SPECTRUM INTENSITIES OF STRONG-MOTION EARTHQUAKES

by G. W. Housner*

The design of structures to resist earthquakes should be based upon a set of rules and procedures which give building designs having the following properties. Each part of the building should have approximately the same factor of safety, buildings of different types should all have approximately the same factors of safety, and the factor of safety should be of such a magnitude that buildings will not be seriously damaged by the strongest earthquake to which they are likely to be subjected. To establish rules which will give such designs, it is necessary to know the stresses that will be produced in structures when they are subjected to earthquakes. This requires a knowledge of the characteristics and intensities of earthquakes and a knowledge of how structures behave during an earthquake. Since no two earthquakes are identical and since there is a wide variation in the size, proportions, mass, rigidity and foundation conditions of structures, it is a difficult problem to determine precisely what happens to buildings during an earthquake.

A rational approach to the problem of determining what happens to a structure during an earthquake is to install, in typical buildings, instruments which will record the motion of the building and the stresses and strains caused by an earthquake. There are at present some instruments installed in some buildings for the purpose of obtaining such measurements and when such data is accumulated for buildings of various types and for a reasonable number of earthquakes, it should be possible to evolve a very satisfactory set of design rules. However, since there is no control over the occurrence of earthquakes it will be many years before this data is available.

An alternate procedure for investigating what happens to buildings during earthquakes is to make use of seismograms. The United States Coast and Geodetic Survey maintains some forty or more accelerometers in the western United States which record the acceleration of the ground during a strong earthquake. Many of these instruments are installed in the basements of buildings and thus record the motion of the base of the building. A typical accelerogram is shown in Figure 1. With such accelerograms available, it is possible to construct a shaking table which reproduces the earthquake ground motions. If then models of buildings are placed on the shaking table and subjected to the reproduced ground motion, it is possible to measure the behavior of the models and determine the stresses and strains produced. There

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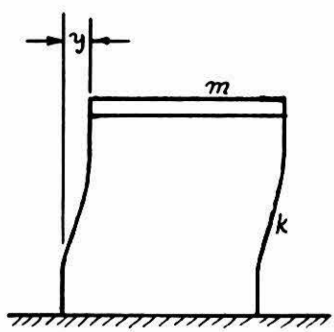
are, however, two difficulties associated with this approach; the first is that it is very difficult to construct a shaking table that will reproduce faithfully such complex ground accelerations as are shown in Figure 1; the second difficulty involves the construction of model buildings since it is not feasible to construct an exact small scale model of an actual building, reproducing all the parts of the buildings and using the same materials. This second difficulty can be avoided if, instead of making models of actual buildings, tests are made with a variety of small scale structures which have properties similar to actual structures but are somewhat simplified. If this is done then the first difficulty also can be avoided for the same results can be obtained without using a shaking table. The method currently being used at the California Institute of Technology involves an electrical analog computor[1] which essentially is a set of electrical circuits whose electrical properties are analogous to the physical properties of the structure which is being simulated. A voltage proportional to the ground acceleration (Figure 1) is applied to the circuits and measurements of the appropriate electrical quantities give the displacements, velocities, shears, bending moments, etc., in the simulated structure. This is thus just an electrical model of the structure and shaking table which can be used more quickly and easily than a physical model. The behavior of structures with various proportions, rigidities and energy losses are currently being investigated under the action of different recorded earthquake ground motions.

Earthquake Spectrum. The properties of an earthquake, as regards its effect on structures, are exhibited in a very informative way by constructing the so-called earthquake spectrum. To explain how this is done, consider a one-degree-of-freedom structure as shown in the accompanying diagram. This structure consists of a mass m supported on elastic columns k and may be considered to be a simplified model of a one-story building. If this structure were placed on a shaking table which reproduced the recorded ground accelerations of an earthquake it would oscillate with displacement y and oscillatory stresses would be produced in the columns. The magnitudes of the displacements and stresses, for a given earthquake, would depend upon the mass of the structure, the rigidity of the columns and the amount of energy loss produced by whatever damping there is in the structure. The natural period of vibration of the structure depends upon these same properties, actual buildings having fundamental periods of vibration ranging from approximately 0.1 seconds to 2.5 seconds or more. If a whole series of model structures of the above type were constructed having periods of vibration ranging from 0.1 to 2.5 seconds, say 97 models were made with the first model having a period of vibration of 0.1 seconds and each succeeding model having a period



1/40 of a second larger than the preceding one and the last model having a period of 2.5 seconds, and these models were placed on the shaking table and subjected to the earthquake ground motion, there would be obtained information on the behavior of structures over the entire range of periods from 0.1 to 2.5 seconds. If, now, the maximum displacement (from which it is possible to compute the maximum stress) were recorded for each structure and a graph were constructed in which each maximum displacement is plotted against the period of vibration of that structure, there could be drawn a curve extending from 0.1 second period to 2.5 second period and the height of the curve at any point would be the maximum displacement of the structure of that period of vibration when subjected to the given earthquake. This curve would be called the displacement spectrum of the earthquake and it sums up the effect of the earthquake in a very concise fashion. If a graph were made of the maximum velocity versus the period of vibration, there would be obtained the velocity spectrum of the earthquake and in a similar fashion there could be constructed an acceleration spectrum which showed the maximum accelerations of the structures or a stress spectrum which showed maximum stresses. In fact, from any one of these the others can be calculated readily.

For reasons which will be given in the following discussion, it is most convenient to work with the velocity spectrum of the earthquake. The mathematical expression defining the velocity spectrum is obtained as follows. If an elastic one-degree-of-freedom structure with small linear damping, of the type described above, is subjected to a base acceleration 'a' the displacement y at any time t may be written in the well known form[2]

$$y = \frac{T}{2\pi} \int_0^t a \; e^{-n\frac{2\pi}{T}(t-\tau)} \sin \frac{2\pi}{T}(t-\tau)\, d\tau \qquad (1)$$

where

y = displacement at time t
T = period of vibration of the structure
a = base acceleration
n = fraction of critical damping

If 'a' is the earthquake ground acceleration, the Equation (1) describes the resulting motion of the structure. The maximum value of Equation (1) which occurs during the time that 'a' is acting is then the maximum displacement of the structure, and this value is the same as would have been measured had the structure been placed on the shaking table and the maximum displacement measured.

If the structure is more complex, as for example having six stories instead of one, the displacement at each story can be written

$$y = \sum_{i=1}^{6} C_i \int_0^t a \; e^{-n_i\frac{2\pi}{T_i}(t-\tau)} \sin \frac{2\pi}{T_i}(t-\tau)\, d\tau \qquad (2)$$

where Ci is a factor dependent on the physical properties of the structure and the point of location where y is measured. The remainder of the symbols have the same significance as in Equation (1) with the subscript i referring to the particular mode of vibration.

The integrals appearing in Equations (1) and (2) are the same and, in general, the linear vibrations of any structure can be expressed in a form similar to Equation (2) with the identical integral appearing and only the factor Ci differing. This means that for any structure of the type being considered here the effect of the earthquake is described by the integral

$$S = \int_0^t a \quad e^{-n\frac{2\pi}{T}(t-\tau)} \quad \sin\frac{2\pi}{T}(t-\tau)\,d\tau \tag{3}$$

Having all the values of this integral for a particular earthquake, it is possible to compute the displacements, shears, bending moments, etc. for any structure. Moreover, except for a slight frequency modulation, Equation (3) is the velocity of the one-degree-of-freedom structure described by Equation (1), so that the velocity spectrum described above in connection with the response of the one-degree-of-freedom structure is just a graph of the maximum values of Equation (3) for a particular earthquake. It may be noted that the acceleration spectrum is obtained if each point on the velocity spectrum is divided by the corresponding period of vibration.

The velocity spectrum thus sums up a great deal of information about the earthquake. For example, in comparing two earthquakes, if it is found that at a period of 1/2 second the ordinate of the velocity spectrum is twice as large for the first earthquake as for the second, then a structure having 1/2 second period would have experienced maximum displacements, accelerations, shears, and stresses twice as great during the first earthquake as during the second, etc. In Figure (2) there are shown a number of velocity spectra computed from the same earthquake record. The curve marked n = 0 is the undamped spectrum, that is, it is applicable to ideal structures with zero energy loss. The curve marked n = 0.2 is the spectrum for structures with 0.2 critical damping, etc.

It will be noted that the effect of damping is very pronounced in reducing the spectrum. This means that the maximum stresses will depend strongly upon the amount of damping there is in the structure. Vibration tests on actual buildings show that the amount of damping varies with the type of construction. A monolithic reinforced concrete warehouse having four stories was vibrated[3] with a large shaking machine and the measured damping was 0.075 critical damping, whereas a similar structure having a reinforced concrete frame but hollow tile filler walls had approximately 0.14 critical damping. Tests on some masonry factory buildings have shown as much as 0.40 critical damping.

It should be noted that when an actual building is subjected to increasingly severe earthquake ground motion it reaches a stage of preliminary failure during which cracking occurs, parts are stressed beyond the yield point, etc. During this preliminary failure large amounts of energy may be dissipated with a consequent relief of the structure. The earthquake spectrum is not applicable to such behavior of the structure, during which the damping and stiffness is changing; it is applicable only to undamaged structures.

Spectrum Intensity. For the structure described by Equation (1) the maximum displacement occurring during an earthquake can be written

$$y_{max} = \frac{T}{2\pi} S \qquad (4)$$

where T is the period of vibration of the structure and S is the maximum value of Equation (3), that is, S is the ordinate of the earthquake spectrum corresponding to the period T. If k is the stiffness of the columns, the maximum shear force in the columns is given by

$$F_{max} = ky_{max} = k \frac{T}{2\pi} S$$

For moderate amounts of damping, say $n \leqq 0.4$, the effect of the damping on the period is negligible and the period of vibration can be taken to be $T = 2\pi\sqrt{\frac{m}{k}}$, where m is the mass of the structure. The maximum shear force may thus be written

$$F_{max} = \sqrt{k\,m} \quad S \qquad (5)$$

It is thus seen that the maximum lateral force, F_{max}, is given by the product of two terms, one of which involves only the mass and stiffness of the structure whereas the other term is the ordinate S of the spectrum. The spectrum may, therefore, be taken as a measure of the severity of the earthquake in the sense that for a given structure, that is, a given k and m, the maximum force is directly proportional to S.

In using the spectrum as a measure of the intensity of an earthquake[4], that is, as a measure of the capability of the earthquake to produce stresses, allowance must be made for the fact that in a city the periods of vibration of the structures will cover a wide range. If the significant range of periods is taken to be from 0.1 seconds to 2.5 seconds, the average value of S over this range is a measure of the intensity of the earthquake. It is a measure of the intensity in the sense that if a city contained a large number of structures having a uniform distribution of periods ranging from 0.1 to 2.5 seconds, and the city were subjected to different earthquakes, then on the average the ratios of the maximum stresses produced would be proportional to the average values of S for the different earthquakes. It is thus seen that this measure of the intensity is an average measure for a range of periods.

Since measures of earthquake intensities are useful only for comparing different earthquakes, it makes no difference whether the average value of S is used or whether the area under the curve is used. Accordingly the spectrum intensity of an earthquake is defined to be the area under the spectrum curve between the periods 0.1 and 2.5 seconds.

Since the stresses in a structure produced by an earthquake depend upon the amount of damping present, it is informative to measure the spectrum intensities of earthquakes for various amounts of damping. The undamped intensity is the area under the spectrum curve computed for zero damping (n = 0.0); the 0.2 damped intensity is the area under the spectrum curve computed for 0.2 critical damping (n = 0.2), etc.

Intensities of Earthquakes. The U. S. Coast and Geodetic Survey has obtained numerous seismograms of earthquakes occurring in the western United States and the most intense motions have been recorded on the dates and locations shown in Table I. Two horizontal components of motion were recorded at each of these locations, thus giving a total of 28 earthquake accelerograms. Spectrum curves were computed for each of these[5], with various amounts of damping and the corresponding spectrum intensities are listed in Table I. The intensity listed for each earthquake is the average of the intensities of the two components; for example, the earthquake recorded at El Centro, May 18, 1940 had computed undamped spectrum intensities of 8.94 for the north-south component and 7.77 for the east-west component, thus giving an average intensity of 8.35; the earthquake recorded at El Centro, December 30, 1934 had computed undamped spectrum intensities of 5.93 for the north-south component and 5.83 for the east-west component, thus giving an average intensity of 5.88.

During a shock three components of ground acceleration are recorded, two horizontal components at 90 degrees to each other and a vertical component. The two horizontal components give slightly different spectrum curves and the spectrum intensities of the two horizontal components differ in magnitude by usually four or five percent. The vertical component of ground acceleration differs somewhat from the horizontal components in that the acceleration is smaller and high frequency components are relatively predominant compared to the horizontal components. There is thus some directional effect involved. The recorded horizontal ground motion will differ somewhat depending upon the direction of the horizontal axis of the recording accelerometer and there will be a marked difference between horizontal and vertical components of ground motion.

TABLE I

SPECTRUM INTENSITIES OF EARTHQUAKES

No.	Location	Date	Damping (fraction critical)		
			0.0	0.2	0.4
1.	El Centro, California	May 18, 1940	8.35	2.71	1.89
2.	El Centro, California	Dec. 30, 1934	5.88	2.09	1.61
3.	Olympia, Washington	Apr. 13, 1949	5.82	2.21	1.77
4.	Vernon, California	Mar. 10, 1933	4.62	1.70	----
5.	Santa Barbara, California	June 30, 1941	3.29	1.80	1.46
6.	Ferndale, California	Oct. 3, 1941	2.99	1.41	1.14
7.	Los Angeles Subway Terminal	Mar. 10, 1933	2.94	0.82	----
8.	Seattle, Washington	Apr. 13, 1949	2.63	1.10	0.84
9.	Hollister, California	Mar. 9, 1949	2.36	1.27	1.00
10.	Helena, Montana	Oct. 31, 1935	1.82	1.02	----
11.	Ferndale, California	Sept. 11, 1938	1.45	0.64	----
12.	Vernon, California	Oct. 2, 1933	1.32	0.69	0.53
13.	Ferndale, California	Feb. 9, 1941	1.10	0.40	----
14.	Los Angeles Subway Terminal	Oct. 2, 1933	0.96	0.45	0.35

The earthquake record that had been obtained at Long Beach, California, March 10, 1933 is not included in Table I because the accelerogram was not complete, the earlier portion of the earthquake not being recorded. It is estimated that the intensities for this record were approximately the same as for El Centro, May 18, 1940. It should also be noted that the accelerometer that recorded the ground motion at Seattle, Washington, April 13, 1949 was located on filled ground adjacent to a sea wall so that the intensities for this record describe the motion of the filled ground and do not describe the intensity of the ground motion over the remainder of the city. It is estimated that the intensities over the other portions of the city were more nearly of the magnitude listed for Olympia, Washington.

A graph of the intensities is shown in Figure 3 where it is seen that there is a rather uniform coverage of intensities ranging from an undamped intensity of 0.96 for Los Angeles Subway Terminal, October 2, 1933 to 8.35 for El Centro, May 18, 1940. The greatest intensity is thus 8.7 times the smallest. In trying to associate these intensities with observed damage to buildings, it must be remembered that the buildings in these cities have an appreciable amount of damping so that it is more reliable to take the 0.2 damped intensities as indicative of damage instead of the undamped intensities. The 0.2 damped intensities range from 0.40 for Ferndale, California, February, 1941 to 2.71 for El Centro, California, May 18, 1940. The greatest intensity is thus 6.8 times the smallest.

Comparison with Modified-Mercalli Intensities. One of the common methods of assessing the intensity of the ground motion is by the Modified-Mercalli scale of intensities. This non-instrumental method of measuring the intensities is based on the observations of persons feeling the ground motion and upon the damage to buildings caused by the earthquake. For convenient reference an abridged form of the Modified-Mercalli scale is given.

Modified-Mercalli Scale

I. Not felt except by a very few under especially favorable conditions.

II. Felt only by a few persons at rest, especially on upper floors of buildings.

III. Felt quite noticeably indoors, but many persons do not recognize it as an earthquake.

IV. During the day felt indoors by many, outdoors by few. Dishes, windows, doors disturbed.

V. Felt by nearly everyone; some dishes, windows and so forth, broken.

VI. Felt by all; some heavy furniture moved, a few instances of fallen plaster or damaged chimneys.

VII. Damage negligible in buildings of good design and construction; slight to moderate in ordinary well-built structures; considerable in poorly built or badly designed structures.

VIII. Damage slight in specially designed structures; considerable in ordinary substantial buildings with partial collapse; great in poorly built structures.

IX. Damage considerable in specially designed structures; great in substantial buildings.

X. Some well-built wooden structures destroyed; most masonry and frame structures destroyed.

XI. Few, if any, masonry structures remain standing.

XII. Damage total.

The Modified-Mercalli intensities, as assessed by the U. S. Coast and Geodetic Survey, are listed in Table II for the 14 localities where the accelerograms were obtained.

TABLE II

MODIFIED-MERCALLI INTENSITIES

		Assessed Intensity
1.	El Centro, 1940	7.5
2.	El Centro, 1934	6
3.	Olympia, 1949	8
4.	Vernon, March 1933	7.5
5.	Santa Barbara, 1941	7
6.	Ferndale, October 1941	6
7.	Subway Terminal, March 1933	6.5
8.	Seattle, 1949	8
9.	Hollister, 1949	7
10.	Helena, 1935	8
11.	Ferndale, 1938	6
12.	Vernon, October 1933	6
13.	Ferndale, February 1941	6
14.	Subway Terminal, October 1933	5.75

These Modified-Mercalli intensities are plotted against the 0.2 damped spectrum intensities in Figure 4. It is seen that there is a considerable scatter of points on this graph as might be expected from the amount of subjective judgment required in assessing M-M intensities. The curve shown in Figure 4 is an approximate ideal relation between the Modified-Mercalli intensities and the 0.2 damped spectrum intensities. Comparing with this curve, it is seen that the relative intensities of points 1 and 2, of points 4 and 7, and of points 12 and 14 are self-consistent. Each pair of these represents two instances where M-M ratings were made in the same locality so that the relative intensities of the two do not reflect subjective judgments as to quality of building construction, etc. Some of the scatter of the points in Figure 4 may be explained by special circumstances which were in effect. For example, point 10, Helena, Montana, October 31, 1935 was preceded by a stronger shock for which accelerograms were not obtained. The first shock caused considerable damage and apparently weakened many buildings so that the shock of October 31 caused more damage than it would have caused had it been the only earthquake to which the city had been subjected. The deviation of point 8, Seattle, Washington, April 13, 1949 is perhaps explained by the fact that the accelerometer that recorded the motion was located on filled ground, adjacent to a sea wall, which apparently had an effect on the ground motion. The spectrum intensity for Seattle is thus indicative of the intensity of ground motion of the filled ground adjacent to the sea wall but it is thought to be less than the intensity of the ground motion in other parts of the city.

Earthquake Magnitudes. The spectrum intensity of an earthquake is a measure of the severity of the ground motion at the point where the seismogram was recorded. The intensity of ground motion will vary over the region affected by the earthquake, being greatest near the center of the shock and diminishing to zero at increasingly greater distances from the center. The intensity is thus not a measure of the magnitude of the earthquake for the intensity may be greater near the center of a small earthquake than it is at a large distance from the center of a great earthquake. A measure of the magnitude was originally proposed by C. F. Richter. He defined the magnitude of an earthquake, for shocks in California, as the logarithm of the maximum trace amplitude expressed in thousandths of a millimeter with which the standard short-period torsion seismometer (free period 0.8 seconds, static magnification 2800, damping nearly critical) would register that earthquake at an epicentral distance of 100 kilometers. The relation between earthquake magnitude, energy and acceleration is discussed by B. Gutenberg and C. F. Richter.[6]

It is shown in the paper by Gutenberg and Richter that when the depth of shock is 18 kilometers the relation between the energy released by the shock and the magnitude of the earthquake may be expressed satisfactorily by the relation

$$E = (10)^{11.3} \quad (10)^{1.8M} \tag{6}$$

where E is the released energy in ergs and M is the magnitude of the earthquake. As shown by this equation the magnitude of an earthquake is a measure of the energy released by the shock. In Table III there are shown the magnitudes of the preceding earthquakes.

It will be noted from Equation (6) that the magnitude of an earthquake varies as the logarithm of the energy released. An earthquake of magnitude 7 thus releases 63 times as much energy as a shock of magnitude 6 and 4000 times as much as a shock of magnitude 5. From Table III it is seen that the earthquake of April 13, 1949, Seattle, Washington had a magnitude of 7.1 whereas the shock of March 10, 1933, Long Beach, California, had a magnitude of 6.25, so that approximately 34 times as much energy was released April 13, 1949 as March 10, 1933. The depth of the Seattle shock was exceptionally large for earthquakes of this type so that there was a corresponding reduction in surface intensity.

TABLE III

EARTHQUAKE MAGNITUDES

	Magnitude	Approximate Depth in km
El Centro, May 18, 1940	6.7	18
El Centro, December 30, 1934	6.5	18
Seattle, April 13, 1949	7.1	60
Santa Barbara, June 30, 1941	5.9	18
Ferndale, October 3, 1941	6.4	18
Long Beach, March 10, 1933	6.25	18
Hollister, March 9, 1949	5.3	18
Helena, October 31, 1935	6.0	18
Ferndale, September 11, 1938	5.5	18
Vernon, October 2, 1933	5.3	18
Ferndale, February 9, 1941	6.6	18

Variation of Intensity. When an earthquake of given magnitude occurs the surface of the ground will move with varying intensity over a region surrounding the center of the shock. This is illustrated in Figure 5 where contour lines of equal intensity are shown for an earthquake of 15 mile depth and 30 mile length of fault. The construction of this graph was based on the observed diminution of amplitudes of seismic waves as they travel from the center of the shock. The diagram is an approximation and is intended to indicate only how the spectrum intensity varies over the region surrounding the center of the shock; the diagram does not take into account local geological conditions which might distort the isoseismal lines. The numbers on the graph indicate the relative intensities at these distances from the center.

As shown by Figure 5, there is a relatively localized area around the center of the shock where the ground motion is severe and a relatively large area where the ground motion has comparatively low intensity. For example, during the earthquake of May 18, 1940 an area of approximately 3000 square miles had a spectrum intensity equal to or greater than that at El Centro, whereas at a distance of 15 miles farther from the center of the shock than the town of El Centro the intensity was diminished to approximately one-half that at El Centro. The question of the probability of earthquake damage thus involves three factors: the magnitudes of earthquakes, the area of destructive intensity per earthquake, and the frequency of occurrence of earthquakes.

Frequency of Occurrence of Strong-Motion Earthquakes. A listing of strong-motion earthquakes is given by Gutenberg and Richter[7] covering the occurrence of world earthquakes of magnitude 7.7 to 8.8 during the period 1904-1946 and of magnitude 7.0 to 7.7 during the period 1918-1946, and also California earthquakes of magnitude 5.2 to 8.7 during the period 1904-1906. When the frequencies of world earthquakes of magnitude 7.0 to 7.7 are multiplied by 43/29 to adjust them to a 43-year period the resulting frequencies of occurrence are as shown in Table IV. A graph of these frequencies is shown in Figure 6.

For the 43-year period there are 629 world earthquakes of magnitude 7.0 or greater and 80 California earthquakes of magnitude 5.2 or greater. During this period the greatest world earthquake had a magnitude of 8.7 and the greatest California earthquake (San Francisco, 1906) had a magnitude of 8.2. It is seen in Figure 6 that there is a very rapid diminution in frequency of occurrence for increasing magnitudes and the graph clearly pinches off at magnitude 8.7. In view of the large number of earthquakes included in the data, this indicates that the probability that an earthquake of magnitude greater than 8.7 will occur is extremely small or, in other words, that there is an apparent upper bound for the magnitude of an earthquake.

The equation of the curve fitted to the world earthquakes in Figure 6 is

$$y = 16x - (3.75x)^2 + (3.11x)^3 \tag{7}$$

In this expression $x = 8.7 - M$, where M is the magnitude. If this expression is multiplied by 10, there is obtained

$$f = 10 \left[16x - (3.75x)^2 + (3.11x)^3\right] \tag{8}$$

with $x = 8.7 - M$. The area under this curve between the magnitudes M_1 and M_2 is equal to the number of earthquakes occurring in a 43-year period with magnitudes between M_1 and M_2.

The number of California earthquakes occurring in the 43-year period is insufficient for making a frequency analysis but, assuming that the number of earthquakes having magnitudes greater than 6.0 is a reasonable estimate of the average number to be expected during a 43-year period and assuming that the relative frequencies of occurrence of California earthquakes follows the same pattern as the frequency of occurrence of world earthquakes, there is obtained the following expression for the frequency of occurrence of California earthquakes.

$$f = \frac{1}{8.6} \left[16x - (3.75x)^2 + (3.11x)^3\right] \tag{9}$$

The area under this curve between $x=8.7 - M_1$ and $x=8.7 - M_2$ is then an estimate of the average number of California earthquakes, having magnitudes between M_1 and M_2 , to be expected during a 43-year interval. This expression may be written in the alternate form, after integration

$$E.N. = \frac{Y}{(43)(8.6)}\left[8x - (3.75)^2 \frac{x^3}{3} + (3.11)^3 \frac{x^4}{4}\right] \qquad (10)$$

which gives the expected number, E.N., of California earthquakes during a period of Y years which have a magnitude greater than M, where $x = 8.7 - M$. Some numerical values computed from this equation are shown in Table IV.

TABLE IV

EXPECTED NUMBER OF CALIFORNIA EARTHQUAKES

Of Magnitude Greater Than	Per Period of Years			
	25	50	100	200
6.0	25	50	99	198
6.2	18	36	73	146
6.4	13	26	53	106
6.6	9.3	19	37	74
6.8	6.4	13	26	51
7.0	4.3	8.6	17	34
7.2	2.6	5.2	10	21
7.4	1.7	3.4	6.7	13
7.6	.97	1.9	3.9	7.8
7.8	.51	1.0	2.0	4.1
8.0	.28	.56	1.1	2.2
8.2	.13	.26	.51	1.0
8.4	.04	.08	.17	.34

Table IV shows that an earthquake of magnitude 8.2 (San Francisco, 1906) or greater can be expected to occur in California on the average of once every 200 years. An earthquake of magnitude 6.7 (El Centro, 1940) or greater can be expected to occur in California on the average 63 times during a 200-year interval. Shocks of magnitude 6.25 (Long Beach, March 10, 1933) or greater can be expected on the average 138 times during a 200-year interval.

With the preceding data it is possible to make an approximate estimate of the frequency with which the average California city can be expected to experience a strong earthquake. To do this it is necessary to make certain assumptions and approximations as follows. Considering only earthquakes of magnitude 6.0 or greater, let it be assumed that on the average for these shocks there will be an area of roughly 2000 square miles subjected to a ground motion corresponding to a 0.2 damped spectrum intensity of 3.0 or greater, that is, a ground motion of severity approximately equal to or greater than that experienced at El Centro, May 18, 1940 or Long Beach, March 10, 1933. Next let it be assumed that earthquakes are equally likely to occur anywhere in the state. Then taking the area of California as approximately 150,000 square miles the probability that the 2000 square mile area will cover any specific point is

$$p = \frac{2000}{150,000} = 0.0133$$

The expected number of times that this will happen during a 200-year interval is, using Table IV, 0.0133 x 198 = 2.6, so that on the average a city in California can expect to experience ground motion of the intensity of El Centro and Long Beach or greater at a rate of approximately 2.6 times per 200 years or 3 times per 230 years. Lesser intensities can be expected more frequently and greater intensities less frequently.

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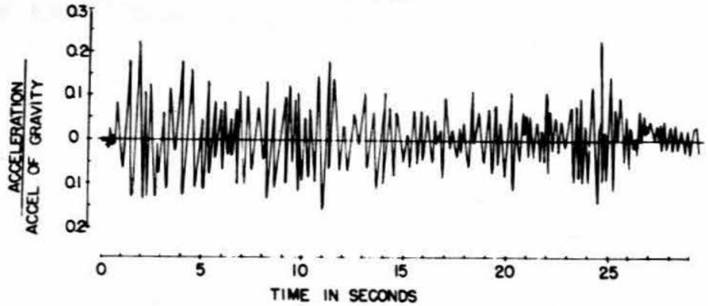
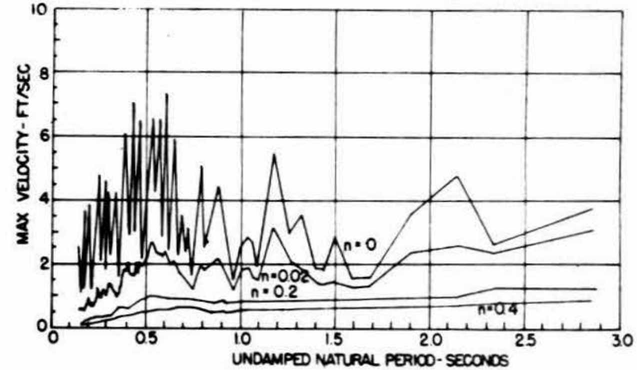
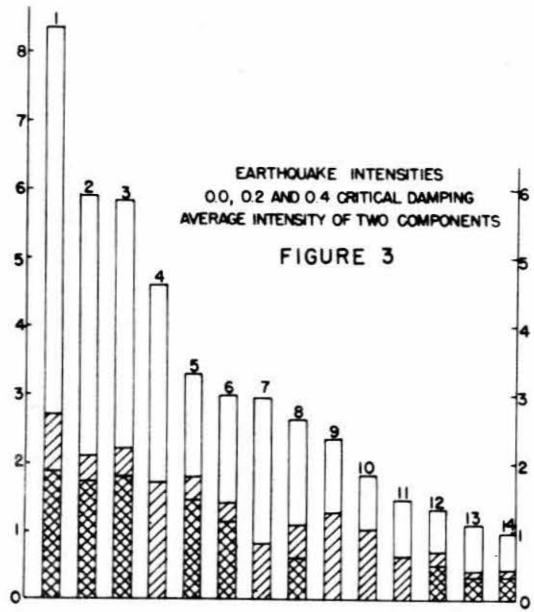


ACCELEROGRAM FOR EL CENTRO, CALIFORNIA EARTHQUAKE OF MAY 18, 1940. COMPONENT E-W FIGURE 1



VELOCITY SPECTRUM FOR EL CENTRO, CALIFORNIA, EARTHQUAKE OF MAY 18, 1940. COMPONENT E-W FIGURE 2



EARTHQUAKE INTENSITIES
0.0, 0.2 AND 0.4 CRITICAL DAMPING
AVERAGE INTENSITY OF TWO COMPONENTS
FIGURE 3

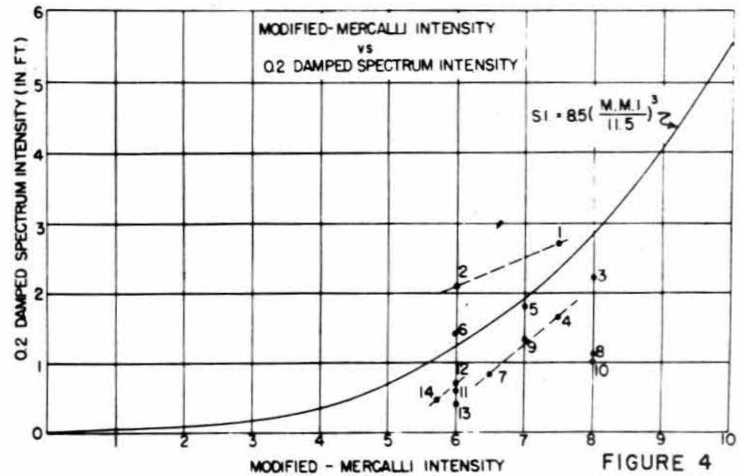


FIGURE 4

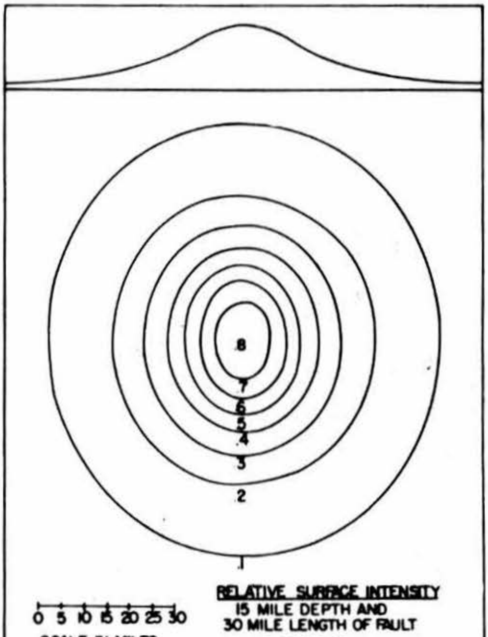


FIGURE 5

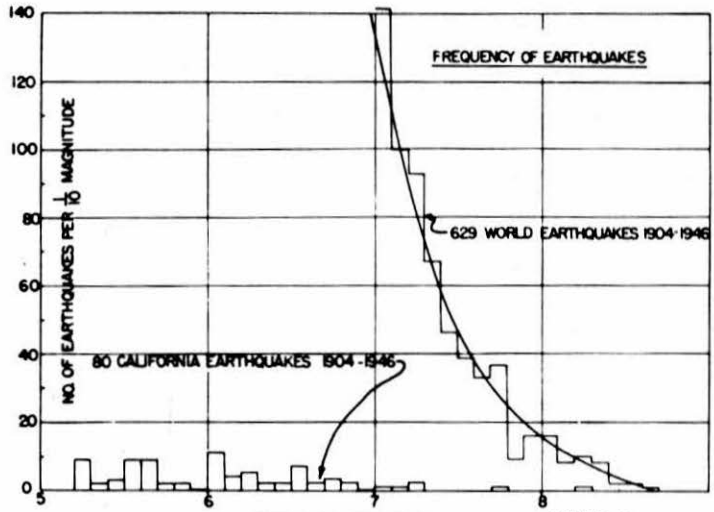


FIGURE 6