglitz (1987) where firms do R&D and compete for a fixed rent in a Bertrand market. The government prefers more than one firm to invest in private R&D and to compete for the production contract. In general, competition among a large number of suppliers in procurements may encourage production efficiency, but discourage R&D. It depends on the structure of the R&D technology.

APPENDIX

Application of the Revelation Principle: In general, the principal first announces a mechanism $[Q(m), P(m), M, x]$, where M is the message space, $m \in M$ is the message that the agent sends to the principal, and x is the principal's proposal to the agent. Given this mechanism, the agent takes an action $\tilde{x}$ and observes his type y. Then the agent sends a message $m = \phi(y) \in M$ to the principal. Finally, the principal asks the agent to produce $Q(\phi(y))$ and pays him $P(\phi(y))$. Since P, Q, and x are continuous variables and are chosen from convex sets, we only need to consider pure strategies of $Q(.)$, $P(.)$, and x. The principal's problem is to design a mechanism $[Q(m), P(m), M, x]$ to

$$\text{Maximize } EW = \int_{\underline{y}}^{\bar{y}} \left[B[Q(\phi(y))] - P(\phi(y)) \right] dH(y \mid x)$$

$$\text{s.t. } [\phi(y), x] \in \text{argmax } EU = \int_{\underline{y}}^{\bar{y}} \left[P(\bar{\phi}(y)) - yQ(\bar{\phi}(y)) \right] dH(y \mid \tilde{x}) - c\tilde{x}. \qquad (*)$$

A mechanism is direct if and only if $M = [\underline{y}, \bar{y}]$. An optimal incentive compatible direct revelation mechanism $[Q(y), P(y), x]$ is the one that

$$\text{Maximizes } EW = \int_{\underline{y}}^{\bar{y}} \left[B(Q(y)) - P(y) \right] dH(y \mid x)$$

$$\text{s.t. } [\phi(y) = y, x] \in \text{argmax } EU = \int_{\underline{y}}^{\bar{y}} \left[P(\bar{\phi}(y)) - yQ(\bar{\phi}(y)) \right] dH(y \mid \tilde{x}) - c\tilde{x}. \qquad (**)$$

The Revelation Principle in this context says that, given any (agent's) optimal strategy $[\phi(y), x]$ in any mechanism $[Q(m), P(m), M, x]$, there exists an incentive compatible direct revelation mechanism $[Q^*(y), P^*(y), x^*]$ in which the principal gets the same expected welfare as in the given optimal strategy of the given mechanism. To see why this is true, we define $Q^*(y) \equiv Q(\phi(y))$, $P^*(y) = P(\phi(y))$, and $x^* = x$ given the mechanism $[Q(m), P(m), M, x]$ and the agent's optimal strategy $[\phi(y), x]$. Then

$$EW^* = \int_{\underline{y}}^{\bar{y}} \left[B(Q^*(y)) - P^*(y) \right] dH(y \mid x^*)$$

$$= \int_{\underline{y}}^{\bar{y}} \left[B[Q(\phi(y))] - P(\phi(y)) \right] dH(y \mid x)$$

$$= EW$$

That is, the principal has the same expected welfare under these two mechanisms. Now, if $[Q^*(y), P^*(y), x^*]$ does not satisfy the incentive compatibility condition $(**)$, then $[\bar{\phi}(y) = y, \tilde{x} = x^*]$

does not maximize

$$EU = \int_{\underline{y}}^{\bar{y}} \left[P^*(\tilde{\phi}(y)) - yQ^*(\tilde{\phi}(y)) \right] dH(y \mid \tilde{x}) - c\tilde{x}$$

$$= \int_{\underline{y}}^{\bar{y}} \left[P\left[\phi(\tilde{\phi}(y))\right] - yQ\left[\phi(\tilde{\phi}(y))\right] \right] dH(y \mid \tilde{x}) - c\tilde{x}$$

$$= \int_{\underline{y}}^{\bar{y}} \left[P(\tilde{\phi}_1(y)) - yQ(\tilde{\phi}_1(y)) \right] dH(y \mid \tilde{x}) - c\tilde{x}$$

where $\tilde{\phi}_1(y) = \phi(\tilde{\phi}(y))$. That is, $[\tilde{\phi}_1(y) = \phi(y), \tilde{x} = x]$ does not maximize

$$\int_{\underline{y}}^{\bar{y}} \left[P(\tilde{\phi}_1(y)) - yQ(\tilde{\phi}_1(y)) \right] dH(y \mid \tilde{x}) - c\tilde{x}$$

That violates the condition (*). Thus, $[Q^*(y), P^*(y), x^*]$ is incentive compatible.

Proof of Lemma 1: First, if $\phi(y) = y$ is the agent firm's optimal strategy, the necessary condition is

$$\frac{\partial R(y, \phi)h(y \mid x)}{\partial \phi} \mid_{\phi = y} = 0$$

for all x and y, i.e. $\dfrac{\partial R(y, \phi)}{\partial \phi} \mid_{\phi = y} = 0$ for all y. The second order condition can be written as $\dfrac{\partial^2 R(y, \phi)}{\partial \phi^2} \mid_{\phi = y} \leq 0$ for all y. Combining these two conditions, we get $Q'(y) \leq 0$ for all $y \in [\underline{y}, \bar{y}]$. Let $R(y) \equiv R(y, y)$, then $P(y) = yQ(y) + R(y)$ and $R'(y) = \dfrac{\partial R(y, \phi)}{\partial y} \mid_{\phi = y} = -Q(y)$.

On the other hand, given that $Q'(y) \leq 0$ and $P(y) = yQ(y) + R(y)$ for all $y \in [\underline{y}, \bar{y}]$, we can show that $R(y) \geq R(y, \phi)$ for any $y, \phi \in [\underline{y}, \bar{y}]$. In fact, since $R(y) = R(\bar{y}) + \int_{y}^{\bar{y}} Q(\tilde{y})d\tilde{y}$, for $\phi > y$ we have

$$R(y) - R(\phi) = \int_{y}^{\phi} Q(\tilde{y})d\tilde{y}$$

$$\geq Q(\phi)(\phi - y) = R(y, \phi) - R(\phi)$$

where the inequality is true because $Q(y)$ is nonincreasing in y. We get $R(y) \geq R(y, \phi)$. The same is true for any $\phi < y$. Thus, for any $y \in [\underline{y}, \bar{y}]$, once $\phi(y) \in [\underline{y}, \bar{y}]$, we have $R(y) \geq R(y, \phi(y))$ which implies

$$\int_{\underline{y}}^{\bar{y}} R(y)h(y \mid x)dy - cx \geq \int_{\underline{y}}^{\bar{y}} R(y, \phi(y))h(y \mid x)dy - cx$$

since $h(y \mid x) \geq 0$ for all x, y. Therefore, truth-telling is the optimal strategy for the agent.

<div align="right">Q.E.D.</div>

Proof of Lemma 2: Suppose the conclusion in Lemma 2 does not hold, i.e. $x^*(Q) < x^*(\tilde{Q})$. From the first order condition, we get

$$\int_{\underline{y}}^{\bar{y}} Q(y) H_x(y \mid x^*(Q)) dy = \int_{\underline{y}}^{\bar{y}} \tilde{Q}(y) H_x(y \mid x^*(\tilde{Q})) dy$$

$$< \int_{\underline{y}}^{\bar{y}} Q(y) H_x(y \mid x^*(\tilde{Q})) dy$$

because $H_x > 0$. This implies that

$$\int_{\underline{y}}^{\bar{y}} Q(y) \left[H_x(y \mid x^*(Q)) - H_x(y \mid x^*(\tilde{Q})) \right] dy < 0.$$

But $H_{xx} < 0$ and $x^*(Q) < x^*(\tilde{Q})$ imply $H_x(y \mid x^*(Q)) \geq H_x(y \mid x^*(\tilde{Q}))$ for all $y \in (\underline{y}, \bar{y})$. The above inequality cannot be true. This is a contradiction. Thus, $x^*(Q) \geq x^*(\tilde{Q})$.

Let EU and $E\tilde{U}$ be the expected utilities for the firm under these two contracts, respectively. When $R(\bar{y}) = \tilde{R}(\bar{y})$ we get

$$E\tilde{U} - EU = \int_{\underline{y}}^{\bar{y}} \left[\tilde{Q}(y) H(y \mid x^*(\tilde{Q})) - Q(y) H(y \mid x^*) \right] dy - cx^*(\tilde{Q})) + cx^*(Q))$$

$$\leq \int_{\underline{y}}^{\bar{y}} Q(y) \left[H(y \mid x^*(\tilde{Q})) - H(y \mid x^*(Q)) \right] dy - c \left[x^*(\tilde{Q})) - x^*(Q)) \right]$$

Using Taylor's expansion, there exists $\xi \in [0, \bar{x}]$ such that

$$H(y \mid x^*(\tilde{Q})) - H(y \mid x^*(Q)) = H_x(y \mid x^*(Q))\delta + H_{xx}(y \mid \xi)\delta^2 / 2,$$

where $\delta = x^*(\tilde{Q}) - x^*(Q)$. Substituting these two equations into the above inequality, we get

$$E\tilde{U} - EU \leq \left[\int_{\underline{y}}^{\bar{y}} Q(y) H_x(y \mid x^*(Q)) - c \right] \delta + \frac{1}{2}\delta^2 \int_{\underline{y}}^{\bar{y}} Q(y) H_{xx}(y \mid \xi) dy.$$

The first term on the right hand side equals zero by the first order condition, and the second term is no larger than zero. Thus, $EU \geq E\tilde{U}$.

<div align="right">Q.E.D.</div>

Proof of Proposition 4: Since $H(y \mid x) = 1 - [1 - F(y)]^x$ for all $x \geq 0$ and $y \in [\underline{y}, \bar{y}]$, then

$$\frac{H(y \mid x)}{h(y \mid x)} = \frac{[1 - F(y)]^{1-x} - [1 - F(y)]}{x f(y)},$$

$$\frac{H_x(y \mid x)}{h(y \mid x)} = -\frac{[1 - F(y)]\log[1 - F(y)]}{x f(y)},$$

$$\frac{\partial\left[H(y \mid x)/h(y \mid x)\right]}{\partial y} = \frac{1}{x}\left[1 - (1-x)[1 - F(y)]^{-x}\right] - \frac{f'(y)}{xf^2(y)}\left[[1 - F(y)]^{1-x} - [1 - F(y)]\right]$$

$$\geq [1 - F(y)]^{-x} \geq 0$$

$$\frac{\partial\left[H_x(y \mid x)/h(y \mid x)\right]}{\partial y} = \frac{1}{x} + \frac{1}{x}\log[1 - F(y)]\left[1 + \frac{f'(y)[1 - F(y)]}{f^2(y)}\right] \geq 0.$$

The above two inequalities hold because of the assumption $\dfrac{f(y)}{1 - F(y)}$ is nonincreasing in y. The conditions in Lemma 3 are satisfied and thus $\hat{\mu} > 0$ by Lemma 3. Now let

$$\phi(y) = H(y \mid \hat{x}) - \hat{\mu}H_x(y \mid \hat{x})$$

$$= 1 - [1 - F(y)]^{\hat{x}} + \hat{\mu}[1 - F(y)]^{\hat{x}}\log[1 - F(y)].$$

Then $\phi(\underline{y}) = 0$, $\phi(\bar{y}) = 1$, and

$$\phi'(y) = [1 - F(y)]^{\hat{x}-1}f(y)\left[\hat{x} - \hat{\mu} - \hat{\mu}\hat{x}\log[1 - F(y)]\right]$$

If $\hat{x} \geq \hat{\mu}$, then $\phi'(y) > 0$ for all $y \in (\underline{y}, \bar{y})$. Thus, $\phi(y) > \phi(\underline{y}) = 0$. That is, $H(y \mid \hat{x}) - \hat{\mu}H_x(y \mid \hat{x}) > 0$ for all $y \in (\underline{y}, \bar{y})$. Since $B''(Q) < 0$, from (1) and (6) we obtain $\hat{Q}(y) < Q^*(y)$ for all $y \in (\underline{y}, \bar{y})$.

If $\hat{x} < \hat{\mu}$, then there exists a $\tilde{y} \in (\underline{y}, \bar{y})$ such that

$$F(\tilde{y}) = 1 - e^{-\frac{\hat{\mu}-\hat{x}}{\hat{\mu}\hat{x}}}$$

Thus, $\phi'(y) < 0$ for $y < \tilde{y}$ and $\phi'(y) > 0$ for $y > \tilde{y}$. Since $\phi(\underline{y}) = 0$, then $\phi(y) < 0$ for some $y > \underline{y}$. But we know $\phi(\bar{y}) = 1$. Thus there exists a $y_0 \in (\underline{y}, \bar{y})$ such that $\phi(y_0) = 0$, $\phi(y) < 0$ for $y < y_0$, and $\phi(y) > 0$ for $y > y_0$. Therefore, from (1) and (6) we have $\hat{Q}(y_0) = Q^*(y_0)$, $\hat{Q}(y) > Q^*(y)$ for $y \in (\underline{y}, y_0)$, and $\hat{Q}(y) < Q^*(y)$ for all $y \in (y_0, \bar{y}]$.

Q.E.D.

Proof of Proposition 7: We show this result for symmetric Nash equilibrium of R&D expenditure. Under condition (A), the optimal quantity $Q(y)$, equilibrium R&D expenditures, and μ are simultaneously determined by (12), (14'), and (15'). If the optimal quantity schedule $Q(y)$ is independent of n for any y, then from (12) we know that the total expenditure x is also independent of n. This together with (15') implies that μ is independent of n. Consider the symmetric equilibrium of R&D expenditure. Let $x(n)$ be the equilibrium individual expenditure. Taking the derivative of both sides of (14') with respect to n, we get

$$0 = \left[ze^z - e^z + 1 + \frac{\mu z}{x(n)} \right] \frac{\partial x(n)}{\partial n} \frac{1}{x(n)^2} \tag{14''}$$

for all $y \in (\underline{y}, \hat{y})$, where $\hat{y}(> \underline{y})$ represents the marginal cost level of the marginal firm and is independent of n, and $z = z(y) = -x(n)\log[1 - F(y)] > 0$ for $y \in (\underline{y}, \bar{y})$. Let $\phi(z) = ze^z - e^z + 1 + \frac{\mu z}{x(n)}$, then

$$\frac{d\phi[z(y)]}{dy} = (ze^z + \frac{\mu}{x_n}) \frac{x_n f(y)}{1 - F(y)}$$

cannot be zero for all $y \in (\underline{y}, \hat{y})$ since $\mu > 0$ by Lemma 6. Thus $\phi(z(y))$ cannot be zero for all $y \in (\underline{y}, \hat{y})$. Since $x = nx(n)$ is independent of n, $\frac{\partial x_n}{\partial n}$ is not zero. Thus, (14'') cannot be true. The contradiction implies that $Q(y)$ does depend on n.

From (12) again, the total expenditure in R&D x will depend on n. Since the expected minimum marginal cost equals $E(y^m | x) = \underline{y} + \int_{\underline{y}}^{\bar{y}} [1 - F(y)]^x dy$, it also depends on n.

Q.E.D.

Proof of Proposition 8: Under technology (B_1), the Nash equilibrium of R&D expenditures is asymmetric by Proposition 5. Under technology (A), we consider symmetric equilibrium only. At the optimum solution $[\hat{Q}(y), \hat{R}(\bar{y}), \hat{x}]$ to (P_n), the buyer's expected welfare can be written as

$$EW(n) = n \int_{\underline{y}}^{\bar{y}} \left[B(\hat{Q}(y)) - y\hat{Q}(y) \right] [1 - H(y | \hat{x})]^{n-1} dH(y | \hat{x}) \tag{8.1}$$

$$- n \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1 - H(y | \hat{x})]^{n-1} H(y | \hat{x}) dy.$$

Viewing n as a continuous variable at this moment, we obtain

$$\frac{dEW(n)}{dn} = \left[\frac{\partial EW}{\partial \hat{Q}(.)} \frac{\partial \hat{Q}(.)}{\partial n} \right] + \frac{\partial EW}{\partial \hat{x}} \frac{\partial \hat{x}}{\partial n} + \frac{\partial EW}{\partial n}, \tag{8.2}$$

where

$$\left[\frac{\partial EW}{\partial \hat{Q}(.)} \frac{\partial \hat{Q}(.)}{\partial n}\right] = n \int_{\underline{y}}^{\bar{y}} [1 - H(y \mid \hat{x})]^{n-1} \frac{\partial \hat{Q}(y)}{\partial n} \left[B'(\hat{Q}(y)) - y - \frac{H(y \mid \hat{x})}{h(y \mid \hat{x})}\right] dH(y \mid \hat{x})$$

$$= -n\hat{\mu} \int_{\underline{y}}^{\bar{y}} [1 - H(y \mid \hat{x})]^{n-1} H_x(y \mid \hat{x}) \frac{\partial \hat{Q}(y)}{\partial n} dy. \tag{8.3}$$

The last equality holds due to (14). Integrating (8.1) by parts, we can rewrite $EW(n)$ as

$$EW(n) = B(\bar{Q}) - \underline{y}\bar{Q} + \int_{\underline{y}}^{\bar{y}} \left[B'(\hat{Q}(y)) - y\right] \hat{Q}'(y)[1 - H(y \mid \hat{x})]^n dy$$

$$- \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1 - H(y \mid \hat{x})]^{n-1} \left[1 + (n-1)H(y \mid \hat{x})\right] dy. \tag{8.4}$$

Then

$$\frac{\partial EW}{\partial \hat{x}} = -n \int_{\underline{y}}^{\bar{y}} \left[B'(\hat{Q}(y)) - y\right] \hat{Q}'(y)[1 - H(y \mid \hat{x})]^{n-1} H_x(y \mid \hat{x}) dy$$

$$- (n-1) \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1 - H(y \mid \hat{x})]^{n-1} H_x(y \mid \hat{x}) dy$$

$$+ (n-1) \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1 - H(y \mid \hat{x})]^{n-2} H_x(y \mid \hat{x}) \left[1 + (n-1)H(y \mid \hat{x})\right] dy$$

$$= -n\hat{\mu} \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1 - H(y \mid \hat{x})]^{n-1} H_{xx}(y \mid \hat{x}) dy$$

$$+ n(n-1) \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1 - H(y \mid \hat{x})]^{n-2} H_x(y \mid \hat{x}) H(y \mid \hat{x}) dy. \tag{8.5}$$

Taking the derivatives of both sides of (11) with respect to n, we get

$$\int_{\underline{y}}^{\bar{y}} \frac{\partial \hat{Q}(y)}{\partial n} [1 - H(y \mid \hat{x})]^{n-1} H_x(y \mid \hat{x}) dy + \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1 - H(y \mid \hat{x})]^{n-1} H_{xx}(y \mid \hat{x}) dy \frac{\partial \hat{x}}{\partial n}$$

$$- (n-1) \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1 - H(y \mid \hat{x})]^{n-2} H_x^2(y \mid \hat{x}) dy \frac{\partial \hat{x}}{\partial n}$$

$$+ \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1 - H(y \mid \hat{x})]^{n-1} H_x(y \mid \hat{x}) \log[1 - H(y \mid \hat{x})] dy = 0. \tag{8.6}$$

Combining (8.3) and (8.5) with (8.6), we obtain

$$\left[\frac{\partial EW}{\partial \hat{Q}(.)}\frac{\partial \hat{Q}(.)}{\partial n}\right] + \frac{\partial EW}{\partial \hat{x}}\frac{\partial \hat{x}}{\partial n} = -n(n-1)\hat{\mu}\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y|\hat{x})]^{n-2}H_x^2(y|\hat{x})dy\frac{\partial \hat{x}}{\partial n}$$

$$+ n\hat{\mu}\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y|\hat{x})]^{n-1}H_x(y|\hat{x})\log[1-H(y|\hat{x})]dy$$

$$+ n(n-1)\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y|\hat{x})]^{n-2}h(y|\hat{x})H_x(y|\hat{x})dy\frac{\partial \hat{x}}{\partial n} \tag{8.7}$$

On the other hand, we have

$$\frac{\partial EW}{\partial n} = \int_{\underline{y}}^{\bar{y}}\left[B'(\hat{Q}(y))-y\right]\hat{Q}'(y)[1-H(y|\hat{x})]^n\log[1-H(y|\hat{x})]dy$$

$$-\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)[1-H(y|\hat{x})]^{n-1}\left[1+(n-1)H(y|\hat{x})\right]\log[1-H(y|\hat{x})]dy$$

$$-\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)[1-H(y|\hat{x})]^{n-1}H(y|\hat{x})dy \tag{8.8}$$

Let $H(y|x) = 1 - u(y)^{\alpha(x)}$, where $u(y) = 1 - F(y)$, $\alpha(x) \geq 0$, $\alpha'(x) > 0$, $\alpha''(x) \leq 0$, and $\alpha(0) = 0$. When $\alpha(x) = x$, $H(y|\hat{x})$ represents technology (A). When $\alpha''(x) < 0$, $H(y|x)$ represents technology (B_1). Substituting this distribution into (8.7) and (8.8), and combining with (8.2), we obtain the following

$$\frac{dEW(n)}{dn} = (n-1)\frac{\partial(n\alpha(\hat{x}))}{\partial n}\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)u(y)^{(n-1)\alpha(\hat{x})}g_1(y)\log u(y)dy$$

$$+ \hat{\mu}\frac{\alpha(\hat{x})\alpha''(\hat{x})}{\alpha'(\hat{x})}\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)u(y)^{n\alpha(\hat{x})}\log u(y)dy$$

$$+ \int_{\underline{y}}^{\bar{y}}\hat{Q}(y)u(y)^{(n-1)\alpha(\hat{x})}g_2(y)dy \tag{8.9}$$

where

$$g_1(y) = u(y)^{\alpha(\hat{x})} - 1 - \hat{\mu}\alpha'(\hat{x})u(y)^{\alpha(\hat{x})}\log u(y)$$

$$g_2(y) = u(y)^{\alpha(\hat{x})} - 1 - \alpha(\hat{x})\log u(y)$$

Let $n = 1$ in (8.9), we have

$$\frac{dEW(n)}{dn}\Big|_{n=1} = \hat{\mu}\frac{\alpha(\hat{x})\alpha''(\hat{x})}{\alpha'(\hat{x})}\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)u(y)^{\alpha(\hat{x})}\log u(y)dy + \int_{\underline{y}}^{\bar{y}}\hat{Q}(y)g_2(y)dy \tag{8.10}$$

Since $\hat{\mu} > 0$ by Lemma 6, $\alpha''(\hat{x}) \leq 0$ by assumptions, and $\log u(y) < 0$ and $g_2(y) > 0$ for all $y \in (\underline{y}, \bar{y})$, (8.10) implies $\dfrac{dEW(n)}{dn}\big|_{n=1} > 0.$

<div align="right">Q.E.D.</div>

Proof of Proposition 9: We consider symmetri equilibrium only. First, since $\sum\limits_{i=1}^{n} x_i = nx(n)$ has a finite limit $x_0 \geq 0$ by the assumption, then when $n \to \infty$ we obtain

$$E(y^m | x(n)) = \underline{y} + \int_{\underline{y}}^{\bar{y}} [1 - F(y)]^{nx(n)} dy \;\to\; \underline{y} + \int_{\underline{y}}^{\bar{y}} [1 - F(y)]^{x_0} dy = y_0 > \underline{y}$$

Second, by Lemma 6, $\mu_n > 0$. Suppose that μ_n has a limit μ_0. Then $\mu_0 \geq 0$. We will show that $\mu_0 = 0$ and $\dfrac{\mu_n}{x_n} \to 1$. Combining (14') with (15'), we obtain

$$\int_{\underline{y}}^{\bar{y}} Q_n'(y)[1 - F(y)]^{x+1}\log[1 - F(y)]\left[\sum_{i=1}^{n} [1 - F(y)]^{-x_i} - n + n\mu_n \log[1 - F(y)] \right] \frac{1}{nx(n)f(y)} dy$$

$$= \mu_n \int_{\underline{y}}^{\bar{y}} Q_n(y)[1 - F(y)]^x \log^2[1 - F(y)] dy$$

Let $n \to \infty$, then

$$\left[\lim \frac{\mu_n}{x(n)} - 1 \right] \int_{\underline{y}}^{\bar{y}} Q_0(y)[1 - F(y)]^{x_0} \log^2[1 - F(y)] dy$$

$$= \mu_0 \int_{\underline{y}}^{\bar{y}} Q_0(y)[1 - F(y)]^{x_0} \log^2[1 - F(y)] dy$$

Since $Q_n(y)$ decreases with y by the assumption, then $Q_n'(y) \leq 0$ for all y and n, and hence $Q_0'(y) \leq 0$. If $\mu_0 > 0$, then the above equation implies that $\lim\limits_{n\to\infty} \dfrac{\mu_n}{x(n)} - 1 \leq 0$. That is, $\mu_n \leq x(n)$ when n is large enough. Thus, when $n \to \infty$, $\mu_n \to 0$ since $x_n \to 0$. Therefore, $\mu_0 = 0$ and hence $\lim \dfrac{\mu_n}{x(n)} = 1$. Thus, (14') implies that $B'(Q_0(y)) = y$.

<div align="right">Q.E.D.</div>

I: R&D | II: PROCUREMENTS

| P announces a menu of contracts | A invests in R&D | A observes the results | A chooses a contract | A produces | P pays A |

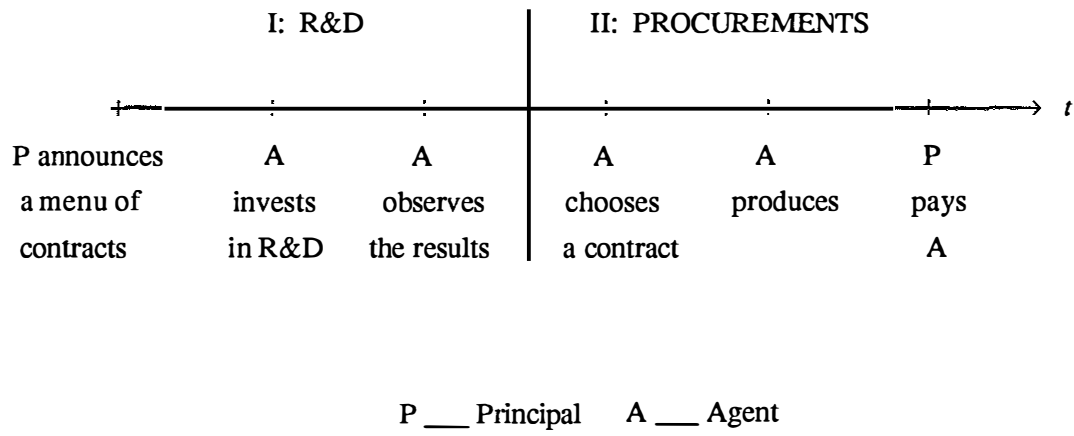P ___ Principal   A ___ Agent

FIGURE 1: The Timing of the two-stage Game
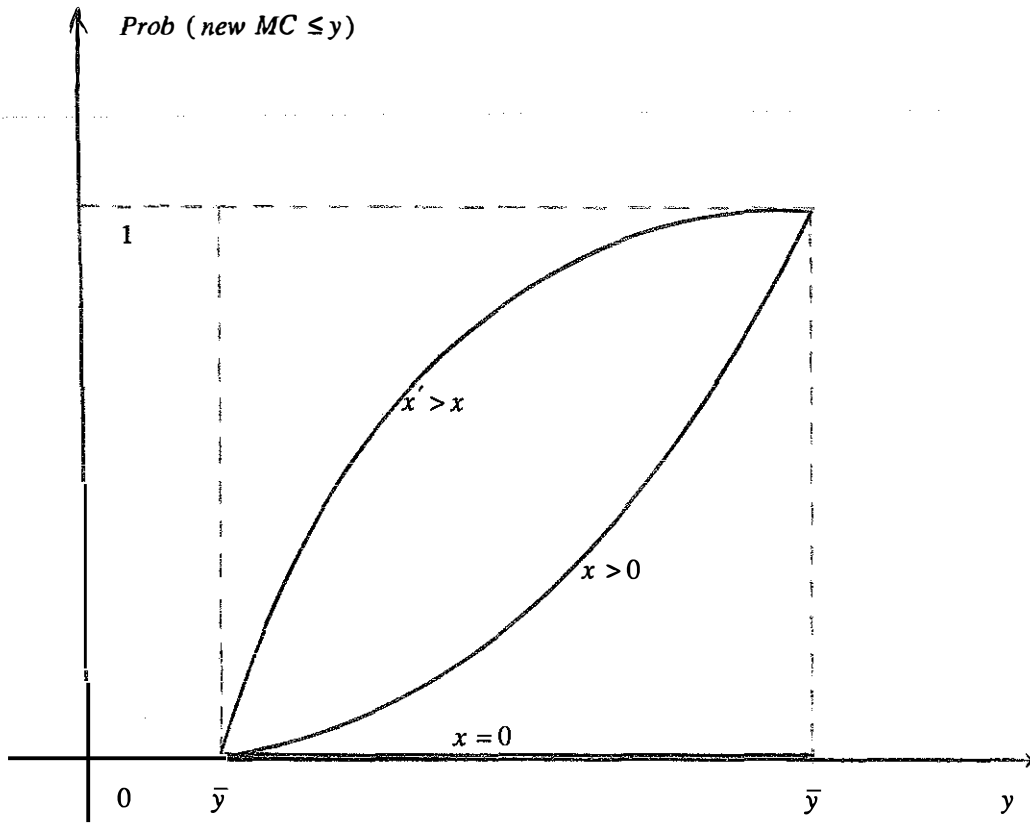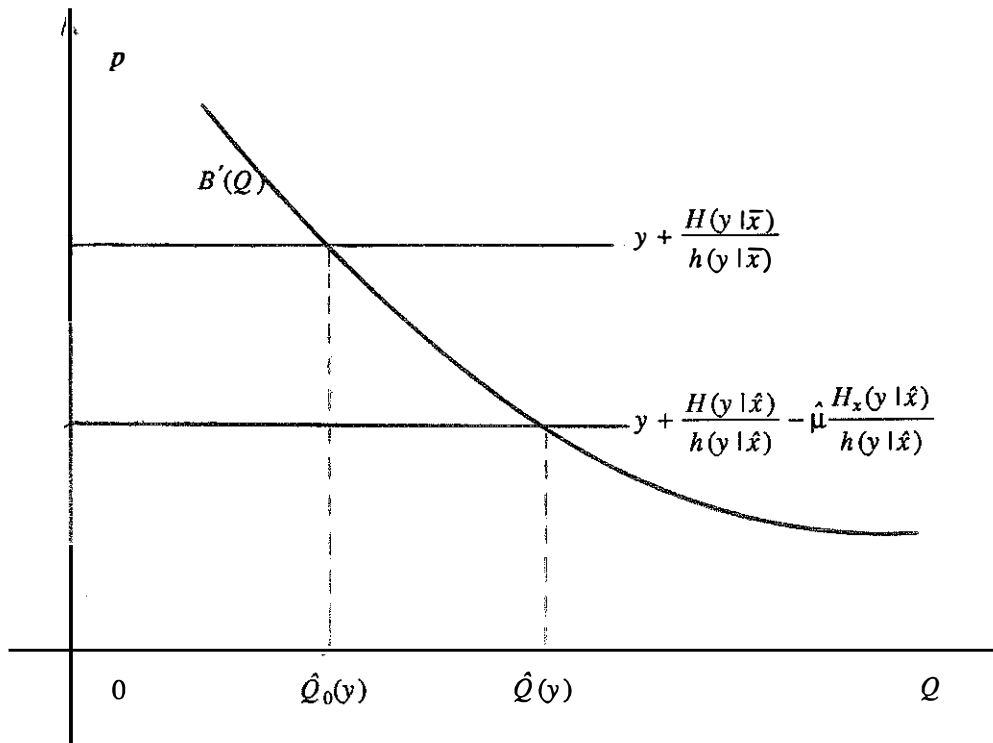

FIGURE 2: R&D Technology
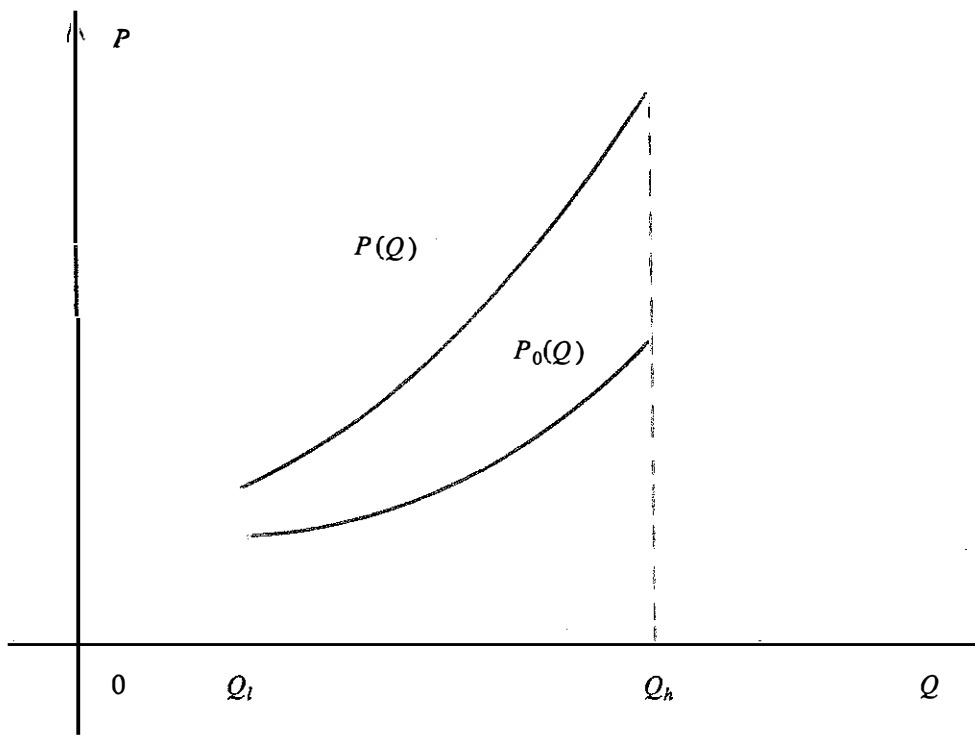
FIGURE 3: Optimal Quantity

FIGURE 4: Nonlinear Payment Schedule

FOOTNOTES

1. The economic theory of procurement has much in common with the economic theory of regulation and auctioning. Because of asymmetric information, designing incentive procurement contracts is similar to regulatory mechanism design. Caillaud, Guesnerie, Rey, and Tirole (1988) review the recent literature on government regulation under asymmetric information.

2. For a more complete survey of the recent literature on research, development, and diffusion, see Reinganum (1988).

3. Research and development in defense procurements is characterized by a particularly high degree of uncertainty. First, the level of innovation is uncertain. Second, the government cannot easily verify the outcome of the innovation activity. Third, R&D decisions or efforts directed towards innovation by the firms are difficult for the government to observe directly. Because of these problems the government has difficulties in rewarding and encouraging innovation activity efficiently. But, one way to reward successful innovation is to create a prize which is related to the production of the item being procured.

4. Laffont and Tirole (1988), and Fudenberg and Tirole (1988) have recently discussed the renegotiation issue in an agency model with moral hazard and in a repeated adverse selection model, respectively. Our model of R&D and production includes both adverse selection and moral hazard problems. Because of this, the design of renegotiation-proof contracts is different from that in Fudenberg and Tirole (1988) and Laffont and Tirole (1988). The optimal production contract here should not only be renegotiation-proof, but also give the firm incentives to reveal its private information and to invest enough in R&D. Therefore, it is important to investigate commitment and renegotiation in procurement and contracting in a two-stage model with both adverse selection and moral hazard.

5. It is often possible for the firm who wins the procurement contract to continue to exert some effort in order to reduce production cost further. If the production cost is ex post observable to the government, incentive contracts could be designed to give the chosen contractor an incentive to reduce production cost in both the R&D and production stage. In this case, incentive contracts can be based on both the R&D outcome and the realized production cost. Similar to Laffont and Tirole (1986), McAfee and McMillan (1987), It can be shown that a menu of linear contracts in both the expected cost and the ex post observed cost is optimal in the case of R&D and production uncertainty. But R&D changes the coefficients of the linear contracts.

6. We can always rescale the variable x such that the R&D cost is linear in x.

7. In the Appendix, we offer a proof that the Revelation Principal applies to our model.

8. If the number of potential firms is endogenous, it can be shown that there exists a free entry equilibrium under which the equilibrium number of firms, the level of investment in R&D, and the break-even level of production cost are simultaneously determined.

9. The innovation processes of different firms may not be the same, but usually have some common elements of technological uncertainty, one of which might be the general difficulty of

cost reduction. Because of these common factors the potential R&D outcomes among different firms may be correlated. It may then be possible for the government to extract the full surplus from the contracting firms by designing appropriate incentive contracts. But this full extraction of surplus may discourage R&D. We do not discuss this issue here in detail.

10. In general, production contracts could be allowed to depend on all bids $(b_1, \ldots, b_n)$ or messages. However, we consider here a variable quantity auction in which the lowest bid firm wins the contract and the quantity depends on the lowest bid only. Without R&D, some regularity conditions are sufficient for this special contract to be optimal within general mechanisms (see Riordan and Sappington 1987). It remains an open question whether, with R&D, offering a contract $[Q_i(b_1, \ldots, b_n), P_i(b_1, \ldots, b_n)]$ to the ith firm improves the govermnent's expected welfare.

11. We should be careful about this implication because some non-primitive conditions have been used in Proposition 9. Certainly, more research is needed to make this point clear.

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