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Multiple Aperture-Based Antihydrogen Parallel Plate Gravity Experiment

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Abstract

An experiment is described that could be carried out at the CERN Antiproton Decelerator facility to determine the direction of acceleration of antihydrogen in the earth's gravitational field. The experiment would use two plates separated by a small distance and oriented parallel with the earth's surface. Multiple cylindrical barriers would be used with gaps that allow the antihydrogen to pass through. Shadow regions are created where, with linear motion, the antihydrogen cannot annihilate. However, with parabolic paths, such as those of objects under the influence of gravity, antihydrogen can annihilate within a shadow region. The probability of an atom annihilating in one of the shadow regions is determined. For simplicity the model considers the antihydrogen source as a point and at a temperature of 4 K.

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1. Introduction

No experimental evidence has been presented that shows whether the gravitational interaction between matter and antimatter is attractive or repulsive. In an experiment including sets of apertures in a circular plane and a source of antihydrogen (the only antimatter atom that can currently be produced) could be used to determine the direction of the gravitational acceleration of antimatter.

The antimatter source would be placed along the axis of symmetry of, and equidistant from, two parallel circular plates. Encircling the antihydrogen source is a set of apertures shown in Fig. 1. A straight path from the source to the edge of an aperture and finally to the circular plate can be used to indicate a 'shadow region' behind

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the aperture where an antihydrogen atom could not annihilate unless influenced by gravity (see Fig. 2). At the end of the first shadow region, another aperture is positioned to create a similar 'shadow' region behind it (see Fig. 3). This is repeated to vary the number of apertures from one to four. The probabilities for an antihydrogen atom to annihilate in the various shadow regions are calculated and compared.

If other sources of forces (such as those due to stray electric fields) are negligible, the experiment could indicate the direction of the antihydrogen gravitational acceleration. If annihilations occur in the shadow regions on the top plate, antimatter and matter are gravitationally repulsive; if annihilations occur on the bottom plate then the opposite is true. The use of circular plates and multiple apertures instead of a previously suggested cylindrical pipe[1, 2], or plates with one aperture[3], will increase the probability for having an antihydrogen atom annihilate in one of the shadow regions, as the surface area of the shadow regions is now considerably larger. This experiment is designed to use a detector, such as the one used by the ALPHA collaboration, to detect the antihydrogen annihilations[4].

Only recently has neutral antimatter been trapped. This has been achieved by the Antihydrogen Laser PHysics Apparatus (ALPHA)[5] collaboration, and more recently by Antihydrogen Trap (ATRAP)[6] collaboration. To determine the gravitational acceleration of antimatter, ALPHA members[7] propose using a light-pulse atom interferometer for a measurement with an accuracy of 1%. The GBAR collaboration plans to observe antihydrogen in free fall at CERN, in order to determine the gravitational acceleration of antihydrogen[8]. Two other collaborations that may study the gravitational interaction between matter and antimatter include ASACUSA and AEGIS [9,10].

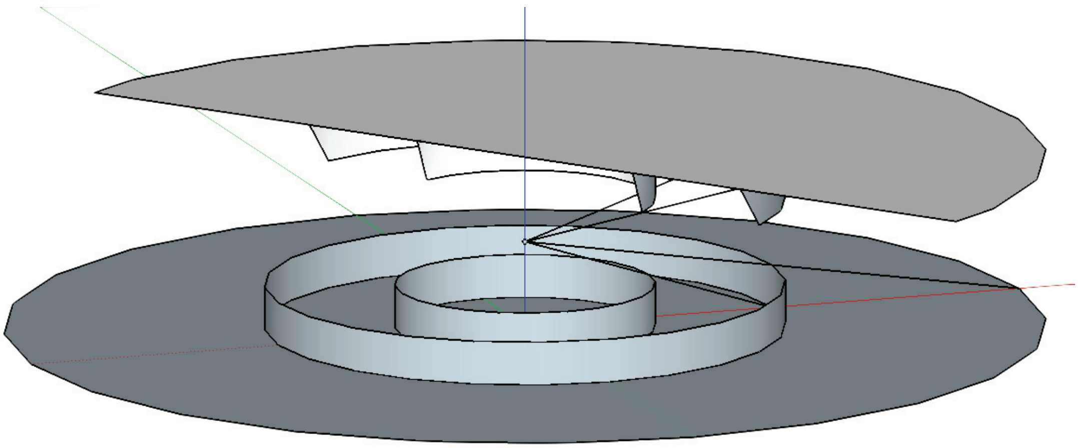

Fig.1 Image of the two circular plates with multiple apertures. For clarity there has been a portion removed from the top and only two apertures are shown.

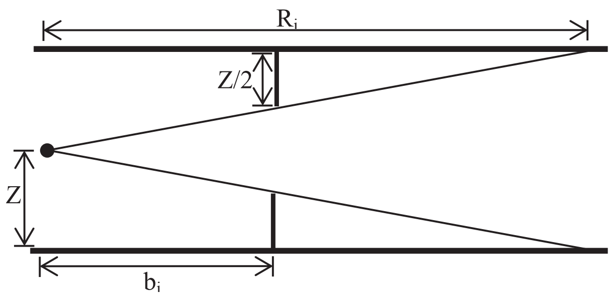

Fig. 2 A vertical cross section showing only the $+x$ direction with one aperture, straight line trajectory and important dimensions labelled.

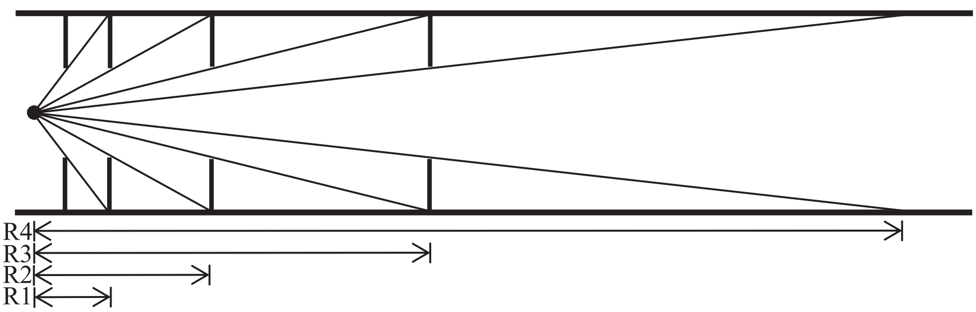

Fig. 3 A vertical cross section showing only the +*x* direction with four apertures, straight line trajectories and the distances to each shadow region labelled.

2. **Analytical model**

For developing an analytical model, a Cartesian coordinate system is defined with the antihydrogen source located at (0,0,0). For this simulation gravity is assumed in the positive *z* direction. Letting the *z* axis coincide with the axis of symmetry of the plates the equations of motion are then,

$$x(t) = x_o + v_{ox}t,$$
$$y(t) = y_o + v_{oy}t,$$
$$z(t) = z_o + v_{oz}t + \frac{gt^2}{2}.$$

Here x_o, y_o, z_o indicate the initial positions and v_{ox}, v_{oy}, v_{oz} the initial velocities. The problem is solved in cylindrical coordinates, and due to the azimuthal symmetry, it can be solved in the r-z plane. Therefore, $r(t)^2 = x(t)^2 + y(t)^2$. Expressing the equations in terms of kinetic energy, $K_o = (mv_o)^2/2$, and then substituting gives

$$K_o = \frac{m\,g\,r^2}{\sin^2(\theta_o)[4\,z - 4\,r\,cot(\theta_o)]},$$

where θ_o is the initial angle from the *x-y* plane.

A minimum and maximum value can be found for K_o such that the antihydrogen annihilates in a selected shadow region. The lower bound corresponds to a trajectory where the antihydrogen passes just above the barrier at R_i. Thus, at this point, $r = aR_i/Z$, where *a* is the height of the barrier in the *z* direction, and *Z* the distance between the plate and antihydrogen. Substitution leads to

$$K_{omin} = \frac{a\,m\,g\,{R_i}^2}{\sin^2(\theta_o)[4\,Z^2 - 4\,Z\,R\,cot(\theta_o)]}.$$

The upper bound on *K* corresponds to a trajectory with the following conditions *z*=*Z* and *r*=R_i:

$$K_{omax} = \frac{m\,g\,\,{R_i}^2}{\sin^2(\theta_o)[4\,Z - 4\,R\,cot(\theta_o)]}.$$

For the straight line trajectories that pass infinitesimally close to an aperture,

$$\theta_{omin} = \arctan\frac{R_i}{Z},$$

and

$$\theta_{omax} = \pi - \arctan\frac{R_i}{Z}.$$

The probability of an antihydrogen atom annihilating in one of the shadow regions, where $K_{omin} < K_o < K_{omax}$ and also $\theta_{omin} < \theta_o < \theta_{omax}$, is to be found. A Maxwellian velocity distribution is assumed, which in Cartesian coordinates is given by

$$f(\boldsymbol{v_o}) = f_o\, e^{(-mv_o^2)/(2k_bT)},$$

where f_o is the normalization constant.

Using the Jacobian transformation matrix, this can be transformed into spherical coordinates. The normalized probability density is given by

$$f(K_o, \theta_o) = \frac{\sin(\theta_0)\sqrt{K_o}}{\sqrt{\pi (k_b T)^3}} e^{(-K_o)/(k_bT)}.$$

Therefore, the probability of an antihydrogen atom annihilating in the i-th shadow region is

$$P_i = \int_{\theta_{omin}}^{\theta_{omax}} \int_{K_{omin}}^{K_{omax}} f(K_o, \theta_o)\, dK_o d\,\theta_o.$$

The apertures are set up in such a way that the shadow region of one ends where the next aperture begins and so $R_n = b_{n+1}$ where n is the maximum number of apertures. It can be shown that

$$R_n = b_1 2^n.$$

Taking R_n=0.6 m, the radial distances from the center to each aperture can be calculated.

Table 1 Aperture distances

n	b_1 (m)	Aperture (b_i) (m)				R_i (m)			
		1	2	3	4	1	2	3	4
1	0.3	0.3	N/A	N/A	N/A	0.6	N/A	N/A	N/A
2	0.15	0.15	0.3	N/A	N/A	0.3	0.6	N/A	N/A
3	0.075	0.075	0.15	0.3	N/A	0.15	0.3	0.6	N/A
4	0.0375	0.0375	0.075	0.15	0.3	0.075	0.15	0.3	0.6

The probability that an antiatom will annihilate on any of the shadow regions is then,

$$P_{total} = \sum_{i=1}^{n} P_i$$

The first integral $\int_{K_{omin}}^{K_{omax}} f(K_o, \theta_o)\, dK_o \equiv K_{sln}$ in solving for P_i can be solved analytically to give

$$K_{sln} = \frac{k_b\, T\left[-\frac{2\sqrt{k_o}\, e^{\frac{-k_o}{k_bT}}}{\sqrt{\pi}} + \sqrt{k_b\, T} erf\left(\sqrt{\frac{k_o}{k_bT}}\right)\right]\sin[\theta_o]}{2\sqrt{{k_b}^3\, T^3}}$$

evaluated from $K_o = K_{omin}$ to $K_o = K_{omax}$ where *erf* is the error function.
The outer integral must be solved numerically.

$$P_i = \int_{\theta_{omix}}^{\theta_{omax}} K_{sln}\, d\,\theta_o$$

A maximum value for P_i is found by applying values from Z=0.01 m to Z=0.7 m in increments of 0.0001 m. This maximum value for the P_i is found at Z=0.02053 m for four apertures. The probabilities are then found for a given number of apertures ranging from one to four. With four apertures the probability increases by about a factor of two in comparison to just having one aperture.

Table 2 P_i with different numbers of apertures

Number or apertures	Value of ΣP_i
4	4.0866×10^{-5}
3	3.8188×10^{-5}
2	3.2702×10^{-5}
1	2.1722×10^{-5}

3. Monte Carlo Simulation

A Monte Carlo simulation is presented that evaluates the path of motion if an antihydrogen atom annihilates in one of the shadow regions. For this simulation, the particles are considered to start at a point on the origin. Due to the azimuthal symmetry, the simulation is carried out in the r-z plane with no loss of generality. The initial positions for the particles are x=y=z=0. The velocity is obtained by sampling a normal (Gaussian) probability density function for each component v_x, v_y, v_z with a mean of 0 and a standard deviation equal to the thermal velocity, $v = \sqrt{\frac{K_B T}{m}}$. Also $v_r = \sqrt{v_X^2 + v_Y^2}$. It is important to note that the trajectory will resemble a section of a parabola.

Two conditions must be met for an antiatom to annihilate in one of the four shadow regions:

1. For each aperture the particle reaches before annihilation, the particle must pass though the aperture.
2. The first time the antiatom reaches the plate at z=Z=0.02053, the x coordinate of the antiatom places it within one of the four shadow regions.

With these conditions accepted the simulation can be graphed.

Condition 1 makes sure the path does not exit the system. If it does, the simulation discontinues the motion when the particle first annihilates without re-entry into the system. The second condition verifies that the particle passes though each aperture that it gets to before annihilating on the disk.

The time t_{max} that a particle's z coordinate is $z = \pm Z$, is found by solving $\pm Z = +v_{oz}\, t + \frac{g\, t^2}{2}$ for t where Z=0.02053. This will yield four possible values for t_{max}, only one of which will be real and positive. The radial coordinate where the antiatom annihilates is $r = v_r t_{max}$.

This also allows the simulation to calculate how many of the apertures the particle will have to pass through before annihilation. For each aperture that the particle reaches, the particle must pass through. The time it takes to reach the ith aperture is $t_i = \frac{R_i}{v_r}$. Here R_i is the radial distance to each aperture. The z coordinate of the particle at t_i is calculate using

$$z = v_{oz}\, t_i + \frac{g\, t^2}{2}$$

If -0.01027 m < z < .01027 m it passes condition one. Figure 4 shows the result of the Monte Carlo simulation of 100,000 trajectories.

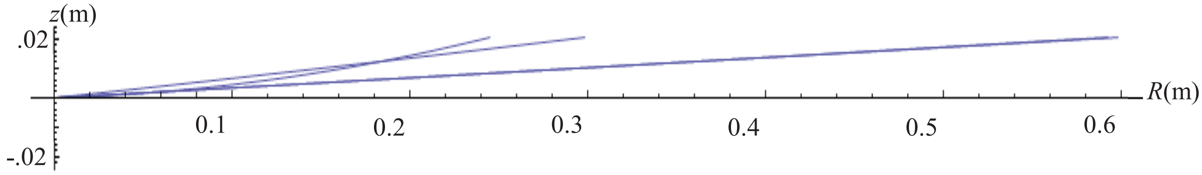

Fig. 4 Simulated annihilations within shadow regions for four apertures, $a=Z/2$, $g=+9.80665$ and $T=4$ K. A total particle count of 100,000 yielded 4 annihilations within the shadow regions. Two annihilations are at about 0.6m, and the associated trajectories are indistinguishable at this size.

4. Conclusion

An analytical model and Monte Carlo simulation have been presented. An increase in the probability of an antihydrogen annihilation within a shadow region, when multiple apertures are used has been found. It is found that additional apertures have decreased the required number of antihydrogen atoms and the associated runtime to indicate the direction of acceleration due to gravity on antiparticles.

5. Acknowledgements

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