Efficient Blind Decoding of Orthogonal Space-Time Block Codes Over Time-Selective Fading Channels

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***Abstract*— In this paper, we consider efficient blind decoder design for orthogonal space-time block codes (OSTBCs). A general decision rule for blind OSTBC decoding is derived assuming a quasi-static flat multiple-input multiple-output (MIMO) Rayleigh fading channel. We use the linear dispersion representation of OSTBCs to derive a blind decoder that results in a quadratic minimization problem, which can be solved efficiently by semi-definite relaxation, sphere decoding or successive interference cancellation. To resolve phase ambiguity problems inherent in blind detectors, rather than using pilot symbols that results in a bandwidth loss, we propose novel totally blind decoders using dual constellations or a superimposed training scheme. To alleviate the computational burden, a minimum mean-square-error (MMSE) channel estimator is also proposed to track the time-varying channel without using the blind decoder.**

I. Introduction

Space-time block codes (STBCs) with orthogonal designs [1], [2] for multi-antenna wireless systems effectively utilize diversity gains. Orthogonal STBCs (OSTBCs) achieve full transmit diversity and are amenable to simple linear maximum-likelihood (ML) decoding if the channel state information (CSI) is known at the receiver. However, estimating a multiple antenna channel may be difficult and the pilot symbols will reduce the effective data rate. Moreover, the linear ML decoder requires that the channel remains static over the length of the entire codeword. The channel variation will destroy the orthogonality of the OSTBC receiver filter, and the linear ML decoder will no longer be valid [3]. These factors have motivated blind space-time detectors.

In [4], a suboptimal blind detector (cyclic detector) has been proposed to approximate blind ML OSTBC decoding; i.e., it does not guarantee global optimization. It also requires multiple pilot symbols. Subspace based blind and semi-blind decoders are proposed in [5], where they are also generalized for redundant linear precoders. However, they do not show ML performance. Recently, an efficient approximate blind ML decoder using semi-definite relaxation (SDR) was given in [6]. This SDR-ML decoder provides a substantially better bit error rate (BER) than the previous blind decoders [4], [5]. However, it is applicable only for binary phase shift keying (BPSK) and also needs pilot symbols to solve the phase ambiguity inherent in all blind decoders. All of these papers assume that the channel remains constant over several block[1] intervals. These studies hence neglect any consideration of the Doppler rate - implicity making it to be zero.

[1]Throughout this paper, the term 'block' refers to an OSTBC matrix codeword.

In this paper, different from [4]–[6], we assume the channel remains constant for a single block only and varies from block to block. We call such a channel a quasi static (QS) fading channel - which is valid for normalized Doppler rates up to 3%. However, for a more realistic assessment, in our simulations, we assume a continuous fading channel with the Jakes' spectrum and a given Doppler. We derive a general decision rule for ML blind OSTBC decoding in a QS fading channel. Using the linear dispersion representation of OSTBCs, we show that the decision rule is a discrete quadratic minimization problem. The resulting detector is only an approximate ML detector over QS channels since the true ML detector cannot result in a simple quadratic form. Rather than exhaustive search, we solve the quadratic problem using sphere decoding (SD) [7], vertical-bell labs layered space-time (V-BLAST) [8] or SDR [9]. To solve the inherent phase ambiguity of blind decoders, pilot symbols may be transmitted as in [4], [6]. However, to improve the bandwidth efficiency, we present two novel approaches for totally blind decoding without explicit pilot symbols. The first scheme uses dual constellations so that the angles between a point in one constellation and any point in the other constellation are different. The second scheme makes use of superimposed training, where pilot symbols are added to data symbols. We also optimize the two blind schemes. To alleviate the computational burden of the blind decoder, we give a minimum mean-square-error (MMSE) channel estimator to estimate the channel for data detection in subsequent blocks.

Notation: $E\{\cdot\}$, $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^\dagger$ denote expectation, complex conjugation, transpose, conjugate transpose and Moore-Penrose pseudo-inverse, respectively. The imaginary unit is $\jmath = \sqrt{-1}$. The trace, determinant and the Frobenius norm of matrix $\mathbf{A}$ are $\text{tr}(\mathbf{A})$, $\det(\mathbf{A})$ and $\|\mathbf{A}\|_F^2 = \text{tr}(\mathbf{A}\mathbf{A}^H)$. A circularly complex Gaussian variable with mean μ and variance σ^2 is denoted by $z \sim \mathcal{CN}(\mu, \sigma^2)$. The sets of real numbers and integers are $\mathbb{R}$ and $\mathbb{Z}$. The $N \times N$ identity matrix is $\mathbf{I}_N$. The Kronecker delta is $\delta_{i,j} = 1$ if $i = j$ and $\delta_{i,j} = 0$ if $i \neq j$ where $i, j \in \mathbb{Z}$.

II. System Model

We consider a MIMO system with N_t transmit and N_r receive antennas. Each block of transmitted symbols has T

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time slots and time interval T_B. The symbols transmitted during the nth block are denoted by the $T \times N_t$ matrix $\mathbf{S}[n] = [s_{t,i}[n]]$, $t = 1, 2, \ldots, T$ and $i = 1, 2, \ldots, N_t$, where $s_{t,i}[n]$ is transmitted by the ith antenna in the $t+(n-1)T$-th time slot. For an OSTBC, P symbols $\mathbf{x}[n] = [x_1[n], x_2[n], \ldots, x_P[n]]^T$ with the same average power $E_s = E\{|x_p[n]|^2\}$ are transmitted in the nth block. The entries of $\mathbf{S}[n]$ are linear in $x_i[n]$ and $x_i^*[n]$, and the block has the orthogonal property

$$\mathbf{S}^H[n]\mathbf{S}[n] = c\left(\sum_{p=1}^{P} |x_p[n]|^2\right)\mathbf{I}_{N_t} \quad (1)$$

where $c = 1/r$ and $r = P/T$ is the rate of the code. For the Alamouti code [1] or the $\mathcal{G}_2$ code in [2], $N_t = 2$, $P = 2$, $T = 2$, $c = 1$. An OSTBC can be alternately represented as [10]

$$\mathbf{S}[n] = \sum_{p=1}^{P} (\alpha_p[n]\mathbf{A}_p + \jmath\beta_p[n]\mathbf{B}_p) = \sum_{p=1}^{P} \left(x_p[n]\mathbf{C}_p + x_p^*[n]\mathbf{D}_p\right) \quad (2)$$

where $x_p[n] = \alpha_p[n] + \jmath\beta_p[n]$ and $\mathbf{A}_q$, $\mathbf{B}_q$ are called dispersion matrices [10], which are constant for a given OSTBC. We will make use of the linear dispersion representation of an OSTBC (2) later.

We consider a frequency-flat Rayleigh fading MIMO channel resulting from a rich scattering environment. The received signal at the jth receive antenna at time slot t in the nth block is

$$r_{t,j}[n] = \sum_{i=1}^{N_t} h_{i,j}[n]s_{t,i}[n] + w_{t,j}[n] \quad (3)$$

where $h_{i,j}[n]$ denotes the path gain from the ith transmit antenna to the jth receive antenna and $w_{t,j}[n]$ is the complex additive white Gaussian noise at the jth receive antenna with zero mean and variance σ_n^2. The fading channel is assumed to be QS, i.e., channel variations within each block are negligible. All path gains are statistically independent ($E\{h_{i,j}[n]h^*_{i',j'}[n]\} = \delta_{i,i'}\delta_{j,j'}$) and have the same time correlation function $R_h(\tau)$. Typically, when classical Jakes' model is used, $R_h[m]$ is given by

$$R_h[m] = E\{h_{i,j}[n]h^*_{i',j'}[n+m]\} = \delta_{i,i'}\delta_{j,j'}\sigma_h^2 J_0(2\pi m f_d T_B) \quad (4)$$

where σ_h^2 denotes the power of the path gain, $J_0(\cdot)$ is the zeroth order Bessel function of the first kind, and f_d is the Doppler frequency due to users' mobility. Note that the QS condition is met when $f_d T_B < 0.03$. Eq. (3) can be written in matrix form as

$$\mathbf{R}[n] = \mathbf{S}[n]\mathbf{H}[n] + \mathbf{W}[n] \quad (5)$$

where $\mathbf{R}[n] = [r_{t,j}[n]]$ is the $T \times N_r$ receive matrix, $\mathbf{H}[n] = [h_{i,j}[n]]$ is the $N_t \times N_r$ channel matrix, and $\mathbf{W}[n] = [w_{t,j}[n]]$ is the $T \times N_r$ noise matrix. The code transmission format and channel are shown in Fig. 1.

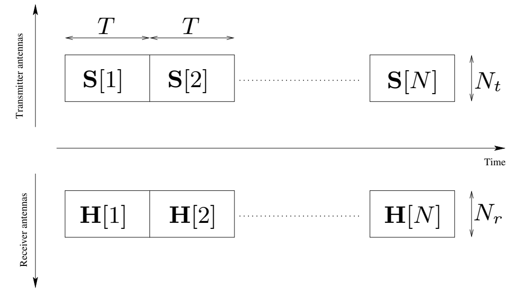

Fig. 1. The transmission diagram of a space time block coded system.

III. Maximum-Likelihood Blind Decoding

This section derives a new general ML metric for blind decoding. Since the blind decoder decodes the transmitted symbols in N consecutive blocks, we consider the received blocks during $n = k+1$ to $n = k+N$. Let $\bar{\mathbf{R}}[k] = [\mathbf{R}^H[k+1], \mathbf{R}^H[k+2], \ldots, \mathbf{R}^H[k+N]]^H$ and $\bar{\mathbf{S}}[k] = [\mathbf{S}^H[k+1], \mathbf{S}^H[k+2], \ldots, \mathbf{S}^H[k+N]]^H$. The ML decision rule for the sequence $\bar{\mathbf{S}}[k]$ can be expressed as

$$\bar{\mathbf{S}}[k] = \arg\max_{\bar{\mathbf{S}}[k]} f(\bar{\mathbf{R}}[k]|\bar{\mathbf{S}}[k]) \quad (6)$$

where $f(a|b)$ is the probability density function (pdf) of a conditioned on b. The conditional pdf (6) can be calculated by averaging the pdf $f(\bar{\mathbf{R}}[k]|\bar{\mathbf{S}}[k], \bar{\mathbf{H}}[k])$ with respect to the channel matrix $\bar{\mathbf{H}}[k]$, which results in

$$\begin{aligned} f(\bar{\mathbf{R}}[k]|\bar{\mathbf{S}}[k]) =& \frac{1}{(\pi^{NN_t}\det(\mathbf{C}_R[k]))^{N_r}} \\ &\times \exp\left(-\mathrm{tr}\left(\bar{\mathbf{R}}^H[k]\mathbf{C}_R^{-1}[k]\bar{\mathbf{R}}[k]\right)\right) \end{aligned} \quad (7)$$

where $\bar{\mathbf{H}} = [\mathbf{H}^T[k+1], \ldots, \mathbf{H}^T[k+N]]^T$ and the conditional covariance matrix $\mathbf{C}_R[k] = E\{\bar{\mathbf{R}}[k]\bar{\mathbf{R}}^H[k]|\bar{\mathbf{S}}[k]\}$ is given by

$$\mathbf{C}_R[k] = \bar{\mathbf{S}}_D[k]\mathbf{C}_H\bar{\mathbf{S}}_D^H[k] + N_r\sigma_n^2\mathbf{I}_{TN} \quad (8)$$

where $\bar{\mathbf{S}}_D[k]$ is a block diagonal matrix

$$\bar{\mathbf{S}}_D[k] = \begin{bmatrix} \mathbf{S}[k+1] & & & \\ & \mathbf{S}[k+2] & & \\ & & \ddots & \\ & & & \mathbf{S}[k+N] \end{bmatrix} \quad (9)$$

and $\mathbf{C}_H$ is the covariance matrix of the vector $\bar{\mathbf{H}}$. $\mathbf{C}_H$ can be represented as

$$\mathbf{C}_H = N_r(\mathbf{C}_h \otimes \mathbf{I}_{N_t}) \quad (10)$$

where $\otimes$ denotes the Kronecker product and $\mathbf{C}_h$ is given by

$$\mathbf{C}_h = \begin{bmatrix} R_h[0] & R_h[1] & \cdots & R_h[N-1] \\ R_h[-1] & R_h[0] & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ R_h[-N+1] & \ldots & \ldots & R_h[0] \end{bmatrix}. \quad (11)$$

If $x_p[n]$'s belong to a unitary constellation, we have

$$\mathbf{S}^H[n]\mathbf{S}[n] = TE_s\mathbf{I}_{N_t}. \quad (12)$$

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However, if $x_p[n]$'s are from a non-unitary constellation, when P is large (for example, $P \geq 4$), $\sum_{p=1}^{P} |x_p[n]|^2 \approx PE_s$ using the large law of numbers in (1) and we have

$$\mathbf{S}^H[n]\mathbf{S}[n] \approx TE_s\mathbf{I}_{N_t}. \tag{13}$$

Since $\det(\mathbf{C}_R[k]) = \det(\mathbf{C}_H\bar{\mathbf{S}}_D^H[k]\bar{\mathbf{S}}_D[k] + N_r\sigma_n^2\mathbf{I}_{N_tN}) \approx \det(TE_s\mathbf{C}_H\mathbf{I}_{N_tN} + N_r\sigma_n^2\mathbf{I}_{N_tN})$ is almost independent of $\bar{\mathbf{S}}_D[k]$ for both unitary and non-unitary constellations, (6) is equivalent to

$$\bar{\mathbf{S}}[k] = \arg\min_{\bar{\mathbf{S}}[k]} \operatorname{tr}\left(\bar{\mathbf{R}}^H[k]\mathbf{C}_R^{-1}[k]\bar{\mathbf{R}}[k]\right). \tag{14}$$

Using the identity $(\mathbf{A}+\mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{C}^{-1} + \mathbf{DA}^{-1}\mathbf{B})^{-1}\mathbf{DA}^{-1}$, (14) becomes

$$\begin{aligned}\hat{\bar{\mathbf{S}}}[k] =& \arg\min_{\bar{\mathbf{S}}[k]} \operatorname{tr}\left(\bar{\mathbf{R}}^H[k](\mathbf{I}_{TN} - \bar{\mathbf{S}}_D[k]\right.\\ &\left.\times(N_r\sigma_n^2\mathbf{C}_H^{-1} + \bar{\mathbf{S}}_D^H[k]\bar{\mathbf{S}}_D[k])^{-1}\bar{\mathbf{S}}_D^H[k])\bar{\mathbf{R}}[k]\right)\\ =& \arg\max_{\bar{\mathbf{S}}[k]} \operatorname{tr}\left(\bar{\mathbf{R}}^H[k]\bar{\mathbf{S}}_D[k]\mathbf{C}\bar{\mathbf{S}}_D^H[k]\bar{\mathbf{R}}[k]\right)\end{aligned} \tag{15}$$

where $\mathbf{C} = (N_r\sigma_n^2\mathbf{C}_H^{-1} + \bar{\mathbf{S}}_D^H[k]\bar{\mathbf{S}}_D[k])^{-1}$. Using (12), $\mathbf{C} = \mathbf{D}\otimes\mathbf{I}_{N_t}$ via Kronecker product properties, and $\mathbf{D} = (N_r\sigma_n^2\mathbf{C}_h^{-1} + TE_s\mathbf{I}_N)^{-1}$ with the (i,j)-th entry $d_{i,j}$. Therefore, (15) can be written as

$$\hat{\bar{\mathbf{S}}}[k] = \arg\max_{\bar{\mathbf{S}}} \sum_{i=1}^{N}\sum_{j=1}^{N} d_{i,j}\operatorname{tr}\left(\mathbf{R}^H[i]\mathbf{S}[i]\mathbf{S}^H[j]\mathbf{R}[j]\right). \tag{16}$$

For brevity, we omit the time index k in (16). We can show that (16) can be expressed as

$$\hat{\mathbf{s}} = \arg\max_{\mathbf{s}} \mathbf{s}^T\mathbf{G}\mathbf{s} \tag{17}$$

where $\mathbf{s} = [\mathbf{s}^T[1],\ldots,\mathbf{s}^T[N]]^T$, $\mathbf{G}$ is a positive semidefinite block matrix with the (i,j)-th block $[\mathbf{G}]_{i,j} = d_{i,j}\mathbf{F}_i^H\mathbf{F}_j$, and $\mathbf{F}_j = [\operatorname{vec}(\mathbf{A}_1^T\mathbf{R}[j]),\ldots, \operatorname{vec}(\mathbf{A}_P^T\mathbf{R}[j]), -\jmath\operatorname{vec}(\mathbf{B}_1^T\mathbf{R}[j]),\ldots,-\jmath\operatorname{vec}(\mathbf{B}_P^T\mathbf{R}[j])]$.

If the channel coherence time is larger than NT_B, the channel remains constant during N blocks. Using M-PSK constellations, all $d_{i,j}$'s are then equal, and hence (16) reduces to the decision metric given in [6]. However, (16) is not limited to BPSK as in [6].

A. Efficient Algorithms

The integer quadratic optimization problem in (17) can be efficiently solved by several algorithms. For brevity, we omit details. For BPSK, (17) can be solved via SDR [9]. In SDR, (17) is relaxed to a convex optimization problem called the semidefinite programming (SDP) [9]. Instead of solving (17), we solve the following so-called SDR problem

$$\begin{aligned}&\max \operatorname{tr}\{\mathbf{SG}\}\\ &\text{s.t. } \mathbf{S}\succeq\mathbf{0}\\ &S_{i,i} = 1.\end{aligned} \tag{18}$$

After obtaining $\mathbf{S}$ from (18) using the algorithm in [9], the blind ML decoding solution $\hat{\mathbf{s}}$ can be found by using the Goemans-Williamson randomization [11], which provides good approximation accuracy with a modest number of randomization operations. The computational complexity of the whole SDR process including randomization is $O((NP)^{3.5})$. However, the SDR decoder is suboptimal. Here, we suggest the use of SD [7] to attain ML performance. If $x_p[k]$'s belong to unitary constellations, $\mathbf{s}^T\mathbf{s} = PN$ and $\eta\mathbf{s}^T\mathbf{s}$ is a constant, where η is a constant. Therefore, the maximization problem (17) becomes

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \mathbf{s}^T\left(\eta\mathbf{I}_{2PN} - \mathbf{G}\right)\mathbf{s}. \tag{19}$$

If η is larger than the maximum eigenvalue of $\mathbf{G}$, $\rho_{\max}$, it can be readily verified that $\eta\mathbf{I}_{2PN} - \mathbf{G}$ is positive definite. There are three possible choices of η: $\rho_{\max}+\sigma_n^2$, $\rho_{\max}+\rho_{\min}$, and $\operatorname{tr}(\mathbf{G})$, where $\rho_{\min}$ is the minimum eigenvalue of $\mathbf{G}$. The third choice is valid since the matrix trace has the property [12]

$$\operatorname{tr}(\mathbf{G}) = \sum_{i=1}^{2PN}\rho_i > \rho_{\max}. \tag{20}$$

We use the first choice in the simulation. Let the Cholesky decomposition of $\eta\mathbf{I}_{2PN} - \mathbf{G}$ be $\mathbf{M}$. Eq. (19) can then be reduced to

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \|\mathbf{Ms}\|^2. \tag{21}$$

The quadratic form (21) is similar to the ML detection rule for a BLAST type MIMO system. Therefore it can be solved by using SD [7] and the V-BLAST detection algorithm [8].

Note that for QPSK each element of $\mathbf{s}$ is chosen from the set $\{-1,1\}$. However, for M-PSK ($M>4$), this does not hold. For any constellation, if $\alpha_p[n]$ is fixed, $\beta_p[n]$ is restricted by the constellation. In SD, when $\alpha_p[n]$ is assigned a value from its candidate set, the candidate set for $\beta_p[n]$ is determined by the bound given by SD and restriction in the constellation. The details of the complex SD are given in [13]. A similar idea can also be applied to the V-BLAST algorithm.

B. Non-Unitary Constellations

When $x_p[k]$'s do not belong to unitary constellation $\mathcal{Q}$, $\det(\mathbf{C}_R[k])$ may not be a constant. From simulation, we find discarding $\det(\mathbf{C}_R[k])$ does not affect the performance. Let $\xi_{\max}$ and $\xi_{\min}$ be the maximum and minimum modulus of the constellation $\mathcal{Q}$, respectively. Eq. (15) is equivalent to minimizing

$$g_1(\mathbf{s}) = \eta PN - \operatorname{tr}\left(\bar{\mathbf{R}}^H\bar{\mathbf{S}}_D(N_r\sigma_n^2\mathbf{C}_H^{-1} + \bar{\mathbf{S}}_D^H\bar{\mathbf{S}}_D)^{-1}\bar{\mathbf{S}}_D^H\bar{\mathbf{R}}\right) \tag{22}$$

where $\mathbf{s}$ is defined in (17). Let $\mathbf{A} = N_r\sigma_n^2\mathbf{C}_H^{-1} + \bar{\mathbf{S}}_D^H\bar{\mathbf{S}}_D$ and $\mathbf{B} = N_r\sigma_n^2\mathbf{C}_H^{-1} + T\xi_{\min}^2\mathbf{I}_N$. It can be readily verified that $\mathbf{A}\succeq\mathbf{B}$, where $\mathbf{A}\succeq\mathbf{B}$ means $\mathbf{A}-\mathbf{B}$ is positive semi-definite. Using Corollary [12, p. 471], we can obtain $\mathbf{A}^{-1}\preceq\mathbf{B}^{-1}$. It can be proven that

$$\operatorname{tr}\left(\bar{\mathbf{R}}^H\bar{\mathbf{S}}_D\mathbf{A}^{-1}\bar{\mathbf{S}}_D^H\bar{\mathbf{R}}\right) \leq \operatorname{tr}\left(\bar{\mathbf{R}}^H\bar{\mathbf{S}}_D\mathbf{B}^{-1}\bar{\mathbf{S}}_D^H\bar{\mathbf{R}}\right). \tag{23}$$

Therefore, we have

$$g_1(\mathbf{s}) \geq \mathbf{s}^T\left(\frac{\eta}{\xi_{\max}^2}\mathbf{I}_{2PN} - \mathbf{G}'\right)\mathbf{s} = g_2(\mathbf{s}). \tag{24}$$

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where the (i,j)-th block of $\mathbf{G}'$ is $[\mathbf{G}']_{i,j} = b_{i,j}\mathbf{F}_i^H\mathbf{F}_j$, $b_{i,j}$ is the (i,j)-th entry of $\mathbf{B}$ and $\mathbf{F}_i$ is defined in (17). When using SD, we solve $g_2(\mathbf{s}) \leq g_1(\mathbf{s}) < r^2$. All the candidates that satisfy $g_2(\mathbf{s}) < r^2$ are found, and the one that makes $g_1(\mathbf{s})$ a minimum is the ML solution. During the search, the bound (or the radius) r^2 can be updated by $g_1(\tilde{\mathbf{s}})$, where $\tilde{\mathbf{s}}$ is a valid candidate within the hyper-sphere. If $\xi_{\max}$ is much larger than $\xi_{\min}$, the bound given by $g_1(\mathbf{s})$ is loose and $g_2(\mathbf{s}) \leq r^2$ contains many points, which makes the algorithm inefficient.

IV. Totally Blind Decoders

In the following, we take the Alamouti code with M-PSK ($Q_M = \{e^{\jmath 2\pi m/M}\}$, $m = 0, \ldots, M-1$) for example. The decoders can also be generalized to other OSTBC's and constellations (details omitted for brevity). Clearly, a phase ambiguity exists in (16). Assuming that the optimal solution of (16) is given by $\hat{\mathbf{S}}[i]$, $i = 1, \ldots, N$, if there exists a unitary matrix $\mathbf{\Theta}$ such that $\tilde{\mathbf{S}}[i] = \hat{\mathbf{S}}[i]\mathbf{\Theta}$ is also a valid codeword, $\tilde{\mathbf{S}}[i]$ is also a set of the optimal solution. The ambiguity can be classified into two classes. First, let us define a rotation matrix

$$\mathbf{\Theta} = \begin{pmatrix} e^{\jmath 2\pi k/M} & 0 \\ 0 & e^{-\jmath 2\pi k/M} \end{pmatrix}, \; k \in \{0, 1, \ldots, M-1\}. \tag{25}$$

If $\mathbf{S}[1]$ or symbols $\{x_1[1], x_2[1]\}$ are transmitted and maximize (17), $\tilde{\mathbf{S}}[1] = \mathbf{S}[1]\mathbf{\Theta}$ is also a valid codeword and the transmitted symbols are identified as $\{x_1[1]e^{\jmath 2\pi k/M}, x_2[1]e^{-\jmath 2\pi k/M}\}$. Another example is, for any OSTBC, $\mathbf{\Theta} = -\mathbf{I}_{N_t}$ is another unitary matrix resulting in the ambiguity. We name it rotational ambiguity to denote the phase rotation on each symbol. Such ambiguity can be resolved using only one pilot, i.e., transmitted at the first symbol $x_1[1]$ as suggested in [6]. The second ambiguity is more complicated. We assume that $\{x_1[1], x_2[1]\}$ is transmitted in the first block and $x_1[1]$ is a pilot symbol. Let

$$\mathbf{\Theta} = \begin{pmatrix} 0 & -e^{\jmath 2\pi k/M} \\ e^{-\jmath 2\pi k/M} & 0 \end{pmatrix}, \; k \in \{0, 1, \ldots, M-1\}. \tag{26}$$

The matrix $\tilde{\mathbf{S}}[1] = \mathbf{S}[1]\mathbf{\Theta}$ is a code matrix given by $\{-x_2[1]e^{\jmath 2\pi k/M}, x_1[1]e^{-\jmath 2\pi k/M}\}$. Therefore if $-x_2[1]e^{\jmath 2\pi k/M} = x_1[1]$, both $\mathbf{S}[1]$ and $\tilde{\mathbf{S}}[1]$ are valid codewords with the first symbol transmitted by $x_1[1]$. This produces the second ambiguity, which cannot be solved by using a single pilot symbol. Since this ambiguity has both rotation on each symbol and permutation of symbols, we name it permutation ambiguity. Since all of the matrices $\mathbf{S}[n]$, $n = 1, \ldots, N$ share the same matrix $\mathbf{\Theta}$ if a $\mathbf{\Theta}$ exists, $\mathbf{\Theta}$ is determined to be $\mathbf{I}_{N_t}$ if a pilot block is transmitted, i.e., $\mathbf{S}[1]$ contains all known symbols. However, the pilot symbols cause a bandwidth loss, which motivates the research for totally blind decoders without any pilots. We present two schemes in the following.

A. Dual-constellation scheme

For the Alamouti code, the two classes of ambiguity include both of the ambiguities. In [14], two different PSK-constellations are used to solve the phase ambiguity in blind OFDM detection, which motivates the totally blind decoder in this paper.

In the first totally blind decoder scheme, we propose the use of dual constellations in N consecutive blocks. The constellation design criterion is:

Criterion 1: The two constellations are chosen such that the angles between a point in one constellation and any point in the other constellation are different.

QPSK ($\mathcal{Q}_4 = \{e^{\jmath m\pi/2+\pi/4}, m = 0, 1, 2, 3\}$) and 3-PSK ($\mathcal{Q}_3 = \{e^{\jmath 2m\pi/3}, m = 0, 1, 2\}$) satisfy this criterion. For example, 3-PSK is used in the $1, 3, \ldots, N-1$, odd blocks and QPSK is used in the remaining even blocks assuming N is an even number. If $\hat{\mathbf{S}}[1]$ and $\hat{\mathbf{S}}[2]$ maximize (16), $\mathbf{\Theta}_1$ is a rotation matrix given by (25) or (26) for $\hat{\mathbf{S}}[1]$, and $\hat{\mathbf{S}}[1]\mathbf{\Theta}_1$ is also a feasible codeword. The rotation angle for $\hat{\mathbf{S}}[1]$ is a multiple of $\pi/2$. However, when it is applied to $\hat{\mathbf{S}}[2]$, it can be verified that $\hat{\mathbf{S}}[2]\mathbf{\Theta}_1$ cannot result in valid codewords for $\hat{\mathbf{S}}[2]$ due to the use of different constellations. Similarly, the rotation matrix $\mathbf{\Theta}_2$ is also not applicable for $\hat{\mathbf{S}}[1]$. Thus, there does not exist a $\mathbf{\Theta}$ that makes both $\hat{\mathbf{S}}[1]\mathbf{\Theta}$ and $\hat{\mathbf{S}}[2]\mathbf{\Theta}$ valid. Therefore, (16) has a unique solution. QPSK with 5-PSK and 8-PSK with 7-PSK also satisfy the property.

The 3-PSK and QPSK constellations pair is not optimized in [14]. We optimize the constellations by maximizing the Euclidean distance between the correct point and the wrong point by additive noise or phase ambiguity. We find that the optimal 3-PSK constellation is $\mathcal{Q}_3 = \{1, e^{\jmath 5\pi/8}, e^{-\jmath 5\pi/8}\}$ and the optimal QPSK is $\mathcal{Q}_4 = \{e^{\jmath k\pi/2+\pi/4}, k = 0, 1, 2, 3\}$.

The binary bits are mapped to 3-PSK via a punctured convolutional encoder in [14]. Here we introduce a mapping scheme similar to a linear block code. 3 binary bits are mapped to two 3-PSK symbols, which consists of 9 tuples. The tuple $(0, 0)$ is not mapped and therefore it has 0.17 bits loss. When performing ML decoding, this tuple is similar to a parity check bit in a linear block code, which can correct the error. Since the gray mapping does not exist for the 3 bits mapping, we develop a quasi-gray mapping scheme by minimizing the neighborhood bit errors. After optimization, we find the suboptimal mapping is given by

$$\begin{aligned} &100 \to (1, e^{\jmath\frac{5\pi}{8}}), \quad 010 \to (1, e^{-\jmath\frac{5\pi}{8}}), \quad 001 \to (e^{\jmath\frac{5\pi}{8}}, 1), \\ &000 \to (e^{\jmath\frac{5\pi}{8}}, e^{\jmath\frac{5\pi}{8}}), \; 011 \to (e^{\jmath\frac{5\pi}{8}}, e^{-\jmath\frac{5\pi}{8}}), \; 111 \to (e^{-\jmath\frac{5\pi}{8}}, 1), \\ &101 \to (e^{-\jmath\frac{5\pi}{8}}, e^{\jmath\frac{5\pi}{8}}), 110 \to (e^{-\jmath\frac{5\pi}{8}}, e^{-\jmath\frac{5\pi}{8}}). \end{aligned} \tag{27}$$

The use of two PSK constellations reduces the minimum Euclidean distance. Alternatively, a semi-blind decoder can be designed by transmitting one pilot (i.e., by fixing one element of s). Then, the ambiguity problems will be eliminated. Compared with the decoder with rotatable codes using a pilot block, the semi-blind decoder is also bandwidth efficient.

Using the dual-constellation scheme, the resulting totally blind decoder (20) can be solved using the modified sphere decoder for PSK in [13].

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B. Superimposed pilots scheme

The superposition of pilot and data symbols has been proposed in [5], [15] for channel estimation. Our key idea is to use superimposed pilots to resolve the phase ambiguity. The p-th transmitted symbol in the n-th block can be represented as

$$x_p[n] = \sqrt{\gamma_{n,p}} t_p[n] + \sqrt{\lambda_{n,p}} u_p[n] \tag{28}$$

where $t_p[n]$ is the known pilot and $u_p[n]$ is a data symbol from $\mathcal{Q}$. We have $\gamma_{n,p} + \lambda_{n,p}$, and $\gamma_{n,p}$ denotes the percentage of the power allocated to training. The average power of $x_p[n]$ is E_s. In fact, (28) is a framework for all of the training schemes in this paper. If $\gamma_{n,p} = 1$ for $p = 1, \ldots, P$, it reduces to the case using a pilot block to solve the ambiguity. When $\gamma_{n,p} = 0$ and the two constellations satisfying Criterion 1 are employed, it becomes the dual constellation scheme. We still call the decoder using (28) a totally blind decoder since if $\gamma_{n,p} \neq 1$, the data rate remains the same as $\gamma_{n,p} = 0$ or full rate. The superimposed pilots can be used only for the first block, i.e., $0 < \gamma_{1,p} < 1$ and $\gamma_{n,p} = 0$ for $n = 2, \ldots, N$ or for all of the blocks $0 < \gamma_{1,p} < 1$ for $n = 1, \ldots, N$. We show next the superimposed pilots scheme is also a necessary condition for the totally blind decoder.

The two constellations scheme resolves the ambiguity by modifying only the phase so that there does not exist a valid rotation matrix $\mathbf{\Theta}$ for all of the OSTBC codewords from the two constellations. For fixed $t_p[n]$, (28) forms a new non-symmetric constellation for $x_p[n]$ with nonzero mean and we denote the new constellation as $\mathcal{Q}_s$. Clearly, either the phase or the amplitude of the point in $\mathcal{Q}_s$ are different from those in $\mathcal{Q}$. Except for BPSK, $\mathcal{Q}_s$ and $\mathcal{Q}$ satisfy Criterion 1. In addition, due to the difference in amplitude, the minimum Euclidean distance between the correct point and the wrong point by additive noise or phase ambiguity may be increased and this leads to performance improvement.

The value for $\gamma_{n,p}$ can be optimized. We take BPSK for example, and we assume that all of the $\gamma_{1,p}$'s are equal and $E_s = 1$. If $\gamma_{n,p} > \lambda_{n,p}$ and $t_p[n] = 1$, $\mathcal{Q}_s = \{\sqrt{\gamma_{n,p}} + \sqrt{\lambda_{n,p}}, \sqrt{\gamma_{n,p}} - \sqrt{\lambda_{n,p}}\}$. Due to additive noise and permutation ambiguity, $\mathcal{Q}_s$ may be treated as $\mathcal{Q}'_s = \{\sqrt{-\gamma_{n,p}} - \sqrt{\lambda_{n,p}}, -\sqrt{\gamma_{n,p}} + \sqrt{\lambda_{n,p}}\}$. To gain the best performance, we should optimize

$$\max \min\{2\sqrt{\lambda_{n,p}}, 2\sqrt{\gamma_{n,p}} - 2\sqrt{\lambda_{n,p}}\}. \tag{29}$$

We can get $\gamma_{n,p} = \frac{4}{5}$ and $\lambda_{n,p} = \frac{1}{5}$. Similarly, if $\gamma_{n,p} < \lambda_{n,p}$, we have $\gamma_{n,p} = \frac{1}{5}$ and $\lambda_{n,p} = \frac{4}{5}$.

For the detection of the superimposed scheme, we note that if $\mathbf{S}[n]$ contains superimposed pilots, $\mathbf{S}^H[n]\mathbf{S}[n]$ cannot be approximated as a diagonal matrix any more and $\mathcal{Q}_s$ is not a unitary constellation. We thus apply the modified SD (22)-(24) for non-unitary constellations. By choosing $\xi_{\max}$ and $\xi_{\min}$ for $\mathcal{Q}_s$, we can define $g_2(\mathbf{s}) = \|\mathbf{Ms}\|^2$ as in (24), where $\mathbf{M}^T\mathbf{M} = \frac{\eta}{\xi^2_{\max}}\mathbf{I}_{2PN} - \mathbf{G}'$. Note that the $\mathbf{s}$ in (21) can be written as

$$\mathbf{s} = \mathbf{\Gamma}_t \mathbf{t} + \mathbf{\Gamma}_u \mathbf{u} \tag{30}$$

where $\mathbf{t}$ and $\mathbf{u}$ are formulated using the real and imaginary parts of $t_p[n]$ and $u_p[n]$ as $\mathbf{s}$ in (17), $\mathbf{\Gamma}_t$ and $\mathbf{\Gamma}_u$ are diagonal matrices with diagonal entries; Thus, $g_2(\mathbf{s})$ is reduced to

$$g_2(\mathbf{u}) = \|\mathbf{y} - \mathbf{M}'\mathbf{u}\|^2 \tag{31}$$

where $\mathbf{y} = -\mathbf{M\Gamma}_t\mathbf{t}$ and $\mathbf{M}' = \mathbf{M\Gamma}_u$. When using SD, we solve $g_2(\mathbf{u}) \leq g_1(\mathbf{s}) < r^2$, $g_1(\mathbf{s})$ is defined in (22) and r^2 is updated using $g_1(\mathbf{s})$.

Since the information bearing symbols at the superimposed pilot blocks have less energy than other pure data symbols, these symbols may not be reliable. To improve the overall performance, we first use the MMSE channel estimator in Section V to estimate the channel in pure data blocks. The channel at superimposed pilot blocks are then predicted and the superimposed data is detected using the linear coherent ML decoder for an OSTBC.

V. Channel estimation and prediction

Even though the data symbols can be efficiently detected using (17) without estimating the channel solved using SD, the computational burden of the blind decoder may also be high for a practical system. To alleviate the burden, the receiver has two modes: blind mode and decision-directed mode. The receiver starts with the blind mode using our blind decoder (17). The detected data are used to estimate and predict the channel. The receiver then reverts to the decision-directed mode, where the decoded symbols are used to predict the channel.

In the blind mode, after the data symbols have been detected using (17), the channel can be MMSE estimated using $\hat{\mathbf{S}}[k]$. If the channel remains constant during N blocks, the MMSE channel estimator is given by

$$\hat{\mathbf{H}} = \left(c \sum_{n=k+1}^{k+N} \sum_{p=2}^{P} |\hat{x}_p[n]|^2 + \frac{\sigma_n^2}{\sigma_h^2} \right)^{-1} \left(\sum_{n=k+1}^{k+N} \mathbf{S}^H[n]\mathbf{R}[n] \right). \tag{32}$$

Further details of the blind and decision-directed mode operations will be provided in a journal version of this paper.

VI. Simulation Results

We now present simulation results for our blind decoder over a flat Rayleigh fading channel. As opposed to the QS fading assumption, the MIMO channel gains are generated by sampling a continuous fading process via the Jakes' model. The SNR is defined to be $E\{\|\mathbf{H}\|_F^2\}/\sigma_n^2$. We have assumed in all the simulations that the receiver has perfect knowledge of channel correlation and noise variance.

We first consider the non-rotatable OSTBC with $N_t = 3$ and $P = 4$ [6]

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & x_4 & -x_3 \\ -x_3 & -x_4 & x_1 & x_2 \end{pmatrix}. \tag{33}$$

The number of receiver antennas is $N_r = 3$ and the number of blocks is $N = 8$. BPSK is used for this code. $x_1[1]$ is transmitted as a pilot to solve the phase ambiguity. The

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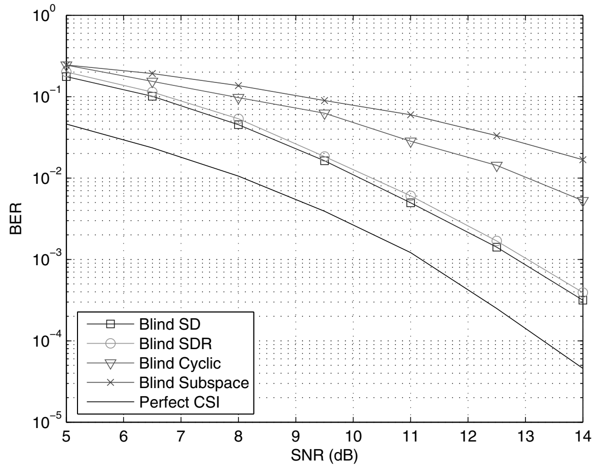

Fig. 2. BER versus SNR for different blind decoders with $N = 8$ and BPSK over a static channel.

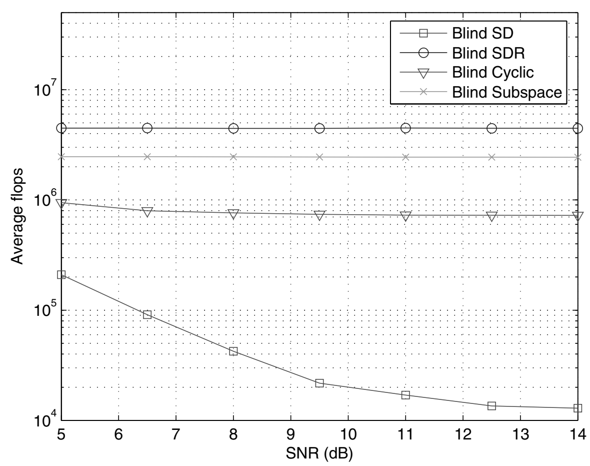

Fig. 3. Average flops versus SNR for different blind decoders with $N = 8$ and BPSK over a static channel.

MATLAB V5.3 command "flops" is used to count the number of flops. The ML decoding with perfect CSI is used as the benchmark. The SDR algorithm follows exactly the one given in [9].

We first consider that the channel remains constant for N blocks. Fig. 2 shows the BER versus SNR of various blind decoders, i.e., (21) with SD (blind SD), blind SDR [6], blind cyclic [4] and blind subspace [5]. The blind SD and blind SDR perform substantially better than the other blind decoders. At BER=10^{-3}, the blind SD has a 0.2-dB gain over blind SDR. Compared with the benchmark, the blind SD performs 2 dB worse, which is due to the differential mechanism behind the blind decoder.

Fig. 3 compares the average complexities of different blind decoders in a static channel. The average flop count is used as the complexity measure. The complexity of the preprocessing stage such as Cholesky decomposition is also counted for blind SD. The complexities of blind SDR and blind subspace are independent of SNR, while the other two depend on SNR. The blind SDR is the most complex one, although its complexity is claimed to be $\mathcal{O}((NP)^{3.5})$. In the observed SNR region, the blind SD achieves the smallest complexity. Therefore, blind SD outperforms the other blind decoders in both BER and complexity in that region.

VII. Conclusion

We have investigated efficient blind OSTBC decoder design. A general ML decision metric for a QS channel has been derived. Our blind decoder results in a quadratic optimization problem that can be efficiently solved using SD, V-BLAST and SDR. We gave two conditions to determine whether an OSTBC can be blindly decoded. For rotatable OSTBCs, pilot symbols are needed to solve the inherent phase ambiguity. To save the bandwidth, we proposed two totally blind decoders via dual constellations and superimposed pilots. We also gave a blind receiver structure for OSTBCs operating in two modes and presented an MMSE channel estimator and predictor.

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