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Cuts, Cancellations and the Closed Time Path: The Soft Leptogenesis Example

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Abstract

By including all leading quantum-statistical effects at finite temperature, we show that no net asymmetry of leptons and sleptons is generated from soft leptogenesis, save the possible contribution from the resonant mixing of sneutrinos. This result contrasts with different conclusions appearing in the literature that are based on an incomplete inclusion of quantum statistics. We discuss vertex and wave-function diagrams as well as all different possible kinematic cuts that nominally lead to CP-violating asymmetries. The present example of soft leptogenesis may therefore serve as a paradigm in order to identify more generally applicable caveats relevant to alternative scenarios for baryogenesis and leptogenesis, and it may provide useful guidance in constructing viable models.

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I. INTRODUCTION

For many extensions of the Standard Model (SM), additional degrees of freedom and parameters entail the possibility of new CP-violating (CPV) phases besides the observed one present in the CKM matrix. In the absence of further modifications, the SM fails to explain the observed baryon asymmetry of the Universe (BAU), so it is interesting to investigate whether the new degrees of freedom and phases may remedy this shortcoming. It would be particularly interesting if the new particles and phases responsible for the BAU were experimentally accessible through high energy collisions close to the electroweak scale (or not too far above it) or through the observation of permanent electric dipole moments (EDMs) (for recent reviews and extensive references, see Refs. [1, 2]).

A particularly rich model with such phenomenological and cosmological prospects is the Minimal Supersymmetric Standard Model supplemented by right-handed singlet neutrinos (νMSSM). New CPV phases can be present within the triscalar couplings and the masses that lead to soft supersymmetry breaking in conjunction with the masses and couplings in the superpotential. This model predicts supersymmetric particles with masses close to electroweak scale, new CPV signals, and possibly an explanation for the emergence of the BAU through the mechanism of soft leptogenesis [3–8].

Of course, the MSSM is of great interest because it offers a solution to the hierarchy problem, provides for radiative electroweak symmetry-breaking, and contains particle candidates for Dark Matter. Moreover, many of its features are paradigmatic for other extensions of the SM. Therefore, it is a highly suitable arena for developing theoretical techniques needed for robust computations of the BAU, identifying related observable CPV effects, and delineating benchmarks for future experimental CPV searches.

These remarks outline the context of the present paper. Using the example of soft leptogenesis in the νMSSM, we study the question whether experimentally accessible CPV may be linked to non-resonant variants of baryogenesis or leptogenesis from out-of-equilibrium decays or inverse decays close to the electroweak scale. Successful baryogenesis from out-of-equilibrium decays, its occurrence at low temperatures (close to the electroweak scale) and experimentally observable CPV are often incompatible requirements. However, it has been proposed that even in absence of resonant enhancement, soft leptogenesis is a viable mechanism for baryogenesis at relatively low temperatures, with the mass of the decaying singlet neutrino and the temperature at which leptogenesis takes place possibly being of order of the electroweak scale [6–8].

At first glance, this may not appear possible, as the diagrams for vacuum decays of the singlet neutrinos that lead to the lepton and slepton asymmetries do not involve (s)lepton number violation in the internal lines. It follows from the CPT theorem that in the vacuum, the produced asymmetries in leptons and sleptons are precisely opposite [6]. Assuming a fast equilibration between particles and sparticles, no net lepton number is produced. It has been argued in Refs. [6–8] that a loophole opens at finite temperature, wherein phase space modifications associated with Fermi suppression (leptons) and Bose enhancement (sleptons) render the vacuum cancellation ineffective. Should this loophole, indeed, prove to be viable, one could anticipate a variety more phenomenologically interesting models of baryogenesis from out-of-equilibrium decays besides soft leptogenesis that link the BAU with experimentally accessible new particles and CPV signals.

Assessing the viability of the proposal requires careful scrutiny of the unitary evolution of the full set of quantum statistical states as well as thermal effects. The quantitative analysis in Ref. [6] only takes account of the quantum statistical corrections for the external states but not those appearing in the loops. However, when calculating the imaginary parts of the loop diagrams using Cutkosky rules, it is immediately obvious (and also well-known) that the imaginary part relevant for the asymmetry arises from momentum regions where particles on internal lines are on-shell. It is, thus, natural to ask whether one must also consider quantum statistical enhancement and suppression factors for the internal propagators as well. In what follows, we demonstrate that it is, indeed, necessary to consider the thermal statistical factors for internal lines and that doing so closes the loophole proposed in Refs. [6–8] .

Although we focus on the model of soft leptogenesis for concreteness, we emphasize that our analysis generalizes to other scenarios, illustrating important features that should be taken into account whenever an asymmetry is supposed to be generated from out-of-equilibrium dynamics in a spatially homogeneous background. For a recent example of the latter, see Ref. [9], wherein a crucial cancellation of the final asymmetry is missed. In this respect, the present work provides a useful guideline for building successful models of baryogenesis and leptogenesis.

To set the stage for our discussion, we note that in the conventional approach to leptogenesis followed in Refs. [6–8] , which combines S-matrix elements from quantum theory with Boltzmann equations from classical physics, no explicit set of rules for correctly including these quantum statistical effects has been worked out. In particular, it is not immediately clear whether the on-shell contributions in internal propagators of loop diagrams are already accounted for through subsequent tree-level scatterings that are described in the Boltzmann equations. This issue is of pivotal relevance for the correct calculation of the asymmetry. In standard leptogenesis with classical statistics (which is a good approximation in the strong washout regime), one may take account of this through the procedure of real intermediate state (RIS) subtraction.

More specifically, when an external particle attaches to a loop of two or more particles, potentially CPV cuts occur from the momentum region where the loop particles are on-shell.[1] In order to ensure a unitary evolution, one must also include in the Boltzmann equations scattering diagrams where the unstable particle appears in an internal line. To avoid double-counting, the RIS must then be subtracted from the scattering rates in such a way that no charge asymmetry is generated in equilibrium. Given the care required in identifying and performing the full set of RIS subtractions, it is perhaps not surprising that its implementations in Refs. [6, 9] were not complete and, as a result, yield spurious, non-vanishing asymmetries.

In performing the RIS subtraction, one encounters two cases, corresponding to whether or not the on-shell intermediate particle is in equilibrium. The present example of soft leptogenesis illustrates both situations. Looking ahead to our detailed calculation in Sections III and IV, we summarize the key physics for each.

[1] At zero temperature, this implies that the sum of the masses of the loop particles must be below the mass of the external particle. At finite temperature, also kinematically allowed crossings of the internal propagators contribute to the cuts.

(1) For the vertex contributions, the intermediate on-shell particle is an out-of equilibrium singlet neutrino N (see Fig. 3). In this case, even when the subtraction of RIS is performed correctly[2], there occurs a cancellation due to opposite asymmetries in leptons ℓ and sleptons $\widetilde{\ell}$. A derivation of this cancellation requires the correct inclusion of all quantum statistical factors for the external states as well as for intermediate on-shell particles (see again Fig. 3). Quantum statistics has only partly been accounted for in Ref. [6], which is why the precise cancellation is missed there.

(2) On the other hand, when the RIS that is to be subtracted corresponds to a particle that is in equilibrium, such as the scalar Higgs doublet H_1 in Section IV (see Fig. 7), no asymmetry is generated in first place. This type of subtraction of equilibrium RIS and the consequent vanishing of the CPV asymmetry has been missed in the context of a different model in Ref. [9]

Alternatively, there exists a way of deriving the leptogenesis kinetic equations following a set of rules that intrinsically respect the unitary evolution of the system while sidestepping the pitfalls of the RIS procedure: the Closed Time Path (CTP) formalism [10–15]. Rather than formulating the problem in terms of S-matrix elements and classical particle distribution functions, the evolution of Green functions of the quantum fields is calculated. In particular, the imaginary parts of self-energies correspond to the inclusive decay and production rates that are necessary in order to track the evolution of the asymmetry. This way, the somewhat heuristic procedure of RIS subtraction can be avoided [16–31].

In the present work, we calculate the source terms for the asymmetry using the CTP formalism. As our main result, we demonstrate that the resulting asymmetry of the lepton number vanishes even when taking into account quantum statistical corrections. In particular, the corrections associated with the internal lines precisely cancel those associated with the final states that are included in Refs. [6–8]. Consequently, the sum

[2] The subtraction is effected either by systematic derivation as performed here or simply by imposing vanishing net CPV rates in equilibrium.

of the lepton and slepton asymmetries is zero[3]. We also note that while we perform the explicit calculations in the context of soft leptogenesis, our findings can be straightforwardly generalized to other conceivable variants of baryogenesis or leptogenesis that exhibit similar CPV diagrammatic cuts.

The plan of this paper is as follows: In Section II, we present the Lagrangian that is relevant for soft leptogenesis. Furthermore, we define the collision terms that enter kinetic equations, which can be used to calculate the lepton asymmetry. The main subject of the present paper are the CPV contributions to the collision terms and the crucial cancellations that these exhibit. In Section III, we start with the general CTP expression for the vertex-type self-energy that may lead to the production of asymmetries. We then describe the strategy for extracting the particular CPV contributions from this self-energy. In the remaining Subsections of Section III, we consider various kinematic cuts and demonstrate that in each case, there is a cancellation of the asymmetries. The corresponding calculations for the wave-function type self-energy are presented in Section IV. As we discuss in Section V, some of our results can be related to the RIS subtraction procedure that, as noted above, is routinely used in standard calculations for leptogenesis . Conclusions are presented in Section VI.

II. KINETIC EQUATIONS IN THE CTP FRAMEWORK

The authors of Refs. [6–8] considered the impact of CPV phases that appear in the soft SUSY-breaking singlet sneutrino ($\widetilde{N}$) and wino ($\widetilde{W}$) mass terms as well as the trilinear interactions involving the slepton and Higgs doublets and $\widetilde{N}$. For pedagogical purposes, we will consider a different source of CPV associated with the relative phase of the bino $\widetilde{B}$ mass term and the supersymmetric μ parameter, though the logic and result (vanishing total lepton number asymmetry) in both cases will be the same. Consequently, in the Lagrangian below, we do not include singlet sneutrinos and winos.

[3] One should mention however, that at temperatures above $10^7\,\mathrm{GeV}$, the equilibration of leptons and sleptons becomes ineffective, because it is suppressed by a helicity flip of the mediating gaugino [32]. As leptons and sleptons suffer different washout rates, a net asymmetry can emerge in such a situation.

Singlet sneutrinos are of particular interest when their b-term is small compared to their lepton-number violating mass and they induce a splitting into two almost degenerate mass eigenstates [3–5]. This opens up the possibility for a variant of resonant leptogenesis. However, in the present paper, we restrict our analysis to the non-resonant regime. Since singlet sneutrinos can play a role for non-resonant CPV that is analogous to the one of the singlet neutrinos, the analysis presented here for fermionic singlets generalizes in a straightforward manner to the bosonic case. Thus, we do not reiterate it here. Similarly, the effects from bino-mediated interactions in soft leptogenesis that we present in this paper are analogous to those mediated by winos. For simplicity, we therefore omit the discussion of the latter.

After suitable field redefinitions through rephasings the νMSSM, mass and interaction terms relevant to our analysis are

$$
\begin{aligned}
\mathcal{L} \supset & -(m_{Hu}^2+\mu^2)H_u^\dagger H_u-(m_{Hd}^2+\mu^2)H_d^\dagger H_d - b\left(H_u^T H_d + H_u^\dagger H_d^*\right)\\
& -\mu\bar\Psi_{\widetilde H^+}\Psi_{\widetilde H^+}-\mu\bar\Psi_{\widetilde H^0}\Psi_{\widetilde H^0}-\frac{1}{2}M_1\bar\Psi_{\widetilde B}\Psi_{\widetilde B}-\frac{1}{2}m_N\bar\Psi_N\Psi_N-\widetilde\ell^\dagger m_{\widetilde\ell}^2\widetilde\ell\\
& -\frac{g_1}{\sqrt{2}}\left[\bar\Psi_{\widetilde H^+}\left(-H_d^{-*}P_{\rm L}+{\rm e}^{{\rm i}\phi_\mu}H_u^+P_{\rm R}\right)\Psi_{\widetilde B}+\bar\Psi_{\widetilde H^0}\left(-H_d^{0*}P_{\rm L}-{\rm e}^{{\rm i}\phi_\mu}H_u^0P_{\rm R}\right)\Psi_{\widetilde B}+{\rm h.c.}\right]\\
& +\frac{g_1}{\sqrt{2}}\left[\widetilde\nu_{\rm L}^*\bar\Psi_{\widetilde B}P_{\rm L}\nu_{\rm L}+\widetilde e_{\rm L}^{-*}\bar\Psi_{\widetilde B}P_{\rm L}e_{\rm L}^-+{\rm h.c.}\right]\\
& -\left[Y{\rm e}^{-{\rm i}\phi_Y}\left(H_u^+\bar\Psi_NP_{\rm L}e_{\rm L}-H_u^0\bar\Psi_NP_{\rm L}\nu_{\rm L}\right)+{\rm h.c.}\right]\\
& +\left[Y{\rm e}^{-{\rm i}\phi_\mu-{\rm i}\phi_Y}\left(\widetilde\nu_{\rm L}\bar\nu_{\rm R}P_{\rm L}\Psi_{\widetilde H^0}+\widetilde e_{\rm L}^-\bar\nu_{\rm R}P_{\rm L}\Psi_{\widetilde H^+}\right)+{\rm h.c.}\right]\,, \qquad (1)
\end{aligned}
$$

where

$$
\Psi_{\widetilde H^+}=\begin{pmatrix}\widetilde H_u^+\\ \widetilde H_d^{-\dagger}\end{pmatrix},\quad \Psi_{\widetilde H^0}=\begin{pmatrix}-\widetilde H_u^0\\ \widetilde H_d^{0\dagger}\end{pmatrix},\quad \Psi_{\widetilde B}=\begin{pmatrix}\widetilde B\\ \widetilde B^\dagger\end{pmatrix},\quad \Psi_N=\begin{pmatrix}N\\ N^\dagger\end{pmatrix}. \qquad (2)
$$

Within the symmetric electroweak phase, the scalar Higgs fields are transformed to a diagonal basis through

$$
\begin{pmatrix}H_u^+\\ H_d^{-*}\end{pmatrix}=\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}H_1^+\\ H_2^+\end{pmatrix},\quad \begin{pmatrix}H_u^0\\ H_d^{0*}\end{pmatrix}=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}H_1^0\\ H_2^0\end{pmatrix}, \qquad (3)
$$

where

$$\tan 2\alpha = \frac{2b}{m_{Hu}^2 - m_{Hd}^2}\,. \tag{4}$$

The mass-square eigenvalues for $H^{0,+}_{1,2}$ are

$$m_{H1,2}^2 = \frac{1}{2}\left(m_{Hu}^2 + m_{Hd}^2 \pm \sqrt{(m_{Hu}^2 - m_{Hd}^2)^2 + 4b^2}\right)\,. \tag{5}$$

For $m_{Hd}^2 - m_{Hu}^2 \gg |b|$, as is often assumed in particular MSSM scenarios, the mixing angle is approximately given by $\sin\alpha \approx 2b/(m_{Hu}^2 - m_{Hd}^2)$, and the mass squares are $m_{H1}^2 \approx m_{Hu}^2$ and $m_{H2}^2 \approx m_{Hd}^2$.

The kinetic equations for soft leptogenesis need to track the distribution of singlet neutrinos, $f_N(\mathbf{k})$ and the asymmetries of leptons $f_\ell(\mathbf{k}) - \bar f_\ell(\mathbf{k})$ and of sleptons $f_{\tilde\ell}(\mathbf{k}) - \bar f_{\tilde\ell}(\mathbf{k})$. The network of kinetic equations can be expressed as

$$\frac{d}{d\eta}\left(f_\ell(\mathbf{k}) - \bar f_\ell(\mathbf{k})\right) = \mathcal{C}_\ell(\mathbf{k}) = \int \frac{dk^0}{2\pi}\mathrm{tr}\left[\mathrm{i}\not\Sigma_\ell^>(k)\mathrm{i}S_\ell^<(k) - \mathrm{i}\not\Sigma_\ell^<(k)\mathrm{i}S_\ell^>(k)\right]\,, \tag{6a}$$

$$\frac{d}{d\eta}\left(f_{\tilde\ell}(\mathbf{k}) - \bar f_{\tilde\ell}(\mathbf{k})\right) = \mathcal{C}_{\tilde\ell}(\mathbf{k}) = -\int \frac{dk^0}{2\pi}\left[\mathrm{i}\Pi_{\tilde\ell}^>(k)\mathrm{i}\Delta_{\tilde\ell}^<(k) - \mathrm{i}\Pi_{\tilde\ell}^<(k)\mathrm{i}\Delta_{\tilde\ell}^>(k)\right]\,, \tag{6b}$$

$$\frac{d}{d\eta}f_N(\mathbf{k}) = \mathcal{C}_N(\mathbf{k}) = \frac{1}{4}\int \frac{dk^0}{2\pi}\mathrm{sign}(k^0)\mathrm{tr}\left[\mathrm{i}\not\Sigma_N^>(k)\mathrm{i}S_N^<(k) - \mathrm{i}\not\Sigma_N^<(k)\mathrm{i}S_N^>(k)\right]\,, \tag{6c}$$

where η is the conformal time and k denotes the conformal momentum. The terms $\mathcal{C}$ are referred to as the collision terms. The propagators iΔ (bosons), iS (fermions) and the distribution functions are introduced in Appendix A. The expansion of the Universe is then implicitly taken into account when obtaining the on-shell conformal four-momentum of a particle of mass m through the relation $k^0 = \pm\sqrt{\mathbf{k}^2 + a^2m^2}$, where a is the scale-factor (see Ref. [22] for more details). For present purposes, it is useful to decompose the self-energies as follows:

$$\mathrm{i}\not\Sigma_\ell = \mathrm{i}\not\Sigma_\ell^{\mathrm{l}} + \mathrm{i}\not\Sigma_\ell^{\mathrm{v}} + \mathrm{i}\not\Sigma_\ell^{\mathrm{w}} + \ldots\,, \tag{7a}$$

$$\mathrm{i}\Pi_\ell = \mathrm{i}\Pi_{\tilde\ell}^{\mathrm{l}} + \mathrm{i}\Pi_{\tilde\ell}^{\mathrm{v}} + \mathrm{i}\Pi_{\tilde\ell}^{\mathrm{w}} + \ldots\,, \tag{7b}$$

$$\mathrm{i}\not\Sigma_N = \mathrm{i}\not\Sigma_N^{\mathrm{l}}\,, \tag{7c}$$

where the superscript "l" indicates the leading order contributions that result from one-loop diagrams, "v" indicates the leading CP-violating contributions from vertex corrections, "w" indicates the leading CP-violating contributions from wave-function corrections, that both result from two-loop diagrams, and the ellipses represent higher order terms that are irrelevant for the calculation of the lepton asymmetry. In an analogous manner, we also decompose the collision terms into $\mathcal{C}_\ell^{\rm l,v,w}$ and $\mathcal{C}_{\widetilde\ell}^{\rm l,v,w}$.

Note that obtaining the solution to Eqs. (6) is not the aim of this paper. They are presented here in order to illustrate the context in which the main objects of our scrutiny – the CPV contributions to the collision terms – occur. For the sake of completeness and setting notation, we list the leading, CP-conserving terms

$$\begin{aligned}
{\rm i}\Pi_{\widetilde\ell}^{{\rm l}ab}(k) =& -Y^2\int\frac{d^4p}{(2\pi)^4}{\rm tr}\left[{\rm i}S_N^{ab}(p+k)P_{\rm L}{\rm i}S_{\widetilde H}^{ba}(p)P_{\rm R}\right] \\
&-\frac{g_1^2}{2}\int\frac{d^4p}{(2\pi)^4}{\rm tr}\left[{\rm i}S_\ell^{ab}(p+k)P_{\rm R}{\rm i}S_{\widetilde B}(-p)P_{\rm L}\right]\,,
\end{aligned} \tag{8a}$$

$$\begin{aligned}
{\rm i}\Sigma_\ell^{{\rm l}ab}(k) =& Y^2\int\frac{d^4p}{(2\pi)^4}P_{\rm R}{\rm i}S_N^{ab}(p+k)P_{\rm L}{\rm i}\Delta_H^{ba}(p) \\
&+\frac{g_1^2}{2}\int\frac{d^4p}{(2\pi)^4}{\rm i}\Delta_{\widetilde\ell}^{ab}(p+k)P_{\rm R}{\rm i}S_{\widetilde B}^{ab}(-p)P_{\rm L}\,,
\end{aligned} \tag{8b}$$

$$\begin{aligned}
{\rm i}\Sigma_N^{{\rm l}ab}(k) =& Y^2\int\frac{d^4p}{(2\pi)^4}\Big[P_{\rm L}{\rm i}S_\ell^{ab}(p+k)P_{\rm R}{\rm i}\Delta_H^{ab}(-p)+C\left[P_{\rm L}{\rm i}S_\ell^{ba}(-p-k)P_{\rm R}\right]^T C^\dagger{\rm i}\Delta_H^{ba}(p) \\
&+{\rm i}\Delta_{\widetilde\ell}^{ab}(p+k)P_{\rm L}{\rm i}S_{\widetilde H}^{ab}(-p)P_{\rm R}+{\rm i}\Delta_{\widetilde\ell}^{ba}(-p-k)C\left[P_{\rm L}{\rm i}S_{\widetilde H}^{ba}(p)P_{\rm R}\right]^T C^\dagger\Big]\,.
\end{aligned} \tag{8c}$$

The superscripts a, b, *etc.* take on values +1 or -1, depending on whether the originating or terminating vertex lies on the forward or backward going branch of the closed time path, respectively.

III. CANCELLATION OF CONTRIBUTIONS FROM VERTEX DIAGRAMS

We now proceed to demonstrate the vanishing of the CPV lepton number asymmetry, organizing the discussion according to different diagram topologies. One may map the latter onto the amplitudes entering conventional asymmetry calculations according to the relevant cuts. The first class, which we denote as "vertex-type" self-energies, correspond to asymmetry contributions generated by the interference of tree-level and one-loop vertex correction amplitudes. The second class, analyzed in Section IV, are equivalent to the interference of tree-level and one-loop wavefunction correction graphs.

A. Vertex-Type Self-Energies in the CTP

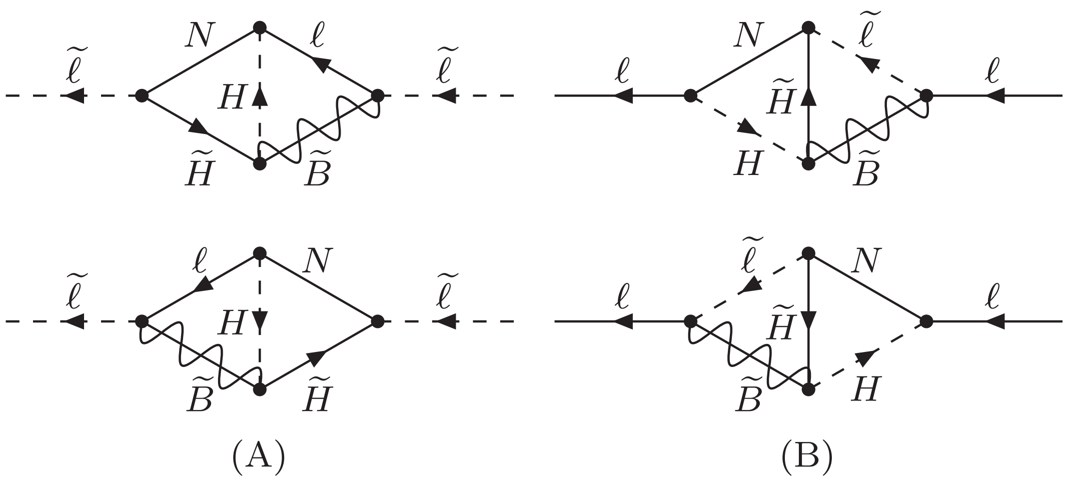

FIG. 1: The vertex-type self-energies $\mathrm{i}\Pi^{\mathrm{v}}_{\widetilde{\ell}}$ (A) and $\mathrm{i}\not{\Sigma}^{\mathrm{v}}_{\ell}$ (B).

The relevant vertex-type self-energies that lead to source terms of asymmetries in ℓ and $\widetilde{\ell}$ are represented diagrammatically in Fig. 1. We evaluate these and extract the contributions that are important when ℓ, $\widetilde{\ell}$, $H_{1,2}$, $\Psi_{\widetilde{H}^{\pm}}$ and $\Psi_{\widetilde{B}}$ are in equilibrium following Ref. [22].

The diagrams in Fig. 1(A) correspond to the CTP self-energy for the slepton $\widetilde{\ell}$

$$\mathrm{i}\Pi^{\mathrm{v}ab}_{\widetilde{\ell}}(k) = - cdY^2 \frac{g_1^2}{2} \mathrm{tr} \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \Big\{ \tag{9}$$

$$P_{\mathrm{L}} \mathrm{i}S^{ac}_{\ell}(-p) \mathrm{i}S^{cb}_{N}(q+k) P_{\mathrm{L}} \mathrm{i}S^{bd}_{\widetilde{H}}(q) P_{\mathrm{L}} \mathrm{i}S^{da}_{\widetilde{B}}(-p-k) \mathrm{i}\Delta^{dc}_{H1}(p+k+q) \sin\alpha\cos\alpha \mathrm{e}^{-\mathrm{i}\phi_\mu}$$

$$+ P_{\mathrm{R}} \mathrm{i}S^{ac}_{N}(-p) \mathrm{i}S^{cb}_{\ell}(q+k) P_{\mathrm{R}} \mathrm{i}S^{bd}_{\widetilde{B}}(q) P_{\mathrm{R}} \mathrm{i}S^{da}_{\widetilde{H}}(-p-k) \mathrm{i}\Delta^{cd}_{H1}(-p-k-q) \sin\alpha\cos\alpha \mathrm{e}^{\mathrm{i}\phi_\mu} \Big\} \,.$$

For simplicity, we assume here and in the following that $m_{H2} \gg m_{H1}$ and $m_{H2} \gg T$, such that contributions form an internal line of H_2 may be neglected. The more general expressions that also include H_2 propagators can easily be inferred from the results presented here. Furthermore, we have only accounted for those contributions that lead to CP-violation (i.e. we have neglected additional terms originating from the mixing of $H_{1,2}$ which do not lead to CP-violation).

The relevant contributions to the self-energy of the lepton ℓ are depicted in Fig. 1(B). Again, we keep only the CPV contribution that is mediated by the lighter Higgs boson H_1. We find

$$
\begin{aligned}
\mathrm{i}\not{\Sigma}_\ell^{\mathrm{v}ab}(k) =& cdY^2\frac{g_1^2}{2}\int\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\Big\{ \qquad (10)\\
& P_\mathrm{R}\mathrm{i}S_{\widetilde{B}}^{ad}(p+k)P_\mathrm{R}\mathrm{i}S_{\widetilde{H}}^{dc}(p+k+q)P_\mathrm{R}\mathrm{i}S_N^{cb}(q+k)P_\mathrm{L}\mathrm{i}\Delta_{\widetilde{\ell}}^{ac}(-p)\mathrm{i}\Delta_{H1}^{bd}(q)\sin\alpha\cos\alpha\mathrm{e}^{\mathrm{i}\phi_\mu}\\
& +P_\mathrm{R}\mathrm{i}S_N^{ac}(-p)P_\mathrm{L}\mathrm{i}S_{\widetilde{H}}^{cd}(-p-k-q)P_\mathrm{L}\mathrm{i}S_{\widetilde{B}}^{db}(-q)P_\mathrm{L}\mathrm{i}\Delta_{\widetilde{\ell}}^{cb}(q+k)\mathrm{i}\Delta_{H1}^{da}(-p-k)\sin\alpha\cos\alpha\mathrm{e}^{-\mathrm{i}\phi_\mu}\Big\}\,.
\end{aligned}
$$

B. Extracting the CPV Contributions

For purposes of simplicity, we assume that those particles charged under the electroweak gauge group are in kinetic equilibrium. In order to extract the leading contributions to the asymmetries in leptons ℓ and sleptons $\widetilde{\ell}$, we substitute equilibrium distributions into the CPV sources $\mathcal{C}^{\mathrm{v,w}}$: $f_B(\mathbf{k}) = 1/[\exp(E/T){-}1]$ and $f_F(\mathbf{k}) = 1/[\exp(E/T){+}1]$ with $E=\sqrt{\mathbf{k}^2+m^2}$ for the bosons $B=\widetilde{\ell},H1,H2$ and fermions $F=\ell,\widetilde{H},\widetilde{B}$. For the bosonic and fermionic equilibrium propagators, one may also apply the Kubo-Martin-Schwinger (KMS) relations

$$\mathrm{i}\Delta^{>}(p) = \mathrm{e}^{p^0/T}\mathrm{i}\Delta^{<}(p)\,, \qquad (11a)$$

$$\mathrm{i}S^{>}(p) = -\mathrm{e}^{p^0/T}\mathrm{i}S^{<}(p)\,, \qquad (11b)$$

where for a general Green's function G^{ab} one has $G^{-+}\equiv G^{>}$ and $G^{-+}\equiv G^{<}$. The singlet neutrino N is in general out-of-equilibrium. We sometimes denote by δS_N or δf_N the difference between the propagator or the distribution function of N and their equilibrium counterparts. We note in passing that in the terms $\mathcal{C}^{\mathrm{l}}$, that describe the washout of the charges, distributions with chemical potential instead of the above equilibrium distributions should be substituted [22].

In order to calculate the leading CPV contribution from vertex diagrams, $\mathcal{C}^{\rm v}$, we may substitute tree-level equilibrium propagators for $\mathrm{i}S_\ell$, $\mathrm{i}\Delta_{\widetilde{\ell}}$, $\mathrm{i}S_{\widetilde{B}}$ and $\mathrm{i}\Delta_H$, while allowing for a non-equilibrium form for $\mathrm{i}S_N$. From Fig. 1 and from the expressions (9) and (10) for the self-energies, we see that for both leptons and sleptons, there occur two diagrammatic contributions to the collision terms that are related by complex conjugation and a reversed fermion flow. (Note that $\mathcal{C}^{\rm v}_{\ell,\widetilde{\ell}}$ may be interpreted as being a result of closing the external lines in the diagrams of Fig. 1.) In order to obtain a result that depends on the CPV phases, at least one of the internal propagators must be imaginary, i.e. off-shell. It is therefore suitable to distinguish the various contributions according to which of the particles is taken to be off-shell, writing $\mathcal{C}^{\rm v}_{\widetilde{\ell}} = \mathcal{C}^{{\rm v}\widetilde{B}}_{\widetilde{\ell}} + \mathcal{C}^{{\rm v}\widetilde{H}}_{\widetilde{\ell}} + \mathcal{C}^{{\rm v}H}_{\widetilde{\ell}} + \mathcal{C}^{{\rm v}\ell}_{\widetilde{\ell}}$ and $\mathcal{C}^{\rm v}_{\ell} = \mathcal{C}^{{\rm v}\widetilde{B}}_{\ell} + \mathcal{C}^{{\rm v}\widetilde{H}}_{\ell} + \mathcal{C}^{{\rm v}H}_{\ell} + \mathcal{C}^{{\rm v}\widetilde{\ell}}_{\ell}$ with the off-shell particle in each component indicated by the superscript. This procedure can also be understood from considering the corresponding diagrams for vacuum decay and their cuts that we provide for each case.

With these remarks in mind, we outline a procedure for extracting the CPV contributions from the self-energies (9) and (10). Essentially, it is the same method that is applied to conventional leptogenesis in Ref. [22]:

- Write down all terms that contribute to $\Sigma^>$ or $\Pi^>$. The $<$ terms follow when replacing all CTP indices $+ \leftrightarrow -$.

- Take one particular propagator off-shell, such that only the (anti)-time ordered components contribute.

- Write down the collision terms $\mathcal{C}^{\rm v}$ or $\mathcal{C}^{\rm wv}$. There are eight individual terms (for both, vertex and wave-function contributions).

- Make use of the relations between the two-point functions, e.g. $G^T + G^{\bar{T}} = G^> + G^<$ is useful. Other identities, that hold under the integrals and will be used for the vertex diagrams are presented in Ref. [22]. (See, for example, relation (13) below.)

- Use KMS relations in order to establish the cancellation or the vanishing of the particular contributions.

- Make use of the behavior of the G^{ab} under momentum reversals, while assuming spatial isotropy of the collision terms: $\mathcal{C}_\ell(\mathbf{k}) = \mathcal{C}_\ell(-\mathbf{k})$ *etc.*

C. Cuts with off-shell $\widetilde{B}$

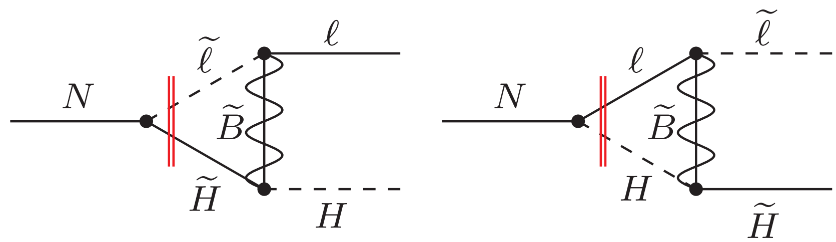

FIG. 2: Cuts in vacuum diagrams with off-shell bino.

In this Section, we take the bino $\widetilde{B}$ to be off-shell. The correspondence with the Boltzmann approach can been seen from the appropriate cuts in the diagrams of Fig. 1. First cut the graphs through the N line and either the $\widetilde{H}$ line in Fig. 1 (A) or the H line in Fig. 1 (B). The result is a contribution to the asymmetries for a final state ($\widetilde{\ell}$, $\widetilde{H}$) or (ℓ, H_1) pair, respectively, corresponding to the interference of the tree-level and one-loop vertex correction amplitudes. The off-shell bino contribution then corresponds to the additional cuts in the vertex correction graphs shown in Fig. 2. For this type of contribution, $\mathcal{C}_{\widetilde{B}}$, it has been proposed that when including thermal corrections, there will be a residual lepton asymmetry [6]. Instead of decays of the singlet fermion N, decays of its superpartner $\widetilde{N}$ are discussed in Ref. [6]. The considerations that are presented here can however be transferred to the decays of scalars straightforwardly.

In the vacuum, the asymmetries from decays of N that are stored in leptons and sleptons are precisely opposite. When calculating the CPV contributions that result from interference of the loop amplitudes in Fig. 2 with the tree-level amplitudes by using Cutkosky's rules, we see that no difference between the integration over internal on-shell particles $\widetilde{\ell}$, $\widetilde{H}$ or ℓ, H and the external phase space integral is made.

Now consider the decays and inverse decays in the finite temperature background of the Early Universe. It is stated in Ref. [6], that the quantum statistics of the external phase space renders the aforementioned cancellation ineffective. In the following, we show that the cancellation is reinstated once the statistical corrections for the internal particles are accounted for as well. As explained above, we assume that all particle number distributions except for the distribution of the right-handed neutrinos N take the equilibrium form. This allows us to frequently apply KMS relations for all propagators except for $S_N^{<,>}$. A calculation that is very analogous to the one in Ref. [22] leads to the CPV contribution to the collision term

$$
\begin{aligned}
\mathcal{C}_{\widetilde{\ell}}^{\mathrm{v}\widetilde{B}}(\mathbf{k}) =& Y^2 \frac{g_1^2}{4} \mathrm{tr} \int \frac{dk^0}{2\pi} \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \Big\{ \qquad (12) \\
& [\mathrm{i}\Delta_{H1}^{<}(p+k+q)\mathrm{i}S_\ell^{<}(-p) - \mathrm{i}\Delta_{H1}^{>}(p+k+q)\mathrm{i}S_\ell^{>}(-p)]\, P_\mathrm{R}\mathrm{i}\delta S_N(q+k) P_\mathrm{L} \\
& \times \left[\mathrm{i}S_{\widetilde{H}}^{<}(q)\mathrm{i}\Delta_{\widetilde{\ell}}^{<}(k) - \mathrm{i}S_{\widetilde{H}}^{>}(q)\mathrm{i}\Delta_{\widetilde{\ell}}^{>}(k)\right] P_\mathrm{L} S_{\widetilde{B}}^{T}(-p-k)\sin\alpha\cos\alpha \mathrm{e}^{-\mathrm{i}\phi_\mu} \\
& - P_\mathrm{R}\mathrm{i}\delta S_N(q+k)\, [\mathrm{i}\Delta_{H1}^{<}(p+k+q)\mathrm{i}S_\ell^{<}(-p) - \mathrm{i}\Delta_{H1}^{>}(p+k+q)\mathrm{i}S_\ell^{>}(-p)] \\
& \times P_\mathrm{R} S_{\widetilde{B}}^{T}(-p-k) P_\mathrm{R} \left[\mathrm{i}S_{\widetilde{H}}^{<}(q)\mathrm{i}\Delta_{\widetilde{\ell}}^{<}(k) - \mathrm{i}S_{\widetilde{H}}^{>}(q)\mathrm{i}\Delta_{\widetilde{\ell}}^{>}(k)\right] \sin\alpha\cos\alpha \mathrm{e}^{\mathrm{i}\phi_\mu} \Big\} \,.
\end{aligned}
$$

Notice that in transforming expression (9) to (12), we have made the replacements

$$
\begin{aligned}
& \mathrm{i}S_\ell^{>}(-p)\mathrm{i}\Delta_{H_1}^{>}(p+k-q) - \mathrm{i}S_\ell^{T,\bar{T}}(-p)\mathrm{i}\Delta_{H_1}^{T,\bar{T}}(p+k-q) \qquad (13a) \\
\to & -\frac{1}{2}\left(\mathrm{i}S_\ell^{<}(-p)\mathrm{i}\Delta_{H_1}^{<}(p+k-q) - \mathrm{i}S_\ell^{>}(-p)\mathrm{i}\Delta_{H_1}^{>}(p+k-q)\right) , \\
& \mathrm{i}S_\ell^{<}(-p)\mathrm{i}\Delta_{H_1}^{<}(p+k-q) - \mathrm{i}S_\ell^{T,\bar{T}}(-p)\mathrm{i}\Delta_{H_1}^{T,\bar{T}}(p+k-q) \qquad (13b) \\
\to & +\frac{1}{2}\left(\mathrm{i}S_\ell^{<}(-p)\mathrm{i}\Delta_{H_1}^{<}(p+k-q) - \mathrm{i}S_\ell^{>}(-p)\mathrm{i}\Delta_{H_1}^{>}(p+k-q)\right)
\end{aligned}
$$

that do not change the integrals, because the dispersive contributions from the product of (anti-) time-ordered propagators cancel, as explained in Appendix B and in in Ref. [22].

The collision term (12) clearly exhibits that in the CTP formalism, there is no distinction between cut particles and external states, because $\widetilde{H}$, $\widetilde{\ell}$ and H, ℓ appear in a symmetric way (exchange of the terms in the square brackets). This may be interpreted as a generalization of Cutkosky's rules from the vacuum background to finite densities.

From the self-energy (10), we find the CPV contribution to the lepton collision term with off-shell $\widetilde{B}$, which is

$$\begin{aligned}
\mathcal{C}_\ell^{\mathrm{v}\widetilde{B}}(\mathbf{k}) =& Y^2 \frac{g_1^2}{4} \sin\alpha\cos\alpha\, \mathrm{tr} \int \frac{dk^0}{2\pi} \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \Big\{ \qquad (14) \\
& \mathrm{i}S^T_{\widetilde{B}}(p+k) P_\mathrm{R} \left[\mathrm{i}S^<_{\widetilde{H}}(p+k+q)\mathrm{i}\Delta^<_{\widetilde{\ell}}(-p) - \mathrm{i}S^>_{\widetilde{H}}(p+k+q)\mathrm{i}\Delta^>_{\widetilde{\ell}}(-p)\right] P_\mathrm{R} \mathrm{i}\delta S_N(q+k) \\
& \times P_\mathrm{L} \left[\mathrm{i}\Delta^<_{H1}(q)\mathrm{i}S^<_\ell(k) - \mathrm{i}\Delta^>_{H1}(q)\mathrm{i}S^>_\ell(k)\right] \mathrm{e}^{\mathrm{i}\phi_\mu} \\
& -\mathrm{i}\delta S_N(q+k) P_\mathrm{L} \left[\mathrm{i}S^<_{\widetilde{H}}(p+k+q)\mathrm{i}\Delta^<_{\widetilde{\ell}}(-p) - \mathrm{i}S^>_{\widetilde{H}}(p+k+q)\mathrm{i}\Delta^>_{\widetilde{\ell}}(-p)\right] P_\mathrm{L} \mathrm{i}S^T_{\widetilde{B}}(p+k) \\
& \times P_\mathrm{L} \left[\mathrm{i}\Delta^<_{H1}(q)\mathrm{i}S^<_\ell(k) - \mathrm{i}\Delta^>_{H1}(q)\mathrm{i}S^>_\ell(k)\right] \mathrm{e}^{-\mathrm{i}\phi_\mu} \Big\}\,.
\end{aligned}$$

When relabeling the momentum variables, it is now easy to see that

$$\int \frac{d^3k}{(2\pi)^3} \mathcal{C}_{\widetilde{\ell}}^{\mathrm{v}\widetilde{B}}(\mathbf{k}) = -\int \frac{d^3k}{(2\pi)^3} \mathcal{C}_\ell^{\mathrm{v}\widetilde{B}}(\mathbf{k})\,. \qquad (15)$$

It is instructive to relate this cancellation to the conventional asymmetry computation and to identify the effects omitted in Ref. [6]. To that end, we show in Figs. 3(A), 3(A)* the cuts in the lepton self energy for an on-shell N, H_1, and ℓ and in Figs. 3(B), 3(B)* the corresponding interfering one-loop and tree-level amplitudes that enter a conventional asymmetry computation. In Fig. 4 we show the corresponding cuts and interfering amplitudes for the slepton self-energies for an on-shell N $\tilde{\ell}$, and $\tilde{H}$. The red cuts (color online) correspond to the interfering tree-level and vertex correction amplitudes for the N decay, while the orange cuts (color online) encompass both the off-shell bino-cuts in the vertex graphs computed above as well as the CPV remainder after the RIS subtraction is implemented for the $2 \leftrightarrow 2$ processes. In the procedure followed in Ref. [6], one would include statistical factors for the particles ℓ and H in Figs. 3(B), 3(B)* [sparticles $\widetilde{\ell}$ and $\widetilde{H}$ in Figs. 4(B), 4(B)*]but omit those for the sparticles $\widetilde{\ell}$ and $\widetilde{H}$ [particles ℓ and H in Figs. 4(B), 4(B)*](no matter whether these appear as external states in the scattering amplitudes or as internal lines in the vertex diagrams). The result would then be a non-vanishing lepton number asymmetry at finite temperature. In contrast, the CTP computation consistently includes the statistical factors for the internal lines as well, leading to the vanishing result of Eq. (15).

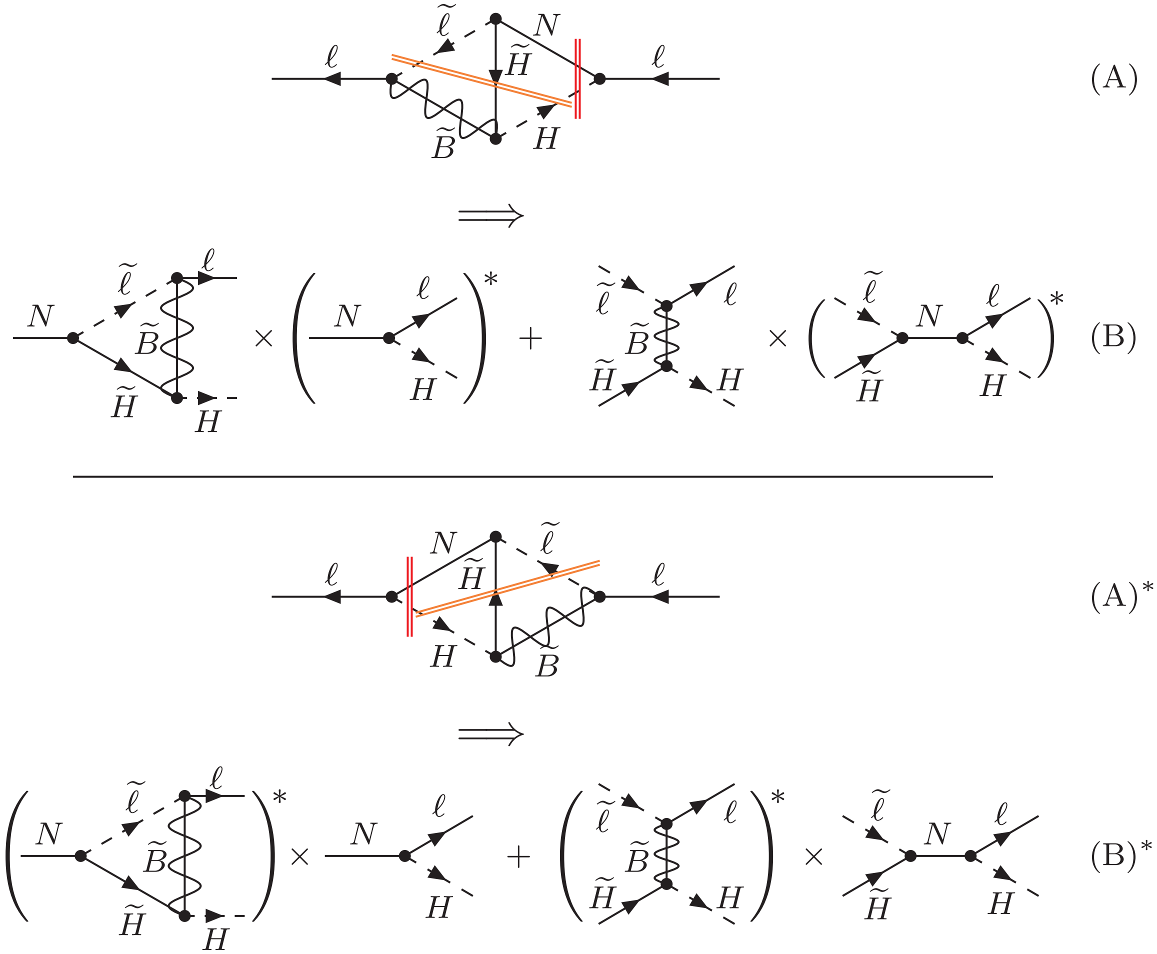

FIG. 3: Relation of the lepton CTP self-energies (A), (A)* to the CPV interference of amplitudes (B), (B)* in conventional approaches, on the example of the contribution from the off-shell $\widetilde{B}$, where the star indicates that the diagrams represent terms that are related by complex conjugation. The CPV contributions to the interference terms occur when $\widetilde{\ell}$ and $\widetilde{H}$ are on-shell. The RIS contribution arises from an on-shell N in the $2 \leftrightarrow 2$ scattering amplitude. In the interference terms, we have suppressed the notation of the integration over the phase space of the external particles other than ℓ.

This cancellation may also be regarded as a consequence of the fact that the d^3k integration in Eq. (15) corresponds to closing the (s)lepton lines in Fig. 1, what leads to identical Feynman diagrams. Therefore, no matter which mass relation $-m_N > m_{\widetilde{\ell}}+m_{H1}$

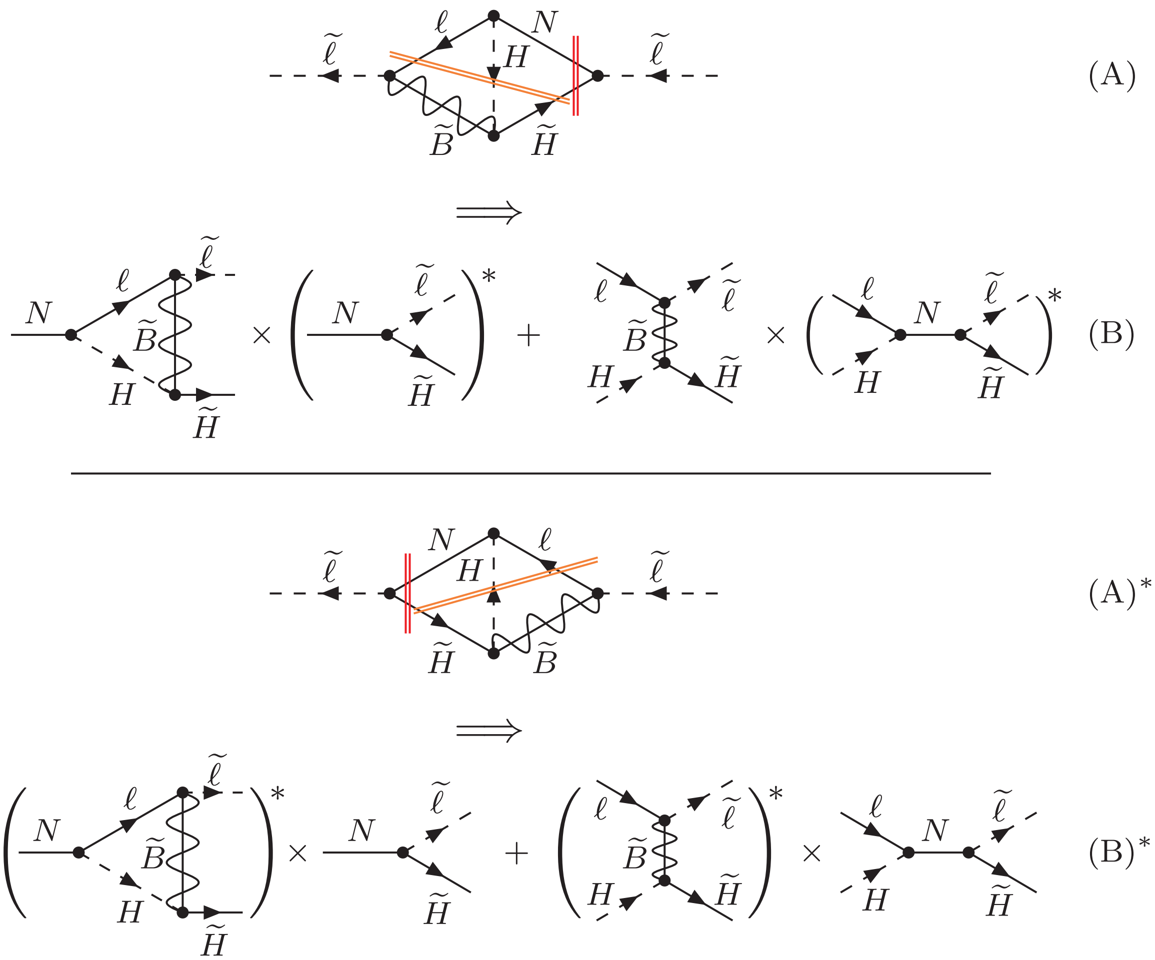

FIG. 4: Relation of the slepton CTP self-energies (A), (A)* to the CPV interference of amplitudes (B), (B)* in conventional approaches, on the example of the contribution from the off-shell $\widetilde{B}$, where the star indicates that the diagrams represent terms that are related by complex conjugation. The CPV contributions to the interference terms occur when ℓ and H are on-shell. The RIS contribution arises from an on-shell N in the $2 \leftrightarrow 2$ scattering amplitude. In the interference terms, we have suppressed the notation of the integration over the phase space of the external particles other than $\widetilde{\ell}$.

or $m_N + m_{H1} < m_{\widetilde{\ell}}$ − holds, the asymmetry produced within the leptons will always cancel the asymmetry produced within the sleptons. The fact that this also holds at finite temperature is in contrast to what is argued in Ref. [6]. The reason for the discrepancy is that here, through the use of the CTP approach, no distinction between internal propagators and external particles is made, and quantum statistical corrections to both are applied. Therefore the the cancellation of lepton and slepton asymmetries present in the vacuum generalizes to finite temperature backgrounds as well.

In Appendix C, we show how $\mathcal{C}_{\widetilde{\ell}}^{\mathrm{v}\widetilde{B}}$ can be further evaluated to take a form that is familiar from the collision term in Boltzmann equations. Such a calculation is however not necessary in order to demonstrate the cancellations, that are a main topic of this work.

D. Cuts with off-shell H_1 and off-shell $\widetilde{H}$

It is also necessary to check whether the cancellation holds for the other possible cuts. We therefore consider contributions arising from terms where the Higgs-boson H_1 is off-shell. From Fig. 1 and Fig. 5, we see that the cuts where $\widetilde{H}$ and those where H_1 are off-shell are qualitatively similar, in the sense that they occur at a vertex with an external particle that carries lepton numbers. Indeed, the expressions for $\mathcal{C}_{\ell}^{\mathrm{v}\widetilde{H}1}$ and $\mathcal{C}_{\widetilde{\ell}}^{\mathrm{v}\widetilde{H}1}$ can be simply inferred from the ones for $\mathcal{C}_{\ell}^{\mathrm{v}H1}$ and $\mathcal{C}_{\widetilde{\ell}}^{\mathrm{v}H1}$ that are derived in this Section by replacing the quantum statistical factors. Consequently, we do not present the calculation for off-shell $\widetilde{H}$ here.

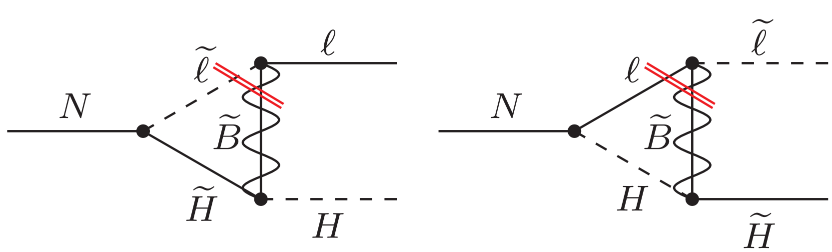

FIG. 5: Cuts in vacuum diagrams with off-shell Higgsino and Higgs boson.

From the slepton self-energy (9), we obtain

$$\mathcal{C}_{\widetilde{\ell}}^{\mathrm{v}H1} = Y^2 \frac{g_1^2}{4} \mathrm{tr} \int \frac{dk^0}{2\pi} \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \mathrm{i}\Delta_{H1}^{T}(p+k+q) \Big\{ \tag{16}$$

$$\Big[P_{\rm L}{\rm i}S^{<}_{\widetilde B}(-p-k)P_{\rm L}{\rm i}S^{>}_{\ell}(-p)-P_{\rm L}{\rm i}S^{>}_{\widetilde B}(-p-k)P_{\rm L}{\rm i}S^{<}_{\ell}(-p)\Big]$$

$$\times\Big[P_{\rm R}{\rm i}S^{>}_{N}(q+k)P_{\rm L}{\rm i}S^{<}_{\widetilde H}(q){\rm i}\Delta^{<}_{\widetilde\ell}(k)-P_{\rm R}{\rm i}S^{<}_{N}(q+k)P_{\rm L}{\rm i}S^{>}_{\widetilde H}(q){\rm i}\Delta^{>}_{\widetilde\ell}(k)\Big]\sin\alpha\cos\alpha{\rm e}^{-{\rm i}\phi_\mu}$$

$$-\Big[P_{\rm L}{\rm i}S^{>}_{\ell}(-p)P_{\rm L}{\rm i}S^{<}_{\widetilde B}(-p-k)-P_{\rm L}{\rm i}S^{<}_{\ell}(-p)P_{\rm L}{\rm i}S^{>}_{\widetilde B}(-p-k)\Big]$$

$$\times\Big[P_{\rm R}{\rm i}S^{<}_{\widetilde H}(q)P_{\rm R}{\rm i}S^{>}_{N}(q+k){\rm i}\Delta^{<}_{\widetilde\ell}(k)-P_{\rm R}{\rm i}S^{>}_{\widetilde H}(q)P_{\rm R}{\rm i}S^{<}_{N}(q+k){\rm i}\Delta^{>}_{\widetilde\ell}(k)\Big]\sin\alpha\cos\alpha{\rm e}^{{\rm i}\phi_\mu}$$

$$+\Big[P_{\rm L}{\rm i}S^{<}_{\widetilde B}(-p-k)P_{\rm L}{\rm i}S^{>}_{\ell}(-p){\rm i}\Delta^{<}_{\widetilde\ell}(k)-P_{\rm L}{\rm i}S^{>}_{\widetilde B}(-p-k)P_{\rm L}{\rm i}S^{<}_{\ell}(-p){\rm i}\Delta^{>}_{\widetilde\ell}(k)\Big]$$

$$\times\Big[P_{\rm R}{\rm i}S^{<}_{N}(q+k)P_{\rm L}{\rm i}S^{>}_{\widetilde H}(q)-P_{\rm R}{\rm i}S^{>}_{N}(q+k)P_{\rm L}{\rm i}S^{<}_{\widetilde H}(q)\Big]\sin\alpha\cos\alpha{\rm e}^{-{\rm i}\phi_\mu}$$

$$-\Big[P_{\rm L}{\rm i}S^{>}_{\ell}(-p)P_{\rm L}{\rm i}S^{<}_{\widetilde B}(-p-k){\rm i}\Delta^{<}_{\widetilde\ell}(k)-P_{\rm L}{\rm i}S^{<}_{\ell}(-p)P_{\rm L}{\rm i}S^{>}_{\widetilde B}(-p-k){\rm i}\Delta^{>}_{\widetilde\ell}(k)\Big]$$

$$\times\Big[P_{\rm R}{\rm i}S^{>}_{\widetilde H}(q)P_{\rm R}{\rm i}S^{<}_{N}(q+k)-P_{\rm R}{\rm i}S^{<}_{\widetilde H}(q)P_{\rm R}{\rm i}S^{>}_{N}(q+k)\Big]\sin\alpha\cos\alpha{\rm e}^{{\rm i}\phi_\mu}\Big\}\,.$$

We note that the last two terms are vanishing due to KMS relations applied to S_ℓ, $S_{\widetilde B}$ and $\Delta_{\widetilde\ell}$. Applying KMS relations to $S_{\widetilde B}$, S_ℓ and S_N would also render the first two terms vanishing. Since we consider however situations when N is out-of-equilibrium, we do not apply KMS in this case, such that these terms remain as contributions to the asymmetry in sleptons $\widetilde\ell$.

The lepton collision term with the off-shell H_1 takes a form that is diagrammatically different from the slepton collision term (16), in the sense that here Δ_{H1} connects to an external vertex of the self-energy $\Sigma^{\rm v}_{\ell}$, while for $\Pi^{\rm v}_{\widetilde\ell}$, it connects two internal vertices. We obtain

$$\int\frac{d^3k}{(2\pi)^3}\mathcal{C}^{{\rm v}H1}_{\ell}=-Y^2\frac{g_1^2}{2}\cos\alpha\sin\alpha\,{\rm tr}\int\frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}{\rm i}\Delta^{T}_{H1}(q)\Big\{\tag{17}$$

$$\big[{\rm i}S^{<}_{\ell}(k)P_{\rm R}{\rm i}S^{>}_{\widetilde B}(p+k)P_{\rm R}{\rm i}S^{<}_{\widetilde H}(p+k+q)P_{\rm R}{\rm i}S^{>}_{N}(q+k)$$

$$-{\rm i}S^{>}_{\ell}(k)P_{\rm R}{\rm i}S^{<}_{\widetilde B}(p+k)P_{\rm R}{\rm i}S^{>}_{\widetilde H}(p+k+q)P_{\rm R}{\rm i}S^{<}_{N}(q+k)\big]$$

$$\times\Big({\rm i}\Delta^{>}_{\widetilde\ell}(-p)-{\rm i}\Delta^{\bar T}_{\widetilde\ell}(-p)\Big){\rm e}^{{\rm i}\phi_\mu}$$

$$-\big[P_{\rm L}{\rm i}S^{>}_{\widetilde B}(p+k)P_{\rm L}{\rm i}S^{<}_{\ell}(k){\rm i}S^{>}_{N}(q+k)P_{\rm L}{\rm i}S^{<}_{\widetilde H}(p+k+q)$$

$$-P_{\rm L}{\rm i}S^{<}_{\widetilde B}(p+k)P_{\rm L}{\rm i}S^{>}_{\ell}(k){\rm i}S^{<}_{N}(q+k)P_{\rm L}{\rm i}S^{>}_{\widetilde H}(p+k+q)\big]$$

$$\times \left(-\mathrm{i}\Delta_{\widetilde{\ell}}^{>}(-p) + \mathrm{i}\Delta_{\widetilde{\ell}}^{T}(-p)\right) \mathrm{e}^{-\mathrm{i}\phi_\mu} \Big\} .$$

Note that when the neutrino N is in equilibrium, the integrand vanishes, as can be easily verified by applying KMS relations to the terms in square brackets. Next, we notice that the terms in square brackets are odd in the momentum variables (this is a consequence of ℓ, $\widetilde{B}$, $\widetilde{H}$ being in equilibrium and N being its own anti-particle), while the terms in round brackets are the retarded/advanced propagators $\mathrm{i}\Delta^R = \mathrm{i}\Delta^> - \mathrm{i}\Delta^{\bar{T}} = -\mathrm{i}\Delta^< + \mathrm{i}\Delta^T$, $\mathrm{i}\Delta^A = \mathrm{i}\Delta^< - \mathrm{i}\Delta^{\bar{T}} = -\mathrm{i}\Delta^> + \mathrm{i}\Delta^T$]. The components $\mathrm{i}\Delta^T$ and $\mathrm{i}\Delta^{\bar{T}}$ are even in the momentum variable and therefore yield a vanishing contribution to the integral. Making use of this observation, we can reexpress

$$\begin{aligned}
\int \frac{d^3k}{(2\pi)^3}\mathcal{C}_\ell^{\mathrm{v}H1} =& - Y^2\frac{g_1^2}{2}\cos\alpha\sin\alpha\,\mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\mathrm{i}\Delta_{H1}^T(q)\Big\{ \qquad (18)\\
&\mathrm{i}S_\ell^>(k)P_\mathrm{R}\mathrm{i}S_{\widetilde{B}}^{<}(p+k)\big[P_\mathrm{R}\mathrm{i}S_{\widetilde{H}}^{<}(p+k+q)P_\mathrm{R}\mathrm{i}S_N^>(q+k)\mathrm{i}\Delta_{\widetilde{\ell}}^{<}(-p)\\
&- P_\mathrm{R}\mathrm{i}S_{\widetilde{H}}^{>}(p+k+q)P_\mathrm{R}\mathrm{i}S_N^<(q+k)\mathrm{i}\Delta_{\widetilde{\ell}}^{>}(-p)\big]\mathrm{e}^{\mathrm{i}\phi_\mu}\\
&- P_\mathrm{L}\mathrm{i}S_{\widetilde{B}}^{<}(p+k)\mathrm{i}S_\ell^>(k)\big[P_\mathrm{R}\mathrm{i}S_N^>(q+k)P_\mathrm{L}\mathrm{i}S_{\widetilde{H}}^{<}(p+k+q)\mathrm{i}\Delta_{\widetilde{\ell}}^{<}(-p)\\
&- P_\mathrm{R}\mathrm{i}S_N^<(q+k)P_\mathrm{L}\mathrm{i}S_{\widetilde{H}}^{>}(p+k+q)\mathrm{i}\Delta_{\widetilde{\ell}}^{>}(-p)\big]\mathrm{e}^{-\mathrm{i}\phi_\mu}\Big\}\\
\underset{(\mathrm{KMS})}{=}& - Y^2\frac{g_1^2}{2}\cos\alpha\sin\alpha\,\mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\mathrm{i}\Delta_{H1}^T(q)\Big\{\\
&\big[\mathrm{i}S_\ell^<(k)P_\mathrm{R}\mathrm{i}S_{\widetilde{B}}^{>}(p+k)P_\mathrm{R}\mathrm{i}S_{\widetilde{H}}^{<}(p+k+q)P_\mathrm{R}\mathrm{i}S_N^>(q+k)\\
&- \mathrm{i}S_\ell^>(k)P_\mathrm{R}\mathrm{i}S_{\widetilde{B}}^{<}(p+k)P_\mathrm{R}\mathrm{i}S_{\widetilde{H}}^{>}(p+k+q)P_\mathrm{R}\mathrm{i}S_N^<(q+k)\big]\mathrm{i}\Delta_{\widetilde{\ell}}^{>}\mathrm{e}^{\mathrm{i}\phi_\mu}\\
&- \big[P_\mathrm{L}\mathrm{i}S_{\widetilde{B}}^{>}(p+k)\mathrm{i}S_\ell^<(k)P_\mathrm{R}\mathrm{i}S_N^>(q+k)P_\mathrm{L}\mathrm{i}S_{\widetilde{H}}^{<}(p+k+q)\\
&- P_\mathrm{L}\mathrm{i}S_{\widetilde{B}}^{<}(p+k)\mathrm{i}S_\ell^>(k)P_\mathrm{R}\mathrm{i}S_N^<(q+k)P_\mathrm{L}\mathrm{i}S_{\widetilde{H}}^{>}(p+k+q)\big]\mathrm{i}\Delta_{\widetilde{\ell}}^{>}\mathrm{e}^{-\mathrm{i}\phi_\mu}\Big\} ,
\end{aligned}$$

where we have applied the KMS relation to S_ℓ, $S_{\widetilde{B}}$ and $\Delta_{\widetilde{\ell}}$. Again, by appealing to the symmetry property in the momentum variables (terms are even under the simultaneous reversal of the momenta and the replacement $< \leftrightarrow >$), we may replace

$$\left(S_\ell^< S_{\widetilde{B}}^> S_{\widetilde{H}}^< S_N^> - S_\ell^> S_{\widetilde{B}}^< S_{\widetilde{H}}^> S_N^<\right)\Delta_{\widetilde{\ell}}^> \to \qquad (19)$$

$$\frac{1}{2}\left(S_\ell^{<}S_{\widetilde{B}}^{>}S_{\widetilde{H}}^{<}S_N^{>}-S_\ell^{>}S_{\widetilde{B}}^{<}S_{\widetilde{H}}^{>}S_N^{<}\right)\left(\Delta_{\widetilde{\ell}}^{>}-\Delta_{\widetilde{\ell}}^{<}\right)$$
$$\underset{\text{(KMS)}}{=}\frac{1}{2}\left(S_\ell^{>}S_{\widetilde{B}}^{<}-S_\ell^{<}S_{\widetilde{B}}^{>}\right)\times\left(S_{\widetilde{H}}^{<}S_N^{>}\Delta_{\widetilde{\ell}}^{<}-S_{\widetilde{H}}^{>}S_N^{<}\Delta_{\widetilde{\ell}}^{>}\right)\,,$$

which then leads us to the observation

$$\int\frac{d^3k}{(2\pi)^3}\left[\mathcal{C}_{\widetilde{\ell}}^{\mathrm{v}H1}(\mathbf{k})+\mathcal{C}_{\ell}^{\mathrm{v}H1}(\mathbf{k})\right]=0\,. \tag{20}$$

Therefore, provided superequilibrium (equilibrium between superpartners) holds, there will be no residual lepton asymmetry.

A useful check is to suppose that instead of a Majorana mass, N would have a Dirac mass, which can be achieved by adding another chiral sterile neutrino degree of freedom. Then the collision term for creation of an N-charge must vanish, which can also be verified explicitly.

E. Cuts with off-shell $\widetilde{\ell}$

Finally, cuts can be applied to the vertex graphs in such a way that they do not go through a line that carries lepton number. We consider here the situation when the slepton $\widetilde{\ell}$ is off-shell. The analogous result for an off-shell lepton ℓ can be easily inferred from what is presented here.

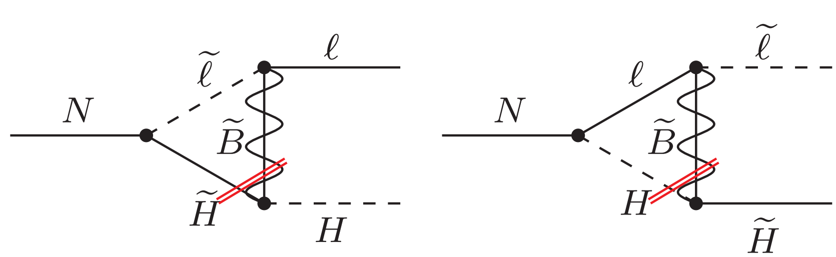

FIG. 6: Cuts in vacuum diagrams with off-shell slepton and lepton.

For the CPV vertex contribution to the lepton collision term, we obtain

$$\int \frac{d^3k}{(2\pi)^3}\mathcal{C}_\ell^{\mathrm{v}\widetilde{\ell}}(\mathbf{k}) = Y^2\frac{g_1^2}{2}\cos\alpha\sin\alpha\int\frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\mathrm{i}\Delta^T_{\widetilde{\ell}}(-p)\Big\{ \tag{21}$$

$$\big[P_\mathrm{R}\mathrm{i}S^>_{\widetilde{B}}(p+k)P_\mathrm{R}\mathrm{i}S^<_{\widetilde{H}}(p+k+q)P_\mathrm{R}\mathrm{i}S^>_N(q+k)P_\mathrm{L}\mathrm{i}S^<_\ell(k)$$

$$-P_\mathrm{R}\mathrm{i}S^<_{\widetilde{B}}(p+k)P_\mathrm{R}\mathrm{i}S^>_{\widetilde{H}}(p+k+q)P_\mathrm{R}\mathrm{i}S^<_N(q+k)P_\mathrm{L}\mathrm{i}S^>_\ell(k)\big]\mathrm{i}\Delta^H_{H1}(q)\mathrm{e}^{\mathrm{i}\phi_\mu}$$

$$+\big[P_\mathrm{L}\mathrm{i}S^<_\ell(k)P_\mathrm{R}\mathrm{i}S^>_N(q+k)P_\mathrm{L}\mathrm{i}S^<_{\widetilde{H}}(p+k+q)P_\mathrm{L}\mathrm{i}S^>_{\widetilde{B}}(p+k)$$

$$-P_\mathrm{L}\mathrm{i}S^>_\ell(k)P_\mathrm{R}\mathrm{i}S^<_N(q+k)P_\mathrm{L}\mathrm{i}S^>_{\widetilde{H}}(p+k+q)P_\mathrm{L}\mathrm{i}S^<_{\widetilde{B}}(p+k)\big]\mathrm{i}\Delta^H_{H1}(q)\mathrm{e}^{-\mathrm{i}\phi_\mu}\Big\}\,,$$

where $\mathrm{i}\Delta^H = \frac{1}{2}(\mathrm{i}\Delta^A + \mathrm{i}\Delta^R) = \frac{1}{2}(\mathrm{i}\Delta^T - \mathrm{i}\Delta^{\bar{T}})$. By KMS again, the integrand vanishes when N is in equilibrium. Similar to the vanishing of $\mathcal{C}_{\widetilde{\ell}}^{\mathrm{v}H1}+\mathcal{C}_\ell^{\mathrm{v}H1}$, the integrated collision term also vanishes when N is not in equilibrium. This is because $\mathrm{i}\Delta^H(k) = \mathrm{i}\Delta^H(-k)$ whereas the terms in the square bracket are odd under momentum reversal.

IV. CANCELLATION OF CONTRIBUTIONS FROM WAVEFUNCTION DIAGRAMS

In the context of soft leptogenesis, another potentially significant contribution arises from CP-violation in the mixing of H_1 and H_2 that is induced through loops with $\widetilde{B}$ and $\widetilde{H}$. While the exclusive vacuum decay rate of a singlet neutrino N to H_1 and ℓ receives CPV contributions through the first diagram in Fig. 7, using the CTP formalism, we can convince ourselves that there is no contribution to leptogenesis, though.

For simplicity, let us assume that $m_{H2} \gg m_{H1}$ and $m_{H2} \gg T$, such that H_2 is always off-shell to a good approximation. The more general case can be easily inferred from the following results. The CPV wave-function contribution to the lepton collision term, that results from the lepton self-energy diagram in Fig. 7, is then given by

$$\mathcal{C}_\ell^{\mathrm{w}}(\mathbf{k}) = \frac{g_1^2}{2}Y^2\sin\alpha\cos\alpha\sin\phi_\mu\int\frac{dk^0}{2\pi}\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{p^2-m_{H2}^2} \tag{22}$$

$$\times\,\mathrm{tr}\left[P_\mathrm{R}\mathrm{i}S^<_N(p+k)\mathrm{i}S^>_\ell(k) - P_\mathrm{R}\mathrm{i}S^>_N(p+k)\mathrm{i}S^<_\ell(k)\right]$$

$$\times\,\mathrm{tr}\Big[\mathrm{i}\Delta^<_{H1}(p)P_\mathrm{R}\mathrm{i}S^<_{\widetilde{B}}(q)P_\mathrm{R}\mathrm{i}S^>_{\widetilde{H}}(p+q) - \mathrm{i}\Delta^>_{H1}(p)P_\mathrm{R}\mathrm{i}S^>_{\widetilde{B}}(q)P_\mathrm{R}\mathrm{i}S^<_{\widetilde{H}}(p+q)\Big]\,.$$

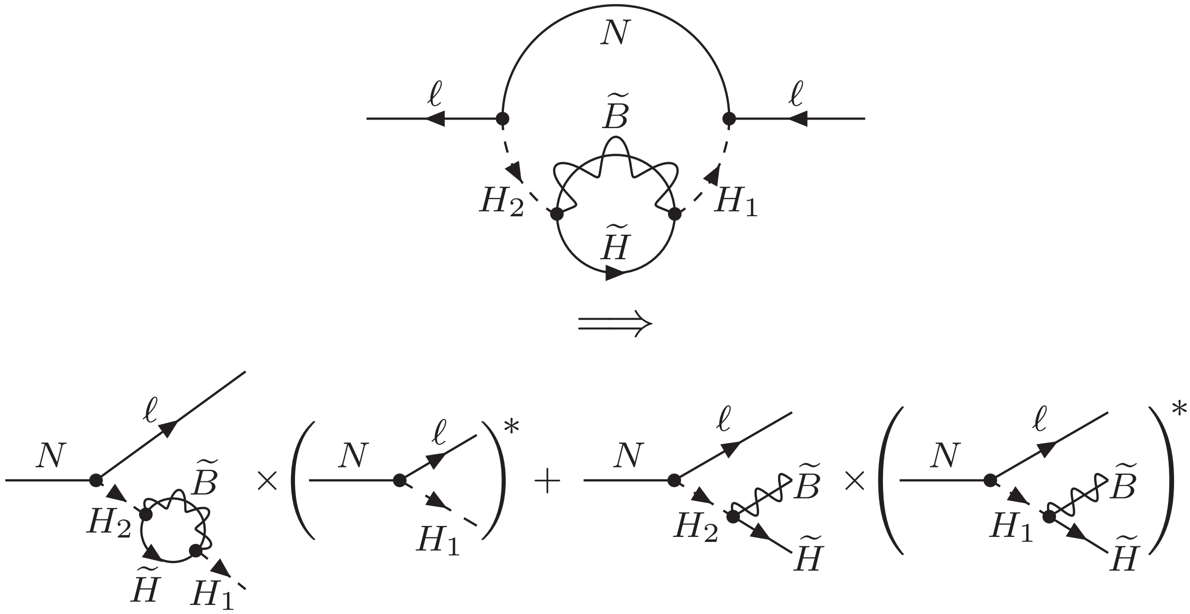

FIG. 7: Diagrams that lead to potential contributions to the lepton asymmetry but that are vanishing when H, $\widetilde{H}$ and $\widetilde{B}$ are in equilibrium. In the interference terms, we have suppressed the notation of the integrals over the phase space of the external particles other than ℓ. Notice that when H_1 corresponds to a RIS, the $1 \leftrightarrow 3$ amplitudes can be cut along H_1 and fused along $\widetilde{B}$ and $\widetilde{H}$, such that they cancel the interference of the $1 \leftrightarrow 2$ amplitudes, as it is explained in Section V.

In order to derive this expression, no use of the KMS relation has been made. However, it is obvious that when applying KMS to the propagators Δ_{H1}, $S_{\widetilde{B}}$ and $S_{\widetilde{H}}$, it immediately follows that $\mathcal{C}_\ell^{\rm w}(\mathbf{k}) \equiv 0$. Essentially, this cancellation is closely related to the fact that in vacuum, the inclusive decay rate of N, which is obtained when considering both $\ell + H_1$ also $\ell + \widetilde{B} + \widetilde{H}$ as final states leads to a vanishing contribution to the lepton asymmetry.

V. RELATION TO RIS SUBTRACTION

The results of Sections III E and IV indicate that the asymmetries in ℓ associated with off-shell $\widetilde{\ell}$ vertex graphs and with wavefunction diagrams readily vanish, even without a cancellation with $\widetilde{\ell}$. When finite temperature effects are neglected, the vanishing results can also be explained using the standard method of Boltzmann equations supplemented by the subtraction of RIS. We first review the salient features of the subtraction procedure for standard leptogenesis and then apply it to the process that is calculated in Section IV.

In leptogenesis calculations that rely on Boltzmann equations in the limit of non-relativistic singlet neutrinos (i.e. $m_N \gg T$), it is useful to employ the averaged decay rate [33]

$$\gamma^{\rm av} = \Gamma_{N\to\ell H,\bar\ell H^*} \frac{\int \frac{d^3p}{(2\pi)^3} \frac{m_N}{\sqrt{\mathbf{p}^2+m_N^2}} {\rm e}^{-\sqrt{\mathbf{p}^2+m_N^2}/T}}{\int \frac{d^3p}{(2\pi)^3} {\rm e}^{-\sqrt{\mathbf{p}^2+m_N^2}/T}} = \frac{K_1(z)}{K_2(z)} \Gamma_{N\to\ell H,\bar\ell H^*}\,, \tag{23}$$

where the factor m_N/E_N accounts for time dilation, $z = m_N/T$ and $\Gamma_{N\to\ell H,\bar\ell H^*}$ is the total vacuum decay rate of a singlet neutrino N into a lepton ℓ and Higgs boson H and their anti-particles. Note that while this expression includes the relativistic time-dilation factors, but it is non-relativistic in the sense that the quantum-statistical distributions are replaced by classical Maxwell distributions.[4]

We furthermore note the standard definition $Y_X = n_X/s$, where n_X is the number density of the particle X and s is the entropy density. Furthermore, $Y_X^{\rm eq}$ denotes the value that Y_X takes in thermal equilibrium. Using the non-relativistic, averaged decay rate (23), the Boltzmann equations for leptogenesis can be expressed as

$$zHs\frac{dY_N}{dz} = -\,\gamma^{\rm av}\frac{Y_N - Y_N^{\rm eq}}{Y_N^{\rm eq}}\,, \tag{24a}$$

$$\begin{aligned} zHs\frac{d(Y_\ell - Y_{\bar\ell})}{dz} =& \frac{Y_N}{Y_N^{\rm eq}}\frac{1+\varepsilon}{2}\gamma^{\rm av} - \frac{Y_N}{Y_N^{\rm eq}}\frac{1-\varepsilon}{2}\gamma^{\rm av} + \frac{Y_{\bar\ell}}{Y_\ell^{\rm eq}}\frac{1+\varepsilon}{2}\gamma^{\rm av} - \frac{Y_\ell}{Y_\ell^{\rm eq}}\frac{1-\varepsilon}{2}\gamma^{\rm av} \\ & -2\gamma_{\ell H\to\bar\ell H^*} + 2\gamma_{\bar\ell H^*\to\ell H} + 2\gamma^{\rm RIS}_{\ell H\to\bar\ell H^*} - 2\gamma^{\rm RIS}_{\bar\ell H^*\to\ell H}\,. \end{aligned} \tag{24b}$$

Here, ε is the usual parameter that describes the relative asymmetry in vacuum decays of N. The relative signs in front of it follow either from direct calculation or can easily be

[4] For completeness, even though this is not essential for the arguments presented in this Section, we note that the relation to the conformal time η used for the collision rates throughout the remainder of this paper is given by $T = 1/\eta$ for a scale factor in the radiation-dominated Universe $a(\eta) = m_{\rm Pl}\eta\sqrt{45/(\pi^3 g_\star)}/2$. The Hubble rate then is $H = (da(\eta)/d\eta)/a^2(\eta)$. In the above expression for the collision terms, one should then multiply the mass terms by $a(\eta)$ and replace the temperature by the constant comoving temperature $T_{\rm com} = (a_{\rm R} m_{\rm Pl}/2)^{1/2}(45/\pi^3 g_\star)^{1/4}$. More details of this parametrization, that is useful in the CTP approach, are given in Refs. [22, 27].

inferred using general arguments of charge conjugation and the CPT-invariance theorem. The factors of 1/2 arise because the individual decay rates into ℓH and $\bar{\ell} H^*$ final states are equal in the absence of CPV, and $\gamma^{\rm av}$ accounts for the averaged total decay rate into both of these states.

The particular terms on the right hand side of Eq. (24b) are explained as follows: The first two describe asymmetric decays of N into ℓ and $\bar{\ell}$, while the third and fourth term describe inverse decays. One might expect that it is sufficient to consider these $1 \leftrightarrow 2$ processes in order to describe leptogenesis at leading order. However, the $2 \leftrightarrow 2$ N-mediated scatterings between ℓH and $\bar{\ell} H^*$ contribute to lepton number violation at leading order when performing the phase space integral over an on-shell intermediate singlet neutrino N in the s-channel. Moreover, for $Y_\ell = Y_{\bar{\ell}} = Y_\ell^{\rm eq}$ the first four terms then add up to $\varepsilon(Y_N/Y_N^{\rm eq}+1)\gamma^{\rm av}$, indicating that an asymmetry would even be generated in equilibrium in conflict with the requirements of CPT invariance. Therefore, the fifth and sixth terms must be included, which correspond to $2 \leftrightarrow 2$ scattering processes mediated by and s-channel N. In other words, the Boltzmann equations are completed by attaching external ℓ and H to the unstable N – which also give rise to the relevant CPV contributions. The $2 \leftrightarrow 2$ rates are CP-invariant, but they include regions of phase space where there is an s-channel divergence of the propagator of N. The rates for reactions via these so-called real intermediate states (RIS) are already accounted in the $1 \leftrightarrow 2$ processes and must be subtracted [33, 34] and can be written in the following suggestive way:

$$\gamma^{\rm RIS}_{\ell H \to \bar{\ell} H^*} = \frac{Y_\ell}{Y_\ell^{\rm eq}} \times \frac{1-\varepsilon}{2}\gamma^{\rm av} \times \frac{1-\varepsilon}{2} \approx \gamma^{\rm av}\frac{1-2\varepsilon}{4}\,, \tag{25a}$$

$$\gamma^{\rm RIS}_{\bar{\ell} H^* \to \ell H} = \frac{Y_{\bar{\ell}}}{Y_\ell^{\rm eq}} \times \frac{1+\varepsilon}{2}\gamma^{\rm av} \times \frac{1+\varepsilon}{2} \approx \gamma^{\rm av}\frac{1+2\varepsilon}{4}\,. \tag{25b}$$

The first factor involving ε is the CPV inverse decay rate of N, while the second factor is the branching ratio of the CPV decays. Substitution into Eq. (24b) leads to the production rate $\varepsilon(Y_N/Y_N^{\rm eq}-1)\gamma^{\rm av}$ for the lepton asymmetry, which vanishes in equilibrium, as it should. Note that the washout terms remain present within the unsubtracted, CP-conserving $2 \leftrightarrow 2$ rates, which are the 5th and 6th term on the right hand side of Eq. (24b).

As emphasized above, a perhaps less heuristic and more controlled way to achieve this result is using the CTP approach, i.e. the same methods employed in the remainder of this paper, and as it is exercised in Ref. [22] (see also Refs. [18, 19] for purely scalar models). Nonetheless, it is instructive to identify the origin of the vanishing contributions of Sections III E and IV in the context of the conventional Boltzmann framework. In so doing, we emphasize that the analysis of Sections III E and IV are more general because the argument based on RIS subtraction assumes a vacuum background, which is an appropriate approximation only when $m_N \gg T$. Moreover, we assume here that $m_N > m_\ell + m_{H1}$ and $m_{H1} > m_{\widetilde{H}} + m_{\widetilde{B}}$, while the results of Sections III E and IV apply to more general kinematic situations as well.

With these comments in mind, we write down the production rate for the lepton asymmetry

$$\begin{aligned} zHs\frac{d(Y_\ell - Y_{\bar\ell})}{dz} =& \frac{Y_N}{Y_N^{\rm eq}}\frac{1+\varepsilon}{2}\gamma^{\rm av} - \frac{Y_N}{Y_N^{\rm eq}}\frac{1-\varepsilon}{2}\gamma^{\rm av} + \frac{Y_{\bar\ell}}{Y_\ell^{\rm eq}}\frac{1+\varepsilon}{2}\gamma^{\rm av} - \frac{Y_\ell}{Y_\ell^{\rm eq}}\frac{1-\varepsilon}{2}\gamma^{\rm av} \\ & -\gamma_{\ell\widetilde{H}\widetilde{B}\to N} + \gamma_{N\to\ell\widetilde{H}\widetilde{B}} + \gamma_{\bar\ell\bar{\widetilde{H}}\widetilde{B}\to N} - \gamma_{N\to\bar\ell\bar{\widetilde{H}}\widetilde{B}} \\ & +\gamma^{\rm RIS}_{\ell\widetilde{H}\widetilde{B}\to N} - \gamma^{\rm RIS}_{N\to\ell\widetilde{H}\widetilde{B}} - \gamma^{\rm RIS}_{\bar\ell\bar{\widetilde{H}}\widetilde{B}\to N} + \gamma^{\rm RIS}_{N\to\bar\ell\bar{\widetilde{H}}\widetilde{B}}\,. \end{aligned} \qquad (26)$$

For the wavefunction contributions (Section IV), the $1 \leftrightarrow 3$ processes are mediated by H_1 or H_2, where H_1 may be on-shell. Interferences between H_1 and H_2 mediated processes correspond to a single cut through the N, $\widetilde{B}$, and $\widetilde{H}$ lines in Fig. 7.[5] Such RIS with on-shell H_1 must be subtracted, and the rates can be written in the suggestive way

$$\gamma^{\rm RIS}_{\ell\widetilde{H}\widetilde{B}\to N} = \gamma_{\widetilde{H}\widetilde{B}\to H_1}\frac{Y_\ell}{Y_\ell^{\rm eq}}\frac{\gamma^{\rm av}}{\gamma_{\widetilde{H}\widetilde{B}\to H_1}}\frac{1-\varepsilon}{2}\,,\quad \gamma^{\rm RIS}_{\bar\ell\widetilde{H}\widetilde{B}\to N} = \gamma_{\widetilde{H}\widetilde{B}\to H_1}\frac{Y_{\bar\ell}}{Y_\ell^{\rm eq}}\frac{\gamma^{\rm av}}{\gamma_{\widetilde{H}\widetilde{B}\to H_1}}\frac{1+\varepsilon}{2}\,, \qquad (27)$$

where $\gamma_{\widetilde{H}\widetilde{B}\to H_1}$ is the rate for the corresponding $2 \leftrightarrow 1$ processes. As we assume supergauge interactions to be in equilibrium, it is equal to the inverse rate, $\gamma_{\bar{\widetilde{H}}\widetilde{B}\to H_1^*} = \gamma_{\widetilde{H}\widetilde{B}\to H_1}$.

---

[5] For the vertex contributions with off-shell $\widetilde{\ell}$ (Section III E), there is no CPV cut in the vertex correction to $N \to \ell H$ at zero temperature. We do not discuss a generalization of the RIS subtraction procedure to include finite temperature corrections (i.e. the quantum statistical distributions of the equilibrium particles ℓ, $\widetilde{\ell}$, H, $\widetilde{H}$ and $\widetilde{B}$ in the present context), as we find the derivation of the CPV rates based on the CTP approach perhaps less heuristic and after all more intuitive and technically simple.

Note that even though the $1 \leftrightarrow 3$ cut in Fig. 7 corresponds to the interference between H_1- and H_2-mediated exchange amplitudes with only H_1 being on-shell, it is nevertheless convenient to express it in terms of the rate for H_1 production, $\gamma_{\widetilde{H}\widetilde{B}\to H_1}$. To see this, we cut the H_1-mediated $1 \leftrightarrow 3$ amplitude along the on-shell H_1 and fuse the $\widetilde{B}$ and $\widetilde{H}$ lines with the H_2-mediated $1 \leftrightarrow 3$ amplitude. This way, we obtain an interference between a tree-level and a one-loop amplitude for $\widetilde{H}\widetilde{B} \to H_1$, cf. Fig. 7. Consequently, in the one-loop integral, only the cut contribution where $\widetilde{B}$ and $\widetilde{H}$ are on-shell is extracted, which is precisely the part that is relevant for CPV. Note that this logic is similar to that of RIS subtraction in conventional leptogenesis, see e.g. Ref. [22]. In that work, the RIS subtraction entails an interference between scattering amplitudes mediated by two different neutrinos N_1 and N_2 where only N_1 may be on-shell. The full rate is nevertheless characterized in terms of the rate for production of the N_1 (inverse) decay, as can again be seen from cutting and fusing the scattering amplitudes.

The second, third and fourth factors on the right hand side combine to the rate for an on-shell H_1 (H_1^*) to inversely decay with ℓ ($\bar{\ell}$) into N, where we assume that $\gamma_{\widetilde{H}\widetilde{B}\to H_1} \gg \gamma^{\rm av}$, such that $\gamma^{\rm av}/\gamma_{\widetilde{H}\widetilde{B}\to H_1}$ approximately is the branching ratio for H_1 inversely decaying with ℓ to N. Moreover,

$$\gamma^{\rm RIS}_{N\to\ell\widetilde{H}\widetilde{B}} = \gamma^{\rm av}\frac{Y_N}{Y_N^{\rm eq}}\frac{1+\varepsilon}{2}\,, \quad \gamma^{\rm RIS}_{N\to\bar{\ell}\bar{\widetilde{H}}\widetilde{B}} = \gamma^{\rm av}\frac{Y_N}{Y_N^{\rm eq}}\frac{1-\varepsilon}{2}\,. \tag{28}$$

Substituting the rates (27) and (28) into Eq. (26), the CPV contributions cancel. The washout of a potentially pre-existing lepton asymmetry is contained within the unsubtracted, CP-conserving $1 \leftrightarrow 3$ rates [the 5th through 8th terms on the right hand side of Eq. (26)]. One should notice that none of the rates is explicitly weighted by factors of $Y_{H_1,\widetilde{H},\widetilde{B}}/Y^{\rm eq}_{H_1,\widetilde{H},\widetilde{B}}$, as we assume that $\widetilde{H}$ and $\widetilde{B}$ are in equilibrium. Recall that this assumption is also crucial in order to establish the results of vanishing asymmetries in Sections III E and IV.

The main conclusion that may be drawn based on the present discussion of RIS subtraction and of the results in the other Sections of this paper is that a complete network of kinetic equations must properly account for all stable asymptotic states. In particular, for standard leptogenesis, diagrams with N as an external state have to be supplemented by corresponding diagrams where ℓ and H attach to N, i.e. the $1 \leftrightarrow 2$ (inverse) decay processes have to be supplemented by $2 \leftrightarrow 2$ scatterings. On-shell ℓ and H yield also the crucial CPV contributions from loop diagrams. Correspondingly, for the processes in Sections III E and IV, the kinetic equations must be augmented by attaching $\widetilde{B}$ and $\widetilde{H}$ to the external H_1 – the same particles that also give rise to CPV when they propagate on-shell within a loop. In the latter case, the asymmetry vanishes, because H_1 (the particle that attaches to the CPV loop involving $\widetilde{B}$ and $\widetilde{H}$) is in thermal equilibrium (due to gauge interactions), while for standard leptogenesis, an asymmetry persists after the subtraction of RIS, provided N (the singlet neutrino that attaches to the CPV loop involving ℓ and H) is out-of-equilibrium.

These conclusions can easily be generalized to additional models. For example, the work of Ref. [9] included no RIS subtraction or any other pertinent completion of the Boltzmann equations by attaching loop particles to the decaying particle. As the loop particles as well as the decaying particle are in thermal equilibrium (all of them are gauged), the resulting asymmetry from a complete set of kinetic equations vanishes.

VI. CONCLUSIONS

In the study of novel, low-scale leptogenesis scenarios that may have interesting phenomenological consequences, it is essential to properly account for the unitary evolution of all the relevant states involved in the possible generation of a net lepton number asymmetry. In principle, one may do so following the conventional Boltzmann equation approach if one properly implements the RIS subtraction procedure and includes the statistical factors for all on-shell particles, whether they appear as explicit external states or as internal lines in loop graphs. The subtleties that this procedure entails makes RIS subtraction fraught with opportunities for error. Moreover, this approach appears somewhat heuristic, and a derivation of a proper inclusion of the statistical factors of the internal line has not yet been reported in the literature. Alternatively, the CTP formulation provides a systematic approach that ensures unitary evolution and avoids the pitfalls one may encounter with the RIS subtraction.

Using soft leptogenesis in νMSSM, where CPV is sourced through the phase $\text{Arg}(\mu M_1 b^*)$, we have shown how the CTP approach with appropriate inclusion of all cuts includes all of the aforementioned requirements. With interactions analogous to those used in Ref. [6], we then obtain a vanishing lepton number asymmetry, in contrast to the conclusion one would reach following the procedures taken in those calculations. Consequently, we argue that the conclusions reached in Ref. [6] are unlikely to apply without extra input. Similar conclusions should apply to the asymmetric freeze-in scenario considered in Ref. [9][6]. The possibilities for low-scale, non-resonant leptogenesis nevertheless remain intriguing. Exploration of the additional physics needed to make such scenarios viable will be the subject of forthcoming work.

Acknowledgments

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Appendix A: Tree Level Propagators

The results that are presented in this paper are obtained using the CTP techniques for calculating CPV rates developed in Ref. [22]. Additional papers aiming for the

[6] Concretely, in the diagrammatic example of Ref. [9], incoming gauge bosons and charginos or neutralinos should be attached to the decaying chargino. From the resulting scattering diagrams, the RIS should then be subtracted, such that the final asymmetry vanishes. To show this, one can follow the calculations presented Sections III E and IV of this present paper. Note that these calculations apply to all kinematic situations, provided the CPV cut is kinematically viable.

formulation of kinetic theory based on the CTP approach include [15, 36–45]. Basic building blocks are the tree-level propagators. For scalar particles, they take the form

$$\mathrm{i}\Delta^{<}(p) = 2\pi\delta(p^2 - m^2)\left[\vartheta(p_0)f(\mathbf{p}) + \vartheta(-p_0)(1 + \bar{f}(-\mathbf{p}))\right] , \tag{A1a}$$

$$\mathrm{i}\Delta^{>}(p) = 2\pi\delta(p^2 - m^2)\left[\vartheta(p_0)(1 + f(\mathbf{p})) + \vartheta(-p_0)\bar{f}(-\mathbf{p})\right] , \tag{A1b}$$

$$\mathrm{i}\Delta^{T}(p) = \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\varepsilon} + 2\pi\delta(p^2 - m^2)\left[\vartheta(p_0)f(\mathbf{p}) + \vartheta(-p_0)\bar{f}(-\mathbf{p})\right] , \tag{A1c}$$

$$\mathrm{i}\Delta^{\bar{T}}(p) = -\frac{\mathrm{i}}{p^2 - m^2 - \mathrm{i}\varepsilon} + 2\pi\delta(p^2 - m^2)\left[\vartheta(p_0)f(\mathbf{p}) + \vartheta(-p_0)\bar{f}(-\mathbf{p})\right] . \tag{A1d}$$

In order to distinguish different fields, in the present context $\widetilde{\ell}$ and H_1 nd H_2, the Green functions Δ, the masses m and the particle and antiparticle distribution functions f and $\bar{f}$ are marked with subscripts. SU(2) gauge group indices are suppressed.

For spin-1/2 fermions, the Green functions are

$$\mathrm{i}S^{<}(p) = -2\pi\delta(p^2 - m^2)(\not{p} + m)\left[\vartheta(p_0)f(\mathbf{p}) - \vartheta(-p_0)(1 - \bar{f}(-\mathbf{p}))\right] , \tag{A2a}$$

$$\mathrm{i}S^{>}_{Ni}(p) = -2\pi\delta(p^2 - m^2)(\not{p} + m)\left[-\vartheta(p_0)(1 - f(\mathbf{p})) + \vartheta(-p_0)\bar{f}(-\mathbf{p})\right] , \tag{A2b}$$

$$\mathrm{i}S^{T}(p) = \frac{\mathrm{i}(\not{p} + M_i)}{p^2 - M_i^2 + \mathrm{i}\varepsilon} - 2\pi\delta(p^2 - m^2)(\not{p} + m)\left[\vartheta(p_0)f(\mathbf{p}) + \vartheta(-p_0)\bar{f}(-\mathbf{p})\right] , \tag{A2c}$$

$$\mathrm{i}S^{\bar{T}}(p) = -\frac{\mathrm{i}(\not{p} + m)}{p^2 - m^2 - \mathrm{i}\varepsilon} - 2\pi\delta(p^2 - M_i^2)(\not{p} + m)\left[\vartheta(p_0)(\mathbf{p}) + \vartheta(-p_0)\bar{f}(-\mathbf{p})\right] . \tag{A2d}$$

Again, subscripts distinguish the fields ℓ, $\widetilde{H}$ and $\widetilde{B}$ and SU(2) indices are suppressed. Majorana fermions observe the constraint $f(\mathbf{p}) = \bar{f}(\mathbf{p})$.

Appendix B: Replacement of Time-Ordered by On-Shell Green Functions

We consider the integral

$$P = \int \frac{d^4k}{(2\pi)^4}\left[\mathrm{i}\Delta^{T}_{X1}(p+k)\mathrm{i}\Delta^{T}_{X2}(q+k) + \mathrm{i}\Delta^{\bar{T}}_{X1}(p+k)\mathrm{i}\Delta^{\bar{T}}_{X2}(q+k)\right] g(k) \tag{B1}$$

$$= \int \frac{d^4k}{(2\pi)^4}\left[\frac{\mathrm{i}}{(p+k)^2 - m_1^2 + \mathrm{i}\varepsilon}\right.$$

$$+ 2\pi\delta\left((p+k)^2 - m_1^2\right)\left[\vartheta(p_0+k_0)f_{X1}(\mathbf{p}+\mathbf{k}) + \vartheta(-p_0-k_0)\bar{f}_{X1}(-\mathbf{p}-\mathbf{k})\right]\Bigg]$$

$$\times\Bigg[\frac{\mathrm{i}}{(q+k)^2 - m_2^2 + \mathrm{i}\varepsilon}$$

$$+ 2\pi\delta\left((q+k)^2 - m_2^2\right)\left[\vartheta(q_0+k_0)f_{X2}(\mathbf{q}+\mathbf{k}) + \vartheta(-q_0-k_0)\bar{f}_{X2}(-\mathbf{q}-\mathbf{k})\right]\Bigg]g(k)$$

$$+\int\frac{d^4k}{(2\pi)^4}\Bigg[-\frac{\mathrm{i}}{(p+k)^2 - m_1^2 - \mathrm{i}\varepsilon}$$

$$+ 2\pi\delta\left((p+k)^2 - m_1^2\right)\left[\vartheta(p_0+k_0)f_{X1}(\mathbf{p}+\mathbf{k}) + \vartheta(-p_0-k_0)\bar{f}_{X1}(-\mathbf{p}-\mathbf{k})\right]\Bigg]$$

$$\times\Bigg[-\frac{\mathrm{i}}{(q+k)^2 - m_2^2 - \mathrm{i}\varepsilon}$$

$$+ 2\pi\delta\left((q+k)^2 - m_2^2\right)\left[\vartheta(q_0+k_0)f_{X2}(\mathbf{q}+\mathbf{k}) + \vartheta(-q_0-k_0)\bar{f}_{X2}(-\mathbf{q}-\mathbf{k})\right]\Bigg]g(k)\,.$$

Integrals of this type are encountered throughout the calculations of the collision terms $\mathcal{C}$ in the present work. In first place, the individual terms include only products of either two time-ordered or two anti-time ordered propagators, not their sum. However, the terms convoluted with the product of two $\bar{T}$-propagators in the integrated two-loop collision terms can generally be brought to the form of the terms convoluted with the two T-propagators by reversing the sign of all momentum variables, making use of $\mathrm{i}\Delta_X^{<,>}(k) = \mathrm{i}\Delta_X^{>,<}(-k)$ and $\mathrm{i}\Delta_X^{T,\bar{T}}(k) = \mathrm{i}\Delta_X^{T,\bar{T}}(-k)$, for isotropic, charge neutral distributions. In this Appendix, we explain the replacement rule for combinations of scalar fields, but corresponding results are easily seen to follow for integrals involving fermionic fields as well (in which case one has also to pay attention to the behavior of the spinor structure under sign reversals).

The terms containing products of finite-density contributions combine to

$$P_1 = 2\int\frac{d^4k}{(2\pi)^4}\Bigg[2\pi\delta\left((p+k)^2 - m_1^2\right)\left[\vartheta(p_0+k_0)f_{X1}(\mathbf{p}+\mathbf{k}) + \vartheta(-p_0-k_0)\bar{f}_{X1}(-\mathbf{p}-\mathbf{k})\right]\Bigg] \tag{B2}$$

$$\times\left[2\pi\delta\left((q+k)^2-m_2^2\right)\left[\vartheta(q_0+k_0)f_{X2}(\mathbf{q}+\mathbf{k})+\vartheta(-q_0-k_0)\bar{f}_{X2}(-\mathbf{q}-\mathbf{k})\right]\right]g(k)\,.$$

For the products of finite density and vacuum terms one obtains

$$P_2=\int\frac{d^4k}{(2\pi)^4}2\pi\delta\left((p+k)^2-m_1^2\right)\left[\vartheta(p_0+k_0)f_{X1}(\mathbf{p}+\mathbf{k})+\vartheta(-p_0-k_0)\bar{f}_{X1}(-\mathbf{p}-\mathbf{k})\right]$$

(B3)

$$\times 2\pi\delta\left((q+k)^2-m_1^2\right)g(k)$$

$$+\int\frac{d^4k}{(2\pi)^4}2\pi\delta\left((q+k)^2-m_2^2\right)\left[\vartheta(q_0+k_0)f_{X2}(\mathbf{p}+\mathbf{k})+\vartheta(-q_0-k_0)\bar{f}_{X2}(-\mathbf{p}-\mathbf{k})\right]$$

$$\times 2\pi\delta\left((q+k)^2-m_2^2\right)g(k)\,.$$

Finally, the products of the vacuum contributions yield

$$P_3=\int\frac{d^4k}{(2\pi)^4}\left[\frac{\mathrm{i}}{(p+k)^2-m_1^2+\mathrm{i}\varepsilon}\frac{\mathrm{i}}{(q+k)^2-m_2^2+\mathrm{i}\varepsilon}+\frac{(-\mathrm{i})}{(p+k)^2-m_1^2-\mathrm{i}\varepsilon}\frac{(-\mathrm{i})}{(q+k)^2-m_2^2-\mathrm{i}\varepsilon}\right]g(k)$$

(B4)

$$=\int\frac{d^4k}{(2\pi)^4}2\pi\delta\left((p+k)^2-m_1^2\right)2\pi\delta\left((q+k)^2-m_2^2\right)g(k)\,.$$

Note that this contribution gives, of course, the same result as obtained by applying the vacuum cutting rules.

In summary, we obtain

$$\begin{aligned}P=&P_1+P_2+P_3=\int\frac{d^4k}{(2\pi)^4}2\pi\delta\left((p+k)^2-m_1^2\right)2\pi\delta\left((q+k)^2-m_2^2\right)\\&\times\Bigg\{1+\left[\vartheta(p_0+k_0)f_{X1}(\mathbf{p}+\mathbf{k})+\vartheta(-p_0-k_0)\bar{f}_{X1}(-\mathbf{p}-\mathbf{k})\right]\\&+\left[\vartheta(q_0+k_0)f_{X2}(\mathbf{p}+\mathbf{k})+\vartheta(-q_0-k_0)\bar{f}_{X2}(-\mathbf{p}-\mathbf{k})\right]\\&+2\left[\vartheta(p_0+k_0)f_{X1}(\mathbf{p}+\mathbf{k})+\vartheta(-p_0-k_0)\bar{f}_{X1}(-\mathbf{p}-\mathbf{k})\right]\\&\times\left[\vartheta(q_0+k_0)f_{X2}(\mathbf{p}+\mathbf{k})+\vartheta(-q_0-k_0)\bar{f}_{X2}(-\mathbf{p}-\mathbf{k})\right]\Bigg\}g(k)\,.\end{aligned}\tag{B5}$$

Now, provided at the point where the on-shell δ-functions are simultaneously fulfilled, $\mathrm{sign}(p_0 + k_0) = \mathrm{sign}(q_0 + k_0)$, we may express

$$P = P_1 + P_2 + P_3 = \int \frac{d^4k}{(2\pi)^4} \left[\mathrm{i}\Delta^{>}_{X1}(p+k)\mathrm{i}\Delta^{>}_{X2}(q+k) + \mathrm{i}\Delta^{<}_{X1}(p+k)\mathrm{i}\Delta^{<}_{X2}(q+k) \right] g(k) \,. \tag{B6}$$

In the other case, $\mathrm{sign}(p_0 + k_0) = -\mathrm{sign}(q_0 + k_0)$, it is

$$P = P_1 + P_2 + P_3 = \int \frac{d^4k}{(2\pi)^4} \left[\mathrm{i}\Delta^{>}_{X1}(p+k)\mathrm{i}\Delta^{<}_{X2}(q+k) + \mathrm{i}\Delta^{<}_{X1}(p+k)\mathrm{i}\Delta^{>}_{X2}(q+k) \right] g(k) \,. \tag{B7}$$

Appendix C: Evaluation of the Collision Terms in Different Kinematic Situations

On the example of the contributions to the collision term from off-shell binos, that is computed in Section III C, we show how to further evaluate it in different kinematic situations, i.e. when $m_N > m_{\widetilde{\ell}} + m_{H1}$ and $m_N + m_{H1} < m_{\widetilde{\ell}}$. (We assume that H_1 is lighter than N and $\widetilde{\ell}$.) While this is not necessary for the main purpose here, which is to show the cancellation $\mathcal{C}^{\mathrm{v}\widetilde{B}}_{\widetilde{\ell}}(\mathbf{k}) + \mathcal{C}^{\mathrm{v}\widetilde{B}}_{\ell}(\mathbf{k}) = 0$, it is instructive to show how the collision terms are related to the usual CPV source terms in the Boltzmann equations that are proportional to δf_N. We therefore concentrate on $\mathcal{C}^{\mathrm{v}\widetilde{B}}_{\widetilde{\ell}}(\mathbf{k})$ and further simplify this expression by performing an integration over d^3k. Then, we need to distinguish the cases $m_N > m_{\widetilde{\ell}} + m_{H1}$ and $m_N + m_{H1} < m_{\widetilde{\ell}}$.[7] This is because the ϑ-functions occurring within the expressions for the finite-density propagators effectively distinguish between these situations. First, for $m_N > m_{\widetilde{\ell}} + m_{H1}$, we find

$$\begin{aligned} \int \frac{d^3k'}{(2\pi)^3} \mathcal{C}^{\mathrm{v}\widetilde{B}}_{\widetilde{\ell}}(\mathbf{k}') = & -Y^2 g_1^2 \sin\phi_\mu \sin\alpha\cos\alpha \int \frac{d^3k}{(2\pi)^3 2\sqrt{\mathbf{k}^2 + m_N^2}} \delta f_N(\mathbf{k}) k^\mu \\ & \times \int \frac{d^3k'}{(2\pi)^3 2\sqrt{\mathbf{k}'^2 + m_{\widetilde{\ell}}^2}} \frac{d^3k''}{(2\pi)^3 2\sqrt{\mathbf{k}''^2 + \mu^2}} (2\pi)^4 \delta^4(k - k' - k'') \left[1 - f_{\widetilde{H}}(\mathbf{k}'') + f_{\widetilde{\ell}}(\mathbf{k}')\right] \end{aligned} \tag{C1}$$

[7] Note that in the latter case, there would be no cut contribution in the vacuum, but there is one present at finite temperature. In the context of standard Leptogenesis this is pointed out and calculated in Refs. [25, 33].

$$\times \int \frac{d^3p}{(2\pi)^3 2\sqrt{\mathbf{p}^2}} \frac{d^3p'}{(2\pi)^3 2\sqrt{\mathbf{p}'^2 + m_{H1}^2}} p_\mu (2\pi)^4 \delta^4(k - p' - p) \left[1 + f_{H1}(\mathbf{p}') - f_\ell(\mathbf{p})\right]$$

$$\times \frac{\mu M_1}{(p + k')^2 - M_1^2} \,.$$

When $m_N + m_{H1} < m_{\widetilde{\ell}}$, the result is

$$\int \frac{d^3k'}{(2\pi)^3} \mathcal{C}_{\widetilde{\ell}}^{\mathrm{v}\widetilde{B}}(\mathbf{k}') = -Y^2 g_1^2 \sin\phi_\mu \sin\alpha \cos\alpha \int \frac{d^3k}{(2\pi)^3 2\sqrt{\mathbf{k}^2 + m_N^2}} \delta f_N(\mathbf{k}) k^\mu \tag{C2}$$

$$\times \int \frac{d^3k'}{(2\pi)^3 2\sqrt{\mathbf{k}'^2 + m_{\widetilde{\ell}}^2}} \frac{d^3k''}{(2\pi)^3 2\sqrt{\mathbf{k}''^2 + \mu^2}} (2\pi)^4 \delta^4(k - k' + k'') \left[f_{\widetilde{H}}(\mathbf{k}'') + f_{\widetilde{\ell}}(\mathbf{k}')\right]$$

$$\times \int \frac{d^3p}{(2\pi)^3 2\sqrt{\mathbf{p}^2}} \frac{d^3p'}{(2\pi)^3 2\sqrt{\mathbf{p}'^2 + m_{H1}^2}} p_\mu (2\pi)^4 \delta^4(k - p' - p) \left[1 + f_{H1}(\mathbf{p}') - f_\ell(\mathbf{p})\right]$$

$$\times \frac{\mu M_1}{(p + k')^2 - M_1^2} \,.$$

The difference between Eq. (C1) and Eq. (C2) is within the statistical weights of $\widetilde{H}$ and $\widetilde{\ell}$. As one should anticipate, Eq. (C2) vanishes in the vacuum, where all distribution functions are *zero*. In the reminder of this work, we do not distinguish between the different kinematic possibilities, because this is not necessary in order to readily see the cancellations that are of relevance for soft leptogenesis.

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