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GUGGENHEIM AERONAUTICAL LABORATORY

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AN INSTRUMENT FOR THE DIRECT MEASUREMENT OF SKIN FRICTION IN HIGH SPEED FLOW

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OF SKIN FRICTION IN HIGH SPEED FLOW

by

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I. INTRODUCTION

In recent years the development of high speed aircraft and missiles has shown the importance of the effects of compressibility and heat transfer on boundary layer flow and hence on drag. The general problem is a highly complex one and involves processes like turbulence and shock waves in a real fluid. Such phenomena pose formidable theoretical difficulties for an analytical solution of the drag problem (for theoretical work on the subject see Refs. 1 and 2). In the case of shock free laminar flow the mechanism of the resistance experienced by a given body is fairly well understood and the difficulties are mainly of a mathematical nature. The case of turbulent flow, however, involves conceptual difficulties in addition to mathematical ones. Under such circumstances the role of experiments is a vital one. They provide evaluation of existing theory and contribute to a general understanding of the phenomenon of drag at high speeds. The quantity of most significance, in many cases, for practical reasons and theoretical analysis, is the viscous shearing stress on the surface of a body moving through a fluid. For reasons of simplicity the case of flow past a thin flat surface is usually singled out for close examination. In experimental terms this involves detailed boundary layer studies on a thin flat plate in a wind tunnel. Now boundary layers in high speed flow are extremely thin. A laminar boundary layer at atmospheric stagnation conditions and at a Mach number $\doteq 1.4$ and Reynolds number $\doteq 10^6$, on a 10 cm. long flat plate, is of the order of 1/2 mm thick and a turbulent boundary layer under similar conditions is about 1 mm thick. Hence accurate determination of skin friction by the measurement of velocity profiles is very difficult and the accuracy questionable. The development of an instrument for direct measurements of the local friction force is reported here.

II. EXPERIMENTAL METHODS OF SKIN FRICTION MEASUREMENT

Under the assumption of continuum flow and no-slip at the solid surface the intensity of skin friction τ_o at any point on a solid surface in a flowing gas is given by

$$\tau_o = \left\{ \mu \frac{\partial u}{\partial y} \right\}_{y=0} \quad \text{gms./ sq. cm.}$$

where

μ = coefficient of viscosity

u = velocity component in the boundary layer parallel to the surface

y = coordinate normal to the surface

For the flow of gases at temperatures and pressures at which the mean free paths of the gas molecules are small compared to the physical dimensions of the system the slip at the solid boundary is negligible. The following discussions will apply to cases where these conditions are realized.

The above expression for skin friction is valid for laminar and, because of the existence of the sublayer, also for turbulent boundary layers. In general, experimental attempts at measurement of the skin friction may be classified into two main types:

1. Those which endeavor to determine the components of the right hand side of the above equation.
2. Those which rely on a bulk measurement of the skin friction itself, or some other physical quantity related to it.

1. Skin Friction by the Velocity Profile Method

In the first type of measurement the physical quantities to be measured are the viscosity μ and the velocity in the boundary layer, u . Kinetic theory and experiment indicate the viscosity coefficient to be a function of the absolute temperature alone. Thus a local measurement of the wall temperature (with, say, a sensitive thermocouple) is sufficient to define $(\mu)_{y=0}$. The measurement of the velocity as a function of the normal coordinate y is however a more complicated procedure. Measurement of the local velocity in moving fluids is extremely difficult and up to the present time no really satisfactory direct methods exist for application to the observation of high speed flows. The only known published measurements of local velocity of interest to fluid mechanics are those of Fage and Townend (Ref. 3) who used an ultramicroscope with a rotating objective to observe boundary layer flow in a water channel. For most aerodynamic investigations the local velocity is not found directly but related quantities like dynamic pressure, mass flow, density etc. are measured. To obtain velocity from such measurements one requires additional information about the flow.

For example, consider the case of the three most extensively used instruments, the pitot tube, the hot-wire anemometer and the interferometer. In order to obtain velocity from the reading of a pitot tube one must know the local static pressure and density. For the hot-wire anemometer the local density is required, while information about the local temperature is necessary for the interferometer. In the case of the boundary layer, theoretical considerations borne out by experiments, show that the variation of pressure normal to the surface is negligible. On this assumption the experimental determination of static pressure in the boundary layer is simplified to a local measurement on the surface. (Since the pressure is measured at the wall and its variation is small the assumption of constant p through the boundary layer introduces only second order errors in a skin friction measurement.) However, for the instruments mentioned above, for each determination of velocity one would still have to make at least one additional measurement apart from the reading of the instrument itself.

In determining skin friction from the slope at the wall of a measured velocity profile, errors in measurement of the profile directly affect τ_o. A method which is not so sensitive to the portion of the velocity profile in the immediate neighborhood of the wall is provided in principle by Kármán's momentum integral. The basic consideration underlying this is the fact that the frictional force on the surface must appear as a momentum defect in the rest of the boundary layer. For the case of steady two-dimensional flow past a flat surface one obtains

$$\frac{\tau_o}{\rho_1 U^2} = \frac{d\theta}{dx} + \theta \left\{ \left(\frac{\delta}{\theta} + 2 \right) \frac{1}{U} \frac{dU}{dx} + \frac{1}{\rho_1} \frac{d\rho_1}{dx} \right\}$$

where ρ_1, U are the density and velocity just outside the boundary layer and θ and δ are defined by:

$$\theta = \text{momentum defect thickness} = \int_0^\infty \frac{\rho u}{\rho_1 U} \left(1 - \frac{u}{U} \right) dy$$

$$\delta = \text{displacement thickness} = \int_0^\infty \left(1 - \frac{\rho u}{\rho_1 U} \right) dy$$

and x is the coordinate in the direction of the flow. Several profiles in the region of determination of τ_o would now have to be measured. This method while not sensitive to errors in u close to the wall involves the added complication of knowing U, ρ_1 and θ as functions of x. Its application is more practical in the case of uniform flow past a flat plate when the pressure gradient in the x-direction is also zero and the above expression reduces to

$$\frac{\tau_o}{\rho_\infty U^2} = \frac{d\theta}{dx}$$

The use of this method does not, however, eliminate the difficulties in the determination of velocity from a pressure or momentum measurement.

Notes on Instruments used in the Measurement of Velocity Profiles:

A. The Pitot (or Total Head) Tube

The specialized form usually used for boundary layer measurements consists of a small bore tube (of the order of 0.05" diameter) with its mouth flattened out to form a narrow rectangular opening. The impact pressure at the mouth is proportional to ρu^2 and is usually measured on a mercury or alcohol manometer. As mentioned before, in order to obtain velocity from the reading of such an instrument one needs a knowledge of either ρ or T. For non-dissipative flows with speeds below the speed of sound the assumption of isentropic deceleration at the tube mouth is closely fulfilled and the velocity u is related to the impact pressure p_o by the relation

$$u = \sqrt{2 C_p T_o \left\{ 1 - \left(\frac{p}{p_o}\right)^{\frac{\gamma-1}{\gamma}} \right\}}$$

here

T_o = stagnation temperature

p = local static pressure

$\gamma = \frac{C_p}{C_v}$, C_p and C_v being the specific heats at constant pressure and constant volume respectively

The stagnation temperature T_o may be measured at some convenient reference point in the flow, for example, in the case of a wind tunnel, in the reservoir or settling chamber. At supersonic speeds a shock wave appears in front of the tube mouth and the entropy loss through this must be taken into account. By assuming the shape of the shock wave to be straight and normal to the flow direction the velocity can still be calculated by use of the well-known Rayleigh pitot tube formula

$$\frac{p}{p_o'} = \frac{\left(\frac{2\gamma}{\gamma+1} M^2 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}}{\left(\frac{\gamma+1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}}$$

where M is the local Mach number just ahead of the tube (in front of the shock wave) p_o' the impact pressure reading of the tube (behind the normal shock wave). The two pitot tube formulas become identical at $M = 1$.

For a skin friction determination with a pitot tube one would then have to know the following:

1. The impact pressure indicated by the tube as a function of y within the boundary layer.
2. The static pressure at the point of measurement.

3. The temperature distribution in the boundary layer.

By assuming the enthalpy to be constant through the boundary layer the temperature measurement is very much simplified but such an assumption causes serious errors in the value of the shearing stress. At high speeds the dissipation due to friction becomes increasingly important and the assumption of constant T_0 in the boundary layer is not valid even approximately. An approximate estimate of the error in τ_0 caused by such an assumption may be readily obtained. For points in the boundary layer very close to the wall the flow may be considered incompressible so that the pitot tube reads $h = p + \frac{1}{2}\rho u^2$. The pressure p is measured on the surface and is approximately constant through the boundary layer. From this

$$\frac{du}{dy} = \frac{1}{p}\sqrt{\frac{RT}{2\left(\frac{h}{p}-1\right)}}\,\frac{dh}{dy} + \sqrt{\frac{R}{2T}\left(\frac{h}{p}-1\right)}\,\frac{dT}{dy}$$

For $y \rightarrow 0$, $\left(\frac{h}{p}-1\right) \rightarrow 0$ and the second term on the right hand side is negligible as compared to the first. Therefore for small values of y

$$\frac{du}{dy} \sim \sqrt{T}$$

Also the viscosity coefficient is approximately

$$\mu \sim \sqrt{T}$$

so that the shear stress at the wall

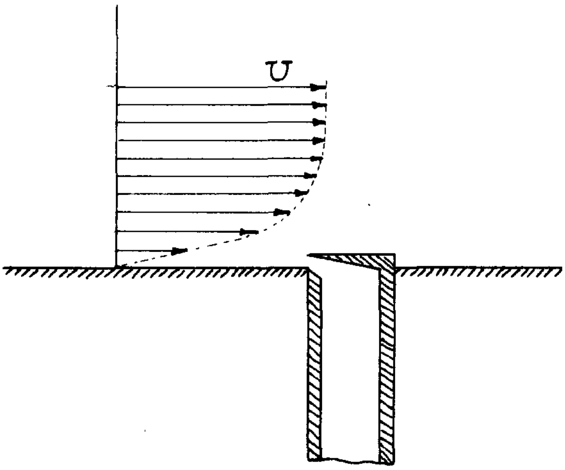
$$\tau_0 = \left(\mu\frac{du}{dy}\right)_w \sim T_w$$

For the typical case of an insulated flat plate with $M \doteq 1.5$ the wall temperature $T_w \doteq 0.85\ T_0$. By using $T_w = T_0$ the wall shearing stress is subject to errors of the order of 15%.

It is interesting to note the significant improvement in accuracy if an actual measured value of the wall temperature T_W is used for the determination of τ_o. The errors in $\left.\frac{du}{dy}\right]_w$ due to temperature are practically eliminated. The main limitation now is in the determination of $\left.\frac{dh}{dy}\right]_w$. Due to the finite size of the pitot tube, h cannot be measured right up to the wall. Another error introduced by the finite size of the pitot tube in boundary layer measurements is due to the transverse total pressure gradient in the boundary layer. The average indicated by the tube does not correspond to the velocity at the center of the tube but to that at some point which is shifted from the center towards the region of higher velocity. In a measurement of the velocity profile in a given boundary layer this error is emphasized for points close to the wall. For a tube with a rectangular opening of 0.005" this shift is of the order of 0.001" in a velocity gradient $\frac{du}{dy} \doteq 10^6$ ft./sec. per foot (representative of a typical laminar boundary layer at Mach number $\doteq$ 1.4 and Reynolds number based on a 10 cm. length $\doteq 10^6$. In actual use the shift can be approximately accounted for (Ref. 4).

B. The Stanton Tube

The Stanton tube is a modification of the pitot tube which gives an indication of the shearing stress on a surface. It consists of a small tube with a hole in its side (see sketch). The tube projects a small distance (of the order of 0.005") from the surface on which the friction is to be measured and the opening in the side of the tube faces the direction of flow. The end of the tube is ground flat



to form a razor sharp lip with the opening. For a distance ϵ very close to the surface (this would have to be within the laminar sublayer in flows with turbulent boundary layers) we have approximately

$$\tau_o \doteq \mu_w \frac{u_\epsilon}{\epsilon} = (\rho_w u_\epsilon^2)\left(\frac{\nu_w}{\epsilon u_\epsilon}\right)$$

where u_ϵ is the effective velocity corresponding to the tube pressure reading and the subscript w denotes conditions at the wall. The surface tube may thus be usefully employed for indications of changes in τ_o (see Ref. 5 for example of use). However, because of the unprecise definition of the quantity u_ϵ as measured by the tube and the extreme sensitivity to ϵ , the height of the tube above the surface, this method is not recommended for absolute measurement of τ_o.

C. The Hot-Wire Anemometer

The hot-wire anemometer depends for its measurement of velocity on the heat loss from a very small diameter (of the order of 0.0005") wire which is heated electrically. For a given temperature difference between the heated wire and the surrounding air the amount of heat transferred is proportional to the square root of the mass flow ρu . In order to find the velocity u from the mass flow one must know the density ρ . For the measurement of mean velocity in boundary layers at low speeds, where the effects of compressibility can be neglected, the hot-wire anemometer is used as a calibrated instrument. The calibration is performed under controlled conditions, for example, in a wind tunnel with a low turbulence level. In this range the sensitivity and accuracy of this instrument are well established. At high speeds the law of heat transfer deviates considerably from the one at low speeds. In particular, the effects of temperature and density gradients on the reading of the instrument (see Ref. 6 for some recent work on this subject) under these conditions are not yet established. Since a hot-wire has to be calibrated under conditions which cannot be exactly reproduced during boundary layer measurements such effects may cause large changes in the calibration constants. Until the behaviour of hot-wires in high speed, particularly supersonic, flow is well established it is felt that this instrument is unsuitable for precise measurements of boundary layer flow at high speeds.

D. Interferometer

This instrument measures directly the change in density with reference to some known value (see Ref. 7 for an account of the theory of operation of this instrument). Since the interferometer leaves the flow undisturbed, it seems, at first sight, to be the ideal instrument for two-dimensional boundary layer measurements. There are, however, serious limitations to its use for such work. These are due to:

1. Errors arising from the refraction of the light traversing the boundary layer.
2. Presence of side wall boundary layers in wind tunnels.
3. Transverse contamination, particularly of laminar boundary layers.

The errors due to refraction are quite complex. As a ray of light traverses the boundary layer it traces a curved path instead of a straight one due to the existence of density gradients in the boundary layer. Thus two types of errors are introduced as a result of refraction. The first is in the optical path length of the ray and the second in the position to which the indicated density corresponds. Further errors result from the fact that due to refraction a ray of light which enters the tunnel at right-angles to the center line leaves the test section at an angle to the exit window or wall. This causes refraction through the window, so that the location of the apparent origin of the ray is further in error. A more complete discussion of these errors may be found in Refs. 8 and 9. It is sufficient to note here that investigations of the magnitudes of the refraction error show that the indicated density profiles in boundary layer measurements with an interferometer may be in error to the extent of approximately 10%.

The second limitation, that of the presence of tunnel wall boundary layers, affects the measured density since the interferometer beam traverses both the wall density field as well as the one being studied. One method of minimizing this error is to pass the reference beam of the interferometer also through the test section and thus cancel the effect of the wall boundary layers (Ref. 10).

The third limitation is of primary consequence in laminar boundary layer measurements. It is well-known (Ref. 11) that external disturbances can cause transition of laminar flow to the turbulent type. Observations of laminar boundary layers on flat plates show that due to transverse contamination of the sides there are always regions of turbulent flow. Again the interferometer beam integrates the density in these regions as part of the laminar boundary layer and hence gives erroneous indications.

The above discussion shows that accurate measurement of density in a boundary layer with an interferometer is far from simple. Approximate methods of correction have been devised by various workers (Refs. 8, 9 and 12). It is felt that the techniques so far developed are inadequate, particularly for taking the rather serious refraction error into account. In addition to these difficulties in the measurement of density one still has the basic problem, mentioned before, of obtaining velocity from the reading of the instrument. Knowing ρ , one still needs the distribution of the static temperature through the boundary layer for a calculation of u . One may proceed to use theoretically calculated temperature distributions in the boundary layer (e.g. in Ref. 13) or alternatively use a pitot tube to supplement the interferometer. In this connection the use of a total head tube is considered to be an improvement over the first method of assuming theoretical distributions of temperature in the boundary layer, and using the interferometer as a primary measuring instrument. The reading of a pitot tube (apart from errors due to finite size) is proportional to ρu^2 and, for all practical purposes, is independent of theoretical assumptions. Hence this, with an accurate independent measurement of ρ , should give a fairly reliable determination of the velocity.

2. Measurement of Skin Friction by Direct Methods

The above discussions show that experimental determination of skin friction by the indirect method of velocity distribution measurement in thin boundary layers is subject to many errors. The shearing stress τ_0 has to be determined by differentiation, either of measured velocity profiles or of slowly varying parameters (the loss of momentum in the boundary layer). Even when the quantities to be differentiated are themselves measured comparatively accurately the results of differentiation can be quite inaccurate.

The second type of skin friction measurement, relying on a direct or bulk measurement, may be performed in principle in two ways.

(a) By a heat transfer measurement.

(b) By a direct force determination.

A. Skin Friction by a Heat Transfer Measurement

The first method depends on the Reynolds' analogy between the transport of momentum and the transfer of heat. If q_0 is the heat flow per unit time from a unit area of the surface, and k is the heat conductivity of the fluid, then

$$q_0 = k\left\{\frac{\partial T}{\partial y}\right\}_{y=0}$$

where T is the local temperature. The analogy with the expression for the intensity of skin friction is at once apparent. For the case of two-dimensional flow with the Prandtl number $\frac{c_p \mu}{k} = 1$ the temperature T is a parabolic function of the velocity u alone. In such a case the heat flow q_0 and the wall shearing stress τ_0 may be explicitly related by an expression (Ref. 1) of the form:

$$\frac{q_o}{\tau_o} = \frac{T_1}{U}\left[\frac{k}{\mu}\right]_{y=0}\left[\frac{\gamma-1}{2}\ M_1^2 - \left(\frac{T_W}{T_1}-1\right)\right]$$

where

T_1 = temperature of free stream, T_W = wall temperature

U = velocity of free stream

M_1 = Mach number of free stream

In the above relation assumptions of laminar flow and equal orders of magnitude for the viscous and thermal boundary layers are inherent. It is easily seen that estimates of skin friction can be obtained by measurement of q_o and the other quantities involved. For cases when the Prandtl number differs from unity but is constant and the flow is turbulent similar relations between the heat transfer and the skin friction may be derived (Ref. 14). These relations, however, are not exact and although they do indicate the existence of a relationship between q_o and τ_o , they are by themselves not reliable enough to form the basis of measurements for τ_o . In such cases calibration procedures have to be relied upon. H. Ludwieg (Ref. 15) has successfully used this principle for the measurement of wall shearing stresses in turbulent flows at low speeds. His instrument consists of a small electrically heated element flush with the surface on which the measurement is to be made and thermally insulated from it. q_o is measured by the amount of electrical energy supplied to the element. The temperatures of the element and the free stream are measured by thermocouples. It must be pointed out that in this case the general relation between q_o and τ_o must be modified to account for the fact that the thermal layer and the friction layer do not originate at the same point. Ludweig calibrated his instrument by measurements of q_o with known values of the friction force, so that his measurements are not affected by theoretical assumptions on the relation between τ_o and q_o. The use of this type of instrument for high speed flow investigations has, however, one serious drawback. It is extremely difficult to provide known shearing stresses for calibration purposes. For instance, it is no longer possible to use the known relation at low speeds between pressure drop in a two-dimensional channel and the shearing stress on the walls. Furthermore, it is open to question whether a heat transfer instrument calibrated in flows with fully developed turbulent boundary layers can be used for the measurement of skin friction in laminar flows, and vice versa. On account of these difficulties it was felt that a method measuring the shear force directly would be superior to the heat transfer method.

B. Direct Force Measurement

In principle this method is very simple. The frictional force is allowed to move a small element of the surface in the direction of the flow, and against some restoring force. This movement is calibrated to indicate the magnitude of the force. This method was used by early investigators like Froude and Kempf in determining the fluid resistance of bodies in water. A more recent application is that of Shultz-Grunow (Ref. 16), who used it for measurements of τ_o in a low speed wind tunnel. Since this method does not rely on any physical assumptions regarding the nature of the boundary layer flow it is inherently very suitable for surface friction measurements. There are, however, several difficulties and possible sources of error in the actual use of this principle, especially for applications to supersonic flow. The moving element has to be separated from the rest of the surface by small gaps. These may cause changes in the velocity and pressure distribution in their immediate vicinity and so falsify the measurements. Again, for local measurements the force element has to be very small compared to the dimensions of the body on which the friction measurements are to be made. This is of special importance in cases where there are large pressure gradients in the direction of flow. The magnitude of the friction forces on a small element are bound to be small (see section IV) so that the force measuring mechanism has to be extremely sensitive. Investigations of these difficulties (see section III) showed that nevertheless the direct force measurement method was quite feasible and practical. Section IV describes an instrument designed on this principle for use in the GALCIT 4" x 10" Transonic Wind Tunnel.

III. INVESTIGATIONS OF THE DIRECT FORCE MEASUREMENT METHOD

1. Measurement of Small Forces (of the order of 100 milligrams)

The common feature of all methods usually employed in the measurement of small forces is that they utilize a small linear or angular movement produced by the force against some mechanical resistance. The available methods are:

1. Mechanical devices utilizing the torsion of a fine wire.
2. Optical Methods
3. Resistance wire gages.
4. Reactance gages.

The mechanical torsion wire is essentially very simple and is utilized extensively in laboratory measurements of small forces (e.g. surface tension of liquids). However, a test set-up showed that it could not be used in the present case without complicated lever systems which would destroy the simplicity and accuracy of the method.

Among the optical methods the use of interferometry for the accurate measurement of small displacements is well-known. The extreme sensitivity of this method, however, requires vibration free supports for the optical elements and makes the practical application, in a case like the present, extremely difficult. Other possible optical methods would include the use of optical levers to magnify small angular changes and, perhaps, the diffraction of monochromatic light through a narrow slit. Actual investigations using these methods showed that although extremely simple to use they were not accurate enough.

The resistance strain gage, extensively used in experimental structural analysis and recently in wind tunnel balance systems, works on the principle that a change in strain in a fine conducting wire is accompanied by a change in the electrical resistance of the wire. Since the percentage change in resistance is of the same order of magnitude as the percentage change in strain and since the resistance is also a function of the temperature rather elaborate electrical equipment is necessary for precision measurements with this type of gage. A test apparatus showed that mainly due to temperature effects the resistance strain gage was not reliable enough for the determination of small friction forces of the order of 100 milligrams.

The reactance type gage is similar to the resistance gage inasmuch as it also employs a mechanical movement to produce a corresponding change in an electrical quantity. However, in this case, it is possible for the percent change in reactance to be several hundred times the percent mechanical change. The change in reactance can thus be easily and accurately measured. The particular form of reactance gage, or pick-up, chosen for the present application is a small variable transformer manufactured commercially. The size and characteristics (Ref. 17) of this transformer make it very convenient for use in the present application.

2. Effect of Gaps

The gaps between the surface of the flat plate and the force element cannot be permitted to have any flow through them since this would cause a serious disturbance in the boundary layer. In the case of supersonic flow the slot disturbances may cause shock waves and, so, errors in the force measurements. In order to investigate these effects tests were made on flat plates with slots. A slot width of 0.01" was used to study the effects in supersonic flow. This slot size was chosen as being a practical one for actual use in the instrument to be developed. Figs. 1 and 2 are high speed spark Schlieren photographs of the flow on the surface of the plate. No disturbances could be detected when the flow was viewed in the Schlieren system by continuous light. In Fig. 1 the plate has a laminar boundary layer. In the case of Fig. 2 the boundary layer was made turbulent with a 0.01" diameter piano wire stretched on the surface of the plate ahead of the slots. The laminar boundary photograph indicates no detectable disturbances due to the presence of the slots. In the turbulent case, there being a steeper velocity gradient at the plate surface, the flow is more susceptible to disturbances and very faint waves can be seen to originate at the location of the slots (Fig. 2). In order to estimate the strength of these waves attempts were made to measure the pressure rise through them by means of a 0.04" diameter static pressure tube and an alcohol manometer. No indication of pressure variation could be obtained. Since previous experience with similar pressure probes (Ref. 18) has shown the instrument to be an extremely sensitive one, it was concluded that the effect of the slots was negligibly small. This effect was further investigated by a detailed exploration of velocity profiles in the vicinity of a sample slot. The profiles on a flat plate with a relatively large slot of about 0.2 cm. were measured with a 0.0005" diameter platinum hot-wire. Since the boundary layers at high speeds are too thin to allow an accurate determination of velocity profiles these measurements were made in a low speed channel so that boundary layers were relatively thick and the hot-wire measurements quite accurate. In order to retain some measure of dynamical similarity with the slot in the high speed tunnel the Reynolds number (approximately 800 referred to free stream velocity and density) based on the slot width was approximately the same in the two cases. The results of these measurements are shown in Fig. 3. The slope of the velocity profiles in the vicinity of the slot and hence the shearing stress is essentially unaltered by the presence of the slot. The small change in the free stream velocity indicated by the profiles is due to the plate's location in a region of slightly retarded flow. This has no effect on the wall shearing stress in the region of measurement. It is interesting to note that the velocity profile measured in the gap shows a small finite velocity at the wall level but the slope of the rest of the profile is not sensibly affected. The profile just behind the slot does not indicate any change of τ_0 as would be the case if the presence of the slot were causing a disturbance. The velocity profiles shown in Fig. 3 are for a turbulent boundary layer on the flat plate. The effect of the gap on a surface with laminar boundary layer would be even smaller.

IV. THE SHEARING STRESS INSTRUMENT

1. Design Requirements

A. The instrument is to be used primarily for two-dimensional measurements in the GLACIT 4" x 10" Transonic Wind Tunnel (see Ref. 18 for description). The small size of the test section demands that the measuring instrument be of extremely small dimensions in order to avoid choking the tunnel.

B. The force measuring system must be sensitive enough to measure the skin friction forces on a flat surface with both laminar and turbulent boundary layers. In the available Mach number and Reynolds number range (Mach number $\doteq$ 0.7 to 1.5 and Reynolds number/cm. $\doteq 10^5$ to 1.7×10^5) this corresponds to friction stresses ranging from about 100 mg./sq. cm. to 10 gm./sq. cm.

C. The moving element and the surrounding plate must at all times be at the same level. Even small inclinations of the element (of the order of 1/50th of a degree) would introduce appreciable errors in the measured forces.

D. The instrument readings must be immune to temperature changes and vibrations from the tunnel.

2. Description

The instrument developed is shown schematically in Fig. 4. The small, 2 mm. x 20 mm., moving element is lapped flush with the surface of the approximately 15 cm. long flat plate on which the measurements are to be made. The element is supported by a flexure linkage. This is specially designed for parallel translation so that during its motion the element remains parallel to the plate surface and at the same level. The rectangular slot in the plate through which the force element is exposed to the flow has accurately lapped sides. The gaps at the leading and trailing edges of the movable element are about 0.005" and 0.01" respectively. The movement of the element is conveyed by the linkage system to the iron core of the small, 5/16" diameter 7/16" long, variable transformer by means of a non-conducting rod. This transformer (available commerically from Schaevitz Engineering, Camden, New Jersey) consists of three co-axial coils; one primary and two secondaries. The primary coil is energized with high frequency (approximately 20 kilocycles) A.C. so that the solenoidal force on the core is effectively eliminated. The output of the secondaries connected in opposition is a function of the core position. The sensitivity is of the order of 0.03 volts per 0.001" core displacement at 5 volts input to the primary. This output is large enough for the detection of core displacements of less than 0.0001". There being no physical contact between the core and the coils there are no errors introduced by friction. Temperature changes of $\pm$ 20°C have no effect on the output of the transformer and the link system is designed so that thermal expansions of various parts do not affect the core position. The possible effect of external vibrations on the instrument is two fold. (a) To make the transformer core oscillate and (b) to effectively displace the core. The first defect is remedied in the design by viscous damping and the second by distributing the mass of the oscillating system so that its center of gravity coincides approximately with its center of percussion. The force range of the instrument can be varied by changing the links in the flexure suspension of the moving element. For the present purpose three sets of links, 0.005", 0.01" and 0.025" thick, provide a force range from about 10 mg. to 10 gm. Calibration of the instrument is obtained by means of a pulley arrangement with jewel bearings and small weights. A sample calibration curve is shown in Fig. 5. The flexure links, moving element, transformer and the damping system are all enclosed in a streamlined windshield which is vented to the lower surface of the flat plate. This serves to equalize the pressure on the two sides of the gaps thus preventing any flow through them. The total frontal area of the instrument, excluding the flat plate, exposed to the flow is 1/4" x 2 1/2".

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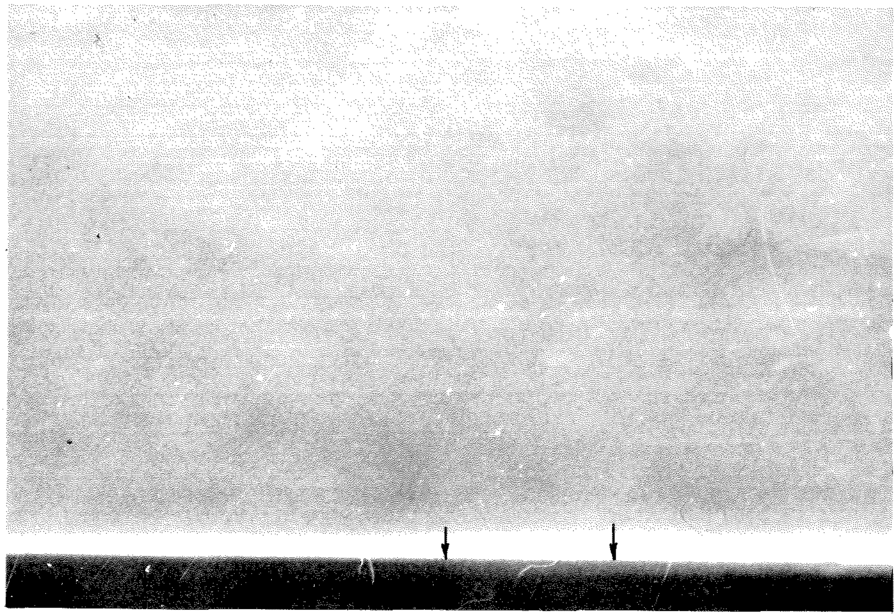
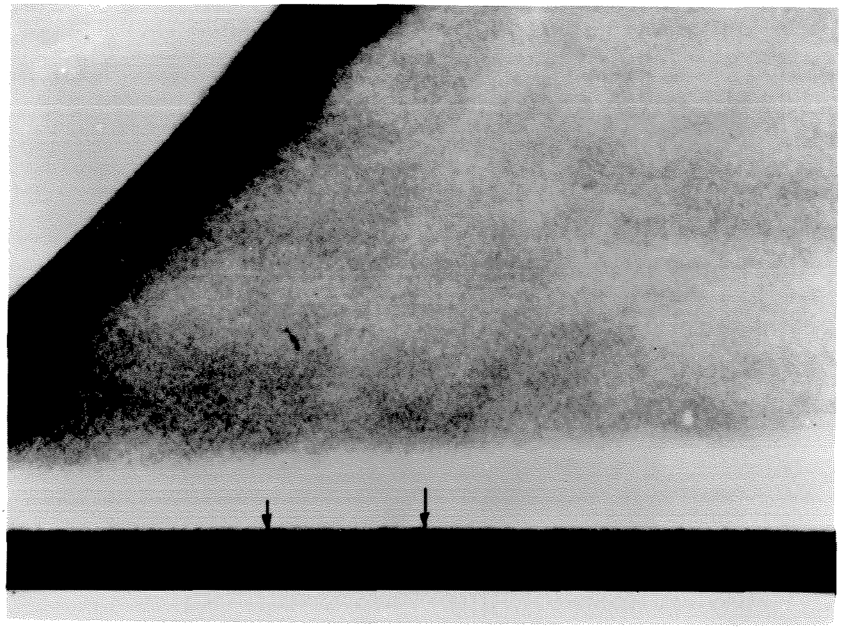


Flow from Left to Right
Laminar Boundary Layer $M \doteq 1.4$
Fig. 1



Flow from Left to Right
Turbulent Boundary Layer $M \doteq 1.4$
Fig. 2

EFFECT OF SLOTS IN PLATE
(Arrows Indicate the Position of Slots)

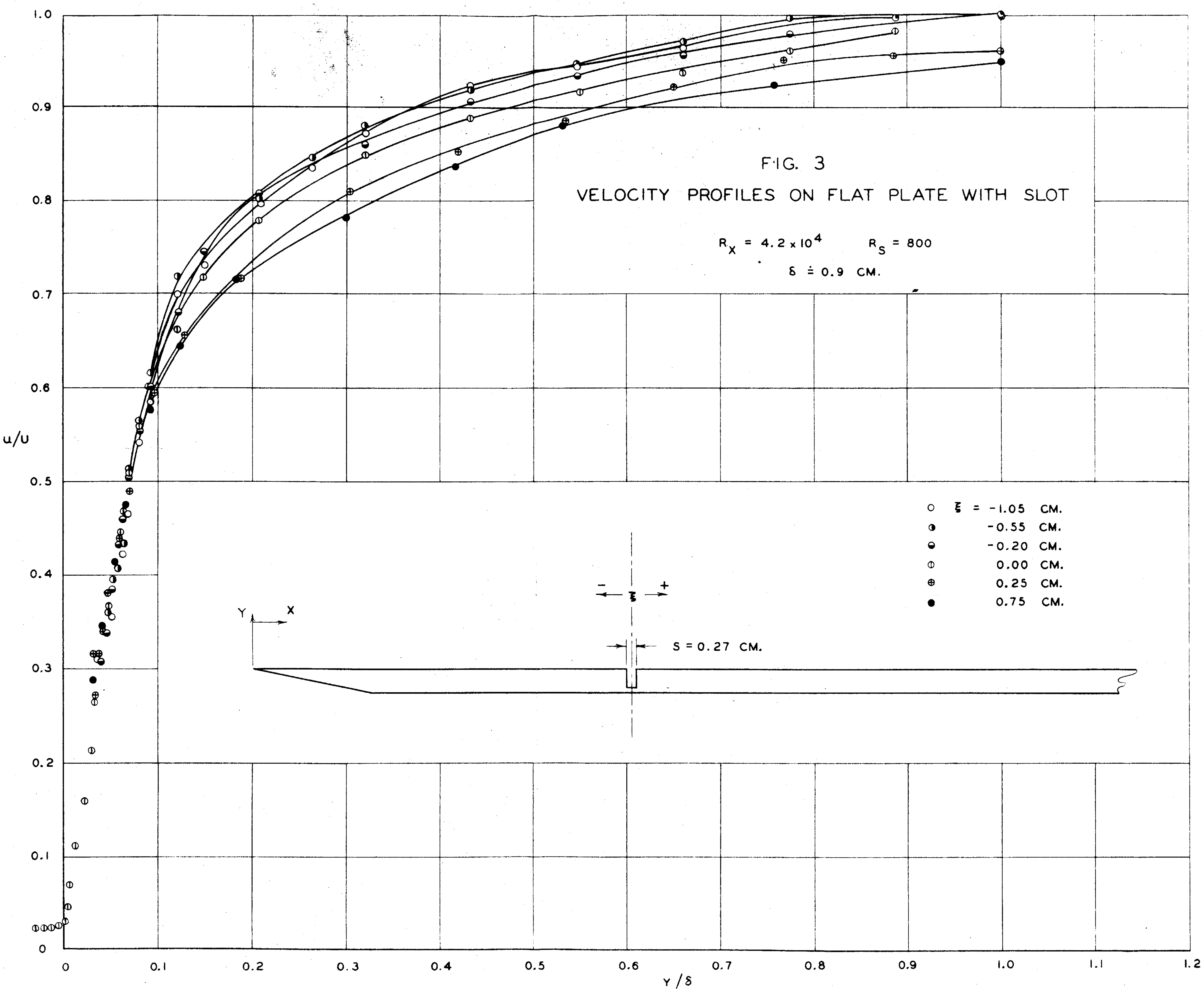
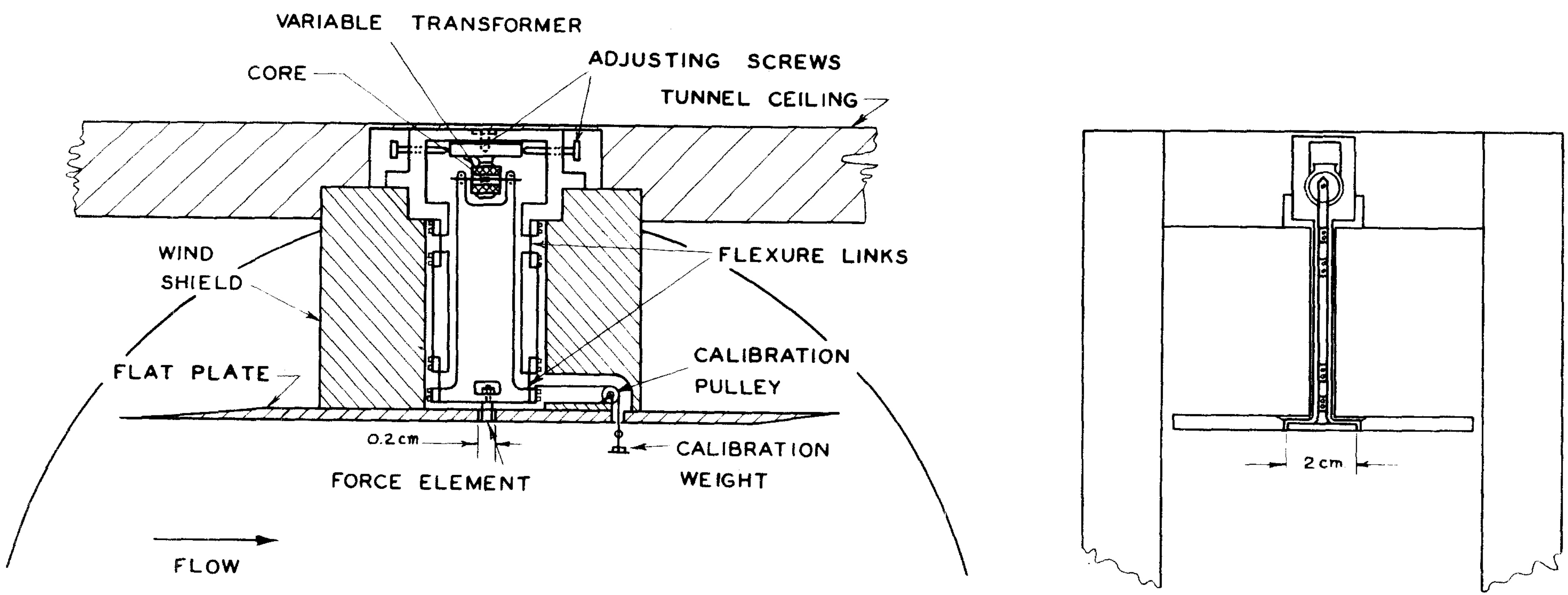


FIG. 3

VELOCITY PROFILES ON FLAT PLATE WITH SLOT

$R_X = 4.2 \times 10^4$ $R_S = 800$

$\delta \doteq 0.9$ CM.



SKETCH OF INSTRUMENT FOR DIRECT MEASUREMENT OF SKIN FRICTION

FIG. 4

FIG. 5

CALIBRATION OF 0.010" LINKS

LINEAR DISPLACEMENT OF CORE FOR 1000 MGM. = 0.007"