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EXPERT OPINIONS AND TAXPAYER COMPLIANCE:
A STRATEGIC ANALYSIS

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ABSTRACT

In this paper we examine the incentives for taxpayers to claim risky deductions and to solicit expert opinions to support their positions, and for the tax agency to distinguish among individuals who do and do not solicit expert opinions for the purposes of auditing. We also consider the implications of an ex ante constraint on the tax agency which requires it to treat all taxpayers who take the deduction alike in terms of audit rates, whether or not they solicit an expert opinion. Finally, we examine the effects of regulations which limit the degree of riskiness for which a supporting opinion can be justified as well as the effects of changes in various penalty rates.

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1. INTRODUCTION

Despite recent attempts to simplify the U.S. Tax Code,[1] it has become increasingly difficult for citizens to fulfill their legal obligations, and more and more taxpayers have turned to paid third parties to help them prepare their tax returns.[2] About half of all individual income tax returns and an even greater proportion of complex returns are now filed with the assistance of a professional income tax preparer (IRS, 1989; Jackson and Milliron, 1987).

The dramatic increase of paid third parties in the revenue collection process has begun to attract the attention of both the IRS and academic researchers. Scotchmer (1989a, 1989b) and Beck, Davis and Jung (1989) analyze theoretical models in which tax advisors reduce taxpayers' uncertainty about true taxable income. In Scotchmer's model the presence of tax advisors results in higher taxpayer welfare but lower compliance, and thus lower net revenue for the government. Beck, Davis and Jung describe a signaling model in which taxpayers have private information regarding their expected tax liabilities. In their model, the decision to consult a practitioner (in order to resolve residual uncertainty) conveys to the government additional information beyond that which would otherwise be conveyed by the individual's reporting behavior. The impact of practitioners on expected government revenue is ambiguous, however, since voluntary payments may either increase or decrease, while post-audit tax and penalty payments decrease with the use of practitioners. Klepper, Mazur, and Nagin (1988) analyze a nonstrategic model in which tax preparers can reduce the penalty rate faced by taxpayers who are discovered to be underreporting "ambiguous" income. In their model percentage noncompliance is greater for "unambiguous" income items on self-prepared returns than professionally prepared returns, while the opposite relationship holds for ambiguous income items. Finally, Reinganum and Wilde (1988, 1989) analyze game-theoretic models which focus on the "service" aspects of tax practitioners, such as reducing the costs of return preparation and providing an alternative to self-representation when a taxpayer is audited. In their basic model (Reinganum and Wilde, 1988), more income generally is reported when practitioners

1. The Tax Reform Act of 1986, for example, increased significantly the proportion of taxpayers who use the standard deduction rather than itemizing deductions and eliminated the distinction between capital gains and ordinary income.
2. An overview of the tax preparer industry in the United States may be found in Dubin, Graetz, and Wilde (1989).

control the reporting decision than if taxpayers did not use a practitioner, and the tax agency devotes less enforcement effort to practitioner-filed returns than to taxpayer-filed returns; the opposite relationships generally hold if taxpayers and the practitioner jointly control the reporting decision.[3] In Reinganum and Wilde (1989), which focuses on the case in which taxpayers control the reporting decision, a variety of outcomes are possible, depending on practitioner penalties and the efficiency gains from using practitioners. In the "policy relevant" case, the tax agency prefers that the taxpayer file his own return while the taxpayer prefers to use a practitioner. In this case, the use of a practitioner again implies less compliance compared to taxpayer-filed returns.

Our analysis here builds on this work and our own earlier work (Graetz, Reinganum and Wilde, 1986) by incorporating the tax agency as a strategic agent into a framework in which tax experts can reduce the penalty rate faced by taxpayers who take risky positions on ambiguous return items.[4] In particular, we analyze a situation in which legal ambiguity exists regarding the legitimacy of a specific deduction. By paying a fee, a taxpayer can solicit an expert opinion that provides a form of insurance against imposition of certain penalties in the event of an adverse ruling by the tax agency on the legitimacy of the deduction should an audit occur; for example, in the presence of a supporting opinion from a qualified tax expert, the taxpayer will be shielded from fraud penalties or certain other penalties that might otherwise apply.

By structuring the model in this manner we have captured one essential role played by tax practitioners in recent years. It has long been the case that an opinion letter from a tax attorney or accountant has served as a practical bar to the imposition of civil and criminal fraud penalties even though "advice of counsel" is not always a technical defense. In addition, professional advice concerning the use and nature of tax return disclosures of doubtful positions can serve to avoid certain penalties, for example, the penalty for "substantial understatements" of tax under § 6661 of the Internal Revenue Code. Indeed, during the heyday of the tax shelter industry in the 1970s, an Assistant Attorney General characterized tax advisers as, in effect, selling "fraud insurance."

In this paper we examine the incentives for taxpayers to claim risky deductions and to solicit expert opinions to support their positions, and for the tax agency to distinguish among individuals who do and do not solicit expert opinions for the purposes of auditing. We also consider the implications of an ex ante constraint on the tax agency which requires it to treat all taxpayers who take the deduction alike in terms of audit rates, whether or not they solicit an expert opinion.[5] Finally, we examine the effects of regulations which limit the degree of riskiness for which a supporting opinion can be justified as well as the effects of changes in various penalty rates.[6]

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3. Some empirical work related to the effects of paid third parties on compliance has also been done (e.g., Klepper and Nagin, 1987; Dubin, Graetz, and Wilde, 1989) although, as is often the case with empirical tax compliance research, the quality of the data limits significantly the usefulness of the results.

4. This description of the function of tax advisors is similar to that found in Klepper and Nagin (1987), but the model we propose is unique in its incorporation of strategic elements.

5. This may occur because the IRS is not able to observe whether such an opinion exists until they audit, although the IRS can know in advanced of audit whether or not the return was prepared by a paid professional preparer.

6. Limitations on the degree of riskiness for which supporting opinions may be given are sometimes imposed by the law or IRS administrative pronouncements (e.g., § 6661 of the Internal Revenue code or IRS Circular 230, 31 C.F.R. Part 10) or by professional standards of conduct (e.g., American Bar Association Formal Opinion 85–352, AICPA, Statement of Responsibility of Tax Practice).

We first establish the existence of equilibria in which (1) some taxpayers take risky positions without supporting opinions, some take risky positions with supporting opinions, and some do not take risky positions at all, and (2) the tax agency audits all returns filed in which taxpayers take risky positions, whether or not a supporting opinion is present. In these equilibria, increases in the penalty on taxpayers for whom the risky position is disallowed in the absence of a supporting opinion increase the number of taxpayers who take risky positions with supporting opinions and decrease expected revenue. To the contrary, increases in the penalty on taxpayers for whom the risky position is disallowed in the presence of a supporting opinion decrease the number of taxpayers who take risky positions with supporting opinions and increase expected revenue.

We further explore properties of the model using as an example the case in which taxpayers are evenly distributed between 0 and 1 with respect to the likelihood that the deduction would be disallowed in an audit. We show that the effects of increases in penalties on taxpayer behavior are stronger when some but not all taxpayers who take risky positions are audited. Increases in the tax rate can increase or decrease taxpayers' incentives to take risky positions but, in general, the less responsive are audit rates to increases in the tax rate, the more likely it is that increases in the tax rate will increase taxpayers' tendencies to take risky positions.

When the tax agency is prohibited from conditioning audit rates on the presence of a supporting opinion, expected revenue generally rises (compared to the unconstrained case) when the tax rate is low *and* the penalty on taxpayers for whom the risky position is disallowed in the absence of a supporting opinion is high relative to the tax rate. Expected revenue generally falls if either the tax rate is high *or* the penalty on unsupported risky positions is low relative to the tax rate. Since the most realistic presumption is one of a high tax rate and a relatively low penalty rate, we conclude that a prohibition on conditioning audit rates on the presence of a supporting opinion generally decreases expected government revenue.

Finally, we consider regulation of tax experts in the form of a limitation on the type of taxpayer for whom supporting opinions can be given. In particular, we assume that there is an upper bound on the likelihood that the deduction would be disallowed in an audit for which supporting opinions can be given. Such an upperbound may be set legislatively by penalties, administratively by rules of practice or through self–regulation by professional organizations. This upper bound captures, for example, mathematically the intention, but not the precise contours, of the ethical obligations of both the accounting and legal professions. (See, e.g., AICPA, Statements of Responsibilities in Tax Practice; ABA, Formal Opinion 85-352). Stricter regulation in terms of a reduction in this upper bound increases the range of equilibria in which tax experts are not used. However, over ranges in which auditing occurs and some taxpayers obtain expert opinions, stricter regulation increases expected revenue.

The paper is organized as follows. In section 2, we lay out the general model and state existence and comparative statics results for equilibria in which some taxpayers use tax experts and for equilibria in which none do. In section 3 we consider in detail the example in which taxpayers are distributed evenly between 0 and 1 with respect to the likelihood that the deduction would be disallowed in an audit. In section 4, using the example, we analyze audit constraints and regulation. Section 5 concludes with a discussion of our results and some of their policy implications.

2. THE MODEL

Our model builds directly on the game-theoretic approach introduced in Graetz, Reinganum and Wilde (1986) and further developed by Beck and Jung (1989a, 1989b). The reader is referred to Graetz, Reinganum and Wilde (1986) for a detailed discussion of the basic framework. In contrast to that paper, however, we herein specifically take the case of the legitimacy of a subtraction-to-income item, a deduction, instead of the underreporting of income. The primary innovation in this paper is the introduction of tax experts into a strategic model of tax compliance.

We consider the following stylized situation. A taxpayer has the option of claiming a particular deduction. If the deduction is taken and the taxpayer is audited, there is some chance that the tax agency will disallow the deduction and require the taxpayer to pay the tax due plus a penalty. The taxpayer, however, can pay a tax expert for a supporting opinion, in which case the penalty associated with an adverse judgement is reduced. We use the following notation:

t = the tax savings associated with taking the deduction,

k = the cost of a supporting opinion,

f_N = the penalty if the deduction is disallowed and *no* supporting opinion is present,

f_O = the penalty if the deduction is disallowed and a supporting opinion *is* present,

π = the probability that the deduction would be disallowed in an audit, and

c = the cost of an audit.

2A. TAX EXPERTS

Tax experts in this model are relatively passive; they simply provide supporting opinions for a fee.

Assumption 1: The cost of providing a supporting opinion is constant at k and is independent of the probability that the deduction would be disallowed in an audit.

Assumption 2: There is no upper bound on the probability that the deduction would be disallowed in an audit, π, for which supporting opinions may be rendered.

Both of these assumptions are relatively strong, but it is the assumption that a supporting opinion can be obtained regardless of the risk associated with the position which is the most unrealistic. However, in section 4B we consider the implications of introducing an upper bound on the degree of riskiness for which supporting opinions may be rendered, and will interpret reductions in this upper bound in terms of the regulation of tax experts. As noted in the introduction, in terms of policy, such limits may be set by legislation, by administrative rules, or by tax experts themselves.

2B. TAXPAYERS

Taxpayers act strategically in deciding whether to take the deduction and whether to purchase a supporting opinion if they do take the deduction. In this subsection we characterize taxpayer behavior, taking tax agency behavior (i.e., audit rates) as given.

Assumption 3: Taxpayers vary according to the probability that the deduction would be disallowed in an audit, which we refer to as their "exposure." The cumulative distribution of exposures is given by $F(\pi)$ with density $f(\pi)$, the latter which we take for expositional convenience to be strictly positive for all $\pi \in [0,1]$.

Assumption 4: A taxpayer's exposure is private information; in particular, the tax agency is unable directly to observe it. The tax agency can, however, condition its audit policy on whether the taxpayer takes the deduction, and whether a supporting opinion is present.

Given Assumption 4, we introduce the following notation:

p = the probability of audit given the deduction is not taken,

p_O = the probability of audit given the deduction is taken with a supporting opinion, and

p_N = the probability of audit given the deduction is taken without a supporting opinion.

Assumption 5: Taxpayers are risk neutral and minimize the sum of taxes plus costs of supporting opinions plus expected penalties; their total tax-related payments.

Define:

T = the tax-related payment when the deduction is *not* taken (independent of taxpayer exposure),

$T_O(\pi)$ = the tax-related payment when the deduction *is* taken by a taxpayer with exposure π, *with* a supporting opinion, and

$T_N(\pi)$ = the tax-related payment when the deduction *is* taken by a taxpayer with exposure π, *without* a supporting opinion.

Remark 1: Given Assumptions 1-5,

(a) $T = t$,

(b) $T_O(\pi) = p_O \pi(t + f_O) + k$,

(c) $T_N(\pi) = p_N \pi(t + f_N)$.

For some combinations of the tax rate, penalties, audit rates, and the cost of a supporting opinion, all taxpayers prefer to take the deduction without a supporting opinion. For other combinations, some of these taxpayers prefer to take the deduction with a supporting opinion, and

for yet other combination some of them prefer not to take the deduction at all. Finally, for still other combinations all three options are preferred by some taxpayers. The next results uses Remark 1 to characterize these cases.

Lemma 1: Given Assumptions 1-5, for arbitrary values of p_O and p_N, the following four choice configurations are exhaustive and mutually exclusive.

(a) If $p_N(t + f_N) \leq \min \{t, p_O(t + f_O) + k\}$ then taxpayers of all exposures find it optimal to take the deduction without a supporting opinion.

(b) If $p_O(t + f_O) + k < \min \{t, p_N(t + f_N)\}$, then taxpayers of all exposures find it optimal to take the deduction. Furthermore,

$\pi \leq k/[p_N(t + f_N) - p_O(t + f_O)]$ implies that no supporting opinion is optimal, and

$\pi > k/[p_N(t + f_N) - p_O(t + f_O)]$ implies that a supporting opinion is optimal.[7]

(c) If $p_O(t + f_O)t \geq (t - k)p_N(t + f_N)$ and $t < p_N(t + f_N)$ then no taxpayer types find it optimal to solicit an opinion. Furthermore,

$\pi \leq t/p_N(t + f_N)$ implies that taking the deduction without a supporting opinion is optimal, and

$\pi > t/p_N(t + f_N)$ implies that not taking the deduction at all is optimal.[8]

(d) If $p_N(t + f_N)k < t [p_N(t + f_N) - p_O(t + f_O)]$ and $t - k < p_O(t + f_O)$ then all three choice options are potentially optimal. In particular,

$\pi \leq k/[p_N(t + f_N) - p_O(t + f_O)]$ implies that taking the deduction with no supporting opinion is optimal,

$k/[p_N(t + f_N) - p_O(t + f_O)] < \pi < (t - k)/p_O(t + f_O)$ implies that taking the deduction with a supporting opinion is optimal, and

$\pi \geq (t - k)/p_O(t + f_O)$ implies that not taking the deduction is optimal.[9]

Inspection of Lemma 1 reveals that taxpayers' optimal decisions depend on their exposure in a very simple way. In particular, taxpayers with low exposures always take the deduction without a supporting opinion, taxpayers with higher exposures may take the deduction with a supporting opinion, and taxpayers with even higher exposures may not take the deduction at all. Thus, we can without loss of generality restrict the taxpayers' strategy to a pair, (π_1, π_2), with the following properties.

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7. If $\pi = k/[p_N(t + f_N) - po(t + f_O)]$ then the taxpayer is indifferent between soliciting and not soliciting a supporting opinion. We adopt the convention in this case that indifference is resolved in favor of not soliciting a supporting opinion.
8. If $\pi = t/p_N(t + p_N)$ then the taxpayer is indifferent between taking the deduction without a supporting opinion and not taking the deduction. We adopt the convention in this case that indifference is resolved in favor of taking the deduction with a supporting opinion.
9. In this case we resolve indifference in favor of *not* soliciting opinions or *not* taking the deduction.

Definition 1: A *strategy* for the taxpayer is given by a pair (π_1,π_2) with $\pi_1 \le \pi_2$ such that

    (a)   the deduction is taken without a supporting opinion if $\pi \le \pi_1$.

    (b)   the deduction is taken with a supporting opinion if $\pi_1 < \pi < \pi_2$

    (c)   no deduction is taken if $\pi \ge \pi_2$,[10]

Figure 1 illustrates the four choice configurations of Lemma 1 in terms of Definition 1.

choice configuration	taxpayer choices by type

a. all take deduction without opinion

b. deduction with or without opinion

c. deduction without opinion or no deduction
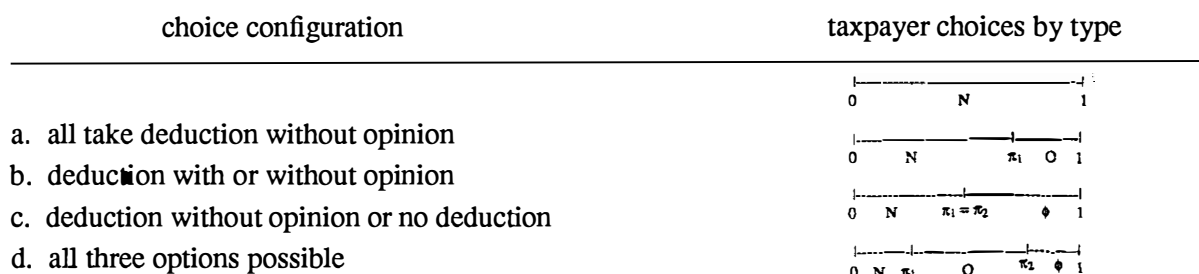
d. all three options possible

Figure 1: Choice configurations by taxpayer exposure

    N =   deduction without supporting opinion

    0   =   deduction with supporting opinion

    ϕ =   no deduction

Since we assume taxpayers minimize their tax related payments, it is necessary to describe an *optimal* strategy pair for any taxpayer given audit strategy on the part of the tax agency. We do this next. Since the optimal taxpayer strategy takes the strategy of the tax agency as given, it is call a "best response."

Definition 2: A *best response* for the taxpayer is a strategy pair (π_1,π_2) the elements of which minimize the taxpayer's tax–related payments.

Remark 2: It follows directly from Lemma 1 that a best response for the taxpayer takes the following form.

    (a)   If $p_O(t + f_O)t \ge (t - k)p_N(t + f_N)$ and $t < p_N(t + f_N)$ then

$$\pi_1 = t/p_N(t + f_N) = \pi_2.$$

    (b)   Otherwise

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10. Here we again resolve indifference in favor of *not* soliciting opinions or *not* taking the deduction at all when $\pi_1 < \pi_2$ (see footnote 9). If $\pi_1 = \pi_2$ we also assume the taxpayer of exposure $\pi = \pi_1$ takes the deduction without a supporting opinion as opposed to not taking the deduction at all (see footnote 8).

$$\pi_1 = k/[p_N(t + f_N) - p_O(t + f_O)]$$

and

$$\pi_2 = (t - k)/p_O(t + f_O).$$

This completes the description of taxpayer behavior, taking audit rates as given. We next describe tax agency behavior, leading up to a definition of a best response for the tax agency, taking taxpayer behavior as given. We then combine these best response definitions in section 2D to define an equilibrium for the game.

2C. THE TAX AGENCY

Our basic specification of the tax agency follows closely that of Graetz, Reinganum and Wilde (1986). Initially we allow the tax agency to condition its audit policy on whether taxpayers who take the deduction obtain a supporting opinion. Later, in section 4A, we analyze the case in which the tax agency is prohibited from conditioning its audit policy on whether a supporting opinion is present (i.e., it must set $p_N \equiv p_O$). In either case, we assume the tax agency acts strategically in setting audit rates.

Assumption 6: The tax agency is risk neutral and maximizes expected revenue (including penalties) net of audit costs. Audit costs are positive.

Assumption 7: The tax agency can set differential audit rates depending on whether the deduction is taken, and if it is, on whether a supporting opinion is present.

Define:

R = expected revenue from a taxpayer who does not take the deduction,
R_O = expected revenue from a taxpayer who takes the deduction with a supporting opinion, and
R_N = expected revenue from a taxpayer who takes the deduction without a supporting opinion.

The tax agency does not know the exposures of individual taxpayers but it does know how exposures are distributed over the population of taxpayers; that is, it knows $F(\pi)$. Because of this, and because the tax agency moves *second*, it can make inferences about the taxpayer's exposure for any return filed by observing whether a supporting opinion was obtained. However, it must behave optimally in light of that information.[11]

We next consider expected revenue for each of the four choice configurations given in Lemma 1 and displayed in Figure 1. Regardless of the choice configuration, if the tax agency sees a return with no deduction it expects revenue of $t - pc$, the tax due minus expected audit costs, since no

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11. It must play a so-called "subgame perfect strategy" (Selten, 1975). This rules out precommitment on the part of the tax agency to particular audit strategies. See Graetz, Reinganum and Wilde (1986) for an extensive discussion of this approach.

additional tax or penalty revenue can result from an audit. If it sees a return that takes the deduction without a supporting opinion, it expects no revenue unless it audits the return, in which case it expects to receive the additional tax and penalty associated with a judgement in favor of the tax agency, times the average likelihood of such a judgement given the exposure of taxpayers who choose to take the deduction without a supporting opinion, minus the cost of an audit. If the tax agency sees a return that takes the deduction with a supporting opinion, it forms a similar expectation of revenue, except in those cases in which it is not part of any taxpayer strategy to choose to take the deduction with a supporting opinion. In these cases the tax agency forms conjectures over what exposures would lead a taxpayer to choose that option even though it doesn't expect them to do so. These conjectures we denote by Π_a and Π_c respectively, for choice configurations a and c in Lemma 1.[12]

These observations are formalized in the next remark.

Remark 3: Given Assumptions 1-6, in all cases $R = t - pc$. Otherwise, for arbitrary values of π_1 and π_2 such that $\pi_1 \leq \pi_2$:

(a) If $p_N(t + f_N) \leq \min \{t, p_O(t + f_O) + k\}$ then

    (i) $R_O = p_O[(t + f_O)E(\pi | \pi \in \Pi_a) - c]$, and

    (ii) $R_N = p_N[(t + f_N)E(\pi) - c]$,

    where $E(\cdot)$ is the expectation operator and $\Pi_a \subset [0,1]$ is an arbitrary set of conjectures.

(b) if $p_O(t + f_O) + k < \min \{t, p_N(t + f_N)\}$ then

    (i) $R_O = p_O[(t + f_O)E(\pi | \pi_1 < \pi \leq 1) - c]$, and

    (ii) $R_N = p_N[(t + f_N)E(\pi | \pi \leq \pi_1) - c]$.

(c) If $p_O(t + f_O)t \geq (t - k)p_N(t + f_N)$ and $t < p_N(t + f_N)$, then

    (i) $R_O = p_O[(t + f_O)E(\pi | \pi \in \Pi_c) - c]$, and

    (ii) $R_N = p_N[(t + f_N)E(\pi | \pi \leq \pi_1) - c]$,

    where $\Pi_c \subset [0,1]$ is an arbitrary set of conjectures.

(d) If $p_N(t + f_N)k < t[p_N(t + f_N) - p_O(t + f_O)]$ and $t - k < p_O(t + f_O)$, then

    (i) $R_O = p_O[(t + f_O)E(\pi | \pi_1 < \pi < \pi_2) - c]$, and

    (ii) $R_N = p_N[(t + f_N)E(\pi | \pi \leq \pi_1) - c]$.

It is immediate from Remark 3 that the tax agency will never audit taxpayers that do not take the deduction (i.e., $p = O$ is always part of an optimal strategy for the tax agency). We can therefore, without loss of generality, describe strategies for the tax agency in terms of just p_N and p_O.

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12. In section 3 we characterize completely all possible equilibrium configurations when F is uniform on $[0,1]$. In that case we restrict attention to conjectures which are "universally divine" (Banks and Sobel, 1987). Universal divinity essentially restricts conjectures to the exposure of the taxpayer with the most to gain by deviating to the particular strategy.

Definition 3: A *strategy* for the tax agency is a pair of audit probabilities (p_N, p_O).

Since we assume that the tax agency maximizes expected revenue net of audit costs, it is necessary to describe an *optimal* strategy pair for the tax agency given any reporting strategy on the part of taxpayers. This optimal strategy is again referred to as a best response since it takes taxpayer strategies as given.

Definition 4: A *best response* for the tax agency is a strategy pair (p_N, p_O), the elements of which maximize R_N and R_O, respectively.

Inspection of Remark 3, which characterizes expected revenue for the tax agency in each of the choice configurations illustrated in Figure 1, reveals that expected revenue from a taxpayer who takes the deduction with a supporting opinion is always equal to the probability of auditing such a taxpayer times a term which depends only on the tax rate, the fine for disallowed deductions when a supporting opinion is present, tax agency expectations about exposures, and audit costs. In other words, it is always equal to the probability of audit times a term which does not depend on the probability of audit. If the term is positive, then an increase in the audit rate always increases expected revenue, if the term is zero then expected revenue is always zero, and if the term is negative then an increase in the audit rate always decreases expected revenue. Thus the optimal audit rate for taxpayers who take the deduction with a supporting opinion will be 1, any value between 0 and 1, or 0 depending on whether the relevant term in Remark 2 is positive, zero, or negative. Similarly, it is clear that a best response for the tax agency is to audit all returns filed with the deduction but no supporting opinion whenever the relevant term which multiplies p_N in Remark 2 is positive, to audit any fraction of such returns whenever it is zero, and to audit none whenever it is negative. These observations are summarized formally as follows.

Lemma 2: Denote the term which multiplies p_O in each of the expressions for R_O in Remark 3 by r_{oi}, and the term which multiplies P_N in each of the expressions for R_N in Remark 3 by r_{Ni}, where $i \in \{a, b, c, d\}$. Then a best response for the tax agency is

$$p_j \begin{cases} = 0 \\ \in [0,1] \\ = 1 \end{cases} \text{ as } r_{ji} \begin{cases} < \\ = \\ > \end{cases} 0$$

for all $j \in \{O, N\}$ and $i \in \{a, b, c, d\}$.

2D. EQUILIBRIUM: EXISTENCE AND GENERAL PROPERTIES

We now define an equilibrium and state some general results. Since many equilibria can involve "corner values" for audit probabilities or critical taxpayer exposures (zeros or ones for p_N, p_O, π_1, or π_2), we will not fully characterize all possible equilibria for the general case. This will be done for a specific example in section 3 below.

Definition 5: An *equilibrium* is a set of strategy pairs $\{(p_N^*, p_O^*), (\pi_1^*, \pi_2^*)\}$ which are best responses to each other.

In terms of the existence of equilibria in the general case, we are most interested in those for which some taxpayer types find it optimal to take the deduction with a supporting opinion. It turns out that if $f_N - f_O > k$, then there always exists an equilibrium in which tax experts are actually used. That $f_N - f_O > k$ is a very weak condition; it merely requires that the savings from taking the deduction with a supporting opinion compared to taking it without a supporting opinion exceeds the cost of soliciting the supporting opinion. If this were not the case, clearly no taxpayers would solicit a supporting opinion.

Proposition 1: Let f_N and f_O be such that $f_N - f_O > k$. Then given Assumptions 1-6, there exists $\tau(f_N, f_O)$ such that an equilibrium with $O < \pi_1^* < \pi_2^* < 1$ and $p_N^* = 1 = p_O^*$ exists for all $t > \tau(f_N, f_O)$. In this equilibrium $\pi_1^* = k/(f_N - f_O)$ and $\pi_2^* = (t - k)/(t + f_O)$.[13]

In the equilibrium of Proposition 1, the tax agency audits all returns that take the deduction and no returns that do not.[14] A change in penalties, if it is small enough, will thus have no effect on audit rates. But a change in penalties can affect which taxpayers take the deduction and which solicit supporting opinions.

Corollary 1: Suppose $f_N - f_O > k$ and $t > \tau(f_N, f_O)$. Then in the equilibrium of Proposition 1, $\partial \pi_1^*/\partial f_N < O = \partial \pi_2^*/\partial f_N$ and $\partial \pi_1^*/\partial f_O > O > \partial \pi_2^*/\partial f_O$.

An increase in the penalty for disallowed deductions in the absence of a supporting opinion has no effect on who takes the deduction, but of those taxpayers who do, it causes more of them to solicit a supporting opinion.[15] An increase in the penalty for disallowed deductions in the presence of a supporting opinion causes fewer taxpayers to take the deduction, and of those who do, it causes fewer to solicit a supporting opinion. Thus, changes in the two kinds of penalties in this model have quite different effects; an increase in the penalty for disallowed deductions in the absence of a supporting opinion increases the range of taxpayers who use a tax expert and an increase in the penalty for disallowed deductions in the presence of a supporting opinion decreases it. Even in the

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13. Assumption 7, that the tax agency can set $p_N \neq p_O$, is not needed here since we consider only equilibria in which $p_N = 1 = p_O$.

14. This need not always be the case—for example, in Section 3 where we assume F is uniform on $[0,1]$, we describe an open set of parameter values for which $O < \pi_1^* < \pi_2^* < 1$, $O < p_N^* < 1$ and $O < p_O^* < 1$.

15. This, for example, may be one of the consequences of the rules for imposition of the 25 percent penalty on substantial understatements of tax under § 6661 of the Internal Revenue Code.

absence of an equilibrium adjustment in audit rates, one might therefore expect that increases in some penalties may potentially reduce average expected revenue. Indeed, denoting average expected revenue by ER, we have the following result.

Corollary 2: Suppose $f_N - f_O > k$ and $t > \tau(f_N, f_O)$. Then in the equilibrium of Proposition 1, $\partial ER / \partial f_N < O$ and $\partial ER / \partial f_O > O$.

In the equilibrium of Proposition 1, increases in the penalty for disallowed deductions when no supporting opinion is present actually decrease expected revenue while, as one might have predicted, increases in the penalty for disallowed deductions when a supporting opinion is present increase expected revenue.

Finally, we note in passing that in the equilibrium of Proposition 1, increases in audit costs have no effect on taxpayer behavior (because audit rates are fixed at 1), and increases in the cost of soliciting a supporting opinion both decrease the range of taxpayers who take the deduction, and of those who still do take the deduction, decrease the range who solicit a supporting opinion.

It is also possible to show that under relatively weak conditions there exist equilibria in which no taxpayers solicit expert opinions, but some still take the deduction and auditing is nontrivial.

Proposition 2: Suppose $c < \min \{(t + f_O)t/(t + f_N), (t + f_N)E(\pi \mid \pi \le t/(t + f_N))\}$ and $k > (f_N - f_O)t/(t + f_N)$. Then there exists a universally divine equilibrium with $O < \pi_1^* = \pi_2^* < 1$ and $p_N^* = 1 = p_O^*$. In this equilibrium $\pi_1^* = t/(t + f_N) = \pi_2^*$.[16]

The conditions for the existence of an equilibrium in which some but not all taxpayers take the deduction but none solicit a supporting opinion are quite natural. To rule out use of tax experts it suffices that the cost of using them is high (k is large) or the benefits of using them low ($f_N - f_O$ is small). At the same time it must be that the cost of auditing is low enough to prevent all taxpayers from taking the deduction (c is small).[17]

3. THE UNIFORM DISTRIBUTION OF EXPOSURES

One of the important aspects of using game-theoretic techniques to analyze tax compliance is that it allows audit rates to be endogenous. We would therefore like to investigate comparative statics similar to those given in Corollaries 1 and 2 for equilibria in which one or both audit rates are less than 1. This is difficult to do in the general case so we now characterize fully all equilibria for a specific distribution of taxpayer exposures, the uniform.[18] This also will allow us to consider the

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16. See footnote 10 for a discussion of universal divinity.

17. Indeed, the assumption here is that audit costs to taxpayers are zero except for the costs of penalties where imposed. As audit costs to taxpayers increase, fewer taxpayers should take the deduction (since only those taxpayers who take the deduction will be subject to these costs), and thus the requirement that c be small is weakened.

18. In the uniform distribution, exposure levels are evenly distributed between 0 and 1. This distribution of exposures is both computationally convenient and "neutral" in the sense that it puts equal weight on all exposures, and thus introduces no particular biases into the analysis.

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effects of prohibiting the tax agency from conditioning its audit policy on whether a supporting opinion is present (i.e., constraining it to set $p_N \equiv p_O$), and the effects of regulation of tax experts in the form of an upper bound on the set of taxpayer exposures for which a supporting opinion can be given (say, $\bar{\pi}$, where $O < \bar{\pi} < 1$). These latter issues we take up in section 4 below.

Assumption 8: $F(\pi) = \pi$ for all $\pi \in [0,1]$.

Assumption 9: $c > k + f_O$.

Assumption 9, that the cost of an audit is greater than the cost of a supporting opinion plus the penalty on disallowed deductions with a supporting opinion, is made for expositional and diagrammatic convenience only. It allows us to solve explicitly for equilibrium values of π_N, π_O, p_N and p_O in all cases, and maximizes the likelihood that equilibria in which tax experts are used will exist. These, of course, are the equilibria in which we are most interested.

Table 1 gives equilibrium values for π_N, π_O, p_N, and p_O for each of 8 distinct equilibrium types, labeled $E\,1$–$E\,8$. It also lists the effective constraints that characterize the set of parameters $\{t, f_N, f_O, c, k\}$ for which each equilibrium type exists.[19] Figure 2 illustrates the range of values of t and f_N for which each equilibrium type exists given $f_O = 0$, $k = 3$, and $c = 5$. As the figure clearly indicates, the eight equilibrium types are exhaustive and mutually exclusive when $f_O = O$. Of particular interest to us, though, are type $E\,1$ and $E\,2$ equilibria when $f_O > 0$. This is because in type $E\,1$ equilibria no endogenous variables (π_N, π_O, p_N and p_O) are equal to 0 or 1. Thus the full range of interactions are possible whenever there is a change in an exogenous parameter. In type $E\,2$ equilibria, the presence of a supporting opinion automatically triggers an audit, yet taxpayers still use tax experts. Indeed, equilibria types $E\,1$–$E\,4$ are all of particular interest in so far as they involve some taxpayers using tax experts.

Proposition 3: Given Assumptions 1-9, there exist nontrivial but disjoint sets of parameter values for which equilibria of type $E\,1$ and $E\,2$ exist, respectively.

Table 2 gives comparative statics results for type $E\,1$ equilibria.

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19. We shall not provide a case-by-case proof of the conditions specified in Table 1. The logic generally follows that of Propositions 1 and 2. A copy of the proofs that these are the relevant constraints and that no other equilibrium types are possible is available from the authors on request.

Table 1: Equilibrium Types, Optimal Strategies and Effective Constraints
When F is Uniform on [0,1] and $k + f_0 < c$.

Equilibrium Type	Optimal Strategies	Effective Constraints	Reason for Constraint
$E1$	$\pi_1^* = \dfrac{2c}{t+f_N}$	1a. $t < f_N - 2f_O$	$\pi_1^* < \pi_2^*$
	$\pi_2^* = \dfrac{2c(f_N - f_O)}{(t+f_N)(t+f_O)}$	1b. $2c(f_N - f_O) < (t+f_N)(t+f_O)$	$\pi_2^* < 1$
	$p_N^* = \dfrac{(t-k)(t+f_O)+k(f_N-f_O)}{2c(f_N-f_O)}$	1c. $(t-k)(t+f_N) < 2c(f_N-f_O)$	$p_O^* < 1$
	$p_O^* = \dfrac{(t-k)(t+f_N)}{2c(f_N-f_O)}$	note: 1a and 1b $\Rightarrow p_O^* > 0$. 1a and 1c $\Rightarrow p_N^* < 1$.	
$E2$	$\pi_1^* = \dfrac{2c}{t+f_N}$	2a. $(t-k)(t+f_N) \geq 2c(f_N-f_O)$	$p_O^* = 1$
	$\pi_2^* = \dfrac{t-k}{t+f_O}$	2b. $2c(t+f_O) < (t-k)(t+f_N)$	$\pi_1^* < \pi_2^*$
	$p_N^* = \dfrac{2c(t+f_O)+k(t+f_N)}{2c(t+f_N)}$	2c. $k(t+f_N) < 2c(f_N-f_O)$	$p_N^* < 1$
	$p_O^* = 1$	note: 2a and 2b $\Rightarrow \pi_2^* < 1$	
$E3$	$\pi_1^* = \dfrac{2c}{t+f_N}$	3a. $(t+f_N)(t+f_O) \leq 2c(f_N-f_O)$	$p_O^* = 0$
	$\pi_2^* = 1$	3b. $k \leq t$	$\pi_2^* = 1$
	$p_N^* = \dfrac{k}{2c}$	3c. $2c < t+f_N$	$\pi_1^* < \pi_2^*$
	$p_O^* = 0$	note: $k < c \Rightarrow p_N^* < 1$.	
$E4$	$\pi_1^* = \dfrac{k}{f_N - f_O}$	4a. $k(t+f_N) \geq 2c(f_N-f_O)$	$p_N^* = 1$
	$\pi_2^* = \dfrac{t-k}{t+f_O}$	4b. $k(t+f_O)+(t-k)(f_N-f_O) \geq 2c(f_N-f_O)$	$p_O^* = 1$
	$p_N^* = 1$	4c. $kt < (t-k)f_N - tf_O$	$\pi_1^* < \pi_2^*$
	$p_O^* = 1$	note: 4c $\Rightarrow \pi_1^* > 0$	
$E5$	$\pi_1^* = \dfrac{2c}{t+f_N} = \pi_2^*$	5a. $t < 2c$	$p_1^* < 1$
	$p_N^* = \dfrac{t}{2c}$	5b. $2c < t+f_N$	$\pi_1^* < 1$
	$p_O^* = 1$	5c. $t \geq f_N - 2f_O$	$p_O^* = 1$
		5d. $(t-k)(t+f_N) \leq 2c(t+f_O)$	no expert opinions

Table 1: Equilibrium Types, Optimal Strategies and Effective Constraints
When F is Uniform on $[0,1]$ and $k + f_0 < c$. (Cont.)

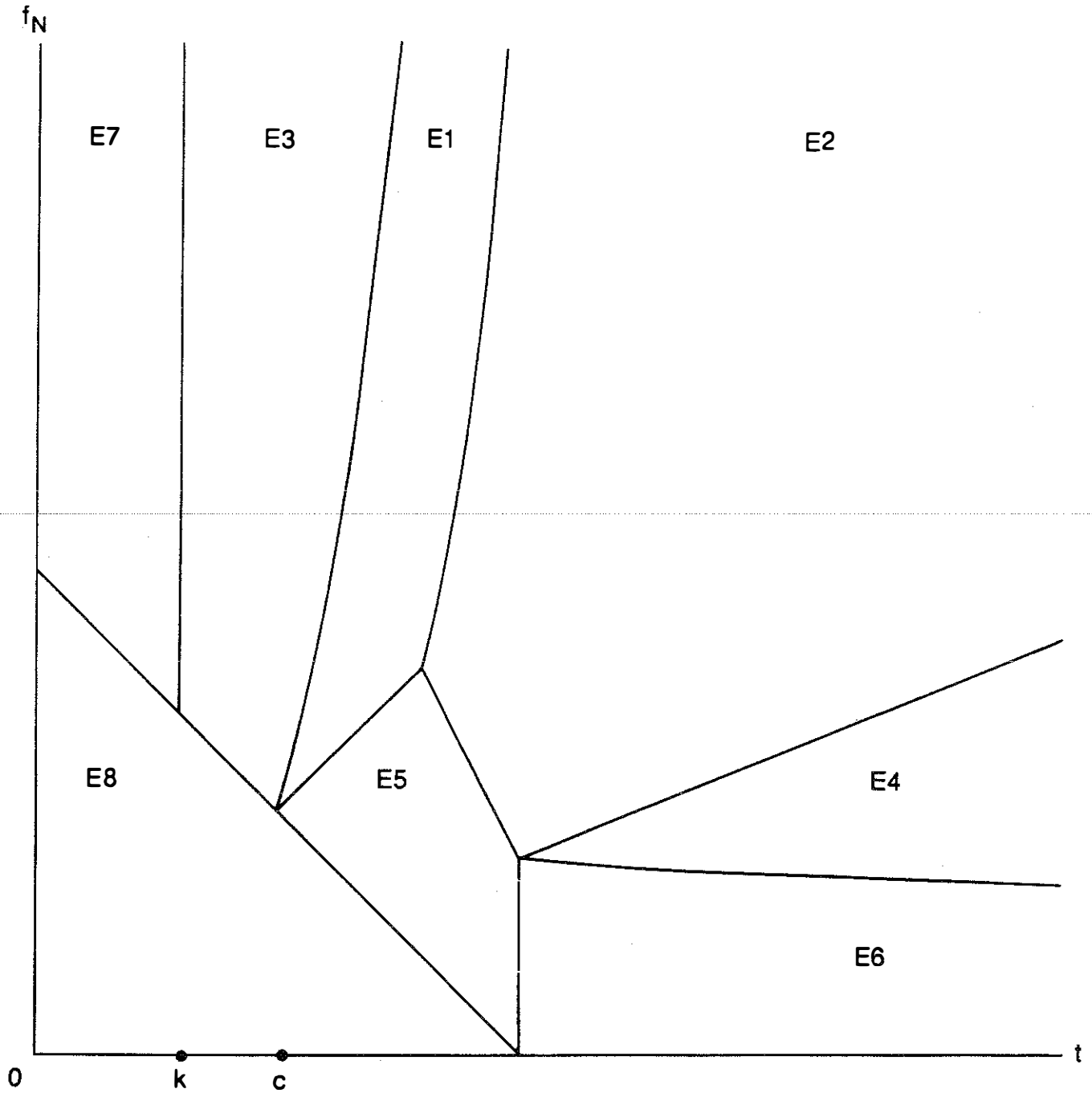| Equilibrium Type	Optimal Strategies	Effective Constraints	Reason for Constraint
$E6$	$\pi_1^* = \dfrac{t}{t + f_N} = \pi_2^*$	6a. $t \geq 2c$	$p_N^* = 1$
	$p_N^* = 1$	6b. $(t + f_O)t \geq c(t + f_N)$	$p_O^* = 1$
	$p_O^* = 1$	6c. $t(t + f_O) \geq (t - k)(t + f_N)$	no expert opinions
$E7$	$\pi_1^* = \dfrac{2c}{t + f_N} = \pi_2^*$	7a. $2c < t + f_N$	$\pi_1^* < 1$
	$p_N^* = \dfrac{t}{2c}$	7b. $t < 2c$	$p_N^* < 1$
	$p_O^* = 0$	7c. $t \leq f_N - 2f_O$	$p_O^* = 0$
		7d. $k \geq t$	no expert opinions
$E8$	$\pi_1^* = 1 = \pi_2^*$	8a. $t + f_N \leq 2c$	$p_N^* = 0$
	$p_N^* = 0$	8b. $t + f_O \leq 2c$	$p_O^* = 0$
	$p_O^* = 0$		

Figure 2: Equilibria when $f_O = O$, $k = 3$, $c = 5$, and F is uniform on $[0, 1]$.
Equilibrium types E1 - E4 involve the use of tax experts, E5 - E8 do not.
In Equilibrium type E8 there is no auditing and all taxpayers take the
deduction without a supporting opinion.

Table 2: Signs of Effects of Changes in Parameters on Equilibrium
Strategies: Type E 1 Equilibrium; F Uniform on $[0,1]$, $k + f_0 < c$.

		parameter			
strategy variable	f_N	f_O	c	k	t
π_1^*	–	O	+	O	–
π_2^*	+	–	+	O	–
p_N^*	–	+	–	+	+
$p_{\mathcal{O}}^*$	–	+	–	–	+
$\pi_2^* - \pi_1^*$	+	–	+	O	–

In the equilibria of Proposition 3 in which $O < \pi_1^* < \pi_2^* < 1$, $O < p_N^* < 1$, and $0 < p_o^* < 1$ (type E 1 equilibria), increases in the penalty for disallowed deduction in the absence of a supporting opinion decrease the range of taxpayer types who take the deduction without a supporting opinion and increase the range who take it with a supporting opinion. The increase in the range of taxpayers who take the deduction with a supporting opinion comes at the expense of both taxpayers who take the deduction without a supporting opinion and taxpayers who do not take the deduction at all. The reason why some taxpayers shift from not taking the deduction to taking it with a supporting opinion when the penalty for disallowed deductions in the absence of a supporting opinion increases, is that the latter causes the audit rate on taxpayers who take the deduction with a supporting opinion to fall. The audit rate falls, in turn, because the average exposure of a taxpayer who takes the deduction with a supporting opinion falls when the penalty for disallowed deduction in the absence of a supporting opinion increases.[20]

On the other hand, increases in the penalty on disallowed deductions in the presence of a supporting opinion have no effect on the range of taxpayers who take the deduction without a supporting opinion and decrease the range who take it with a supporting opinion, the latter solely at the expense of taxpayers who do not take the deduction at all.[21]

Increases in the cost of an audit increase the range of taxpayer types who take the deduction, with and without a supporting opinion, and decrease audit rates for both types of return. Increases in the cost of a supporting opinion have no effect on equilibrium taxpayer behavior at all due to balancing increases in p_N^* and decreases in $p_{\mathcal{O}}^*$. Finally, increases in the tax rate *decrease* the range of taxpayer types who take the deduction, with and without a supporting opinion, and *increase* audit rates for both types of return. In other words, for equilibria in which some taxpayers do not take the

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20. Compare, for example, this result to Corollary 1 where $p_N^* = 1 = p_o^*$.
21. Again, compare this result to Corollary 1.

deduction, some take it with a supporting opinion, and some take it without a supporting opinion, and auditing is stochastic, with respect to those taxpayers who do take the deduction, increases in the tax rate *decrease* taxpayers' tendencies to take risky positions on ambiguous deductions.[22]

One troublesome feature of Proposition 3 is that existence of Type $E1$ equilibria requires $f_N - 2f_O > t$ (see Table 1, constraint 1a). Even if $f_O = O$ this condition is strong. Table 3 gives comparative statics results for type $E2$ equilibria, in which $O < \pi_1^* < \pi_2^* < 1$ and $O < p_N^* < 1 = p_{\partial}^*$. Given Assumptions 1-9, this type of equilibrium can be shown to exist over a range of values of f_N less than t.

Table 3: Signs of Effects of Changes in Parameters on Equilibrium
Strategies: Type $E2$ equilibria; F Uniform on $[0,1]$, $k = f_O < c$.

strategy variable	f_N	f_O	c	k	t
			parameter		
π_1^*	–	O	+	O	–
π_2^*	O	–	O	–	+
p_N^*	–	+	–	+	+
p_{∂}^*	O	O	O	O	O
$\pi_2^* - \pi_1^*$	+	–	–	–	+

Comparative statics for type E2 equilibria are identical to type $E1$ equilibria for π_1^* and p_N^*. And in type $E2$ equilibria $p_{\partial}^* = 1$ by definition; i.e., all taxpayers who take the deduction with a supporting opinion are audited. Thus the significant differences between the comparative statics of type $E1$ and type $E2$ equilibria are all with respect to π_2^*. In particular, in type $E2$ equilibria, increases in the cost of a supporting opinion decrease π_2^*, and increases in the tax rate increase it. Thus, increases in the tax rate can *increase* taxpayers' tendencies to take risky positions on ambiguous deductions, depending on the type of equilibrium. In general, the *less* responsive are audit rates to increases in tax rates, the more *likely* that increases in the tax rate will increase taxpayers' tendencies to take risky positions.

Finally, we note from Table 1 that there is no particular relationship between audit rates across equilibrium types. In type $E2$ equilibria $p_N^* < p_{\partial}^*$ by definition, in type $E3$ equilibria $p_N^* > p_{\partial}^*$ by definition, and in type $E4$ equilibria $p_N^* = p_{\partial}^*$. In all of these equilibrium types some taxpayers

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22. This result is similar to the finding in Graetz, Reinganum and Wilde (1986) that increases in the tax rate *increase* compliance. To the extent that audit rates do not respond to increases in the tax rate, the result may fail to hold in either context.

actually solicit expert opinions.

4. ADDITIONAL ISSUES: AUDIT CONSTRAINTS AND REGULATION

In this section, working in the context of the uniform distribution example presented in Section 3, we explore two policy-related issues. In subsection 4A we consider the effects of prohibiting the tax agency from conditioning its audit policy on whether a supporting opinion is solicited and in subsection 4B we consider the effects of the regulation of tax experts by setting an upper bound on the exposure for which supporting opinions can be given.

4A. AUDIT CONSTRAINTS

In this subsection we replace Assumption 7, that the tax agency can set differential audit rates, with the following.

Assumption 10: The tax agency must set $p_N = p_O$.

We denote the constrained audit rate by p_c. Table 4 gives equilibrium values for π_N, π_O and p_c for each of 5 distinct equilibrium types, three of which are present in the unconstrained case ($E4$, $E6$, and $E8$—see Table 1), and two of which are new ($E9$ and $E10$).[23] Like Table 1, Table 4 also lists the effective constraints that characterize the set of parameters $\{t, f_N, f_O, c, k\}$ for which each equilibrium type exists. These equilibrium types are exhaustive and mutually exclusive. This is true in general but can also be seen in Figure 3, which illustrates the case of $f_O = O$, $k = 3$ and $c = 5$.

We are primarily interested in analyzing the revenue effects of replacing Assumption 7 with Assumption 10, i.e., constraining the tax agency to set $p_N = p_O$. To do this we must be able to link equilibrium types before and after the regime switch. It turns out that when $f_O = O$ and $2k > c$, the qualitative configuration of the equilibrium types illustrated in Figures 2 and 3 are "generic," and we can readily link the various equilibrium types.

Proposition 4: Given Assumptions 1-6 and Assumptions 8-9, if $f_O = O$ and $2k > c$, then expected revenue with constrained audits (Assumption 10) can be greater than, equal to, or less than expected revenue with unconstrained audits (Assumption 7). In particular,

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23. In fact, the equilibrium type labeled $E10$ in Table 4 actually is very similar to equilibrium type $E7$ in Table 1. The difference is that the tax agency is constrained to set $p_O^* = p_N^*$ in the former case so sequential rationality is not an issue— the tax agency never expects to see a return taking the deduction with a supporting opinion so given that it cannot set p_O independently of p_N, it sets p_c^* according to R_N only. If it sees a return taking the deduction with a supporting opinion it must audit it with probability p_c^*. Thus constraint 10c is different than constraint 7d, both of which stem from the requirement that taxpayers must not want to take the deduction with a supporting opinion. Equilibrium types $E6$ and $E8$ each have one less constraint in the constrained case compared to the unconstrained case for similar reasons.

Table 4: Equilibrium Types, Optimal Strategies and Effective Constraints
When F is Uniform on $[0,1]$, $k + f_0 < c$ .and $p_N = p_O$ by constraint

Equilibrium Type	Optimal Strategies	Effective Constraints*	Reason for Constraint
$E4$	$\pi_1 = \dfrac{k}{f_N - f_O}$	4c. $kt < (t - k)f_N - tf_O$	$\pi_1^* < \pi_2^*$
	$\pi_2^* = \dfrac{t - k}{t + f_O}$	4d. $k^2(t + f_O) \geq (t - k)(f_N - f_O)(2c - t + k)$	$p_c^* = 1$
	$p_c^* = 1$	note: $4c \Rightarrow \pi_1^* > 0$	
$E6$	$\pi_1^* = \dfrac{t}{t + f_N} = \pi_2^*$	6a. $t \geq 2c$	$p_c^* = 1$
	$p_c^* = 1$	6c. $t(t + f_O) \geq (t - k)(t + f_N)$	no expert opinions
$E8$	$\pi_1^* = 1 = \pi_2^*$	8a. $t + f_N \leq 2c$	$p_c^* = 0$
	$p_c^* = 0$		
$E9$	$\pi_1^* = \dfrac{2ck(t - k)}{k^2(t + f_N) + (f_N - f_O)t(t - 2k)}$	9a. $kt < (t - k)f_N - tf_O$	$\pi_1^* < \pi_2^*$
	$\pi_2^* = \dfrac{2c(f_N - f_O)(t - k)^2}{(t + f_O)[k^2(t + f_N) + (f_N - f_O)t(t - 2k)]}$	9b. $k^2(t + f_O) < (t - k)(f_N - f_O)(2c - t + k)$	$p_c^* < 1$
	$p_c^* = \dfrac{k^2(t + f_N) + (f_N - f_O)t(t - 2k)}{2c(f_N - f_O)(t - k)}$	note: $9a \Rightarrow \pi_1^* > 0$	
$E10$	$\pi_1^* = \dfrac{2c}{t + f_N} = \pi_2^*$	10a. $2c < t + f_N$	$\pi_1^* < 1$
	$p_c^* = \dfrac{t}{2c}$	10b. $t < 2c$	$p_c^* < 1$
		10c. $t(t + f_O) \geq (2c - t)(t + f_N)$	no expert opinions

* Constraint labels match Table 1 whenever they are the same.
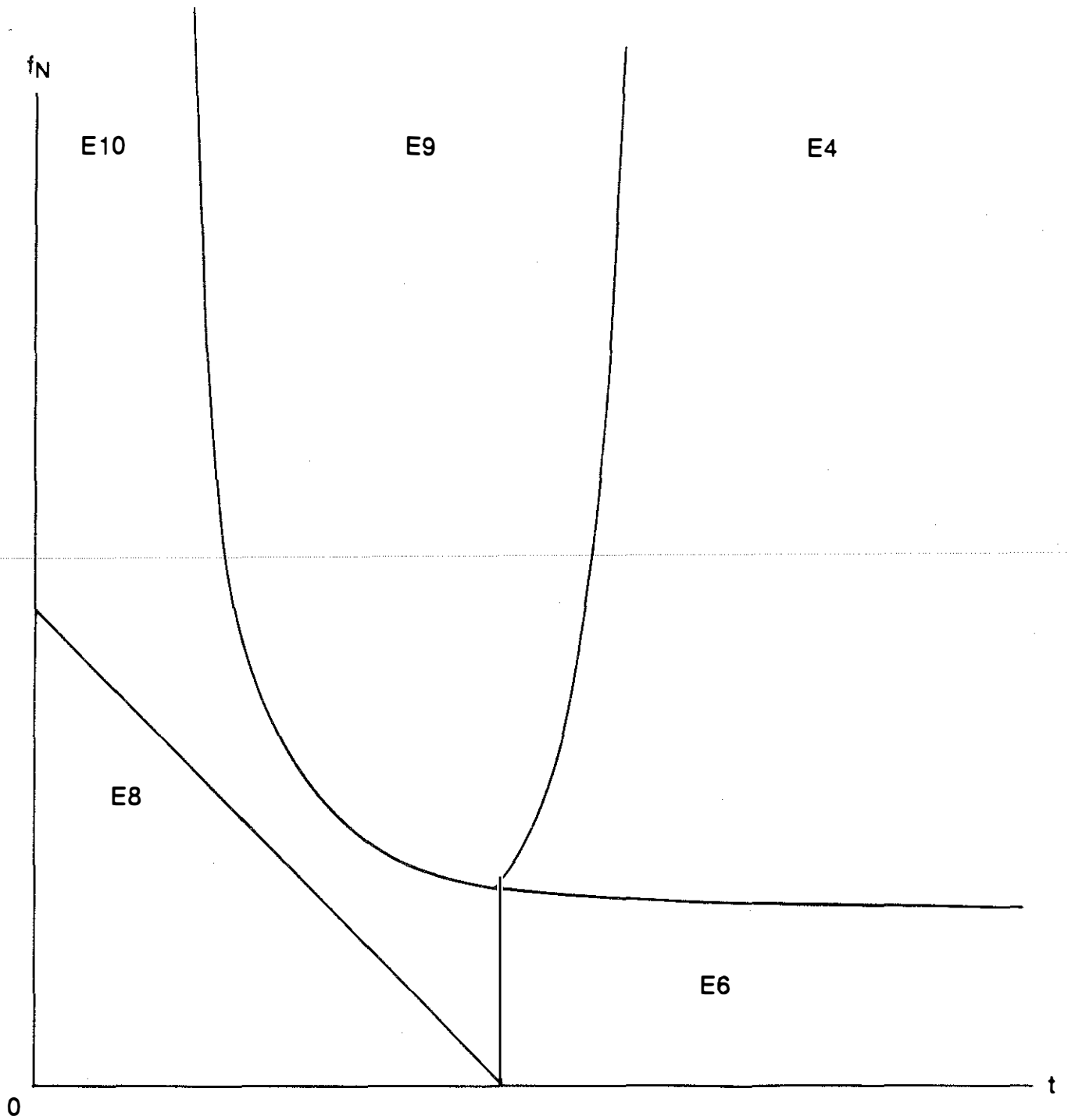
Figure 3: Equilibria when $f_O = 0$, $k = 3$, $c = 5$, F is uniform on [0, 1] and $p_N = p_O$ by assumption. Equilibrium types E4 and E9 involve the use of tax experts; E6, E8 and E10 do not.

(a) For initial type $E\,3$ equilibria, expected revenue always increases under the regime switch,

(b) For initial type $E\,1$ equilibria, expected revenue increases, stays the same, or decreases under the regime switch as t is less than, equal to, or greater than $2k$.

(c) For initial type $E\,2$ equilibria, expected revenue always decreases under the regime switch; and

(d) For initial type $E\,4$ equilibria, expected revenue is constant under the regime switch.

Figure 4 shows the regions where expected revenue increases, decreases or stays the same after audits are constrained for those equilibria in which tax experts are used in the unconstrained case. The reason why expected revenue does not always fall is that the constraint that $p_N = p_O$ can allow a form of precommitment. In some cases, the ability to precommit in this form can allow the tax agency actually to increase its expected revenue.[24]

Corollary 3: Under the conditions of Proposition 4, compared to unconstrained equilibria in which some taxpayers solicit supporting opinions, expected revenue generally will rise when the tax agency is constrained not to base its audit policy on whether a supporting opinion is present if the tax rate is low ($t < 2c$) *and* the penalty rate for disallowed deductions in the absence of a supporting opinion is high relative to the tax rate ($f_N > t$). Expected revenue generally will fall (or stay constant) when the tax rate is high ($t > 2c$) *or* the penalty rate is low relative to the tax rate ($f_N < t$).

4B. REGULATION OF TAX EXPERTS

As we noted in Section 2A, the assumption that tax experts can render supporting opinions which protect their clients from some penalties for any degree of riskiness is quite strong. In this subsection we relax that assumption by introducing a fixed upper bound on the exposure for which supporting opinions may be rendered. We then investigate the effects of decreases in that upper bound, which we interpret in terms of regulation of tax experts, either through self–regulation or by legislation or administrative rules. This analysis permits us to consider the existence of limits on the riskiness of taxpayer positions for which supporting opinions can be obtained and the effects of shifts in that degree of riskiness, issues which in recent years have been in the forefront of debates concerning both the structure and level of legislative tax penalties and the proper administrative or ethical constraints on the legal and accounting professions.

Assumption 11: There is an upper bound, $\bar{\pi}_2$, where $0 < \bar{\pi}_2 < 1$ such that no supporting opinion may be given for taxpayer exposures greater than $\bar{\pi}_2$.

Proposition 5: Suppose Assumptions 1, 3-9, and 11 hold. Suppose also that $f_O = 0$, $2c > k$, and $\bar{\pi}_2 > (2c - k)/2c$. Then the following equilibrium types in which some taxpayers solicit supporting opinions are possible.
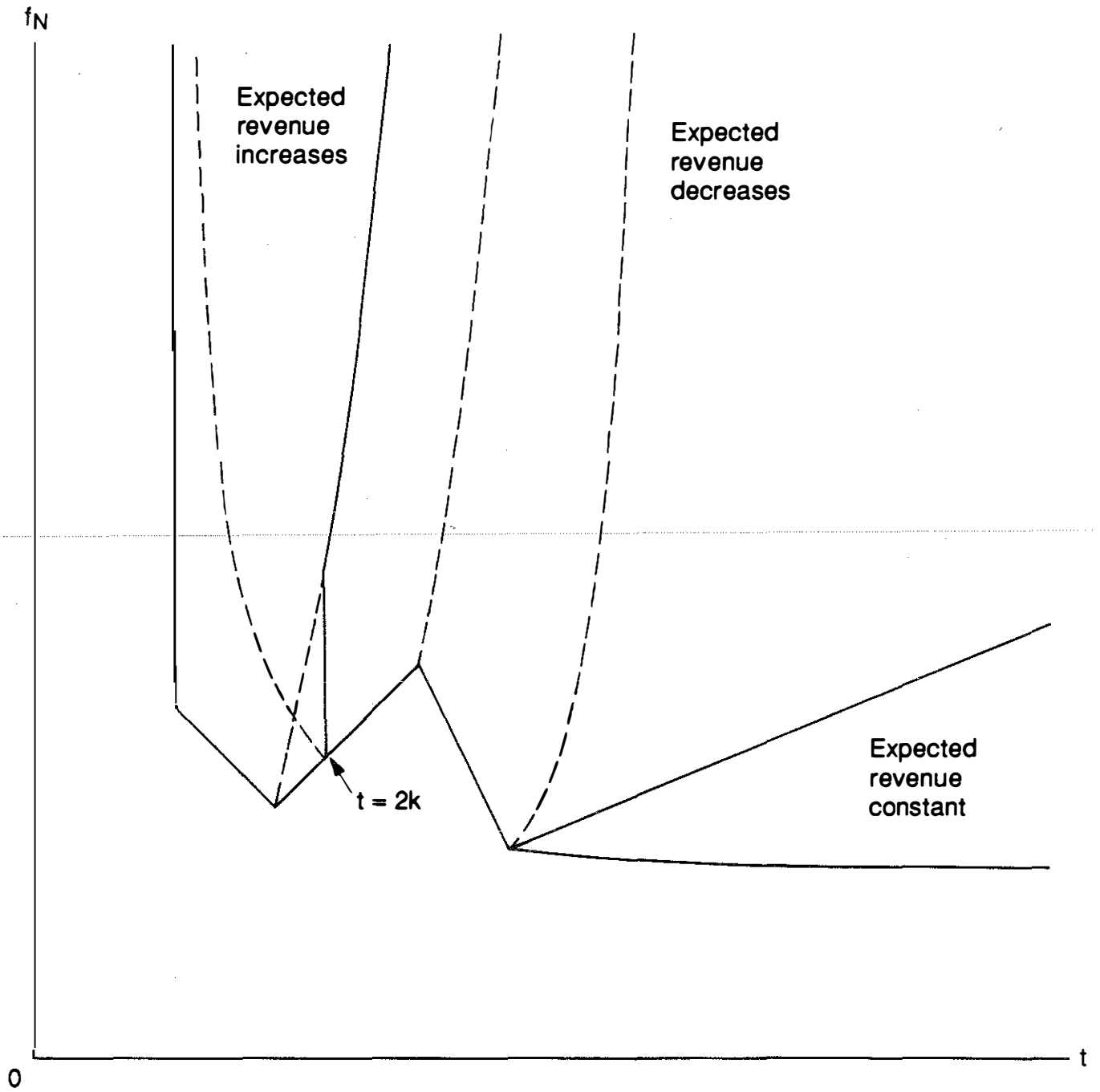
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24. See footnote 11 and text supra.

Figure 4: Effects on expected revenue of constrained audits ($p_N = p_O$) in cases where tax experts play a nontrivial role in the initial equilibrium.

(a)   Initial (unconstrained) equilibrium types $E\,1, E\,2$, and $E\,4$.

(b)   Constrained versions of initial equilibrium types $E\,2$ and $E\,4$ in which $\pi_2^* = \bar{\pi}_2$,

(c)   A new equilibrium type (E11) in which $p_N^* = k/2c$, $p_\sigma^* = 0$, $\pi_1^* = 2c/(t + f_N)$ and $\pi_2^* = \bar{\pi}_2$.

Proposition 5 is illustrated in Figure 5. As $\bar{\pi}_2$ falls, the range of values of t and f_N for which tax experts are used decreases. However, in cases where decreasing the upper bound on the degree of riskiness for which supporting opinions can be given does not eliminate the use of tax experts altogether, it tends to increase expected revenue.

5. POLICY IMPLICATIONS

It has long been true that tax experts have been able to protect their clients from certain kinds of penalties, especially those requiring a showing of fraud or willful misconduct. Recent years have witnessed significant legislative changes in penalties on taxpayers and tax advisors alike. Policy debate continues over both the appropriate level of penalties and the proper conditions under which taxpayers may be exculpated from the imposition of penalties. During this same period, both the American Bar Association and the American Institute of Certified Public Accountants have reviewed their standards governing the conduct of tax practitioners. While this latter activity has not resulted, nor could it result, in the kind of mathematically precise upperbound considered here, it does reflect an effort to produce a meaningful upperbound and to increase the probability that an uncertain deduction would be sustained on audit or through litigation.[25]

Likewise, the IRS in August, 1986 proposed amendments to Circular 230 (31 C.F.R. Part 10), which contains regulations governing the practice of attorneys, certified public accountants, and enrolled agents before the Internal Revenue Agents. These proposed amendments would introduce a new "due diligence" requirement for practitioners in advising clients about "positions taken with respect to the tax treatment of all items on returns." In general, the IRS would require that representations in tax returns must accurately reflect facts, that positions on tax returns must accurately reflect law and that practitioners have an affirmative duty to assure that these obligations are met. These IRS proposals have proved very controversial, having generated considerable opposition from both the accounting and legal professions and have not been made effective.

We have demonstrated here the interrelationships among penalty policies, audit practices and the ethical or other regulatory constraints on tax advisers. The presence of third party experts, who are able to protect taxpayers from certain types of penalties, has a systematic (if sometimes ambiguous) effect on the game played between taxpayers and the tax enforcement agency.

For example, experts may serve to deflect attempts to discourage taxpayers from taking risky tax return positions on ambiguous issues through increases in penalties on taxpayers for whom the risky position is disallowed in the absence of a supporting opinion. The principal effect of increases in such penalties will be to increase the number of taxpayers who solicit supporting opinions.

25. See ABA Formal Opinion 85-352 (issued July 7, 1985, revising Formal Op. 314 issued April 27, 1965) and Statement of Responsibilities in Tax Practice and the Federal Taxation Executive Committee of the AICPA in August, 1988.
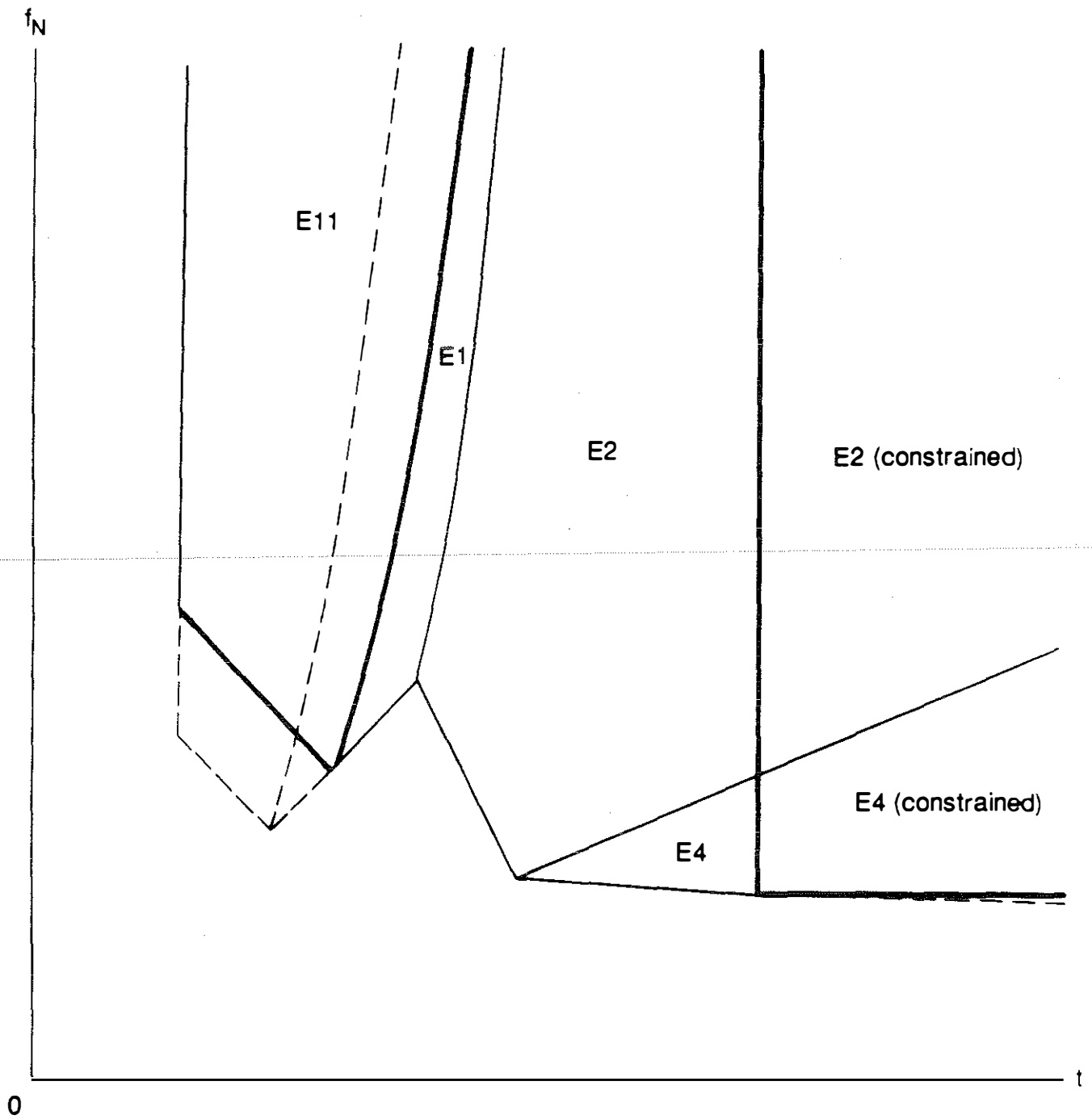
Figure 5: Equilibria under [self—regulation when $f_0 = 0$, $k = 3$, $c = 5$, $\pi_2 = 8$ and F is uniform on [0, 1]. Heavy solid lines indicate new constraints compared to the base case (Figure 2).

Indeed, the differential in penalties which depends upon the presence of absence of supporting opinions is an important component of the demand for supporting opinions. The structure of penalties, therefore, affects not only taxpayers' incentives to claim risky deductions but also their incentives to seek supporting opinions. Moreover, the existence and nature of any upperbound limiting the kinds of risks that allow supporting opinions to be issued by tax professionals plays a significant role both in terms of potential revenues to the government and as a determinant of demand for expert opinions.

We have demonstrated here that experts play a significant role in the taxing process, a role that demands attention to the interrelationships between IRS audit practices, the structure of tax penalties and the ethical or regulatory constraints on tax practitioners. Much more work remains to be done theoretically, empirically and in terms of public policy. Our work here is primarily theoretical and focuses on only one aspect of the role of expert third parties, but it offers a substantial beginning.

APPENDIX

Proof of Lemma 1:

There are 4 cases to consider.

Case a: All prefer the deduction with no supporting opinion. Thus, for all π, we need $p_N \pi (t + f_N) \leq t$ and $p_N \pi (t + f_N) \leq p_O \pi (t + f_O) + k$. If the first inequality holds for $\pi = 1$ it holds for all π. Thus we need

$$p_N (t + f_N) \leq t. \tag{1}$$

The second inequality can be written as $\pi [p_N(t + f_N) - p_O(t + f_O)] \leq k$. If the bracketed term is negative then the inequality always holds. Otherwise it is sufficient if it holds for $\pi = 1$:

$$p_N (t + f_N) - p_O (t + f_O) \leq k. \tag{2}$$

Case b: Some take the deduction with no supporting opinion, some take it with a supporting opinion. Define π_1 as the point of indifference; i.e., $p_N \pi_1 (t + f_N) \equiv p_O \pi_1 (t + f_O) + k$, or

$$\pi_1 = k / [p_N (t + f_N) - p_O (t + f_O)]. \tag{3}$$

We need $O < \pi_1 < 1$, or

$$p_N (t + f_N) - p_O (t + f_O) > k. \tag{4}$$

We also need all taxpayers to prefer taking the deduction. It suffices again to check $\pi = 1$:

$$p_O (t + f_O) + k < t \tag{5}$$

Case c: Some take the deduction with no supporting opinion, some do not take it at all. Define π_1 as the point of indifference; i.e., $p_N \pi_1 (t + f_N) = t$, or

$$\pi_1 = t / p_N (t + f_N). \tag{6}$$

We need $O < \pi_1 < 1$, or

$$t < p_N (t + f_N). \tag{7}$$

We also need no taxpayer to want a supporting opinion; for $\pi \leq \pi_1, p_N \pi (t + f_N) \leq p_O \pi (t + f_O) + k$, and for $\pi \geq \pi_1, t \leq p_O \pi (t + f_O) + k$. It suffices that both should hold at π_1:

$$p_N (t + f_N)(t - k) \leq t p_O (t + f_O). \tag{8}$$

Case d: Some take each option. Define π_1 as the point of indifference between taking the deduction without and with a supporting opinion, and π_2 as the point of indifference between the latter and not taking the deduction: i.e., $p_N \pi_1(t + f_N) \equiv p_O \pi_1(t + f_O) + k$ and $p_O \pi_2(t + f_O) + k \equiv t$, or

$$\pi_1 = k/[p_N(t + f_N) - p_O(t + f_O)], \tag{9}$$

$$\pi_2 = (t - k)/p_O(t + f_O). \tag{10}$$

We need $O < \pi_1 < \pi_2 < 1$, or

$$p_N(t + f_N)k < t[p_N(t + f_N) - p_O(t + f_O)], \tag{11}$$

$$(t - k) < p_O(t + f_O). \tag{12}$$

Finally, it can be shown these cases are exhaustive and mutually exclusive except for boundary points where the taxpayer is indifferent. Let $a = p_N(t + f_N)$ and $b = p_O(t + f_O)$. Then we have six cases and four optimal choice configurations:

	choice configuration			
parameter case	(a)	(b)	(c)	(d)
1. $a < b < t$	□	*	*	*
2. $a < t < b$	□	*	*	*
3. $b < a < t$	□	□	*	*
4. $b < t < a$	*	□	□	□
5. $t < a < b$	*	*	□	*
6. $t < b < a$	*	*	□	□

$* \equiv$ not possible

$□ \equiv$ exists for some parameter values

Cases 1, 2 and 5 yield unique optimal choice configurations; (a), (a) and (c), respectively. Case 3 yields (a) or (b) as $a - b \underset{>}{\overset{<}{}} k$ and Case b yields (c) or (d) as $a(t - k) \underset{>}{\overset{<}{}} bt$. Case 4 can yield (b), (c) or (d), but these can easily be shown to be mutually exclusive.

Q.E.D.

Proof of Proposition 1: Since $p_1 = 1 = p_2$, we have immediately from Definition 2 that $\pi_1 = k/(f_N - f_O)$ and $\pi_2 = (t - k)/(t + f_O)$. Thus $\pi_1 < \pi_2$ requires

$$t > kf_N/t(f_N - f_O - k). \tag{13}$$

We need only to guarantee that $p_N = 1 = p_O$. Define $h(\pi_1) = E(\pi \mid \pi \le \pi_1)$ and $g(\pi_1, \pi_2) = E(\pi \mid \pi_1 < \pi < \pi_2)$. Then $p_N = 1$ requires

$$t \ge [c - f_N h(\pi_1)]/h(\pi_1). \tag{14}$$

This is easily satisfied for t large enough since π_1 is independent of t. Finally, $p_O = 1$ requires $tg(\pi_1, \pi_2) \ge c$. But

$$\lim_{t \to \infty} g(\pi_1, \pi_2) = \lim_{t \to \infty} \int_{\pi_1}^{\pi_2} xf(x)dx/[F(\pi_2) - F(\pi_1)] = \int_{\pi_1}^{1} xf(x)dx/[1 - F(\pi_1)]$$

since $\lim_{t \to \infty} \pi_2 = 1$. Furthermore, $g(\pi_1, \pi_2)$ is monotonically increasing in t. Hence for t large enough all needed constraints are met.

Q.E.D.

Proof of Corollary 2: Average expected revenue is

$$ER = t[1 - F(\pi_2^*)] + (t + f_O)\int_{\pi_1^*}^{\pi_2^*} xf(x)dx - cF(\pi_2^*) + (t + f_N)\int_O^{\pi_1^*} xf(x)dx.$$

Hence

$$\frac{\partial ER}{\partial f_N} = f_N f(\pi_1^*)\pi_1^* \frac{\partial \pi_1^*}{\partial f_N} < 0$$

since $\partial \pi_1^*/\partial f_N < 0$, and

$$\frac{\partial ER}{\partial f_O} = f(\pi_2^*)[(t + f_O)\pi_2^* - t]\frac{\partial \pi_2^*}{\partial f_O} + \int_{\pi_1^*}^{\pi_2^*} xf(x)dx - cf(\pi_2^*)\frac{\partial \pi_2^*}{\partial f_O} + (f_N - f_O)\pi_1^* f(\pi_1^*)\frac{\partial \pi_1^*}{\partial f_O} > 0$$

since $\partial \pi_1^*/\partial f_O > O$, $\partial \pi_2^*/\partial f_O < O$, and $(t + f_O)\pi_2^* - t = -k$ by the definition of π_2^*.

Q.E.D.

Proof of Proposition 2: Since $p_N = 1 = p_O$, it is immediate that $\pi_1^* = t/(t + f_N) = \pi_2^*$. To keep $p_O^* = 1$ we need $(t + f_O)t/(t + f_N) > c$, given that universal divinity implies $\Pi_c = \pi_1^*$. To keep $p_N^* = 1$ we need $E(\pi \mid \pi \leq t/(t + f_N)) > c/(t + f_N)$ and to keep all taxpayers types from wanting to take the deduction with a supporting opinion we need $(f_N - f_O)t/(t + f_N) < k$.

<div align="right">Q.E.D.</div>

Proof of Proposition 3:

(a) Existence of type $E\,1$ equilibria: There are three relevant constraints for type $E\,1$ equilibria, 1a, 1b, 1c in Table 1. Given A9, $O < p_N^* < 1$ and $p_O^* > O$ are implied by these. The proof consists of showing that the set of (t, f_N) pairs which satisfy constraints 1a, 1b and 1c look qualitatively like that illustrated in Figure 2; i.e., in (t, f_N) space, the boundary of constraint 1b lies to the left of the boundary of constraint 1c, above the lower bound of constraint 1a.

Note first that constraint 1a is bounded below by $f_N = t + 2f_O$. Next, let the point of intersection of this line with the boundary of constraint 1b be denoted by (t^b, f_N^b) and with the boundary of constraint 1c by (t^c, f_N^c). Then t^b is given by

$$\frac{2cf_O + t^b(t^b + f_O)}{2c - f_O - t^b} = t^b + 2f_O,$$

or $t^b = c - f_O$. Similarly, t^c is given by

$$\frac{2cf_O + t^c(t^c - k)}{2c - t^c + k} = t^c + 2f_O$$

or $t^c = \{c + k - f_O + [(f_O - c - k)^2 - 4(c + k)f_O]^{1/2}\}/2$. Thus straightforward algebra shows $t^b > t^c$. It remains to show that the boundary of constraint 1b lies to the left of the boundary of constraint 1c for all t such that $c < t < 2c$, the latter since the boundary of constraint 1b asymptotes to $2c$. This follows directly.

(b) Existence of type $E\,2$ equilibria: We again show that the qualitative configuration of the boundaries of constraints 2a, 2b, and 2c illustrated in Figure 2 are generic. This is trivial. The boundary of constraint 2a is identical to the boundary of constraint 1c; it has positive slope and asymptotes to $2c + k$. The boundary of constraint 2c is a straight line with positive slope and nonnegative f_N-intercept. Thus it must intersect the boundary of constraint 2a, say at t^{ab}. The boundary of constraint 2b has negative slope and goes to infinity as t approaches k from above. If it intersects the boundaries of constraints 2a and 2c below t^{ab} then it is redundant. Otherwise it takes the form shown in Figure 2, intersecting the boundary of constraint 2c at t^{bc} where

$$t^{bc} = \{(2c - f_O) + [(2c - f_O)^2 + 8cf_O]^{1/2}\}/2.$$

Thus a large set of values of (t, f_N) yield type $E2$ equilibria. In particular, for $t > \max \{t^{ab}, t^{bc}\}$ all f_N such that $f_N > (tk + 2cf_O)/(2c - k)$ yield type $E2$ equilibria. Clearly for many of these equilibria $t < f_N$ since the slope of the boundary of constraint 2c is less than 1.

Q.E.D.

Proof of Proposition 4: The proof takes two steps. First we show that when $f_O = O$ and $2k > c$, the qualitative relationship of the equilibrium types illustrated in Figures 2 and 3 is generic. This allows us to link equilibrium types under A7 and A10 and thus make revenue comparisons.

Step 1:

We start with the unconstrained case—Figure 1. The proof of Proposition 3 already establishes the form of type $E1$ equilibria.

(a) The boundary of constraint 2b intersects the boundary of constraint 2a ($\equiv$ 1c) at the same point as the boundary of constraint 1a. The former is given by

$$\frac{t(t - k)}{2c - t + k} = \frac{t[2c - t + k]}{t - k}$$

and the latter by

$$\frac{t(t - k)}{2c - t + k} = t$$

Both equations yield $t = c + k$.

(b) The boundary of constraint 2b ($\equiv$ 5d) intersects the boundary of constraint 4c ($\equiv$ 6c) at the same point as the boundary of constraint 4a ($\equiv$ 2c). This, of course, also equals the boundary of constraint 6a ($\equiv$ 5a). The boundaries of constraints 2b and 4c, intersect at

$$\frac{2ct - t(t - k)}{t - k} = \frac{kt}{t - k}$$

and those of constraints 4c and 2c at

$$\frac{kt}{t - k} = \frac{tk}{2c - k}$$

Both equations yield $t = 2c$ which is identical to the boundary of constraint 6a. Since $k < c$, $c + k < 2c$, so that constraint 2b is indeed nontrivial (see the proof of Proposition 3).

(c) The boundary of constraint 4c ($\equiv$ 6c) can easily be shown to be below the boundary of constraint 2b ($\equiv$ 5c) for all $t < 2c$. We wish to show it lies above the boundary of constraint 8a ($\equiv$ 5b). This requires

$$\frac{kt}{t - k} > 2c - t$$

or

$$t^2 - 2ct + 2ck > 0.$$

If we try to set $t^2 - 2ct + 2ck = O$, it has real solutions if and only if $c > 2k$. To say that $t^2 - 2ct + 2ck > O$ thus means that $c < 2k$.

(d) Finally, we show that the boundary of constraint 8a ($\equiv$ 5b) intersects the boundaries of constraints 1b ($\equiv$ 3a) and 5c ($\equiv$ 1a) at the same point. We know from the proof of Proposition 3 that the boundaries of constraints 1b and 5c intersect at $t = c$, so it suffices to that $t = c$ solves $t - 2c = t$.

(e) Arguments (a) - (d) establish the genericity of Figure 2 when $f_O = O$ and $c < 2k$. With respect to Figure 3, the only additional element is the boundary of constraint 9b ($\equiv$ 4d). But the boundary of constraint 9b intersects that of constraint 4c whenever

$$\frac{tk}{t - k} = \frac{tk^2}{(t - k)(2c - t + k)}.$$

This occurs uniquely at $t = 2c$. Above $t = 2c$, the boundary of constraint 9b is increasing and asymptotes to $2c + k$. Thus Figure 3 is also generic.

PART II

Based on Part I, we can relate equilibrium types in the two regimes according to the following.

Possible Equilibrium Types Before and After Audit Constraint

unconstrained	$E1$	$E2$	$E3$	$E4$	$E5$	$E6$	$E7$	$E8$
constrained	$E9, E10$	$E4, E9$	$E9, E10$	$E4$	$E9, E10$	$E6$	$E10$	$E8$

In the unconstrained case, equilibria $E1, E2, E3$, and $E4$ involve the solicitation of expert opinions. We analyze the revenue effects of the regime shift on these equilibria.

a) type $E1$ equilibria:

(i) $E1 \rightarrow E9$: Here $ER_1 \leq ER_9$ if and only if

$$t(1 - \pi_2^1) \leq t(1 - \pi_2^9)$$

where ER_i = expected revenue in type Ei equilibrium and $\pi_2^i = \pi_2^*$ for equilibria of type Ei, $i = 1,9$. Since

$$\pi_2^1 = 2cf_N/t(t + f_N), \text{ and}$$

$$\pi_2^9 = 2cf_N(t - k)^2/t[k^2(t + f_N) - tf_N(t - 2k)],$$

this condition reduces to $ER_1 \leq ER_9$ if and only if $2k \geq t$. The constraint is nontrivial if and only if $(2ck)^{1/2} < 2k < 2c$. The first inequality follows from $c < 2k$ and the latter from A9.

(ii) $E1 \rightarrow E10$: Here $ER_1 \leq ER_{10}$ if and only if $t(-\pi_2^1) \leq t(1 - \pi_2^{10})$. Since $\pi_2^1 = 2cf_N/t(t + f_N)$ and $\pi_2^{10} = 2c/(t + f_N)$, this reduces immediately to $f_N \geq t$, which always holds by constraint 1a.

b) *type $E2$ equilibria:*

(i) $E2 \rightarrow E4$: Here $ER_2 \geq ER_4$ if and only if

$$[tE(\pi \mid \pi_1^2 \leq \pi \leq \pi_2^2) - c](\pi_2^2 - \pi_1^2) + t(1 - \pi_2^2)$$

$$\geq [(t + f_N)E(\pi \mid \pi \leq \pi_1^4) - c]\pi_1^4 + [tE(\pi \mid \pi_1^4 \leq \pi \leq \pi_2^4) - c](\pi_2^4 - \pi_1^4) + t(1 - \pi_2^4).$$

Since $\pi_1^2 = 2/(t + f_N)$, $\pi_1^4 = k/f_N$, and $\pi_2^2 = (t - k)/t = \pi_2^4$, this constraint reduces to $2cf_N \geq k(t + f_N)$, which always holds by constraint 2c.

(ii) $E^2 \rightarrow E9$: Here $ER_2 \geq ER_9$ if and only if

$$[tE(\pi \mid \pi_1^2 \leq \pi \leq \pi_2^2) - c](\pi_2^2 - \pi_1^2) + t(1 - \pi_2^2) \geq t(1 - \pi_2^9).$$

A sufficient condition for $ER_2 \geq ER_9$ is therefore $\pi_2^2 \leq \pi_2^9$. Since $\pi_2^2 = (t - k)/t$ and $\pi_2^9 = 2cf_N(t - k)^2/t[k^2(t + f_N) - tf_N(t - 2k)]$, this condition reduces to constraint 9b, and thus always holds.

c) *Type $E3$ equilibria:*

Since ER_9 and ER_{10} are both positive and $ER_3 = 0$, it is immediate that $ER_9 > ER_3$ and $ER_{10} > ER_3$.

d) *Type $E4$ equilibria:*

Type E4 equilibria remain type E4 equilibria under the regime switch so there is no revenue effect.

<div align="right">Q.E.D.</div>

Proof of Proposition 5:

Given the hypotheses of the proposition, Figure 2 is generic and we can work from Table 1. Initial equilibrium types will still occur whenever $\pi_2^* \le \bar{\pi}_2$. The following table gives new constraints and the resulting redundant constraint for each initial equilibrium types that still exists (only type E3 vanishes in this case).

equilibrium type	new constraint	redundant constraint
E1	$f_N \le t^2\bar{\pi}_2/(2c - t\bar{\pi}_2)$	$\pi_2^* < 1$: $f_N < t^2/(2c - t)$
E2	$t \le k/(1 - \bar{\pi}_2)$	none
E4	$t \le k/(1 - \bar{\pi}_2)$	none

Initial equilibrium types E2 and E4 exist in modified form with $\pi_2^* = \bar{\pi}_2$. The following table gives new constraints that bind under the hypotheses of the proposition and the resulting redundant constraints.

equilibrium type	new constraints	redundant constraints
E2	$\pi_2^* = \bar{\pi}_2$: $t \ge k/(1 - \bar{\pi}_2)$	$p_{\partial}^* = 1$: $f_N \le t(t - k)/(2c - t + k)$
	$p_{\partial}^* = 1$: $f_N \le t^2\bar{\pi}_2/(2c - t\bar{\pi}_2)$	$\pi_1^* < \pi_2^*$: $f_N > [2ct - t(t - k)]/(t - k)$
E4	$\pi_2^* = \bar{\pi}_2$: $t \ge k/(1 - \bar{\pi}_2)$	$\pi_1^* < \pi_2^*$: $f_N > kt/(t - k)$
	$\pi_1^* < \bar{\pi}_2$: $f_N > k/\bar{\pi}_2$	

Initial equilibrium types E5 - E7 do not involve the use of tax experts. Initial equilibrium type E8 continues to exist as before. Some tedious calculations also show that Figure 5 is generic. This leaves two zones unaccounted for. In one a "new" equilibrium type, E11, exists. It has the form given in the proposition and exists for all (t, f_N) such that $f_N > (2c - t\bar{\pi}_2)/\bar{\pi}_2$, $t > k$, and $f_N > t^2\bar{\pi}_2/(2c - t\bar{\pi}_2)$. These constraints are associated respectively with $\pi_1^* < \bar{\pi}_2$, $p_{\partial}^* = 0$, and the requirement that for all $\pi \in (\pi_1, \bar{\pi}_2)$ it is preferable to take the deduction with an expert opinion rather

than not take the deduction.

The other "unaccounted for" zone does not include the use of tax experts. To see this, note first that $t\bar{\pi}_2 < c$ on this range. Hence if $\pi_1 < \bar{\pi}_2$ then $p_0^* = 0$ since $t\pi_2 < c$ implies $tE(\pi \mid \pi_1 < \pi \le \bar{\pi}_2) < c$ whenever $\pi_1 \subset \bar{\pi}_2$. Also, we have $(t + f_N)\bar{\pi}_2/2 < c$. Hence, if $\pi_1 < \bar{\pi}_2$ then $p_N = 0$ by a similar argument. Thus no third parties are used but $p_0^* = 0$ anyway since $t\bar{\pi}_2 < c$. Since it is impossible for $p_N = 1$ this is just equilibrium type 7, which we know requires $f_N < (2ck - t^2\bar{\pi}_2)/t\bar{\pi}_2$, which lies inside the zone.

Q.E.D.

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