## MODEL FOR THE HADRONS\*

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A particle spectrum is obtained from duality.

A particle spectrum in the low-mass region is obtained from the spin structure of the dominant Regge exchange. Since we work in the low-energy region, diffraction is negligible and resonances dominate the scattering.¹ Our calculational procedure exploits the fact that the dominant Regge exchange forces certain helicity amplitudes to vanish in the forward or backward directions.

The spectrum is an extension of the familiar quark model. The high-lying meson trajectories are those in the quark model (36, all L) plus nonets of subsidiary exchange degenerate trajectories containing particles with the quantum numbers  $J^{PC} = (0^{++}, 1^{--}, 2^{++}, \cdots)$  and  $(0^{-+}, 1^{+-}, 2^{-+}, \cdots)$ . Unless the  $(0^{-+}, 1^{+-}, \cdots)$  trajectory produces particles with the quantum numbers  $1^{+-}$  and  $2^{-+}$  in the low-mass region, an additional exchange-degenerate trajectory  $(1^{-+}, 2^{+-}, \cdots)$  must be included.

In addition to the trajectories in the baryon quark model  $[(\underline{56}, \text{ even } L), (\underline{70}, \text{ odd } L)]$ , our model requires the presence of at least a singlet and octet of trajectories. There are candidates for all the predicted low-mass particles lying on the new trajectories.

The basic assumption is that there is a region of s and t within which the spin structure of the scattering amplitude is given by the cross-channel exchange of a leading, factorizable, exchange-degenerate trajectory  $(\rho - A_2)$ , while in the direct channel the scattering is given by resonant contributions. Diffraction scattering is expected to be unimportant in this region. Our basic assumption provides us with two alternate calculations of the imaginary part of the scattering amplitude which we relate through crossing. This is expressed through

$$\sum_{\substack{b,i\\\lambda',\mu',\sigma',\tau'}} \chi_{\lambda\lambda'}\chi_{\mu\mu'}\chi_{\sigma\sigma'}\chi_{\tau\tau'}X^{ab}\beta_i{}^b\xi_{\lambda'\sigma'}{}^i\xi_{\mu'\tau'}{}^i\delta_{\lambda'-\sigma'}\delta_{-(\mu'-\tau')}{}^i = \sum_i g_i{}^ag_i{}'^a\xi_{\lambda\mu}{}^i\xi_{\sigma\tau}{}^id_{\lambda-\mu,\sigma-\tau}{}^{ji},$$

where  $\chi$  is the spin crossing matrix; X is the SU(3) crossing matrix;  $\beta$  describes the strength of cross-channel exchanges;  $\xi$ , the polarization of their couplings;  $\delta$ , a factor associated with their angular momentum, is

$$\delta_{\lambda}^{i} = e^{-i\pi\lambda/2} \left(\frac{2\alpha_{i}}{\alpha_{i}-\lambda}\right)^{1/2};$$

and g is a direct-channel resonance coupling.

When a labels an exotic SU(3) representation, the right-hand side vanishes, leaving us with null equations<sup>5</sup> which lead to patterns of exchange-degenerate trajectories.<sup>5,6</sup> Here we exploit the fact that the dominance of the leading Regge exchange together with factorization results in the vanishing of the left-hand side in specific helicity states in the forward or backward directions. The resonances must cancel in these states, leading to

$$\sum_{i} g_{i}^{a} g_{i}^{\prime a} \xi_{\lambda \mu}^{i} \xi_{\sigma \tau}^{i} d_{\lambda - \mu, \sigma - \tau}^{j} (0 \text{ or } \pi) = 0.$$

We implement these equations in all scattering reactions involving the  $\pi$ ,  $\rho$ , N, and  $\Delta$  SU(3) multiplets. Although we cannot precisely determine the masses of the resonances we predict, we would expect that they populate a 1- to 2-BeV² region above threshold. One may also consider reactions involving the predicted resonances; they will require additional trajectories. The results of that calculation will be published elsewhere. This process of including ever more resonances as external particles is expected to fail for reactions with sufficiently high thresholds.

The equations do not lead to a unique spectrum. All solutions, however, contain states which do not appear in the quark model. The most economical solution for mesons has, in addition to  $(\underline{36}, L=1)$  and  $(\underline{36}, L=2)$ ,  $0^{++}$  and  $1^{+-}$  (or  $2^{-+}$ ) nonets in the low-mass region. Another natural solution would add, instead,  $0^{++}$ ,  $0^{-+}$ , and  $1^{-+}$  nonets to the quark-model states. These two solutions are also distinguished by their helicity couplings.

There may be a candidate for the  $1^{-+}$  in the lower  $A_2$  region. In addition, this same region may

very well contain a candidate for the  $0^{++}$ . A natural candidate falling in the  $0^{-+}$  nonet is, of course, the E.

For baryons, our model requires at least four decuplets, three singlets, and eight octets, four of whose F/D ratios for coupling to pseudoscalar-meson-baryon (PB) are 1 and four of whose F/D ratios are  $-\frac{1}{3}$ . Although most of these can be accommodated in the quark model [(70, L=1), (56, L=2)], we predict at least an additional singlet and octet with an F/D for coupling to PB of  $-\frac{1}{3}$ . An obvious candidate for the octet is the  $N_{1/2}$ +(1470), i.e., the "Roper". Since its F/D is  $-\frac{1}{3}$ , the isosinglet member of this octet decouples from  $\overline{K}N$ , and therefore the  $\Lambda_{1/2}$ +(1750) should not belong to the Roper octet<sup>11</sup> and is a natural candidate for the singlet.

Further details of the model will be published elsewhere.

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## ERRATUM

TUNNELING-ASSISTED NUCLEAR SPIN-LAT-TICE RELAXATION. P. S. Allen and S. Clough [Phys. Rev. Letters 22, 1351 (1969)].

The definition of y in the line following Eq. (7) should read

$$y = (3J\tau_t)^{-1}$$
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<sup>&</sup>lt;sup>1</sup>K. Igi, Phys. Rev. Letters <u>9</u>, 76 (1962); A. Logunov, L. D. Soloviev, and A. A. Tavkhelidze, Phys. Letters <u>24B</u>, 181 (1967); K. Igi and S. Matsuda, Phys. Rev. Letters <u>18</u>, 625 (1967); R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).

<sup>&</sup>lt;sup>2</sup>M. Gell-Mann, Phys. Letters <u>8</u>, 214 (1964); G. Zweig, CERN Reports Nos. Th 401 and Th 412, 1964 (unpublished), and in <u>Proceedings of the International School of Physics "Ettore Majorana", Erice, Italy, 1963</u>, edited by A. Zichichi (W. A. Benjamin, Inc., New York, 1964).

<sup>&</sup>lt;sup>3</sup>The model does not determine the point at which particles first appear on these trajectories.

<sup>&</sup>lt;sup>4</sup>If the leading trajectory is parity doubled, all details of the model are altered. We do not consider this case here.

<sup>&</sup>lt;sup>5</sup>J. Mandula, C. Rebbi, R. Slansky, J. Weyers, and G. Zweig, Phys. Rev. Letters 22, 1147 (1969).

<sup>&</sup>lt;sup>6</sup>J. L. Rosner, Phys. Rev. Letters <u>21</u>, 950 (1968); H. J. Lipkin, to be published; M. Kugler, Phys. Rev. (to be published); R. H. Capps, Phys. Rev. Letters <u>22</u>, 215 (1969).

<sup>&</sup>lt;sup>7</sup>A solution consists of a spectrum and coupling patterns. Only the spectrum is discussed here.

<sup>&</sup>lt;sup>8</sup>J. Mandula, J. Weyers, and G. Zweig, Phys. Rev. Letters <u>23</u>, 266 (1969).

 $<sup>^9</sup>$ D. J. Crennell, U. Karshon, K. W. Lai, J. S. O'Neall, and J. M. Scarr, Phys. Rev. Letters <u>22</u>, 1327 (1969).  $^{10}$ M. Aguilar-Benitez, J. Barlow, L. D. Jacobs, P. Malecki, L. Montanet, M. Tomas, M. Della Negra, J. Cohen-Ganouna, B. Lorstad, and N. West, Phys. Letters <u>29B</u>, 62 (1969), have found two  $K\bar{K}$  peaks in the  $A_2$  region in  $\bar{p}p$  annihilation although only one peak is observed in  $\pi p$  reactions. Since C forbids the  $K\bar{K}$  decay of a  $1^{-+}$ , these enhancements may only be  $0^{++}$  or  $2^{++}$  mesons. An I=1  $0^{++}$ , however, could not be produced peripherally in  $\pi p$  reactions.

<sup>&</sup>lt;sup>11</sup>H. Harari, in <u>Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968</u>, edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968).