Comment on "Heat-Flow Induced Anomalies in Superfluid ^{4}He near T_λ"

In a recent Letter [1], Haussmann and Dohm (HD) presented a renormalization group treatment of the ^{4}He lambda transition in a heat current Q. In this Comment, we use simple arguments that yield the same critical point exponent for the depressed T_λ, and nearly the same critical velocity, but indicate that HD may not have calculated the proper specific heat anomaly.

Near T_λ, the heat current is given by $Q = -\rho_s v_s S T$ in standard notations of the two-fluid model. Of the terms in Q, only ρ_s and v_s may be singular, so for the purpose of computing exponents, we write $Q_c \sim (\rho_{sc} v_{sc}^2/2)/v_{sc}$. The numerator is a singular term in the free energy density, and every such term goes to zero inversely as the correlation volume, i.e., $\rho_{sc} v_{sc}^2 \sim \xi^{-d}$. The denominator is given by [2]

$$v_{sc} = -i(\hbar/m)|\nabla\psi|/\psi \sim |\nabla\psi|/\psi\,, \tag{1}$$

where m is the atomic mass of ^{4}He. Thus v_{sc} has the character of an inverse length. Since the correlation length is the only relevant length at a critical point, $v_{sc} \sim \xi^{-1} \sim t^{\nu}$, where $t = (T_\lambda - T)/T_\lambda$. Thus $Q_c \sim \xi^{-d} t^{-\nu}$, or

$$T_\lambda(0) - T_\lambda(Q) \sim Q^{1/\nu(d-1)}, \tag{2}$$

which is the same result arrived at by HD.

Equation (1) envisions a wave-function-like order parameter which, in uniform flow has the form $\psi = \psi_0 e^{i\vec{k}\cdot\vec{r}}$, where $\vec{r}$ is a space vector and $\vec{k}$ is related to v_s by $\vec{v}_s = \hbar\vec{k}/m$. The order parameter is governed by a differential equation [3]

$$\xi^2\nabla^2\psi = (|\psi|^2 - 1)\psi\,, \tag{3}$$

which has a solution $|\psi|^2 = 1 - (k\xi)^2$. Thus $|\psi|^2$ is driven to zero at superfluid velocity.

$$v_{sc} = \hbar/m\xi = 112t^{\nu} \qquad [\text{m/sec}]. \tag{4}$$

This justifies the argument in Eq. (1) that $v_{sc} \sim \xi^{-1}$. Fluctuations are taken into account by using the experimental value of ν rather than that predicted by mean field theory.

Equation (4) may be compared to the results of HD

$$v_{sc} = [1/\sqrt{6} - 0.0112]2^{\nu}\hbar/m\xi = 70.3t^{\nu} \qquad [\text{m/sec}]. \tag{5}$$

The difference is due almost entirely to the fact that HD's critical velocity is the consequence of a stability criterion, $\partial Q/\partial v_s \geq 0$, rather than simply the velocity that drives $|\psi|^2$ to zero. The same criterion gives a factor $2^{\nu}/\sqrt{6}$ in Eq. (4).

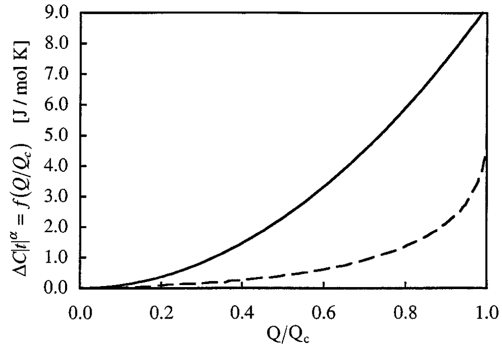

FIG. 1. The scaling function discussed in the text.

We now turn to the heat capacity anomaly. Under superfluid flow the free energy per unit volume is increased by [4] $\Delta F(T, v_s) = \rho_s v_s^2/2$. At constant Q, the proper free energy to use is $\Phi(T, \vec{q}) = F - \vec{v}_s\vec{q}$, where $\vec{q} = \rho_s\vec{v}_s$. The molar heat capacity change is

$$\Delta C = -(TV\partial^2\Delta\Phi/\partial T^2)_{\vec{q}} = -[TV\partial^2(-q^2/2\rho_s)/\partial T^2]_{\vec{q}}$$
$$= \zeta(\zeta + 1)Q^2 V t^{-(\zeta+2)}/(2\rho_0 S^2 T_\lambda^3) = f(Q/Q_c)t^{-\alpha}$$
$$f(Q/Q_c) = 9.2(Q/Q_c)^2 \qquad [\text{J/mole K}], \tag{6}$$

where $\rho_s = \rho_0 t^{\zeta}$, $\rho_0 = 0.37$ g/cm^3, $S = 1.58$ J/g K, $\zeta = (2 - \alpha)/3 = \nu$, α is the heat capacity exponent, $V = 27.38$ cm^3/mole is the molar volume, and $Q_c = 7580t^{2\nu}$ [W cm^{-2}] [1]. The dashed line in Fig. 1 is the scaling function $f(Q/Q_c)$ of HD. The solid line is our result which is based on the two-fluid model neglecting any dependence of ρ_s on $\vec{v}_s$. It is not clear to us why the HD calculation differs so little from these standard arguments in its other principal results, and so much in the predicted heat capacity.

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