Swarms of Femtosats for Synthetic Aperture Applications

S.-J. Chung[1], F. Y. Hadaegh[2]
[1]*University of Illinois at Urbana-Champaign, Urbana, USA;*
[2]*Jet Propulsion Laboratory, California Institute of Technology, Pasadena, USA*

Abstract
The Silicon Wafer Integrated Femtosatellites (SWIFT) Swarm Project presents a new paradigm-shifting definition of spacecraft technology that can enable flight of swarms of fully capable femtosats. One of the most important applications of SWIFT is a distributed aperture array. New swarm Golay array configurations are introduced and shown to dramatically increase the effective diameter derived from optical performance metrics. A system cost analysis based on this comparison justifies deploying a large number of spacecraft for sparse aperture applications.

1. Introduction

In this paper, “swarm" refers to a collection of hundreds to thousands of spacecraft, while “femtosat" implies a 100-gram class spacecraft. In contrast with prior work such as the Cubesats [18], the satellite-on-a-printed circuit board (PCBSat) [1], and the passive silicon-chip spacecraft [2], the SWIFT represents a fully capable 100-gram satellite built on novel 3-D silicon wafer fabrication and integration techniques. Each femtosat can be actively controlled in all six degrees of freedom (6DOF) such that a desirable synergetic behavior emerges from the interactions among spacecraft and between the spacecraft and the environment.

The femtosat swarm will push the frontier of the existing formation flying spacecraft concepts [3-6] by one or two orders of magnitude in two major technological drivers: the enormous number (1000 or more) of spacecraft compared with the previous 2-10 spacecraft formation concepts; and a tiny size and “miniaturized” capability of 100-gram femtosats. As a result, the feasibility of SWIFT-Swarm is predicated on two enabling technological developments: fabrication of 100-gram femtosats and the individual and synergetic guidance, navigation, and control (GN&C) capabilities of femtosats. The synergetic GN&C capabilities of the swarm permit coordinated maneuvers of femtosats so that the swarm can collectively exhibit or exceed a capability of much more complicated monolithic space systems.

A substantial amount of literature exists in the areas of multi-agent coordination and cooperative control for robotics, ground vehicles, and formation flying spacecraft. Nevertheless, none of the existing GN&C technologies can simultaneously address (a) such enormous number of spacecraft in swarms, (b) “relatively” limited control, sensing, and communication capabilities of femtosats, and (c) the complex three-dimensional (3-D) 6-DOF motions governed by Earth’s gravity field and various disturbances. In particular, the latter distinguishes the SWIFT swarm from other mobile sensor networks and the relevant dynamic models and control techniques are further discussed in a series of our papers [7,8].

One of the most important applications derived from such synergetic behaviors includes a sparse aperture sensing or stellar interferometry. By employing the interferometric imaging metrics such as Point Spread Functions (PSFs) and Modulation Transfer Functions (MTFs), we present a unique performance and cost analysis that illustrates the cost benefits of the femtosat swarm architecture. The purpose of this paper is to present a compelling reason behind an enormously large number of femtosats deployed in the swarm architecture.

2. Random Sparse Aperture Arrays and Performance Analysis

We can spread out radar or optical telescopes with an aim to achieve a resolution that is comparable to a large monolithic aperture. Due to the stringent requirement on interferometric beam combination [10], that is, the beams should be combined within a fraction of wavelength, telescopes for longer wavelengths, such as radio or submillimeter wavelengths, appear to be a more promising application of sparse aperture sensing for the SWIFT swarm.

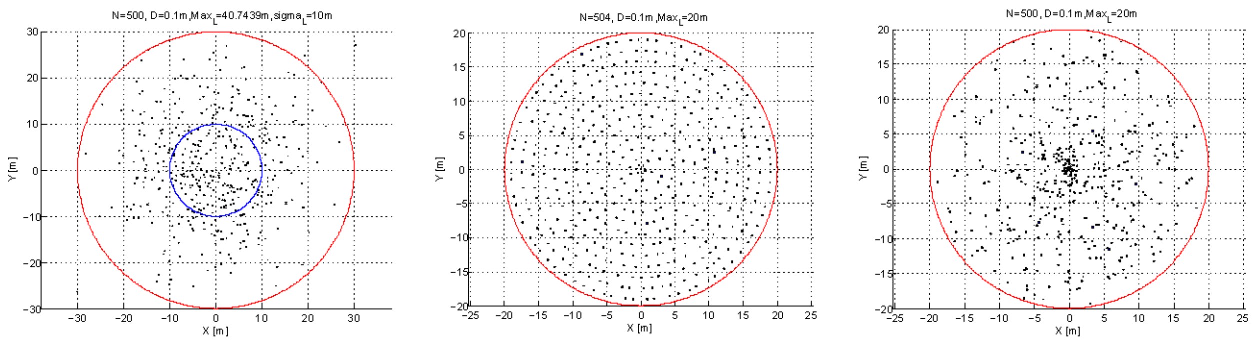
Three possible swarm configurations can be considered depending on the distribution characteristic of femtosats, as shown in Fig. 1. Previously, random arrays have been studied for communication relay [11], antennas [12], and space-based radars [13]. All the configurations in consideration are assumed to be on a projected circular plane. The passive relative orbits (PROs) of femtosats in the relative orbital frame can be of any arbitrary orientation such that the projected plane is always circular and normal to the line of sight (see [7]).

In a Gaussian Random Array shown in Fig. 1a, the locations of femtosats on the projected circular plane follow the Gaussian distribution with a variance σ^2. On the other hand, each spacecraft in a Structured Disk Array is distributed with a prescribed radial and angular separation, thereby ensuring a certain separation distance between spacecraft. This can be interpreted in the sense of the discrete uniform distribution. This array can also be randomly perturbed slightly due to sensor and control errors of each femtosat, as shown Fig. 1b. The third configuration, shown in Fig. 1c, is called a Uniform Disk Array, since femtosats are spread by the continuous uniform distribution. In contrast with the Gaussian Random Array, we can prescribe the maximum radius of the array for both Structured and Uniform Disk Arrays.

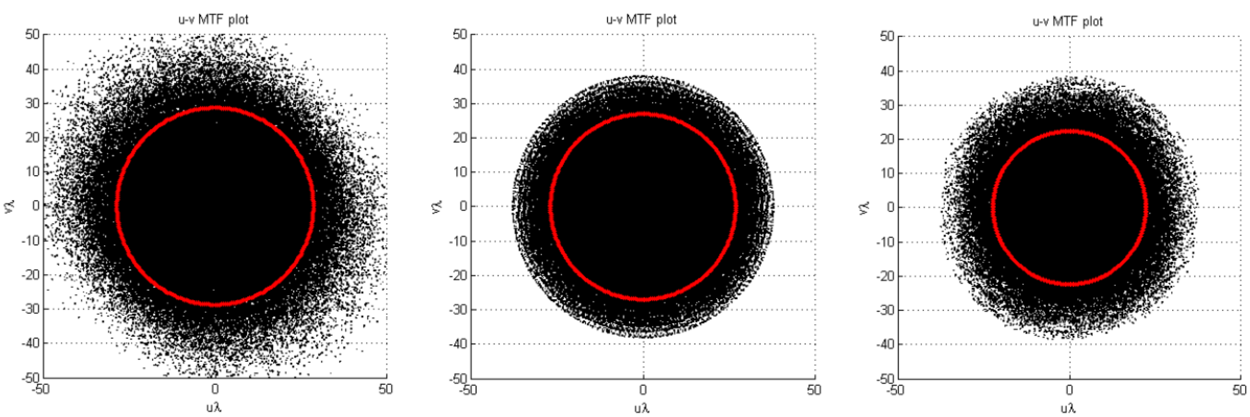
The femtosat swarm can be used to employ a sparse aperture radar interferometry in space to achieve a fine angular resolution, which is determined by the Rayleigh's criterion $\theta_r = 1.22\lambda/D$. For sparse aperture arrays, the diameter D of a monolithic aperture for the wavelength λ of interest should be replaced by the effective diameter D_{eff}. Then, how can we determine the effective diameter of the random array configurations shown in Fig. 1? The angular resolution alone is inadequate for many sparse-aperture or interferometric array applications. When looking at extended objects such as Earth's surface or faint distant nebulae, evaluation of an optical or radar system involves more than simply looking at a point source response point spread function (PSF), which determines the angular resolution by the method of full width at half maximum (FWHM). The modulation transfer function (MTF) is a better metric to evaluate the contrast (modulation) transfer characteristic of snapshot imaging of extended objects [10]. The PSF is the squared modulus of the Fourier transform of the complex pupil function. The optical transform function (OTF) is a Fourier transform of the PSF, and the MTF is an absolute value of the OTF. Alternatively, a 2-D version of the MTF plot can be determined by taking the auto-correlation points, commonly called u-v points defined as

$$u = \pm(x_i - x_j)/\lambda, \qquad v = \pm(y_i - y_j)/\lambda \qquad (1)$$

where (x_i, y_i) and (x_j, y_j) are any possible pair of points within apertures, and λ is the wavelength of interest.



(a) Gaussian Random, a=10m (blue) (b) Structured Disk, L=20m (c) Uniform Disk, L=20m 3a=30m (red)



(d) MTF of (a), D_{eff} =28m (e) MTF of (b), D_{eff} =27m (f) MTF of (c), D_{eff} =21m

Figure 1. Random sparse array configurations (a)-(c) and their MTF plots (d)-(f), with 500 spacecraft. The diameter of each aperture is 10 cm, while the effective diameter (D_{eff}) for instantaneous u-v filling is given in (d)-(f).

In order to properly compare with monolithic filled apertures, we choose to define the effective diameter as the maximum radius of the MTF plots without singular (zero) u-v points. The D_{eff} of each configuration is denoted by the red circles in Fig. 1(d)-(f). We assume that each femtosat carries a radar aperture of 10cm in diameter. By determining the array size that yields an instantaneous filled u-v coverage in the MTF with 500 apertures, the effective diameters for each configuration are computed as 28m, 27m, and 21m, respectively. Hence, we can see that the Gaussian Random and Structured Disk Arrays can achieve a finer angular resolution for an instantaneous full u-v coverage.

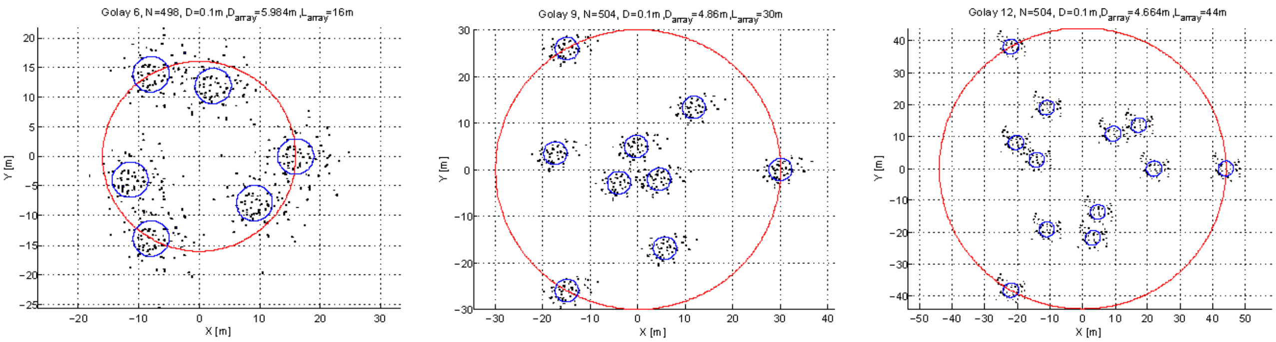
This result does not imply that a sparse aperture array with 500 femtosats can only achieve the effective diameter of 28m. If we can integrate images for a longer period of time, similar to Synthetic Aperture Radar (SAR) and Very Large Baseline Interferometry (VLBI), the finest angular resolution achieved by the array is determined by the largest separation distance between the apertures, In other words, D_{eff} becomes the max separation distance. For example, similar to the Air Force Research Laboratory's TechSat21 [6], a swarm of femtosats can be spread over a distance of 1-5 km, thereby yielding a much finer angular resolution. This is technologically feasible since the apertures distributed on the PROs will be rotating with respect to the center of the relative frame, as discussed in [7]. Such a large separation distance can be beneficial for longer wavelengths, since the angular resolution is inversely proportional to the wavelength. However, such a large baseline length will inevitably lead to much more sparse u-v filling with many singular points (zero contrast), thereby decreasing the sensitivity or Signal-to-Noise Ratio (SNR) [9,10]. Also, a non-compact u-v coverage cannot be used for snapshot imaging. Hence, the array design of sparse apertures is a trade-off between the angular resolution of a point target and the sensitivity or contrast characteristics of a filled aperture. In order to properly compare with a fully-filled monolithic aperture, in particular for the cost analysis in Section 4, we proceed to use an instantaneous u-v coverage.

3. Swarm Golay Arrays and Performance Analysis

By examining the u-v coverage, we can find that there are many redundant u-v points that could have been saved to increase the effctive diameter. In this paper, we introduce new random sparse arrays that can further optimize the number of spacecraft needed for a target effective diameter. Let us recall an optimal imaging configuration designed for a small number of apertures (N=3–12), proposed by Golay [14]. These arrays are all non-redundant and optimized for compactness in the arrays' u-v plot.

Since we can construct a filled u-v coverage by using the proposed random arrays shown in Fig. 1, a nice extension of Golay arrays is to spread multiple femtosats within the fractionated aperture diameters defined by the original Golay configuration. The proposed Swarm Golay-6, Swarm Golay-9, and Swarm Golay-12 are shown in Fig. 2. The MTF characteristic of a Swarm Golay-3 Array turns out to be similar to that of a Gaussian Random or Structured Disk Array, hence the Golay-3 is omitted here. By computing the MTF without discontinuous singular u-v points, the effective diameter D_{eff} is determined as shown in Fig. 2(d)-(f). By using the same number of femtosats (N=500), these three Swarm Golay Arrays achieve much larger effective diameters, thereby implying that we can further reduce the system mass or cost by utilizing swarm

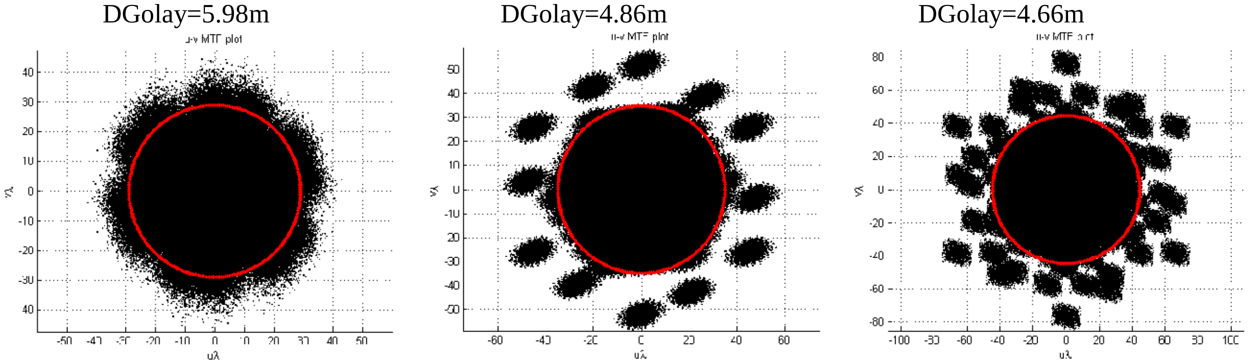
Golay arrays.



(a) Swarm Golay-6, L=16m, DGolay=5.98m

(b) Swarm Golay-9, L=30m, DGolay=4.86m

(c) Swarm Golay-12, L=44m, DGolay=4.66m



(d) MTF of (a), Deff=29m

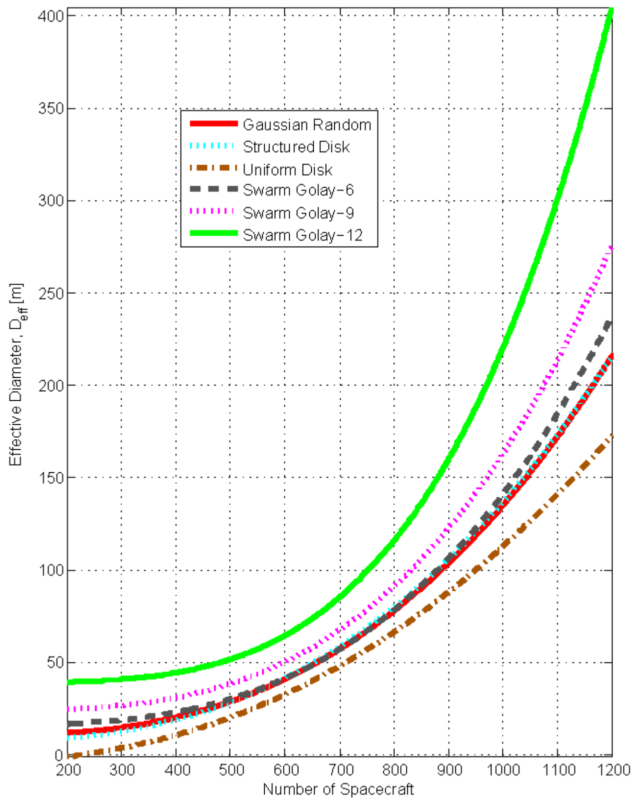
(e) MTF of (b), Deff=35m

(f) MTF of (c), Deff=44m

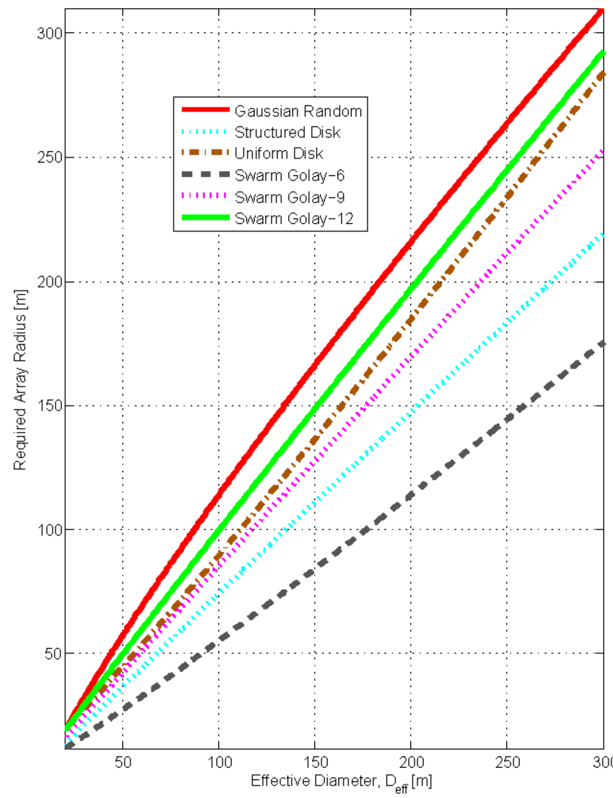
Figure 2. Swarm Golay configurations (a)-(c) and their MTF plots (d)-(f) for 500 spacecraft with the aperture diameter of 10 cm. The effective diameters (D_{eff}) for instantaneous u-v filling are larger than those in Fig. 3.

The results from Figs. 1 and 2 are summarized in Fig. 3 for various aperture sizes and numbers. In general, the results are in excellent agreement with prior work [12] indicating a smaller number of apertures are needed in random arrays. What is new here is that we can further reduce the number of apertures required to achieve a comparable aperture performance (e.g., angular resolution). As shown in Fig. 5a, we can more rapidly improve the angular resolution of the swarm aperture arrays by increasing the number of spacecraft. This result can be viewed as a compelling rationale behind the swarm architecture that employs thousands of femtosats.

For the same number of femtosats, the Golay-12 Array achieved a larger effective diameter, followed by the Golay-9, the Golay-6, and the Gaussian Random Arrays. Another information we can extract here is the size of the array needed to achieve the desired D_{eff} based on the full u-v coverage requirement. For example, the Gaussian Random Array needs the largest array size, hence possibly imposing more demanding communication requirements. According to our preliminary point design, the long-distance communication sub-system can drive the system mass and cost. However, in the Swarm Golay Arrays, a swarm of femtosats can be divided to 6, 9, or 12 subset groups, and this fractionated grouping can be advantageous in swarm controls and communications.



(a) Effective diameter as a function of number of apertures



(b) Array size as a function of effective diameter

Figure 3. Effective diameters from swarm aperture configurations

4. System Cost Analysis

We can hypothesize from Fig. 3 that the 400-m effective diameter constructed from 1200 femtosats can be manufactured at a fraction of the manufacturing cost of an immense monolithic spacecraft carrying a 400-m diameter telescope, which in fact cannot be launched and built with the existing technology. In this section, we present an important cost analysis corroborating this dramatic cost saving of the spacecraft swarm architecture. At a much smaller scale, MIT's ARGOS project [10,15] concluded that an optical sparse aperture is much more cost-effective even after considering the additional cost associated with controls and interferometric beam combination.

In order to compare the system cost of building and launching a monolithic aperture radar with that of sparse aperture arrays presented in the previous section, we modify the NASA Advanced Mission Cost Model [16] by multiplying it with $1/\lambda^{-0.5}$

$$\text{Cost}=\$2.25\ \text{billion}\times(\text{mass}/10{,}000\text{kg})^{0.654}\times(1.555^{\text{difficulty level}})\times N^{-0.406}\times 1/\sqrt{\lambda} \qquad (2)$$

where N is the number of flight systems; and difficulty levels are: -2=very low, -1=low, 0=average, 1=high, and 2=very high.

The cost model given in Eq. (2) computes the cost of a 1-kg CubeSat to be $2 million, which is close to the maximum launch cost of a CubeSat. This model also correctly predicts the project cost of infra-red (IR) space telescopes such as the Hubble Space Telescope (HST) and the James Webb Space Telescope (JWST) to be about $4 billion by substituting the mass and the difficulty level (2 for JWST and 1 for HST). However, how can we predict a cost for a large monolithic spaceborne radio telescope? Here, we consider the impact of Wavelength of Diffraction-Limited Performance (WDLP) by adding a factor of $1/\lambda^{-0.5}$ where λ is in μm. The additional factor $\lambda^{-0.5}$, proposed by Stahl [17], well matched the existing cost data of ground based telescopes. In a nutshell, this additional factor indicates that a radar space telescope with the wavelength 1m is 1000 times less expensive than a telescope of 1 μm IR wavelength.

The project cost of a large monolithic space radar can be predicted, as shown in Fig. 4, to be a function of the diameter by assuming that the mass of the telescope is proportional to D^{2}. Consequently, the cost of a large monolithic space-based radar is proportional to $D^{1.308}$. The exponent of 1.308 agrees relatively well with the exponent 1.12 of the space telescope parametric cost model by Stahl [16]. Note that this number is smaller by a factor of two than the popular Meinel's cost model $D^{2.7}$ for ground telescopes, which includes the cost of telescope mount and dome. The key point here is not the exact amount of the predicted cost, but the exponent of the diameter that indicates a cost trend as a function of the diameter size. By identifying such Cost Estimation Relationship (CER), we can determine significant cost drivers in the design process.

Similarly, we can compute the cost of building a 100-gram femtosat from Eq. (2). The total cost of a swarm array can be computed by multiplying the number of femtosats required in Fig. 3a for a given effective diameter. The result is shown in Fig. 4. As summarized in Table 1, we can conclude that the cost exponent of the monolithic aperture is much steeper than that of the proposed swarm array configurations. As a result, we can dramatically save the system cost of launching a large radar aperture by deploying swarms of much smaller apertures. If the cost of fabricating a single femtosat is higher than expected, the results shown in Fig. 4 still indicates that there exists a break-even point between a monolithic telescope and a swarm array. In other words, in order to take advantage of the cost savings from the swarm, we need to increase the number of apertures or femtosats.

This conclusion justifies the rationale behind a large number of femtosats. Also, it is expected that the benefits from the swarm architecture is more substantial when we factor in the learning cost saving as well as risk-reduction effects that are not captured in this cost analysis.

Table 1 Comparison of cost as a function of D or D_{eff}

Meinel's law Monolithic	Monolithic Aperture	Gaussian Random	Structured Disk	Uniform Disk	Swarm Golay-6	Swarm Golay-9	Swarm Golay-12
$D^{2.7}$	$D^{1.308}$	$D_{\text{eff}}^{0.217}$	$D_{\text{eff}}^{0.236}$	$D_{\text{eff}}^{0.26}$	$D_{\text{eff}}^{0.157}$	$D_{\text{eff}}^{0.132}$	$D_{\text{eff}}^{0.116}$

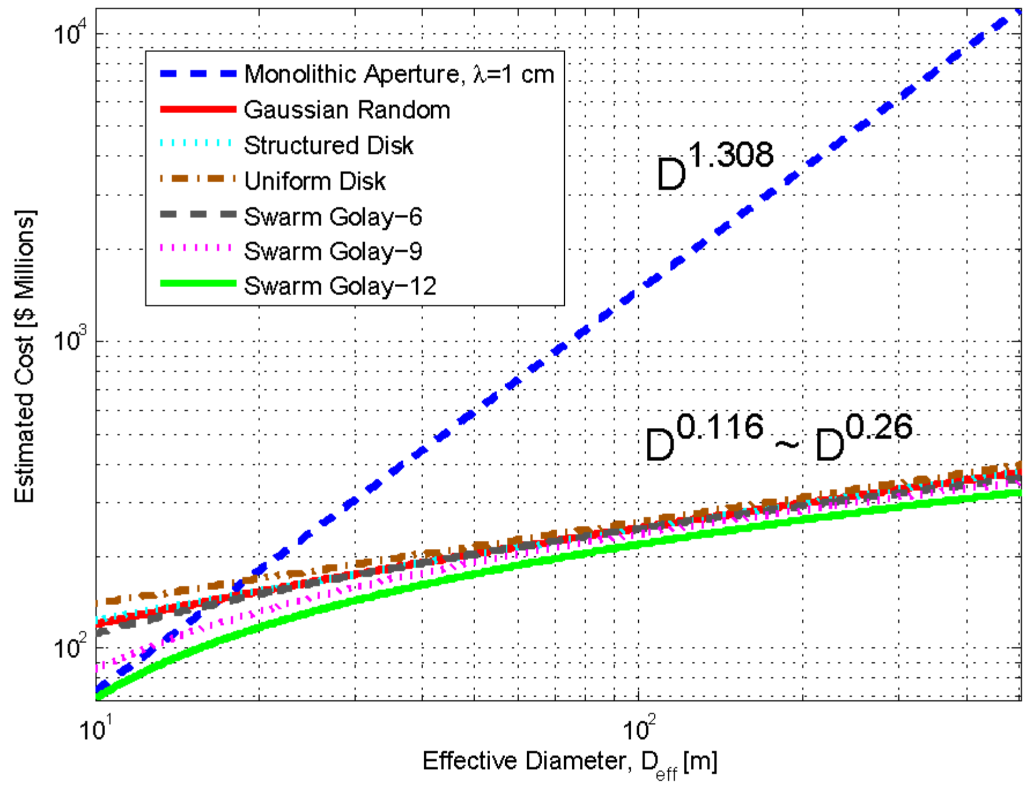


Figure 4. The system cost analysis shows that the swarm configurations, defined in Figs. 1 and 2, are much more cost-effective than a monolithic telescope even without considering the learning curve saving.

5. Conclusion

The SWIFT project aims to deliver hundreds or thousands of small femtosatellites for potential distributed aperture or sensing applications. We introduced new random sparse arrays that can further optimize the number of spacecraft needed for a target effective diameter for an instantaneous imaging purpose. By using the same number of femtosats, the proposed Swarm Golay Arrays achieve much larger effective diameters, thereby implying that we can dramatically reduce the system mass or cost by utilizing swarm Golay arrays.

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