

# Rate Equations Analysis of Phase-Locked Semiconductor Laser Arrays Under Steady State Conditions

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**Abstract**—Rate equations analysis of phase-locked semiconductor laser arrays has been carried out. It was found that for given (laser) current densities, the photon density distribution in the array elements is that particular one which maximizes the total photon density. The results of this analysis were then combined with the waveguiding properties of the laser array waveguide, yielding a basic model of phase-locked diode laser arrays. This model explains the effects of the variation of the current combination through the array elements on its mode structure that were observed recently.

**P**HASE locking of semiconductor injection lasers has been the subject of widespread research effort recently, with most of the work implemented in various monolithic configurations of one-dimensional arrays [1]–[10]. The few theoretical investigations of such arrays, to date, involve the evaluation of the array far-field pattern using an ad hoc presumed near-field pattern [4], and more recently, the construction of its optical field in terms of the array supermodes [11] (i.e., the eigenmodes of the array waveguide). A more complete analysis of the array properties, however, should include the effect of the gain distribution among the different array elements, as determined by the carrier and photon densities, rather than considering just the “cold” cavity of the array. The effect of the saturated gain distribution is of particular importance in the case of multiple-stripe lasers. Whereas single-stripe lasers are designed mostly to support a single spatial mode,  $N$ -channel laser arrays are characterized by  $N$  (lateral) supermodes [11]. Since each of these supermodes exhibits, generally, different excitations of the different array channels [11], it is clear that gain saturation effects are important in determining the modal gain of the supermodes and, hence, their relative excitation at various pumping levels. This, in turn, determines the array far-field pattern [11] and its longitudinal mode structure [12]. Furthermore, as the control of the gain distribution among the array channels can be realized by providing each laser with a separate contact [10], the results of this more complete

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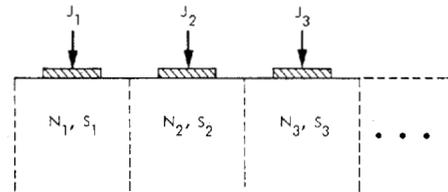


Fig. 1. Schematic illustration of the laser array. The carrier and photon density in each laser are  $N_i$  and  $S_i$ , respectively, and the current density flowing through it is  $J_i$ .

analysis can be utilized to find the conditions under which a single supermode can be excited. Such mode of operation is most desirable, since it results in narrow far-field pattern and narrow spectral linewidth of the array.

In this paper, we present a rate equation analysis of injection laser arrays, which yields the most favorable optical power distribution associated with a given current distribution across the array. This information is then combined with the optical analysis of the array waveguide. The resulting basic model of phase-locked arrays serves to explain their observed mode structure.

The laser array is depicted schematically in Fig. 1. It is assumed that the array operates in a single longitudinal, phase-locked mode. (Such mode of operation is feasible, as demonstrated experimentally in [12].) Each element of the array is characterized by its photon density  $S_i$ , which is the portion of the intensity of the array supermode [11] in the  $i$ th channel, and its carrier density  $N_i$ , and is fed with current density  $J_i$ . [The array lasers are treated as discrete elements; i.e., the spatial (lateral) distributions of  $S$  and  $N$  are assumed to have been eliminated by an appropriate integration.] The interplay among these variables can be described by means of the laser rate equations [13]. Consider first the simplest case of two coupled lasers. The steady state laser rate equations are

$$\frac{dN_1}{dt} = \frac{J_1}{qd_1} - A(N_1 - N_{om})S_1 - \frac{N_1}{\tau_s} = 0 \quad (1a)$$

$$\frac{dN_2}{dt} = \frac{J_2}{qd_2} - A(N_2 - N_{om})S_2 - \frac{N_2}{\tau_s} = 0 \quad (1b)$$

$$\frac{d(S_1 + S_2)}{dt} = A[(N_1 - N_{om})S_1 + (N_2 - N_{om})S_2] - \frac{S_1 + S_2}{\tau_{ph}} + \beta \frac{N_1 + N_2}{\tau_s} = 0 \quad (2)$$

where  $d_i$  is the active region thickness of the  $i$ th laser,  $A$  is the gain constant [13],  $N_{om}$  is the carrier density required for transparency,  $q$  is the electron charge, and  $\tau_s$  and  $\tau_{ph}$  are the carrier and photon lifetimes, respectively. The parameter  $\beta$  denotes the fraction of spontaneous emission coupled into the lasing mode, and is of the order of  $10^{-4}$ . For practical values of the parameters, it was found that the last term in (2) can be neglected in all subsequent calculations. It is also worth mentioning that (1) and (2) do not include any coupling terms between the two cavities. This is due to the fact that when the array is phase-locked, it is basically a single composite device operating in a single supermode [11], and, thus, the concept of coupling is irrelevant (i.e., there is no such thing as coupling within a mode).

Using the following normalizing transformations for the pumping current densities, photon densities and carrier densities,

$$p_i = A\tau_{ph} \frac{J_i \tau_s}{qd_1} - A\tau_{ph} N_{om} \equiv n_{oi} - n_{om} \quad i = 1, 2 \quad (3a)$$

$$s_i = A\tau_s S_i \quad i = 1, 2 \quad (3b)$$

$$n_i = A\tau_{ph} N_i \quad i = 1, 2 \quad (3c)$$

and analyzing the resulting set of equations, it is found (see Appendix A) that the only stable solution (see below) to (1) and (2) is for photon density ratio which satisfies the following quadratic equation:

$$\frac{s_2}{s_1} - \frac{s_1}{s_2} = \frac{p_1 - p_2}{\sqrt{p_2 p_1} - 1}. \quad (4)$$

The solution of (4) is shown in Fig. 2 in terms of the current densities flowing through the lasers. Parameter values used for the calculations are  $A = 1.5 \cdot 10^{-6} \text{ cm}^3 \cdot \text{s}^{-1}$ ,  $N_{om} = 7.5 \cdot 10^{17} \text{ cm}^{-3}$ ,  $d = 0.2 \text{ } \mu\text{m}$ ,  $\tau_s = 3 \cdot 10^{-9} \text{ s}$  and  $\tau_{ph} = 1 \cdot 10^{-12} \text{ s}$  which describe typical GaAs laser arrays. As expected,  $s_2$  approaches  $s_1$ , as  $J_1$  approaches  $J_2$ . For operation far above threshold (i.e.,  $p_i \gg 1$ ,  $i = 1, 2$ ),  $s_2/s_1$  approaches the asymptotic value of  $\sqrt{p_2/p_1}$ . This square root dependence is a direct result of gain saturation in the lasers.

It is interesting to note that the total photon density  $s = s_1 + s_2$  attains its maximum at the operating point given by (4). As shown in Fig. 3, possible solutions where the modal gain equals the modal loss exist for many values of  $s$ . However, the only stable solution is that in which the modal gain equals the modal loss at a single point, indicated in Fig. 3 by  $P_0$ . All other solutions are unstable with respect to fluctuations in  $s$ —for example, due to spontaneous emission—and therefore will gravitate toward  $P_0$ .

In the more general case of an  $N$ -element array, the pertinent normalized equations are also derived using the transformations of (3). The resulting set of equations is:

$$0 = n_{oi} - (n_i - n_{om})s_i - n_i, \quad i = 1, 2 \cdots N \quad (5)$$

$$0 = \sum_{i=1}^N (n_i - n_{om} - 1)s_i, \quad (6)$$

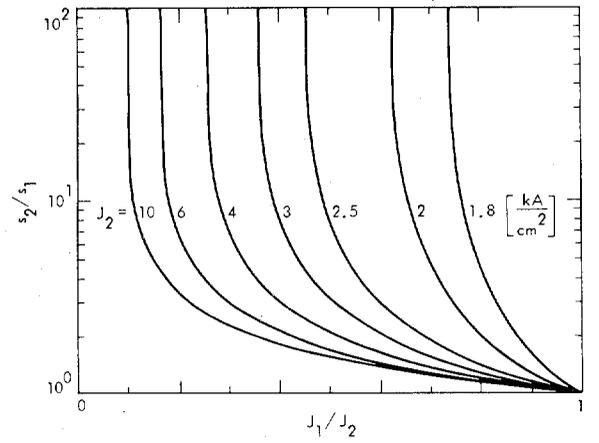


Fig. 2. Photon density distribution ( $s_2/s_1$ ) in a two-element array versus current density in the first laser (with the current density in the second laser as a parameter).

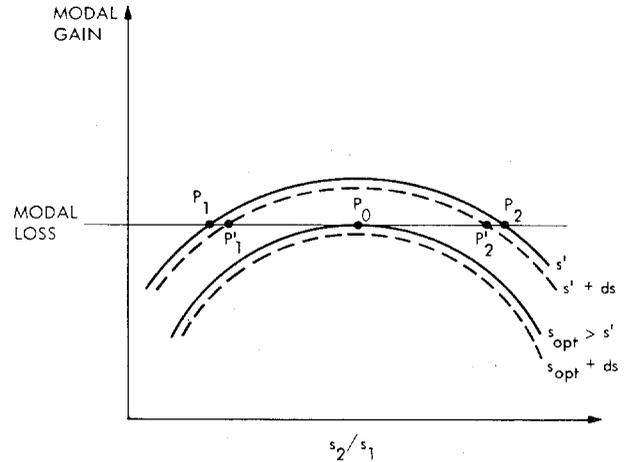


Fig. 3. Schematic dependence of the modal gain  $s_2/s_1$ , with  $s = s_1 + s_2$  as a parameter.

Following a similar procedure as in the case of the two coupled lasers, we find that the fraction of photon density in the  $i$ th laser,  $\rho_i$ , is given by

$$\rho_i \equiv \frac{s_i}{\sum_{j=1}^N s_j} \equiv \frac{s_i}{s} = \frac{C\sqrt{p_i} - 1}{C \sum_{j=1}^N \sqrt{p_j} - N} \quad (7)$$

where  $C$  is a parameter that depends on the pumping of the different lasers:

$$C = \frac{\left[ \sum_{i=1}^N p_i + N \right] + \sqrt{\left[ \sum_{i=1}^N p_i + N \right]^2 - 4 \left[ \sum_{i=1}^N \sqrt{p_i} \right]^2}}{2 \sum_{i=1}^N \sqrt{p_i}} \quad (8)$$

The total (normalized) photon density of the array is given by

$$s = C \sum_{i=1}^N \sqrt{p_i} - N. \quad (9)$$

(The derivation of (7)–(9) is outlined in Appendix B.) It is interesting to note that sufficiently above threshold (i.e.,  $C\sqrt{p_i} \gg 1$ ), we can see from (7) that the photon density ratio between any two lasers in the array does not depend on the number of array elements. In some symmetrical cases, this is true regardless of the magnitude of the pumping currents; for example,  $\rho_2/\rho_1$  is the same in a 2-element array with normalized pumping currents  $p_1$  and  $p_2$  as in a 4-element array with  $p_4 = p_1$  and  $p_3 = p_2$ .

The importance of the information contained in (7) [or (4), for the  $N = 2$  case] becomes obvious when it is combined with the waveguiding properties of the array. The intensity patterns of the array supermodes, and, therefore, the photon density fractions  $\rho_i$  of the different channels are determined by the propagation constants  $\beta_i$  and the gains  $\gamma_i$  in the various array channels as well as by the coupling coefficients  $K_{ij}$  between the array elements [11]. The (field) gains  $\gamma_i$  are related to the current densities  $J_i$  by

$$\gamma_i = \frac{n_i - n_{om}}{2v_i\tau_{ph}} \quad i = 1, 2, \dots, N \quad (10)$$

where  $v_i$  is the phase velocity of the laser mode in the  $i$ th channel.

The formal procedure leading to (7), on the other hand, enables us to determine the individual channel photon densities  $s_1$  and  $s_2$  given the currents  $J_1$  and  $J_2$  in the case of 2 elements. It follows that in order to excite a pure supermode in the total  $N$  channel structure, the individual currents  $J_1, J_2, \dots$  must be such that the resulting photon densities  $s_1 \dots s_N$ , as determined from (7), are the same as those determined from the supermode analysis. Or, to summarize, at given total output  $s$ , a given supermode requires a specific set of channel currents.

We have determined qualitatively in our separate contact laser array experiments that, indeed, a single supermode depended in a sensitive fashion on an independent adjustment of the channel currents [12].

A peculiar property of multichannel waveguides is the significant sensitivity of their supermode intensity patterns with respect to frequency [12]. Formally, this frequency dependence enters mainly through the dispersion of the propagation constants  $\beta_i$ . Therefore, it is clear that changing the current density combination  $J_i$ , yielding a change in the intensity pattern for  $\rho_i$  [via (7)], would result in tuning of the lasing wavelength, as was indeed observed experimentally [12], [14]. For example, in the simple case of a two-element array the phase-velocity mismatch  $\Delta\beta \equiv \beta_1 - \beta_2$  and the gain difference  $\Delta\gamma \equiv \gamma_1 - \gamma_2$  are related to the photon density ratio  $\rho \equiv \rho_2/\rho_1 = s_2/s_1$  through [15]

$$\left[ \frac{2\rho}{\rho^2 - 1} \frac{\Delta\beta}{2K} \right]^2 + \left[ \frac{2\rho}{\rho^2 + 1} \frac{\Delta\gamma}{2K} \right]^2 = 1 \quad (11)$$

where  $K$  is the coupling coefficient of the two laser waveguides. Equation (11) is obtained by straightforward algebraic manipulations from the coupled mode formalism [16] which is modified to take into account the gain difference  $\Delta\gamma$ , between the two waveguides by replacing  $\Delta\beta$  with  $\Delta\beta - i\Delta\gamma$  [15].  $\rho$  is

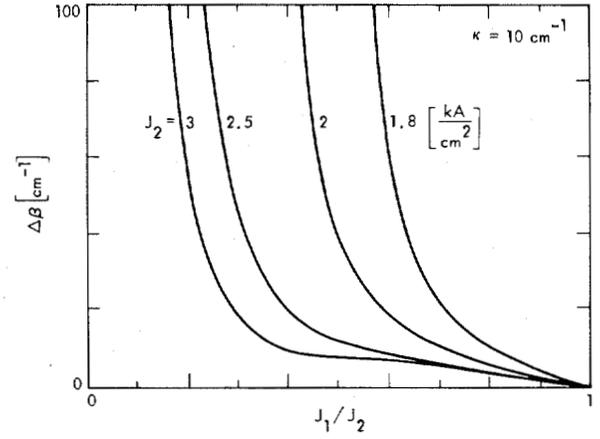


Fig. 4. Schematic depiction of current induced wavelength tuning of a two-laser array.

the squared magnitude of the admixture factor of the fields in the two waveguides. Using (4), (10), and (11), one can relate the deviation from phase matching  $\Delta\beta$  to the current combination through the coupled lasers. Fig. 4 shows an example of such calculations, employing parameters which correspond to GaAs lasers. For given values of  $J_1/J_2$ , the coupled lasers would tend to oscillate with detuning  $\Delta\beta$  in which their modal gain is maximized, taking into account the photon density ratio between the two coupled channels. Since  $\Delta\beta$  is readily related to the wavelength deviation from the phase-matching wavelength [15], Fig. 4 can be used in order to evaluate the wavelength tuning which results from the currents variation. Such a current-induced wavelength tuning in a coupled cavity laser has been recently demonstrated [14], with a tuning range of  $\sim 30$  Å.

In conclusion, we have presented a basic analysis for the photon density distribution among lasers in a semiconductor phase-locked laser array. The results of this analysis, when combined with the waveguiding properties of the array waveguide, give insight into the observed mode structure of laser arrays and yield useful guidelines in optimum array design.

#### APPENDIX A DERIVATION OF (4)

The modal gain of the two element laser array mode is

$$G = A(N_1 - N_{om}) \frac{S_1}{S_1 + S_2} + A(N_2 - N_{om}) \frac{S_2}{S_1 + S_2}. \quad (A1)$$

In normalized units [see (3)] the modal gain can be expressed as

$$g = \frac{p_1}{1 + s + \Gamma} + \frac{p_2}{1 + s + \frac{1}{\Gamma}} \quad (A2)$$

where  $s \equiv s_1 + s_2$ ,  $\Gamma \equiv s_2/s_1$ , and use has been made of (1a) and (1b). From (A2) it is clearly seen that for  $s' > s$ ,  $g(\Gamma; s) < g(\Gamma; s')$ , as depicted schematically in Fig. 3. For a given  $s$  it

can be found by simple differentiation that  $g(\Gamma)$  has a maxima at

$$\Gamma_{\text{opt}} = \frac{\sqrt{\frac{p_2}{p_1}}(1+s) - 1}{1+s - \sqrt{\frac{p_2}{p_1}}} \quad (\text{A3})$$

and that the value of the modal gain at this point is

$$g_{\text{opt}} = \frac{(1+s)(p_1 + p_2) - 2\sqrt{p_1 p_2}}{s(s+2)} \quad (\text{A4})$$

when the array is lasing, the modal gain equals to the modal loss whose normalized value is unity. As explained in the second paragraph following (4), the only stable solution is when there is a single solution to  $g = 1$ , which occurs for  $g_{\text{opt}}$  and  $\Gamma_{\text{opt}}$  given by (A4) and (A3), respectively. Equating  $g_{\text{opt}} = 1$  in (A4) yields a quadratic equation for  $s$  which has only one physical ( $s > 0$ ) solution. Inserting that solution in (A3) yields an expression for  $\Gamma_{\text{opt}}$  that depends only on  $p_1$  and  $p_2$ . By straightforward algebra it can be shown that

$$\Gamma_{\text{opt}} - \frac{1}{\Gamma_{\text{opt}}} = \frac{p_2 - p_1}{\sqrt{p_1 p_2} - 1} \quad (\text{A5})$$

which is identical to (4).

#### APPENDIX B DERIVATION OF (7)-(9)

From the results of the two-element array we deduce that arrays distribute the photon densities among their elements such that the total photon density is maximized. We will consider next an  $N$ -element array. Defining

$$\rho_i = \frac{s_i}{s} \quad i = 1, \dots, N \quad (\text{B1})$$

we want to maximize

$$s = \sum_{i=1}^N s_i \quad (\text{B2})$$

subject to the constraint that the normalized modal gain is unity (see Appendix A):

$$\sum_{i=1}^N \frac{p_i}{1+s_i} \rho_i = \frac{1}{s} \sum_{i=1}^N \frac{p_i}{1+s_i} s_i = 1. \quad (\text{B3})$$

The constraint of (B3) can be alternatively expressed as

$$\sum_{i=1}^N s_i - \sum_{i=1}^N \frac{p_i s_i}{1+s_i} = 0. \quad (\text{B4})$$

Using Lagrange's multipliers method, we want to maximize the following function:

$$F = \sum_{i=1}^N s_i - \lambda \left[ \sum_{i=1}^N s_i - \sum_{i=1}^N \frac{p_i s_i}{1+s_i} \right] \quad (\text{B5})$$

where  $\lambda$  is the Lagrange multiplier. Differentiating with respect to  $s_i$  yields:

$$\frac{\partial F}{\partial s_i} = 1 - \lambda + \lambda p_i \frac{1}{(1+s_i)^2}. \quad (\text{B6})$$

Solving  $\partial F / \partial s_i = 0$  yields

$$s_i = C\sqrt{p_i} - 1 \quad (\text{B7})$$

with

$$C \equiv \sqrt{\frac{\lambda}{\lambda - 1}}. \quad (\text{B8})$$

(It is clearly seen that  $\partial^2 F / \partial s_i^2 < 0$ .) To solve for  $C$ , we insert (B7) in (B4), resulting in a quadratic equation for  $C$ , whose solution is (8). Finally, (9) is obtained by summing (B7) over  $i = 1 \dots N$ , and (7) is obtained by dividing (B7) by (9).

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Eli Kapon, photograph and biography not available at the time of publication.

S. Margalit, photograph and biography not available at the time of publication.

Amnon Yariv (S'56-M'59-F'70), for a photograph and biography, see p. 30 of the January 1984 issue of this JOURNAL.

# Lateral Mode Discrimination and Control in High-Power Single-Mode Diode Lasers of the Large-Optical-Cavity (LOC) Type

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**Abstract**—A comprehensive study of lateral mode discrimination and control in weak-laterally-confining large-optical-cavity (LOC)-type structures is presented. The analysis centers on two types of CDH-LOC laser structures: type A, which supports only the fundamental lateral mode in both the passive and active regimes; and type B, which supports several lateral modes in the passive regime and only the fundamental mode in the active regime. The transverse confinement factor  $\Gamma$  is peaked in the center of the structure and varies significantly across the lasing region for both device types. In the passive regime it is found that the effective-index (lateral) profile is a W-shaped waveguide for type A devices and a positive-index waveguide for type B devices. A discussion and analysis of losses in W-guides is also presented. Under carrier injection (i.e., active regime) the evolution of W-guides in CDH-LOC structures is presented as a function of increasing current density up to lasing threshold. For both type A and type B devices the effective-index profiles and corresponding lateral far-field patterns are analyzed as a function of threshold mode gain. Carrier-induced bulk-index depressions are found to be in the  $-0.02$  to  $-0.04$  range depending on the value of the threshold mode gain. The corresponding antiguiding parameter,  $R = k_o \delta n / \delta g$ , takes values in the  $-3$  to  $-4$  range, which imply values between 6 and 8 for the linewidth enhancement factor  $\alpha$ . It is found that by controlling the threshold mode gain (i.e., changing the device length and/or its facet(s) reflectivity) devices of the same cross-sectional geometry can be made to lase either multimode (spatially) or in the fundamental mode.

MODE-stabilized diode lasers of the large-optical-cavity (LOC) type have received much attention in the literature [1]–[13]. Generally, the LOC concept [14], [15] con-

sists of introducing between the active and cladding layer(s) one or two layers of material with refractive index intermediate between those of the active and cladding layers. The additional layer(s) also called “guide” layer(s), allow a larger transverse spot size than for “standard” 3-layer double-heterojunction (DH) structures and implicitly increased capability for high-power laser operation. Indeed, LOC-type mode-stabilized devices have provided the highest CW output power from a single semiconductor diode laser [16], and the highest power in the fundamental spatial mode under 50 percent duty cycle drive conditions [17].

Aside from increasing the transverse spot size, the presence of guide layers in LOC structures has two important consequences: 1) increasing the laser's threshold-current temperature sensitivity as a result of carrier leakage between the active and guide layer(s) [15] and 2) making the optical-mode spot size sensitive to the index depressions induced by injected carriers in the active layer [11], [12], since the refractive index step between active and guide layer(s) is relatively small (0.10–0.15). Most previous analyses [5], [6], [8] of transverse- and lateral-mode control in LOC-type structures *have not* considered the effect of carrier-induced index depressions, since they were thought to be negligible. However, recently it has been pointed out by several workers [11], [18]–[21] that carrier-induced index depressions reach relatively large values (0.02–0.05). Thus, they become of concern when the refractive-index differential between adjacent layers is relatively small (0.10–0.15) as is the case for practical LOC structures. We have previously reported briefly in a letter [11] on the ef-

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