Coupling of Highly Nonlinear Waves with Linear Elastic Media

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ABSTRACT

This paper reports a fundamental study of the coupling between highly nonlinear waves, generated in a one dimensional granular chain of particles, with linear elastic media, for the development of a new Non Destructive Evaluation and Structural Health Monitoring (NDE/SHM) paradigm. We design and use novel acoustic actuators to excite compact highly nonlinear solitary waves in a one-dimensional linear elastic rod and investigate the pulse propagation across the interface. To model the actuator and rod system we use Finite Element Analysis (Abaqus) and obtain excellent agreement between the experimental observations and the numerical results. We also study the response of the system to the presence of defects (cracks) in the steel rod, by comparing the wave propagation properties in pristine and cracked test objects. The obtained results encourage the use of highly nonlinear waves as an effective tool for developing a new, viable NDE/SHM method.

Keywords: Highly Nonlinear Waves, Nondestructive Evaluation, Novel Actuators

1. INTRODUCTION

Non Destructive Evaluation (NDE) and Structural Health Monitoring (SHM) techniques are testing procedures performed to detect, locate and quantify structural damage, material pre-stress and working conditions without affecting the object's future usefulness. NDE/SHM techniques are needed to ensure the performance and the proper response of engineering systems, avoiding failures as well as keeping maintenance costs at a minimum. One of the most common techniques used for NDE/SHM of civil structures is based on impact testing.[1–3] In this approach, the material or structure undergoing testing is typically struck with an impacting device such as an instrumented hammer. The hammer is used as a mean to convert mechanical energy into kinetic energy thereby it generate propagating pulses in the specimen or otherwise, it is used to excite natural frequencies of the system. Sensors are then placed in selected locations in the system to detect reflections, frequency shifts and other wave propagation characteristics. This information is then used to provide some indication about the presence of defects or the alteration of the structures' properties. In the last few decades several stress wave based inspection/monitoring techniques have been developed for damage detection and material characterization in metallic and composite structures.[1, 3–6] Such detailed information on the nature of the damage comes at the cost of extended time of interrogation and heavy data processing. Thus, there is a widespread need for NDE/SHM techniques, which can perform continuous monitoring of the structure using an automated non-intrusive remotely controlled actuator. We propose the use of a granular system to generate highly nonlinear solitary waves which will be used to interrogate structures, as an alternative approach to the classical instrumented hammers.[7, 8]

In this paper we study experimentally and numerically the formation and propagation of highly nonlinear solitary waves in a granular chain and their interactions with a linear elastic rod (Fig 1). We study the impulse's behavior at and across the interface between the nonlinear and the linear system. We compare results obtained

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by modeling the nonlinear system in Finite Elements (Abaqus), discrete particle simulations (Matlab) and continuum theory. To verify experimentally the effectiveness of the new actuators for damage detection, we compare the use of our new highly nonlinear actuators with typical commercial hammers by performing tests on pristine and damaged rods.

2. SOLITARY WAVES: BACKGROUND

Highly nonlinear acoustic solitary waves were first reported numerically, experimentally and analytically in one-dimensional granular crystals (i.e. chains of spherical particles)[9, 10] and have, since then, gained increasing attention in the scientific community.[7, 11–17] Highly nonlinear media support acoustic pulses that have intrinsically different properties than those found in conventional linear acoustic signals, allowing for a stronger degree of tunability.[18] By a small variation of the particles assembling, for example, it is possible to significantly alter the properties (wave length, speed, amplitude, and frequency) of the excited pulse and the number of pulses in a given train. The use of solitary waves for the detection of defects and impurities in granular media was discussed by Sen, et al.[14] and by Hong, et al.[15] Solitary waves have been demonstrated to be sensitive to the granular materials properties,[13] such as the elastic modulus, as well as to the applied static preload.[13, 18] In addition, the dependence of the backscattered signal's velocity and shape on the presence of light and heavy impurities in a granular chain has also been noted.[15] Highly nonlinear solitary pulses have been studied numerically and experimentally in various one-dimensional highly nonlinear systems assembled from chains of stainless-steel, glass, brass, nylon, polytetrafluoroethylene (PTFE) and Parylene coated steel beads.[12, 13, 16–18]

To describe the formation and propagation of the highly nonlinear solitary waves in the granular chain, we consider particles interacting according to the nonlinear Hertzian contact law.[19] From this discrete system, a continuum approach based on long wavelength approximation was derived for uniform[9, 10] and heterogeneous[12] materials that show a nonlinear contact interaction law relating the force (F) and displacement δ:

$$F = k\delta^n \qquad (1)$$

where k is function of the particle's and material's parameter (radius of contact, Poisson ratio and elastic constant), n is the nonlinear exponent (with $n > 1$), and the displacement δ is considered positive in compression. In tension ($\delta < 0$) the force becomes zero (particles can freely move apart from each other). A more detailed description of highly nonlinear waves is given in Ref 9. The recent derivations and generalizations of the nonlinear wave theory in the continuum limit[9, 12] offer a significant expansion to the classical methods of acoustic signal generation by providing means to obtaining a broader degree of tunability and pulse shaping control over conventional strikers available in commercial acoustic systems. In particular, the tunability reported for the solitary waves[7, 11–13, 18] provides complete control over tailoring: i) the choice of the wave's width (spatial and temporal length) for defects investigation, ii) the composition of the excited train of waves (i.e. number and separation of the waves used for testing) and iii) their amplitude and velocity.

Figure 1: Experimental set up consisting of 20 vertically aligned stainless steel particles (diameter of 4.76 mm), positioned on top of a 4.76 mm diameter steel rod. The rectangular elements (in orange) indicate the position of the piezo gauges used for monitoring the signal propagation in the system [as shown in Ref 7].

The input solitary waves can be tuned for specific application by varying one or more of the field parameters (i.e. static and dynamic force) or by changing the material property of the system (diameter of the bead, elastic properties, etc).[18] The analytical expression for the tunability of the solitary waves speed derived from the discretization of the particles in the chain can be described as follows:[18]
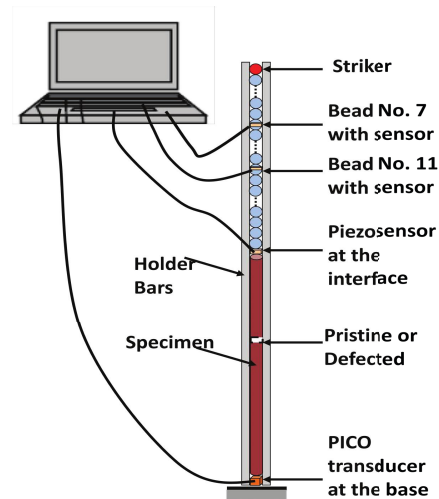
$$V_s = 0.6802 \left(\frac{2E}{a\rho^{3/2}(1-\nu^2)} \right)^{1/3} F_d^{1/6} \tag{2}$$

$$V_s = 0.9314 \left(\frac{4E^2 F_0}{a^2 \rho^3 (1-\nu^2)^2} \right)^{1/6} \frac{1}{(f_r^{2/3}-1)} \left\{ \frac{4}{15} \left[3 + 2f_r^{5/3} - 5f_r^{2/3} \right] \right\}^{1/2} \tag{3}$$

where F_0 represents the static pre-stress (pre-compression) added to the system, $f_r = F_d/F_0$ and F_d is the maximum contacts force between the particles in the discrete chain. The dependence of the solitary wave properties on the materials parameters is shown in Eq. 2 for a non-prestressed system and in Eq. 3 for a prestressed system. Another interesting feature of the highly nonlinear solitary waves is determined by the fact that the system is size independent and the solitary waves can therefore be in principle scalable to smaller or larger dimensions, according to the needs of each specific application.

Since the pulses generated by our nonlinear actuator system are single impulses, we define a "pseudo-frequency" of the signal as the inverse of the temporal pulse length excited in the different structures. We used Eqs. 2 and 3, to plot the experimental range of achievable pseudo frequencies varying selected values of average maximum dynamic force (F_m) between $[0-200N]$ and static load (F_0) between $[0-30N]$ in our actuator. To calculate such ranges, we considered a mono dispersed chain of 20 stainless steel (316 types), 4.76 mm diameter beads, which are used for the nonlinear actuation system in all the experiments described in this report. From Fig. 2, we see that by changing the dynamic or static force, we can change the pseudo frequency of the input pulse over a wide range. This is of special importance for NDE techniques as it allows a dramatic improvement over currently available devices (i.e. miniature hammers) in which the working frequency is fixed for a given striker size and limited to relatively low ranges.
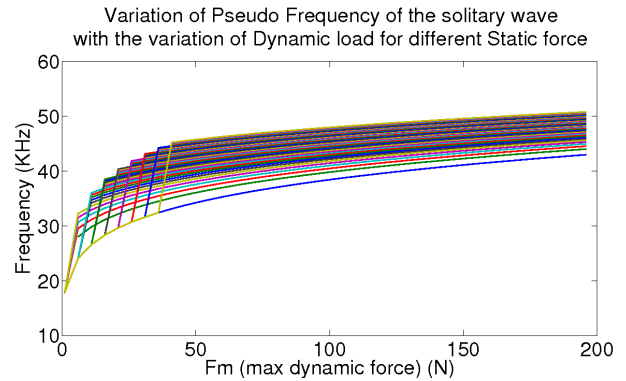

Figure 2: Plot showing the tunability achievable with the highly nonlinear granular actuator used in experiments for different ranges of pseudo frequencies. Each line represents a different value of static precompression applied to the system varied between $0-30N$. An even wider range of pseudo frequency can be obtained by optimizing the particles size, materials and shape within the chain.

3. EXPERIMENTAL SETUP AND PROCEDURE

We assembled a highly nonlinear actuator by aligning a granular chain of 20 stainless steel beads (316-type, provided by McMaster-Carr) confined into a Delrin holder. The particles, each 4.76 mm in diameter, had a mass of 0.45g, Elastic modulus equal to 193 GPa and Poisson Ratio equal to 0.3. In this system, single solitary pulses were generated impacting the top particle of the chain with a similar bead at different velocities. Two calibrated piezosensors (RC$\sim 10^3 \mu s$, Piezo Systems Inc.) made of lead zirconate titanate square plates, with thickness 0.5 mm and 3 mm side, with nickel-plated electrodes and customized micro-miniature wiring (supplied by Piezo Systems, Inc.) were used to study the formation and propagation of the solitary waves in the actuator (a schematic representation of the piezosensors placed in the actuators is shown in Fig. 3 of Ref 18). An additional piezo gauge was embedded inside two thin stainless steel discs (316-type) and glued to one end of the rods to visualize the solitary waves propagation at the interface between the nonlinear and the linear media.

Three linear elastic rods 4.76 mm diameter each made from stainless steel (316-type, length 254 mm) were used for the investigations. We tested one pristine rod to study the coupling of the nonlinear signal with a linear elastic medium, and two damaged rods to assess the validity of the novel nonlinear actuator system for damage detection. The first damaged rod had a machined notch ($\sim 1\ mm$ wide and $\sim 2\ mm$ deep) located at the middle section along the length of the rod. The second damaged rod had three machined notches (each notch of same dimension as used for first damaged rod), located at equal distances along its length. Physical Acoustics PICO

sensors were used as end receivers bonded onto the bottom of the rod specimens using PCB Petro Wax. All the transducers were connected to a Tektronix oscilloscope to monitor the force-time responses of the traveling excitations.

Impulses in the linear elastic rods were also excited using a commercial PCB 086D80 miniature hammer for benchmark comparisons. The entire system was positioned vertically inside a stand composed of four garolite rods to guarantee alignment (a schematic diagram of the overall experimental set up is shown in Fig. 1).

4. DATA ANALYSIS

We used Fast Fourier Transforms (FFT) to characterize the propagating pulses in the frequency domain. In addition, time information preserving continuous wavelet transforms were used to examine the change in the wave forms between their propagation in the pristine and defected systems (results not shown here). In order to compare the pulses' differences quantitatively, a signal processing statistical technique was used. The root mean square deviation (RMSD) method interprets the difference between the signature of the pristine state and damaged /altered states. The RMSD index as adopted by Giurgiutiu and Rogers[20] is presented here:

$$RMSD(\%) = \sqrt{\frac{\sum_{i=1}^{N}(y_i - x_i)^2}{\sum_{i=1}^{N}(x_i)^2}} * 100 \tag{4}$$

Where, x_i and y_i $(i = 1, 2, 3 \ldots N)$ are signatures obtained from the PZT bonded to the structure before and after the damage is incurred.

5. FINITE ELEMENT MODEL

We modeled the experimental set up using a finite element approach to describe the stress wave propagation and localization in the system and particularly around interfaces. The same approach could be also useful to understand the effect of dissipation in the granular system and to provide a more accurate phenomenological description of the problem.[21]

Finite element models of the specimens and of the granular chain were generated in Abaqus/CAE and the problem was solved in Abaqus/Explicit. Three different geometrical components were used in the numerical analysis: the beads as spheres, the rods as cylinders and a base plate on which the rod rested as a rigid plate. The entire sphere were modeled with solid (continuum) elements. To get a denser mesh in the vicinity of the contact point (in order to capture the point contact effects with high accuracy) and a coarser mesh far from it (to reduce computational costs), tetrahedral elements of second order (modified 10-node tetrahedral (C3D10M)) were used.

Two types of mesh elements were selected in the finite element modeling of the rods. One was a 10-node second order tetrahedral element in the vicinity of the contact point and the other was an 8-node linear brick element away from the contact point. The base plate was modeled as a rigid body. The total number of elements generated was 334 for the beads, 709 for the rods and 100 for the base plate. The contact interaction between two bodies was defined using the surface to surface interaction (Explicit) in Abaqus, where one of the surfaces was selected as master and the other as slave. The constraints applied on the contact were of small-sliding kinematic and the contact properties used were no friction in the tangential direction and contact pressure-over closure in the normal direction. Such properties were selected to make sure that the two bodies in contact would not overlap with each other. More about the Abaqus element and contact details can be found in Ref 22.

All the material's properties and the dimensions of the parts modeled in Abaqus were kept the same as the experimental set up described in section 3.

6. EXPERIMENTAL AND NUMERICAL RESULTS

6.1 Comparison of Pulses Generated with a Commercial Hammer and a Nonlinear Wave Actuator

A comparison between the pulses generated by a standard hammer and the nonlinear wave generator was performed on the pristine and damaged rods. The input signal generated on the top cross section of the rod by both the nonlinear actuator and the hammer are shown in Fig 3(a, b). To compare the shape of the two pulses, we normalized their amplitude with respect to their peak force. From the force-time curves as well as the FFT of the waves, it is evident that the waves generated in the two cases are similar (as per the selection of the given nonlinear actuator experimental set up). It is important to mention that the nonlinear actuator provides an impulse of the form:$\sim cos^4\left((x - V_s t)/A\right)$; while the classical linear excitation is of the form:$\sim \Psi(x - c_0 t)$, where V_s and c_0 represent the speed of the solitary wave in a highly nonlinear medium and the sound velocity in a linear medium respectively and Ψ is a function describing a linear wave.

6.2 Defect Detection in a Linear Elastic Rod Using the Highly Nonlinear Actuator

We studied the highly nonlinear pulse propagation behavior across the interface between the highly nonlinear and the linear materials and the dynamic response of the system in the presence or absence of defects. The data obtained from the experiments conducted on the pristine and defected rods using the highly nonlinear waves as input signal (Fig. 3(c,d)) showed evidence of impulse modification in the transmitted pulses, indicating that the system is suitable for being used in damage identification and detection. Earlier investigations described a comparison of the performance of a linear actuator (hammer) and our highly nonlinear striker on a rod with a single damage point (one notch).[7, 11] An increased effectiveness and accuracy of the nonlinear actuator was reported. The pulse excited with the nonlinear actuation system was shown to be very repeatable, and the presence of the defect(s) significantly altered the transmitted wave's amplitude and shape. In the present paper Fig. 3(c) shows the wave propagation in the pristine, one notch and three notch rod and Fig. 3(d) shows the respective FFTs of the pulses. From Fig. 3(d) we see that the frequency peaks tend to shift toward a lower range as the number of defects in the system increases.
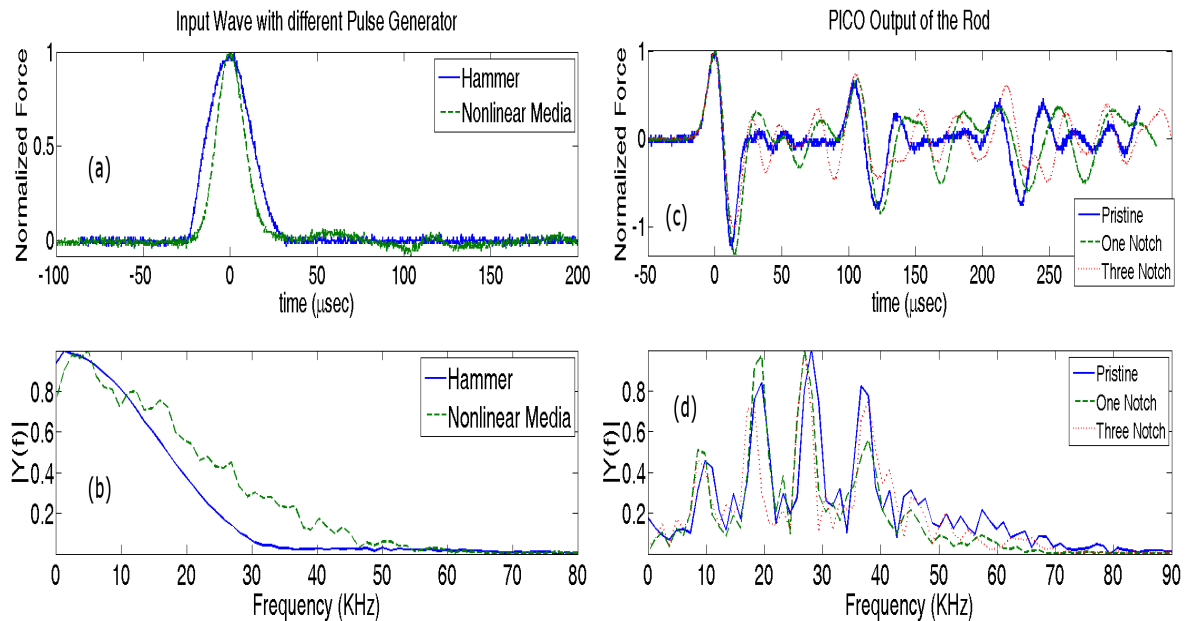

Figure 3: a) Comparison of a pulse obtained with a PCM hammer and a Nonlinear Wave. b) Comparison of frequency spectrum obtained from a PCB hammer pulse and the nonlinear wave. c) Comparison of the output obtained in a pristine, one notch and three notch rods utilizing a nonlinear system as an input. d) Comparison of the frequency spectra output in a pristine, one notch and three notch rod corresponding to (c).

A more detailed interpretation of the results can be understood from the RMSD (%). As described in section 4, the root mean square deviation (RMSD) method allows to quantitatively measure the difference between the acoustic signature obtained from the testing of the pristine rod and the one measured from the damaged/altered samples. We obtained a RMSD value of 38.86% for the one notch rod in comparison with the pristine rod, and a RMSD (%) value of 49.28% for the three notch rod in comparison to the pristine rod. As to be expected an accretion in the RMSD (%) value was reported as the number of defects in the specimen increases.

6.3 Numerical Benchmark

A finite element model of only two spheres was generated to validate the contact interaction used in the numeric with the prediction from the Hertz Theory. In these simulations the lower sphere was held firm by applying fixed boundary conditions and the upper sphere was given a displacement boundary condition. The contact and mesh details for the two sphere were the same as the one described in section 5. The displacement applied on the upper sphere was equal to = 1% of the sphere's radius (the 1% value was selected in order to maintain small displacements, according to the assumptions used in the Hertz Theory).

Initially, we modeled the bead elements with a fully elastic constitutive response. In this case, we extrapolated the contact behavior as a Force-displacement power law similar to the Hertz model (Eq. 1)[19] and obtained a value of the contact exponent equal to $n = 1.65$. This value differs by 10% as compared to the value predicted by the Hertz theory ($n = 1.5$). When the material's response was changed from a fully elastic constitutive behavior to elasto-plastic, the power law value obtained was $n = 1.506$. Figure 4 shows the power law fitting in logarithmic scales of the Force-displacement response for the elasto-plastic model. A more detailed analysis, to understand the model is required to correctly predict the stress behavior in the vicinity of the contact point and its corresponding value in experiments.
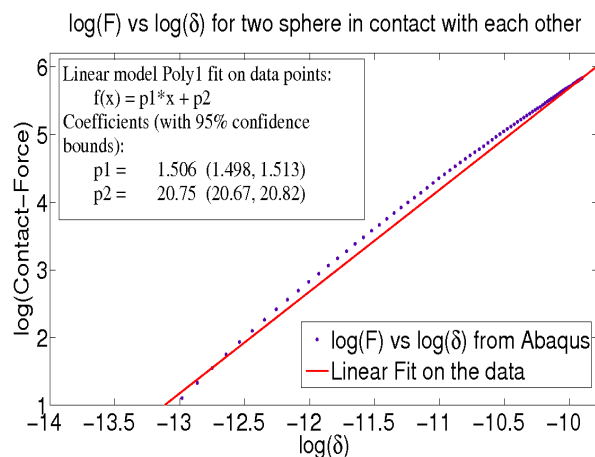

Figure 4: Linear fit on the Abaqus data for two spheres in contact.

6.4 Modeling of the Granular Chain and Validation

We modeled the actuator chain (composed of 20 particles, a striker and a bottom wall) in Abaqus and compared the output of the Finite Element approach with the classical discrete element model reported in the literature.[9, 12] We later compared both numerical results with experiments for validation. For the Finite Element approach the contact and mesh details for the 20 spheres and the rigid base were the same as described in section 5. The rigid plate representing the bottom wall at one end of the chain was assigned fixed boundary conditions, and the first bead in the chain was assigned a predefined velocity of 0.626 m/s, which corresponds to the velocity obtained by dropping the striker from a 2 cm height (same as in experiments). To match the experimental setup the chain of beads was assigned a gravity field in the direction of its axis. To compare the dynamic forces obtained in the numerical models with the experimental values measured by the piezosensors at the center of the beads, we averaged the contact forces as explained in Ref 17. Abaqus simulation results for the 10th and 15th beads from the top of the chain are shown in Fig. 5(a).

Force-time plots corresponding to the waves propagating in the granular chain were also generated using a discrete particles modeling in Matlab by solving for a system of discrete nonlinear elastic springs.[9, 12] In this case we modeled particles as rigid bodies connected by nonlinear springs excited by a striker (the velocity and particles material properties were kept the same as in experiments in both modeling approaches). Results obtained in this case are shown in Fig. 5(b).

The corresponding force-time plots obtained in experiments, with sensors positioned inside the 10th and 15th particles from the top, are shown in Fig. 5(c).
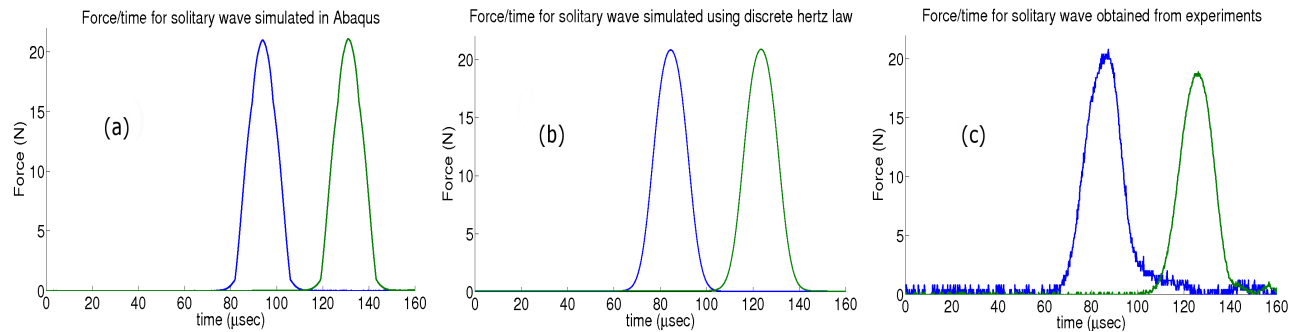
Figure 5: Force time plot for the solitary wave propagation in chain of 20 beads using a) Abaqus simulation. b) Discrete Hertz Law c) Experiments. The curves represent the wave propagation in particles number 7 and 11 from the top of the chain.

It is evident that the results obtained in the three cases present a very good agreement both in terms of qualitative pulse shape, amplitudes and lengths. A comparison of the wave speed in the three different cases also show a quantitative agreement: $641m/s$, $635m/s$ and $610m/s$ for Abaqus, the discrete modeling and experiments respectively.

Snapshots of the Von Mises stress wave propagation obtained in the finite element analysis for the chain of beads is shown in Fig. 6 (a).

6.5 Excitation of Acoustic Signals in the Linear Elastic Rod

Preliminary simulations and experiments on the acoustic wave transmission from the nonlinear medium to the linear elastic rods were performed. In the finite element modeling, twenty three different parts were used for the complete system (the base plate, the rod and the 21 beads). These are described in section 5. The entire system was vertically oriented by applying a gravity field along the direction of chain. The force-time output for this system was taken from the 7th and 11th beads from the top of the chain, from the interaction between the 20th bead and the rod and from the interaction between the rod and the rigid base plate. The data was gathered from these selected surfaces in order to maintain consistency with the sensors location in the experimental set-up. A force-time plot of the waves generated in this system is shown in figure 7. A cross-section of the system, which shows the nonlinear pulse interacting with the interface is shown in figure 6(b).
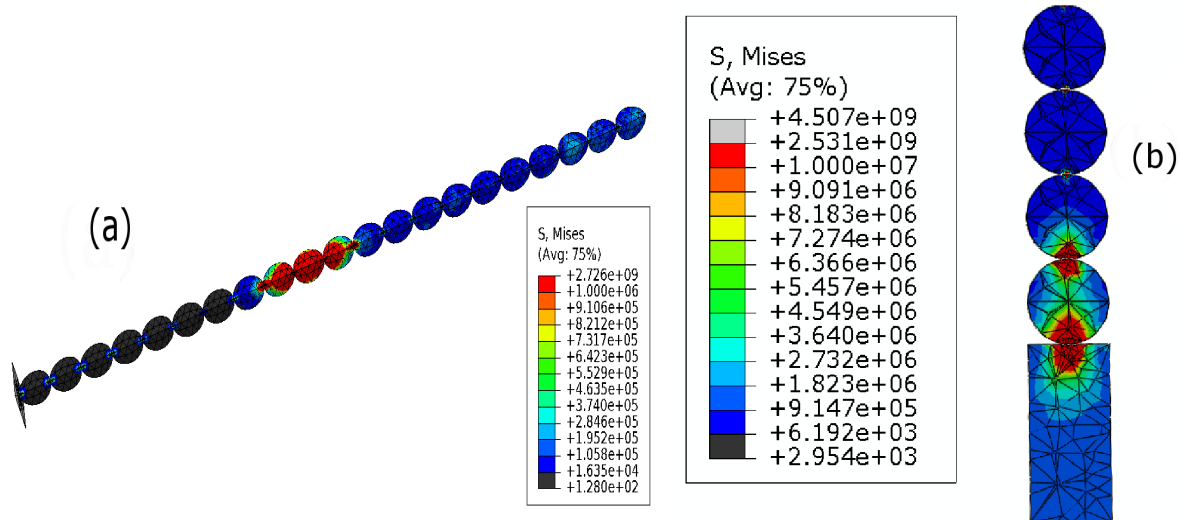

Figure 6: Stress wave propagation (a) in the chain of 20 beads and a striker bead, (b) at the intersection between the chain of beads and linear elastic rod.
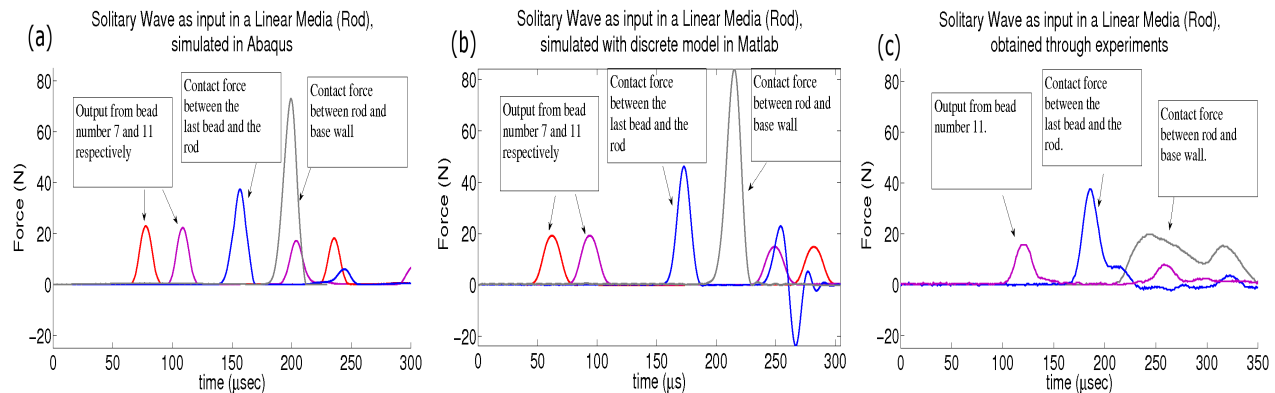
Figure 7: Solitary wave as input in linear rod using a) Abaqus simulation. b) Discrete Hertz Law c) Experiments. The curves represent the wave propagation in the particles number 7 and 11 from the top of the chain.

7. NONLINEAR ACTUATOR DESIGN

All the experiments described in this manuscript were performed by using a manually controlled nonlinear actuator, in which the pulses were generated by dropping a striker on top of the vertically oriented granular chain enclosed in a holder (shown in Ref 7, 18).

A fully automated device (Fig. 8) which is able to excite highly nonlinear pulses and operate independently in different orientations (not only vertically), has been developed in our laboratory.[11] Such stand-alone system may be used for nondestructive evaluation (NDE) and in particular for Structural Health Monitoring (SHM) due to its low power requirements and independent/remote operation. The system designed is composed of a spring loaded adjustable striker, a chain of stainless steel 316-type particles of diameter 4.76 *mm* (provided by McMaster-Carr), a trigger and a cyclically loading system (linear actuator) for the automated operation. Precompression can be added by tightening or loosening the chains base cap to control the pulse pseudo-frequency (Fig. 2). Through this combination of elements, the system can generate pulses to be directed into a structure or material to be evaluated. The system can be modified in the future to operate it remotely through wireless transmission (i.e. via Bluetooth) and for controlling the input signal.
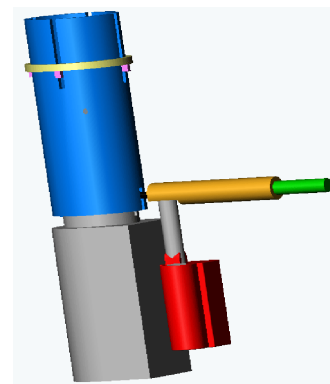

Figure 8: Schematic of fully automated nonlinear actuator.

8. CONCLUSION

In this paper we presented preliminary results on the design of a new nonlinear actuator system based on tunable granular chains, its achievable operating range and its application to the testing of linear systems. From a fundamental prospective, the project represents a first step towards studying the dynamic coupling of a linear and a highly nonlinear system (and vice-versa). We developed a finite element model for the granular chain and the tested rods and compared results obtained in Abaqus with a discrete element model and experiments. We obtained preliminary results for the testing of 1-D systems (pristine and defected metallic rods) excited by our highly nonlinear actuators and found excellent agreement between the three approaches. We showed that the granular system used for impulse excitation behaves as a self modulated filter/pulse shaper, inputting very precise, short and sharp waveforms capable of detecting defects in the studied structures. We designed and built a novel automated, spring loaded actuator capable of cyclically exciting and detecting highly nonlinear waves for in-situ autonomous operations. Advantages reported for such actuators include the reproducibility of the excited pulses (i.e. the possibility of repeating the exact same acoustic excitation without large variations between subsequent strikes), the ability to tune the wave properties in-situ, and its automated operation with

low power requirements. All this hints at the development of novel and improved devices for Non Destructive Evaluation and Structural Health Monitoring (NDE/SHM).

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