Towards the Laboratory Search for Space-Time Dissipation

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It has been speculated that gravity could be an emergent phenomenon, with classical general relativity as an effective, macroscopic theory, valid only for classical systems at large temporal and spatial scales. As in classical continuum dynamics, the existence of underlying microscopic degrees of freedom may lead to macroscopic dissipative behaviors. With the hope that such dissipative behaviors of gravity could be revealed by carefully designed experiments in the laboratory, we consider a phenomenological model that adds dissipations to the gravitational field, much similar to frictions in solids and fluids. Constraints to such dissipative behavior can already be imposed by astrophysical observations and existing experiments, but mostly in lower frequencies. We propose a series of experiments working in higher frequency regimes, which may potentially put more stringent bounds on these models.

I. INTRODUCTION

Gravity has not yet been unified with other fundamental forces, which have been put into the Standard Model of particle physics. General relativity, currently the most successful theory for gravity, describes gravity as the classical geometry of space-time, which interacts only with the classical energy-momentum content of matter. No fully consistent quantum theory of gravity has been found — nor is it completely clear how quantum gravity can be experimentally probed.

General relativity has been systematically verified with increasing accuracy in the classical domain, with more opportunities arising from higher precision laboratory experiments and gravitational-wave detection [1, 2]. On the other hand, having achieved great success in the microscopic world, quantum mechanics are now going to be tested in systems involving macroscopic mechanical objects, where gravity may start playing a role [3–14].

In this paper, we shall still start off as testing general relativity, namely, testing the speculation that general relativity is not a fundamental theory, but an emergent one that appears to take place only in macroscopic spatial and temporal scales. In this work, we propose an experimental strategy to search for evidence of emergent gravity. We will soon see that the tests here will be intimately connected to tests of quantum mechanics, in systems where gravity plays a role.

Emergent gravity has been motivated from at least two types of reasoning [15]. The *hydrodynamical* point of view, proposed by Sakharov [16, 17], argued that Einsteinian gravity can naturally arise at large scales due to the dependence of vacuum energy (of non-gravitational

fields) on spacetime geometry. The *thermodynamical* point of view focuses on the role played by horizons, initially from the fact that the area of an event horizon never decreases [18] and the creation of particles by black holes [19]. Later, the work of Jacobson made a more general extension to Rindler horizons of particles that can exist anywhere in a spacetime [20], and made the connection between Einstein's equation and an equilibrium equation of state. Recent work further considered the microscopic origin of the horizon entropy — motivated by the holographic principle [21] and by loop quantum gravity [22].

A macroscopic dynamics emerging from an averaging over microscopic dynamics often has two features: (i) *fluctuations* can arise from the imperfect averaging over the microscopic degrees of freedom, (ii) the macroscopic equations of motion can be non-conservative (often time asymmetric, or *dissipative*) due to excitations of the microscopic degrees of freedom that do not instantaneously couple back to the macroscopic dynamics. An instructive example is the emergence of classical continuum dynamics from molecular dynamics and (quantum) atomic physics. The macroscopic properties of solids and liquids, for example: (i) density, elastic moduli, compressibility, as well as (ii) thermal conductivity, friction coefficient, viscosity all in fact emerge from the microphysics of molecules. Because fundamental physical laws are usually considered to be time symmetric, those properties in group (ii), which are associated with time asymmetric physical processes, are direct signatures of the underlying molecular dynamics. In fact, physical processes associated with these properties had been discovered and well characterized long before the underlying microphysics was well understood — which took place after more direct observations of molecular structure became feasible. The above analogy generates hope that even though the microphysics that underlies gravity may well be taking

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place at very high energies and very small spatial and temporal scales, astrophysical observations and laboratory experiments in the currently accessible regime might be able to reveal their existence.

Even without specifically considering the concept of emergence, testing the validity of general relativity has long been a major research effort [1, 2]. However, most of these tests were performed in astrophysical or cosmological settings, for example in the solar system [23, 24] and in relativistic binaries [25, 26]. In the context of laboratory tests, deviation from Newton's inverse square law at small distances has been proposed as a possible signature of models with extra dimensions [27–29]. Experiments have been performed to put bounds on such deviations [30]. Experimental bounds on Lorentz invariance (of many kinds) have also evolved substantially [31–34] since the days of the Michelson-Morley experiment and now tightly constrain several alternative gravity theories [35, 36]. All such tests focus on low frequencies — since the theories being tested for mainly have consequences in these regimes. In this work, we shall consider families of theories motivated by emergence, in particular those that have signatures toward higher frequencies and smaller length scales. We will consider a variant of *non-local* models [37, 38], which were indeed motivated by space-time having "memories" in the past. In these models, the gravitational field lags the motion of the source, even in the near zone. We will focus on tests on spatial and temporary scales accessible in the lab.

From a different starting points, experiments have also been proposed for testing quantum mechanics involving macroscopic objects [3–14]. Some of the theoretical modifications of quantum mechanics were motivated by gravity [5, 9, 39, 40], some are motivated by the determinism of the classical world [41, 42], some both [39, 40]. Most of these experiments focused on the stochastic aspect of the modifications, although dynamical effects were also speculated [5, 43, 44].

In this work, we will recognize that our variant of non-local gravity [37, 38] and the Diosi-Penrose (DP) gravity-induced decoherence [39, 40] can be unified — both can arise from an infinite family of fields that couple with the energy-momentum content of space-time: non-local gravity as the dynamical consequence of the coupling, while DP decoherence as the stochastic back action. Such a connection has been hinted by Diosi [43, 44], but we shall make a more general argument. We will then explore possible experimental configurations that will test this unified model.

This paper is organized as follows: In Section II, we briefly explain how dissipation might be incorporated theoretically, and set up a phenomenological model to test in the weak-field regime, which is parametrized by phase-lag $\phi(\omega)$ (see the discussion in Sec.II A), similar to the loss-angle used to characterize material losses [45]. In Section IV, we briefly discuss how astrophysical processes might be used to constrain $\phi(\omega)$, and argue that there exists a new regime that is particular suitable for lab experiments. In Sec. V, we propose experimental strategies and explore their potential performance.

II. DISSIPATION IN SPACE-TIME

In this section, we set up a strawman theory to fit in our intuition about spacetime dissipation. In particular, we would like to explore dissipative modifications to general relativity where the dynamics of spacetime is changed. This could be due to tracing out unknown "environmental" or microscopic degrees of freedom, but here we focus only on a effective theory for the spacetime itself. In this sense, these strawman models are *phenomenological*, as the unknown microphysics no longer enters the dynamics, but rather affects the parameters of these models. It is also possible that different micro-physical theories lead to similar phenomenological models at low energies and large spatial scales.

In addition, we expect dissipative effect leads to a distinctive experimental signature in the linearized gravity regime: Newton's constant will effectively become complex in the frequency domain, so that for an oscillatory source, the phase of the Newtonian gravitational potential will lag behind that of the oscillatory of the motion, with a finite phase difference, *even in the near zone*. We shall discuss the phenomenology of such models, illustrating how they differ from traditional modified gravity models.

A. Non-local Einstein's Equation and its relation to existing modifications to GR

Let us assume that the presence of unknown microphysics introduces a delayed-response for the spacetime geometry with respect to its matter source. This delayed-response effectively makes the Einstein equations to be non-local, which may be written as

$$G_{\mu\nu}(t, \mathbf{x}) = 8\pi \int_{-\infty}^{+\infty} K_{\mu\nu\alpha\beta}(t - t') T^{\alpha\beta}(t', \mathbf{x}) dt'. \quad (2.1)$$

This is a generalization of non-local gravity models proposed by Hehl and Mashhoon [37, 38]. The form of Eq. (2.1) is already written into a 3+1 form, which indicates that the Einstein tensor at one space-time event is determined as a weighted average over the worldlines of a family of fiducial observers. The existence of these special observers implied a preferred frame — as a consequence of the unspecified miscrophysics, similar to the effect of extra degrees of freedom in other modified gravity theories — for example the Einstein-aether theory [46], where the configuration of the aether field effectively defines a preferred frame. [1]

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[1] Some other approaches to nonlocal gravity include [47, 48] non-commutative geometry effects.

The filter $K_{\mu\nu\alpha\beta}$ is a *bi-tensor*, and parametrizes possible delayed-responses. It is symmetric in its first and second pairs of indices (i.e. $K_{\mu\nu\alpha\beta} = K_{\nu\mu\alpha\beta} = K_{\mu\nu\beta\alpha}$), in order to respect the symmetry of $G_{\mu\nu}$ and $T_{\mu\nu}$. Since we have already chosen a preferred set of observers for the non-local integral, let us also perform a 3+1 decomposition of different components of the tensor $K_{\mu\nu\alpha\beta}$. To illustrate properties of such models, we choose a filter function of

$$
\begin{aligned}
K_{\mu\nu\alpha\beta}(t - t') = \, & g_{\mu\alpha}g_{\nu\beta}\delta(t - t') \\
& + [K(t - t') - \delta(t - t')]g_{\alpha 0}g_{\beta 0}g_\mu{}^0 g_\nu{}^0
\end{aligned}
\tag{2.2}
$$

so that only the 00 component of the Einstein equations is changed. In the vacuum case we recover the source-free Einstein equations $G_{\mu\nu} = 0$, which suggests that the propagation of gravitational waves is unchanged in this modified gravity theory. Therefore in order to experimentally test its effect, we need to study the cases where the stress energy tensor of matter is nonzero. As explained in Sec. II C, the matter equation of motion is modified, which can be derived from Eq. 2.1 and the Bianchi identity.

In addition, Eq. 2.1 shows that the Einstein tensor $G_{\mu\nu}$ depends non-locally on the stress-energy tensor $T_{\mu\nu}$; in other words, space-time curvature depends non-locally on its energy-momentum content. In order for this dependence to be causal, we must impose

$$
K(t) = 0\,, \quad t < 0\,.
\tag{2.3}
$$

Let us write the Fourier transformation of K as

$$
\int_{-\infty}^{\infty} dt\, e^{i\omega t}\, K(t) \equiv e^{i\phi(\omega)}\,,
\tag{2.4}
$$

where $\phi(\omega)$ is expected to be small. If $\phi(\omega)$ is a rational function, condition Eq. 2.3 can be achieved by requiring $\phi(\omega)$ to have no poles on the upper half complex plane. Furthermore, in order to keep $G_{\mu\nu}$ and $T_{\mu\nu}$ real-valued in the time domain, we need to impose $K(t) \in \mathbf{R}$, or

$$
\phi(\omega) = -\phi^*(-\omega)\,, \quad \omega \in \mathbf{R}\,.
\tag{2.5}
$$

In particular, $\phi(0) \in \mathbf{R}$, which can be absorbed into the definition of the Newton's constant, leading to

$$
\phi(0) = 0\,.
\tag{2.6}
$$

Furthermore, in order for ϕ to represent a phase lag (instead of a variation in the magnitude of the Newton's constant), we shall require $\phi(\omega)$ to be real-valued for low frequencies — although in general requiring $\phi(\omega)$ to be real-valued for all frequencies may conflict with the requirement that K be causal. One prescription is to choose

$$
e^{i\phi(\omega)} = \frac{1}{1 - i\omega\tau_*}\,, \quad \text{or} \quad \phi(\omega) \approx \omega\tau_*\,, \quad \tau_* > 0
\tag{2.7}
$$

which leads to

$$
K(t) = \frac{1}{\tau_*} e^{-t/\tau_*}\Theta(t)\,,
\tag{2.8}
$$

with Θ the Heaviside step function. With this filter function, the space-time geometry has a (short) response time of τ_*.

We assume that the dynamics of microscopic degrees of freedom only takes place at high frequencies, while in low frequencies accessible by astrophysical processes and our experiments, ϕ can be simply Taylor expanded as

$$
\phi(\omega) = \omega\phi'(0) + \frac{\omega^2}{2}\phi''(0) + \dots\,.
\tag{2.9}
$$

Since the leading order term of the above expansion is the same as Eq. 2.7, we can identify

$$
\tau_* \approx \phi'(0) \approx \frac{\phi(\omega)}{\omega}\,,
\tag{2.10}
$$

which all have the physical meaning of time-lag.

B. Linearized Einstein's Equation and its solution for periodic source

In the case where gravity is weak, we can simplify Eq. 2.1 by considering a perturbed flat metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, with $|h| \ll 1$. Within such limit, Eq. 2.1 can be linearized, while the only different component from linearized Einstein's equation is (C.f. Eq. 2.1 and Eq. 2.2)

$$
\left[-\omega^2 + \nabla^2\right]\bar{h}^{00} = -16\pi e^{i\phi(\omega)}T^{00}\,.
\tag{2.11}
$$

Here $\bar{h}_{\mu\nu}$ is the trace-reversed spacetime metric perturbation,

$$
\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{h}{2}\eta_{\mu\nu}\,,
\tag{2.12}
$$

which satisfies the gauge condition (with 4-dimesional covariant derivative)

$$
\nabla^\mu \bar{h}_{\mu\nu} = 0\,.
\tag{2.13}
$$

Note that in order to obtain Eq. (2.11) we have performed a 3+1 split of coordinates, transformed t into the frequency domain, and used $\mathbf{x}$ to denote spatial coordinates. As discussed in Sec. II A, $\phi(\omega)$ is a real-valued function characterizing phase-lag as functions of frequency, and $T_{\mu\nu}(\omega, \mathbf{x})$ is the stress-energy tensor of the source. The gauge condition described in Eq. (2.13) considerably simplifies the form of linearized Einstein's equation, and it is still compatible with dissipative-gravity modifications, as the left hand side of Eq. (2.1) is unmodified.

We now solve the wave-equation Eq. (2.11) and obtain a retarded solution

$$
\bar{h}_{00}(f, \mathbf{x}) = 4\int d^3\mathbf{x}'\, \frac{T_{00}(f, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} e^{i[2\pi f|\mathbf{x} - \mathbf{x}'| + \phi(\omega)]}\,.
\tag{2.14}
$$

Suppose the source is oscillating at a constant frequency $f_0 = \omega_0/2\pi$. In the near zone ($2\pi f_0 R/c \ll 1$) we have

$$\bar{h}_{00}(t,\mathbf{x}) \approx 4 \int d^3\mathbf{x}' \frac{T_{00}(t,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} e^{i\phi(\omega_0)}. \qquad (2.15)$$

Again, the form of Eq. (2.11) together with the requirement that $T_{\mu\nu}$ and $\bar{h}_{\mu\nu}$ be real-valued in the time domain, require that $\phi(0) = 0$. The power series expansion in Eq. 2.9 then allows us to switch freely between ϕ and ϕ' at low frequencies. Physically ϕ carries the interpretations of a phase lag while ϕ' is a time lag. We also note that this expansion does not necessarily hold at high frequencies, where microphysics of the underlying "environment" may contribute to a very different ϕ. For concreteness, subsequent sections detailing existing constraints and experimental sensitivity, which are supposedly targeting much lower frequencies comparing to underlying microscopic motions, will have final results phrased in terms of ϕ.

To see how this model modifies near zone interactions, we work with the gravitational potential U_{DG} of our model, defined as

$$g_{00} = \eta_{00} + h_{00} \qquad (2.16)$$
$$= -1 + 2U_{\mathrm{DG}}. \qquad (2.17)$$

Using this with Eq. 2.15 and Eq. 2.12, for a periodic source moving at frequency f_0, we see that

$$U_{\mathrm{DG}} = \int d\mathbf{x}' \frac{\rho(t,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} e^{i\phi(\omega_0)}, \qquad (2.18)$$

where $\rho(t,\mathbf{x}') = T_{00}(t,\mathbf{x}')$. Note that this differs from the usual Newtonian potential, U, simply by a phase, i.e., $U_{\mathrm{DG}} = U e^{i\phi(\omega_0)}$.

C. Bianchi Identity and equations of motion in the Newtonian limit

Now let us consider the equation of motion for matter based on Eq. (2.1). Unlike last section where we focus on the metric in the wave-zone, here we are mainly interested in the matter motion in the Newtonian near-zone. The Bianchi identity $G^{\mu\nu}{}_{|\nu} = 0$ requires that

$$G^{\mu\nu}{}_{,\nu} + \Gamma^{\mu}{}_{\nu\alpha}G^{\alpha\nu} + \Gamma^{\nu}{}_{\nu\alpha}G^{\mu\alpha} = 0, \qquad (2.19)$$

where the Einstein tensor $G^{\mu\nu}$ is to be replaced by the right hand side of Eq. 2.1, and the above equation becomes the modified equation of motion for matter. Because of the modification, the matter motion is generically non-conservative. As a simple example, applying Eq. 2.19 and Eq. 2.1 to a point mass in the Newtonian limit, the equation of motion is just

$$0 \approx \left[T^{i0}(t,\mathbf{x})\right]_{,0} + \Gamma^{i}{}_{00} \int_{-\infty}^{\infty} dt' K(t-t') T^{00}(t',\mathbf{x})$$
$$\approx m \left[v^i(t)\right]_{,0} - m\frac{1}{2}\partial^i h_{00} \int_{-\infty}^{\infty} dt' K(t-t'). \qquad (2.20)$$

As we anticipate that the theory recovers the Einstein gravity in the static limit, $\int_{-\infty}^{\infty} K(t-t')dt'$ should be set to one. As a result, the point mass equation of motion becomes

$$a^i(t) = \frac{f^i(t)}{m}, \qquad (2.21)$$

where f^i is the Newtonian gravitational force acting on the point mass. As we can read from Eq. 2.1 and discussion in later sections, the metric is affected not only by local matter stress-energy, but also contributions of the past light-cone. In particular,

$$\frac{f_i(t)}{m} = \int_{-\infty}^{\infty} dt' K(t-t')\Psi_{,i}(t',\mathbf{x}(\mathbf{t}')), \qquad (2.22)$$

and here Ψ is the Newtonian potential. Comparing the above equation with the point mass equation of motion, we find that

$$a_i(t) = \int_{-\infty}^{\infty} dt' K(t-t')\Psi_{,i}(t',\mathbf{x}(\mathbf{t}')). \qquad (2.23)$$

This means that within the Newtonian limit, this dissipative gravity model leads to an different equation of motion from General Relativity.

D. Relation between non-local gravity and spontaneous wave-function collapse

Let us turn to collapse models of quantum mechanics. The initial motivation for these models were from the randomness of quantum-state reduction, and its incompatibility with classical determinism. The assumption of these models are that quantum states of macroscopic objects spontaneously become localized due to an *intrinsic* collapse process [39–42]. As was later realized, these collapses can all be modeled in general as a continuous measurement on matter density [8, 10] — with spatially distributed measuring devices that have different mutual correlations.

In a continuous quantum-measurement process, back action takes the form of back-action noise, but also sometimes in the form of dissipation. This has lead Diosi to further propose that the collapse models may also cause the Newtonian gravitational potential to have a delay τ_* when responding to matter density changes [44]. This is the same as Eq. (2.9) as we consider the lowest-frequency contribution from non-local gravity. Diosi argued that current experimental data can constrain τ_* to around 1 ms. However, in this paper we shall remain flexible about the function form of $\phi(\omega)$.

Let us further argue that a more general $\phi(\omega)$ can also arise from the quantum-measurement consideration. More specifically, a continuous measurement process can be modeled as coupling the observable we need to measure with a field degree of freedom, which has an incoming state, and the out-going state contains information

about that degree of freedom. Quantum or thermal fluctuations in the incoming state provides the stochastic back-action, while information contained in the out-going state corresponds to dissipation.

Now suppose we apply this to space-time geometry. If $h_{00}(t, \mathbf{x})$ is being continuously monitored, we need to introduce an extra dimension η, and an additional field n — for each $\mathbf{x}$ — that propagates on the t-η plane. The field couples to h_{00} as η approaches 0 — it therefore gains information about h_{00}, and acts back to the dynamics of h_{00}. In this way, depending on the propagation law of n along the extra dimension, and the detailed way it is couple to h_{00}, arbitrary shapes of the phase delay $\phi(\omega)$ between h_{00} and energy density can be constructed.

In general, as we consider different $\mathbf{x}$, coupling to different components of the metric, as well as the above dependence on η, we can recover general non-local theories of gravity.

III. EFFECTS ON ONE AND TWO-BODY MOTIONS

In this section, we shall discuss the dynamical effect of emergent gravity on one and two bodies.

A. Effects on Single Body Motion

Consider a spherical, homogeneous object (with mass M, radius R) moving with non-relativistic velocity v relative to the preferred frame where Eq. (2.1) holds. According to Eq. (2.23), there is a force generated by the "past" gravitational field of the same object. This self-gravitational force is dissipative for the object's motion, as it generates acceleration anti-parallel to the direction of motion:

$$\mathbf{a} = -\frac{GM}{R^3}\mathbf{v}\int_0^\infty t\, K(t)dt\,, \qquad (3.1)$$

where G has been restored in this equation.

If we assume that the kernel function is described by Eq. (2.8), the acceleration evaluates to

$$\mathbf{a} = -\frac{GM}{R^3}\tau_*\mathbf{v} = -\frac{GM}{R^3}\phi'(0)\mathbf{v}\,. \qquad (3.2)$$

The phase-lagging feature of the nonlocal Einstein equation inevitably leads to a dissipative self-interacting gravitational field.

B. Effects on Two-Body Motion

In order to calculate effects on two-body interactions, we take a step back and look at the "full" theory. Consider

$$\begin{aligned} G_{00}(f, \mathbf{x}) &= 8\pi e^{i\phi(2\pi f)}T_{00}(f, \mathbf{x}) \\ &\approx 8\pi[1 + i\phi(2\pi f)]T_{00}(f, \mathbf{x}). \end{aligned} \qquad (3.3)$$

Writing the Fourier transform of $i\phi(\omega)$ as $\Phi(t)$ (which is $K(t) - \delta(t)$), the time-domain version becomes

$$G_{00} = 8\pi\left[T_{00} + \int_{-\infty}^0 \Phi(t-t')T_{00}(t')dt'\right]. \qquad (3.4)$$

Assuming $\Phi(t-t')$ is a causal kernel implies $\Phi(t-t') = 0$ for $t < t'$, allowing us to extend the limits of the integral to $\pm\infty$, resulting in our final expression

$$G_{00} = 8\pi\left[T_{00} + \int_{-\infty}^\infty \Phi(t-t')T_{00}(t')dt'\right]. \qquad (3.5)$$
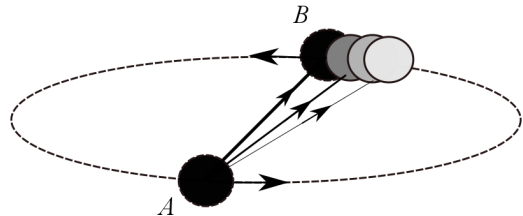

Figure 1. Binary motion taking into account the dissipative gravity effect. In the Newtonian limit, star A feels the instantaneous gravitational attraction from star B, as well as the contribution form "past" gravitational field due to the dissipative gravity effect, according to Eq. (3.5).

To estimate the effect in binary motion, we consider two equal mass stars A and B in orbit around each other as shown in Fig. (1). In the Newtonian limit, star A and B feel the instantaneous attraction force from each other, which are orthogonal to their velocities. If we further take into account the dissipative gravity effect, as shown by Eq. (3.5), the external gravitational field acting on star A (and vice versa for star B) contains a piece that originates from the past of star B. These "past" gravitational fields exert a tangential force on star A's orbital motion, which in turn introduces additional orbital energy loss/gain (depending on the sign of $\phi(\omega)$) with respect to the gravitational radiation. For example, if $\phi(\omega) > 0$, although the gravitational field lags behind the source motion, the tangential force is along the star's direction of motion and and the binary motion actually gains energy according the positive phase lag. In this case we see that the rate of energy gain is given by

$$\begin{aligned} \dot{E}_{\mathrm{MG}} &\sim q\frac{M^2}{r^2}\int_{-\infty}^0 \Phi(t-t')\sin[2\pi f(t-t')]dt' \\ &\sim q(Mf)^{4/3}\phi(2\pi f), \end{aligned} \qquad (3.6)$$

where $q = m_1 m_2/(m_1+m_2)^2$ is the symmetric mass ratio, the post-Newtonian parameter $v^2 = (2\pi Mf)^{2/3}$, and the subscript MG stands for mutual gravity. As we require that the standard Newtonian result to be recovered in the static limit, this constrains the DC phase-lag such that $\phi(f = 0) = 0$. Therefore if the frequency is sufficiently low, on can Taylor-expand the phase and obtain

$$\dot{E}_{\mathrm{MG}} \sim qM^{-1}(Mf)^{7/3}\phi'_0. \qquad (3.7)$$

Experiment	Upper limit on $\phi(2\pi f_0)$	f_0 (Hz)	Reference
PSR J0737-3039	7.9×10^{-36}	1.1×10^{-4}	[49]
PSR B1534+12	4.3×10^{-38}	2.8×10^{-5}	[50]
PSR B1913+16	2.4×10^{-39}	3.6×10^{-5}	[51]
PSR B2127+11C	1.6×10^{-37}	3.5×10^{-5}	[52]
Earth Orbital Motion	3×10^{-17}	3.2×10^{-8}	[53]
Shapiro delay	0.014	3.2×10^{-8}	[54–56]
Torsion Pendulum	$0.1^\dagger$	0.084	[57]
Quadrupole Antenna	$0.1^\dagger$	60.5	[58]
Cantilever	1×10^{-3}	324.1	[59]
Rotor/Cantilever	0.1	353	[60]

Table I. Constraints on ϕ derived from existing measurements at different frequencies. Quantities with a † superscript are estimated by assuming a 0.1 rad uncertainty in the phase at the specified frequency. The phase lag in the Shapiro delay measurement is obtained by converting the constraint on the time-lag estimate on the same system.

However, this is not the leading order dissipative gravity effect, which comes from the self-dissipative dragging force as shown in the previous section. In the binary system, such a damping mechanism dissipates energy at rate

$$\dot{E}_{\rm SG} = -\left(\frac{m_1^2}{R_1^3}v_1 + \frac{m_2^2}{R_2^3}v_2\right)\phi_0' = -qMv\phi_0'\left(\frac{m_1}{R_1^3} + \frac{m_2}{R_2^3}\right),$$
(3.8)

which is about a factor of r^3/R^3 larger than $\dot{E}_{\rm MG}$. To help further understand this result, recall that quadrupole radiation of gravitational waves has power

$$\dot{E}_2 \sim -\frac{32}{\pi}q^2(Mf)^{10/3},$$
(3.9)

which is one and a half post-Newtonian order higher than $E_{\rm MG}$, and consequently even smaller comparing to $E_{\rm SG}$! A direct consequence of this observation is that binary pulsars should provide the best observational constraints on the magnitude of ϕ at low frequencies, because of the high compactness of neutron stars.

IV. EXISTING OBSERVATIONAL AND EXPERIMENTAL CONSTRAINTS

A. Binary pulsars

According to our previous analysis, if we turn on the dissipative gravity effect, the change of fractional energy emission rate for a binary pulsar system is

$$\frac{\Delta\dot{E}}{\dot{E}} = \frac{\dot{E}_{\rm SG}}{\dot{E}_2} = \frac{(2\pi)^{2/3}\pi\phi_0'M}{32q(Mf)^3}\left(\frac{m_1}{R_1^3} + \frac{m_2}{R_2^3}\right)$$
$$\approx \frac{(2\pi)^{2/3}\pi\phi_0'M^2}{32q(Mf)^3R^3}.$$
(4.1)

and, denoting P_b as the period of the binary motion, we have

$$\frac{\Delta\dot{P}_b}{\dot{P}_b} = \frac{3}{2}\frac{\Delta\dot{E}}{\dot{E}}.$$
(4.2)

Therefore the accuracy of period measurement constrains the magnitude of possible dissipative gravity effect. We can then use the fact that $\phi_0' = \phi(2\pi f_0)/(2\pi f_0)$ to phrase our results in terms of the phase angle. The compactness of neutron stars $M_{1,2}/R_{1,2}$ can be determined from different models of nuclear equation of state, and here for simplicity the neutron star radius R is estimated as $13km$. Constraints from a representative sample of binary pulsars appear in Table I. We can see that pulser systems give really stringent upper bound on ϕ at low frequencies (recall the $1ms$ constraint on τ_* in [44]), as nuclear densities are much higher than normal matter. On the other hand, it remains interesting to constrain ϕ at higher frequencies, using table-top experiments.

B. Solar system test

Solar system test of gravity theories generally involves measuring perihelion precession, spin precession measurement (such as "Gravity Probe B") and test of the "Weak Equivalence Principal". None of the above experiments seems to constrain a friction-type force well [61]. We can nevertheless expect the period change after one orbital period of earth:

$$\frac{\Delta T}{T} = \frac{3}{2}\frac{\Delta E}{E} = 3\frac{GM_ET}{R_E^3v}\phi_0',$$
(4.3)

where M_E, R_E are the earth mass and average radius respectively. Plugging in the numbers, we find

$$\phi_0' \sim 2 \times 10^2 \frac{\Delta T}{T}s \sim 1.4 \times 10^{-10}s,$$
(4.4)

where we have used the orbital period change of earth due to tidal friction ~ 2.3ms/century as an estimate [53]. Obviously this is a much looser constraint than the binary pulsar tests, because normal matter density is much lower than the nuclear matter density. As the earth orbital period is longer than the periods of binary pulsars, this test belongs to the low-frequency regime which is ruled out by binary pulsar tests.

C. Shapiro Delay

Shapiro delay is the time delay of light due to the local gravitational field of a nearby massive body [56, 62]. For a static source, our model predicts the same static gravitational field as general relativity, so there is no observable Shapiro delay effect. However, as we have shown above, this is not the case for a moving source.

In the solar system, the time delay of light for a moving source has been measured in 2002 [54, 55], as Jupiter moved by the line of sight to quasar J0842+1835. The timing sequence of the delay signal was applied to constrain the speed of gravity, which was measured to be consistent with the speed of light, with a relative uncertainty up to 20%. We notice that the phase-lag due to dissipative gravity, can be effectively translated to a shift in the "speed of gravity" in this case. More specifically, if the closest distance between the trajectory of the light is d and the velocity component of Jupiter moving towards the light trajectory is v_d, then the time lag is roughly bounded by

$$\frac{\phi' v_d}{d} \leq 20\% \, , \text{or} \quad \phi' \leq \frac{d}{5 v_d} \, . \tag{4.5}$$

According to the parameters presented in [54, 55], $v_d \sim 14 \, km/s$ and $d \sim 10^6 km$, and consequently we obtain $\phi' \leq 7 \times 10^4 s$. Because the moving-body correction to the Shapiro delay is already a second order effect, in terms of v_d/c, this measurement a gives much weaker constraint on ϕ' than the test based on planetary motion.

D. Near field experiments

Although astrophysical tests provide the tightest constraints on our model, laboratory-scale experiments designed to test the nature of near-field gravity provide constraints at much higher frequencies. Examples include torsion pendulua [57] and the quadruple antenna [58], which look for deviations in the amplitude and do not report phase information. In these cases, we assume the experiment has observed no phase lag greater than .1 radians. Other experiments involving cantilevers [59] and improvements thereof [60] explicitly measure the phase angle. Table I summarizes these constraints.

The self-damping effect for moving bodies described in Sec. III A has the consequence that there is an intrinsic damping force experienced by any harmonic oscillator involving moving masses. From Eq. (3.2), and assuming a harmonic oscillator with natural angular frequency ω_0, the mechanical quality factor is limited by

$$Q^{-1} \geq \frac{G\rho}{\omega_0} \tau_*(\omega_0) = \frac{G\rho}{\omega_0^2} \phi(\omega_0), \tag{4.6}$$

where $\rho = M/R^3$, and we have allowed the gravitational delay, $\phi(\omega)$, to be frequency dependent. Here we see that the damping effect is greater for low frequency oscillators with dense masses. Depending on the value of ϕ, this damping could exceed other mechanical dissipation effects. If this is not the case, the mechanical quality factor will only be slighly modified by gravitational self-damping and estimating the non-gravitational mechanical damping could lead to large systematic uncertainty in the estimation of the gravitational phase lag. For the oscillators described in Sec. V, we find that the contraint
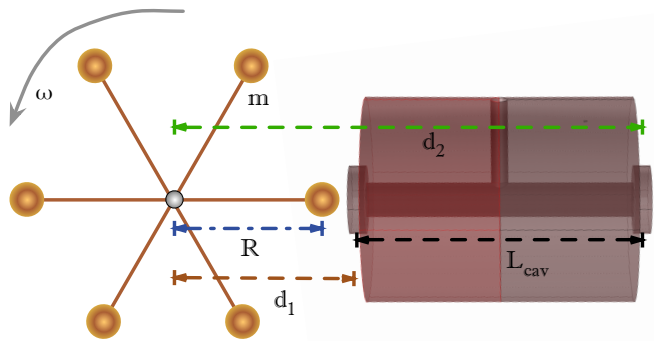

Figure 2. Plan view of a schematic diagram of the experiment. In this example, a rotor with 6 spokes and masses spins at an angular frequency ω near a rigid Fabry-Perot cavity with an angular eigenfrequency $6\,\omega$. The laser beam passes underneath the masses.

placed by direct measurement of the phase lag to be more stringent than the constraint set by measuring the quality factor.

E. Gravitational wave experiments

It is clear from Sec. III B that our model has implications for gravitational wave detection, effecting the coalescence of compact binaries. In general, the phase evolution of the waveforms will be modified and a careful study of the full implications of Eq. (2.1) is required. These issues will be explored in future studies. However, for the model under consideration here, binary pulsars should provide a stronger constraint for low frequency motions.

V. EXPERIMENTAL PROBE OF GRAVITATIONAL DISSIPATION

A. High Frequency Eotvos Experiment

Here we discuss one possible experimental approach for testing the existence of dissipative gravity, this approach roughly models the paradigm of the classic experiments of Baron von Eotvos [63, 64] while incorporating the experience gained over the past several decades of short scale gravity experiments [30, 57, 59, 65, 66] The experiment consiststs of an active mechanical Attractor and a high sensitivity gravitational Responder. They are constructed such that their oscillations cause tidal gravitational interactions. By measuring the response of the Respnder, we can look for the possible phase lag predicted by a dissipative gravity model.

The outline of this Section goes as follows: in Section V A 1, we estimate the magnitude of the gravitational acceleration induced by our periodic attractor system. in Section V A 2, we estimate the sensitivity limit of a mechanical responder which is limied by Brownian

thermal noise. in Section V B, we calculate the minimum detectable phase lag using the system we prescribe.

1. Tidal gravitational field of the periodic attractor

Our aim is to probe the frequency dependence of gravitational forces and how they may differ, in the non-relativistic limit, from Newtonian gravity. To this end we aim to use a system of moving masses that create a periodic tidal gravitational attraction and will use a sensitive mechanical system to measure the mechanical response to the gravitational force. An example of such a system is shown in Figure 2.

The scaling of the tidal gravitational acceleration of the periodic attractor system is approximated by

$$g_{\text{tidal}} = G\rho L, \tag{5.1}$$

where G is Newton's constant, ρ is the density of the material used in the Attractor, and L is a characteristc length scale of the attractor.

2. Acceleration sensitivity of the responder

To maximize the signal amplitude, we propose to use a mechanical system with a high quality factor resonance as our probe of the gravitational force. The motion of the mechanical system will be sensed using interferometric metrology to maximize readout sensitivity. With such a narrow mechanical resonance, and in addition we will integrate for long times to accumulate larger signal to noise ratio, for a given resonator system the measurement is essentially single frequency.

A detailed noise analysis will be presented in Section V B, and it will show that in most cases the dominant noise source is Brownian thermal fluctuations of the mechanical resonator.

Near the mechanical resonance, the force power spectral density of thermal fluctuations seen by the mechanical responder is given by

$$S_F^{\text{th}} = 4mk_BT\frac{\omega_0}{Q}, \tag{5.2}$$

where k_B is Boltzmann's constant, m is the mass of the responder, T is the temperature, ω_0 is the resonant frequency, and Q is the mechanical quality factor.

Because we want to measure the sensitivity to gravitational acceleration, in units of acceleration power spectral density, the thermal noise is

$$S_g^{\text{th}} = 4k_BT\frac{\omega_0}{mQ}. \tag{5.3}$$

3. Sensitivity to phase lag

The behavior of the mechanical resonance forces the Newtonian signal component to be in the orthogonal

Parameter	Cantilever	Diluted Cantilever	Rigid Cavity	Pendula
Responder Mass	1 mg	1 mg	300 g	1 kg
Responder Q	10^6	10^{10}	10^8	10^8
Lowest Eigenfrequency	200 Hz	40 kHz	10 kHz	2 Hz
Temperature	100 mK	100 mK	1 K	120 K
Stored Power	10 mW	60 W	1 kW	10 W

Table II. Physical parameters of proposed gravitational responders.

quadrature with respect to the signal of the periodic attractor. And, ignoring systematic effects which will be discussed in Section V E, any in-phase signal will be due to the dissipative gravity effect. The magnitude of the signal in acceleration units is

$$g_\phi = \phi(\omega)g_{\text{tidal}}, \tag{5.4}$$

and this is being compared to the RMS thermal acceleration of the oscillator due to thermal noise,

$$\sqrt{\langle g_{\text{th}}^2 \rangle} = \sqrt{\frac{S_g^{\text{th}}}{\tau}}, \tag{5.5}$$

where τ is the integration time, and we assume that S_g is approximately constant near the mechanical resonance. Therefore the minimum detectable dissipation phase angle detectable with an SNR of unity after a time τ is

$$\phi(\omega_0) = \frac{1}{G\rho L}\sqrt{\frac{4k_BT\omega_0}{mQ\tau}}. \tag{5.6}$$

Given the calculation shown in Figure 3, we can estimate that $g_{\text{tidal}} \approx 1\,\text{nm/s}^2$, and assuming $\omega_0 = 2\pi \times 2\,\text{kHz}$, $Q = 10^8$, $T = 1\,\text{K}$, $m = 0.3\,\text{kg}$, and $\tau = 24\,\text{hours}$, the minimim detectable phase angle is $\phi = 3.7 \times 10^{-7}$ radians.

B. Experimental Designs and Sensitivity Limits

The example given in Section V A 1 is illustrative in showing the general order of magnitude of an expected signal. For a more detailed analysis including a noise budget estimate, it is necessary to define a specific resonator cavity geometry and readout scheme.

Here we describe a few potential Responder configurations that are common in the gravitational-wave and opto-mechanics communities. All take the form of a high finesse Fabry Perot cavity but vary in the configuration and geometry of the cavity mirrors.

Schematic diagrams of the configurations are given in Figure 4. Here we will briefly discuss the features of each
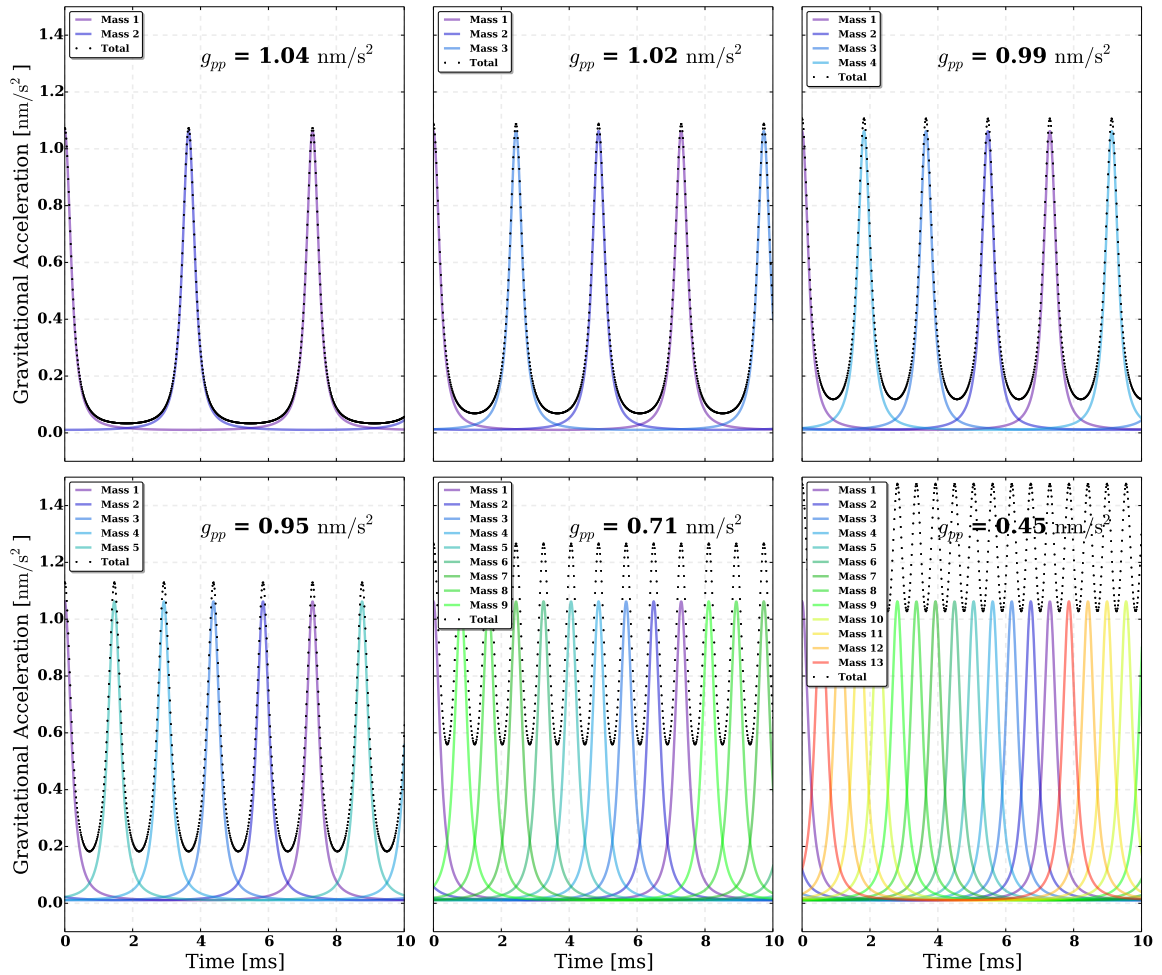
Figure 3. Differential acceleration (peak-peak) between the two ends of a Fabry-Perot cavity with a multi-mass periodic Attractor (cf. Fig. 2). As the number of masses is increased, the signal frequency increases, although the peak strain is correspondingly reduced. In all 6 plots, $m = 0.01\,kg$, $R = 0.095\,m$, and $d_1 = 0.12\,m$.

(a) Cantilever Cavity
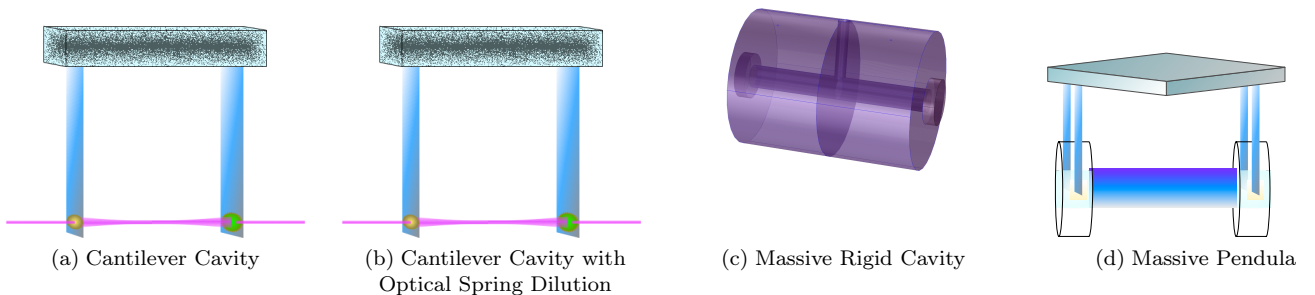
(b) Cantilever Cavity with Optical Spring Dilution

(c) Massive Rigid Cavity

(d) Massive Pendula

Figure 4. Schematic diagrams of four possible gravitational responders.

configuration and provide a noise budget estimate assuming feasible experimental parameters. These parameters for each configuration are given in Table II.

A pendulum suspended test mass configuration would be similar to the arm cavities used in large scale gravitational-wave observatories such as LIGO, however with reduced physical dimensions.

In this configuration, the sensing cavity is similar to the rigid kind used for stabilization of lasers. A solid piece of single crystal silicon with optically contacted mirrors
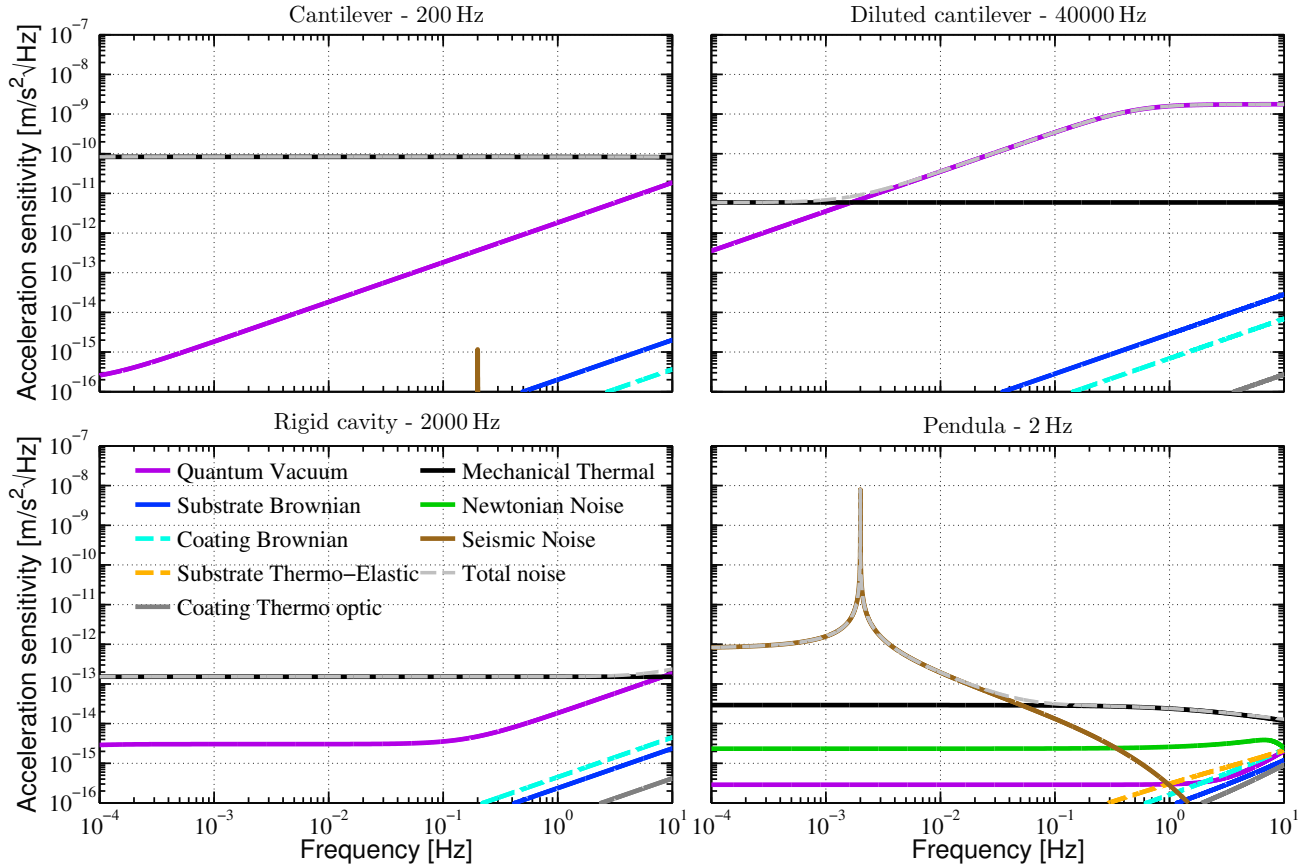
Figure 5. Shown here are the detailed noise budgets for the four proposed cavity configurations. The horizontal axis denotes positive sideband frequencies separated from the cavity mechanical resonance. In most cases the total noise is dominated by thermal noise of the mechanical oscillator. In the case of the suspended pendulum, the low frequency noise is dominated by seismic vibrations.

is used to measure the gravitational accelerations.

In the past few decades, microcantilever force sensors have become widely deployed as high sensitivity force and acceleration sensors [67], especially in the biological agent detection fields and radiological. Microcantilevers have also been used for sensitive fundamental physics experiments[68], such as Casimir force experiments [69] and the searches for fifth force and extra dimensions by looking for short range deviations in the Newtonian $1/r^2$ law. They are recently also candidates to measure quantum backaction noise [70, 71].

C. Components of the Noise Model

Our noise model consists of several noise terms which limit the measurement of the gravitational coupling of the periodic attractor to the responder. Each noise term is explained below.

To characterize the Responder, in particular, we are interested in the sensitivity to tidal gravitational accelerations. To calculate the acceleration sensitivity, we first compute the displacement noise contribution from

all terms, then this is multiplied by $|\chi^{-1}/m|$ to produce the acceleration noise. As described in Section V A 3, the sensitivity is maximized at the mechanical resonance, and the resonance is very narrow, thus we make logarithmic plots in Figure 5, where the origin of the horizontal axes are at the mechanical resonance frequencies.

1. Force Noises

a. *Seismic* Vibrations of the laboratory due to external seismic fluctuations and nearby vibrating machinery will produce motion of the support point of the Responder cavity. This will be mostly rejected below the Responder's mechanical resonance frequency. Near the resonance the rejection will be imperfect due to imperfect matching of the pendular resonant frequencies (we assume a mismatch of $\sim 0.1\%$) of the two ends of the cavity.

b. *Newtonian Gravitational Noise* Fluctuations in the Newtonian gravitational potential due to local sources (e.g. seismic, anthropogenic, air pressure, etc.) can limit the acceleration sensitivity due to their spatial
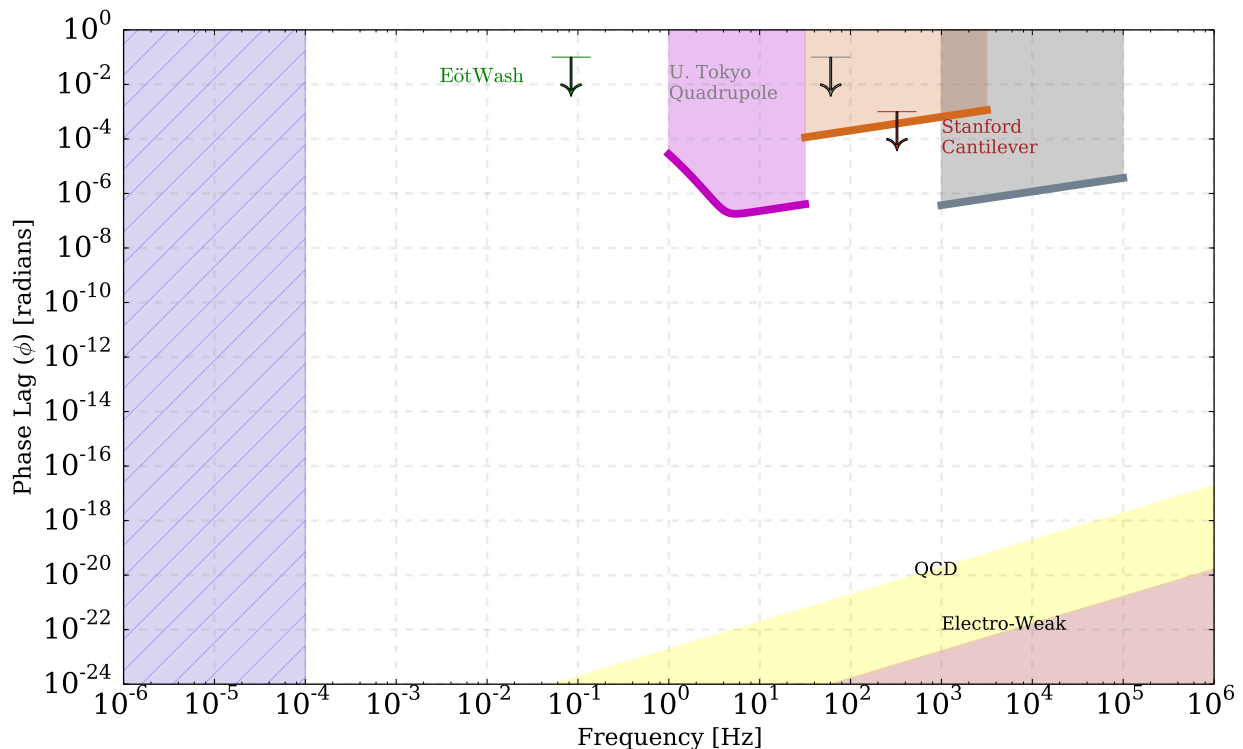
Figure 6. **Comparison of Experimental limits and Theoretical Predictions**. The arrows represent existing upper limits on the phase lag angle (cf. Table I). The shaded blue region is excluded due to pulsar observations. To give a sense of scale, the shaded regions at the bottom of the plot give estimates for ϕ assuming a dissipation of unity at the Electro-Weak and QCD length scales. The solid lines are expected upper limits from the experiments described in Sec. V B, where each point in the line represents an oscillator resonant at that frequency. The thick purple line (from 1-30 Hz) represents the massive pendulum cavity. The thick orange line (from 30-3000 Hz) represents the small cantilever cavity. The thick grey line (from 1-100 kHz) represents the rigid spacer cavity.

gradients across the ends of the Responder cavity [72, 73]. As most of these fluctuations are sourced by mechanical vibrations (and have a rather red power spectrum), they are most serious for the massive suspended cavity and least serious for the rigid, monolithic cavity (cf. Fig. 4).

    c.   *Thermal Noise*   Several different kinds of thermal noise are relevant for these experiments. Internal friction of the suspension support elements, cantilever flex joints, and rigid cavity spacer result in Brownian forces which move the mirrors. The same kind of internal friction in the mirror coatings also produces Brownian [45] strain fluctuations of the coating phase. Thermodynamic temperature fluctuations [74, 75] can lead to apparent optical cavity length fluctuations through a finite thermal expansion coefficient of the mirror substrate (Substrate Thermo-Elastic) and coatings, as well as through the temperature dependence of the index of refraction of the coating materials [76, 77]. We refer to the coherent expansion and refractive fluctuations in the coating as 'Coating Thermo-Optic'.

    d.   *Quantum Vacuum*   Fluctuations of the vacuum electro-magnetic field limit a variety of high precision measurements [78, 79]. In the particular case of a

Fabry-Perot interferometer, the fluctuations in the phase quadrature limit the resolution of the optical phase measurement and is usually referred to as 'shot noise'. The fluctuations in the amplitude quadrature produce radiation pressure 'shot noise' in the circulating field within the Fabry-Perot cavity; this fluctuating pressure moves the cavity mirrors. This Quantum Vacuum noise limits these experiments in different frequency bands. If the thermal noise can be reduced, then squeezed light techniques can be employed to further constrain the spacetime dissipation.

D. Limits on complex phase angle for oscillators of different frequencies

    The sensitivity to tidal accelerations given from our noise model can be combined with the acceleration induced by the periodic attractor to give a final measurement limit of the complex gravitational phase angle. Generalizing Equation 5.5 for a frequency varying S_g, the measurable limit on ϕ is

$$\phi_{\text{lim}} = \frac{1}{g_{\text{att}}} \sqrt{\int_{\omega_0 - 1/2\tau_{\text{avg}}}^{\omega_0 + 1/2\tau_{\text{avg}}} S_g \frac{d\omega}{2\pi}} \qquad (5.7)$$

where S_g is the *power* spectral density of the acceleration noise of the receiver, ω_0 is the receiver resonance frequency, and g_{att} is the differential acceleration caused by the periodic attractor. We estimate an acceleration on the order of $1\,\text{nm/s}^2$ for g_{att}, as illustrated in Figure 3. The limits for the various types of oscillator, for various possible resonant frequencies, after 24 hours of integration, are shown in Figure 6.

E. Elimination of Systematic Effects

The sensitivity shown above in Figure 6 includes only the limits due to Gaussian random noise. In reality, experiments where the signal is so weak are likely to be limited by systematic effects. In these searches for a gravitational anomaly, we should be especially concerned about an anomalous signal showing up due to imperfections in the experimental apparatus, rather than new physics.

There are several systematic effects including time delay, mechanical response limits, mis-estimation of source oscillator phase, clock / timing errors in the digitizer, timing error in the interferometric measurement of the source motion, etc.

We will estimate and subtract these effects in the following ways:

1. Electro-magnetic pickup of the electrical driving signal in the readout electronics at f_{drive} will be rejected since it is not at the gravitational perturbation frequency (f_{signal}). High frequency harmonics of the drive signal will be attenuated by usual grounding and shielding practices [80].

2. Patch charges on the Attractor or Responder can produce a fluctuating electrostatic force at f_{motion}. Both the Attractor and Responder can be coated with a slightly conductive coating to minimize charge gradients and the Attractor will be enclosed within a Faraday cage to ground these fields.

3. Light scattered from the cavity mirrors can backscatter from the Attractor back into the cavity mode to produce a signal at f_{drive}. To mitigate this, we will use super polished optics for low scatter and blacken the nearby surfaces. To calibrate the scatter signal we will apply a few Lambertian marks to the Attractor at irregular intervals.

4. In order to make a high precision estimate of ϕ, one must know the mechanical admittance precisely. The phase angle of the admittance will be measured using non-gravitational forces (e.g. with electrostatic force actuators and radiation pressure on the laser beam). The difference between the electromagnetic force admittance and the gravitational admittance will ultimately limit the sensitivity of this search.

VI. CONCLUSIONS

The confrontation between quantum mechanics and general relativity points to a modification of one or both of the theories. Previously, two communities have been tackling these problems: modification of gravity, and possible modifications of quantum mechanics. In this work, we intend to bring up the connection between these two different approaches. In order to do so, we have written down model theories that are both be viewed as modifications to GR and to QM. Experimentally, these can plausibly be probed by precision measurement performed in the laboratory scale. While it is certain that the proposed experimental approaches are extremely challenging, they should be able to, at the very least, place interesting upper limits on these types of alternative theories of gravity and point us towards new physics.

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