GALCIT SM REPORT 79-15 November 1979

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INFORMAL PROGWSS REPORT NO. 3

ON

THE INCORPORATION OF BENDING INTO THE BUCKLING DELAMINATION ANALYSIS

In a previous report* a one dimensional model of buckling-delamination in a column was summarized. The main assumption made in that report was that the unbuckled portions of the column can be assumed to remain straight. The purpose of this report is to investigate the more general problem in which bending effects are taken into account. We deal here with the case of a single off center delamination in a column. (The case of multi-delaminations in a column can be worked out too under slight modifications.) Following the general procedure outlined in previous reports, consider the column of unit width shown in fig, la, State I represents the unstressed column while state II denotes the axially and uniformally compressed column. State III differs from 11 by allowing the delamination to buckle. Our aim here is to find an expression for the strain energy release rate of state III.

In order to do this, we assume that the deformed column of state TTT can be divided into three beam-column portions having the same angle of rotation (θ_o) at their ends, as shown in fig. lb. Under this assumption, we now proceed as follows:

The beam-column differential equation is given by

$$EI \frac{d^4y}{dx^4} + k^2 \frac{d^2y}{dx^2} = 0 \ , \quad k^2 = P/EI$$

P is the axial load, E is Young's modulus and I is the cross section moment of inertia.

The general solution of this equation is given by

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*Informal Progress Report No. 2

$$y = A_1 \cos kx + A_2 \sin kx + A_3 x + A_4 \qquad (1)$$

where A_i are constantsof integration

Referring to fig. 1b we have the following boundary conditions

$$\begin{aligned} &\text{Portion (1):} \quad y_1(0) = y_1(\ell_1) = 0 \quad , \quad y_1'(0) = -y_1'(\ell_1) = \theta_o \\ &\text{Portion (2):} \quad y_2(0) = y_2'(0) = y_2'''(0) = 0 \quad , \quad y_2'(\ell_2) = \theta_o \qquad (2) \\ &\text{Portion (3):} \quad y_3(0) = y_3(\ell_3) = 0 \quad , \quad y_3'(0) = -y_3'(\ell_3) = \theta_o \end{aligned}$$

Inserting (2) in (1), we found

$$y_i - \frac{\theta_o \ell_i}{2u_i \sin u_i} \left[\cos\left(u_i - \frac{2u_i x_i}{\ell_i}\right) - \cos u_i\right] \quad , \quad i = 1,3 \qquad (3a,b)$$

$$y_2 = \frac{\theta_o \ell_2}{2u_2 \sin 2u_2} \left[1 - \cos \frac{2u_2 x_2}{2}\right]$$

where

$$u_j = \frac{k_j \ell_j}{2} = \sqrt{\frac{P_j}{EI_j}} \quad \frac{\ell_j}{2} \quad , \quad j = 1,2,3 \qquad (3c)$$

Now, regarding portion (3), we observe that $y_3(\ell_3/2) > 0$ when $u_3 < n$ and $y_3(\ell_3/2) < 0$ when $2\pi < u_3 > \pi$

Referring to figs-la, 1b **it** is clear that under physical consideration $y_3(\ell_3/2) < 0$, i.e., $\pi < u_3 < 2\pi$

Remembering that $u_3 = \pi$ is the Euler buckling parameter for clamped-clamped beam ($\theta_o = 0$), and having in mind that we are seeking solution eo a problem where portion **(3)** is pose buckle, we set $u_3 = \pi + 6$, $\delta > 0$. Under the assumption that θ_o is small, we assume that 6 is also small ($\delta << \pi$) (this assumption will be confirmed later on). Putting $u_3 = \delta + \pi$ in the expression for y_3 and collecting terms up to order δ , we found

$$Y3 = A_3 \left[1 - \cos\left(\frac{2\pi + 2\delta}{\ell_3} x - \delta\right)\right] \qquad (3d)$$

where $A_3 = A(1 - \delta/\pi)$, $A = \frac{-\theta_o \ell_3}{2\pi\delta}$

Now we find θ_o from the condition that $M_2 = M_1 + M_3 + P_3(\frac{T - H}{2}) - P_1 \frac{H}{2}$ (see fig. 1b). Upon using $M_1 = -EI_1 y_1''(0)$, $M_2 = -EI_2 y_2''(\ell_2)$, $M_3 = -EI_3 y_3''(0)$ and equations (3) we found

$$\theta_o = \frac{P_1 \frac{H}{2} - P_3(\frac{T - H}{2}) + E\frac{H^3 A \pi^2}{3\ell_3^2}(1 - \frac{\theta_o \ell_3}{2\pi^2 A})}{\frac{P_2 \ell_2}{2u_2} \operatorname{ctan} 2u_2 + \frac{P_1 \ell_1}{2u_1} \operatorname{ctan} u_1} \tag{4}$$

The distributions of axial strains in the three column portions can be found by the fact that the column ends are fixed during the passing of the system from state II to III. This gives (see figs. 1a, 1b)

$$2\varepsilon_2 \ell_2 + 2\frac{1}{2}\int_o^{\ell_2} y_2'^2\, dx + \varepsilon_1 \ell_1 + \frac{1}{2}\int_o^{\ell_1} y_1'^2\, dx + H\theta_o = \varepsilon_o L \tag{5}$$

Compatibility between portion (1) and portion (3) is governed by

$$\varepsilon_3 \ell_3 + \frac{1}{2}\int_o^{\ell_3} y_3'^2\, dx = \varepsilon_1 \ell_1 + \frac{1}{2}\int_o^{\ell_1} y_1'^2\, dx + T\theta_o \tag{6}$$

Balance of axial forces between the column portions gives

$P_2 = P_1 з P_3$ or, since E is common

$$T\varepsilon_2 = (T - H)\varepsilon_1 + H\varepsilon_3 \tag{7}$$

where we made use of the relations $P_1 = (T - H)E\varepsilon_1$, $P_2 = TE\varepsilon_2$, $P_3 = HE\varepsilon_3$. Using the assumption that $u_3 = \pi + \delta$, we found ε_3 by using (3c) to be

$$\varepsilon_3 = \frac{\pi^2}{3}\frac{H^2}{\ell_3^2}(1 + 2\delta/\pi). \tag{8}$$

Using (3) and (8), the system (5) - (7) can be expressed as follows

$$\bar{\varepsilon}_1 = \frac{\bar{\varepsilon}_o - (1-\bar{\ell})\bar{H}^3/\bar{\ell}^2}{(1-\bar{H}+\bar{\ell}\bar{H})} + \frac{(1-\bar{\ell})\alpha_2 + \alpha_3}{(1-\bar{H}+\bar{\ell}\bar{H})}$$

$$\bar{\varepsilon}_2 = \frac{(1-\bar{H})\bar{\varepsilon}_o + \bar{H}^3/\bar{\ell}}{(1-\bar{H}+\bar{\ell}\bar{H})} + \frac{(1-\bar{H})\alpha_3 - \alpha_2\bar{\ell}}{(1-\bar{H}+\bar{\ell}\bar{H})} \qquad (9)$$

$$\bar{c}^2 = \frac{\bar{\varepsilon}_o - \bar{H}^2/\bar{\ell}^2}{(1-\bar{H}+RH)} + \frac{(1-\bar{\ell})\alpha_2 + a_3}{(1-\bar{H}+\bar{\ell}\bar{H})} + \alpha_1$$

where

$$\bar{\varepsilon}_o = \frac{\varepsilon_o}{\varepsilon_L} \qquad \bar{\varepsilon}_1 = \varepsilon_1/\varepsilon_L \ , \ \bar{\varepsilon}_2 = \varepsilon_2/\varepsilon_L \ , \ \bar{c} = \frac{A\pi}{\ell}/(\varepsilon_L)^{1/2}$$

$$\bar{H} = H/T \ , \quad \bar{\ell} = \ell/L \ , \ \delta = \frac{-\sqrt{3}\ \bar{\theta}_o}{2\pi\bar{c}} \qquad \bar{\theta}_o = \frac{L}{T}\theta_o \qquad (10)$$

and $\varepsilon_L = \frac{\pi^2}{3}\frac{T^2}{L^2}$ (Euler buckling strain for clamped-clamped column of length L and thickness T). Note that we have taken $\ell_1 = 2$, $\ell_2 = (L - \ell)/2$, $\ell_3 = \ell$

also

$$\alpha_1 = \frac{3\bar{\theta}_o}{\pi^2\bar{\ell}}\left(1 - \frac{\bar{c}\ \bar{\ell}}{2\sqrt{3}} + \frac{\bar{H}^2}{\sqrt{3}\ \bar{\ell}\bar{c}}\right) + \frac{3\bar{\theta}_o^2}{8\pi^2}\,\frac{(2u_1 - \sin 2u_1)}{u_1 \sin^2 u_1}$$

$$\alpha_2 = \frac{\sqrt{3}\ \bar{H}^3\bar{\theta}_o}{\pi^2\bar{c}\ \bar{\ell}^2} \qquad (11)$$

$$\alpha_3 = \frac{-3\bar{H}\bar{\theta}_o}{\pi^2} - \frac{3\bar{\theta}_o^2}{8\pi^2}\left[\frac{\bar{\ell}}{u_1}\,\frac{(2u_1 - \sin 2u_1)}{\sin^2 u_1} + (1-\bar{\ell})\,\frac{(4u_2 - \sin 4u_2)}{2u_2 \sin^2 2u_2}\right]$$

with

$$u_1 = \frac{\pi\bar{\ell}}{(1-\bar{H})}\sqrt{\bar{\varepsilon}_1} \ , \quad u_2 = \frac{\pi(1-\bar{\ell})}{2}\sqrt{\bar{\varepsilon}_2}$$

and θ_o is found from equ. (4) to be

$$\bar{\theta}_o = \frac{(\bar{\varepsilon}_1 - \bar{\varepsilon}_2)(1 - \bar{H}) + \frac{2\bar{H}^3\bar{c}}{\sqrt{3}\,\bar{\ell}}}{\frac{\bar{\varepsilon}_1\bar{\ell}(1 - \bar{H})}{u_1 \tan u_1} + \frac{\bar{\varepsilon}_2(1 - \bar{\ell})}{2u_2 \tan 2u_2} + \frac{\bar{H}^3}{\frac{-2}{\ell\pi}}} \tag{12}$$

The system (9) - (12) composed of three basic unknowns, namely $\bar{\varepsilon}_1$, $\bar{\varepsilon}_2$ and $\bar{c}$. A solution of this system is carried out by means of a numerical iteration scheme. The method can be summarized as follows:

Step zero: Choose $\bar{H}$, $\bar{\ell}$, $\bar{\varepsilon}_o$

Step one: Solve (9) with $\alpha_1 = \alpha_2 = \alpha_3 = 0$

Step two: Use these values to calculate $\bar{\theta}_o$ (equ. 12)

Step three: Calculate α_1, α_2, α_3 (equ. 11)

Step four: Calculate again $\bar{\varepsilon}_1$, $\bar{\varepsilon}_2$, $\bar{c}$ equ. (9) by using the values just obtained for θ_o, α_1, α_2, α_3.

Step five: Repeat this process until convergency of the system variables is achieved. It was found that about five iterations are needed in order to provide satisfactory convergence.

The assumption that $\delta \ll 1$ (equ. 10) (so that equ. (3b) could be approximated by (3d)) is checked out in fig. 2 and found valid. Figure 3 shows the dependence of $\bar{\theta}_o$ on geometrical and loading parameter.

Strain energy calculations

The strain energy of state III is given by

$$U_{III} = \frac{2P_2}{2}\varepsilon_2\ell_2 + \frac{P_1}{2}\varepsilon_1\ell_1 + \frac{P_3}{2}\varepsilon_3\ell_3 + E\frac{I_3}{2}\int_0^{\ell_3} y_3''^2\,dx + \frac{EI_1}{2}\int_0^{\ell_1} y_1''^2\,dx + 2\frac{EI_2}{2}\int_0^{\ell_2} y_2''^2\,dx$$

where the first three terms are associated with compression, while the last three are due to bending.

Upon making use of equ. **(3)**, we found

$$\frac{U_{III}}{\frac{E\pi^4 T^5}{18L^3}} = (1 - \bar{\ell})\bar{\varepsilon}_2^2 + (1 - \bar{H})\bar{\ell}\bar{\varepsilon}_1^2 + \bar{H}^5/\bar{\ell}^3 + \frac{2\bar{H}^3\bar{c}^2}{\bar{\ell}} - \frac{2\sqrt{3}\,\bar{H}^3}{\pi^2\bar{\ell}}\bar{c}\bar{\theta}_o\left(\frac{3}{2}+\left(\frac{\bar{H}}{\bar{\ell}\bar{c}}\right)^2\right)$$

$$+ \frac{3}{4}\frac{(1 - \bar{H})^3}{\pi^4}\frac{u_1}{\bar{\ell}}\bar{\theta}_o^2\frac{(2u_1 + \sin 2u_1)}{\sin^2 u_1} + \frac{3}{2\pi^4}\frac{u_2}{(1 - \bar{\ell})}\bar{\theta}_o^2 \qquad (13)$$

$$\frac{(4u_2 + \sin 4u_2)}{\sin^2 2u_2} \equiv F$$

where again we set $\ell_1 = \ell$, $\ell_3 = \ell$, $\ell_2 = (L - \ell)/2$

Once the values of $\bar{\varepsilon}_1$, $\bar{\varepsilon}_2$, $\bar{c}$, $\bar{\theta}_o$ are obtained using the numerical procedure outlined earlier, U_{III} can be calculated, The strain energy release rate can be found from (13) by

$$\bar{G} \equiv \frac{\frac{-\partial U_{III}}{\partial \ell}}{\frac{ET^5}{L^4}} = \frac{-\pi^4}{18}\frac{\partial F}{\partial \bar{\ell}}$$

The derivative is then approximated numerically as follows;

$$\bar{G} = \frac{-\pi^4}{18}\,\frac{F(\bar{\ell}) - F(\bar{\ell} + \Delta\bar{\ell})}{\Delta\bar{\ell}} \quad , \Delta\bar{\ell} \text{ is small} \qquad (15)$$

$\bar{G}$ has been calculated numerically and the results are shown in fig. 4 and 5. Also shown in fig. 4,5 are the results for the beam-column analysis which does not include the bending effect. Note that this limit case is obtained by setting the number of iteration to zero,

<u>Discussion of results.</u>

In order to interpret the fracture energy criterion using these results, a plot of experimentally obtained fracture energy for graphite epoxy composite laminates is plotted in fig ,5 for the special case

$E = 1.04 \times 10^7$ p.s.i, $L = 7''$ and $T = .24''$ By considering the results of fig. 5, it can be concluded that the arresting capability of initial crack is greater in the current analysis over the previous one (no bending effect, i.e., $\theta_o = 0$) provided $\bar{\ell} > .5$, roughly. This observation is important, since they tend to back up our experimental results. Furthermore, the deviation of the current analysis from the previous one ($\theta_o \equiv 0$) is not affected much by the thickness ratio H/T, but is affected largely by the loading parameter $\bar{\varepsilon}_o$. Roughly speaking, both analyses produced close results for $\bar{\varepsilon}_o < .5$ and deviate considerably for larger $\bar{\varepsilon}_o$. In order to gain more information regarding this matter, consider the following problem:

<u>A thin delaminated layer attached to a halif-space.</u>

Consider a uniformly compressed isotropic-elastic half-space containing a thin delamination of thickness H, length ℓ and unit width, as shown in fig.6 . State I represents the unstressed half-sapce, while state II denotes the axially and uniformally compressed hall' space. State III differs from II by allowing the delamination to split. Our- aim here is to find an expression for the strain energy of states III. For simplicity let $\nu = 0$ (ν is the Poisson's ratio). Regarding the thin buckle as a D.C.B. having zero slope at both ends it is easy to show, using strength of material approximation that

$$\omega(x) = A(1 + \cos 2\pi x/\ell)$$
$$\varepsilon_{cr} = \frac{\pi^2}{3}\frac{H^2}{\ell^2} \qquad (16)$$

where $\omega(x)$ is the deflection and ε_{cr} is the Euler buckling strain of a clamped-clamped column of thickness H and length R.

Assuming that after buckling, the state of uniform compression remains unaltered in the unbuckled medium, the compatability condition before and after buckling of this delamination can be expressed as

$$\varepsilon_{cr}\ell + \frac{1}{2}\int_{-\ell/2}^{\ell/2}\left(\frac{\partial\omega}{\partial x}\right)^2 dx = \varepsilon_o \ell \qquad (17)$$

The strain energy occupied by the buckled column is

$$U_b = \frac{EH}{2}\varepsilon_{cr}^2 \ell + \frac{EI}{2}\int_{-\ell/2}^{\ell/2}\left(\frac{\partial^2\omega}{\partial x^2}\right)^2 dx \quad , \qquad I = \frac{H^3}{12}$$

by making use of (16) and (17), we find

$$U_b = \frac{EH}{2}\ell\varepsilon_{cr}(2\varepsilon_o - \varepsilon_{cr})$$

The strain energy in the unbuckled portion (the half-space less a void of thickness H and length ℓ) is

$$U_2 = \frac{-EH}{2}\varepsilon_o^2\ell + U_o$$

Note that U_o is the strain energy of a half-space under uniform compression ε_o. (U_o = const.)

Then the total strain energy in state III is

$$U_{III} = U_b + U_2 = \frac{EH}{2}\ell\varepsilon_{cr}(2\varepsilon_o - \varepsilon_{cr}) - \frac{EH}{2}\varepsilon_o^2\ell + U_o = \frac{-EH}{2}\ell(\varepsilon_o - \varepsilon_{cr})^2 + U_o \qquad (18)$$

also

$$G_\infty \equiv \frac{-\partial U_{III}}{\partial \ell} = \frac{EH}{2}(\varepsilon_o - \varepsilon_{cr})(\varepsilon_o + 3\varepsilon_{cr}) \qquad (19a)$$

or

$$G_\infty = \frac{EH}{2}\left(\varepsilon_o - \frac{\pi^2}{3}\frac{H^2}{\ell^2}\right)\left(\varepsilon_o + \frac{\pi^2 H^2}{\ell^2}\right) \qquad (19b)$$

Introducing fictitious parameter T and L, (4b) can be written as

$$\bar{G}_\infty \equiv \frac{G_\infty}{ET^5/L4} = \frac{\pi^4}{18}\bar{H}(\bar{\varepsilon}_o - \bar{H}^2/\bar{\ell}^2)(\bar{\varepsilon}_o + 3\bar{H}^2/\bar{\ell}^2), \quad \bar{\varepsilon}_o = \frac{\varepsilon_o}{\frac{\pi^2 T^2}{3L^2}} \qquad (19c)$$

where $\overline{H} = H/T$, $\overline{\ell} = \ell/L$. Now recall eq. (8) of P.R. No. 2 with N = 1 (a column of thickness T and length L containing a single delamination of thickness H and length ℓ). This equation can he written as

$$\overline{G} \equiv \frac{-\partial U/ET^5}{\partial \ell / L4} = \frac{\pi^4 \overline{H}}{18} \frac{(1 - \overline{H})}{(1 - \overline{H} + \overline{\ell}\overline{H})^2} (\overline{\varepsilon}_o - \frac{\overline{H}^2}{\overline{\ell}^2})(\overline{\varepsilon}_o + \frac{3\overline{H}^2}{\overline{\ell}^2} + \frac{4\overline{H}^2}{\overline{\ell}^2} \frac{\overline{\ell}\overline{H}}{(1 - \overline{H})}) \quad (20)$$

Note that (20) reduces to (19) as $\overline{H} \to 0$, as might be expected.
To see the rule of $\overline{H}$, we use (19) and (20) to get

$$\frac{G - G_\infty}{G_\infty} = \frac{\overline{H}}{(1 - \overline{H} + \overline{\ell}\overline{H})^2} (1 - \overline{H} - \overline{\ell}^2\overline{H} - 2\overline{\ell} + 2\overline{H}\overline{\ell} + \frac{4\overline{\ell}}{3(1 + \frac{1}{3} \frac{\varepsilon_o \overline{\ell}^2}{\overline{H}^2})}) \quad (21)$$

Relation (19b) is plotted in fig, 7, relation (21) is plotted in fig. 8.

So far we have made three approximations to the problem of a single delamination in a column, The more accurate one is that given by eq. (15) (a delamination in a column in which the unbuckled portions are allowed to bent,) The second is the less accurate problem given by eq. (8) of P.R. No. 2 (a delamination in a column in which the unbuckled portions assumed to remain straight). The third approximation, given by eq. (19b) or (19c) of this report (a delamination in a half-sapce) is the limit case of the above two as T $\to\infty$, fig. 9 shows a comparison between the three methods. It is seen that the error in the approximation of a real problem of delamination in a column as delamination attached to a half space is on the order of $\overline{H}$, provided the load is sufficiently lower than the Euler buckling load of the undelarninated column.

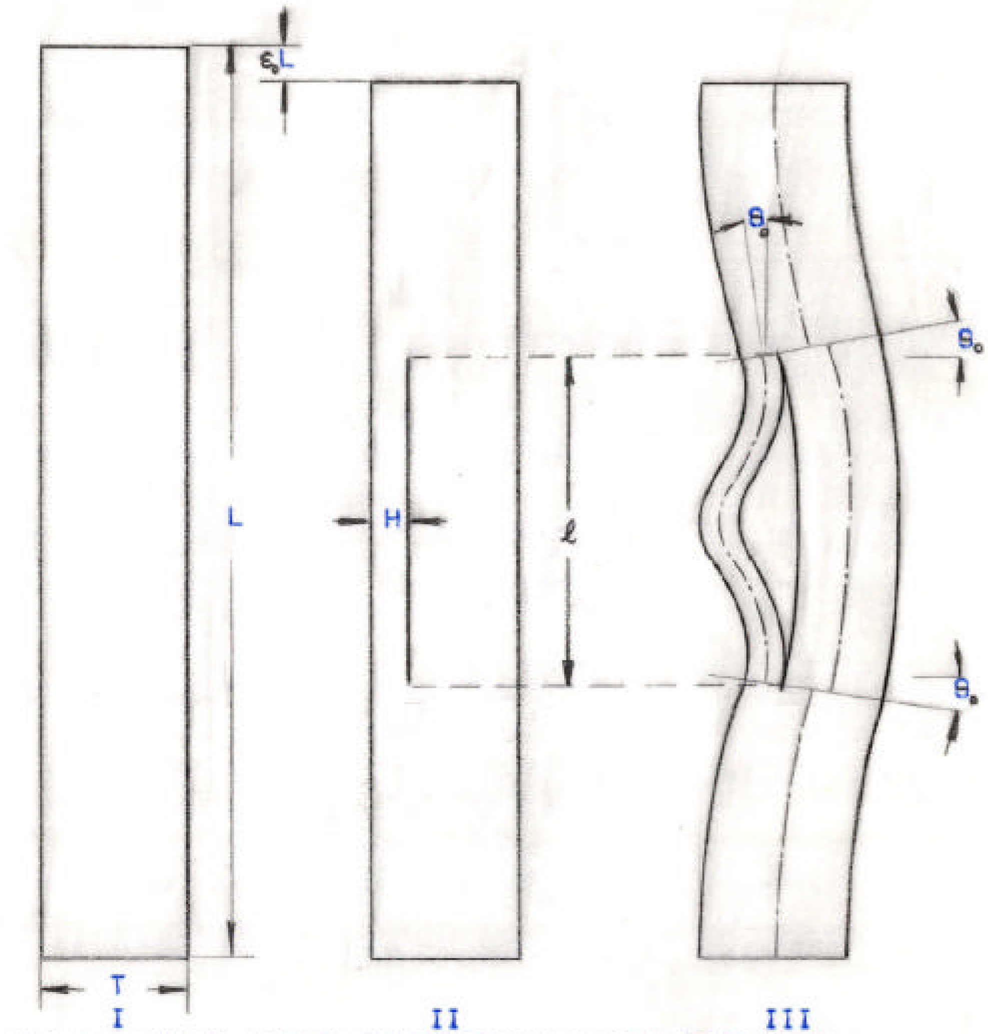


FIG. 1a. THREE STATES IN THE DEFORMATION OF COLUMN

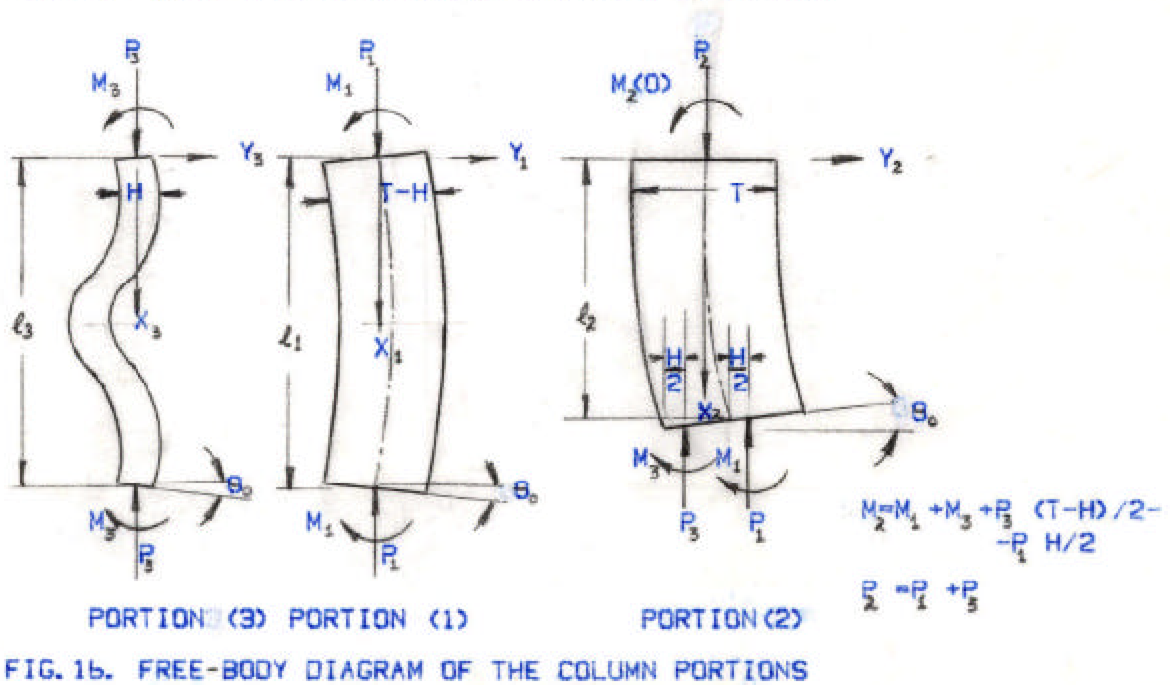


FIG. 1b. FREE-BODY DIAGRAM OF THE COLUMN PORTIONS

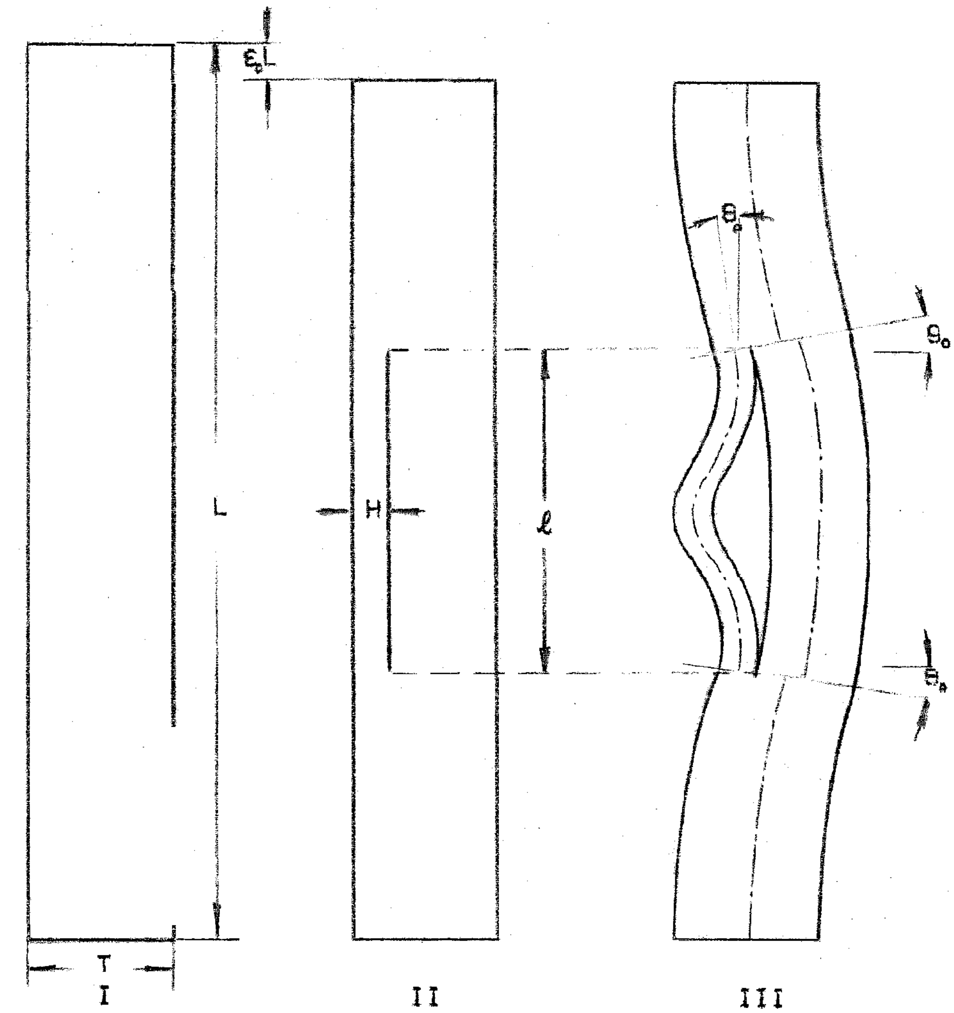


FIG. 1a. THREE STATES IN THE DEFORMATION OF COLUMN

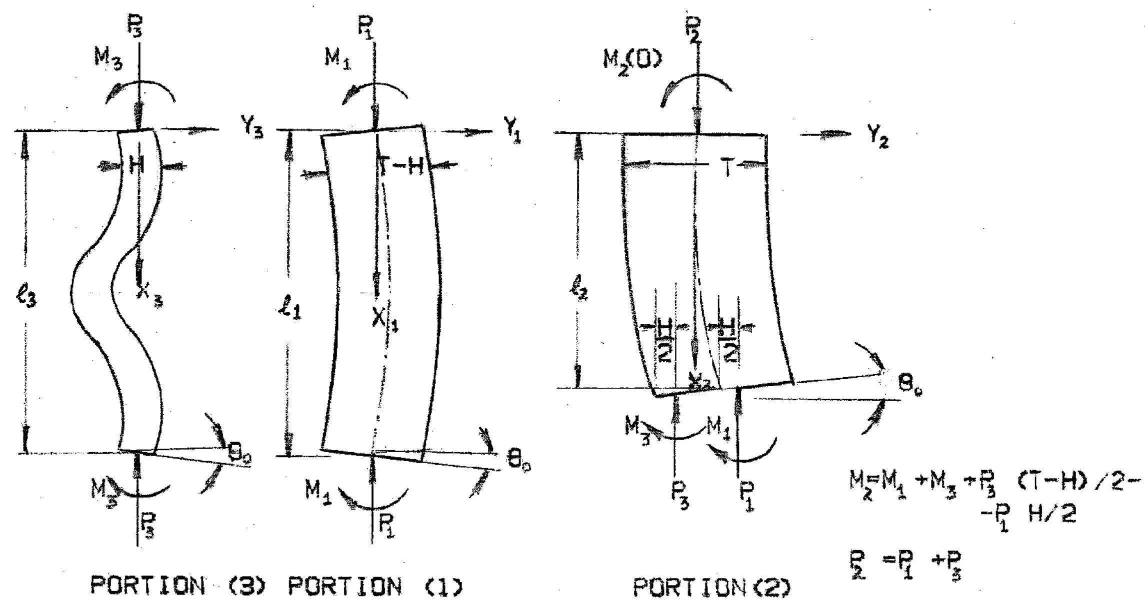


FIG. 1b. FREE-BODY DIAGRAM OF THE COLUMN PORTIONS

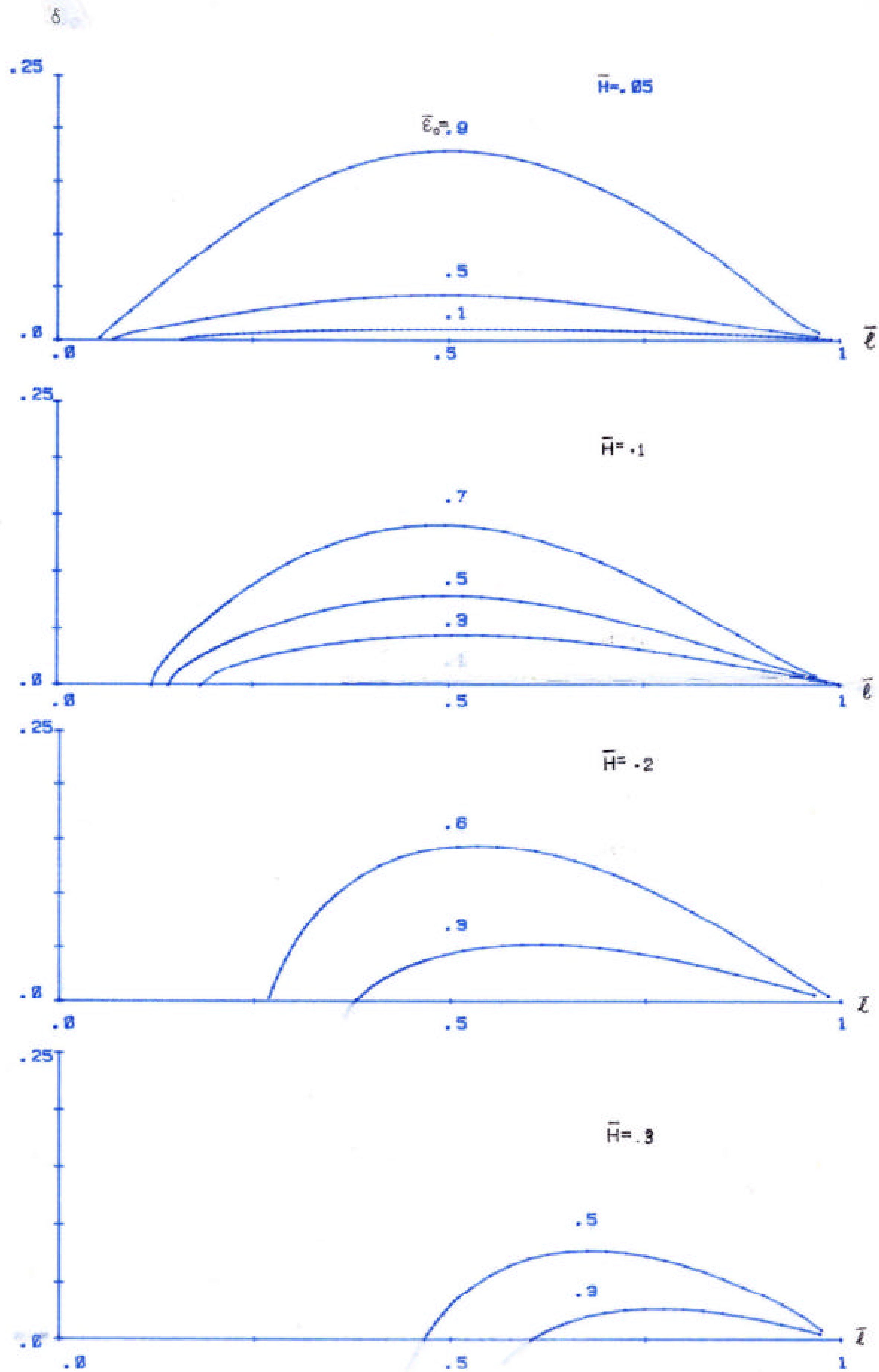
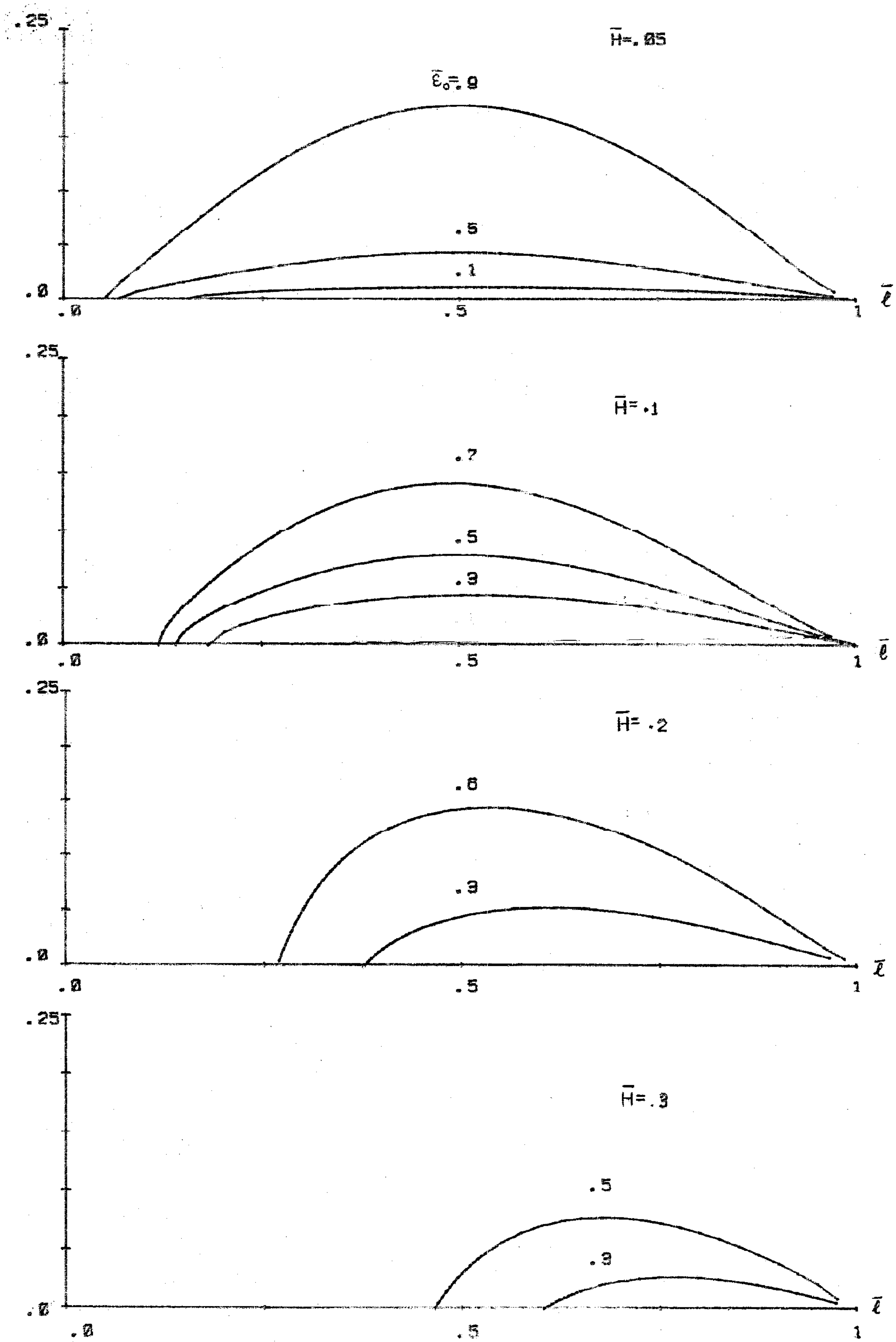


Fig. 2. THE DEPENDENCE OF δ ON LOADING



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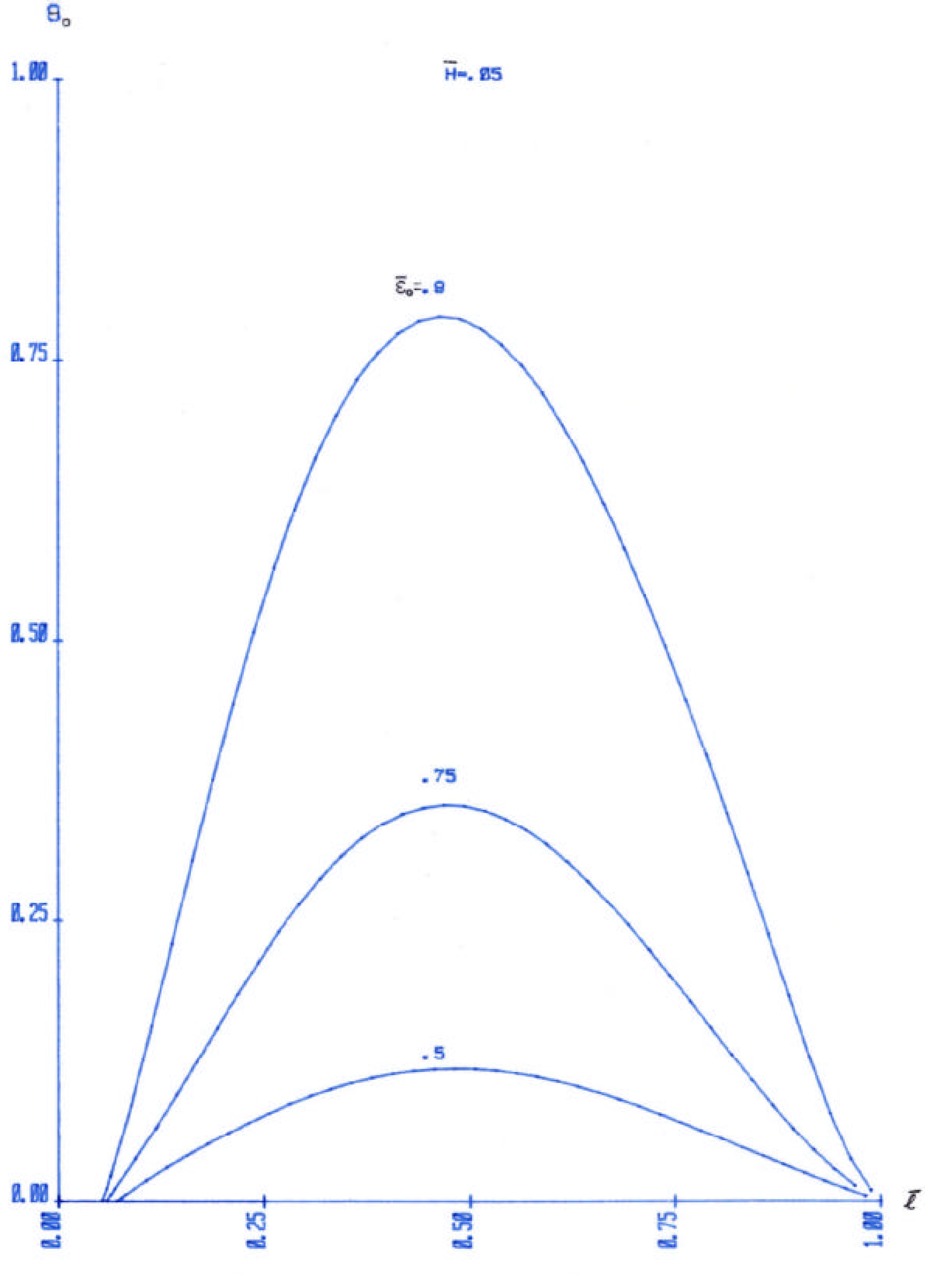


FIG. 3 THE DEPENDENCE OF ROTATION ANGLE θ_o ON LOADINGO

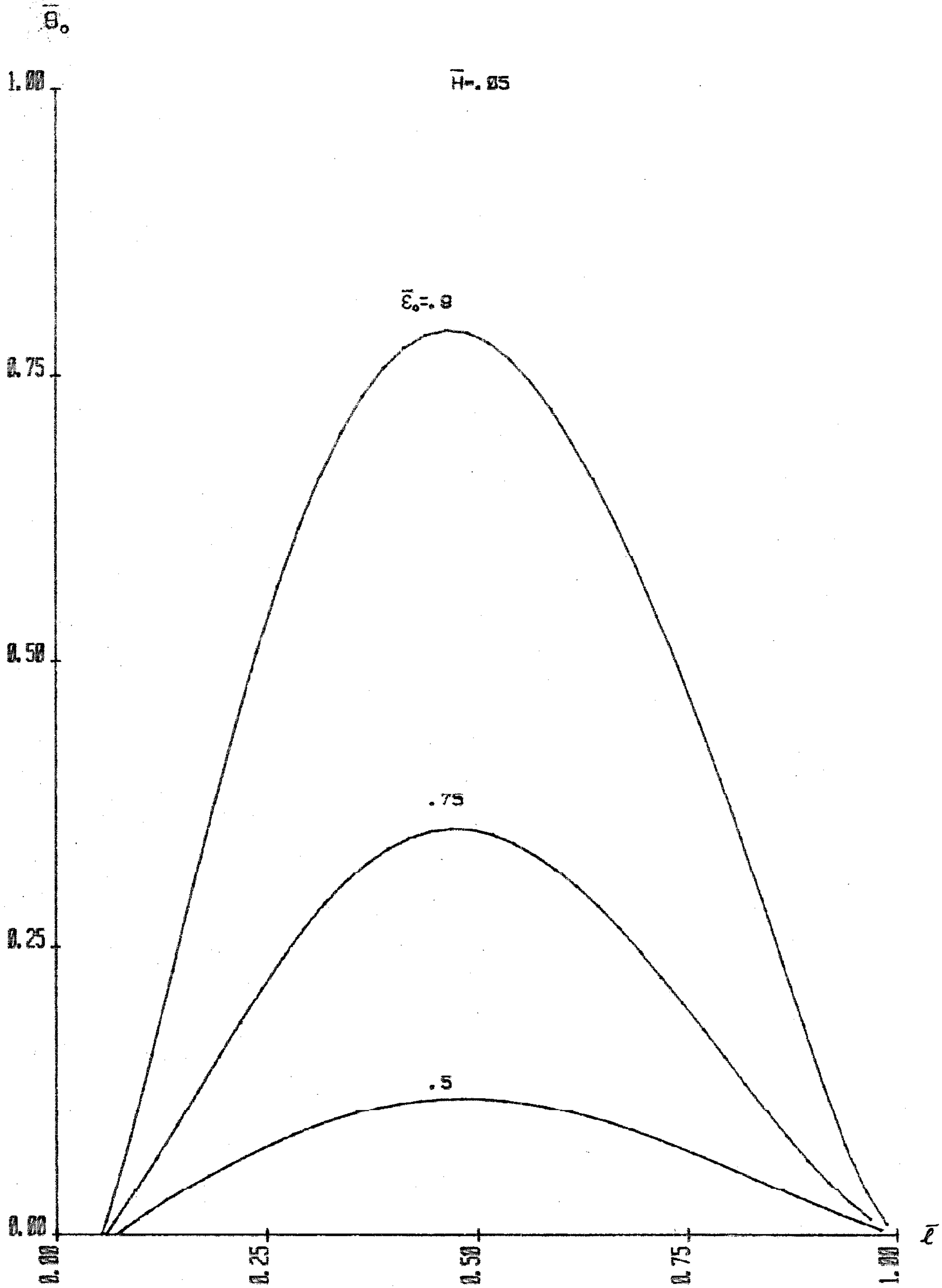
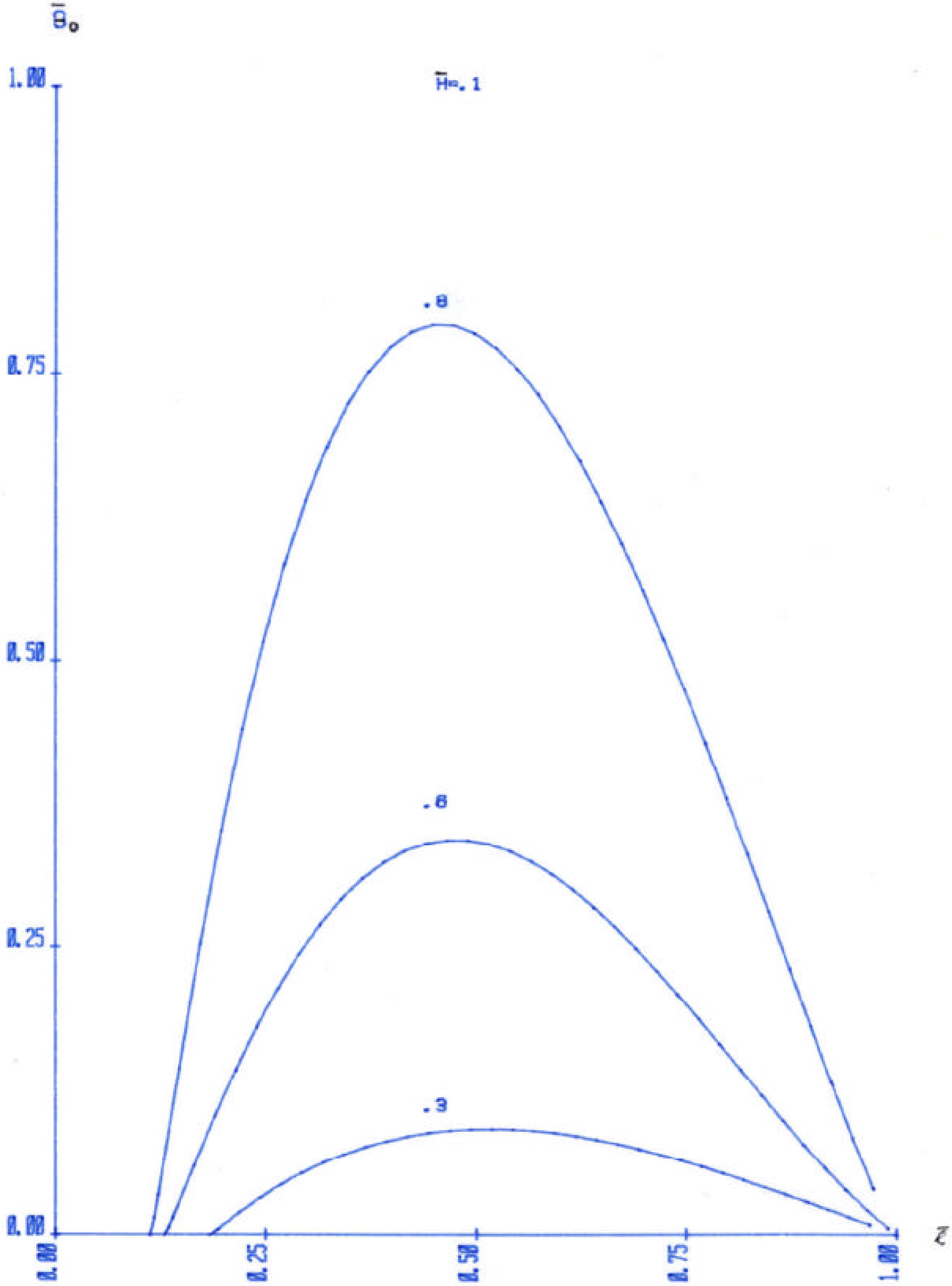
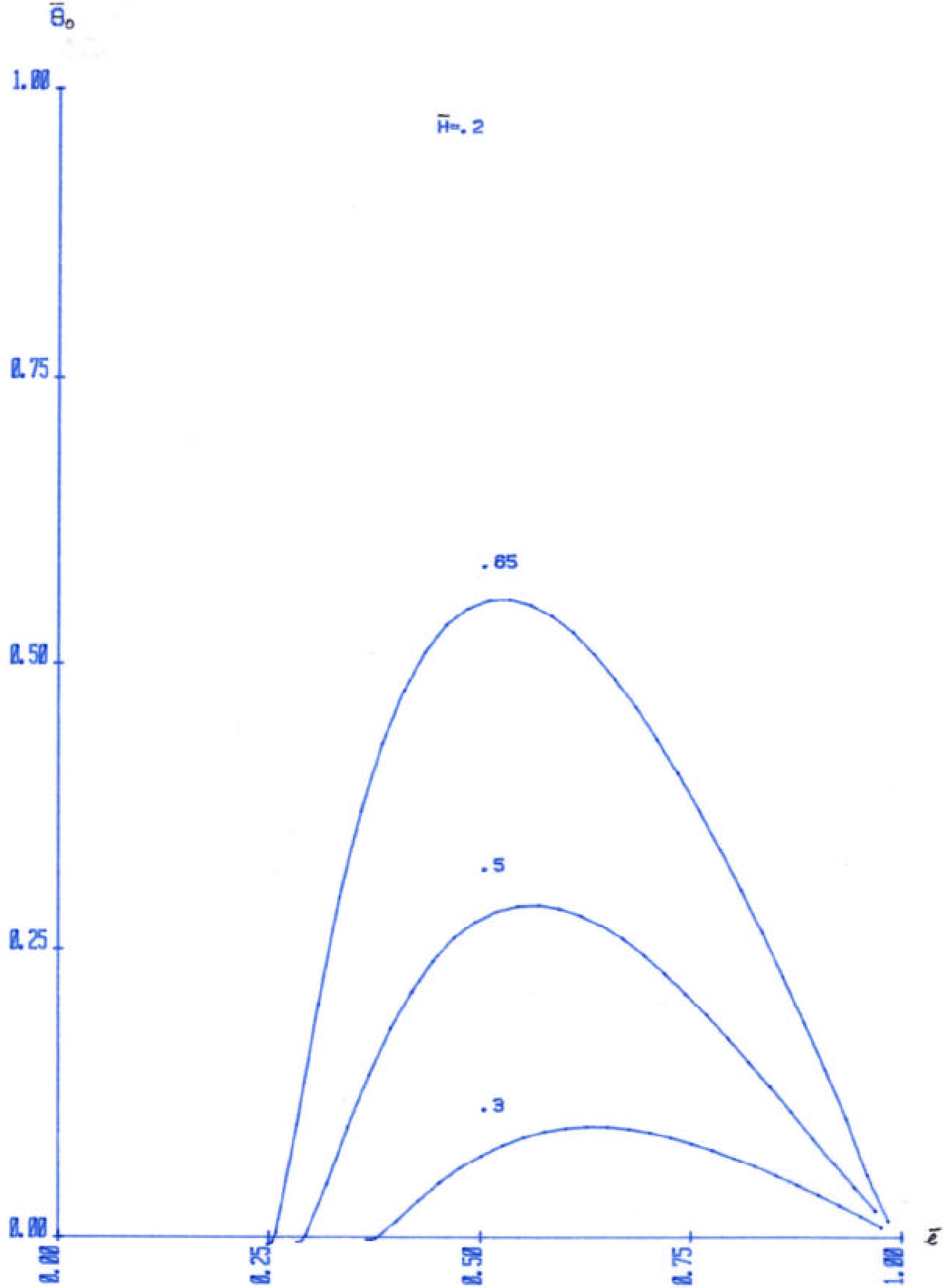


FIG. 3 THE DEPENDENCE OF ROTATION ANGLE Θ_0 ON LOADINGO



$\bar{\theta}_o$
1.00
0.75
0.50
0.25
0.00
$\bar{H}$=.1
.8
.6
.3
0.00
0.25
0.50
0.75
1.00
ζ

FIG. 3. CONT.



$\bar{\theta}_o$
1.00
0.75
0.50
0.25
0.00
$\bar{H}$=.2
.65
.5
.3
0.00
0.25
0.50
0.75
1.00
$\bar{\ell}$

FIG. 3. CONT.

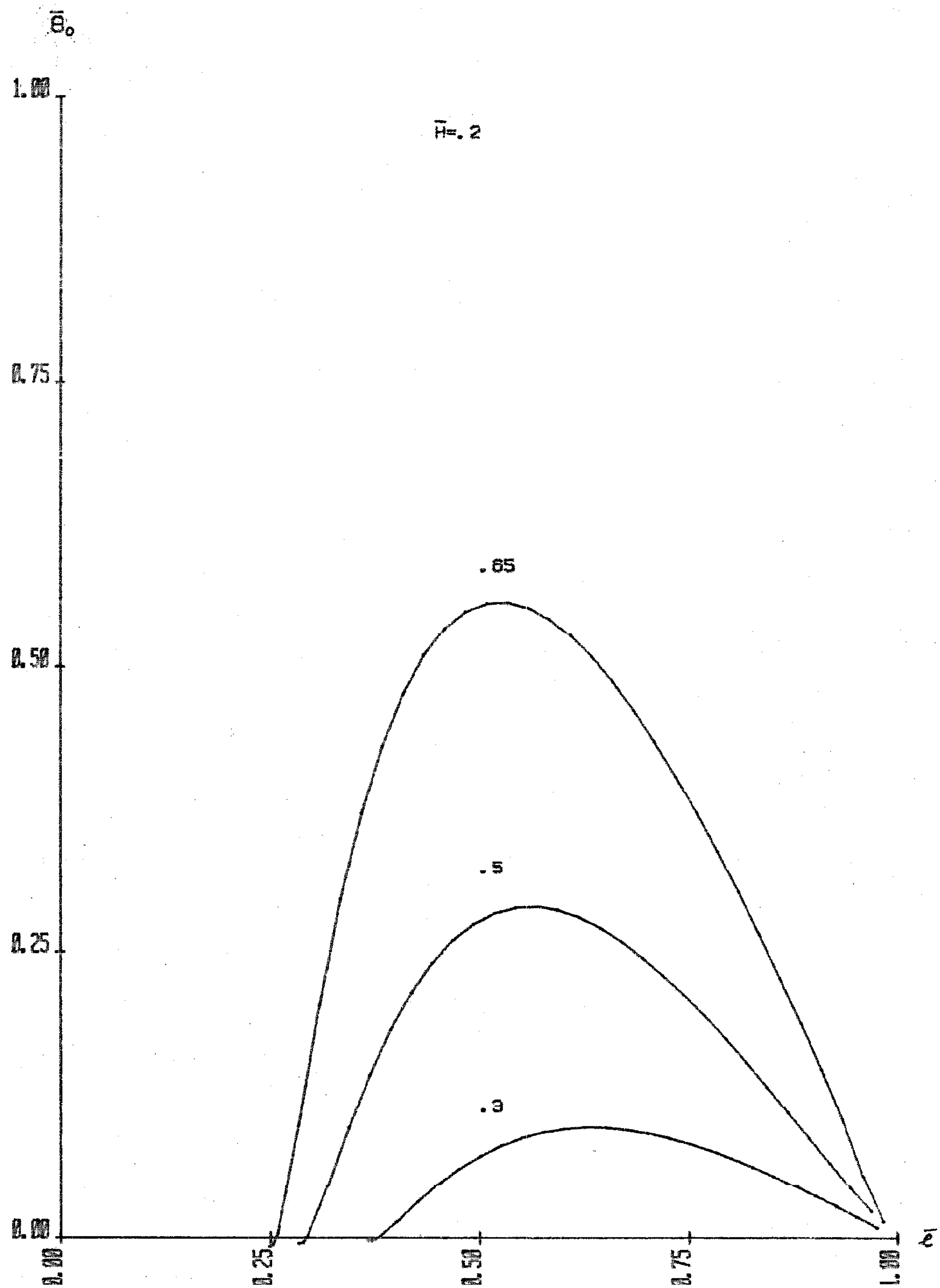
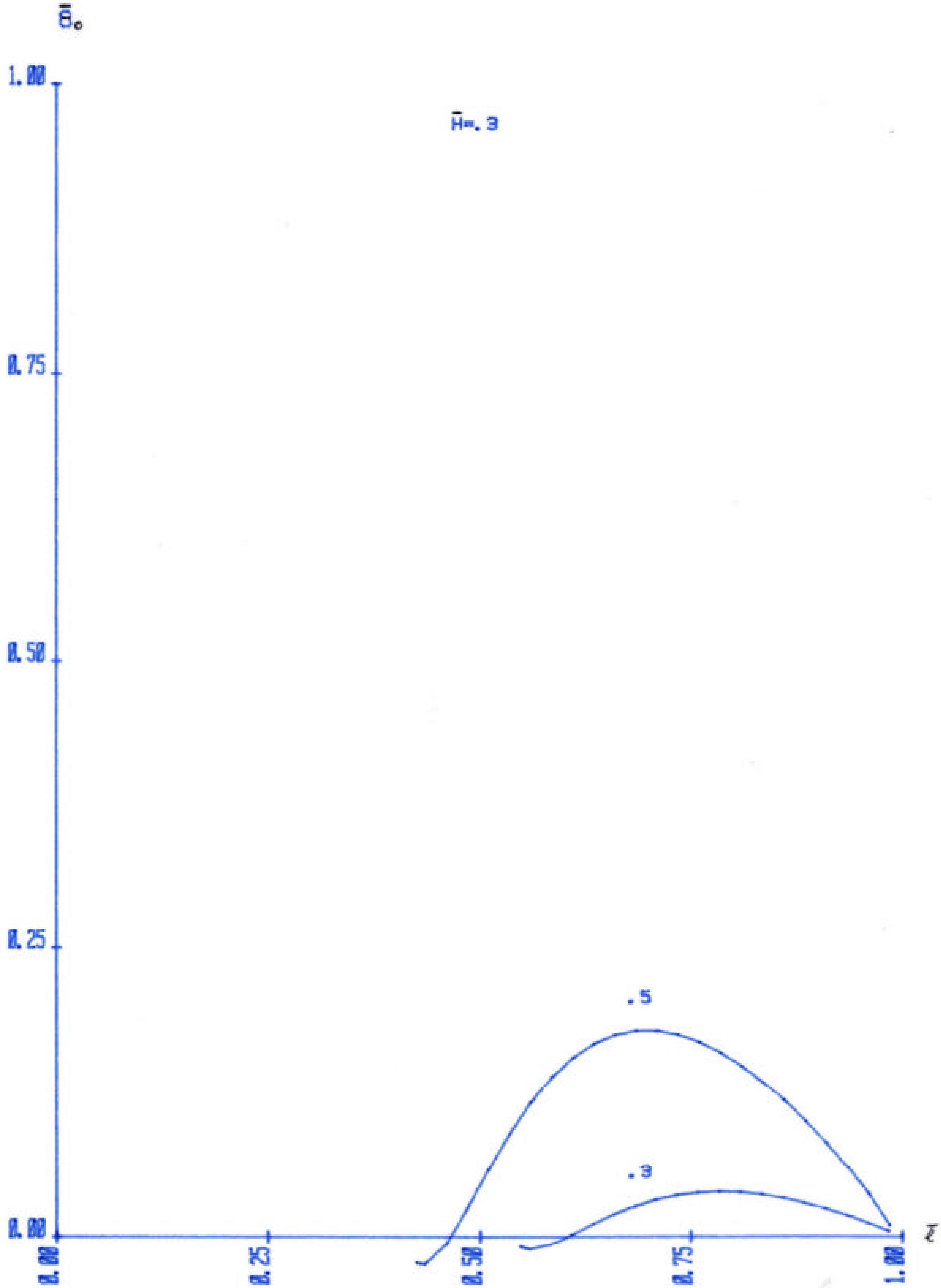
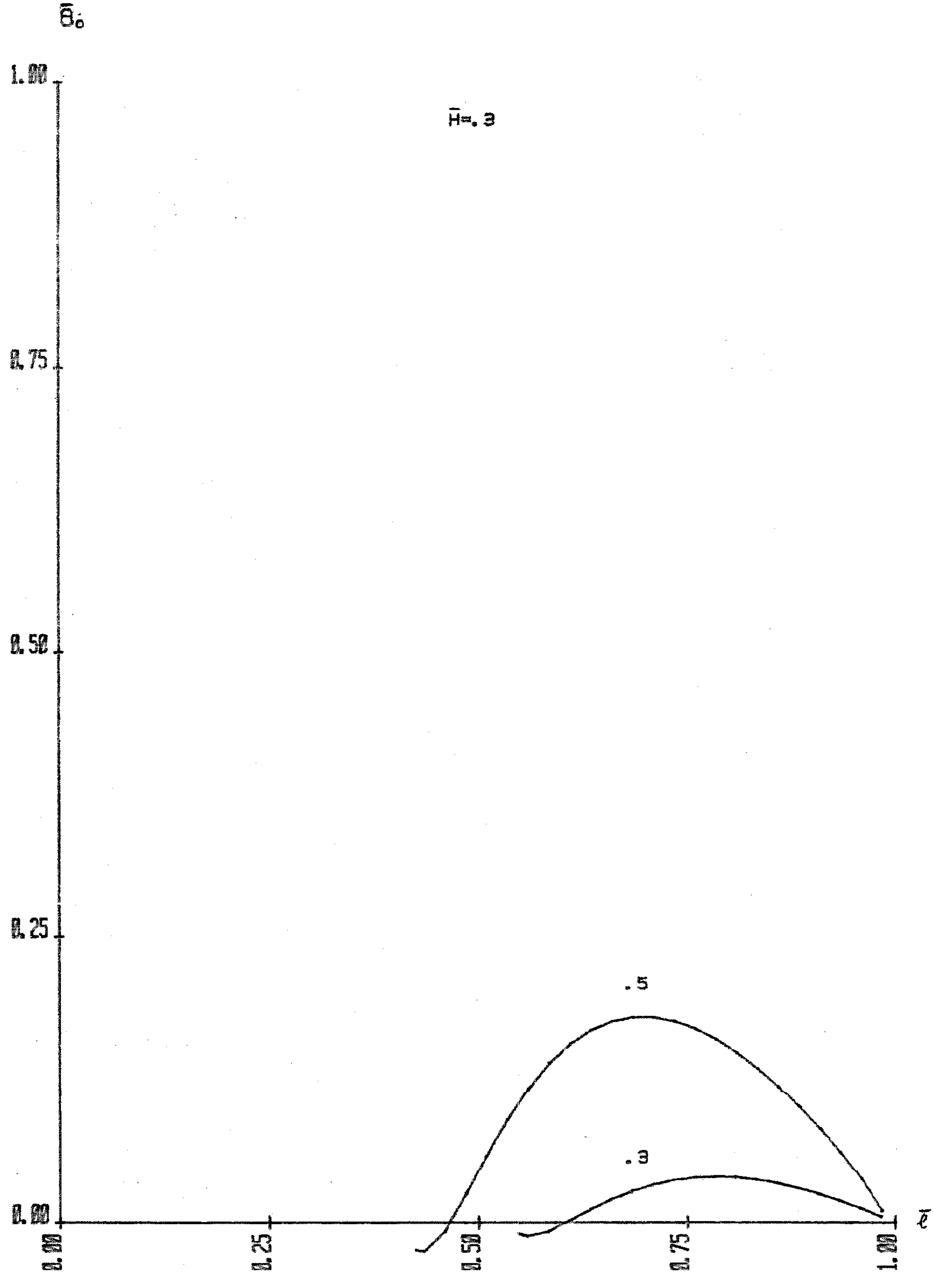


FIG. 3. CONT.



$\bar{\theta}_o$
1.00
0.75
0.50
0.25
0.00
$\bar{H}$=.3
.5
.3
0.00
0.25
0.50
0.75
1.00
$\bar{\ell}$

FIG. 3. CONT.



$\bar{\theta}_{\dot{o}}$
1.00
0.75
0.50
0.25
0.00
H̄=.3
.5
.3
0.00
0.25
0.50
0.75
1.00
$\bar{\ell}$

FIG. 3. CONT.

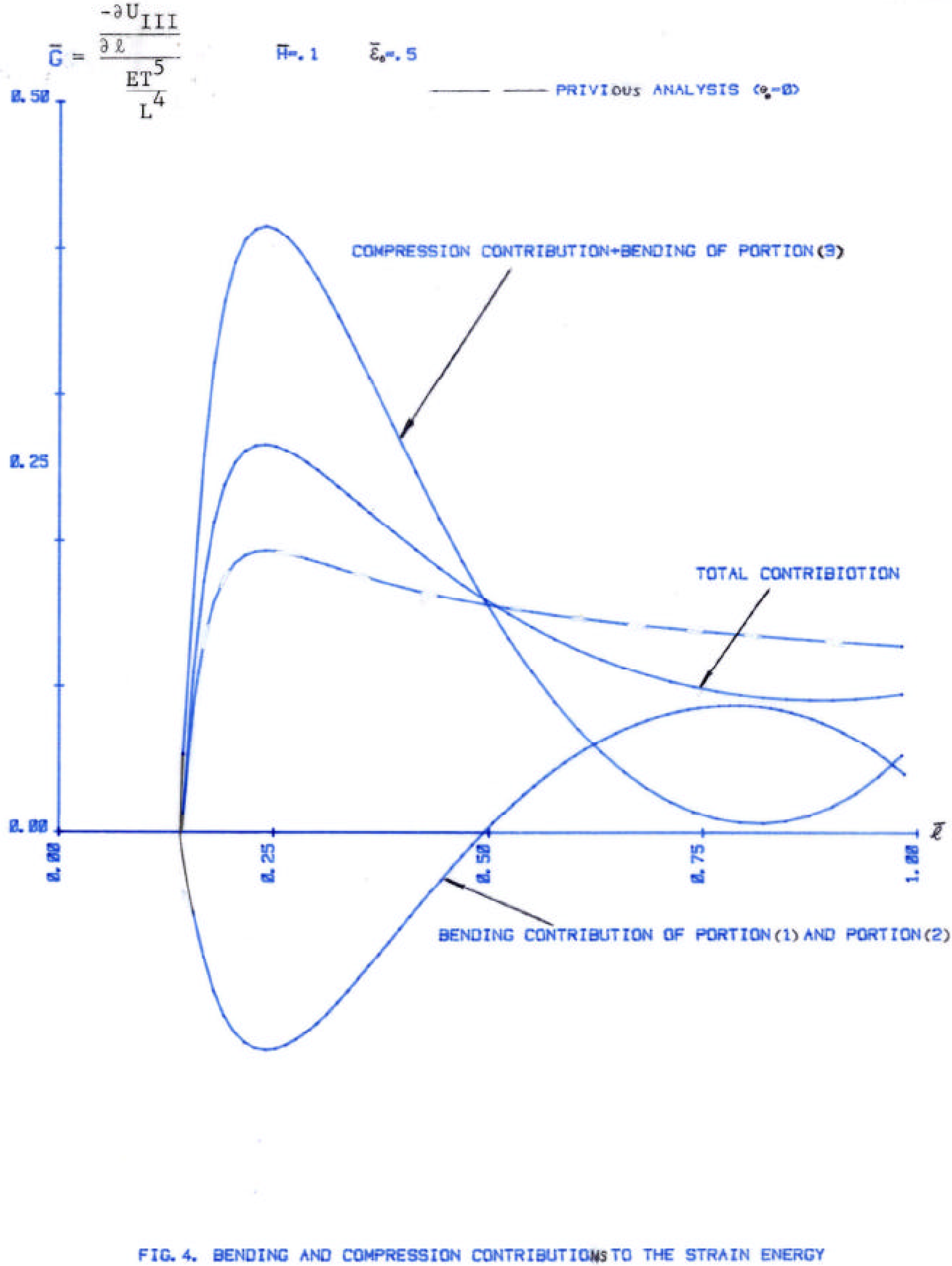


FIG. 4. BENDING AND COMPRESSION CONTRIBUTIONS TO THE STRAIN ENERGY RELEASE RATE

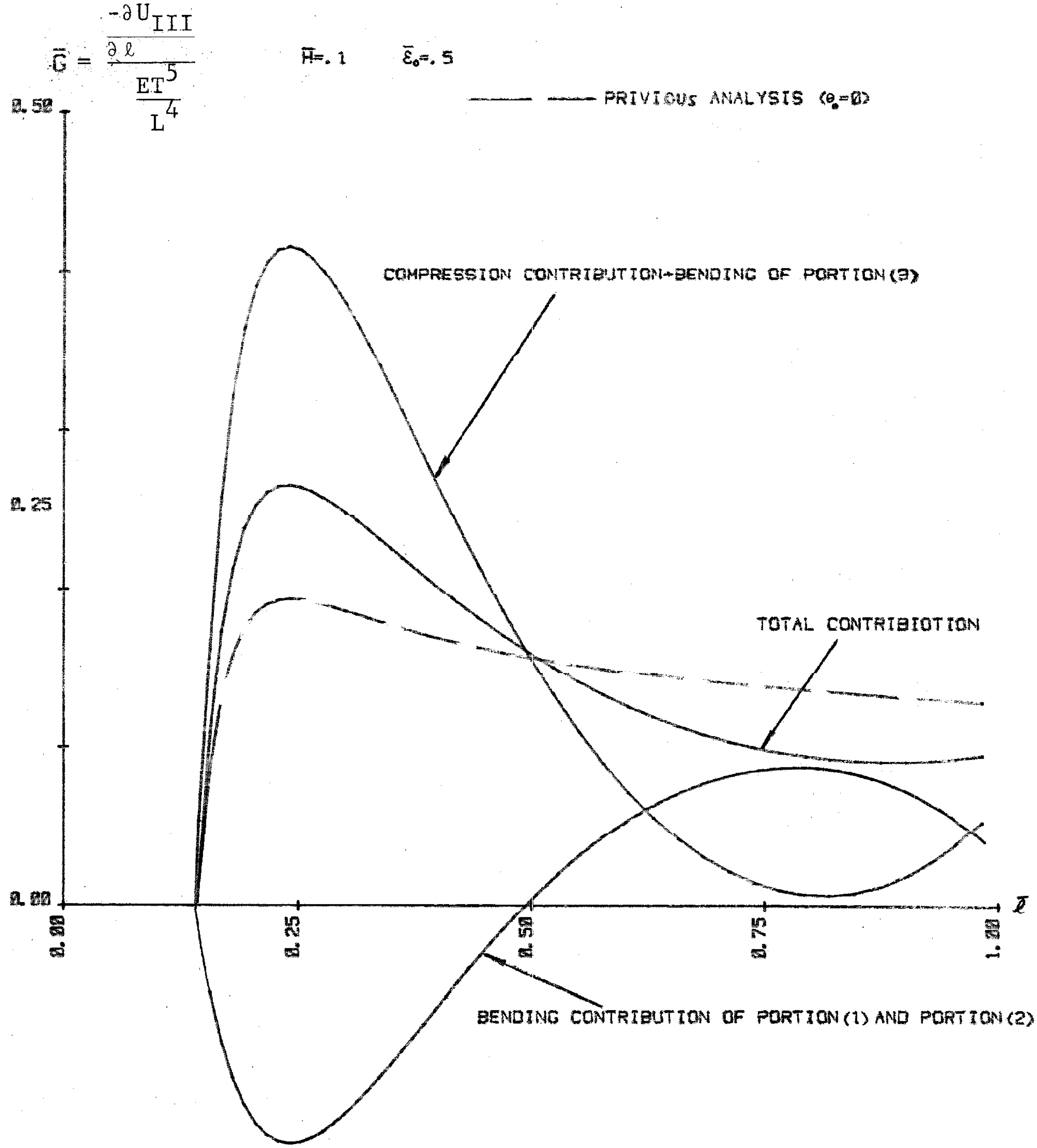


FIG. 4 BENDING AND COMPRESSION CONTRIBUTIONS TO THE STRAIN ENERGY RELEASE RATE

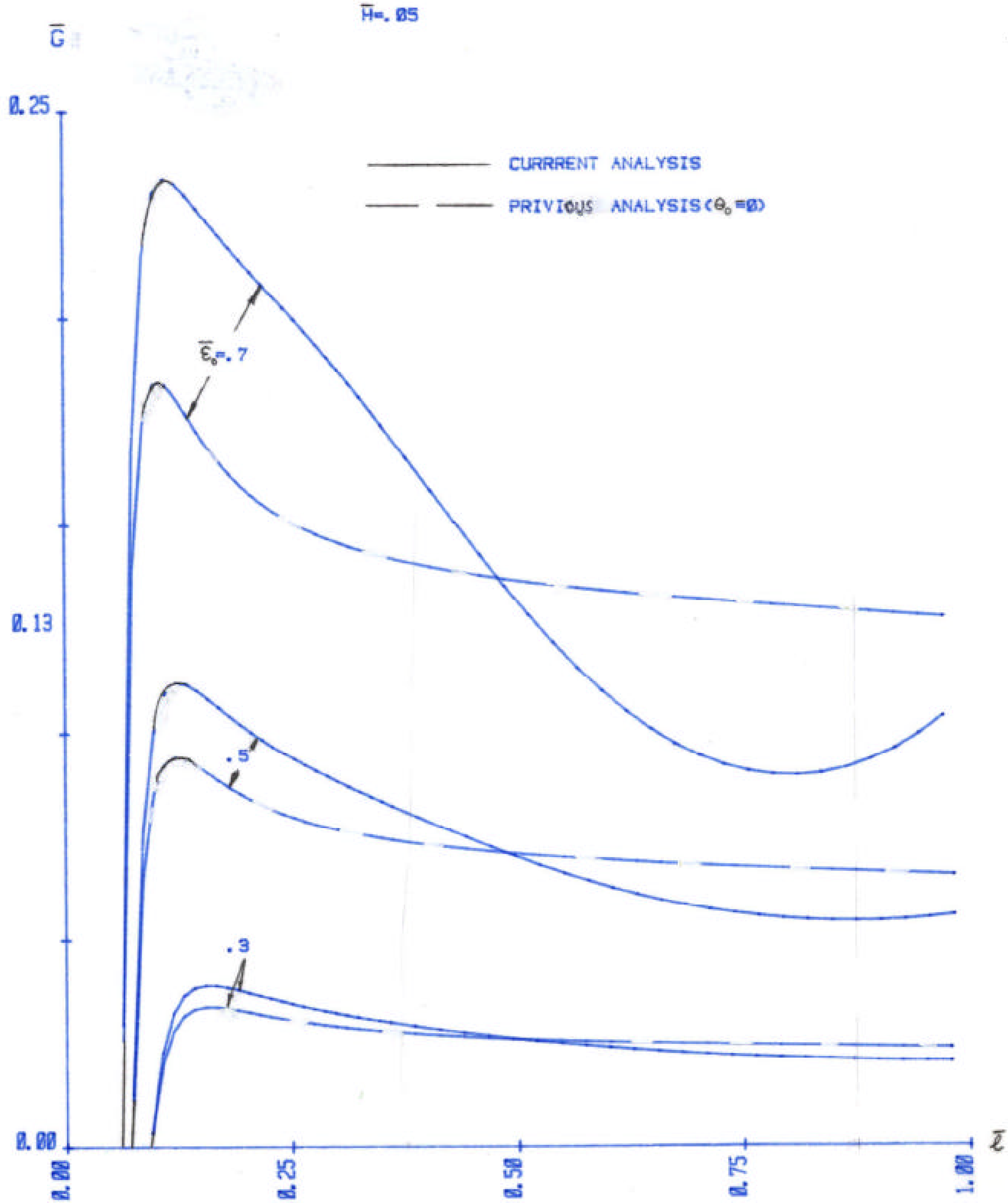


FIG. 5. DEPENDENCE OF STRAIN ENERGY RELEASE RATE ON LOADING

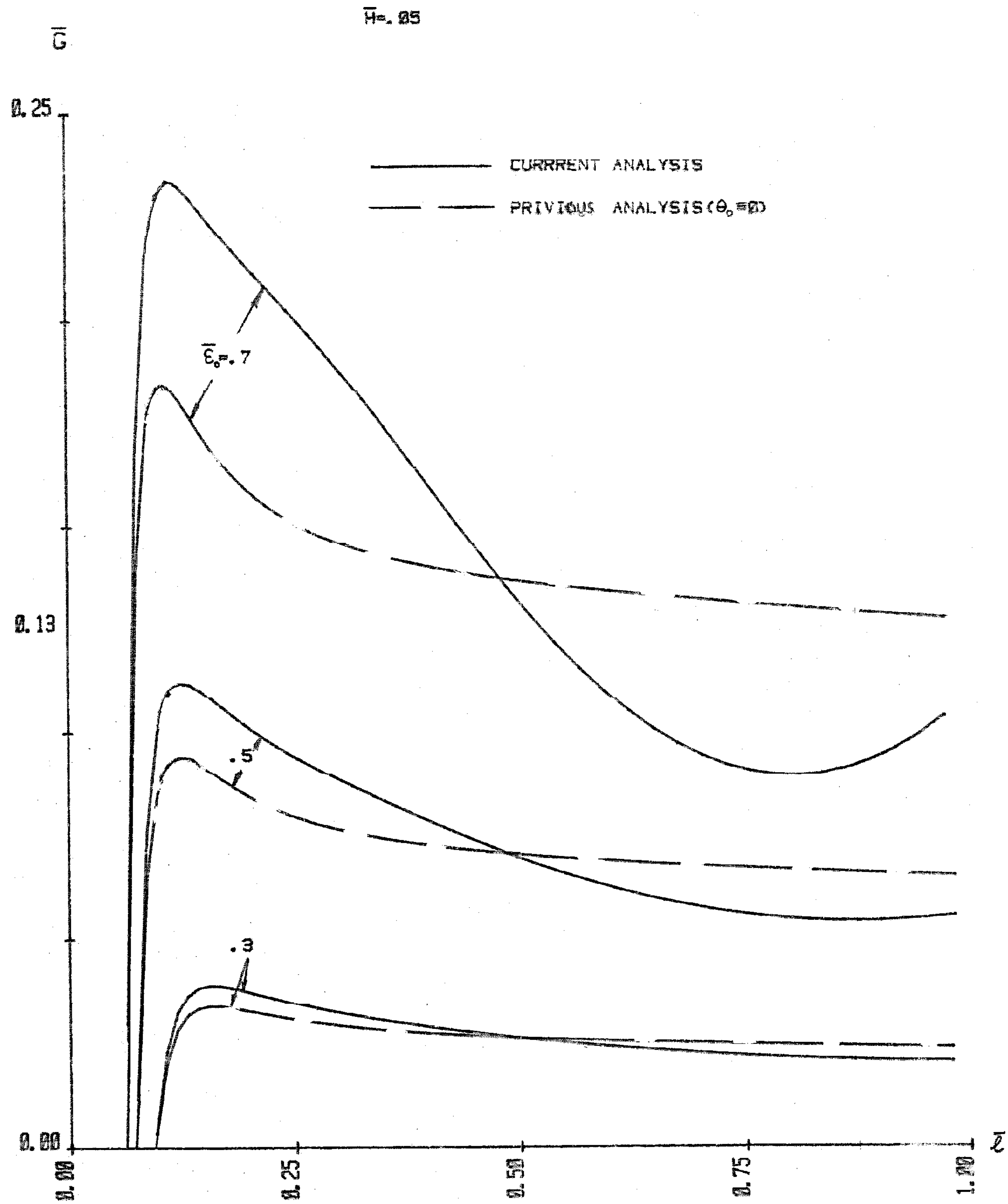
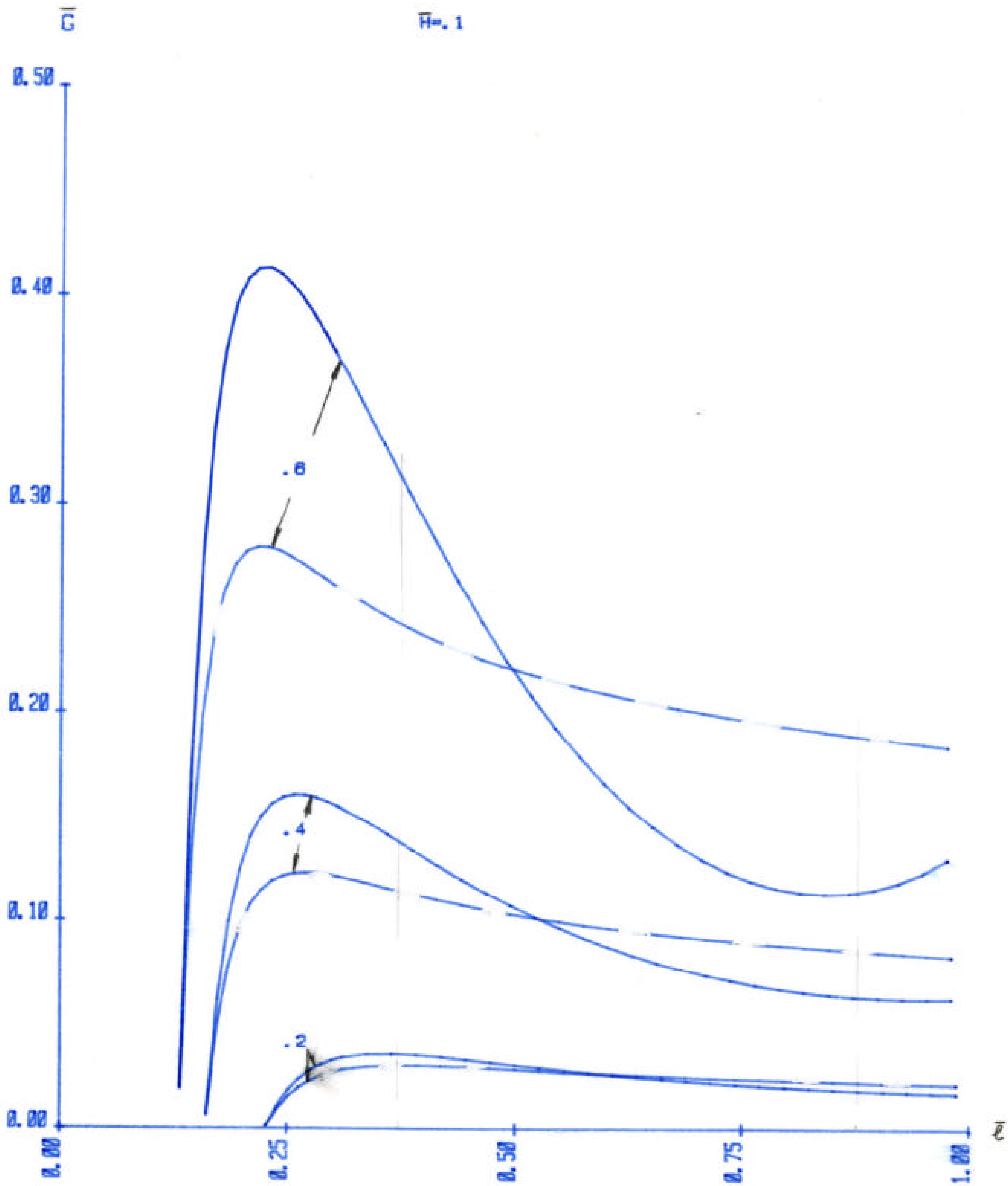


FIG. 5. DEPENDENCE OF STRAIN ENERGY RELEASE RATE ON LOADING



$\overline{G}$
$\overline{H}$=.1
0.50
0.40
0.30
0.20
0.10
0.00
.6
.4
.2
0.00
0.25
0.50
0.75
1.00
$\overline{\ell}$

FIG. 5. CONT.

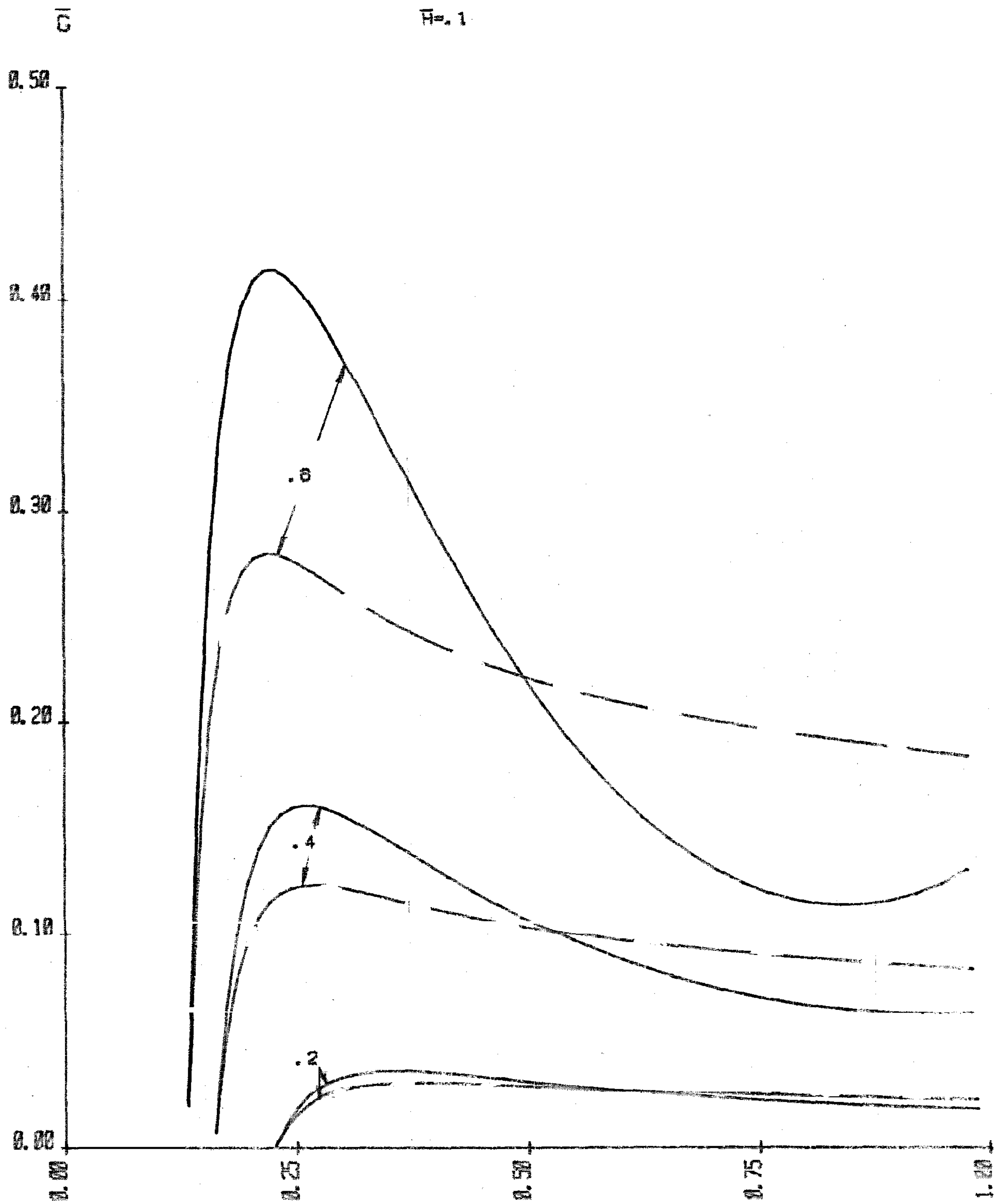


FIG. 5. CONT.

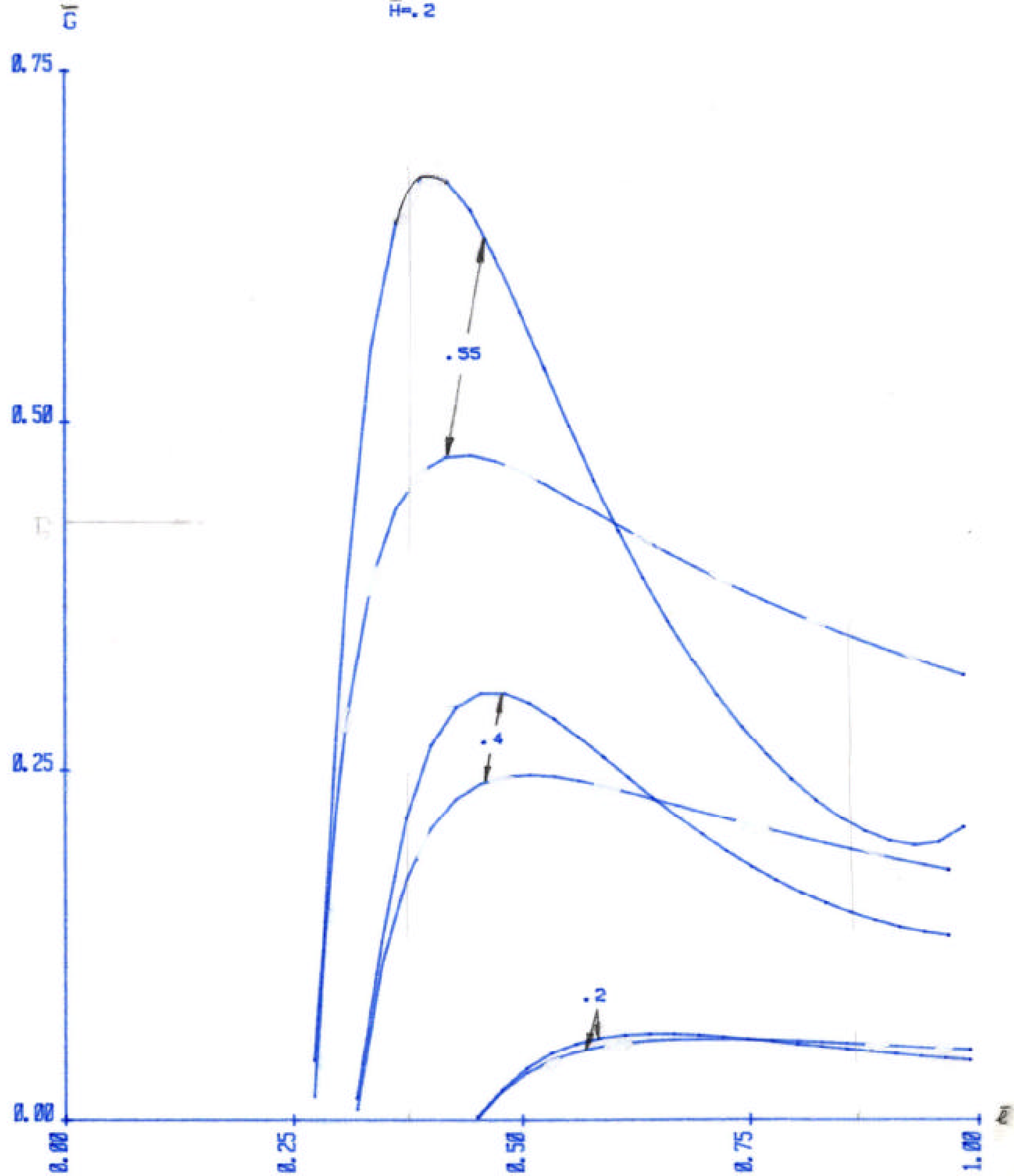


FIG. 5. CONT.

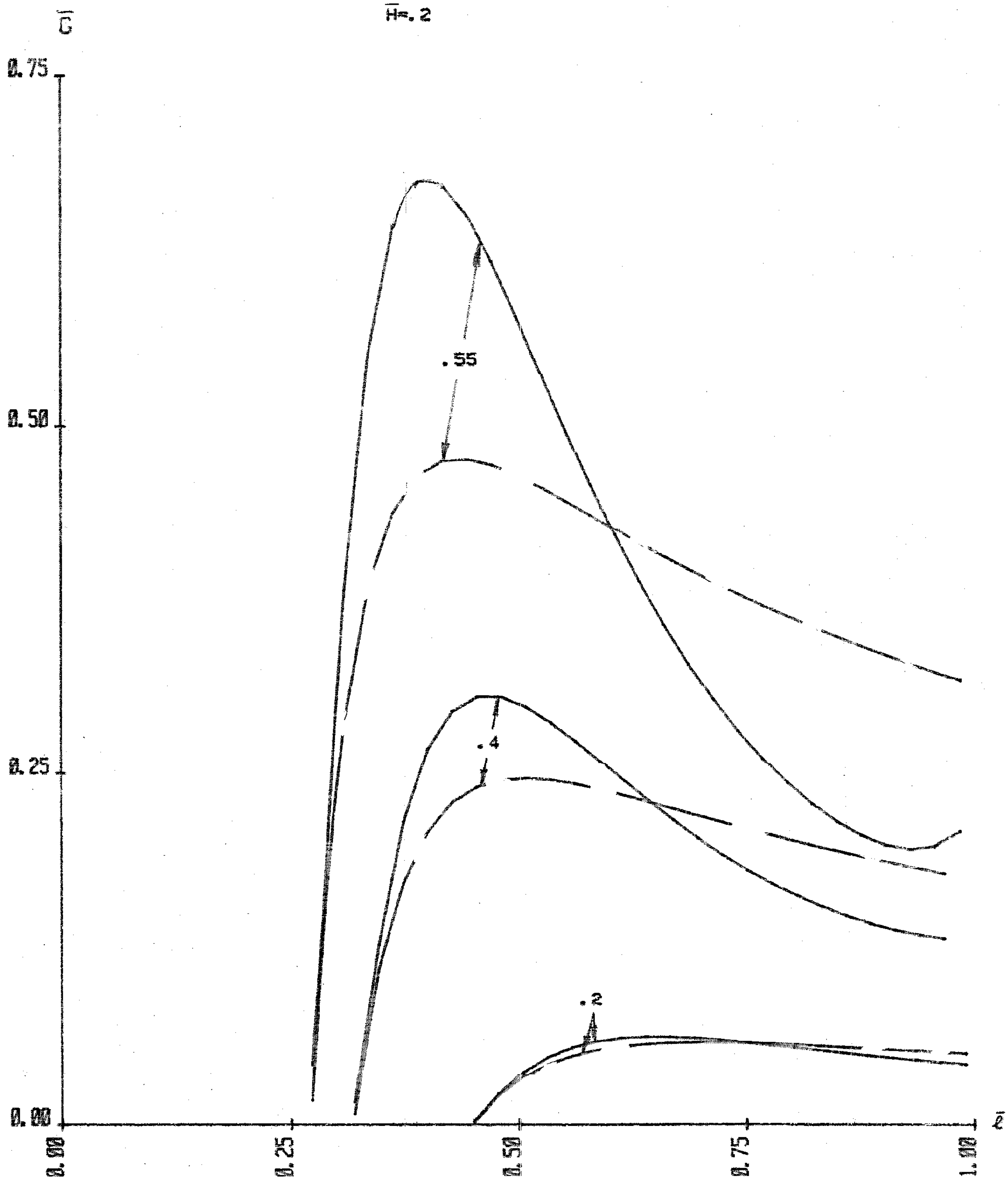


FIG. 5. CONT.

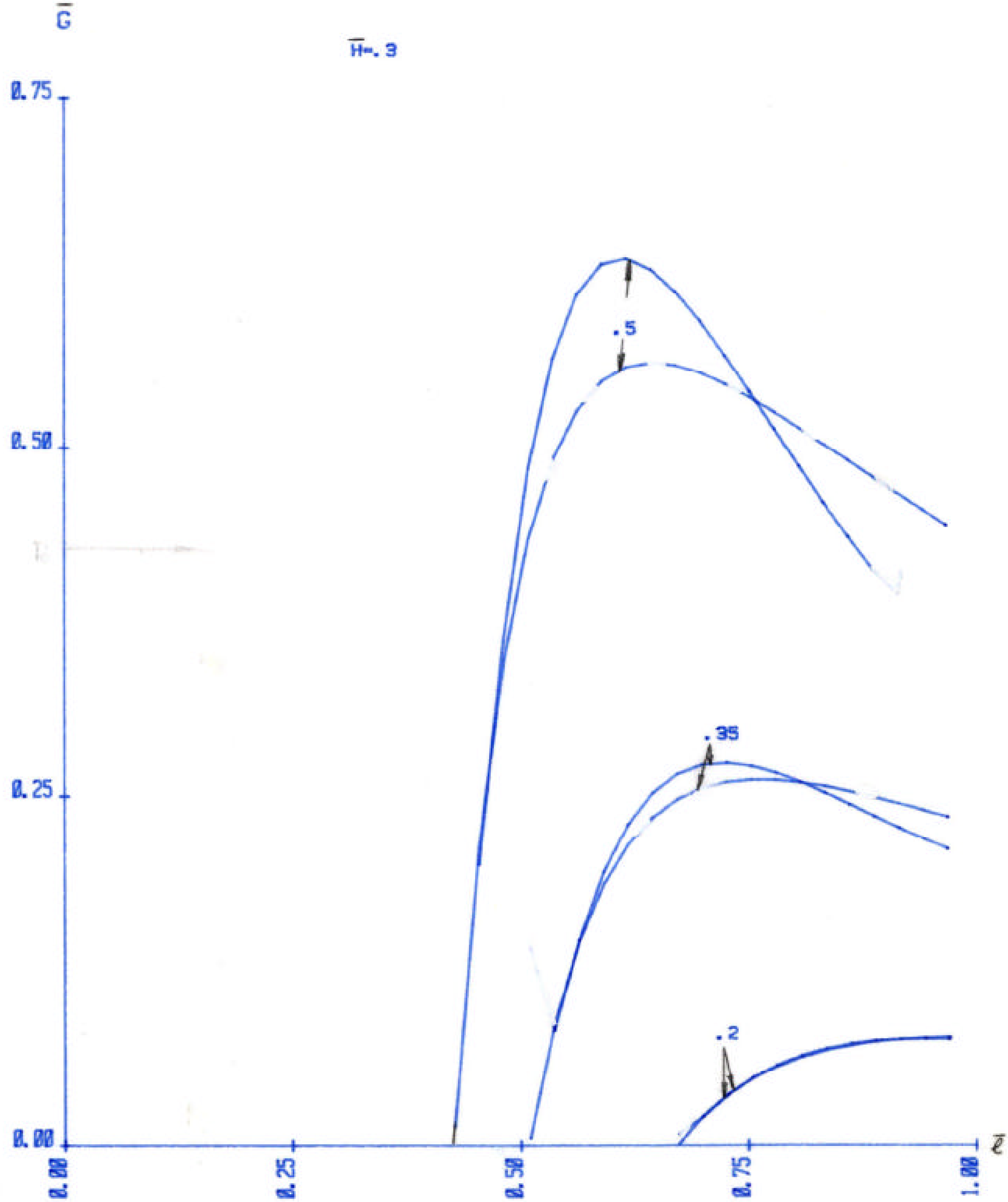
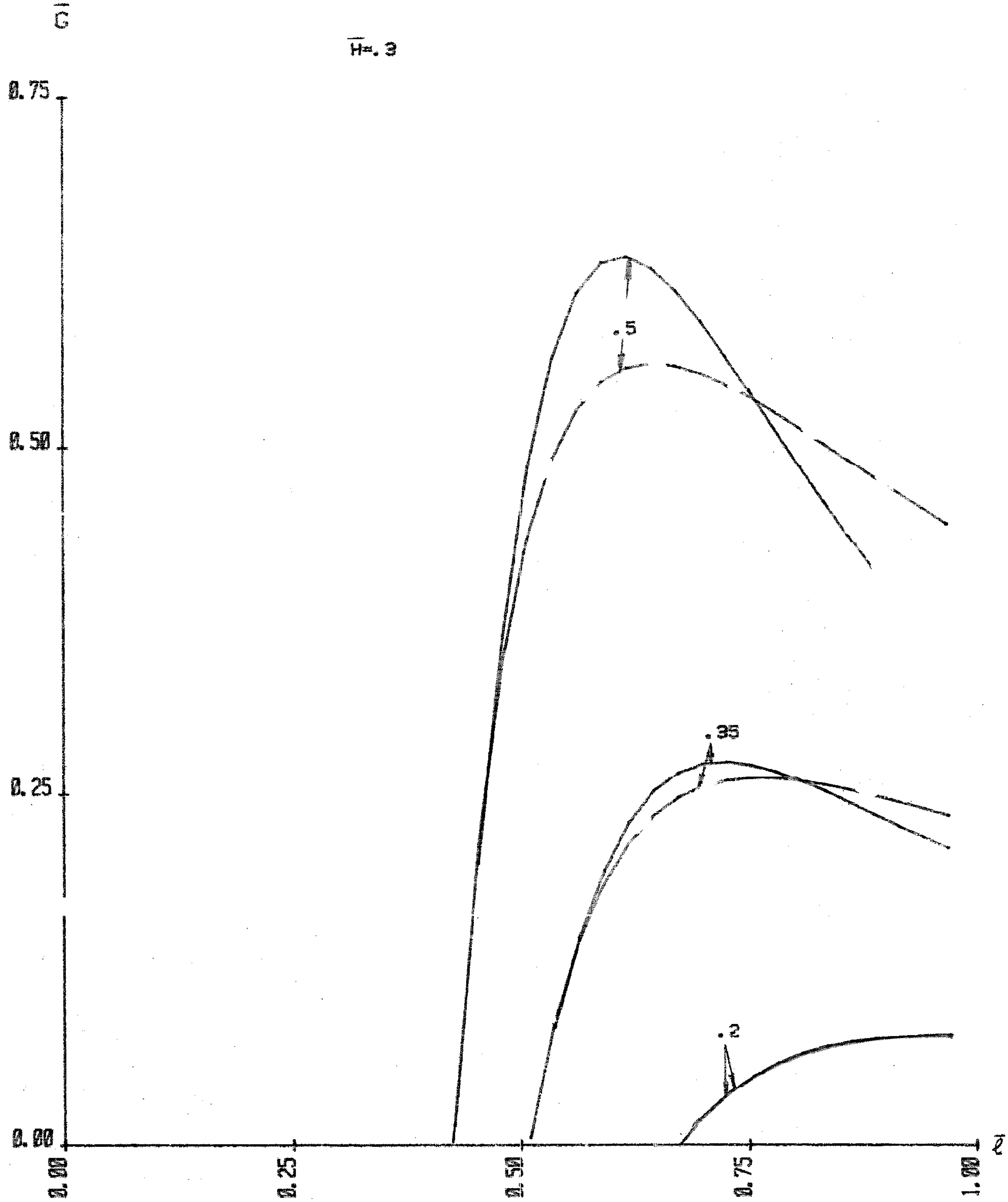


FIG. 5. CONT.



$\overline{G}$
$\overline{H}$=.3
0.75
0.50
0.25
0.00
.5
.35
.2
0.00
0.25
0.50
0.75
1.00
ℓ

FIG. 5. CONT.

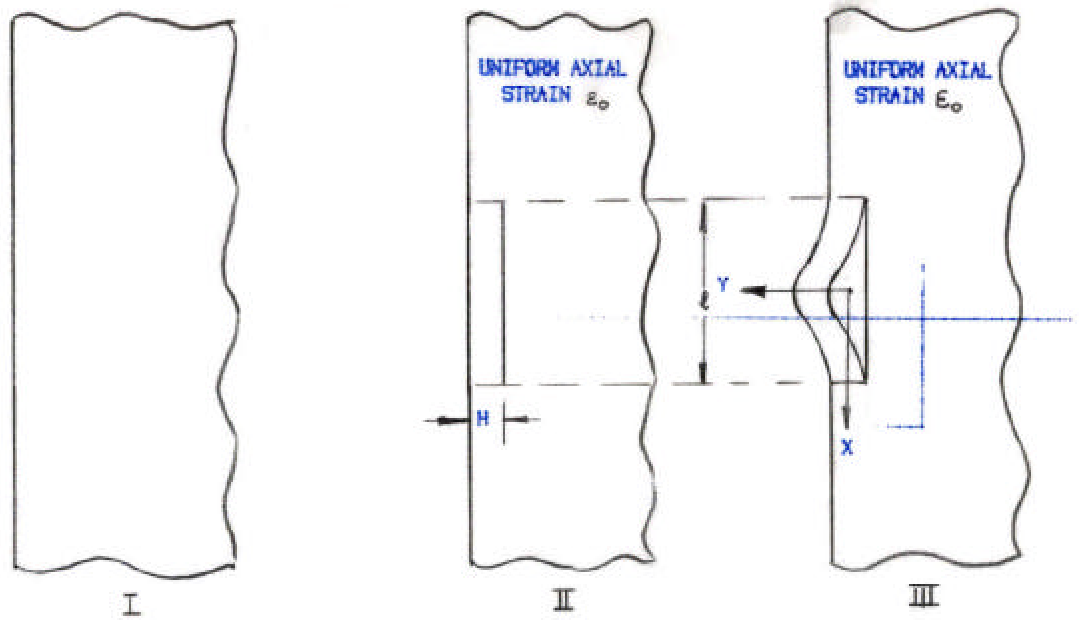


FIG. 6 THREE STATES IN THE DEFORMATION OF A COLUMN

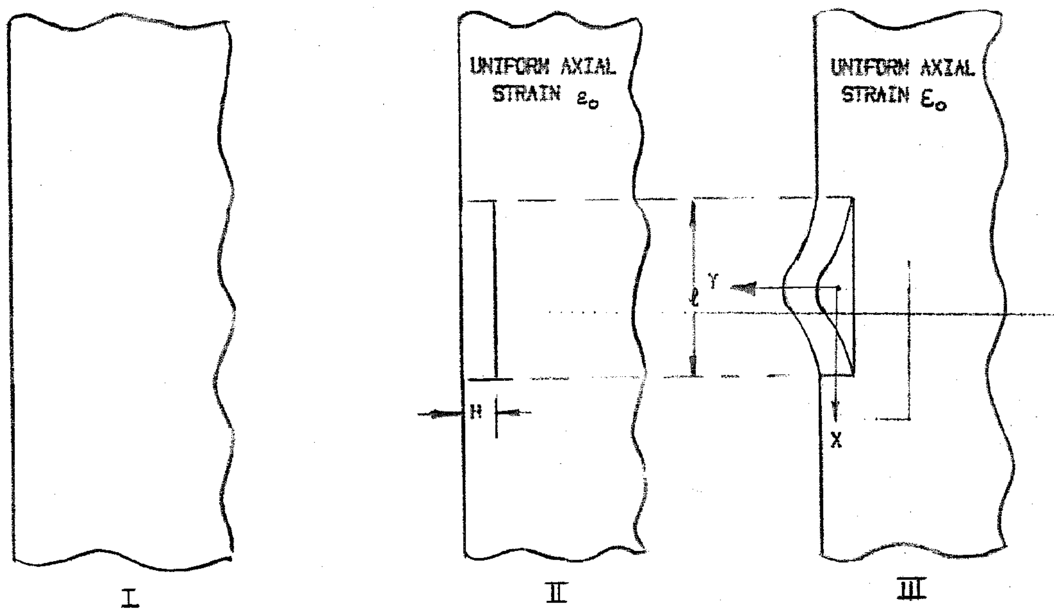


FIG. 6 THREE STATES IN THE DEFORMATION OF A COLUMN

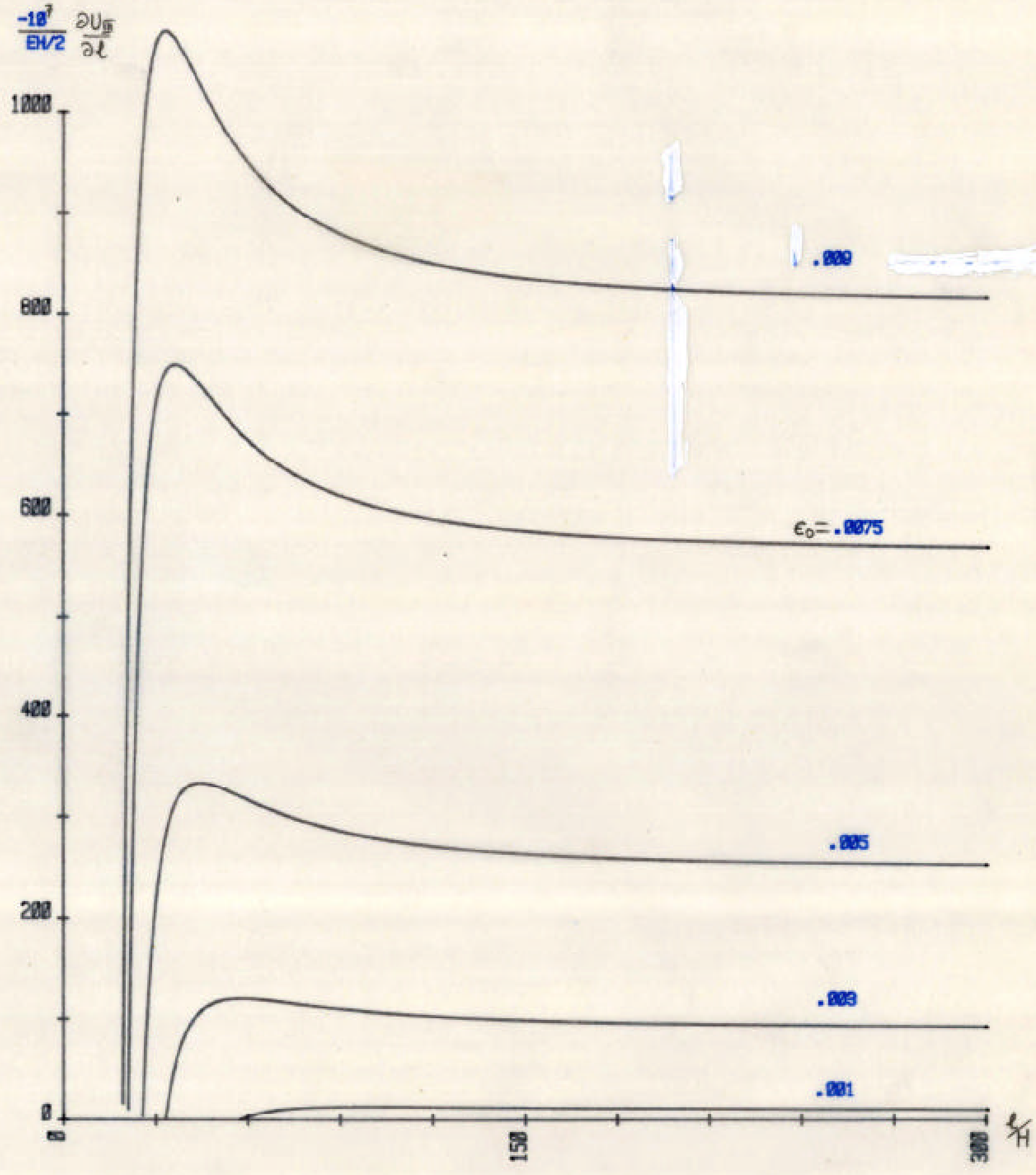


FIG. 7 STRAIN ENERGY REALEASE RATE OF A DELAMINATION ATTACHED TO A HALF SPACE

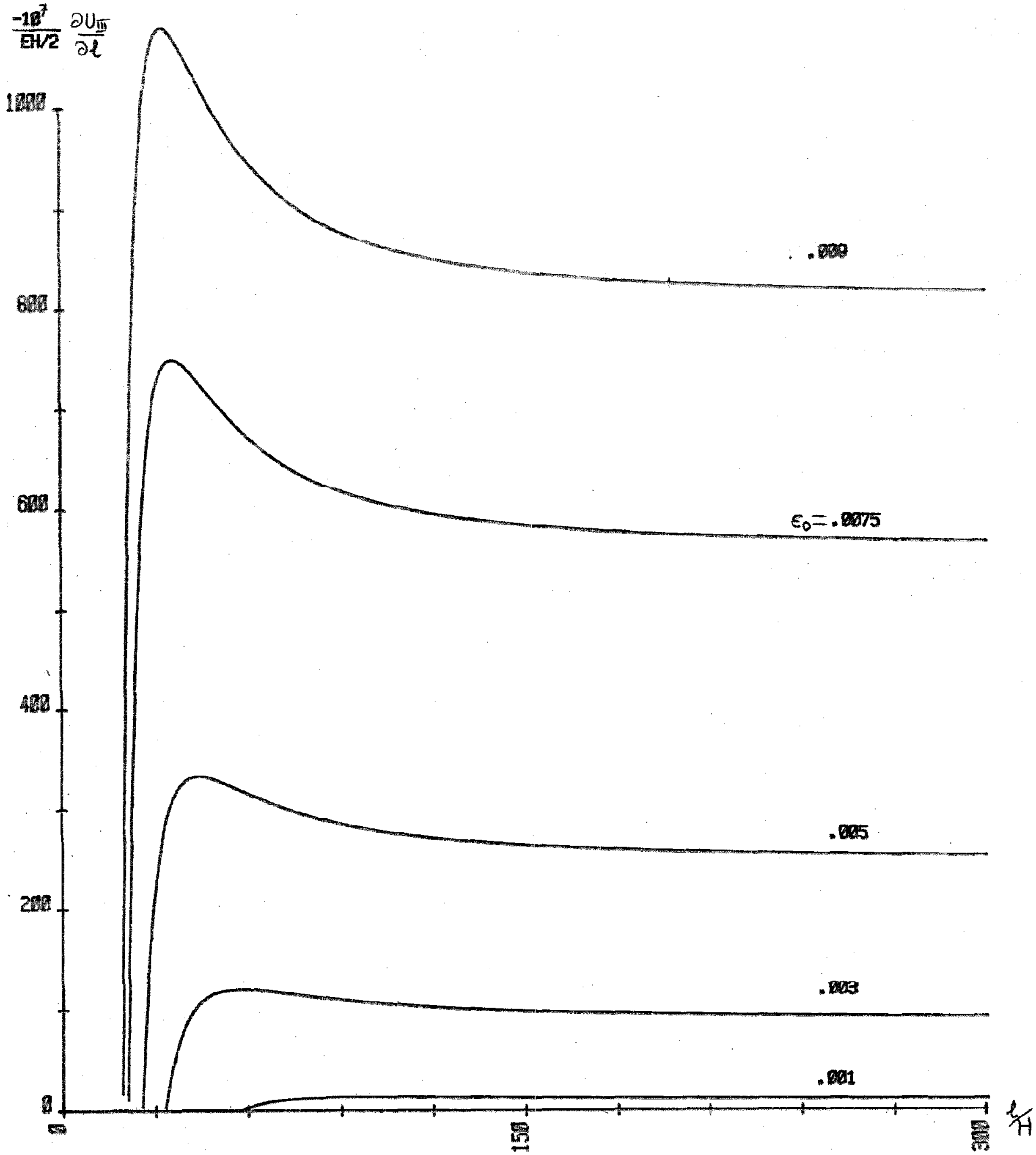


FIG. 7 STRAIN ENERGY REALEASE RATE OF A DELAMINATION ATTACHED TO A HALF SPACE

File 11

```
0: gto "six"
1: pclr
2: wrt 705,"VS4"
3: fxd 0
4: scl 0,300,0,
 1000;wrt 705,
 "VS2"
5: xax 0,30,0,
 300,5
6: yax 0,100,0,
 1000,2
7: "six":
8: ent E
9: .5→R
10: (π↑2/3/E)↑.5
 →L
11: -.45+L→L
12: 0→K
13: "one":
14: K+1→K
15: L+R→L
16: (E-(π/L)↑2/
 3)*(E+(π/L)↑2)→
 G
17: 10000200*G→G
18: if K>1;gto
 "ten"
19: plt L,G,1
20: "ten":
21: wrt 705,"VS2
 "
22: plt L,G,2
23: if L<300;
 gto "one"
*5880
```

File 11

```
0: gto "six"
1: pclr
2: wrt 705,"VS4"
3: fxd 0
4: scl 0,300,0,
 1000;wrt 705,
 "VS2"
5: xax 0,30,0,
 300,5
6: yax 0,100,0,
 1000,2
7: "six":
8: ent E
9: .5→R
10: (π↑2/3/E)↑.5
 →L
11: -.45+L→L
12: 0→K
13: "one":
14: K+1→K
15: L+R→L
16: (E-(π/L)↑2/
 3)*(E+(π/L)↑2)→
 G
17: 10000000*G→G
18: if K>1;gto
 "ten"
19: plt L,G,1
20: "ten":
21: wrt 705,"VS7
 "
22: plt L,G,2
23: if L<300;
 gto "one"
*5980
```

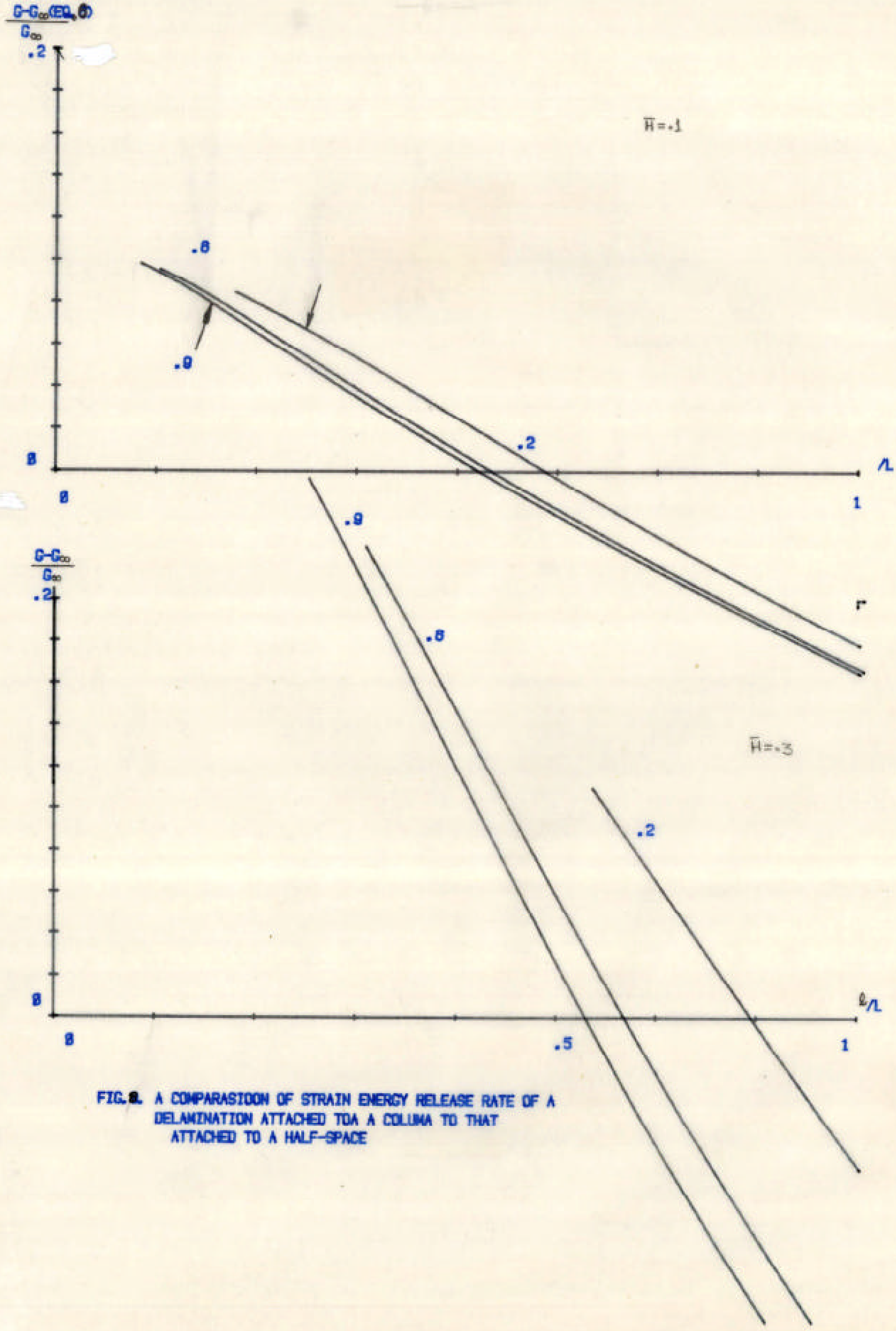


FIG.8. A COMPARASIOON OF STRAIN ENERGY RELEASE RATE OF A DELAMINATION ATTACHED TOA A COLUMA TO THAT ATTACHED TO A HALF-SPACE

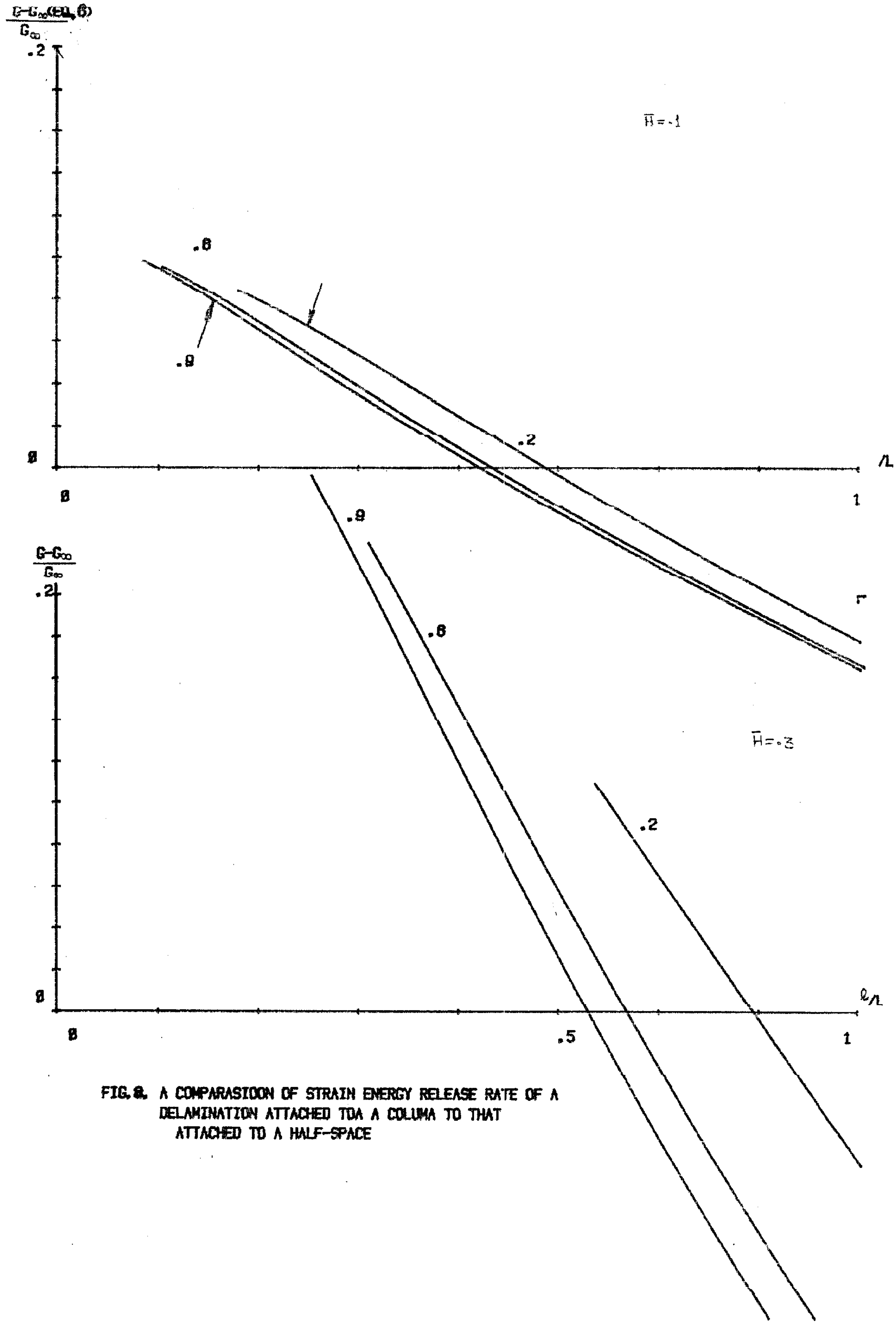


FIG.8. A COMPARASIOON OF STRAIN ENERGY RELEASE RATE OF A DELAMINATION ATTACHED TOA A COLUMA TO THAT ATTACHED TO A HALF-SPACE

```
0: gto "six"
1: pclr
2: fxd 2
3: wrt 705,"VS4"
4: scl 0,1,0,.2;
 wrt 705,"VS2"
5: wrt 705,"VS2"
6: xax 0,.125,0,
 1,20
7: yax 0,.02,0,
 .2,20
8: "six":
9: ent H
10: ent E
11: 0→R
12: (H↑2/E)↑.5→L
13: 0→K
14: "one":
15: K+1→K
16: R+.00005→R
17: L+R→L
18: 4*L/3/(1+1/
 3*E*(L/H)↑2)→G
19: G+1-H-L*L*H-
 2*L+2*H*L→G
20: G*H/(1-H+L*
 H)↑2→G
21: if K>1;gto
 "ten"
22: plt L,G,1
23: "ten":
24: wrt 705,"VS2
 "
25: plt L,G,2
26: if L>1;gto
 "six"
27: gto "one"
*20828
```

```
0: gto "six"
1: pclr
2: fxd 2
3: wrt 705,"VS4"
4: scl 0,1,0,.2;
 wrt 705,"VS2"
5: wrt 705,"VS2"
6: xax 0,.[illegible],0,
 1,20
7: yax 0,.02,0,
 .2,20
8: "six":
9: ent H
10: ent E
11: 0→R
12: (H↑2/E)↑.5→L
13: 0→K
14: "one":
15: K+1→K
16: R+.00005→R
17: L+R→L
18: 4*L/3/(1+1/
 3*E*(L/H)↑2)→G
19: G+1-H-L*L*H-
 2*L+2*H*L→G
20: G*H/(1-H+L*
 H)↑2→G
21: if K>1;gto
 "ten"
22: plt L,G,1
23: "ten":
24: wrt 705,"VS2
 "
25: plt L,0,2
26: if L>1;gto
 "six"
27: gto "one"
*20828
```

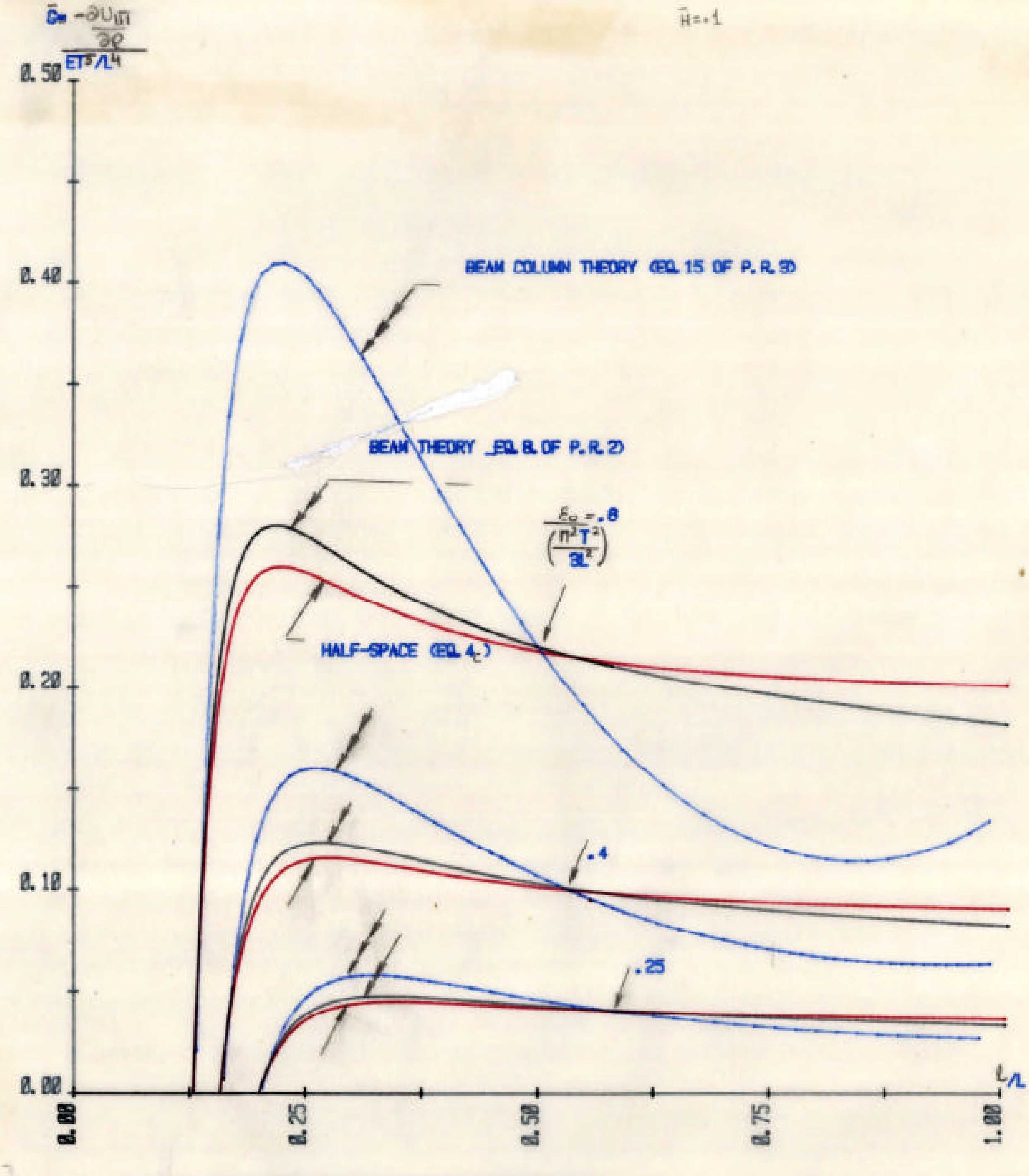


FIG. 9A COMPARASION OF STRAIN ENERGY RELEASE RATE RESULTED USING THREE DIFFERENT APPROACHED

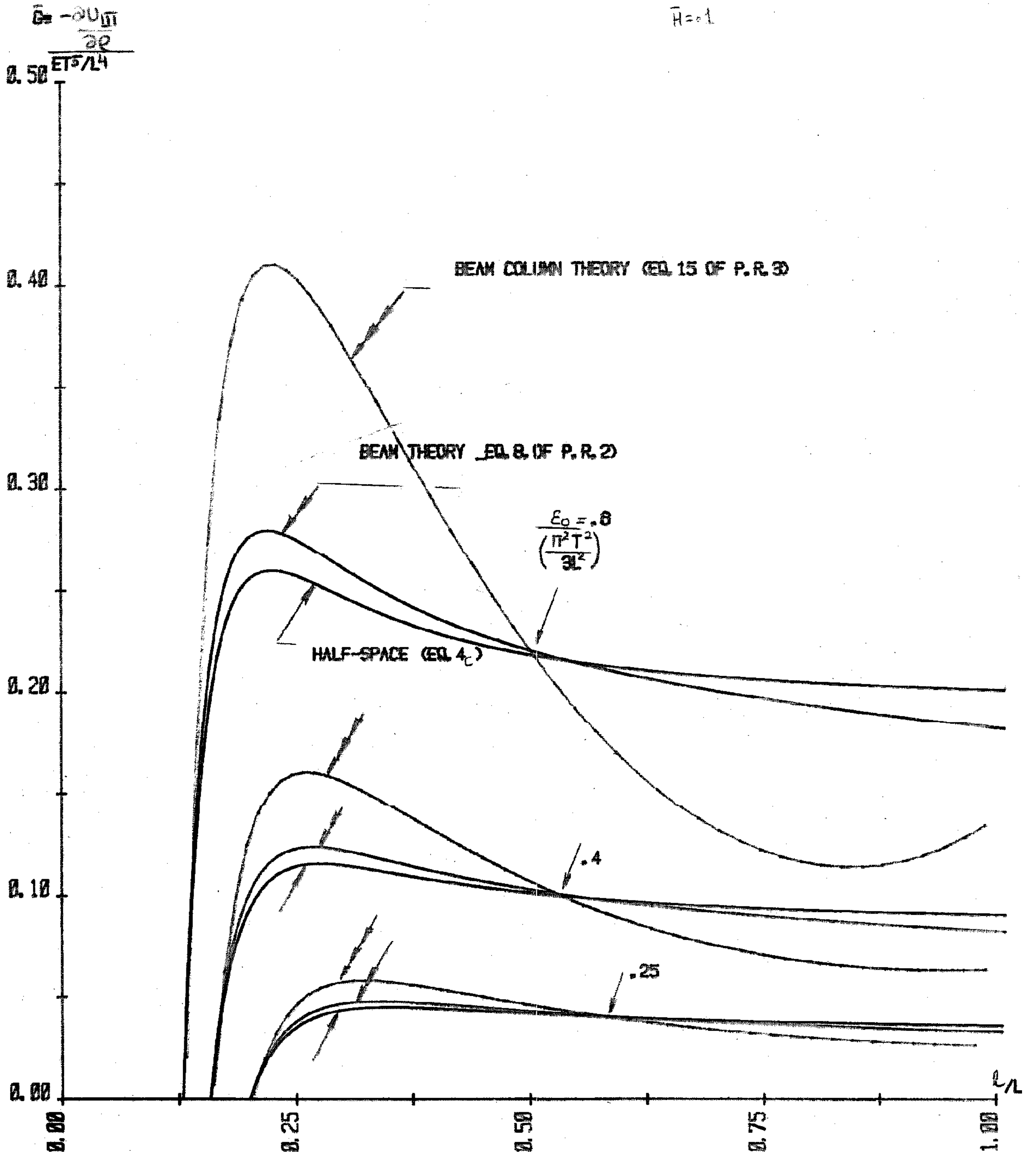


FIG. 9A COMPARASION OF STRAIN ENERGY RELEASE RATE RESULTED USING THREE DIFFERENT APPROACHED case of P.R. No 4

```
0: gto "six"
1: pclr
2: fxd 2
3: wrt 705,"VS4"
4: scl 0,1,0,.5;
 wrt 705,"VS2"
5: wrt 705,"VS2"
6: xax 0,.125,0,
 1,2
7: yax 0,.05,0,
 .5,2
8: "six":
9: ent H
10: ent E
11: 0→R
12: (H↑2/E)↑.5→L
13: 0→K
14: "one":
15: K+1→K
16: R+.00005→R
17: L+R→L
18: π↑4/18*H→G
19: (E-(H/L)↑2)*
 (E+3*H*H/L/L)*
 G→G
20: if K>1;gto
 "ten"
21: plt L,G,1
22: "ten":
23: (4/(1-L)↑2*
 (1+N*H*L-N*H)-
 N*H↑3/L)/(1-N*
 H)→Z
24: wrt 705,"VS2
 "
25: if E>(1/L)↑2
 *((1-N*H)↑2*(1-
 N*H+L*N*H)+(1-
 L)*N*H↑3);pen#
 2;line 2,1;gto
 29
26: 1+N*H*L/(1-
 N*H)-H↑3*N/L/
 (1-N*H)→S;if
 E>S;line 1,.5;
 pen# 3
27: if E<S;pen#
 1;line
28: if E>Z;pen#
 4
29: plt L,G,2
30: if L>1;gto
 "six"
31: gto "one"
*27474
```

case of P.R. No2

```
0: gto "six"
1: pclr
2: fxd 2
3: wrt 705,"VS4"
4: scl 0,1,0,.5;
 wrt 705,"VS2"
5: wrt 705,"VS2"
6: xax 0,.125,0,
 1,2
7: yax 0,.05,0,
 .5,2
8: "six":
9: ent N
10: ent H
11: ent E
12: 0→R
13: (H↑2/E)↑.5→L
14: 0→K
15: "one":
16: K+1→K
17: R+.00005→R
18: L+R→L
19: π↑4/18*N*H*
 (1-N*H)/(N*H*L+
 1-N*H)↑2→G
20: (E-(H/L)↑2)*
 (E+3*H*H/L/L*
 (1+4/3*N*H*L/
 (1-N*H)))*G→G
21: G/N→G
22: if K>1;gto
 "ten"
23: plt L,G,1
24: "ten":
25: (4/(1-L)↑2*
 (1+N*H*L-N*H)-
 N*H↑3/L)/(1-N*
 H)→Z
26: wrt 705,"VS2
 "
27: if E>(1/L)↑2
 *((1-N*H)↑2*(1-
 N*H+L*N*H)+(1-
 L)*N*H↑3);pen#
 2;line 2,1;gto
 31
28: 1+N*H*L/(1-
 N*H)-H↑3*N/L/
 (1-N*H)→S;if
 E>S;line 1,.5;
 pen# 3
29: if E<S;pen#
 1;line
30: if E>Z;pen#
 4
31: plt L,G,2
32: if L>1;gto
 "six"
33: gto "one"
*13462
```

case of P.R. No 4

```
0: gto "six"
1: pclr
2: fxd 2
3: wrt 705,"VS4"
4: scl 0,1,0,.5;
 wrt 705,"VS2"
5: wrt 705,"VS2"
6: xax 0,.125,0,
 1,2
7: yax 0,.05,0,
 .5,2
8: "six":
9: ent H
10: ent E
11: 0→R
12: (H↑2/E)↑.5→L
13: 0→K
14: "one":
15: K+1→K
16: R+.00005→R
17: L+R→L
18: π↑4/18*H→G
19: (E-(H/L)↑2)*
 (E+3*H*H/L/L)*
 G→G
20: if K>1;gto
 "ten"
21: plt L,G,1
22: "ten":
23: (4/(1-L)↑2*
 (1+N*H*L-N*H)-
 N*H↑3/L)/(1-N*
 H)→Z
24: wrt 705,"VS2
 "
25: if E>(1/L)↑2
 *((1-N*H)↑2*(1-
 N*H+L*N*H)+(1-
 L)*N*H↑3);pen#

 29
26: 1+N*H*L/(1-
 N*H)-H↑3*N/L/
 (1-N*H)→S;if
 E>S;line 1,.5;
 pen# 3
27: if E<S;pen#
 1;line
28: if E>Z;pen#
 4
29: plt L,G,2
30: if L>1;gto
 "six"
31: gto "one"
*27474
```

case of P.R. No 2

```
0: gto "six"
1: pclr
2: fxd 2
3: wrt 705,"VS4"
4: scl 0,1,0,.5;
 wrt 705,"VS2"
5: wrt 705,"VS2"
6: xax 0,.125,0,
 1,2
7: yax 0,.05,0,
 .5,2
8: "six":
9: ent N
10: ent H
11: ent E
12: 0→R
13: (H↑2/E)↑.5→L
14: 0→K
15: "one":
16: K+1→K
17: R+.00005→R
18: L+R→L
19: π↑4/18*N*H*
 (1-N*H)/(N*H*L+
 1-N*H)↑2→G
20: (E-(H/L)↑2)*
 (E+3*H*H/L/L*
 (1+4/3*N*H*L/
 (1-N*H)))*G→G
21: G/N→G
22: if K>1;gto
 "ten"
23: plt L,G,1
24: "ten":
25: (4/(1-L)↑2*
 (1+N*H*L-N*H)-
 N*H↑3/L)/(1-N*
 H)→Z
26: wrt 705,"VS2

27: if E>(1/L)↑2
 *((1-N*H)↑2*
 N*H+L*N*H)+(1-
 L)*N*H↑3);pen#
 2;line 2,1;gto
 31
28: 1+N*H*L/(1-
 N*H)-H↑3*N/L/
 (1-N*H)→S;if
 E>S;line 1,.5;
 pen# 3
29: if E<S;pen#
 1;line
30: if E>Z;pen#

31: plt L,G,2
32: if L>1;gto
 "six"
33: gto "[illegible]"
*[illegible]
```