Determining the $^{12}C(\alpha,\gamma)^{16}O$ cross section from Coulomb dissociation

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We estimate the $E1$ and $E2$ contributions to the Coulomb dissociation reaction $^{16}O+Pb\rightarrow\alpha+^{12}C+Pb$ using semiclassical Coulomb excitation theory. For projectile energies below 300 MeV/nucleon and scattering angles greater than 1°, we find that the process is dominated by the $E2$ component. This is in contrast to the astrophysically interesting $^{12}C(\alpha,\gamma)^{16}O$ cross section, which is dominated by the $E1$ multipole at the most effective energy of 300 keV. The $E2$ sensitivity of Coulomb dissociation would usefully complement forthcoming ^{16}N β-decay data, which will constrain only the $E1$ component.

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The $^{12}C(\alpha,\gamma)^{16}O$ cross section at $E_{c.m.}=300$ keV is of great importance in the later stages of stellar evolution [1]. To date, it has been estimated [2–8] only by extrapolation from measurements at higher energies ($\gtrsim 1.0$ MeV), since Coulomb repulsion makes the cross section at 300 keV far too small ($\approx 3\times 10^{-17}$ b) to be measured directly. The dominant effect of Coulomb repulsion can be factored out of the cross section by considering the S factor, $S(E)=E\sigma(E)e^{2\pi\eta(E)}$ where $\eta(E)=Z_1Z_2e^2/\hbar v$ is the Sommerfeld parameter for the initial nuclei.

Both $E1$ and $E2$ multipolarities contribute to the $^{12}C(\alpha,\gamma)^{16}O$ cross section at the energies of interest. The $E1$ cross section at 300 keV is poorly determined by higher energy measurements because it is expected to be influenced strongly by the barely bound 7.12 MeV level of ^{16}O, while exeriments at the higher energies are dominated by a broad 9.6 MeV level. Current best estimates for $S_{E1}(300\text{ keV})$ range from 0.010 to 0.050 MeV b, although values from 0.00 to 0.20 MeV b can fit the current data [3]. Measurements of the ^{16}N β-decay spectrum could substantially reduce the uncertainty in the $E1$ component [5].

The $E2$ contribution, which is expected to result from direct capture only, is likely to be a significant (but minority) fraction of the total cross section [2–3]. Best values of $S_{E2}(300\text{ keV})$ range from 0.007 to 0.014 MeV b, but a range of 0.000 to 0.028 MeV b is permitted by the data.

It has been proposed [9] that the $^{12}C(\alpha,\gamma)^{16}O$ cross section at $E_{c.m.}=300$ keV could be determined by measuring the dissociation of an ^{16}O beam in the Coulomb field of a high-Z target. The Coulomb dissociation cross section is related to the $^{16}O(\gamma,\alpha)^{12}C$ photoabsorption cross section, which in turn is directly proportional to the cross section for the time-reversed radiative capture process of interest. The importance of adequately controlling possible excitation by the target's nuclear field is clear [10]. However, less obvious is the fact that the $E1$ and $E2$ multipoles are weighted differently in the Coulomb dissociation than they are in the radiative capture. In this Brief Report we use extrapolations of the measured $E1$ and $E2$ components of the radiative capture cross section to estimate the corresponding Coulomb dissociation cross sections under a wide range of experimental conditions. We find that the enhanced importance of the $E2$ multipole in Coulomb dissociation implies that it will be very difficult to determine the $E1$ radiative capture cross section from such experiments.

In general, the Coulomb dissociation cross section depends on the angle between the breakup particles [11]. Here, we consider only the integral of the cross section over this angle, so that the contributions from the different multipoles add incoherently. As only $E1$ and $E2$ multipoles are important, we can write

$$\sigma^{\text{capt}}=\sum_{\lambda=1}^{2}\sigma_{E\lambda}^{\text{capt}}\ ,\quad \sigma^{\text{photo}}=\sum_{\lambda=1}^{2}\sigma_{E\lambda}^{\text{photo}}\ ,\quad \sigma^{\text{CD}}=\sum_{\lambda=1}^{2}\sigma_{E\lambda}^{\text{CD}} \tag{1}$$

for the capture, photoabsorption, and Coulomb dissociation cross sections, respectively. The radiative capture and photoabsorption cross sections are related through detailed balance: $\sigma_{E\lambda}^{\text{photo}}=(k^2/2k_\gamma^2)\sigma_{E\lambda}^{\text{capt}}$, where the quotient of the wave numbers in the two channels is given by $k^2/2k_\gamma^2=\mu c^2E_{c.m.}/E_\gamma^2$. Here E_γ is the energy of the photon, while μ and $E_{c.m.}$ are the reduced mass and center-of-mass energy of the $^{12}C+\alpha$ channel. In using the principle of detailed balance, we are assuming that the ^{12}C is observed to emerge from Coulomb dissociation only in its ground state. Energy conservation requires $E_\gamma=E_{c.m.}+Q$, where the Q value for $^{12}C(\alpha,\gamma)^{16}O$ is 7.162 MeV.

The double differential cross section for Coulomb dissociation through each multipole component is related to the corresponding photodissociation cross section by

$$\frac{d^2\sigma_{E\lambda}^{\text{CD}}}{d\Omega\,dE_\gamma}=\frac{1}{E_\gamma}\frac{dn_{E\lambda}}{d\Omega}\sigma_{E\lambda}^{\text{photo}}\ , \tag{2}$$

where Ω is the direction of projectile scattering. The virtual photon number $dn_{E\lambda}/d\Omega$ depends upon the multipole order λ, the excitation energy E_γ, and the trajectory of the ^{16}O projectile.

For a nonrelativistic projectile moving beyond the range of nuclear forces, the trajectory is that appropriate to Rutherford scattering. For a projectile of mass m_p

and charge $Z_p e$ passing a massive target nucleus of charge $Z_t e$, the trajectory can be characterized by the projectile's energy per nucleon (W) and its Rutherford scattering angle (θ). These are related to the impact parameter b by $b = Z_p Z_t e^2 / 2 W m_p \tan(\theta/2)$ if b is large enough so that the nuclear force can be ignored. For high projectile velocities and impact parameters large compared to the distance of closest approach in a head-on collision $2a = Z_t Z_p e^2 / W(A_1 + A_2)$, the path will be nearly a straight line.

Calculations of the virtual photon numbers $dn_{E1}/d\Omega$ and $dn_{E2}/d\Omega$ for the nonrelativistic Rutherford [12] and straight-line [13] orbits are straightforward. Relativistic effects are expected to be of order v^2/c^2 [14]. For the energies considered here ($W \leq 400$ MeV/nucleon) these relativistic corrections are smaller than the present uncertainty in σ^{capt} and will be ignored in computing σ^{CD}.

Because the $E1$ and $E2$ contributions to the capture cross section are both significant and carry different weights $dn_{E\lambda}/d\Omega$ in the Coulomb dissociation cross section, it is necessary to separate the dissociation cross section into its multipole components to find the capture cross section. It has been previously noted [13] that because the virtual photon numbers $dn_{E1}/d\Omega$ and $dn_{E2}/d\Omega$ have different dependences on experimental conditions such as scattering angle and incident projectile energy, the two components σ_{E1}^{CD} and σ_{E2}^{CD} can be extracted from measurements of σ^{CD}, the total Coulomb dissociation cross section. Such an extraction will be very difficult, however, if one of the components is always much larger than the other.

To investigate this possibility, we plot (Fig. 1) the ratio

$$R = \frac{d^2\sigma_{E2}^{\text{CD}}}{d^2\sigma_{E1}^{\text{CD}}} = \frac{\sigma_{E2}^{\text{capt}}}{\sigma_{E1}^{\text{capt}}} \frac{dn_{E2}}{dn_{E1}} \tag{3}$$

for a Pb target as a function of W for different values of b and θ at $E_{\text{c.m.}} = 300$ MeV. The solid line is the result of numerical calculations of $dn_{E2}/d\Omega$ and $dn_{E1}/d\Omega$ using the Rutherford orbit; the dashed line shows the straight-line approximation using the analytic expressions [13]

$$\frac{dn_{E2}}{d\Omega} = \frac{Z^2\alpha}{\pi^2\xi^2}\left[\frac{c}{v}\right]^4 x^2\{K_1^2(x) + x^2[K_1^2(x) + k_0^2(x)] + xK_0(x)K_1(x)\},$$

$$\frac{dn_{E1}}{d\Omega} = \frac{Z^2\alpha}{4\pi^2}\epsilon^2\left[\frac{c}{v}\right]^2 x^2[K_0^2(x) + K_1^2(x)],$$

where $\epsilon = 1/\sin(\theta/2)$ is the orbital eccentricity, $\xi = E_\gamma a/\hbar v$ is the adiabaticity parameter, and $x = \epsilon\xi$. Although we have chosen $E_{\text{c.m.}} = 0.3$ MeV to construct this plot, the ratio of virtual photon numbers remains nearly constant for $0.3 \lesssim E_{\text{c.m.}} \lesssim 2.0$ MeV because the relevant quantity $E_\gamma = E_{\text{c.m.}} + 7.162$ MeV varies little. Extrapolations of laboratory capture data (Fig. 2) show that the ratio $\sigma_{E2}^{\text{capt}}/\sigma_{E1}^{\text{capt}}$ is also expected to be roughly constant over this same range. We have used $\sigma_{E2}^{\text{capt}}/\sigma_{E1}^{\text{capt}} = 0.5$ for illustrative purposes, but the best extrapolations of current analyses [2,3,5] predict $0.15 \lesssim \sigma_{E2}^{\text{capt}}/\sigma_{E1}^{\text{capt}} \lesssim 0.8$ at $E_{\text{c.m.}} = 300$ keV.

Figure 1 shows that for $10 \leq W \leq 200$ MeV/nucleon and $\theta > 1°$, the ratio R is expected to be greater than 11, so that any measurement of the Coulomb dissociation cross section would be dominated by the $E2$ contribution. However, Fig. 2 shows that the capture cross section of interest is largely (65–90%) due to the $E1$ contribution. The $E1$ cross section could be determined from a sufficiently accurate measurement of the angular dependence of the Coulomb dissociation. For example, if a determination of $\sigma_{E1}^{\text{capt}}$ was desired to a precision of 50% and $R = 11$, then the error in σ^{CD} would have to be less than 4%. Unfortunately, in the region where the relative $E1$ contribution is largest, the experimental difficulties (measurements at very small scattering angles and a very small opening angle for the $^{12}C+\alpha$ pair because of the high projectile energy) would result in the largest experimental uncertainties.

An examination of the angular distribution predicted by an extension of the semi-classical model used here [11]

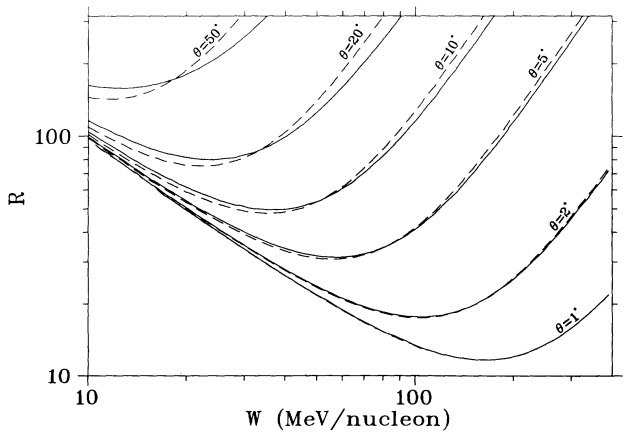

FIG. 1. Calculations of R (the ratio of $E2$ to $E1$ Coulomb dissociation cross sections at $E_{\text{c.m.}} = 300$ keV) using the Rutherford orbit appropriate to a Pb target (solid line) and the straight-line approximation (dashed line).

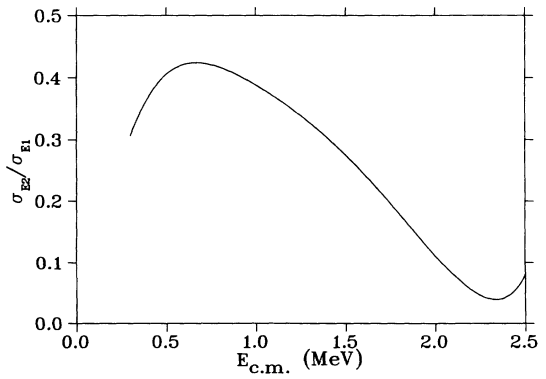

FIG. 2. The ratio $\sigma_{E2}^{\text{capt}}/\sigma_{E1}^{\text{capt}}$ calculated from fits by Filippone *et al.* [3] to the $E1$ capture and scattering data of Kremer *et al.* [7] and the $E2$ capture data of Redder *et al.* [8].

shows non-negligible interference effects between the $E1$ and $E2$ multipoles. In principle, an accurate model of this interference, combined with angular correlation Coulomb dissociation data, would be quite sensitive to the $E1$ component. However, even in the absence of such correlations, the dominance of the $E2$ multipole in Coulomb dissociation usefully complements other experiments that are sensitive mainly to the $E1$ part. For example, forthcoming ^{16}N β-decay data should strongly constrain the $E1$ component of the $^{12}C(\alpha,\gamma)^{16}O$ cross section at 300 keV, but they will not help in determining the $E2$ component [5]. This suggests that the total capture cross section might be determined by combining good measurements of both processes with existing data on $^{12}C(\alpha,\gamma)^{16}O$.

It is important to note that the problem of mixed multipolarities is peculiar to $^{12}C(\alpha,\gamma)^{16}O$ among cross sections of astrophysical interest. Reactions such as $^{13}N(p,\gamma)^{14}O$, $^{3}He(\alpha,\gamma)^{7}Be$, and $^{3}H(\alpha,\gamma)^{7}Li$ are dominated by $E1$ processes in both the capture and Coulomb dissociation processes and hence could be determined more easily from measurements of the latter.

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