A PHYSICAL MODEL OF LIQUID HELIUM

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Abstract

A new physical model of liquid He^4 based on the hypothesis that rotons behave like tiny quantized circular vortex rings is presented. It is shown that the energy of a state will not only depend on the distribution in numbers of rotons with various momenta, but also on the arrangements and orientations of the rotons. The λ-transition then can be interpreted to reveal two aspects: T_λ is both the lowest temperature at which all helium atoms partake in excitation, and the point of the initiation of the general destruction of order, i.e. the general randomization of the orientation of the rotons. Other implications from the theory are also discussed.

A Physical Model of Liquid Helium

I. Introduction

In this paper, a new physical model for liquid He^4 is presented. From the outset, it should be emphasized that it is far from a complete theory. Rather, it is a first step towards a complete theory. Therefore it is liable to serious objections and criticisms. But when we are facing a difficult problem and cannot make significant progress for a long time, every avenue of some promise should be explored. This is the motive behind the publishing of the theory in this preliminary form. To the author, the existing physical theory of liquid helium, stripped down to its essentials, only consists of the following few ideas: Landau's postulated spectrum of excitations[1], which he identified one part to represent phonons and the other part as rotons, and Feynman's[2] proposition of quantized vortices. Most of the voluminous arguments are just sophisticated dressings. They do make the basic hypotheses of Landau and Feynman more plausible, but they really do not improve the validity of the theories. This opinion may not be shared by eveyone. However, it is with this assessment that the author ventures to present his somewhat naive theory, which he hopes may at least offer some food for thought to other workers in this field.

In the physical theory as originated by Landau[1], excitations corresponding to different parts of a single spectrum are identified with phonons and rotons. Our experience in solid state physics makes it easy for us to visualize the phonons. Not very many persons have a clear idea as to what rotons are. The nomenclature suggests that rotons are

associated with rotation. Indeed, the ordinary fluid motion can be de-
composed into the dilatational and rotational parts. With phonons taking
care of the dilatational aspect, we need other excitations to take care of
the rotational motion; hence we have rotons. Landau borrowed the analogy
that angular momenta are quantized in quantum mechanics to infer the
energy gap for the roton spectrum. The argument is of doubtful validity,
since Feynman[3] argued later that the energy gap results from the Bose
statistics that helium atoms have to obey. Feynman[4], in constructing
the wave function for rotons, found them to behave very much like tiny
classical vortex rings. If they can be identified with vortex rings, they
should of course be quantized vortex rings; and in Landau's original
analogy, the quantization of the angular momentum should correspond to
the quantization of circulation. In this sense, the quantized vortices of
macroscopic size may now be interpreted as large rotons. The physical
model developed in the following is based on the postulate that rotons be-
have essentially like quantized circular vortex rings. It should be em-
phasized here that this is really an independent postulate, not an exten-
sion of Feynman's ideas, and the author is solely responsible for its
drawbacks and imperfections.

II. Excitations in Liquid Helium

In this model, liquid He^4 is considered to consist of two distinct
types of excitations, i.e. phonons and rotons, each having its own spec-
trum. For phonons, we have the dispersion relation

$$\epsilon^{(p)} = cp \quad , \tag{1}$$

where p is the momentum and c, the sound speed. For rotons, the

dispersion relation is taken to be

$$\epsilon^{(r)} = Ap^{\frac{1}{2}} \quad .$$

(2)

The last dispersion relation is borrowed from classical hydro-dynamics. For a classical vortex ring, if its radius R is much greater than its core radius a, the energy and momentum associated with it are given by[5],[6]:

$$\epsilon = \frac{\rho}{2} \kappa^2 R\left(\eta - \frac{7}{4}\right) \quad ,$$

(3)

and

$$p = \pi\rho\kappa R^2 \quad ,$$

(4)

while the velocity of the vortex ring is

$$v = \frac{\kappa}{4\pi R}\left(\eta - \frac{1}{4}\right) \quad ,$$

(5)

where κ is the circulation around the core of the vortex ring, and

$$\eta = \ell n\left(\frac{8R}{a}\right) \quad .$$

Then, for this case, we have

$$A = \left(\eta - \frac{7}{4}\right)\left(\frac{\rho\kappa^3}{4\pi}\right)^{\frac{1}{2}} \quad .$$

(6)

The parameter η is a slowly varying function of R, thus A may be treated approximately constant. The extrapolation from a classical vortex ring down to a vortex ring of interatomic dimension raises the most serious objections. So far we can not justify this extrapolation. As we stated in Section I, the nature and the dispersion relation of rotons in this theory should be taken as a hypothesis, whose merit is to be judged by whether it leads to any better understanding of the phenomena.

Now the circulations are quantized[6]. Therefore, for He^4 with

atomic mass m, we have

$$\kappa = \frac{h}{m} = 0.997 \times 10^{-3} \text{ cm}^2 \text{ sec}^{-1} \quad .$$

Most rotons would have only one unit of circulation, since for the same momentum, to have two units of circulation would increase the energy by about three-fold. Formally, for an assembly of multitudes of phonons and rotons, the energy of a given state may be schematically written as

$$E = E^{(p)} + E^{(r)} + E^{(pr)} \quad , \tag{7}$$

where $E^{(p)}$ is the energy due to phonons if no rotons are present; $E^{(r)}$, that due to rotons if no phonons are present; and $E^{(pr)}$, the remaining part which may be called the phonon-roton interaction energy. Let us neglect $E^{(pr)}$ as a first approximation. In the same approximation, we shall neglect the interactions among phonons, then

$$E^{(p)} = \sum_{i} n_i^{(p)} \epsilon_i^{(p)} = \sum_{i} n_i^{(p)} c p_i \quad , \tag{8}$$

where $n_i^{(p)}$ is the number of phonons with momentum p_i.

The expression of $E^{(r)}$ will not be as simple as that of $E^{(p)}$. J. J. Thomson, more than eighty years ago, with a view to constructing a kinetic theory of fluids, had investigated in detail the motion of vortex rings in the Adam Prize essay[7]. Among others, it is found that the energy of a system of circular vortex rings is

$$T = \sum_{i} \left[2 p_i V'_i - R_i \cdot \frac{d p_i}{dt} \right] + \frac{\rho}{2} \iint_{S} V^2 \, r \cdot n \, dS \tag{9}$$

where V is the velocity of the fluid, R_i is the position vector of the

center of the ith vortex ring, $\underset{\sim}{p}_i$ is the momentum of the ith vortex ring, as if it is single, and V_i' the average velocity of the vortex ring in the direction of $\underset{\sim}{p}_i$.

The last term of (9) will yield a term like $\frac{1}{2} M\overline{V^2}$, where M is the total mass of the fluid and $\overline{V^2}$ is the average of V^2 over the boundary. This term will ordinarily not contribute to the internal energy of the system. The term $\sum_i \underset{\sim}{R}_i \cdot \dfrac{d\underset{\sim}{p}_i}{dt}$ may be interpreted as that due to collision-al interactions, which we shall neglect also as a first approximation. Then, taking rotons to behave like these vortex rings, we have

$$E^{(r)} = \sum_j 2p_j(v_j + w_j) \quad ,$$

where v_j is the velocity of the jth roton as if it is single, and w_j the average velocity in the direction of v_j induced by all the rest of the rotons. We may rewrite the last equation as

$$E^{(r)} = \sum_i n_i^{(r)} [Ap_i^{\frac{1}{2}} + 2 p_i u_i] \quad , \tag{10}$$

where $n_i^{(r)}$ is the number of rotons with momentum p_i, and u_i is the average of w's over these $n_i^{(r)}$ rotons.

From (10), we see that the energy of a state will not only depend on the distribution in numbers of rotons with various momenta, $\{n_i^{(r)}\}$, but also on the arrangements and orientations, $\{P\}$, of the rotons. We may thus write the energy of a state as

$$E\{n_i^{(p)}, n_j^{(r)}, P\} = E_o + \sum_{P_i} n_i^{(p)} cp_i + \sum_{P_j} n_j^{(r)} Ap_j^{\frac{1}{2}} + \sum_{P_j} 2n_j^{(r)} p_j u_j(P) \quad . \tag{11}$$

The partition function Q is thus

$$Q = \sum_{\{n_i^{(p)}, n_j^{(r)}, P\}} \exp[\, -E\{n_i^{(p)}, n_j^{(r)}, P\}/kT\,] \quad . \quad (12)$$

Let us denote

$$q = \sum_{\{P\}} \exp\left[-2 \sum_{P_j} 2n_j^{(r)} p_j u_j(P)/kT\right] \quad . \quad (13)$$

In general, q will depend on $\{n_j^{(r)}\}$. But it is conceivable that q may not depend on $\{n_j^{(r)}\}$ sensitively. Rather it may only depend on the total number of rotons present, which is related directly to the density and temperature of the system. If that is the case, then q may be factored out, and (12) becomes

$$Q = q\, e^{-E_o/kT} \cdot \prod_{P_i} \frac{1}{1-e^{-cp_i/kT}} \cdot \prod_{P_j} \frac{1}{1-e^{-Ap_j^{\frac{1}{2}}/kT}} \quad . \quad (14)$$

The range of p_i and p_j in (14) can be determined by arguments like those in Debye's theory of solids. Let N be the number of atoms in the system, then we have

$$3N = \frac{V}{h^3}\left[\int_o^{p^{(p)}(\epsilon_m)} 4\pi p^2 dp + \int_{P_o}^{p^{(r)}(\epsilon_m)} 4\pi p^2 dp\right] = \frac{4\pi V}{3h^3}\left[\left(\frac{\epsilon_m}{c}\right)^3 + \left(\frac{\epsilon_m}{A}\right)^6 - P_o^3\right],$$

$$(15)$$

where ϵ_m is the maximum cut-off energy and p_o is the minimum cut-off momentum for rotons, since the radius of the rotons are bounded below by the atomic dimension. This minimum cut-off momentum or energy is to be interpreted as the energy gap which Landau proposed for rotons. Denote $\theta = \dfrac{\epsilon_m}{k}$ as the cut-off temperature, then θ is related to

particle density through the equation (15). When the system is fully
excited, it turns out that roton modes are dominant; thus it is a good ap-
proximation that

$$\theta = \left(\frac{9N}{4\pi V}\right)^{1/6} \frac{h^{\frac{1}{2}}A}{k} \quad .$$

The parameter A can now in principle be determined from thermo-
dynamic data through θ.

III. The Nature of λ-transision

Despite all the advancement of the understanding of the superfluid
helium, the nature of the λ-transition is still largely an unsolved prob-
lem. Landau's theory starts from the absolute zero and is not good as
the λ-point is approached. The Einstein condensation of Bose gases
leads to a discontinuity of slope on the specific heat curve at the λ-point,
while observations[8] indicate a logarithmic singularity in the neighbor-
hood of λ-point. These are the indications that we really only have a
partial knowledge about liquid helium so far. The present theory, with
all its unsatisfactory features, nevertheless attempts to construct a
complete picture of liquid helium. The term q in equation (14) plays a
vital role in the understanding of the nature of the λ-transition.

Qualitatively, the term q shows the existence of an order-disorder
transition quite analogous to that of the Ising problem[9]. For the two-
dimensional Ising problem with nearest neighbor interaction only, it is
well known that the transition is marked by a logarithmic singularity on
the specific heat curve. The same kind of singularity at the λ-transition
in liquid helium is also due to similar mechanisms as we can see from
the expression for q. At the present stage, it seems futile to attempt

quantitative correlations. The computation of q is vastly more complex than the two-dimensional Ising problem. It is a three-dimensional problem. The interactions are not limited to nearest neighbors. Moreover, the rotons with varying strength, are not fixed in space and their orientations are not necessarily quantized. However, it is still possible to see what should be the configuration that has the lowest energy. This is the configuration in which the rotons will have the greatest possible induced velocity opposite to its natural velocity; and most desirably, all of them. After trying a few, we can convince ourselves that the lowest energy configuration is the case that all rotons are aligned in the same direction. If indeed they are all lined up, a flow will appear in that direction. The bulk of the fluid can be stationary because the fluid region is divided into many domains, and rotons in different domains are lined up in different directions, just like the case of ferromagnetic materials. When different domains line up in the same direction, then there is a flow of rotons or normal component of the fluid.

We can obtain the thermodynamic quantities from (14). With neglect of the minimum cut-off momentum, the free energy of the system is given by:

$$F = E_o - kT \ln q + NkT \left[3\ln(1 - e^{-\theta/T}) - \frac{1}{2(1+\tau)} D_6 \left(\frac{\theta}{T}\right) - \frac{\tau}{1+\tau} D_3 \left(\frac{\theta}{T}\right) \right] ,$$

$$(17)$$

where

$$D_n(x) = \frac{n}{x^n} \int_o^x \frac{y^n dy}{e^y - 1}$$

and

$$\tau = \left(\frac{A^2}{ck\theta}\right)^3 ,$$

which is small in comparison with unity.

The information contained in (17) is very limited, since we do not yet know how to compute q. However, the comparison with experimental thermodynamic data for T far from T_λ, the region where the contribution from q is relatively not very important can give us a rough estimate of θ. The value of θ turns out to be of the order of 4 or 5 times T_λ. If we take $\theta \approx 4 T_\lambda$, the number of rotons at temperature T, is given by the following equation:

$$N_r = \frac{V}{h^3} \int_{P_o}^{P_r(\epsilon_m)} \frac{4\pi p^2 dp}{e^{Ap^{\frac{1}{2}}/kT} - 1} \approx \frac{18N}{5} \left(\frac{T}{\theta}\right) D_5\left(\frac{\theta}{T}\right) \quad ,$$

which will be about $\frac{1}{10}$ of N at T_λ. On the other hand, if we define the mass of the roton by the relation

$$m_r = \frac{p}{v} \quad ,$$

we obtain roughly

$$m_r \approx \frac{\pi \rho \kappa R^2}{\frac{\kappa}{4\pi R}\left(\eta - \frac{1}{4}\right)} = \frac{3\pi}{\left(\eta - \frac{1}{4}\right)} \cdot \frac{4}{3} \pi \rho R^3 \quad .$$

Thus m_r is about 10 times the mass of helium, if R is taken to be about interatomic dimension.

Therefore the λ-transition reveals two aspects: T_λ is the lowest temperature at which all the helium atoms partake in excitation, i.e. when $\rho_n = \rho$. Also since for further increase of temperature, no new excitation could be created due to the using up of all the unexcited helium atoms, the general destruction of order, i.e. the general randomization of the orientation of the rotons, will commence.

IV. Further Thoughts and Discussion

It would be most desirable to be able to calculate q; then the validity of this model could be decided at once. For the time being, we shall be content with exploring other aspects of the present theory.

We mentioned that when different domains of ordered rotons line up in the same direction. there will be a flow of normal component, or heat flow. To cut off the heat flow means the disruption of the alignments of the domains. So there is a natural tendency to maintain the heat flow, once it is set up. In a torus, then, a persistance of heat flux can be expected. It would be interesting to explore in this direction experimentally.

The collisional interaction between rotons as well as the collisional interaction between the rotons and the macroscopic quantized vortices can now be cast in a more definite version. Some rough classical calculations yield results quite consistent with the existing experimental data. For instance, we may take the collision time between rotons

$$t_r = \frac{1}{N_r v_r \pi \overline{\sigma}^2}$$

where v_r is the average roton velocity, and $\pi \overline{\sigma}^2$ is the collision cross section. The result agrees very well with the established expression by Landau and Khalatnikov[10], if we take $\overline{\sigma} \approx 10^{-7}$ cm. This value is reasonable, since each roton consists of about 10 atoms and its radius of influence is presumably somewhat larger.

The same value of the collision diameter applies well also for the derivation of the mutual friction coefficient in the theory of Hall and

Vinen[11],[12]. The nature of the collision is essentially similar to the change of the state of motion of a material particle in the flow field of a vortex line. Then it is also easy to see that the collision diameter for momentum exchange perpendicular to the relative velocity, $\overline{\sigma}_\perp$, is very small, since the net transfer of the transverse momentum is negligible after the roton has traversed through the entire region of influence of the vortex line. In our model, the rotons are actually identified with material particles, so its interaction with macroscopic vortex lines can be visualized quite clearly, and the extrapolation from the classical hydrodynamics also presents not too much difficulty. We may not be so at ease with phonons or like excitations.

There is tension in the vortex element. The tension in the roton is

$$\frac{\epsilon^{(r)}}{2\pi R} \approx \frac{\rho \kappa^2}{4\pi} \sim 10^{-8} \text{ dyne.}$$

The existence of tension can be attributed to the tendency to transfer the kinetic energy of the neighboring rotating atoms to the core atoms. We may note that the Van der Waals force between the atoms in liquid helium is also of the order of 10^{-8} dyne[13]. So the molecular force is just about enough to prevent the splitting of the core. In this connection, we may mention the still unsolved problem regarding the nuclei of ultra-sonic cavitation in liquid helium[14],[15]. Based on the present model, then we can interpret the tensile strength not as the force needed to over-come the Van der Waals force, but the force which together with the vortex tension will overcome the Van der Waals force. This force could be much smaller than the Van der Waals force, and this could explain the low tensile strength which is observed.

In He I, the energies of the prevailing rotons become larger. The

energy could be increased either by increasing the radius or by increasing the circulation. Since all the atoms have partaken in the formation of rotons. the increase in size of one roton has to do it at the expense of other rotons. Therefore, when the temperature gets higher the increase of energy will preferably be achieved through the increase of circulations; and more and more rotons with more than one quantum of circulation will appear. Now as the temperature increases, the interatomic distance also increases, hence the Van der Waals force weakens. On the other hand, the tension of the rotons increases with the units of circulation they carry. Eventually the Van der Waals force can no longer hold the tensions in the majority of the rotons, and then vaporization starts.

If this picture applies to He I, it may also apply to ordinary simple liquids. It is indeed very intriging to ask whether the rotons are the primary excitations even in the ordinary liquid. If we can by any means find that the result of interactions among the rotons are not very important, then it will enhance greatly our understanding of the liquid state.

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A new physical model of liquid He^4 based on the hypothesis that rotons behave like tiny quantized circular vortex rings is presented. It is shown that the energy of a state will not only depend on the distribution in numbers of rotons with various momenta, but also on the arrangements and orientations of the rotons. The λ-transition then can be interpreted to reveal two aspects: T_λ is both the lowest temperature at which all helium atoms partake in excitation, and the point of the initiation of the general destruction of order, i.e. the general randomization of the orientation of the rotons. Other implications from the theory are also discussed.

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