



Fig. 1. Asymptotic behavior of input impedance as function of terminating impedance. The image impedance is the midpoint X between the breaks in the curve.

are made.) The values of R_{t1z} , R_{t1p} , R_{t2z} , and R_{t2p} for this problem are, with signs reversed, 1.1×10^3 , 10^3 , 1.1×10^2 , and 10^2 . These values are the coordinates of the asymptotic behavior, A and B , and the midpoint X represents the image impedance of the network viewed from either pair of terminals. Also note that $F=1.1$ if W is a_{21} , an active element, and this value is the same as the ratio between the open- and short-circuit impedances at either terminal [3].

The portrayal of the variation in input impedance as a function of terminating impedance shown in Fig. 1 has been of great value as a teaching aid in addition to supplying information about mismatching of impedances.

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- [2] H. K. Bode, *Network Analysis and Feedback Amplifier Design*. Princeton, N. J.: Van Nostrand, 1945, ch. 1.
- [3] *Ibid.*, Ch. 5.

Comment on "FET Input Capacitance"

In a recent correspondence,¹ a formula for the input capacitance of a field-effect transistor (FET) was derived. The formula is incorrect because of an error in theory, the identical error having been committed by several other authors.² The source of the error has previously been noted,³ and this correspondence repeats some of the pertinent arguments of Richer.³

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¹ F. Spangbu, "FET input capacitance," *Proc. IEEE (Correspondence)*, pp. 1156-1157, August 1965.

² See, e.g., H. A. R. Wegener, "The cylindrical field-effect transistor," *IRE Trans. on Electron Devices*, vol. ED-6, pp. 442-449, November 1959, and S. Teszner, "Sur les nouvelles structures du transistor," *Onde Elect.*, vol. 41, pp. 307-320, April 1961.

³ I. Richer, "Properties of an arbitrarily doped field-effect transistor," Tech. Rept., Div. Engrg. Appl. Sci., California Institute of Technology, Pasadena, Calif., 1964.

Under a given set of assumptions, there can be only one correct expression for the input capacitance of an FET. If two different expressions are obtained by two different methods, then one method must be invalid. The procedure used in Spangbu¹ was to divide the depletion region lying between the channel and the gate into differential sections, and then to apply the formula for the capacitance of a planar reverse-biased $p-n$ junction to each section; the total input capacitance was obtained by summing—i.e., by integrating—the differential capacitors along the length of the gate channel junction. For this integration to be valid the differential capacitors must all be connected in parallel. Although the gate is a common point for one terminal of each capacitor, the current flowing through the resistance of the channel results in a difference of potential between the other terminals. Thus, the differential capacitors are not in parallel and cannot be summed to obtain the desired formula.

The correct expression for the input capacitance C_{ii} may be obtained most easily by taking the voltage derivative of the charge in transit between the source and the drain. The result may, in fact, be derived for arbitrary (one-dimensional) impurity distributions. For a symmetrical step-junction FET, for example, C_{ii} is given by

$$C_{ii} = \left| \frac{\partial Q}{\partial V_g} \right|$$

$$= 6KZ \frac{L}{a} (\sqrt{d} + \sqrt{s}) \frac{(1-\sqrt{d})^2 + 4(1-\sqrt{d})(1-\sqrt{s}) + (1-\sqrt{s})^2}{[3(\sqrt{d} + \sqrt{s}) - 2(d + \sqrt{ds} + s)]^2}$$

where d and s are, respectively, the normalized drain-gate and source-gate potentials, the other symbols having the same significance as in Spangbu.¹

This input "charge-capacitance" has been shown^{3,4} to be identical at low frequencies to the "real" input capacitance, i.e., the capacitance that is calculated by considering the FET structure to be a distributed RC network.

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⁴ I. Richer, "The equivalent circuit of an arbitrarily doped field-effect transistor," *Solid-State Electron*, vol. 8, pp. 381-393, April 1965.

An Electronic Half-Tone Image Recording Technique

In magazine and newspaper printing, different levels of gray are presented in terms of variations in the diameter of small black dots arrayed in a fixed pattern. In one sense, the storage is binary; i.e., there are no variations of color within the dot itself. The technique described here uses an electronic storage tube to generate a mosaic of diameter modulated dots to record the intensity of two-dimensional antenna patterns.

The Tektronix 564 recording oscilloscope was used with auxiliary circuits, as shown in Fig. 1, to provide a uniform, medium resolution, reliable display on which images can be stored for about one hour with no degradation in either image quality or intensity. The video signal is sampled at regular time intervals to provide about 60 samples per fast scan line. Each sample pulse initiates one set of the waveforms shown in Fig. 2. Two of the waveforms are decaying sinusoids in phase quadrature which are added to the sweep signals at the "X" and "Y" inputs to the oscilloscope. They cause the beam to trace out a small logarithmic spiral pattern at a point on the screen determined by the values of the sweep voltages at the sampling instant.

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