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Generation-recombination noise of junction-gate field-effect transistors

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Abstract

The generation-recombination noise of junction-gate field-effect transistors is calculated taking into account the variable mobility. The field dependence of mobility suggested by Trofimenkoff is used, and the resultant spectral intensity of the drain-noise fluctuations shows no signs of a logarithmic singularity at saturation. The need for any cutoff procedure to remove the logarithmic singularity at saturation is therefore removed, and it is thus an improvement over earlier methods.

List of symbols

a = metallurgical halfchannel thickness
b = normalised halfchannel thickness of undepleted portion of channel
$= 1 - (x_{n1}/a)$
$= 1 - (W/W_0)$
b_s, b_d = value of b at the source and drain ends of channel; the saturation value is indicated by a prime
$c = -W_0/\epsilon_0 L$
$C(x)$ = capacitance between channel and gate
E_t = electron energy of trap
E_F = electron energy at Fermi level
ϵ = electric field
ϵ_0 = electric-field constant
ϵ_c = critical electric field
F_t = occupation probability of an s.r.h. centre
$g(W)$ = conductance
g_0 = junction-gate-f.e.t. maximum transconductance; i.e. when $W_{GD} = W_0$, $W_{GS} = 0$
I = normalised drain current
I_D = direct drain current, prime indicates saturation value
I_0 = junction-gate-f.e.t. saturation drain current for $W_{GS} = 0$
L = metallurgical channel length
n_t = occupied s.r.h. centre density
$= N_t F_t$
$N = N_D - N_A$
N_D = acceptor density
N_A = donor density
N_n = net ionised donor density in an n region
N_t = density of s.r.h. centres
q = magnitude of electronic charge
$S_{i_d}(f)$ = spectral intensity due to a fluctuation δi_d
$S_{n_t}(f)$ = spectral intensity due to a fluctuation δn_t
t = time
V_{DS} = direct drain–source voltage
V_{GS} = direct gate–source voltage
V_{po} = pinchoff voltage
W = direct component gate–channel potential
W_{GS}, W_{GD} = direct components of gate–source and gate–drain potentials
W_0 = pinchoff potential
$= V_{po} + \psi$
$W(y)$ = gate–channel potential
x = axis normal to plane of channel
x_{n1}, x_{n2} = x co-ordinate of depletion boundaries in n region of channel
y = co-ordinate axis along channel
y_0 = y co-ordinate of equilibrium
Z = metallurgical channel width
$\Delta\Lambda$ = volume element
$= 2Z\Delta x \Delta y$
ϵ_s = semiconductor permittivity
ζ = normalised gate–channel potential
$= W/W_0$
μ = carrier mobility
μ_c = constant low-field value of mobility
μ_n = drift mobility of electrons
μ_o = low-field carrier mobility
τ_t = lifetime or time constant of trap
ψ = barrier potential
ω = angular frequency

1 Introduction

The problem of noise in junction-gate field-effect transistors has been analysed by several suthors.$^{1-3}$ It is proposed to discuss, the generation-recombination noise in the transistor, taking into effect the variable mobility. It will be seen that the generation-recombination noise loses all signs of a logarithmic singularity at saturation.

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2 Theoretical considerations

It is assumed that the direct drain current is constant along the channel. This results from the fact that the gate–channel p–n junctions are reverse-biased, and the effect of diffusion or other currents flowing across the junction is neglected. If $\zeta = W/W_0$ is the normalised gate–channel potential, the conductance per unit channel length in terms of the drain current I_D is given by

$$g(\zeta) = g_0 L(1 - \zeta^{\frac{1}{2}}) - (I_D/\epsilon_0) \tag{1}$$

ϵ_0 is a constant giving a better fit to experimental observations. Prior[4] and Gibbons[5] have given data yielding values of this constant. The differentiation of eqn. 1 with respect to ζ gives

$$\Delta g(\zeta) = -g_0 L(1/2)\zeta^{-\frac{1}{2}}\Delta\zeta \tag{2}$$

If δi_d denotes the drain-current fluctuation under short-circuit conditions, it satisfies the following differential equation:

$$\delta i_d = -\frac{d}{dy}\{\Delta g(W)\Delta W\} \tag{3a}$$

This arises from the equation

$$\Delta i_d = -\frac{d}{dy}\{g(W_0)\Delta W\} \tag{3b}$$

where we now consider $\Delta g(W)$ to be the conductance per unit channel length due to a small section of thickness Δx. If eqn. 2 is substituted into eqn. 3*a* and the integration is carried out over the ranges 0 to y_0 and $(y_0 + \Delta y)$ to L, where y_0 denotes the equilibrium value with no noise fluctuations,

$$\delta i_d y = (1/2)g_0 L\zeta^{-\frac{1}{2}}\Delta\zeta\Delta W \quad 0 < y < y_0 \tag{4a}$$

$$\delta i_d(y - L) = (1/2)g_0 L\zeta^{-\frac{1}{2}}\Delta\zeta\Delta W \quad y_0 + \Delta y < y < L \tag{4b}$$

By subtracting eqn. 4*a* from eqn. 4*b*, it is found that, at y_0

$$\delta i_d = -(1/2)g_0\zeta^{-\frac{1}{2}}\Delta\zeta\Delta W \tag{5}$$

with

$$\delta W = \Delta W(y_0 + \Delta y) - \Delta W(y_0) \tag{6a}$$

The fluctuation δn_t causes a charge fluctuation $q\delta n_t\Delta\Lambda$ in a volume $\Delta\Lambda$ where $\Delta\Lambda$ is taken to be equal to $2\Delta x\Delta yZ$ around the point (x, y_0). This fluctuation results in a gate-channel potential δW and a change in the depletion-layer thickness. If $C(x)$ is the capacitance between the channel and the gate,

$$C(x) = 2\epsilon_s Z\Delta y/x \tag{6b}$$

and

$$\delta W = -q\delta n_t\Delta\Lambda/C(x) \tag{6c}$$

3 Calculation of spectral intensity

A Fourier analysis of eqn. 5 may now be made. If δn_t is assumed to have an exponential decay such that $\delta n_t = \delta n_{t0}\exp(-t/\tau_t)$ and the Wiener–Khinchine theorem for the spectral intensity is applied, it is found that

$$S_{\delta n_t}(f) = K/\Delta\Lambda \tag{7a}$$

where

$$K = 4N_t F_t(1 - F_t)\tau_t/(1 + \omega^2\tau^2_t) \tag{7b}$$

N_t is the s.r.h. centre density, F_t is the occupation probability of a centre given by a Fermi distribution of the form $\{1 + \exp(E_F - E_t)/kT\}^{-1}$ where E_t is the energy level of the trap and $(1 - F_t)$ is the probability that the trap is empty. E_F is the Fermi energy level, k is the Boltzmann constant and T is the temperature. It is then seen that

$$\begin{aligned} S_{i_d}(f) &= \left(\frac{g_0 qx\Delta x\Delta\zeta}{2\epsilon_s\zeta^{\frac{1}{2}}}\right)^2\overline{\delta n_t^2} \\ &= \frac{q^2 g_0^2 Kx^2\Delta x\Delta\zeta(-I_D)}{8\epsilon_s Z\zeta g_0 W_0 L\{(1 - \zeta^{\frac{1}{2}}) + cI/3\}} \end{aligned} \tag{8a}$$

This follows from Van der Ziel's[2] expression for Δy in terms of $\Delta\zeta$ in the case where there is a fluctuation in the gate charge. The expression below has been modified to include variable mobility effects:

$$\Delta y = -\frac{g_0 W_0 L}{I_D}[(1 - \zeta^{\frac{1}{2}}) + cI/3]\,\Delta\zeta \tag{8b}$$

Here I represents the normalised drain current.

The variable mobility is assumed to be given by

$$\mu = \mu_0/(1 + \epsilon/\epsilon_0) \tag{8c}$$

This empirical equation was proposed by Trofimenkoff.[6] A modified version of eqn. 8*c* was later proposed by Caughey and Thomas[7] to give a better approximation for electrons in silicon. In eqn. 8*c*, μ_0 is the low-field mobility and ϵ_0 is the constant which, if appropriately chosen, gives a much better fit to experimental observations. The Trofimenkoff approximation is found to be better than that of Dacey–Ross:[8]

$$\mu = \mu_c(\epsilon_c/\epsilon)^{\frac{1}{2}} \quad \epsilon > \epsilon_c \tag{8d}$$

where ϵ_c is the critical field at which μ starts to deviate from its constant low-field value μ_c. The Dacey–Ross approximation has been used to modify the current, voltage and noise characteristics of the junction-gate f.e.t., by Halladay and Van der Ziel.[3] However, the weakness of the approximation lies in the fact that one has to assume $\epsilon > \epsilon_c$ throughout the channel. This is not true, since ϵ is much lower near the source region than near the expop. The alternative approach would be to divide the channel into two regions bounded by the condition $\epsilon = \epsilon_c$ with a different solution for each region with appropriate boundary conditions. The Trofimenkoff approximation avoids these difficulties. The calculations in this paper are for a silicon channel of the n type with ϵ_0 equal to 8·5 kV/cm.

If eqn. 8*a* is integrated over the whole depletion region, first with respect to x from $x = 0$ to $x = x_{n1} = a\zeta^{\frac{1}{2}}$ then converted to an integration with respect to the normalised halfthickness b of the channel, the following expression is obtained:

$$S_{i_d}(f) = \frac{2a^3}{3}A\int_{b_d}^{b_s}\frac{(1 - b^2)db}{b + cI/3} \tag{9a}$$

This integral may be solved by a suitable change of variable to yield the following result:

$$S_{i_d}(f) = a^3 A' G_1(b_s, b_d) \tag{9b}$$

with

$$A' = q^2 g_0^2 K/24\epsilon_s^2 ZL \tag{9c}$$

$$A = A' I_D/I_0 \tag{9d}$$

The function $G_1(b_s, b_d)$ is given by

$$\begin{aligned} G_1(b_s, b_d) = \frac{2I}{3}\Big\{&(1 + cI/3)^2\ln\left(\frac{b_s + cI/3}{b_d + cI/3}\right) \\ &+ (1/2)(b_s^2 - b_d^2) \\ &- (2 + cI/3)(b_s - b_d)\Big\} \end{aligned} \tag{9e}$$

At saturation, $I = I'$, which is finite. Furthermore, the value of the normalised gate–drain potential ζ_d' is given by

$$\zeta_d' = (1 - cI'/3)^2 \tag{10}$$

which is less than one, since c and I' are positive. The value of b'_d at saturation is given by

$$\begin{aligned} b'_d &= (1-\zeta'_d)^{\frac{1}{2}} \\ &= cI'/3 \end{aligned} \tag{11}$$

The spectral intensity at saturation is therefore given by

$$S'_{i_d}(f) = a^3 A' G'_1(b_s, b_d) \tag{12a}$$

with

$$\begin{aligned} G'_1(b_s, b_d) = \frac{2I}{3}\bigg[&(1+cI'/3)^2 \ln\left(\frac{b_s + cI'/3}{2cI'/3}\right) \\ &+ (1/2)\{b_s^2 - (cI'/3)^2\} \\ &- (2+cI'/3)(b_s - cI'/3)\bigg] \end{aligned} \tag{12b}$$

The normalised spectral intensity of the noise $S_{i_d}(f)/a^3A'$ is shown as a function of the drain–source potential $-V_{DS}/W_0$ in Fig. 1. Different values of $\zeta_s = W_{GS}/W_0$ have been used in the calculation. The noise increases with the drain bias until it reaches a certain finite value at saturation. This is given by the intersection of the spectral intensity curve at a particular value of the gate bias, with the broken line. For a given value of the drain bias, the generation-recombination noise increases with the gate bias.

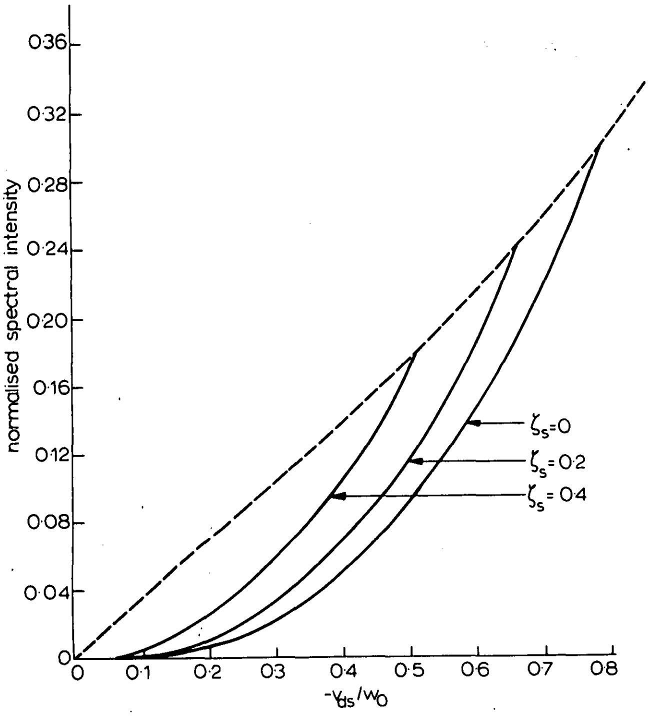

Fig. 1
Normalised spectral intensity of the generation-recombination noise of a junction-gate field-effect transistor plotted as a function of the normalised drain bias for various values of the gate bias

The broken line marks the onset of saturation

4 Conclusion

The logarithmic singularity at saturation has been removed from the expression for the spectral intensity of the generation-recombination drain-noise current. This results in a continuous noise/bias characteristic. The introduction of the high-field-mobility effect removes the singularity over the whole channel length without recourse to the channel-shortening effect. Halladay and Van der Ziel used the Dacey–Ross approximation and the singularity could be removed only by a cutoff procedure. The use of the Trofimenkoff approximation results in a removal of the singularity without any such procedure, and hence it could be considered to be an improvement.

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