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A Precision Method of Generating Circular Cylindrical Surfaces of Large Radius of Curvature for Use in the Curved-Crystal Spectrometer*

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A method is here described for generating circular cylindrical surfaces of large radius of curvature on blocks of steel or other material with a close approach to optical precision utilizing an ordinary machine shop surface grinder. Convex and concave surfaces about 3×5 inches in dimensions with radii of curvature of 79 inches (two meters) have been ground by this method both on cast iron and on stainless-steel blocks with a precision of about 0.0002 inches as regards surface imperfections. A very moderate amount of subsequent lapping sufficed to give surfaces of optical precision. The surfaces are used for clamping lamina of crystalline quartz for use in a curved crystal focusing gamma ray spectrometer. A companion paper describes the gamma ray spectrometer.

THE PROBLEM AND ITS REQUIREMENTS

IN the construction of a large focusing spectrograph of the transmission type[1] for short wave-length x-rays and gamma-rays, it was necessary to generate on stainless-steel blocks true circular cylindrical surfaces (both concave and convex) of large radius of curvature (2 meters) with very high precision. Relative to the size of the block this radius is so large that the curvature is barely perceptible on casual inspection. These curved surfaces form a vise in which a crystalline quartz plate, 1 mm in thickness, is clamped so as to deform it elastically to cylindrical form. A rubber gasket is placed between the plate and the concave block so that only the convex block accurately defines the curvature. The atomic planes used for the x-ray and gamma-ray reflection (in the present case the (310) planes) are thus made to converge so as to focus monochromatic radiation, reflected from all elements of the crystalline plate, quite accurately in a fine spectral line. The totality of all such spectral lines forms the elements of a circular cylindrical focal surface of diameter equal to the radius of curvature of the crystal. In the present instance where the distance from the curved crystal to the focal cylinder (on which the spectral lines appear) is two meters, and where the focusing must be sufficiently precise from all parts of the crystal plate to give line widths of the order of 0.1 mm, it is clear that the surface of the clamping block must have optical precision.

The problem involves more than just the generation of a true circular arc of large radius. The generators of the circular cylinder must all be rigorously straight, truly parallel to each other, and normal to the plane of the base circle (base plane of the cylinder), i.e., the surface must have *no warp* and *no conical error*. Also, the base plane of the cylinder and the direction of the cylindrical generators must be *known*, preferably by generating the curved surface on a block provided with plane ground edge surfaces to serve as reference faces, one of which shall be accurately parallel to the base of the cylinder, and the other accurately parallel to the generators and the axis of the cylinder.

To meet these rigid requirements it was felt that the use of a simple two-meter radius bar or sweep might be inadequate to insure the parallelism of the cylindrical generators and their perpendicularity to the base plane of the cylinder. Some method involving *a well-defined, very stable axis of rotation* was needed. A large vertical boring mill, with sufficient radius to permit mounting a grinding wheel at two meters from the center, would doubtless serve the purpose, provided its bearings and tool carriage were sufficiently reliable, but no such machine was readily available in the present instance. At best, the boring mill would be a very cumbersome and awkward solution, requiring a mammoth machine tool to do a miniscule job.

* This research is now being conducted under Navy Contract N6onr-244 Task Order IV, dated March 1, 1947.
[1] A companion paper in the present issue of this journal describes this instrument.

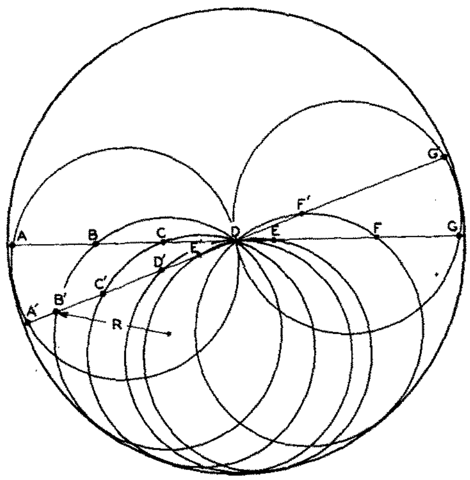

FIG. 1. Illustrating how points on a circle of radius R, which rolls without slipping inside a fixed circle of radius $2R$, describe straight lines which are diameters of the larger circle. Conversely, the same rolling motion may be imposed on the smaller circle by constraining two points on its circumference to move along the two straight diameters.

METHOD OF GENERATING THE CYLINDRICAL SURFACES

The method we have adopted is perhaps most easily understood by considering the kinematics of a circle which rolls without slipping inside and in contact with another circle of twice its diameter. Both circles lie in the same plane. It is a well-known and easily proved fact that for this case any point fixed in the circumference of the smaller rolling circle executes a straight line which is one of the diameters of the larger fixed circle. In Fig. 1, D is the center of the large fixed circle. On the smaller rolling circle one point describes the diameter, AG, and simultaneously another point describes the diameter $A'G'$. The successive positions of the rolling circle are shown, as are also the successive positions AA', BB', CC', etc. to GG' of the chord on the rolling circle which connects the two points which, respectively, generate the fixed diameters AG and $A'G'$. From this figure it is clear that *the motion of the rolling circle can equally well be generated by constraining the two points on its circumference to slide along the two straight diameters while maintaining the length of the chord between the points fixed.* If the acute angle between the two diameters is θ, the length of the chord of the rolling circle L, and the radius of the rolling circle R, then by elementary considerations of plane geometry

$$R = L/2\sin\theta. \tag{1}$$

Imagine now, as shown in Fig. 2, that a rapidly rotating grinding wheel of radius r is centered concentric with the large fixed circle and that a block B is rigidly attached to the rolling circle in a position *inside* its circumference. The figure should make it clear that the grinding wheel will sweep out on the block B a *convex* circular profile of radius $R-r$. In Fig. 2 the block B is shown for two positions of the rolling circle as B and B', respectively. If, on the other hand, a block C is rigidly attached to the rolling circle in a position *outside* the circumference of the latter, then the grinding wheel will sweep out a *concave* circular profile of radius $R+r$ on it. The two positions of this block are shown as C and C'. This is the principle upon which our method of generating the cylindrical surfaces is based. The rolling and fixed circles however have no physical existence, but the *motion* of the rolling circle is imparted to a rigid mechanical system by a chord of fixed length L whose two end points are constrained to slide along straight lines making an acute angle θ with each other. From Eq. (1) it is clear that with apparatus of very moderate dimensions, extremely large radii $R-r$ and $R+r$, can be generated. Thus, for example, suppose that the desired convex radius is 2 meters. If we make

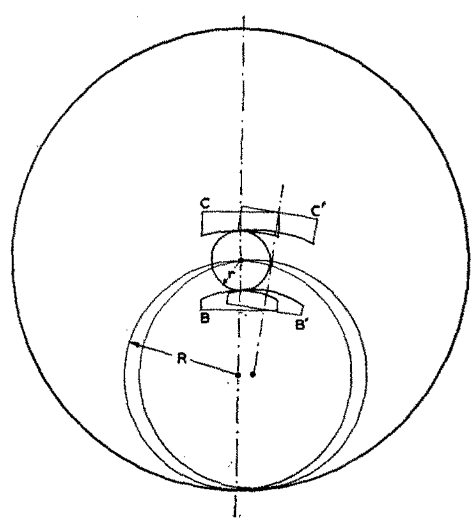

FIG. 2. Illustrating how the rolling motion of Fig. 1 is used to cause a grinding wheel of radius r to generate convex and concave curved cylindrical surfaces.

$L=0.424$ meter (about $16\frac{3}{4}$ inches) and $\sin\theta = 0.100$, then a radius $R=2.12$ meters will result. If the radius r of the grinding wheel is 12 cm, the radius of the convex ground surface $R-r$ will then be just 2 meters. The radius of the convex and concave ground surfaces can be varied by varying either L or $\sin\theta$. In practice we have found it convenient to use an arm, L, of fixed length and to vary $\sin\theta$ so as to give the desired convex and concave radii.

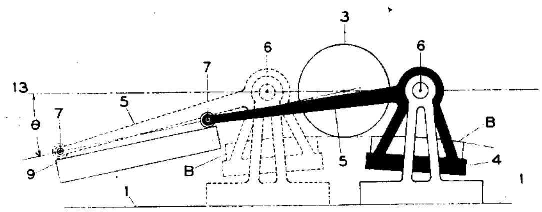

FIG. 3A. Schematic view illustrating in principle the method of grinding a convex cylindrical surface of large radius on the surface grinder. For the actual set-up see Fig. 3B. This view is not to scale and the curvature of the block is greatly exaggerated for clarity.

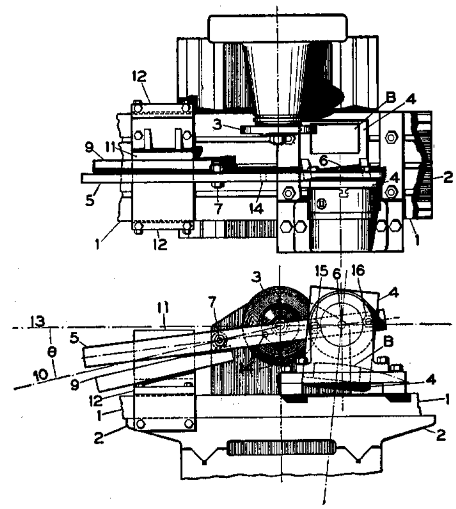

FIG. 3B. Upper view is a plan and lower view an elevation of the set-up on a surface grinder for profiling a convex cylindrical surface (on block B). The curvature of the surface and the angle θ of the inclined plane 9 are exaggerated for better clarity and the drawing is not to exact scale.

ADAPTATION OF THE METHOD TO A SURFACE GRINDER

The line drawings, Fig. 3A and B show the general arrangement we have adopted for doing this work in a surface grinder. Figure 3A is intended to convey in simple schematic form the kinematics of the method. In this figure the parts are highly idealized and simplified to aid in illustrating the principle. Figure 3B shows both in plan and elevation the actual set-up on the surface grinder with reference numbers for the parts. The surface grinder happened to be a Brown and Sharpe No. 5, but any make of similar construction could be used equally well. Such machines have a carriage, 1 (Fig. 3B), on which flat work to be ground is usually held on a magnetic chuck. This carriage has an automatic reciprocating motion which feeds the work transversely back and forth under the grinding wheel. There is also a slower automatic cross motion of a carriage, 2, at right angles to the motion of 1 (normal to the plane of the paper in the elevation view of Fig. 3B) which we have indicated by showing the profile of the V-shaped ways. A third feed (not shown in Figs. 3) is provided on such machines which raises and lowers the grinding wheel, 3.

For our present purpose the magnetic chuck was removed and a circular dividing head (from a milling machine) was mounted on carriage 1 with its axis of rotation parallel to the cross-feed of carriage 2 and *at the same height as the axis of the grinding wheel at its finish cut.* This dividing head should have a true bearing very free from either radial or axial lost motion. Care must be exercised to insure that the axis of rotation of this dividing head shall be *truly parallel to the cross-feed motion* of the carriage 2. This can be established with the aid of a cylindrical bar (not shown) centered truly in a chuck mounted on the dividing head so that when the latter is rotated on its axis the bar exhibits no eccentricity at either near or far end when tested with a dial indicator. The carriage 2 is then displaced along the cross-feed direction and a dial indicator, clamped on the grinding head of the machine with its feeler point in contact with first the top, and then the side of the cylindrical bar, is used to check the parallelism of the axis of the dividing head with the cross-feed ways. It is the axis of rotation of the dividing head which in part determines the direction of the generators of the cylindrical surface to be ground. This surface can be defined no better than the quality of this axis (freedom from lost motion, etc.) will permit.

A rigid bracket, 4, on which the work is to be mounted, is now provided on the dividing head. This bracket can be screwed either to a face plate on the spindle of the dividing head, or, as shown in Fig. 3B, it can be secured directly to the face of the dividing head chuck. Our dividing head was provided with means for rotating it into accurately indexed positions where it could be locked. While locked in this way the grinding wheel was fed down and the surface of the bracket was ground off true with the two feeds (carriages 1 and 2) of the grinder.

The rolling motion described in Figs. 1 and 2 is imparted to the system consisting of the dividing head, the bracket, and the block mounted thereon whose surface is to be profiled. The motion is imparted by means of an arm, 5, on which the length L is accurately laid out as the distance between the centers 6 and 7, namely the centers of the dividing head axis and the small, ball-bearing roller, respectively. The arm is rigidly screwed to the dividing head and forms part of the rolling system (arm, 5, dividing head, bracket, 4, and work B) which we shall refer to as system 8 (not labelled in the drawing), *the system relative to which the block B to be profiled is fixed.* By means of a hardened and ground straight edge, 9, the center 7 of the ball-bearing roller is constrained to follow an oblique straight line 10 as the carriage 1 reciprocates on its ways. *This oblique straight line (projected) must pass through the center of the grinding wheel when making the final cut.* Our straight edge was simply clamped by means of "C" clamps (not shown in Figs. 3) to the true vertical surface of a bracket 11, supported on a bridge structure 12, which in turn was bolted to the carriage 2. In this way the straight edge *partakes of the cross-feed motion of 2 but not of the transverse motion of carriage 1.* The acute angle of obliquity of line 10 with line 13 (along which latter line the center 6 of the dividing head travels) is the angle θ of Eq. 1. In fact, lines 10 and 13 are the lines AF and $A'F'$ of Figs. 1 and 2. The bridge structure 12 was so designed as not to interfere with the motion of carriage 1.

We have used the following method for adjusting the angle θ and correctly locating the straight edge 9 on the bracket 11. The dividing head is locked in the position in which the surface of bracket 4 was previously ground. The arm 5 is provided with an auxiliary hole 14 (accurately located on the same straight line as the centers 6 and 7) into which a dummy disk of exactly the same diameter as the roller, 7, can be centered. (The straight edge 9 is to be held in contact with these two rollers while it is being located on bracket 11.) The bolts 15 and 16 holding arm 5 to the dividing head are loosened and arm 5 is allowed to rotate about center 6 on a pin which passes through an accurately located hole in arm 5 and which is itself truly centered on the axis 6 of the dividing head. Bolts 15 and 16 pass with ample clearance through oval holes in the arm 5 to permit this adjustment of the angle θ. This angle is readily adjusted by applying the "sine bar" principle. The height of the axis 6 above the bed of carriage 1 is measured and this, together with the length L between points 6 and 7, gives the data necessary to compute the correct height to set 7 above the bed of 1 in order to give any desired value of $\sin\theta$. This operation is best performed with carriage 1 displaced to the right to facilitate the height measurements without interference from the bridge structure 12. When this setting has been correctly adjusted the bar 5 is clamped by the bolts 15 and 16 to the dividing head, and *carriage 1 is displaced to a horizontal position in which the axis 6 of the dividing head coincides exactly with the axis of the grinding wheel at final cut position,* (as shown by a dial indicator test). With the straight edge 9 held in contact with the two rollers at 7 and 14 the former can now be clamped to the bracket 11 (by means of "C" clamps) in the correct position it must occupy. The dividing head is now unlocked so as to be free to rotate on its axis 6, and the dummy roller on the bar at 14 is removed. Carriage 1 can now be allowed to reciprocate freely on its ways. In order to insure contact between the roller and the straight edge, a light weight can be hung on the end of the bar 5 or it can be spring loaded. *The surfaces of roller and straight edge should be protected from abrasive dust and other foreign matter with appropriate shielding.*

We wish to emphasize particularly the importance of designing all the parts of the above described mechanism *with adequate rigidity* to avoid elastic deflections which might vitiate the accuracy of the generated surface. This is particularly true as regards the bar 5 and the roller at 7. Since the latter roller bears on the straight edge in a different vertical plane from the plane of the bar 5, the spring or weight loading which holds the roller in contact with the straight edge may exert a couple which may (1) deflect the axle of the roller or (2) twist the bar 5 sufficiently to introduce small errors in the motion.[2] In the design of the dividing head and its axis and supporting structure and also the bracket supporting the work, great care must also be exercised to minimize elastic deflections.

If the cylindrical arc generated on the block B is to be symmetrically located so that the block is the same thickness at the two extreme ends of the arc then a perpendicular bisector from axis 6 to the ground surface of bracket 4 must exactly bisect the block. The block B should preferably be prepared by grinding its back and four edge surfaces accurately plane with true right angles between all intersecting surfaces. The block is secured to the bracket 4 by screws from below, not shown in Figs. 3A or B. In mounting the block on the bracket 4, the four edge surfaces should be adjusted by dial-indicator test so that they stand parallel to the motions of carriages 1 and 2. These edge surfaces then furnish the necessary references regarding the base plane of the cylinder and the direction of the cylindrical generator lines. If the surfaces are to be further corrected after grinding by a process of lapping, *these reference edge faces are of great value for correctly aligning the concave and convex laps relative to each other.*

In the case of the convex cylindrical surface the grinding wheel is fed downward against the work, starting with its axis in a position somewhat higher than the final position it should occupy during the finishing cut. This final position, as we have pointed out, should coincide with the height of the axis 6 of the dividing head. However, small deviations from this position only introduce small errors in the cylindrical figure which we shall presently evaluate. A minor difficulty comes from the change in radius r of the grinding wheel, owing to wear. The best way to handle this is to estimate the wear to be expected by a preliminary trial and to make the final setting of $\sin\theta$ such that the desired radius $R-r$ of the convex

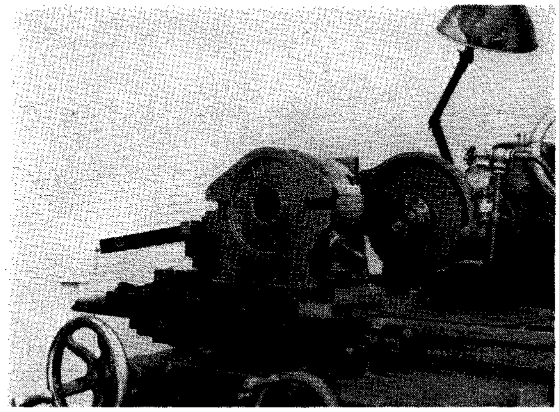

FIG. 4. Photographic view of surface grinder set-up for profiling a cylindrical surface of two-meter radius of curvature. View is from the front of the machine. The carriage 1 of Figs. 3A and B is displaced to the extreme left, and the block being profiled can just be seen below and to the left of the grinding wheel. Parts of the bracket supporting the work (numbered 4 in Figs. 3A and B), the bar 5, the straight edge 9, and the bridge structure 12, are also visible.

cylindrical surface is calculated for the *expected* value of r allowing for wear. Just before the last finishing cuts are taken it may be well to check the radius of the grinding wheel and readjust $\sin\theta$ accordingly if a very accurate result as regards the radius of curvature of the finished surface is required.

In the case of generating the concave surface the dividing head with its bracket supporting the work is turned through 180° so as to bring the work *above* the grinding wheel. The angular setting of the bar 5 to establish $\sin\theta$ correctly is made with the dividing head clamped in this inverted position and with the ground surface of the bracket (which holds the work) truly parallel to the motions of carriages 1 and 2. The adjustment of $\sin\theta$ is otherwise entirely similar to that described for generation of the convex surface. It must not be forgotten, however, that in the case of the concave surface the radius generated is $R+r$, not $R-r$, and hence a different value of $\sin\theta$ must be used even if the convex and concave blocks are to have the same radius.

In the case of the concave surface the grinding is started with the grinding wheel somewhat *below* the final position for the finishing cut; this position, as before, should be such that the center of the grinding wheel and the center of the dividing head axis coincide. Here again small

[2] A valuable improvement might indeed be made by using a well-fitted dovetail slide instead of the roller and straight edge. The sliding carriage would then be pivoted to the bar 5 at the point 7.

deviations from this coincidence only introduce small errors in the truth of the circular figure.

An alternative way of grinding the concave surface is to adjust the bar 5 and straight edge 9 so that the angle θ lies above the line 13 instead of below it. The work can then be situated below the grinding wheel and the grinding wheel fed downward against it.

Figures 4 and 5 are photographic views of the entire surface grinder set-up.

An analysis of the errors resulting from small displacements of the grinding wheel is given in an appendix to this article.

DESCRIPTION OF THE CLAMPING BLOCKS

Two stainless-steel blocks were originally prepared, hardened, and ground by the above described method to radii of curvature of two meters, one convex, the other concave. Tapering openings were provided in these blocks to permit the passage of the x-rays or gamma-rays. A line drawing of these blocks, showing the horizontal and vertical ribs across the opening in the convex block, is given in the companion article describing the spectrometer.

CORRECTION OF THE SURFACES BY LAPPING SUBSEQUENT TO GRINDING

The surfaces were tested after the grinding process, both by optical and by x-ray means. The quartz crystal plate, 1 mm in thickness and 77×72 mm in its other dimensions, was prepared with two optically flat and parallel surfaces,[3] and this was clamped in direct contact against the *convex* stainless-steel block but with a rubber gasket between it and the *concave* steel block. Thus only the figure of the *convex* block is vitally important in determining the curved figure of the crystal. The figure was tested (1) by reflecting light from the quartz surface, and (2) by reflecting $WK\alpha$ x-rays from the internal (310) planes of the crystal. In the optical test a straight-line filament lamp, placed on the axis of the cylinder of curvature of the crystal, was used as a source and the image of this, also on the axis of curvature, was examined with a microscope placed just above the lamp. The x-ray and optical tests were found to agree as to the errors of the curved figure. The errors of focus of the surfaces generated in the surface grinder were found in this way to be too large to be acceptable for our purpose, and it was decided to correct the figure of the convex stainless-steel block by lapping. It would not, however, have been satisfactory to use the concave and convex stainless-steel blocks themselves as laps to be rubbed together because of the holes and the ribs on the convex block. The procedure followed, therefore, was to prepare two solid blocks of cast iron somewhat larger than the stainless-steel blocks, $3\frac{1}{2} \times 5$ inches being the dimensions of the curved cast-iron surfaces. By precisely the same procedure as described above these cast-iron "laps" were profiled in the surface grinder, one convex, the other concave, both radii being as nearly as possible 2 meters. They were provided with truly ground plane reference edge faces. A simple lapping machine shown in the photograph of Fig. 6 was constructed. This device constrains the upper movable member of the two lapping blocks to move in such a way that the generators of the two cylindrical surfaces always remain strictly parallel to each other without placing any other constraint on the motion. To accomplish this it is necessary that each of the two rods of the parallelogram (clearly

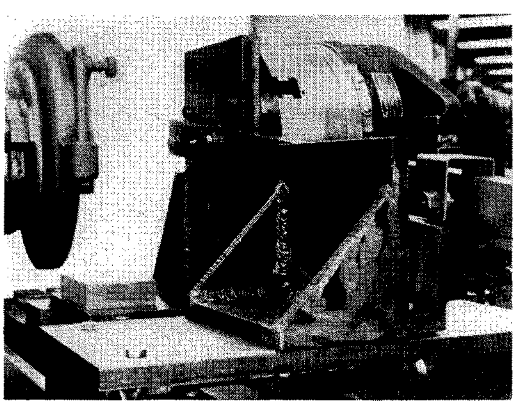

FIG. 5. Photographic view of surface grinder set-up for profiling a cylindrical surface of two-meter radius of curvature. This is a view looking from a position behind and to the left of the surface grinder. The bridge structure 12 and bracket 11, together with the straight edge 9, which they support, are visible in the foreground. The bracket 4 mounted on the dividing head chuck and the block being profiled can be seen further back. Lever arm 5 is also clearly visible. The reference numbers of Figs. 3A and B are used in this caption.

[3] We wish to mention with praise the careful work done in preparing the quartz plates by the Penn Optical Company of Pasadena to whom we are also indebted for suggesting to us the procedure (to be explained later) using a pair of auxiliary cast-iron laps to correct the stainless-steel blocks.

visible at the top of the machine) shall be provided with universal joints at either end, the inter-center distances between these joints (18 inches) being exactly the same on each rod. These joints were easily made out of small ball bearings having races of the "self-aligning" type since the amplitude of motion of the joints is small in all directions save for the lapping motion in the horizontal plane. The axle holes in these self-aligning ball bearings fit smoothly without lost motion (but not tightly) over cylindrical pins, two of which are mounted on the "duralumin" oscillating plate (visible at the left), and the other two on the plate to which the movable lap is bolted. Each pair of pins has a separation of 6 inches. At frequent intervals during the lapping process we disengaged the pins on the plate bolted to the movable lap from their ball-bearing holes and turned the movable lap around through exactly 180° relative to the fixed lap. We believe that a much better result is obtained by these frequent reversals.

The stationary, cast-iron lap was bolted down to the bed of the lapping machine with care to adjust its reference edges strictly parallel to those of the movable block. This was done with precision by holding a hardened and ground steel "parallel" bar against the edges of both laps, with a small amount of "ultramarine" blue spread on the surface of the bar to act as a marker on the cast-iron edges. The lapping was then started, using as abrasive grade No. 305-A lapping compound prepared by the U. S. Products Company of Pittsburgh, Pennsylvania, in their special grease medium.[4] The lapping was entirely by hand. A circular stroke of diameter about half an inch was used, and the movable block was displaced at random into many positions relative to the fixed block. In no case, however, was this displacement (measured from the position of register of all four edges) more than about one-quarter the dimension of the block, in order to avoid gravitational effects due to overhang which might falsify the profile. In the surprisingly short time[5] of about five hours lapping the abrasive was cutting at all points of both surfaces. The two laps were then carefully cleaned with oil and solvents and lapping was continued with U. S. Products Corporation No. 38–1200 compound. This is their finest lapping compound, the particles being stated to be only about 2μ in size. Again the process was continued until cutting was indicated over all of both surfaces; this result required only about three hours. When cleaned, the two cast-iron surfaces now exhibited a brilliant mirror finish which permitted an optical test of the concave member by examining the image of a line filament lamp with a microscope. We were delighted to find a *very perfect image* whose width, utilizing the whole cast-iron surface, and taking account of the diameter of the line filament source, was practically the same as one would expect from diffraction theory for a perfect cylindrical surface!

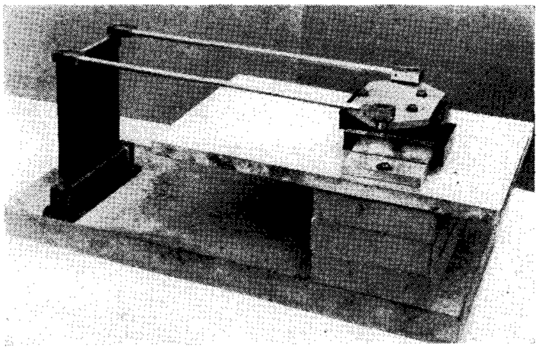

FIG. 6. Hand-lapping machine for insuring parallelism of cylindrical generators while convex and concave blocks are being rubbed together.

We now proceeded to correct the convex stainless-steel block by lapping it against the concave cast-iron block in the lapping machine of Fig. 6, using the finest compound No. 38–1200. This work proceeded very rapidly, requiring about an hour and a half to obtain contact over the whole surface. Here again frequent reversals of the steel block were made. A preliminary optical test of the convex stainless-steel block (by reflecting light from the crystal plate when the latter was clamped against it), and also of the

[4] We are much indebted to Professor W. Hansen of Stanford University for recommending the use of the lapping compounds of this firm and to the U. S. Products Corporation, 518 Melwood Street, Pittsburgh, who very kindly and ably advised us regarding the proper compounds for this job and generously presented us with samples more than adequate for our use. These compounds have proved extraordinarily satisfactory for the purpose.

[5] The times of lapping stated here are in every case the net periods during which the surfaces were actually being rubbed together, excluding time out for cleaning, testing, etc.

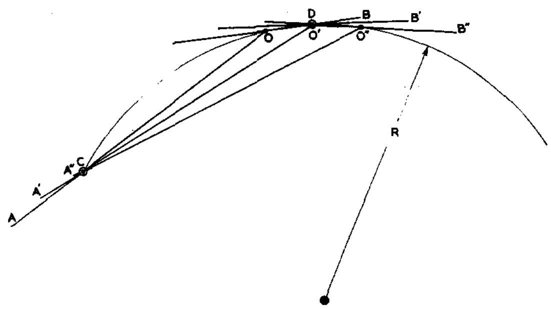

FIG. 7. Alternate method of regarding the kinematics of generating the cylindrical surfaces of large radii. Here the work is regarded as *fixed*. The legs of a rigid angle AOB are each constrained to pass through one of two fixed points C, D, and the vertex O then describes the circle of radius R.

concave cast-iron lap, showed that the perfection of figure of the cast-iron lap had been slightly impaired by the lapping of the stainless-steel block. This was corrected very quickly by lapping the two cast-iron laps together once again, using the finest compound. The stainless-steel block was then corrected once more by a brief lapping with the same compound against the retouched cast-iron lap. This time no appreciable error showed in the cast-iron lap after complete contact had been obtained. To our great satisfaction a very fine optical test was now indicated when the quartz plate was finally pressed into intimate contact against the stainless-steel holder.

It should be stated, however, that the problem of springing the surface of the quartz plate into true optical contact with the curved stainless-steel surface is not as easy as might at first be supposed.[6] This was investigated by placing the quartz plate against the polished surface of the *convex* cast-iron lap and studying the pattern of interference fringes. By exerting pressure on the quartz plate with a stick the fringes could be displaced at all save a few isolated points where the surfaces, were undoubtedly held apart by invisible foreign particles. It was found that extraordinary precautions must be taken to exclude foreign matter of all kinds by extremely careful and repeated washing of the surfaces with acetone in a dust-free room. Even then, great patience and many repeated trials may be necessary before good contact is obtained. This is especially true of the stainless-steel block since the present construction is such that the optical fringes cannot be studied anywhere save on the ribs, and one must repeat the cleaning process until the optical-reflection test of the crystal surface shows that the image is perfect.

In the optical-reflection tests of the curved quartz plate held in the stainless-steel clamp, an interesting effect worthy of description was observed. *The image of the line filament lamp in this case always appeared double* in the field of the microscope, each individual member of the pair being about the width we had expected from the optical tests of the cast-iron lap. The doublet separation was about 0.2 mm. With a polarizing Nicol prism in front of the lamp and an analyzing Nicol in front of the microscope, we were able to prove that the two lines of the doublet are differently polarized. Since no such effect of birefringence is detected with the quartz plate before it is curved, we conclude that the effect is probably a birefringence due to strain from curving the plate. The effect is of course not present in the x-ray or gamma-ray reflection, but was quite disturbing to us in the optical tests until we understood it.

Subsequent x-ray tests of the perfection of this curved quartz-crystal plate, in which x-rays are reflected from ten different positions uniformly spaced over the aperture in the stainless-steel block, show that the tungsten K α-lines focus

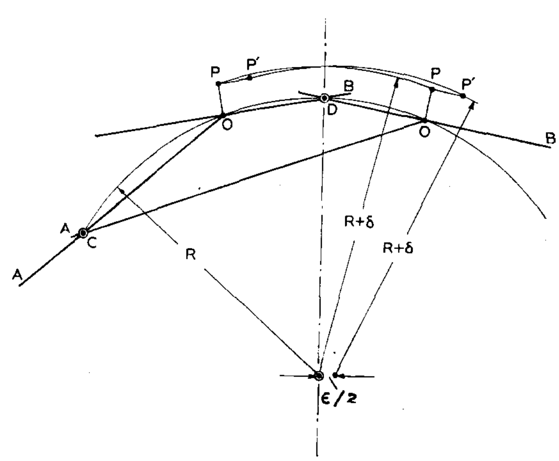

FIG. 8. Geometry for analysis of errors coming from small displacements of the grinding-wheel center away from its correct location at the vertex of the angle AOB.

[6] The problem of obtaining contact between two *flat* surfaces is somewhat easier because the surfaces can be *slid* into contact so that minute particles are pushed off by the advancing edges.

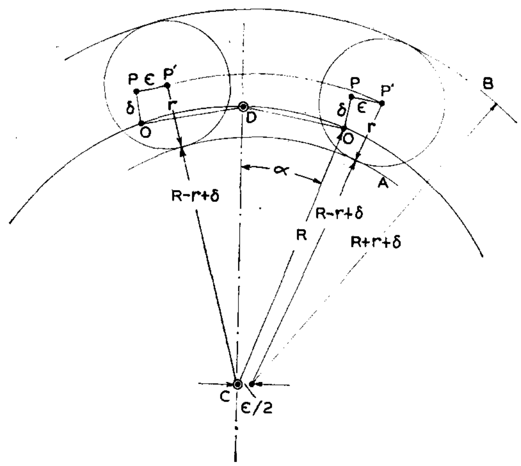

FIG. 9. Geometry of errors resulting from displacements ϵ and δ of the grinding wheel from its correct location.

from all ten regions to within less than 0.1 mm of the same focal position at two meters distance from the crystal. A full-sized reproduction of the fluorescent K-spectrum of silver made with this crystal is shown in the companion article on the spectrometer.

APPENDIX

Analysis of the Errors Resulting From Small Displacements of the Grinding Wheel

There is an alternative way of regarding the kinematics of the process of generating both convex and concave surfaces which has some advantages from the point of view of analysis of the errors coming from small displacements of the grinding wheel away from its correct location. If we regard the rigid rolling system 8 (consisting of the block to be profiled, the dividing head, and the arm 5) *as our fixed system of reference*, then the lines 10 and 13 of Figs. 3A and B, which include the fixed acute angle θ between them, can be thought of as a rigid angle (see Fig. 7) whose two legs are constrained to pass, respectively, through the two points C and D. Note that the motion may continue so that D may lie either on OB or as in the position $A''O''B''$ *on the extension of OB beyond the vertex O*. It is obvious then that the vertex O will describe a circle relative to system 8 (the system in which the work is stationary), and since the vertex O is the center of the grinding wheel, the circumference of the grinding wheel will envelop another circle which is the profile generated on the work.

Now, however, if at the final cut the grinding-wheel center does not coincide in height with the center of the dividing head, then instead of the circle generated by the vertex O of the rigid angle AOB of Fig. 7 there will be generated a curve of higher degree by the motion of the slightly offset point P, situated at the extremity of a short perpendicular to OB erected at O as shown in Fig. 8. Again if the line 10 in Figs. 3A and B (defined by the roller and straight edge) does not intersect the line 13 of those figures exactly at the center of the grinding wheel there may also be introduced an error of positioning of the generating point *along* the line OB as at P' in Fig. 8.

It becomes necessary, therefore, to analyze the departures from a circle of the higher degree curves generated by the motions of points such as P or P' slightly offset from the vertex O of the rigid angle AOB. We have to bear in mind that the arc generated by this method is that part for which the vertex O lies on either side and not far distant from the fixed point D. We have also to bear in mind that the profile generated by the grinding wheel is not the curve generated by P or P' but the envelope of circles of radius r whose centers lie on the latter curve.

Since the point O describes a circle of radius R we may for the purpose of analysis dispense with the line AO and the point C and utilize the simplified construction of Fig. 9. In this figure O is the end of the radius R which rotates about the fixed center C. OD is a chord (of variable length) of the circle of radius R passing through the point D in the circumference. The points P and P' are rigidly related to this chord, being, respectively, at the apex P of the perpendicular to this chord (of height δ) erected at O and at a point a small distance ϵ offset from P in a direction parallel to the chord DO. ϵ and δ may be either positive or negative. They are represented in the positive sense in Fig. 9.

We are interested in computing the errors in the generated arcs A and B as regards their departures from a true circle, the errors being measured in the direction of the normals to the curve. We shall take as the true circle with which these arcs are compared the circles of radii $R-r+\delta$ and $R+r+\delta$ which (for small ϵ) are substantially identical with the arcs A and B, respectively, when O is close to D. The short line OP is parallel to the bisector of the angle $\alpha(<DCO)$, and hence OP makes an acute angle with CO equal to $(\alpha/2)$. Because ϵ and δ are small in comparison to R we can write with good approximation that the errors in the arcs A and B, namely E_A and E_B, are

$$E_B = E_A = \epsilon \sin(\alpha/2) - \delta[1-\cos(\alpha/2)]. \qquad (2)$$

This formula gives the error positive when the arcs A or B lie *outside* the circle to which they are compared. Note that since $\sin(\alpha/2)$ is an odd function of α, the displacement ϵ causes the arc to lie *outside* the circle when O is on the one side of D and *inside* the circle when O is on the other side of D. The displacement δ, on the contrary, always causes the arc to lie inside the circle (so long as δ is positive as shown), and hence introduces an error symmetrically distributed about D.

Formula 2 can be simplified if we replace $\sin(\alpha/2)$ by $x/(2R)$ and $(1-\cos(\alpha/2))$ by $x^2/(8R^2)$, approximations which are very accurate if α is small. Here x is the distance along the chord of the cylindrical arc measured from the center of the block on which the profile is generated.

$$E_B = E_A = (\epsilon x/2R) - (\delta x^2/8R^2) \qquad (3)$$

As an example, let us take $R = 200$ cm, $x = \pm 6$ cm, $\delta = 0.2$ cm, and $\epsilon = 0.2$ cm. We find for the two terms above (written in the same order as in Eq. 3) the following values:

$$E_A = E_B = \pm 0.003 - 0.0000225 \text{ cm}.$$

Clearly, vertical displacements of the grinding wheel axis of the order of two millimeters from coincidence with the height of the axis of the dividing head, are tolerable. Longitudinal displacements of the grinding wheel axis produce somewhat more serious errors. Actually however even these errors vanish if, instead of comparing the generated profile with a reference circle whose center is exactly in the perpendicular bisector of the block, we compare it with another true circle whose center is displaced in the x direction by an amount $\epsilon/2$ in the sense PP' of Fig. 8, i.e., in the direction of positive ϵ. In fact, the errors introduced by the first term in Eq. 3 are a linear function of x; the whole profile is tilted through an angle $\epsilon/(2R)$ counterclockwise. This is the same as though the center of the circle were displaced to the right a distance $\epsilon/2$. Thus a horizontal error in the location of the center of the grinding wheel of two millimeters merely throws the profile of the generated arc off center on the block by 0.5 mm. As far as the *circularity* of the profile is concerned, the term in ϵ introduces no error whatever to the high order of approximation calculated here. We have found these predictions verified in practice. Actually the ordinary errors of a grinding machine from variable oil-film thicknesses, vibration, and elastic deflections are larger than those above computed.

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A High Resolving Power, Curved-Crystal Focusing Spectrometer for Short Wave-Length X-Rays and Gamma-Rays*

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Description is given of a transmission-type, curved-crystal focusing spectrometer for short wave-length x-rays, and gamma-rays having a dispersion of 1.186 x.u. per mm at short wave-lengths. The spectrometer utilizes the (310) planes of quartz in a crystalline plate of dimensions 80×70 ×1.0 mm curved cylindrically to a radius of two meters. High luminosity is obtained since the useful aperture in the crystal holder has an area of 10 cm² and subtends 0.00025 stereradians at the focus. It also affords high resolution since by photographic tests with x-rays the curved plate has been shown to focus a specified x-ray wave-length to within 0.06 mm of the same position on the focal circle for all parts of its useful aperture and over the entire operating wave-length range. The geometry of the mechanism permits absolute measurements with a precision screw of the sine of the Bragg angle on both sides of the reflecting planes, affording a wave-length range which includes at longest wave-lengths the K-spectrum of silver and goes down to zero wave-lengths. For short wave-length gamma-rays the source is placed at the focus. A multiple-slit collimator of tapering die-cast lead partitions spaced apart with tapering separators, is used at short wave-lengths to transmit the monochromatic diffracted beam and absorb the directly transmitted and scattered heterogeneous beam. The present collimator limits the spectrum that can be studied to a shortest wave-length of 7. x.u. corresponding to 1.75 Mev. The intensity of the diffracted beam is to be measured with a special multi-cellular G. M. counting tube of high efficiency, provided with a number of thin lead partitions through which the beam passes successively. In photographic spectra made with this instrument the tungsten and also the silver $K\beta_1\beta_3$ doublet is completely and clearly resolved. Reproductions of such photographic x-ray spectra are shown in which the line breadths have substantially the natural breadth. Fluorescence spectra of silver have been made in 10-minute exposures. A companion paper gives the all-important precision technique of generating the curved cylindrical stainless steel clamping blocks for the crystal.

I. THE GEOMETRICAL OPTICS OF THE CURVED-CRYSTAL FOCUSING SPECTROMETER

TWO types of curved-crystal focusing spectrometer may conveniently be distinguished: the transmission and the reflection types. The exact solution of this problem for both these cases was conceived by the author in 1927, and it was first stated in the literature[1] with explicit diagrams showing the exact geometrical solution for both cases in a paper describing a multiple crystal x-ray spectrograph, an instrument constructed for the study of the structure of the Compton line which utilized the same geometry applied, however, to many little crystals

* This research is now being conducted under Navy Contract N6onr-244 Task Order IV, dated March 1, 1947.

[1] J. W. DuMond and H. A. Kirkpatrick, "Multiple crystal x-ray spectrograph," Rev. Sci. Inst. 1, 88 (1930), see Fig. 2.