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ON THE DAMPING OF SCREW DISLOCATION MOTION IN FCC CRYSTALS BY PHONON VISCOSITY *

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ABSTRACT

The phonon viscosity mechanism, as it applies to screw dislocations moving on the {111} planes of face-centered-cubic crystals, is examined. Formulas are derived for changes in the elastic stiffnesses of cubic crystals with the impression of an arbitrary elastic strain. The result is specialized to the case of the strain field of a screw dislocation in a face-centered-cubic crystal. Lattice energy absorption from a moving screw dislocation is then considered through the relaxation of the elastic stiffnesses. Using the formula for energy absorption, a dislocation damping coefficient, B, is found. B is independent of temperature above the Debye temperature, and at lower temperatures has different forms for anisotropic and isotropic crystals. The damping coefficient for an edge dislocation is qualitatively similar to that of the screw dislocation.

The results are consistent with recent measurements of the damping coefficient for dislocations in copper,

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I. INTRODUCTION

Dislocation mobility in pure copper single crystals [1] as well as in zinc [2, 3] and aluminum [4, 5] is characterized by two important features: (a) the attainment of relatively high velocities for small applied stresses, and (b) decreasing interaction with the crystal lattice with decreasing temperature in the temperature range from about 70°K to 300°K. This dislocation behavior was first proposed theoretically by Leibfried [6], nearly twenty years before direct experimental evidence was available. His model postulated that, in the absence of a large Peierls' barrier, dislocation motion was damped by the interaction of lattice thermal waves with the elastic properties of dislocations. Leibfried specifically considered how dislocation mobility is affected by the stress field of a thermal phonon. This, however, is not the only mechanism to be considered. Since 1950, theories have been put forward to explain thermal phonon-dislocation core and thermal phonon-dislocation strain field interactions. The inherent complexity of the problem has required that many simplifying assumptions be made to obtain estimates of the dislocation drag coefficient as a closed form function of lattice and dislocation properties. While the various theories predict the proper order of magnitude of damping in copper, the experimentally observed temperature dependence of the damping coefficient of edge dislocations in the temperature range from 66°K to 373°K could not be closely approximated by the predictions of one or a combination of mechanisms.

The phonon viscosity theory developed by Mason and others is attractive because it alone predicts the high-temperature behavior

(temperatures near the Debye temperature) of the damping coefficient that is experimentally observed. However, the theory has not been worked out in great detail, and there exists some confusion as to the validity of several of its arguments.

The theory itself stems from the original work of Akhieser, who, in a fundamental paper [7], clearly demonstrated the lattice absorption of energy from acoustic (or strain) waves with long wavelengths relative to the thermal phonon mean free path. He reasoned as follows: the thermal phonons are highly localized with respect to the varying strain field; hence, they can be considered as traveling through a uniform medium with a slowly modulating strain field. The lattice thermal frequencies are changed as the result of the impression of the strain, creating temperature differences between phonon modes at any spatial point. The phonons relax to a new thermal equilibrium at some common temperature (adiabatic temperature) through phonon-phonon collisions. Akhieser considered 3-phonon N-processes due to anharmonic lattice terms and showed, using the Debye approximation for phonon wave velocities, that the relaxation process was accompanied by an increase of entropy of the system and a corresponding energy absorption from the strain wave.

In a series of papers, Mason [8, 9], Mason and Bateman [10], and Mason and Rosenberg [11] showed that the Akhieser effect was applicable to the absorption of energy from the strain field of a dislocation traveling through an otherwise defect-free crystal lattice. Rather than calculate the entropy produced during the relaxation process described above, they demonstrated that the elastic stiffnesses

of the crystal were altered by the perturbation from equilibrium. They noted that a change in an elastic stiffness multiplied by the time required for the material relaxation was equal to a viscosity, which in turn could be related to energy absorption. In equation form:

$$\delta C \tau = \eta \ , \qquad (1.1)$$

where δC is the difference between the unrelaxed and relaxed modulus, τ is the relaxation time, and η is the equivalent viscosity. A proof of equation (1.1) is given in Appendix A. The complexity of the situation was therefore reduced to calculation of changes in the elastic stiffnesses at all points in the lattice caused by the strain field of a moving dislocation, and concurrently to find for each an appropriate relaxation time.

Mason and Bateman [10] derived formulas for increases in the elastic stiffnesses of cubic crystals due to the impression of longitudinal or shear strain in a cubic axis direction. However, in their derivation, an erroneous assumption was made, leading to overestimation of these changes. In Section II the problem is reconsidered for an arbitrary applied strain in cubic crystals. In Section IV, employing the results of Section II, changes in specific moduli due to the motion of a screw dislocation in an fcc crystal are determined.

The relaxation time for the process has been considered by several authors [8, 11, 12]. These investigators have concluded that the appropriate relaxation time constants are those derived from lattice thermal conductivity measurements. In Section III it is argued that the range of applicability of the above is limited to the temperature region close to the Debye temperature.

The formula for the dislocation damping coefficient for a screw dislocation is determined in Section V. However, limited knowledge of the temperature dependence of the relaxation time involved precludes a precise calculation of the damping coefficient except near the Debye temperature.

Finally, in Section VI predictions of the theory are compared with the experimental evidence for damping forces on dislocations in copper [1].

II. EFFECT OF ELASTIC STRAIN ON THE ELASTIC STIFFNESSES OF CUBIC CRYSTALS

In the following, the conduction electrons (in the case of metals) are not considered. Electronic effects become important at very low temperatures.

The internal energy per unit volume of a single crystal in thermal equilibrium is given by:

$$U = W + E \ , \tag{2.1}$$

where W is the elastic strain energy and E is the energy of the thermal phonons. This elastic strain energy is:

$$W = \tfrac{1}{2} \sum_{i,j} C_{ij} S_i S_j \tag{2.2}$$

where the C_{ij} are the isothermal elastic stiffnesses, the S_i are the elastic strains, and i and j are summed from 1 to 6.

$$E = \sum_{\underline{q}, p} \left(\langle n_{\underline{q}, p}(T) \rangle + \tfrac{1}{2} \right) \hbar \omega_{\underline{q}, p} \tag{2.3}$$

where the summation includes each possible lattice wavevector $\underline{q}$ with polarization p. $\omega_{\underline{q}, p}$ is the frequency of mode $(\underline{q}, p)$, and $\langle n_{\underline{q}, p}(T) \rangle$ is the phonon occupancy number of that mode at temperature T. $\hbar$ is Planck's constant.

Since the density of points in reciprocal space is large, the sum over $\underline{q}$ in equation (2.3) can be replaced by an integral. Then:

$$E = \sum_{p} \int \left(\langle n_{q, p}(T) \rangle + \tfrac{1}{2} \right) \hbar \omega_{q, p} \, S(q, \theta, \varphi) \, q^2 \sin\theta \, d\theta d\varphi dq \ , \tag{2.4}$$

where θ and φ are polar angles and $S(q, \theta, \varphi)$ is the density of points

in reciprocal space per unit volume. Equation (2.4) can be simplified by the following Debye approximation:

(a) $S(q, \theta, \varphi) = 1/(2\pi)^3$

(b) $\omega_{\underline{q}, p} = q v_{\underline{q}, p}$, where $v_{\underline{q}, p}$ is the Debye velocity of mode $(\underline{q}, p)$

(c) the phase velocities in the $\langle 100 \rangle$, $\langle 110 \rangle$, and $\langle 111 \rangle$ directions only are considered. For these directions, propagating waves can be either purely longitudinal or transverse. Reciprocal space is divided into equal volumes about each of these 26 directions, and for each volume the phase velocity is assumed to be equal to that in the principal direction it surrounds.

Equation (2.4) then reduces to:

$$E = \sum_{k, p} v_{k, p} \int_k \left(\langle n_{q, p}(T) \rangle + \frac{1}{2} \right) \frac{\hbar}{(2\pi)^3} q^3 \sin\theta \, d\theta \, d\varphi \, dq$$

$$= \sum_{k, p} E_{k, p} \quad , \tag{2.5}$$

where k extends from 1 to 26.

Consider a small instantaneous change of the elastic strains in the crystal. The process is adiabatic in the sense of no spatial heat flow. In addition, since the change is instantaneous, it occurs in the absence of phonon-phonon collisions. Hence the phonon occupation numbers remain unperturbed. In equation form:

$$d\langle n_{k,p}(T)\rangle = \frac{\partial\langle n_{k,p}(T)\rangle}{\partial\omega}\, d\omega + \frac{\partial\langle n_{k,p}(T)\rangle}{\partial T}\, dT = 0 \ . \qquad (2.6)$$

$\langle n_{k,p}(T)\rangle$ is given by Planck's distribution function. That is:

$$\langle n_{k,p}(T)\rangle = \frac{1}{\exp[\hbar\omega_{k,p}/k_B T]-1} \ , \qquad (2.7)$$

where k_B is Boltzmann's constant.

Substitution of Planck's function for $\langle n_{k,p}(T)\rangle$ in equation (2.8) gives:

$$\frac{d\omega_{k,p}}{\omega_{k,p}} = \frac{dT}{T} \ . \qquad (2.8)$$

The Grüneisen relations [13] require that the phonon frequencies change according to:

$$\omega_{\underline{q},p} = \omega^{o}_{\underline{q},p}\left(1 + \sum_j \gamma^{j}_{q,p} S_j\right) \ , \qquad (2.9)$$

where $\omega^{o}_{\underline{q},p}$ is the frequency of mode $(\underline{q}, p)$ for the state of no strain, and the $\gamma^{j}_{\underline{q},p}$ are the appropriate Grüneisen numbers. The temperature change for the mode (k, p) is then:

$$dT = T\,\frac{\left(\sum_j \frac{\partial\omega_{k,p}}{\partial S_j}\, dS_j\right)}{\omega_{k,p}} = T\,\frac{\sum_j \gamma^{j}_{k,p}\, dS_j}{1 + \sum_j \gamma^{j}_{k,p} S_j} \ , \qquad (2.10)$$

where the dS_j are the small perturbations in strain. Because of the variety of the Grüneisen numbers [10], it is obvious that each phonon mode, in general, has a different temperature change.

Using equations (2.5) and (2.9), one finds the change in thermal energy to be:

$$\Delta E = \sum_i \frac{\partial E}{\partial S_i} dS_i$$

$$= \sum_i \sum_{k,p} \frac{\partial v_{k,p}}{\partial S_i} \frac{E_{k,p}}{v_{k,p}} dS_i$$

$$= \sum_i \sum_{k,p} \frac{\gamma^i_{k,p}}{1 + \sum_i \gamma^j_{k,p} S_j} E_{k,p} dS_i$$

$$\approx \sum_i \sum_{k,p} \gamma^i_{k,p} E_{k,p} dS_i - \sum_{i,j} \sum_{k,p} \gamma^i_{k,p} \gamma^j_{k,p} S_j E_{k,p} dS_i \ . \qquad (2.11)$$

The first term on the right-hand-side of equation (2.11) represents the variation with strain of the lattice thermal expansion. Accordingly, the second term is an addition to the elastic strain energy, since the total energy remains unchanged for the process. The total change in elastic strain energy is then found from equations (2.2) and (2.11) and is given by:

$$\Delta W = \sum_{i,j} C_{ij} S_j dS_i + \sum_{i,j} \sum_{k,p} \gamma^i_{k,p} \gamma^j_{k,p} S_j E_{k,p} dS_i$$

$$= \sum_{i,j} \left[C_{ij} + \sum_{k,p} \gamma^i_{k,p} \gamma^j_{k,p} E_{k,p} \right] S_j dS_i$$

$$= \sum_{i,j} C'_{ij} S_j dS_i \ , \qquad (2.12)$$

where the C'_{ij} are the effective elastic stiffnesses for the process, and are defined by:

$$C'_{ij} = C_{ij} + \sum_{k,p} \gamma^i_{k,p} \gamma^j_{k,p} E_{k,p} \ . \qquad (2.13)$$

The $C^{!}_{ij}$ are to be distinguished from the so-called adiabatic stiffnesses, which can be derived from thermodynamic arguments on the basis of no spatial heat flow with the impression of strain, but with the assumption of thermal equilibrium between phonon modes at any point [15, 16].

III. RELAXATION OF THE ELASTIC STIFFNESSES

The elastic stiffnesses altered by the small change in strain relax to their isothermal values through a two-stage process. The first step is the equilibration to a common temperature of the phonon modes at each point. This occurs through phonon-phonon transitions.

If the strain perturbation is spatially dependent, then, according to equation (2.10), the temperature change of each phonon mode is a function of position in the crystal. Hence, in the general case, the common equilibration temperature of the relaxed phonons depends on position in the crystal. This is the adiabatic state mentioned in Section II. Return to the isothermal condition is now determined by the macroscopic parameters involved in heat flow between spatial points. The energy absorbed from a dislocation strain field in the adiabatic-isothermal transition has been determined exactly [17, 18], and is negligibly small for metals.

In the following, only the initial transition is considered.

It is reasonable to assume that the elastic moduli will have a common relaxation time, because equilibration of the phonon modes will affect all the elastic stiffnesses simultaneously. The relaxation time, in turn, will be determined by the mean free path of thermal phonons. As understood in [7], the relaxation must include phonon N-processes; hence, the phonon-phonon contribution cannot be identified directly with the lattice thermal conductivity relaxation time, which is most influenced by phonon U-processes [19]. This is especially true at about one-half the Debye temperature and below, where the occupancy numbers of long-wavevector phonons (those re-

sponsible for U-processes) are much smaller than the corresponding values for phonons of lower energy. In the range of the Debye temperature, the occupancy numbers of all phonons become classical, the probability of occurrence of U-processes approaches that of N-processes, and thus the lattice thermal conductivity relaxation time becomes a meaningful measure of that which can be applied to the Akhieser effect. The high-temperature value is proportional to the inverse temperature [20].

IV. THE EFFECT OF THE STRAIN FIELD OF A SCREW DISLOCATION ON THE ELASTIC STIFFNESS OF FCC CRYSTALS

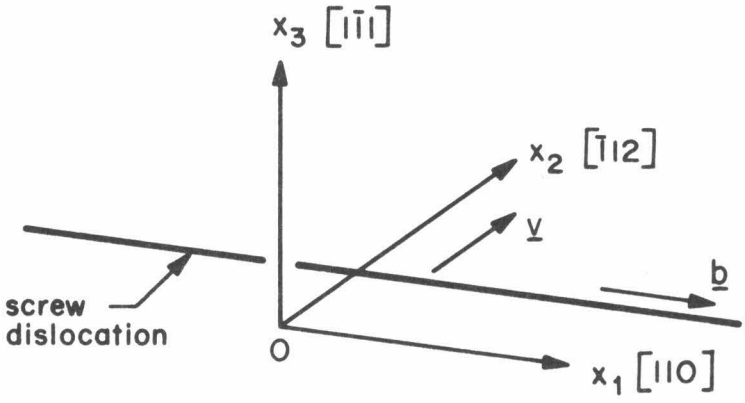
In Figure 1a, a screw dislocation moving uniformly with velocity v on a (111) plane of a fcc crystal is depicted. It is assumed that v is sufficiently small that the dislocation carries its static strain field with it. A moving cylindrical coordinate system (r, θ, z) is fixed to the dislocation, with the positive z-axis in the direction of the Burgers vector $\underline{b}$ (Figure 1b). In terms of (r, θ, z), and using isotropic elasticity, the dislocation strain field at any point is given by:

$$S_5 = \frac{b \cos \theta}{2\pi r}$$
$$S_6 = -\frac{b \sin \theta}{2\pi r} \qquad (4.1)$$

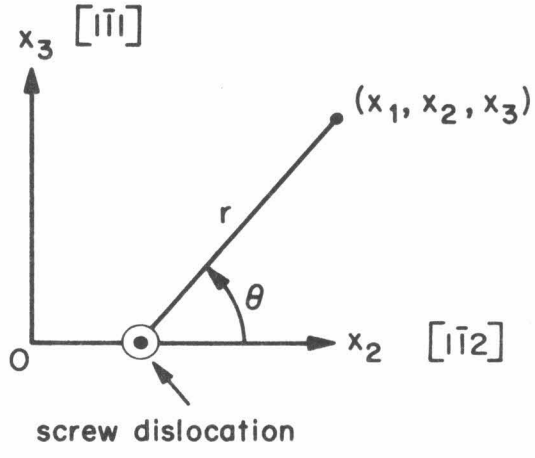
The elastic stiffness matrix for the coordinate orientation of Figure 1a is:

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 & 0 \\ C_{12} & C_{11} & C_{13} & -C_{14} & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ C_{14} & -C_{14} & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & C_{56} \\ 0 & 0 & 0 & 0 & C_{56} & C_{66} \end{pmatrix}$$

The reader is referred to [21] for the explicit expressions of the C_{ij}'s in terms of the three independent elastic stiffnesses. From equation (4.1) it is seen that the elastic moduli affected by the dis-



(a)



(b)

Figure 1. Schematic Representation of a Screw Dislocation Gliding on the (111) Plane of a Fcc Crystal. $\underline{b}$ is the Dislocation Burgers Vector. $\underline{v}$ is the Dislocation Velocity.

location shear strains are C_{55}, C_{56}, and C_{66}. Using equation (2.13) one finds:

$$\begin{aligned} \delta C_{55} &= \sum_{k,p} (\gamma^5_{k,p})^2 E_{k,p} \\ \delta C_{56} &= \sum_{k,p} \gamma^5_{k,p} \gamma^6_{k,p} E_{k,p} \\ \delta C_{66} &= \sum_{k,p} \gamma^5_{k,p} \gamma_{k,p} E_{k,p} \end{aligned} \tag{4.2}$$

V. THE DISLOCATION DAMPING COEFFICIENT FOR SCREW DISLOCATIONS IN FCC CRYSTALS

The rate of energy dissipation per unit volume due to a modulus relaxation is:

$$\dot{Q} = \eta \dot{S}^2 = (\delta C \tau)\dot{S}^2 \tag{5.1}$$

where $\dot{S}$ is the time rate of change of strain and η is the appropriate viscosity. For a screw dislocation, equation (5.1) becomes:

$$\dot{Q} = \tfrac{1}{2}\tau \int_V [(\delta C_{55} + \delta C_{56})\dot{S}_5^2 + (\delta C_{66} + \delta C_{56})\dot{S}_6^2]\,dV \ , \tag{5.2}$$

where V is the total crystal volume where the mechanism applies. The rate of energy dissipation per unit length of dislocation is:

$$\dot{Q} = \tfrac{1}{2}\tau \int_{r=a_o}^{\infty} \int_{\theta=0}^{2\pi} [(\delta C_{55}+\delta C_{56})\dot{S}_5^2 + (\delta C_{66}+\delta C_{56})\dot{S}_6^2]\,r\,dr\,d\theta \ , \tag{5.3}$$

where a_o is an inner cut-off radius. For values of $r < a_o$, as shown below, the viscosity mechanism is inapplicable. The strain rates $\dot{S}_5$ and $\dot{S}_6$ are given by:

$$\begin{aligned} \dot{S}_5 &= -\frac{bv}{2\pi}\frac{\cos 2\theta}{r^2} \ , \\ \dot{S}_6 &= \frac{bv}{2\pi}\frac{\sin 2\theta}{r^2} \ . \end{aligned} \tag{5.4}$$

Substituting equations (5.4) into equation (5.3) and performing the integration yields:

$$\dot{Q} = \frac{b^2 v^2 \tau}{16\pi a_o^2} \sum_{k,p} (\gamma_{k,p}^5 + \gamma_{k,p}^6)^2 E_{k,p} \tag{5.6}$$

with the aid of equations (4.2).

A dislocation drag force, F, is defined by:

$$F = Bv \ , \tag{5.7}$$

where B is the damping coefficient. Then:

$$\dot{Q} = Fv = Bv^2 \quad . \tag{5.8}$$

Comparison of equation (5.7) with equation (5.8) provides:

$$B = \frac{b^2\tau}{16\pi a_o^2} \sum_{k,p} (\gamma^5_{k,p} + \gamma^6_{k,p})^2 E_{k,p} \quad . \tag{5.9}$$

For crystals with elastic isotropy, the expression for B can be somewhat simplified. Here, the transverse Debye velocity can be assumed independent of polarization, and both the longitudinal and transverse velocities are independent of crystallographic direction. From equation (2.19) it is seen that $E_{k,p}$ has only two unique values, one for each of the transverse and longitudinal modes. Equation (5.9) reduces to:

$$B = \frac{b^2\tau}{16\pi a_o^2} \left[E_1 \sum_k (\gamma^5_{k,1} + \gamma^6_{k,1})^2 + E_2 \sum_k \{(\gamma^5_{k,2} + \gamma^6_{k,2})^2 + (\gamma^5_{k,3} + \gamma^6_{k,3})\}\right] , \tag{5.10}$$

where E_1 and E_2 are the values of elastic energy in the longitudinal and transverse modes. The temperature dependence of B, calculated from equation (5.10), can be expected to differ from that of (5.9) in the general anisotropic case, where the lattice energy depends strongly on the crystallographic direction. At high temperatures, as is shown below, this effect is unimportant, because here the lattice energies are independent of the Debye velocities.

Near the Debye temperature:

$$E_{k,p} \rightarrow \frac{N}{26} k_B T \quad , \tag{5.11}$$

where N is the number of atoms per unit volume, and k_B is Boltzmann's constant. In this case, equation (5.9) reduces to:

$$B = \frac{b^2 N k_B \tau T}{416\pi a_o^2} \sum_{k,p} (\gamma^5_{k,p} + \gamma^6_{k,p}) \ , \qquad (5.12)$$

which, according to the argument of Section IV, is independent of temperature.

A similar procedure can be followed to determine B for an edge dislocation. Again, the strain rates are proportional to the dislocation velocity. The terms in the summation are different, but calculable, and at high temperature B is independent of temperature, as above.

The viscosity concept, as discussed here, is valid only for those regions with small elastic strains. For example, in equation (2.11), the perturbations in the elastic stiffnesses were derived on the basis of small applied strains. For a dislocation, there is a core region within which elastic strains are large and the phonon - dislocation interaction cannot be described by a linear approximation. If one assumes that linear elasticity is valid for strains less than 0.1, then equation (4.1) provides a core radius equal to 2b, where b is the magnitude of the dislocation Burgers vector. In this case, $a_o = 2b$. There is general agreement in the literature with respect to this effect [9, 12, 22].

However, a second consideration must be understood. The basis postulate of the viscosity theory requires that the relaxation time of the thermal phonons be less than the period of the applied strain field. It is difficult to determine a period (or frequency) for a moving dislocation strain field until it is Fourier analysed in terms of har-

monic waves. Then the strain field at any point can be considered as a superposition of plane waves, with periods ranging from 0 to ∞. The harmonic spectrum useful for the viscosity concept includes only those waves with periods greater than the phonon relaxation time.

Several authors [9, 12, 22] have considered the problem, although not on the basis of a Fourier decomposition of the moving dislocation strain field. The analyses postulate that an inner radius a_o can again describe the so-called cut-off radius, which can be greater than the radius of the dislocation core. While the interpretation is not strictly correct, it is useful because it provides an estimate for the critical velocity, above which B is velocity-dependent. Suzuki, et al. [22] defined the period of the strain field, τ_s, at any point to be equal to two times the distance r from that point to the dislocation center divided by the dislocation velocity. This is a reasonable assumption, for it is seen that the total shear strain for any point goes from a particular value at (r, θ) to the same value at $(r, 180^o-\theta)$. Then:

$$\tau_s = \frac{2r}{v} \quad . \tag{5.13}$$

The viscosity theory is valid in the region:

$$\tau_s > \tau \quad , \tag{5.14}$$

where τ is the appropriate relaxation time for the elastic moduli. That is:

$$a_o = \tfrac{1}{2} v\tau \quad . \tag{5.15}$$

Since τ increases with decreasing temperature, so will a_o, according to equation (5.15). However, the effect on B will be apparent

only if a_o becomes greater than 2b. Equation (5. 15) defines a critical dislocation velocity, v_c, above which a_o exceeds the radius of the non-linear core;

$$v_c = \frac{4b}{\tau} \quad . \tag{5. 16}$$

For velocities greater than v_c, B decreases in correspondence with the increase in a_o.

VI. THE COMPATIBILITY OF PHONON VISCOSITY WITH THE MEASUREMENTS OF DISLOCATION VELOCITY IN COPPER

Recent dislocation mobility measurements in copper [1] in the temperature range 66°K to 373°K have lent qualitative support to the phonon viscosity theory. These measurements indicated that:

(a) the damping coefficient becomes insensitive to temperature above the Debye temperature (~340°K).

(b) the damping coefficient is velocity dependent at low temperatures (<100°K), and decreases with increasing velocity (the maximum velocity was about 9000 cm/sec at 66°K).

Observation (a) is in agreement with the prediction of equation (5.12). This is, of course, at variance with the other proposed theories for dislocation damping by thermal phonons. Observation (b) is in accord with a cut-off radius that increases with increasing velocity (equation 5.9). However, the existence of a velocity-dependent cut-off radius will be confirmed theoretically only when a suitable phonon relaxation time at low temperatures is found (equation 5.15).

VII. SUMMARY

The concept of phonon viscosity as it applies to dislocations in fcc crystals was discussed in some detail. First, a general formula for changes in the elastic stiffnesses of cubic crystals caused by an arbitrary applied strain was derived. Then the formula was specialized to the specific case of the strain field of screw dislocations in an fcc crystal.

With this knowledge, and following the work of Mason, a damping coefficient for screw dislocations was found. The weak point of the analysis was lack of knowledge of the elastic stiffness relaxation time well below the Debye temperature. However, at higher temperatures, a more precise expression for the magnitude and temperature dependence of B was determined. B was shown to be independent of temperature above the Debye temperature, as was predicted by Mason. However, the low-temperature dependence of B for anisotropic materials deviated from that of Mason's original analysis.

The results were applied to recent direct mobility measurements in copper [1], and qualitative agreement with the viscosity concept was found in that the experimentally-determined damping coefficient behaved as predicted at high temperature (in sharp contrast to the predictions of other phonon mechanisms).

APPENDIX A

VISCOSITY OF THE STANDARD LINEAR SOLID

Consider the standard linear solid shown in the figure.

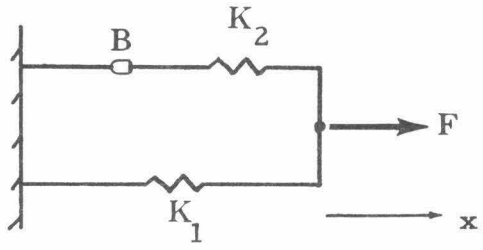


Figure 2. Standard Linear Solid.

K_1, K_2 are spring constants, and B is a dashpot viscosity.

The equation of motion for this solid is:

$$\frac{dF}{dt} + \frac{K_2}{B} F = (K_1+K_2) \frac{dx}{dt} + \frac{K_1 K_2}{B} x \qquad \text{(A-1)}$$

Zener [23] showed that this model corresponds to an anelastic solid with modulus relaxation. Zener defined:

M_R = relaxed modulus

M_u = unrelaxed modulus

τ_ε = relaxation time of stress at constant strain

τ_σ = relaxation time of strain at constant stress

In terms of Zener's notation:

$$\begin{aligned} B/K_2 &= \tau_e \\ K_1 &= M_R \\ \frac{B(K_1+K_2)}{K_2} &= M_R \tau_\sigma \end{aligned} \qquad \text{(A-2)}$$

Using equations (A-2), we find that:

$$B = (M_u - M_R)\tau_\varepsilon \ .$$

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