PHYSICAL REVIEW D **73,** 053008 (2006)

Systematic limits on $\sin^2 2\theta_{13}$ in neutrino oscillation experiments with multireactors

H. Sugiyama*
Theory Group, KEK, Tsukuba, Ibaraki 305-0801, Japan

O. Yasuda†
Department of Physics, Tokyo Metropolitan University, Hachioji, Tokyo 192-0397, Japan

F. Suekane‡
Research Center for Neutrino Science, Tohoku University, Sendai, Miyagi 980-8578, Japan

G. A. Horton-Smith§
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125, USA
(Received 17 September 2004; revised manuscript received 20 December 2005; published 17 March 2006)

Sensitivities to $\sin^2 2\theta_{13}$ without statistical errors (''systematic limit'') are investigated in neutrino oscillation experiments with multiple reactors. Using an analytical approach, we show that the systematic limit on $\sin^2 2\theta_{13}$ is dominated by the uncorrelated systematic error σ_u of the detector. Even in an experiment with multidetectors and multireactors, it turns out that most of the systematic errors including the one due to the nature of multiple sources is canceled as in the case with a single reactor plus two detectors, if the near detectors are placed suitably. The case of the KASKA plan (7 reactors and 3 detectors) is investigated in detail, and it is explicitly shown that it does not suffer from the extra uncertainty due to multiple reactors.

DOI: 10.1103/PhysRevD.73.053008 PACS numbers: 14.60.Pq, 25.30.Pt, 28.41.−i

I. INTRODUCTION

Recently the possibility to measure θ_{13} by a reactor experiment has attracted much attention [1–9]. To achieve sensitivity $\sin^2 2\theta_{13} \sim 0.01$, reduction of the systematic errors is crucial, and near and far detectors seem to be necessary for that purpose. On the other hand, it appears to be advantageous to do an experiment at a multireactor site to gain statistics and high signal to noise ratio, and in fact in the most of cases considered in [1–9] there are more than one reactor. In this paper we discuss the systematic errors in reactor neutrino oscillation experiments with multireactors and multidetectors in an analytical way. In Sec. II we discuss the cases with a single reactor to illustrate our analytical method. In Sec. III we consider the cases with n_r reactors and show that the larger n_r gives totally the smaller contribution to the sensitivity from the uncorrelated errors of the fluxes. Irrespective of the number of reactors, if there are more than one detectors, we can cancel the correlated errors which includes the error of the fluxes. We emphasize in this paper that the sensitivity to $\sin^2 2\theta_{13}$ with vanishing statistical errors (we refer to the sensitivity as the systematic limit) is dominated by the uncorrelated error of the detectors in most cases. It is also emphasized that a lot of caution has to be exercised to estimate the uncorrelated error. In the appendix we give some details on how to derive the analytic results used in the main text, using the equivalence of the pull method and the covariance matrix approach [10–13]. Throughout this paper we do not use the binning of the numbers of events because the discussions on the uncorrelated bin-to-bin systematic errors are complicated. Also we will discuss only the systematic errors, i.e., we will consider the case where the statistical errors are negligibly small.

II. SYSTEMATIC ERRORS

To discuss the systematic limit on $\sin^2 2\theta_{13}$ in neutrino oscillation experiments with multiple detectors and reactors, we have to introduce the systematic errors of the detectors and the reactors (fluxes). There are two kinds of systematic errors among the numbers of events at the detectors, namely, the uncorrelated error σ_u and the correlated error. The former is identified with the uncorrelated error among detectors ($\delta_u^{(d)}$), and the latter is made from the correlated error among the detectors ($\delta_c^{(d)}$) and the error of the reactors which is composed of the uncorrelated error $\rho_u^{(r)}$ and the correlated one $\rho_c^{(r)}$. Examples of origins of those errors are listed below:

$\sigma_u (= \delta_u^{(d)})$: the uncertainty in the baseline lengths, a portion of measuring the detector volume, a part of the detection efficiency, etc.

$\delta_c^{(d)}$: the theoretical uncertainty in the cross section of the detection, etc.

$\rho_u^{(r)}$: the uncertainties in the composition of the fuel, etc.

*Electronic address: hiroaki@post.kek.jp
†Electronic address: yasuda@phys.metro-u.ac.jp
‡Electronic address: suekane@awa.tohoku.ac.jp
§Electronic address: gahs@phys.ksu.edu
Present address: Kansas State University, Manhattan, KS 66506, USA

$\rho_c^{(r)}$: the theoretical uncertainties in the spectrum of the $\bar{\nu}_e$ flux, etc.

In this paper we adopt the reference values for $\sigma_u(=\delta_u^{(d)})$ and $\delta_c^{(d)}$ used in [2], where basically the same reference values as in the Bugey experiment [14] were assumed. $\delta_c^{(d)}$ and σ_u can be estimated to be

$$\begin{aligned}\sigma_u &= \delta_u^{(d)} = 0.8\%/\sqrt{2} = 0.6\% \\ \delta_c^{(d)} &= \sqrt{(2.7\%)^2 - (2.1\%)^2 - (0.8\%/\sqrt{2})^2} = 1.6\%,\end{aligned} \tag{1}$$

where the factor $\sqrt{2}$ appears because the relative normalization error σ_{rel} between two detectors is related to σ_u by $\sigma_{rel} = \sqrt{2}\sigma_u$ in [14]. In the estimation of $\delta_c^{(d)}$, we used 2.7% total error and 2.1% error of the flux which are the values in the CHOOZ experiment. As for the correlated and uncorrelated errors of the the flux from the reactors, we adopt the same reference values as those used by the KamLAND experiment [15]

$$\rho_c^{(r)} = 2.5\%, \qquad \rho_u^{(r)} = 2.3\%. \tag{2}$$

Note that the word ''correlated'' means just the type of the error, and then correlated errors exist even if there is no partner.

III. ONE REACTOR

To explain our analytical approach, let us start with the simplest case, namely, the case with one reactor.

A. One detector

Let m be the measured number of events at the detector, and t be the theoretical predictions (hypothesis) to be tested. Our strategy in this paper is to assume no neutrino oscillation for the theoretical predictions t's and assume the number of events with oscillations for the measured values m's. Then, we examine whether a hypothesis with no oscillation is excluded or not, say at the 90% CL, from the value of χ^2. In the context of neutrino oscillation experiments, we have

$$\frac{m}{t} - 1 = -\sin^2 2\theta \left\langle \sin^2\left(\frac{\Delta m^2 L}{4E}\right)\right\rangle, \tag{3}$$

in the two flavor framework, where θ is the mixing angle, Δm^2 is the mass squared difference,[1] E is the neutrino energy, L is the distance of the reactor and the detector, and

$$D(L) \equiv \left\langle \sin^2\left(\frac{\Delta m^2 L}{4E}\right)\right\rangle \equiv \frac{\int dE \epsilon(E) f(E) \sigma(E) \sin^2(\frac{\Delta m^2 L}{4E})}{\int dE \epsilon(E) f(E) \sigma(E)}. \tag{4}$$

[1]Throughout this paper, we use the two flavor framework. To translate it into the three flavor notation, θ and Δm^2 should be interpreted as θ_{13} and $|\Delta m^2_{31}|$, respectively.

$\epsilon(E)$, $f(E)$, $\sigma(E)$ stand for the detection efficiency, the neutrino flux, and the cross section, respectively.

χ^2 is defined as

$$\chi^2 = \min_{\alpha' s}\left\{\left[\frac{m - t(1 + \alpha_c^{(d)} + \alpha_c^{(r)} + \alpha_u^{(r)})}{t\sigma_u}\right]^2 + \left(\frac{\alpha_c^{(d)}}{\delta_c^{(d)}}\right)^2 + \left(\frac{\alpha_c^{(r)}}{\rho_c^{(r)}}\right)^2 + \left(\frac{\alpha_u^{(r)}}{\rho_u^{(r)}}\right)^2\right\} \tag{5}$$

$$\begin{aligned}&= \frac{(\frac{m}{t} - 1)^2}{\sigma_u^2 + (\delta_c^{(d)})^2 + (\rho_c^{(r)})^2 + (\rho_u^{(r)})^2} \\ &= \frac{\sin^4 2\theta D(L)^2}{\sigma_u^2 + (\delta_c^{(d)})^2 + (\rho_c^{(r)})^2 + (\rho_u^{(r)})^2},\end{aligned} \tag{6}$$

where $\alpha_c^{(d)}$, $\alpha_c^{(r)}$ and $\alpha_u^{(r)}$ are the variables of noises to introduce the systematic errors $\delta_c^{(d)}$, $\rho_c^{(r)}$ and $\rho_u^{(r)}$, respectively. We give an easier way to derive (6) in the Appendix A, where integration over the α variables as those of Gaussian, instead of minimizing with respect to these variables, do the same job. Equation (6) shows that the square of the total systematic error is given by the sum of the squares of all the systematic errors.

B. Two detectors

Next let us discuss a less trivial example with a single reactor, one near and one far detectors. Let m_n and m_f be the measured numbers of events at the near and far detectors, t_n and t_f be the theoretical predictions, respectively. Then χ^2 is given by

$$\begin{aligned}\chi^2 = \min_{\alpha' s}\Bigg\{&\left[\frac{m_n - t_n(1 + \alpha_c^{(d)} + \alpha_c^{(r)} + \alpha_u^{(r)})}{t_n\sigma_u}\right]^2 \\ &+ \left[\frac{m_f - t_f(1 + \alpha_c^{(d)} + \alpha_c^{(r)} + \alpha_u^{(r)})}{t_f\sigma_u}\right]^2 \\ &+ \left(\frac{\alpha_c^{(d)}}{\delta_c^{(d)}}\right)^2 + \left(\frac{\alpha_c^{(r)}}{\rho_c^{(r)}}\right)^2 + \left(\frac{\alpha_u^{(r)}}{\rho_u^{(r)}}\right)^2\Bigg\},\end{aligned} \tag{7}$$

where we have assumed that the uncorrelated errors for the two detectors are the same and are equal to σ_u. Equation (7) can be evaluated also by integrating over the variables $\alpha_c^{(d)}$, etc. as Gaussian instead of minimizing with respect to these variables. After some calculations (See Appendix A for details), we obtain

$$\chi^2 = \left(\frac{m_n}{t_n} - 1, \frac{m_f}{t_f} - 1\right) V^{-1} \begin{pmatrix} \frac{m_n}{t_n} - 1 \\ \frac{m_f}{t_f} - 1 \end{pmatrix},$$

where

$$V \equiv \sigma_u^2 I_2 + [(\delta_c^{(d)})^2 + (\rho_u^{(r)})^2 + (\rho_c^{(r)})^2]H_2 = \begin{pmatrix} \sigma_u^2 + (\delta_c^{(d)})^2 + (\rho_u^{(r)})^2 + (\rho_c^{(r)})^2 & (\delta_c^{(d)})^2 + (\rho_u^{(r)})^2 + (\rho_c^{(r)})^2 \\ (\delta_c^{(d)})^2 + (\rho_u^{(r)})^2 + (\rho_c^{(r)})^2 & \sigma_u^2 + (\delta_c^{(d)})^2 + (\rho_u^{(r)})^2 + (\rho_c^{(r)})^2 \end{pmatrix} \quad (8)$$

is the covariance matrix; I_2 represents 2×2 identity matrix and H_2 does a 2×2 matrix whose elements are all unity. It is seen that only the covariant matrix in the χ^2 depends on the errors. Note that any liner transformation of V does not change the value of χ^2. Diagonalization of V is, however, worthwhile to investigate analytically the behavior of χ^2. After diagonalizing V we have

$$\chi^2 = \frac{[(m_n/t_n - 1) + (m_f/t_f - 1)]^2}{4(\delta_c^{(d)})^2 + 4(\rho_u^{(r)})^2 + 4(\rho_c^{(r)})^2 + 2\sigma_u^2} + \frac{[(m_n/t_n - 1) - (m_f/t_f - 1)]^2}{2\sigma_u^2} \quad (9)$$

$$= \sin^4 2\theta \Bigg[\frac{(D(L_f) + D(L_n))^2}{4(\delta_c^{(d)})^2 + 4(\rho_u^{(r)})^2 + 4(\rho_c^{(r)})^2 + 2\sigma_u^2} + \frac{(D(L_f) - D(L_n))^2}{2\sigma_u^2} \Bigg], \quad (10)$$

where L_n and L_f are the distances from the reactor to the near and far detector, respectively; The first term on the right-hand side in Eq. (9) stands for the contribution from the sum of the yields at the near and far detectors, while the second term corresponds to the difference between them. The first term determines the normalization of flux, namely, the sensitivity to $\sin^2 2\theta$ at very large $|\Delta m^2|$ where all $D(L)$ becomes $0.5\sin^2 2\theta$. On the other hand, the second term gives the main sensitivity at the concerned value of $|\Delta m^2|$ (e.g. 2.5×10^{-3} eV^2) as we see below.

Putting the reference values (1) and (2) together, we have

$$2\sigma_u^2 = (0.8\%)^2,$$
$$4(\delta_c^{(d)})^2 + 4(\rho_u^{(r)})^2 + 4(\rho_c^{(r)})^2 + 2\sigma_u^2 = (7.6\%)^2.$$

We can ignore the contribution from $(4(\delta_c^{(d)})^2 + 4(\rho_u^{(r)})^2 + 4(\rho_c^{(r)})^2 + 2\sigma_u^2)^{-1}$ in Eq. (10) because that is only 1% compared to that from $(2\sigma_u^2)^{-1}$.[2] Hence, χ^2 is given approximately by

$$\chi^2 \simeq \sin^4 2\theta \frac{(D(L_f) - D(L_n))^2}{2\sigma_u^2}. \quad (11)$$

We see that the main sensitivity is determined indeed by the relative normalization error $\sigma_{rel} = \sqrt{2}\sigma_u$. By comparing (6) and (11), it is clear that the sensitivity is improved significantly by virtue of near detector. The origin of the improvement is the fact that the minimum eigenvalue of (8) is σ_u^2. We refer to that the minimum eigenvalue of the covariance matrix V becomes σ_u^2 as ''the near-far cancellation''. The hypothesis of no oscillation is excluded at the 90%CL if χ^2 is larger than 2.7, which corresponds to the value at the 90% CL for 1 degree of freedom. This implies that the systematic limit on $\sin^2 2\theta$ at the 90% CL, namely, the sensitivity in the limit of infinite statistics, is given by

$$(\sin^2 2\theta)_{\text{limit}}^{\text{sys only}} \simeq \sqrt{2.7}\frac{\sqrt{2}\sigma_u}{D(L_f) - D(L_n)}. \quad (12)$$

Equation (12) also tells us that, in order to optimize $(\sin^2 2\theta)_{\text{limit}}^{\text{sys only}}$ for a given value of σ_u, we have to maximize $D(L_f) \equiv \langle \sin^2(\Delta m^2 L_f/4E)\rangle$ while minimizing $D(L_n) \equiv \langle \sin^2(\Delta m^2 L_n/4E)\rangle$. Note that $D(L_f)$ can not be unity because of neutrino energy spectrum; The possible maximum value of $D(L_f) - D(L_n)$ is 0.82, which is attained for $\Delta m^2 = 2.5 \times 10^{-3}$ eV^2, $L_f = 1.8$ km, and $L_n = 0$. Then, we can estimate analytically the highest possible sensitivity to $\sin^2 2\theta$ (the lower bound of $(\sin^2 2\theta)_{\text{limit}}^{\text{sys only}}$) at a single reactor experiment, assuming that the uncorrelated error σ_u is smaller enough than other errors:

$$(\sin^2 2\theta)_{\text{limit}}^{\text{sys only}} \gtrsim \frac{\sqrt{2.7}\sqrt{2}\sigma_u}{0.82} = 2.8\sigma_u \simeq 0.016, \quad (13)$$

where we use $\sigma_u = (0.8/\sqrt{2})\%$ for example. In practice, however, $D(L_n)$ will not be able to vanish. Assuming $\Delta m^2 = 2.5 \times 10^{-3}$ eV^2, $L_f = 1.7$ km, and $L_n = 0.3$ km, we have $D(L_f) \simeq 0.82$ and $D(L_n) \simeq 0.07$. Then, $\sigma_u = (0.8/\sqrt{2})\%$ gives the sensitivity

$$(\sin^2 2\theta)_{\text{limit}}^{\text{sys only}} \simeq \sqrt{2.7}\frac{\sqrt{2}\sigma_u}{D(L_f) - D(L_n)} \simeq 3.1\sigma_u \simeq 0.018.$$

This sensitivity corresponds to and agrees with the value obtained numerically in [2].

Note that the systematic limit (or a value of the sensitivity) itself is not a good measure of the power of a reactor experiment because it depends on the assumption of the values of errors, especially of σ_u. The factor $(\sin^2 2\theta)_{\text{limit}}^{\text{sys only}}/\sigma_u$ is, however, a good measure for the setups with one reactor and two detectors because it is almost independent of assumptions of error sizes. Actually, we see that it is a good measure for more complicate setups also because the systematic limit is dominated by σ_u again as we see in the following sections.

IV. n_r REACTORS

It is straightforward to generalize the argument in the previous section to a general case with multireactors and multidetectors. The covariance matrix V is given by $\sigma_u^2 \times$ (unit matrix) + (the rest), and in most cases, as long as the

[2]This is more or less the derivation of χ^2 used in [2].

near detectors are placed properly, the determinant of (the rest) is zero or very small compared to σ_u^2. The minimum eigenvalue of the covariance matrix, which gives main contribution to χ^2, is approximately given by σ_u^2. Therefore, the systematic limit is dominated by the uncorrelated error σ_u also in general cases.

A. One detector

As in Sec. III, as a warming up, let us consider the case with one detector and multiple reactors. When there are $n_r(>1)$ reactors, the total number m of the measured events is a sum of contributions m_a $(a = 1, \cdots, n_r)$ from each reactor, and this is also the case for the theoretical predictions t and t_a $(a = 1, \cdots, n_r)$. So we have

$$m = \sum_{a=1}^{n_r} m_a, \qquad t = \sum_{a=1}^{n_r} t_a.$$

Assuming for simplicity that the size of the uncorrelated error in the flux from the reactors is common ($\rho_{ua}^{(r)} = \rho_u^{(r)}$), we get (See Appendix A)

$$\chi^2 = \frac{(\frac{m}{t} - 1)^2}{\sigma_u^2 + (\delta_c^{(d)})^2 + (\rho_c^{(r)})^2 + (\rho_u^{(r)})^2 \sum_{a=1}^{n_r} (\frac{t_a}{t})^2} = \frac{\sin^4 2\theta (\sum_{a=1}^{n_r} \frac{t_a}{t} D(L_a))^2}{\sigma_u^2 + (\delta_c^{(d)})^2 + (\rho_c^{(r)})^2 + (\rho_u^{(r)})^2 \sum_{a=1}^{n_r} (\frac{t_a}{t})^2}, \quad (14)$$

where L_a is the baseline length from a-th reactor. By comparing (14) with (6), we find that $\rho_u^{(r)}$ is the error that controls the effect of the multiple reactor nature on χ^2 (the sensitivity to $\sin^2 2\theta$). Since there is only one detector in this case, the near-far cancellation does not occur and the systematic limit on $\sin^2 2\theta$ is affected by $\delta_c^{(d)}$, $\rho_c^{(r)}$, and $\rho_u^{(r)}$. However, if the yield from each reactor is equal, i.e., if

$$\frac{t_a}{t} = \frac{1}{n_r}, \quad (15)$$

then the contribution of the uncorrelated error of the reactors is minimized as

$$(\rho_u^{(r)})^2 \sum_{a=1}^{n_r} \left(\frac{t_a}{t}\right)^2 = \frac{1}{n_r} (\rho_u^{(r)})^2. \quad (16)$$

Comparing Eqs. (6) and (14), we observe that the contribution of the uncorrelated error of the reactors decreases as the number of the reactors increases, as long as the condition (16) is satisfied.[3] This is because the average of independent n_r fluctuations is smaller than a single fluctuation. This reduction of the contribution from $\rho_u^{(r)}$ is a potential merit of the multireactor case.

[3]One can show from the Cauchy-Schwarz inequality that $\sum_{a=1}^{n_r} (t_a/t)^2 \leq 1$ always holds even if the condition (16) is not satisfied. Hence, the contribution of the uncorrelated error of the reactors decreases always.

On the other hand, $\sum_{a=1}^{n_r} t_a D(L_a)/t$ in the numerator of (14) can be maximized for the case of equal baseline length [See Fig. 1(a)]. The reactors for an experiment to measure $\sin^2 2\theta_{13}$ have the same (or similar) power usually and then the condition (15) means the case of equal baseline length. Therefore, the condition (15) gives an ideal setup for the case with one detector and multiple reactors to maximize the oscillation and minimize the contribution from $\rho_u^{(r)}$. The condition will be a guideline to optimize the setup even for general case.

B. n_d detectors

Let us now discuss more general cases with n_r reactors and n_d detectors. For simplicity we assume again that the size of the uncorrelated errors for the detectors are the same, and the size of the uncorrelated errors in the flux from the reactors are also the same: $\sigma_{uj} = \sigma_u$, $\rho_{ua}^{(r)} = \rho_u^{(r)}$. Let t_{aj} (m_{aj}) be the theoretical prediction (measured value) for the number of events of neutrinos from the a-th reactor $(a = 1, \cdots, n_r)$ at the j-th detector $(j = 1, \cdots, n_d)$ and $t_j = \sum_{a=1}^{n_r} t_{aj}$ $(m_j = \sum_{a=1}^{n_r} m_{aj})$ be the theoretical (measured) total number of events at the j-th detector. Then generalizing the discussions in the previous sections, we have

$$\chi^2 = \left(\frac{m_1}{t_1} - 1, \cdots, \frac{m_{n_d}}{t_{n_d}} - 1\right) V^{-1} \begin{pmatrix} \frac{m_1}{t_1} - 1 \\ \vdots \\ \frac{m_{n_d}}{t_{n_d}} - 1 \end{pmatrix},$$

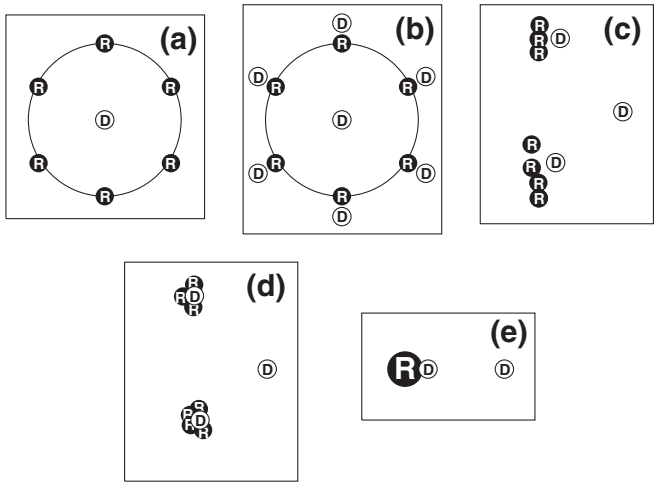

FIG. 1. Configurations of the experiments: (a) n_r reactors + one detector. (b) n_r reactors $+(n_r + 1)$ detectors. (c) The KASKA plan with 7 reactors + 3 detectors. (d) The ideal limit of the KASKA plan. (e) One reactor + 2 detectors to be compared with (c).

where elements of the covariance matrix V are given by

$$V_{jk} = \delta_{jk}\sigma_{\rm u}^2 + (\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 + (\rho_{\rm u}^{(\rm r)})^2 \sum_{a=1}^{n_r} \frac{t_{aj}}{t_j}\frac{t_{ak}}{t_k}. \tag{17}$$

1. n_r reactors and $(n_r + 1)$ detectors

In Sec. III we have seen that the near-far cancellation (the reduction of the minimum eigenvalue of V to $\sigma_{\rm u}^2$) occurs in the case of a single reactor experiment with one near and one far detectors. Now we would like to ask the following question: what happens to this cancellation in the case of an experiment with multiple reactors and detectors? To answer this question, let us consider the ideal case with n_r reactors and $(n_r + 1)$ detectors, where each reactor has a near detector in its neighborhood and each reactor produces the same number of events at a far detector [See Fig. 1(b)]:

$$\text{near detectors: } \frac{t_{aj}}{t_j} = \delta_{aj} \quad (j = 1, \cdots, n_r; a = 1, \cdots, n_r)$$
$$\text{far detector: } \frac{t_{an_r+1}}{t_{n_r+1}} = \frac{1}{n_r} \quad (a = 1, \cdots, n_r). \tag{18}$$

We can expect naively that the number of near detectors (n_r) is sufficient to cancel the errors in the fluxes from n_r reactors. The condition (18) enable us to diagonalize V analytically and we find that the minimum eigenvalue of V is $\sigma_{\rm u}^2$. It means that the near-far cancellation occurs for this case. Here we assume the following conditions:[4]

$$\left|\frac{\Delta m^2 L_{\rm n}}{4E}\right| \ll 1, \qquad \left|\frac{\Delta m^2 L_{\rm f}}{4E}\right| \simeq \frac{\pi}{2}. \tag{19}$$

These conditions have to be satisfied in an experiment which aims to measure θ_{13}. The systematic limit on $\sin^2 2\theta$ at the 90% CL becomes simple with the condition (19) and is given by

$$(\sin^2 2\theta)_{\rm limit}^{\rm sys\ only} = \sqrt{2.7}\sqrt{1 + \frac{1}{n_r}}\frac{\sigma_{\rm u}}{D(L_{\rm f})}\left\{1 + \frac{\sigma_{\rm u}^2/n_r}{\sigma_{\rm u}^2 + (n_r + 1)[(\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 + (\rho_{\rm u}^{(\rm r)})^2/n_r]}\right\}^{-1/2} \tag{20}$$

$$\simeq \sqrt{2.7}\sqrt{1 + \frac{1}{n_r}}\frac{\sigma_{\rm u}}{D(L_{\rm f})}. \tag{21}$$

As in the case with one reactor, the dominant contribution to the systematic limit comes from the uncorrelated error $\sigma_{\rm u}$ because of the near-far cancellation. Then, the factor $(\sin^2 2\theta)_{\rm limit}^{\rm sys\ only}/\sigma_{\rm u}$ becomes a good measure of the power of the setup; For example, we obtain 2.1 by assuming seven reactors and $L_{\rm f} = 1.7$ km. This value is smaller than 2.8 of (13) for the optimal case with one reactor and two detectors. The factor $\sqrt{1 + 1/n_r}$ which appears in the dominant contribution by $\sigma_{\rm u}$ indicates that the effective systematic error decreases as the number $(n_r + 1)$ of the detectors increases, since more information is obtained with more detectors. The contribution of the uncorrelated error of flux, $\rho_{\rm u}^{(\rm r)}$, is reduced in (20) by a factor of n_r due to the averaging over the independent n_r fluctuations; Although this reduction is a potential merit of the multiple reactor complex as we have seen in Sec. IVA, it is usually negligible in the multidetector system because the contribution of $\rho_{\rm u}^{(\rm r)}$ can be irrelevant to the sensitivity by the near-far cancellation. To conclude, the answer to the question at the beginning of this subsection is that the near-far cancellation can occur even for the case with n_r reactors if there are $(n_r + 1)$ detectors, and then the systematic limit is dominated by $\sigma_{\rm u}$. It should be noted that the number n_r of the near detectors in this case is sufficient but not necessary to guarantee this reduction of errors, as we will see below in the case of the KASKA plan.

2. The case of the KASKA plan

The Kashiwazaki-Kariwa nuclear power station consists of two clusters of reactors, and one cluster consists of four reactors while the other consists of three [See Fig. 1(c)]. According to the discussion in the previous section, we understand that near-far cancellation can occur for the KASKA case if we have seven near detectors. In the KASKA plan, however, not each reactor but each cluster of reactors is assumed to have a near detector. In this subsection we would like to clarify the following questions on the KASKA plan [4]: (a) Is the number of near detectors sufficient for the reduction of the minimum eigenvalue of V to $\sigma_{\rm u}^2$ (the near-far cancellation)? (b) What are the disadvantages of multiple sources? (c) Is the KASKA plan optimized with respect to the sensitivity to $\sin^2 2\theta$? Here we again assume that the size of the uncorrelated error in the flux from the reactors is common and the size of the uncorrelated errors of the three detectors are the same.

Before we discuss the systematic limit for the actual KASKA plan, let us consider the ideal limit, in which all the reactors in each cluster shrinks to one point as is shown

[4]For simplicity we assume here that the distance between the a-th reactor and its near detector is equal to $L_{\rm n}$ for $a = 1, \cdots, n_r$. In order for (18) to be satisfied, Eq. (19) is necessary. So in this ideal situation which we are considering, the dependence on $\langle \sin^2(\Delta m^2 L_{\rm n}/4E)\rangle$ cannot be discussed in a manner consistent with the assumption (18).

in Fig. 1(d); The ideal limit is similar to the case discussed in the section IV B 1, namely, the case of two reactors ($n_r = 2$) and three detectors. In this ideal limit, we have

$$\text{near 1: } \frac{t_{a1}}{t_1} = \begin{cases} \frac{1}{4} & (a = 1, \cdots, 4) \\ 0 & (a = 5, 6, 7) \end{cases} \tag{22}$$

$$\text{near 2: } \frac{t_{a2}}{t_2} = \begin{cases} 0 & (a = 1, \cdots, 4) \\ \frac{1}{3} & (a = 5, 6, 7) \end{cases} \tag{23}$$

$$\text{far: } \frac{t_{a3}}{t_3} = \frac{1}{7} \qquad (a = 1, \cdots, 7). \tag{24}$$

In this case, we obtain the systematic limit analytically (See Appendix C.)

$$(\sin^2 2\theta)^{\text{sys only}}_{\text{limit}} \simeq \sqrt{2.7}\sqrt{1 + \left(-\frac{3}{7}\right)^2 + \left(-\frac{4}{7}\right)^2}\,\frac{\sigma_{\text{u}}}{D(L_{\text{f}})} = \sqrt{2.7}\,\frac{\sqrt{74}}{7}\,\frac{\sigma_{\text{u}}}{D(L_{\text{f}})}. \tag{25}$$

Only σ_{u} gives the dominant contribution to the systematic limit in Eq. (25) due to the near-far cancellation. The reason why we have the factor $\sqrt{74/49}$ instead of $(1 + 1/n_r)^{1/2}|_{n_r=2} = \sqrt{3/2}$ is because the ratio of the $\bar{\nu}_e$ yield at the first cluster to that at the second one is 4:3 instead of 1:1 assumed in (18).

In reality, however, the conditions (22)–(24) are not exactly satisfied in the setting of the actual KASKA plan [4]. Let us evaluate the exact eigenvalues of the covariance matrix by taking into account the actual parameters in [4]. Table I shows the power of the reactors and the distance between the seven reactors and the three detectors. From this we can calculate the fraction t_{aj}/t_j ($a = 1, \cdots, 7$, $j = 1, 2, 3$) which is given in Table II. In this case, the eigenvalues of the covariance matrix V can be obtained only numerically for given values of $\delta_{\text{c}}^{(\text{d})}$, $\rho_{\text{c}}^{(\text{r})}$, and $\rho_{\text{u}}^{(\text{r})}$; We use the reference values of (1) and (2). We find that the minimum eigenvalue is

$$\sigma_{\text{u}}^2 + (0.12\%)^2. \tag{26}$$

The value is very close to σ_{u}^2 usually although the multi-reactor nature makes the near-far cancellation imperfect with an extra 0.12% error which vanishes for $\rho_{\text{u}}^{(\text{r})} = 0$. It means that the near-far cancellation occurs with a very good approximation in the actual KASKA plan with two near detectors; This is the answer to the question (a) at the beginning of this subsection. The systematic limit on $\sin^2 2\theta$ is approximately given by the contribution from the minimum eigenvalue (See Appendix C):

$$(\sin^2 2\theta)^{\text{sys only}}_{\text{limit}} \simeq \frac{\sqrt{2.7} \times \sqrt{\sigma_{\text{u}}^2 + (0.12\%)^2}}{|0.81\overline{D(L_3)} - 0.5\overline{D(L_1)} - 0.31\overline{D(L_2)}|} \simeq 3.9\sigma_{\text{u}} \simeq 0.022, \tag{27}$$

where $\overline{D(L_j)}$ is the average of each contribution $D(L_{aj})$:

$$\overline{D(L_j)} \equiv \sum_{a=1}^{7} \frac{t_{aj}}{t_j} \left\langle \sin^2\left(\frac{\Delta m^2 L_{aj}}{4E}\right) \right\rangle \qquad (j = 1(\text{near}), 2(\text{near}), 3(\text{far})). \tag{28}$$

Here L_{aj} is the distance between the a-th reactor and the j-th detector, and t_{aj}/t_j is the fraction of the yield from the a-th reactor at the detector $j = 1, 2, 3$. When the near-far cancellation occurs sufficiently, the value of $(\sin^2 2\theta)^{\text{sys only}}_{\text{limit}}/\sigma_{\text{u}}$ gives a good measure for the power of a reactor experiment almost independently of assumptions of error sizes; The smaller value means the better setup of reactor experiments.

To see how effectively the contributions from errors to the sensitivity are reduced in the actual KASKA plan, comparison is given in Fig. 2 between the sensitivities to $\sin^2 2\theta$ of the actual KASKA plan [Fig. 1(c)] and of a hypothetical experiment with a single reactor and two detectors (300 m and 1.3 km baselines) depicted in Fig. 1(e); Exact χ^2 including all eigenvalues of V is used for each case with the 20 ton yr data size and the values of systematic errors in (1) and (2). We observe that there is

TABLE I. The powers of the the reactors in GW_{th} and the distance L_{aj} in meters from the three detectors ($j =$ 1(near), 2(near), 3(far)) to each reactor ($a = 1, \cdots, 7$). The powers of reactors are listed in http://www.tepco.co.jp/kk-np/index-j.html. The positions of the reactors were read from a map and are subject to a few meters of inaccuracy. However, such inaccuracy hardly affects the estimation of the systematic errors in the text. Take care about the positions of the reactor #5 and #7.

Reactors (a)	Power/GW_{th}	L_{a1}/m	L_{a2}/m	L_{a3}/m
1	3.293	482	1663	1309
2	3.293	401	1504	1224
3	3.293	458	1374	1233
4	3.293	524	1149	1169
5	3.293	1552	371	1484
6	3.926	1419	333	1397
7	3.926	1280	340	1306

TABLE II. The fractions t_{aj}/t_j of the yields at each detector j from the reactor a in the KASKA plan.

Reactors (a)	t_{a1}/t_1	t_{a2}/t_2	t_{a3}/t_3
1	0.208	0.012	0.133
2	0.301	0.015	0.152
3	0.231	0.017	0.149
4	0.176	0.025	0.166
5	0.020	0.239	0.103
6	0.029	0.353	0.139
7	0.035	0.339	0.159

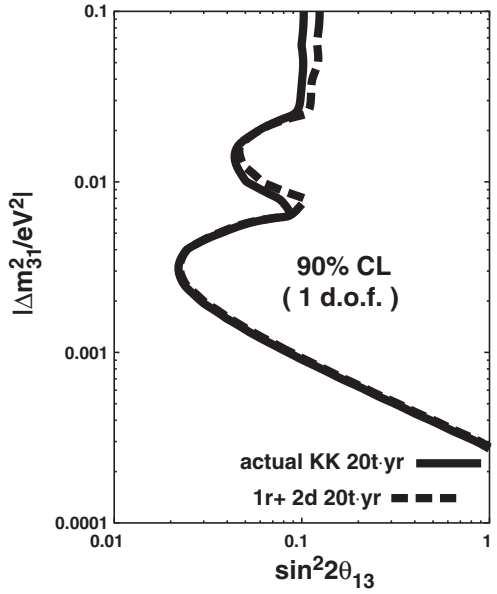

FIG. 2. The comparison of the sensitivity to $\sin^2 2\theta_{13}$ for the actual KASKA plan [the solid line; with the configuration depicted in Fig. 1(c)] and a hypothetical case [the dashed line; with the configuration depicted in Fig. 1(e)]. The statistical errors as well as all the systematic errors are taken into account.

little difference between the sensitivities at $\Delta m^2 = 2.5 \times 10^{-3}$ eV^2. Also it is remarkable that the sensitivity of the actual KASKA plan for higher value of Δm^2 is better than of the single reactor experiment. This is exactly because of the reduction of the uncorrelated error from due to the nature of multireactors (cf. Eq. (16)), where the near detectors play a role as far detectors in this case. Here, it should be mentioned that we see in Fig. 2 that the sensitivity in KASKA changes only to $\sin^2 2\theta \simeq 0.03$ even for $\Delta m^2 = 2 \times 10^{-3}$ eV^2.

Once we know that two near detectors are sufficient for the approximate near-far cancellation with the Kashiwazaki-Kariwa nuclear power station, we should investigate the optimal locations of the detectors for the sensitivity. Note that the systematic limit (27) becomes (25) of the ideal limit approximately if we substitute zero for $\overline{D(L_1)}$ and $\overline{D(L_2)}$. $\overline{D(L_j)} = 0$ is realized for the case that the detector is very close to a reactor like the ideal limit [Fig. 1(d)]. In the actual KASKA case, however, each of the two near detectors should not be too close to a reactor because it makes impossible to cancel the uncorrelated error of the flux from other reactors, namely $\rho_{\mathrm{u}}^{(\mathrm{r})}$. It is a potential disadvantage of the case of $n_d < n_r + 1$ that we can not maximize the difference between the oscillation probabilities at the far detector and near detectors keeping the near-far cancellation; This is the answer to the question (b) at the beginning of this subsection. Hence, it is nontrivial to find the optimal locations of two near detectors at the Kashiwazaki-Kariwa site and to see the dependence of the sensitivity on the near detector positions. The dependence of the sensitivity on the far detector position will be rather simple because the spread of reactors in each cluster is small compared with the baseline length of the far detector.

To do the analysis, we first obtain the optimized positions of the detectors with the reference values of errors in Eqs. (1) and (2) as the answer to the question (c) at the beginning of this subsection. Then, we examine the sensitivity to $\sin^2 2\theta$ by varying the position of each detector, leaving the locations of the remaining detectors in the optimized ones. The results are given by the contour plots in Fig. 3 without statistical errors and in Fig. 4 with the data size of 20 ton yr. In these figures the locations of the detectors are also depicted for the optimized case and for the currently planned case. From these two figures we observe that the distance between each near detector and the reactors in each cluster is approximately (300 ± 130) m in the optimized case. The optimal positions of near detectors are not so different between those figures because the statistical error is negligible at near detectors even for the data size of 20 ton yr. On the other hand, the optimal position of the far detector in Fig. 4 is closer to reactors than that in Fig. 3 in order to make the statistical error small. From Figs. 3 and 4, we see that the positions of the near detectors in the KASKA plan are almost optimized for the reference values of errors while the baseline length for the planned position of the far detector is not sufficient even for the 20 ton yr case. In order to see the dependence of the optimal positions on Δm^2, Figs. 5 and 6 are made for $\Delta m^2 = 2 \times 10^{-3}$ eV^2. The optimal positions of near detectors are almost independent of Δm^2 because they observes fluxes without the oscillation, and the optimized

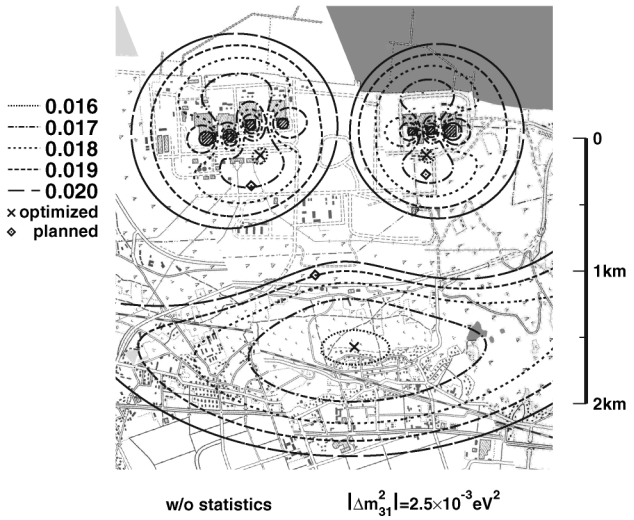

FIG. 3. The contour plots of the systematic limit on $\sin^2 2\theta_{13}$ in the KASKA experiment. The optimized and currently planned positions of the detectors are also depicted. When the contour for each detector is plotted, it is assumed that other detectors are located in the optimized positions. The map in the background was taken from Ref. [17].

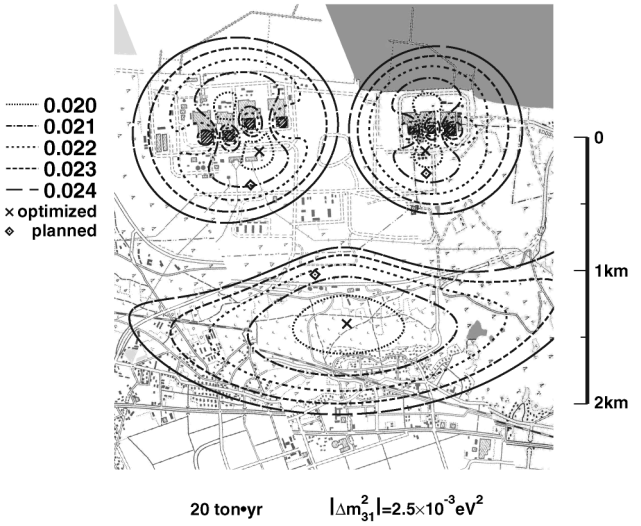

FIG. 4. The same contour plot of the sensitivity to $\sin^2 2\theta_{13}$ as Fig. 3 with the data size of 20 ton yr. The map in the background was taken from Ref. [17].

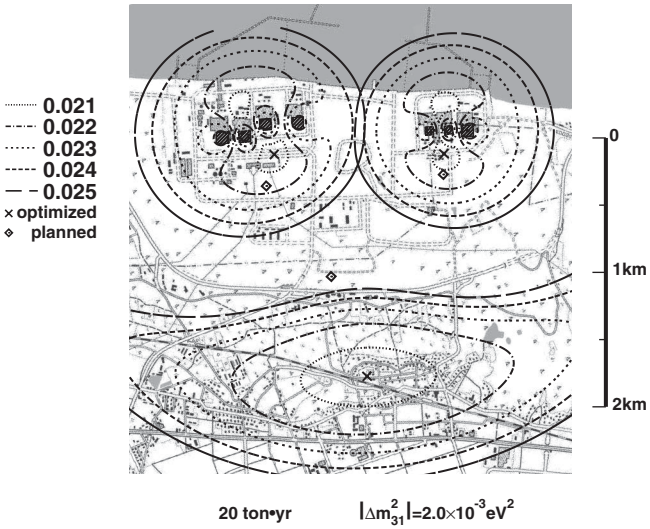

FIG. 6. The same contour plot of the sensitivity to $\sin^2 2\theta_{13}$ as Fig. 5 with the data size of 20 ton yr. The map in the background was taken from Ref. [17].

position of the far detector becomes farer from reactors for smaller Δm^2 as is expected by the simple oscillation behavior.

Since Fig. 3 shows $(\sin^2 2\theta)^{\text{sys only}}_{\text{limit}} \simeq 0.016$ for the optimal setup with $\sigma_{\text{u}} = 0.8/\sqrt{2}\%$, we obtain

$$\frac{(\sin^2 2\theta)^{\text{sys only}}_{\text{limit}}}{\sigma_{\text{u}}} \simeq 2.8, \tag{29}$$

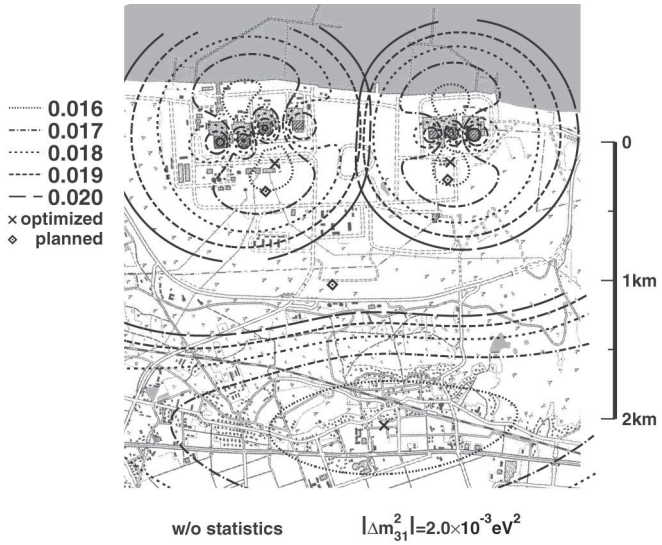

FIG. 5. The contour plots of the systematic limit on $\sin^2 2\theta_{13}$ in the KASKA experiment for a different value of $|\Delta m^2_{31}|$ from the value used in Fig. 3. The optimized and currently planned positions of the detectors are also depicted. When the contour for each detector is plotted, it is assumed that other detectors are located in the optimized positions. The map in the background was taken from Ref. [17].

as a good measure of the optimal power of the KASKA experiment. Note that the value is equal accidentally to (13) of the optimal case with one reactor and two detectors. The disadvantage of the nonzero baseline length for near detectors in the KASKA experiment is compensated by the advantage that two near detectors give more information than that given by one near detector; Roughly speaking, the advantage is that $\sqrt{74}\sigma_{\text{u}}/7$ in (25) is smaller than $\sqrt{2}\sigma_{\text{u}}$ in (13). Therefor, the KASKA plan is very powerful potentially to measure $\sin^2 2\theta_{13}$.

V. DISCUSSION AND CONCLUSION

Using the analytical method, we estimated the systematic limits (sensitivity without statistical error) on the neutrino oscillation parameter $\sin^2 2\theta_{13}$ in various setups of reactor experiments at 90% CL for 1 degree of freedom. In the simplest case, where there is one reactor and two detectors, the minimum eigenvalue of the covariance matrix V becomes σ^2_{u} (the near-far cancellation); The minimum eigenvalue dominates the systematic limit. In the case of multiple n_r reactors, we showed analytically that the near-far cancellation is possible with $(n_r + 1)$ detectors as a naive expectation. We found that the setup with multiple detectors has an advantage of the reduction of the remaining contribution to the systematic limit from σ^2_{u} if the setup is appropriate for the near-far cancellation. On the other hand, we explicitly showed that the contribution to the sensitivity to $\sin^2 2\theta_{13}$ from the the uncorrelated error of the flux, which controls the multireactor nature, is negligibly small in the KASKA plan and the near-far cancellation occurs with a very good approximation although there are only three detectors for seven reactors $(n_d < n_r + 1)$. The only disadvantage of experiments with

$n_d < n_r + 1$ is that one cannot put the near detectors arbitrarily close to 1 of the reactors (even if one neglects the technical difficulties), because that would ruin the cancellation of the uncorrelated error of the flux, as we have seen explicitly in the KASKA case. We presented also the optimal positions of detectors in the KASKA plan; The planned positions of near detectors are close to the optimal ones although it is better if the baseline length for the far detector becomes longer beyond the bound of the power station site. In all cases studied here, it is the uncorrelated error $\sigma_{\rm u}$ that dominates the systematic limit on $\sin^2 2\theta_{13}$, and hence it is quite important to estimate $\sigma_{\rm u}$ carefully. The factor $(\sin^2 2\theta)^{\rm sys\ only}_{\rm limit}/\sigma_{\rm u}$ seems to be a good measure of the power of a reactor experiment almost independently of assumptions of error sizes; For example, the value is about 2.8 for the KASKA experiment with the optimal detector positions presented in Fig. 3 and it is so good as to be equal (accidentally) to the value for the optimal case with one reactor and two detectors.

In this paper, we dealt with total numbers of events for simplicity. If we want to utilize the spectral information also, we must consider the correlation of errors between bins. An error that uncorrelates between bins and between detectors controls the sensitivity with the spectral information unless the error is very large. Then, the optimal positions of detectors are very different from the ones for the rate analysis (See [16] for the detail).

ACKNOWLEDGMENTS

This work was supported in part by Grants-in-Aid for Scientific Research No. 11204015, No. 16540260 and No. 16340078, Japan Ministry of Education, Culture, Sports, Science, and Technology, and U.S. Department of Energy Grant No. DE-FG-88ER40397.

APPENDIX A: DERIVATION OF THE COVARIANCE MATRIX

In this appendix we first show that the form of χ^2 which is expressed as the minimum of the function of the α variables with respect to these variables leads to the form of χ^2 which is bilinear in the ratio m_j/t_j of the measured value m_j divided by the theoretical prediction t_j. This has been known in the literature [10–13] as the equivalence between the so-called pull approach and the covariance matrix approach. And then we show that the same job can be done by integration of $\exp(-\chi^2/2)$ over the variables m_j/t_j.

In the cases which we are considering, the correlated systematic errors $t_1^2\sigma_{\rm u1}^2, \cdots, t_n^2\sigma_{{\rm u}\ell}^2$ are introduced by the variables

$$\vec{\alpha} \equiv \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_\ell \end{pmatrix}.$$

Introducing the notation

$$y_j \equiv \frac{m_j}{t_j} - 1, \qquad \vec{y} \equiv \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix},$$

χ^2 can be written as

$$\begin{aligned}\chi^2 &= \min_{\vec{\alpha}}[(\vec{y} - B\vec{\alpha})^T \mathcal{D}_{\rm u}^{-1}(\vec{y} - B\vec{\alpha}) + \vec{\alpha}^T \mathcal{D}_{\rm c}^{-1}\vec{\alpha}] \\ &= \min_{\vec{\alpha}}[(\vec{\alpha} - A^{-1}B\mathcal{D}_{\rm u}^{-1}\vec{y})^T A(\vec{\alpha} - A^{-1}B\mathcal{D}_{\rm u}^{-1}\vec{y}) \\ &\quad + \vec{y}^T(\mathcal{D}_{\rm u}^{-1} - \mathcal{D}_{\rm u}^{-1}BA^{-1}B^T\mathcal{D}_{\rm u}^{-1})\vec{y}] \\ &= \vec{y}^T(\mathcal{D}_{\rm u}^{-1} - \mathcal{D}_{\rm u}^{-1}BA^{-1}B^T\mathcal{D}_{\rm u}^{-1})\vec{y}, \end{aligned} \tag{A1}$$

where B is an $n \times \ell$ matrix that determines how the errors affect on $\vec{y}$,

$$\mathcal{D}_{\rm u} \equiv {\rm diag}(\sigma_{\rm u1}^2, \cdots, \sigma_{{\rm u}n}^2)$$

is an $n \times n$ diagonal matrix whose element is the normalized uncorrelated systematic error $\sigma_{{\rm u}j}^2$ for the variable m_j $(j = 1, \cdots, n)$,

$$\mathcal{D}_{\rm c} \equiv {\rm diag}(\sigma_{\rm c1}^2, \cdots, \sigma_{{\rm c}\ell}^2)$$

is an $\ell \times \ell$ diagonal matrix whose element is the normalized correlated systematic error $\sigma_{{\rm c}j}^2$ for the variable α_j $(j = 1, \cdots, \ell)$, and we have defined

$$A \equiv \mathcal{D}_{\rm c}^{-1} + B^T\mathcal{D}_{\rm u}^{-1}B.$$

Note that we can incorporate the effect of the statistical errors in our formalism by redefining $\mathcal{D}_u \to {\rm diag}(\sigma_{\rm u1}^2 + 1/\sqrt{t_1}, \cdots, \sigma_{\rm u1}^2 + 1/\sqrt{t_n})$, although we do not discuss the statistical errors in the present paper. From Eq. (A1) we see that the covariance matrix V is given by

$$V = (\mathcal{D}_{\rm u}^{-1} - \mathcal{D}_{\rm u}^{-1}BA^{-1}B^T\mathcal{D}_{\rm u}^{-1})^{-1}.$$

We could prove by brute force that V can be written as

$$V = \mathcal{D}_{\rm u} + B\mathcal{D}_{\rm c}B^T, \tag{A2}$$

but it is much easier to prove it by expressing the matrix element V_{ij} as the integral of $\exp(-\chi^2/2)$ over the variables y_j.

First of all, let us prove that the matrix element V_{ij} can be written as

$$V_{ij} = \mathcal{N}_y \int d\vec{y} y_i y_j \exp\left(-\frac{1}{2}\vec{y}^T V^{-1}\vec{y}\right), \tag{A3}$$

where $\mathcal{N}_y$ is the normalization constant defined by

$$\mathcal{N}_y^{-1} \equiv \int d\vec{y} \exp\left(-\frac{1}{2}\vec{y}^T V^{-1}\vec{y}\right).$$

Proof of Eq. (A3) goes as follows. Diagonalizing the covariance matrix V, which is real symmetric, by an orthogonal matrix $\mathcal{O}$

$$V = \mathcal{O}^T \mathcal{D} \mathcal{O} \equiv \mathcal{O}^T \mathrm{diag}(v_1, \cdots, v_n) \mathcal{O},$$

the exponent can be rewritten as

$$\vec{y}^{\,T} V^{-1} \vec{y} = \vec{y}^T \mathcal{O}^T \mathcal{D}^{-1} \mathcal{O} \vec{y} \equiv \vec{y'}^T \mathcal{D}^{-1} \vec{y'},$$

so that we have

$$\begin{aligned}
&\mathcal{N}_y \int d\vec{y} y_i y_j \exp\left(-\frac{1}{2}\vec{y}^T V^{-1} \vec{y}\right) \\
&= \mathcal{N}_y \int d\vec{y'} (\mathcal{O}^T \vec{y'})_i (\mathcal{O}^T \vec{y'})_j \exp\left(-\frac{1}{2}\vec{y'}^T \mathcal{D}^{-1} \vec{y'}\right) \\
&= \mathcal{N}_y \int d\vec{y'} (\mathcal{O})_{ki} y'_k (\mathcal{O})_{lj} y'_l \exp\left(-\frac{1}{2}\sum_{i=1}^{n} \frac{y'^2_j}{v_j}\right) \\
&= (\mathcal{O})_{ki} (\mathcal{D})_{kl} (\mathcal{O})_{lj} = (\mathcal{O}^T \mathcal{D} \mathcal{O})_{ij} = V_{ij}.
\end{aligned}$$

Thus Eq. (A3) is proved.

Now Eq. (A3) can be simplified by expressing as the integral over the variables $\vec{\alpha}$ of the original χ^2:

$$\begin{aligned}
V_{ij} &= \mathcal{N}_y \int d\vec{y} y_i y_j \exp\left(-\frac{1}{2}\vec{y}^T V^{-1} \vec{y}\right) \\
&= \mathcal{N}_y \mathcal{N}_\alpha \int d\vec{y} \int d\vec{\alpha} y_i y_j \\
&\quad \times \exp\Big\{-\frac{1}{2}[(\vec{\alpha} - A^{-1} B \mathcal{D}_{\mathrm{u}}^{-1} \vec{y})^T A (\vec{\alpha} - A^{-1} B \mathcal{D}_{\mathrm{u}}^{-1} \vec{y}) \\
&\quad + \vec{y}^T (\mathcal{D}_{\mathrm{u}}^{-1} - \mathcal{D}_{\mathrm{u}}^{-1} B A^{-1} B^T \mathcal{D}_{\mathrm{u}}^{-1}) \vec{y}]\Big\} \\
&= \mathcal{N}_y \mathcal{N}_\alpha \int d\vec{y} \int d\vec{\alpha} y_i y_j \\
&\quad \times \exp\Big\{-\frac{1}{2}[(\vec{y} - B\vec{\alpha})^T \mathcal{D}_{\mathrm{u}}^{-1} (\vec{y} - B\vec{\alpha}) + \vec{\alpha}^T \mathcal{D}_{\mathrm{c}}^{-1} \vec{\alpha}]\Big\},
\end{aligned} \tag{A4}$$

where we have used Eq. (A1), the normalization constant $\mathcal{N}_\alpha$ is defined by

$$\mathcal{N}_\alpha^{-1} \equiv \int d\vec{\alpha} \exp\Big\{-\frac{1}{2}[(\vec{\alpha} - A^{-1} B \mathcal{D}_{\mathrm{u}}^{-1} \vec{y})^T A (\vec{\alpha} - A^{-1} B \mathcal{D}_{\mathrm{u}}^{-1} \vec{y})]\Big\},$$

and $\mathcal{N}_y$ and $\mathcal{N}_\alpha$ are related by

$$\mathcal{N}_y \mathcal{N}_\alpha = (2\pi)^{(n+\ell)/2} (\det \mathcal{D}_{\mathrm{u}})^{1/2} (\det \mathcal{D}_{\mathrm{c}})^{1/2}.$$

Equation (A4) can be easily calculated by shifting the variable $\vec{y} \to \vec{y''} \equiv \vec{y} - B\vec{\alpha}$:

$$\begin{aligned}
V_{ij} &= \mathcal{N}_y \mathcal{N}_\alpha \int d\vec{y''} \int d\vec{\alpha} (\vec{y''} - B\vec{\alpha})_i (\vec{y''} - B\vec{\alpha})_j \\
&\quad \times \exp\Big\{-\frac{1}{2}[(\vec{y} - B\vec{\alpha})^T \mathcal{D}_{\mathrm{u}}^{-1} (\vec{y} - B\vec{\alpha}) + \vec{\alpha}^T \mathcal{D}_{\mathrm{c}}^{-1} \vec{\alpha}]\Big\} \\
&= (\mathcal{D}_{\mathrm{u}})_{ij} + (B)_{ik} (B)_{jl} (\mathcal{D}_{\mathrm{c}})_{kl} = (\mathcal{D}_{\mathrm{u}} + B \mathcal{D}_{\mathrm{c}} B^T)_{ij}.
\end{aligned}$$

Hence Eq. (A2) is proved.

For example, in the case of one reactor with one detector, we can obtain very easily the covariance matrix (6)

$$\begin{aligned}
V &= \mathcal{N}_y \mathcal{N}_\alpha \int d(m/t - 1) \int d\vec{\alpha} \left(\frac{m}{t} - 1\right)^2 \exp\left(-\frac{1}{2}\chi^2\right) \\
&= \sigma_{\mathrm{u}}^2 + (\delta_{\mathrm{c}}^{(\mathrm{d})})^2 + (\rho_{\mathrm{c}}^{(\mathrm{r})})^2 + (\rho_{\mathrm{u}}^{(\mathrm{r})})^2,
\end{aligned}$$

where we use the unminimized-version of (5) as the χ^2. Note that the values of $\mathcal{N}_y$ and $\mathcal{N}_\alpha$ are not necessary for the calculation. On the other hand, the χ^2 gives

$$\mathcal{D}_{\mathrm{u}} = \sigma_{\mathrm{u}}^2, \qquad \mathcal{D}_{\mathrm{c}} = \mathrm{diag}((\delta_{\mathrm{c}}^{(\mathrm{d})})^2, (\rho_{\mathrm{c}}^{(\mathrm{r})})^2, (\rho_{\mathrm{u}}^{(\mathrm{r})})^2), \qquad B = (1, 1, 1),$$

and we have the same result

$$V = \mathcal{D}_{\mathrm{u}} + B \mathcal{D}_{\mathrm{c}} B^T = \sigma_{\mathrm{u}}^2 + (\delta_{\mathrm{c}}^{(\mathrm{d})})^2 + (\rho_{\mathrm{c}}^{(\mathrm{r})})^2 + (\rho_{\mathrm{u}}^{(\mathrm{r})})^2.$$

In the case of one reactor with two detectors (cf. Eq. (7)), we have

$$\mathcal{D}_{\mathrm{u}} = \sigma_{\mathrm{u}}^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathcal{D}_{\mathrm{c}} = \mathrm{diag}((\delta_{\mathrm{c}}^{(\mathrm{d})})^2, (\rho_{\mathrm{c}}^{(\mathrm{r})})^2, (\rho_{\mathrm{u}}^{(\mathrm{r})})^2), \quad B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

so that we obtain the covariance matrix (8)

$$V = \mathcal{D}_{\mathrm{u}} + B \mathcal{D}_{\mathrm{c}} B^T = \begin{pmatrix} \sigma_{\mathrm{u}}^2 + (\delta_{\mathrm{c}}^{(\mathrm{d})})^2 + (\rho_{\mathrm{u}}^{(\mathrm{r})})^2 + (\rho_{\mathrm{c}}^{(\mathrm{r})})^2 & (\delta_{\mathrm{c}}^{(\mathrm{d})})^2 + (\rho_{\mathrm{u}}^{(\mathrm{r})})^2 + (\rho_{\mathrm{c}}^{(\mathrm{r})})^2 \\ (\delta_{\mathrm{c}}^{(\mathrm{d})})^2 + (\rho_{\mathrm{u}}^{(\mathrm{r})})^2 + (\rho_{\mathrm{c}}^{(\mathrm{r})})^2 & \sigma_{\mathrm{u}}^2 + (\delta_{\mathrm{c}}^{(\mathrm{d})})^2 + (\rho_{\mathrm{u}}^{(\mathrm{r})})^2 + (\rho_{\mathrm{c}}^{(\mathrm{r})})^2 \end{pmatrix}.$$

In the case of n_r reactors with one detector (cf. Equation (14)), errors are introduced to χ^2 as follows

$$\chi^2 = \min_{\alpha' s} \left\{ \frac{1}{t^2 \sigma^2} \left[m - t\left(1 + \alpha_{\mathrm{c}}^{(\mathrm{d})} + \alpha_{\mathrm{c}}^{(\mathrm{r})} + \sum_{a=1}^{n_r} \frac{t_a}{t} \alpha_{\mathrm{u}a}^{\mathrm{r}}\right)\right]^2 + \left(\frac{\alpha_{\mathrm{c}}^{(\mathrm{d})}}{\delta_{\mathrm{c}}^{(\mathrm{d})}}\right)^2 + \left(\frac{\alpha_{\mathrm{c}}^{(\mathrm{r})}}{\rho_{\mathrm{c}}^{(\mathrm{r})}}\right)^2 + \sum_{a=1}^{n_r} \left(\frac{\alpha_{\mathrm{u}a}^{(\mathrm{r})}}{\rho_{\mathrm{u}}^{(\mathrm{r})}}\right)^2 \right\},$$

where $\alpha_{\mathrm{u}a}^{(r)}$ is the variable to introduce the uncorrelated of the flux from the a-th reactor. We have again assumed for simplicity that the size of the uncorrelated error in the flux from the reactors is common: $\rho_{\mathrm{u}a}^{(\mathrm{r})} = \rho_{\mathrm{u}}^{(\mathrm{r})}$. We find

$$\mathcal{D}_{\rm u} = \sigma_{\rm u}^2,$$
$$\mathcal{D}_{\rm c} = {\rm diag}[(\delta_{\rm c}^{\rm (d)})^2, (\rho_{\rm c}^{\rm (r)})^2, (\rho_{\rm u}^{\rm (r)})^2, \cdots, (\rho_{\rm u}^{\rm (r)})^2],$$
$$B = \left(1, 1, \frac{t_1}{t}, \cdots, \frac{t_{n_r}}{t}\right),$$

so that we obtain

$$V = \mathcal{D}_{\rm u} + B\mathcal{D}_{\rm c}B^T = \sigma_{\rm u}^2 + (\delta_{\rm c}^{\rm (d)})^2 + (\rho_{\rm c}^{\rm (r)})^2 + (\rho_{\rm u}^{\rm (r)})^2 \sum_{a=1}^{n_r}\left(\frac{t_a}{t}\right)^2.$$

In the case of n_r reactors with n_d detector (cf. Equation (17)), χ^2 is defined by

$$\chi^2 = \min_{\alpha' s}\left\{\sum_{j=1}^{n_d}\frac{1}{t_j^2\sigma_{\rm u}^2}\left[m_j - t_j\left(1 + \alpha_{\rm c}^{\rm (d)} + \alpha_{\rm c}^{\rm (r)} + \sum_{a=1}^{n_r}\frac{t_{aj}}{t_j}\alpha_{{\rm u}a}^{(r)}\right)\right]^2 + \left(\frac{\alpha_{\rm c}^{\rm (d)}}{\delta_{\rm c}^{\rm (d)}}\right)^2 + \left(\frac{\alpha_{\rm c}^{\rm (r)}}{\rho_{\rm c}^{\rm (r)}}\right)^2 + \sum_{a=1}^{n_r}\left(\frac{\alpha_{{\rm u}a}^{\rm (r)}}{\rho_{\rm u}^{\rm (r)}}\right)^2\right\}.$$

We have

$$\mathcal{D}_{\rm u} = \sigma_{\rm u}^2 I_{n_d},$$
$$\mathcal{D}_{\rm c} = {\rm diag}[(\delta_{\rm c}^{\rm (d)})^2, (\rho_{\rm c}^{\rm (r)})^2, (\rho_{\rm u}^{\rm (r)})^2, \cdots, (\rho_{\rm u}^{\rm (r)})^2],$$
$$B = \begin{pmatrix} 1 & 1 & \frac{t_{11}}{t_1} & \cdots & \frac{t_{n_r1}}{t_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \frac{t_{1n_d}}{t_{n_d}} & \cdots & \frac{t_{n_rn_d}}{t_{n_d}} \end{pmatrix},$$

where I_{n_d} is an $n_d \times n_d$ unit matrix, so that we obtain the covariance matrix (17)

$$V_{jk} = [\mathcal{D}_{\rm u} + B\mathcal{D}_{\rm c}H^T]_{jk} = \delta_{jk}\sigma_{\rm u}^2 + (\delta_{\rm c}^{\rm (d)})^2 + (\rho_{\rm c}^{\rm (r)})^2 + (\rho_{\rm u}^{\rm (r)})^2 \sum_{a=1}^{n_r}\frac{t_{aj}}{t_j}\frac{t_{ak}}{t_k}.$$

APPENDIX B: DERIVATION OF THE SYSTEMATIC LIMIT (20)

In the ideal case with n_r reactors and $n_r + 1$ detectors [Fig. 1(b)], the covariance matrix becomes

$$V = \sigma_{\rm u}^2 I_{n_r+1} + [(\delta_{\rm c}^{\rm (d)})^2 + (\rho_{\rm c}^{\rm (r)})^2]H_{n_r+1} + (\rho_{\rm u}^{\rm (r)})^2 \times \begin{pmatrix} 1 & 0 & \cdots & 0 & 1/n_r \\ 0 & \ddots & & \vdots & \vdots \\ \vdots & & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 & \vdots \\ 1/n_r & \cdots & \cdots & \cdots & 1/n_r \end{pmatrix}, \tag{B1}$$

where I_{n_r+1} is an $(n_r + 1) \times (n_r + 1)$ unit matrix, and H_{n_r+1} is an $(n_r + 1) \times (n_r + 1)$ matrix defined by

$$H_{n_r+1} \equiv \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ 1 & \cdots & 1 \end{pmatrix}. \tag{B2}$$

This covariance matrix (B1) can be diagonalized as

$$U^{-1}VU = {\rm diag}\left\{(n_r + 1)\left[(\delta_{\rm c}^{\rm (d)})^2 + (\rho_{\rm c}^{\rm (r)})^2 + \frac{\sigma_{\rm u}^2}{n_r} + \frac{(\rho_{\rm u}^{\rm (r)})^2}{n_r}\right], \sigma_{\rm u}^2 + (\rho_{\rm u}^{\rm (r)})^2, \cdots, \sigma_{\rm u}^2 + (\rho_{\rm u}^{\rm (r)})^2, \sigma_{\rm u}^2\right\},$$

where U is a unitary matrix defined by

$$U = (\vec{u}^{(1)}, \cdots, \vec{u}^{(n)}), \qquad \vec{u}^{(1)} \equiv \frac{1}{\sqrt{n_r + 1}}\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix},$$
$$\vec{u}^{(2)} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \qquad \vec{u}^{(3)} \equiv \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \qquad \cdots,$$
$$\vec{u}^{(n_r+1)} \equiv \frac{1}{\sqrt{n_r(n_r + 1)}}\begin{pmatrix} 1 \\ \vdots \\ 1 \\ -n_r \end{pmatrix}. \tag{B3}$$

Introducing the notation

$$\vec{y} \equiv \begin{pmatrix} m_1/t_1 - 1 \\ \vdots \\ m_{n_r}/t_{n_r} - 1 \\ m_{n_r+1}/t_{n_r+1} - 1 \end{pmatrix} \simeq -\sin^2 2\theta D(L_{\rm f})\begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix},$$

χ^2 can be written as

$$\chi^2 = \frac{(\vec{y}\cdot\vec{u}^{(1)})^2}{\sigma_{\rm u}^2 + (n_r + 1)[(\delta_{\rm c}^{\rm (d)})^2 + (\rho_{\rm c}^{\rm (r)})^2 + (\rho_{\rm u}^{\rm (r)})^2/n_r]} + \frac{\sum_{j=2}^{n_r}(\vec{y}\cdot\vec{u}^{(j)})^2}{\sigma_{\rm u}^2 + (\rho_{\rm u}^{\rm (r)})^2} + \frac{(\vec{y}\cdot\vec{u}^{(n_r+1)})^2}{\sigma_{\rm u}^2}.$$

From Eq. (B3), we have

$$\vec{y}\cdot\vec{u}^{(1)} \simeq -\frac{\sin^2 2\theta D(L_{\rm f})}{\sqrt{n_r + 1}}, \qquad \vec{y}\cdot\vec{u}^{(j)} \simeq 0$$
$$(j = 2, \cdots, n_r), \qquad \vec{y}\cdot\vec{u}^{(n_r+1)} \simeq \sqrt{\frac{n_r}{n_r + 1}}\sin^2 2\theta D(L_{\rm f}),$$

and we finally get

$$\chi^2 \simeq \sin^4 2\theta [D(L_{\rm f})]^2 \left\{ \frac{n_r}{n_r+1}\frac{1}{\sigma_{\rm u}^2} + \frac{1}{n_r+1}\frac{1}{\sigma_{\rm u}^2 + (n_r+1)[(\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 + (\rho_{\rm u}^{(\rm r)})^2/n_r]} \right\}.$$

APPENDIX C: DERIVATION OF THE SYSTEMATIC LIMIT IN THE KASKA PLAN

In the ideal limit of KASKA [Fig. 1(d)], the covariance matrix is given by

$$\begin{aligned} V &= \sigma_{\rm u}^2 I_3 + [(\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2] H_3 + (\rho_{\rm u}^{(\rm r)})^2 \begin{pmatrix} 1/4 & 0 & 1/7 \\ 0 & 1/3 & 1/7 \\ 1/7 & 1/7 & 1/7 \end{pmatrix} \\ &= \begin{pmatrix} (\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 + \frac{(\rho_{\rm u}^{(\rm r)})^2}{4} + \sigma_{\rm u}^2 & (\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 & (\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 + \frac{(\rho_{\rm u}^{(\rm r)})^2}{7} \\ (\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 & (\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 + \frac{(\rho_{\rm u}^{(\rm r)})^2}{3} + \sigma_{\rm u}^2 & (\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 + \frac{(\rho_{\rm u}^{(\rm r)})^2}{7} \\ (\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 + \frac{(\rho_{\rm u}^{(\rm r)})^2}{7} & (\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 + \frac{(\rho_{\rm u}^{(\rm r)})^2}{7} & (\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2 + \frac{(\rho_{\rm u}^{(\rm r)})^2}{7} + \sigma_{\rm u}^2 \end{pmatrix}, \end{aligned} \quad \text{(C1)}$$

where I_3 is a 3×3 unit matrix and H_3 is a 3×3 matrix defined in Eq. (B2). It is easy to show that the diagonalized matrix out of (C1) is

$$\text{diag}\,(\sigma_{\rm u}^2,\ \sigma_{\rm u}^2 + \Lambda_-,\ \sigma_{\rm u}^2 + \Lambda_+),$$

where

$$\begin{aligned} \Lambda_\pm \equiv{}& \frac{3}{2}[(\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2] + \frac{61}{168}(\rho_{\rm u}^{(\rm r)})^2 \pm \frac{1}{2}\Big\{ 9[(\delta_{\rm c}^{(\rm d)})^2 \\ &+ (\rho_{\rm c}^{(\rm r)})^2]^2 + \frac{5}{6}[(\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2](\rho_{\rm u}^{(\rm r)})^2 \\ &+ \left(\frac{13}{84}\right)^2 (\rho_{\rm u}^{(\rm r)})^4 \Big\}^{1/2}. \end{aligned}$$

For example, the values of errors in (1) and (2) give $\Lambda_- = (1.2\%)^2$ and $\Lambda_+ = (5.4\%)^2$. The corresponding eigenvectors are

$$\vec{u}^{\,(1)} = \frac{1}{\sqrt{74}}\begin{pmatrix} -4 \\ -3 \\ 7 \end{pmatrix}, \qquad \vec{u}^{(2)} = \mathcal{N}_- \begin{pmatrix} -\Lambda_- + 46\eta \\ -\Lambda_- + 53\eta \\ -\Lambda_- + 49\eta \end{pmatrix},$$

$$\vec{u}^{(3)} = \mathcal{N}_+ \begin{pmatrix} -\Lambda_+ + 46\eta \\ -\Lambda_+ + 53\eta \\ -\Lambda_+ + 49\eta \end{pmatrix},$$

where

$$\eta \equiv \frac{1}{84}\frac{(\rho_{\rm u}^{(\rm r)})^2}{(\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2},$$

and $\mathcal{N}_\pm$ are the normalization constants. Hence we get

$$\begin{aligned} \chi^2 &= \sin^4 2\theta \left\{ \frac{49}{74}\frac{[D(L_{\rm f})]^2}{\sigma_{\rm u}^2} + \frac{[u_3^{(2)} D(L_{\rm f})]^2}{\sigma_{\rm u}^2 + \Lambda_-} + \frac{[u_3^{(3)} D(L_{\rm f})]^2}{\sigma_{\rm u}^2 + \Lambda_+} \right\} \\ &\simeq \sin^4 2\theta \frac{49}{74}\frac{[D(L_{\rm f})]^2}{\sigma_{\rm u}^2}. \end{aligned}$$

In the actual KASKA case [Fig. 1(c)], the covariance matrix is given by

$$\begin{aligned} V = {}& \sigma_{\rm u}^2 I_3 + [(\delta_{\rm c}^{(\rm d)})^2 + (\rho_{\rm c}^{(\rm r)})^2] H_3 \\ &+ (\rho_{\rm u}^{(\rm r)})^2 \begin{pmatrix} 0.221 & 0.042 & 0.149 \\ 0.042 & 0.298 & 0.138 \\ 0.149 & 0.138 & 0.145 \end{pmatrix}. \end{aligned} \quad \text{(C2)}$$

Diagonalization of (C2) can be done numerically with reference values of $\delta_{\rm c}^{(\rm d)}$, $\rho_{\rm c}^{(\rm r)}$, and $\rho_{\rm u}^{(\rm r)}$. Using the values in Eqs. (1) and (2), we obtain the diagonalized covariance matrix

$$\begin{aligned} &\text{diag}\,(\lambda_1, \lambda_2, \lambda_3) = \text{diag}(\sigma_{\rm u}^2 + (0.12\%)^2, \\ &\quad \sigma_{\rm u}^2 + (1.2\%)^2 - (0.62\%)^2, \\ &\quad \sigma_{\rm u}^2 + (5.4\%)^2 - (0.73\%)^2) \end{aligned}$$

and the corresponding three eigenvectors

$$\vec{u}^{(1)} \equiv \begin{pmatrix} u_1^{(1)} \\ u_2^{(1)} \\ u_3^{(1)} \end{pmatrix} = \begin{pmatrix} -0.4975 \\ -0.3114 \\ 0.8097 \end{pmatrix},$$

$$\vec{u}^{(2)} = \begin{pmatrix} 0.6503 \\ -0.7516 \\ 0.1105 \end{pmatrix}, \qquad \vec{u}^{(3)} = \begin{pmatrix} 0.5741 \\ 0.5815 \\ 0.5764 \end{pmatrix}.$$

χ^2 can be written as

$$\chi^2 = \frac{(\vec{y}\cdot\vec{u}^{(1)})^2}{\lambda_1} + \frac{(\vec{y}\cdot\vec{u}^{(2)})^2}{\lambda_2} + \frac{(\vec{y}\cdot\vec{u}^{(3)})^2}{\lambda_3} \simeq \frac{(\vec{y}\cdot\vec{u}^{(1)})^2}{\lambda_1},$$

where $\vec{y}$ in this case is given by

$$\vec{y} \equiv \begin{pmatrix} \frac{m_1}{t_1} - 1 \\ \frac{m_2}{t_2} - 1 \\ \frac{m_3}{t_3} - 1 \end{pmatrix} = -\sin^2 2\theta \begin{pmatrix} \sum_{a=1}^{7} \frac{t_{a1}}{t_1} D(L_{a1}) \\ \sum_{a=1}^{7} \frac{t_{a2}}{t_2} D(L_{a2}) \\ \sum_{a=1}^{7} \frac{t_{a3}}{t_3} D(L_{a3}) \end{pmatrix}$$

$$\equiv -\sin^2 2\theta \begin{pmatrix} \overline{D(L_1)} \\ \overline{D(L_2)} \\ \overline{D(L_3)} \end{pmatrix},$$

using the definition of $D(L)$ in Eq. (4) and $\overline{D(L_j)}$ in Eq. (28). Hence we get

$$\chi^2 \simeq \frac{\sin^4 2\theta}{\sigma_u^2 + (0.12\%)^2} [\overline{D(L_3)} u_3^{(1)} + \overline{D(L_1)} u_1^{(1)} + \overline{D(L_2)} u_2^{(1)}]^2,$$

from which Eq. (27) follows.

[1] Y. Kozlov, L. Mikaelyan, and V. Sinev, Yad. Fiz. **66**, 497 (2003) [Phys. At. Nucl. **66**, 469 (2003)]; V. Martemyanov, L. Mikaelyan, V. Sinev, V. Kopeikin, and Y. Kozlov, Phys. At. Nucl. **66**, 1934 (2003 [Yad. Fiz. **66**, 1982 (2003)].

[2] H. Minakata, H. Sugiyama, O. Yasuda, K. Inoue, and F. Suekane, Phys. Rev. D **68**, 033017 (2003).

[3] P. Huber, M. Lindner, T. Schwetz, and W. Winter, Nucl. Phys. **B665**, 487 (2003).

[4] M. Kuze, 6th International Workshop on Neutrino Factories and Superbeams (NuFact04), Osaka, Japan, 2004.

[5] F. Ardellier *et al.*, hep-ex/0405032.

[6] M. H. Shaevitz and J. M. Link, hep-ex/0306031.

[7] T. Bolton, 6th International Workshop on Neutrino Factories and Superbeams (NuFact04), Osaka, Japan, 2004.

[8] Y. Wang, 3rd Workshop on Future Low-Energy Neutrino Experiments, Niigata, Japan, 2004.

[9] K. Anderson *et al.*, hep-ex/0402041.

[10] D. Stump *et al.*, Phys. Rev. D **65**, 014012 (2002).

[11] M. Botje, J. Phys. G **28**, 779 (2002).

[12] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. Nadolsky, and W. K. Tung, J. High Energy Phys. 07 (2002) 012.

[13] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo, Phys. Rev. D **66**, 053010 (2002).

[14] Y. Declais *et al.*, Nucl. Phys. **B434**, 503 (1995).

[15] K. Eguchi *et al.* (KamLAND Collaboration), Phys. Rev. Lett. **90**, 021802 (2003).

[16] H. Sugiyama and O. Yasuda, hep-ph/0508090.

[17] Digital Map 25 000 (map image), No. 36, *Nagaoka*, Geographical Survey Institute, 2002.