Instability and Subsequent Evolution of Electroweak Bubbles

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Bubbles in a first-order electroweak phase transition are nucleated with radii R_0 and expand with velocity v. If v is subsonic, a bubble becomes unstable to nonspherical perturbations when its radius is roughly $10^4 R_0$. These perturbations accelerate the transition, and the effective velocity of bubble growth rapidly becomes supersonic. The transition should subsequently proceed spherically via detonation. If for some reason the onset of detonation is postponed, the surface area of the bubbles may be enhanced by 10^5. We discuss consequences for electroweak baryogenesis.

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There has been much interest in the dynamics of a possible first-order electroweak phase transition (EWPT) recently. The motivation is clear: One of the fundamental problems in particle physics and cosmology is the origin of the baryon asymmetry of the Universe (BAU). While C and CP violations [1], as well as baryon-number-violating instanton effects [2], have been known to exist in the standard model for quite some time, only recently has it been suggested that the rate for baryon-number-violating interactions may become appreciable at high temperatures [3]. If the EWPT is first order, the third of Sakharov's [4] criteria for baryogenesis, out-of-equilibrium processes, may also be found in electroweak (EW) physics. Thus, the BAU may be explained in the era of the Superconducting Super Collider.

In the standard picture of a first-order EWPT, spherical bubbles are nucleated with microphysical radii $R_0 \sim 10^{-17}$ cm and then expand spherically with velocity v to macroscopic radii $R_{\rm perc} \sim 10^{14} v R_0$ before they collide. The bubble-wall velocity v is still uncertain, but recent estimates suggest that a wall may propagate subsonically (i.e., as a deflagration front) [5]. In this Letter we show that shape instability of the bubble wall rapidly causes the propagation to become turbulent and proceed more quickly, and probably instigates the onset of detonation; then the bubbles expand spherically to fill space at supersonic velocities.

A deflagration front is unstable to perturbations with wavelengths in the range $\lambda_c \lesssim \lambda \lesssim \lambda_{\rm max}$ where $\lambda_c \sim R_0 v^{-2}$ is set by the surface tension in the wall [6–8], and $\lambda_{\rm max}$, which is proportional to the radius R of the bubble, is set by the underlying expansion of the bubble [7]. After the bubbles are nucleated, they expand spherically until $\lambda_{\rm max}$ reaches λ_c; then $R_{\rm inst} \sim 100 v^{-2} R_0$ and hydrodynamic instabilities set in. The subsequent bubble shape will be roughly spherical; however, instead of a smooth surface, the wall is highly wrinkled with distortions that enhance the surface area of the wall and thereby accelerate the transition. Although the details are far from understood (in any fluid dynamic system), the onset of turbulence and corresponding acceleration of the transition should result in a detonation front shortly after the bubbles are nucleated, when roughly a fraction 10^{-21} of the Universe has been converted to the low-temperature phase. On the other hand, if for some reason the transition continues as a deflagration, then the surface area of the walls is enhanced by 5 orders of magnitude by the time the bubbles percolate. In either case, the dynamics of the transition assumed in models where baryon number is produced in a first-order EWPT could be significantly altered.

We limit ourselves to the case where the transition occurs at temperatures near the critical temperature T_c, the latent heat is small compared to the thermal energy density, and $v \ll 1$; under these assumptions the calculations simplify considerably. Such conditions are possible in the standard model [5]. We feel that a more general analysis should result in similar conclusions. We neglect the expansion of the Universe, since the time scale for the EWPT is much smaller than the expansion time scale [5,9].

First we review some results from the theory of combustion of relativistic fluids [10] in the case where fluid velocities are nonrelativistic. Consider a planar interface in the y-z plane that propagates in the $-x$ direction. Then in the rest frame of the wall, matter in the symmetric phase enters the interface with a velocity v_1, and matter in the broken-symmetry phase leaves the interface with velocity v_2. Conservation of energy and momentum across the interface leads to the conditions

$$w_1 v_1 = w_2 v_2 \quad \text{and} \quad p_1 = p_2 , \tag{1}$$

where $w = e + p$ is the enthalpy density, e is the energy density, and p is the pressure. Throughout, the subscript 1 refers to the symmetric phase and the subscript 2 refers to the broken-symmetry phase.

The e_i and p_i may be obtained from finite-temperature field theory. Near T_c the effective potential may be written [5,9,11] as

$$V(\phi,T)=\tfrac{1}{2}\gamma(T^2-T_0^2)\phi^2-\tfrac{1}{3}\alpha T\phi^3+\tfrac{1}{4}\lambda\phi^4\,, \qquad (2)$$

where ϕ is the Higgs field, $T_0^2=(\gamma/\lambda)\phi_0$, $\phi_0=250$ GeV is the Higgs vacuum expectation value, and α, λ, and γ are parameters that depend on the W, Z, and top-quark masses, and on the Higgs structure of the theory. The critical temperature T_c is defined as the temperature at which there exists a second minimum of V degenerate with the minimum at the origin; for the effective potential above, $T_c^2=T_0^2/(1-\frac{2}{9}\alpha^2/\gamma\lambda)$ [9].

Although the difference in free energies $B(T)$ between the two phases is in general a complicated function of T, if the transition occurs near T_c then

$$B(T)\simeq(L/4)[1-(T^4/T_c^4)]$$

where

$$L=-T_c(\partial B/\partial T)|_{T_c}=(4\alpha^2\gamma/9\lambda^2)T_0^2T_c^2$$

is the latent heat of the transition [9]. This leads to the rather simple equation of state (which mimics the QCD bag model),

$$p_1(T)=[w_1(T)-L]/4,\quad e_1(T)=[3w_1(T)+L]/4\,, \qquad (3)$$

$$p_2(T)=w_2(T)/4,\quad e_2(T)=3w_2(T)/4\,, \qquad (4)$$

where $w_1(T)=a_1T^4$ and $w_2(T)=a_2T^4$, $a_1\simeq a_2\sim 100$, and $a_1-a_2=L(4T_c^4)^{-1}$ (see Ref. [9]).

We now study the hydrodynamic stability of a spherical bubble to small nonspherical perturbations. For distortions with $\lambda\ll R$, it is valid to treat the wall as a planar interface. The stability of a planar front for combustion of a relativistic gas was recently discussed by Link [8]. Consider a small perturbation to the planar discontinuity of the form $x_f=d\exp(iky+\omega t)$. This distortion in the wall surface will be accompanied by perturbations to the velocity and pressure. If $\mathbf{v}$ and p are unperturbed quantities, then the perturbations p' and $\mathbf{v}'$ must satisfy [8]

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right]p'+wc_s^2\nabla\cdot\mathbf{v}'=0\,,\quad \left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right]\mathbf{v}'+\frac{1}{w}\nabla p'=0\,, \qquad (5)$$

where $c_s=1/\sqrt{3}$ is the speed of sound. There are four boundary conditions on the discontinuity that the perturbed quantities must satisfy. The first,

$$p_1'=p_2'-\sigma\left(\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial t^2}\right)x_f\,, \qquad (6)$$

follows from Eq. (1) and includes the effects of surface tension and finite mass density of the wall; these two effects favor a flat surface. The boundary condition on $(wv_x)'$ (to lowest order in v_1 and v_2), from Eq. (1), is

$$w_1(v_{x1}'-\partial x_f/\partial t)=w_2(v_{x2}'-\partial x_f/\partial t)\,. \qquad (7)$$

Requiring that the tangential velocities on both sides be equal leads to

$$v_{y1}'+v_1\partial x_f/\partial y=v_{y2}'+v_2\partial x_f/\partial y\,. \qquad (8)$$

We make the ansatz that the enthalpy flux across the interface is proportional to the net blackbody energy flux across the interface. Then the perturbed enthalpy flux is [8]

$$(wv)'=\tfrac{3}{4}a(p_1'-p_2')\,, \qquad (9)$$

where a is a fudge factor, $0\le a\le 1$, and $a=1$ in case the enthalpy flux *is* the blackbody energy flux. (Our final results will not depend on a.) Equating Eqs. (7) and (9) gives us our fourth boundary condition.

If nontrivial solutions that satisfy Eq. (5) and the boundary conditions can be found for some $\omega>0$, then there are growing modes, and the wall is unstable to small perturbations. To satisfy the equations of motion and the boundary conditions, ω must satisfy [8]

$$\omega^2(v_1+v_2)+2\omega v_1v_2+\left[k^2(v_1-v_2)+\frac{\sigma k^3}{w_1v_1}\right]v_1v_2=0\,. \qquad (10)$$

If $v_2>v_1$ (i.e., if the phase transition proceeds via deflagration), then Eq. (10) has a positive root for wave numbers $k<k_c=(v_2-v_1)w_1v_1/\sigma$, and there are growing modes. For larger wave numbers the system is stabilized by surface tension.

From Eqs. (1), (3), and (4), we find $v_2-v_1\simeq(L/w_1)v_1$; thus the wall is unstable to small perturbations with

$$\lambda\gtrsim\lambda_c\equiv k_c^{-1}\simeq\sigma/Lv_1^2\,. \qquad (11)$$

In the limit of small supercooling the surface tension is given by $\sigma=(2^{3/2}\alpha^3/3^4\lambda^{5/2})T_c^3$ [9]. Determining v_1 is much more difficult and requires an investigation of the microscopic interactions of the particles in the thermal bath with the advancing wall. Recent estimates suggest that for the minimal standard model, the wall velocity may be in the range $0.01\lesssim v_1\lesssim 0.3$ [5]. Note that $\lambda_c\sim R_0v_1^{-2}\sim 10^{-15}$ cm (for $v_1\sim 0.1$).

As long as $\lambda\ll R$, the analysis assuming a planar interface should be valid; however, for $\lambda\sim R$, one should take the expansion of the bubble into account. As the bubble expands, the wavelength of the perturbation increases. If the amplitude of a perturbation grows more slowly than the wavelength, then the distortion is smoothed out in time. For our case of a weak transition, $\delta\equiv(v_1-v_2)/v_1\simeq L/w\sim 0.01\ll 1$, the growth rate for an instability with a large λ is found from Eq. (10) to be $\omega\simeq\delta v_1k/2$, while the growth rate for the bubble is roughly v_1/R. For a perturbation to be unstable it must have $k\gtrsim 2/R\delta$, while perturbations with $\lambda\gtrsim\lambda_{\max}\simeq R\delta/2$ will be stabilized. Although this derivation is heuristic, for $\delta\ll 1$ it reproduces the results of the exact analysis for the case of spherical combustion of a nonrelativistic gas, and it should be a good approximation in the case of a slowly moving relativistic gas as considered here. So, perturbations with $\lambda \lesssim \lambda_c$ will be stabilized by surface tension, and those with $\lambda \gtrsim \lambda_{\max}$ will be stabilized by the growth of the bubble.

In the standard picture of a first-order EWPT, bubbles are nucleated with radii $R_0 \sim \sigma/L$ and then grow spherically with velocities v until the bubbles percolate. The radius of the bubble at this time is (again, assuming typical parameters [9]) $R_{\rm perc} \sim 10^{-4} v (M_{\rm Pl}/T_c) T_c^{-1} \sim 10^{14} v R_0$. On the other hand, when $\lambda_{\max}$ reaches λ_c, $R_{\rm inst} \simeq \sigma w / L^2 v^2 \simeq 100 v^{-2} R_0$; at this point the bubble becomes unstable to nonspherical perturbations. Since $R_{\rm perc}$ is many orders of magnitude larger than $R_{\rm inst}$ the perturbations have plenty of time to mature and the standard picture of bubble evolution may be drastically altered.

If the bubble volume is V, the bubble will look somewhat spherical with a nominal radius R given by $V = \frac{4}{3}\pi R^3$; however, instead of a smooth surface, the wall is highly wrinkled on scales $\lambda_c \lesssim \lambda \lesssim \lambda_{\max}$, and the surface area of such a bubble is actually much larger than $4\pi R^2$. Perturbations to the fluid flow accompany those in the bubble surface, so that the normal flow velocity of fluid across the interface is v_1 at every point on the surface [cf. Eqs. (7) and (9)]; therefore, the rate of the transition is enhanced. A similar situation arises in supernova theory, where the burning of a carbon-oxygen white dwarf proceeds via deflagration and the rate at which burning occurs is proportional to the surface area of the wrinkled flame [12]. The surface area is enhanced roughly by a factor

$$\frac{\text{surface area}}{4\pi R^2} \simeq \left(\frac{\lambda_{\max}}{\lambda_c}\right)^{D-2}, \tag{12}$$

where the fractal dimension D [13] is some number between 2 and 3 but most likely near 2.6 [12]. The effective velocity $v_{\rm eff} \equiv dR/dt$ at which a sphere of comparable volume to the bubble would expand becomes

$$v_{\rm eff} \simeq (\lambda_{\max}/\lambda_c)^{D-2} v_2 . \tag{13}$$

Although the exact fractal dimension is uncertain, the qualitative form of Eq. (13) is correct; $v_{\rm eff}$ might differ from our estimate by an order of magnitude or so, but this has little effect on our conclusions. There is a possibility that once the perturbation goes nonlinear (i.e., its amplitude becomes comparable to its wavelength) it becomes stabilized and the resulting flow is not turbulent [7]. However, in this case, the wall would still be wrinkly on length scales from λ_c to $\lambda_{\max}$. The resulting surface-area enhancement and $v_{\rm eff}$ would still be comparable to those given in Eqs. (12) and (13). Shortly after instabilities set in, the transition accelerates and when $R \sim \lambda_c \delta^{-1} v_1^{1/(2-D)} \sim R_0 v^{-4} \delta^{-1} \ll R_{\rm perc}$, $v_{\rm eff}$ becomes supersonic. At this point only a fraction $(R/R_{\rm perc})^3 \sim 10^{-21}$ (for $v_1 \sim 0.1$) of the Universe has been converted to the new phase. Therefore, if baryogenesis occurs at the EWPT, it takes place after $v_{\rm eff}$ becomes supersonic.

The most likely scenario is that when $v_{\rm eff}$ increases past c_s, a detonation wave sets in. Simply stated, the reason is that the deflagration front is preceded by a fluid flow, and it is hard to see how the appropriate fluid flow can be maintained in front of a deflagration wave itself moving supersonically. In the frame of the deflagration front, the flow velocity of fluid into the interface is smaller than the flow velocity of fluid out of the interface (i.e., $v_2 > v_1$). In the "rest" frame of the Universe, the fluid is at rest far away from the transition; furthermore, by symmetry arguments, the fluid inside the bubble must be at rest. There is a piston effect as the wall pushes the fluid outside the bubble with a speed $v_2 - v_1 = \delta v_1$. A precompression shock precedes the deflagration front and accelerates the fluid which is initially at rest, radially outward to a velocity δv_1 [10]. If the wall is distorted and the transition is accelerated, the wall pushes the fluid outside the bubble with a velocity near $\delta v_{\rm eff}$. Since $\delta v_{\rm eff} \ll 1$, only a weak shock is needed. Weak shocks travel at velocities only slightly larger than c_s [14]; once $v_{\rm eff} \gtrsim c_s$, the deflagration front will merge with the shock to form a detonation wave. Although the exact mechanism for onset of detonation from deflagration is still under investigation in fluid systems and is not entirely understood even in the nonrelativistic case [7,15], the onset of detonation from a shock preceding an accelerating turbulent deflagration front is observed in laboratory experiments [7], and appears in the theory of type-Ia supernovae [15]. In order to satisfy the hydrodynamical equations of motion with the boundary conditions that the fluid far from the bubble as well as at the center of the bubble be at rest, the Chapman-Jouget condition must be satisfied, and the bubble expands at a velocity $v \simeq \sqrt{1/3} + \sqrt{2\delta/9}$ (for $\delta \ll 1$) slightly larger than c_s [14]. Since perturbations cannot propagate faster than c_s, they should be smoothed out and the bubble expands spherically to fill all space.

A detonation wave heats the gas as it passes, so one might worry that if the gas is heated to a temperature above T_c the phase transition cannot continue. A detonation will certainly propagate if the supercooling of the Universe is greater than the subsequent heating, as is found in some models (though not all). Further work should investigate the details of the phase transition at a detonation front in the case that the gas is heated above T_c.

On the other hand, if for some unforeseen reason, a terminal $v_{\rm eff}$ smaller than c_s is reached, then the subsequent evolution could continue as a deflagration with a distorted surface. This distortion can be quite dramatic: By the time the bubbles percolate, the surface area of the bubbles is enhanced by roughly $(R_{\rm perc}\delta/\lambda_c)^{D-2} \sim 10^5$ [cf. Eq. (12)]. If this is the case it might play a role in EW baryogenesis. However, the baryon number in recently proposed existing models where baryogenesis occurs at the phase boundary [16,17] should be unaltered. Although the transition in this case would be accelerated, the resultant baryon number is generally proportional to the amount of fluid that passes through the wall and this remains unaltered by turbulence. In some models such as that in Ref. [17], the rate of baryogenesis depends on transport of particles near the wall and the final baryon number depends on the wall velocity; in such models, the resulting baryon asymmetry depends not on $v_{\rm eff}$ but only on the *local* velocity of fluid through the wall, which remains unchanged (in the nonrelativistic limit) by turbulence. We can only speculate that relativistic corrections could actually alter the flow velocity across the wall. Another possible effect of the wall convolution is that in models where transport near the wall is crucial, particles could multiple scatter off one wall into another wall; however, this would require that λ_c be smaller than the particle mean free path.

Throughout we assumed that latent heat is transported from the surface hydrodynamically. If, on the other hand, radiative transport is important and bubble growth is limited by diffusion of latent heat from the wall, then the wall may become unstable on length scales larger than the mean free path of radiation as shown by Freese and Adams [18] for the case of a first-order QCD phase transition. If so, instabilities may set in even earlier than we found (as soon as the bubbles nucleate), and the surface-area enhancement could possibly be even larger than we estimated. The resulting bubble shape in this case may deviate drastically from spherical; the bubble looks like seaweed. Multiple scatter would also become more important in this case.

To summarize, the propagation of a deflagration front in a weakly first-order EWPT becomes turbulent, the transition is accelerated, and the effective propagation velocity of the walls rapidly becomes supersonic. Under these conditions the deflagration front could turn into a detonation shortly after the bubble is nucleated, and the macroscopic growth of the bubbles should occur via a detonation wave traveling near c_s. Our results suggest that, due to hydrodynamic effects, macroscopic bubble propagation may be significantly different from what one would expect from detailed studies of the microscopic kinetics [5]. This should come as no surprise; it has long been known that the propagation velocity of a spherical detonation wave is determined by hydrodynamics (the Chapman-Jouget condition [6,10,14]) and not by the microscopic kinetics of the reaction.

Strictly speaking, our analysis is valid only for nonrelativistic propagation velocities and for transitions with small latent heat, but a more general analysis under less restrictive assumptions should result in similar conclusions. For example, as long as the deflagration velocity v is subsonic, small perturbations could propagate ahead of the detonation front, and the hydrodynamic instability should exist; in addition, as v is increased (while still subsonic), λ_c is decreased [cf. Eq. (11)] so the instability should set in sooner. If the latent heat is increased, λ_c decreases and $\lambda_{\rm max}$ becomes larger, so turbulence should set in sooner. Also, for larger latent heats, the detonation front propagates at a larger velocity [14]. The transition from deflagration to detonation may also be important for the dynamics of the QCD phase transition if it is first order.

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