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#### HIGH ALTITUDE TESTS ON THE GEOGRAPHICAL, DIRECTIONAL, AND SPECTRAL DISTRIBUTION OF COSMIC RAYS

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#### ABSTRACT

Intensity of cosmic rays in the high Andes.—Depth-ionization readings taken in Lake Miguilla, Bolivia (alt. 4570 m) and on Lake Titicaca (3820 m) agreed closely with observations in the northern hemisphere previously reported.

Intensity of cosmic rays at sea-level.—Three electroscopes which differed considerably in form, wall material and volume, all agreed in giving at sea-level an ionization of 1.4 ions/cc/sec.

Effect of Milky Way on cosmic ray intensity.—Two sets of day and night observations, each of three days duration, in a deep valley of the Andes at 4700 m elevation failed to bring to light any difference between the radiation coming in from the plane of the Milky Way and the plane normal thereto.

Spectral distribution of the cosmic rays.—A new curve which includes the readings in South America and those published in 1926, analyzed by the method before used, yields absorption coefficients which vary from  $\mu = 0.25$  per meter of water to  $\mu = 0.15$ . The corresponding wave-lengths, using the Compton equation, are .000525A and .00032A.

Ionization at extreme altitudes.—Under the assumption that the largest absorption coefficient, viz.  $\mu = 0.25$ , is valid for the upper regions of the atmosphere, the ionization at any altitude may be calculated, with the aid of the Gold tables, taking the sea-level value as 1.4 I/cc/sec. The total ionization, computed from this data by a graphical integration, for the Millikan and Bowen sounding-balloon observations reported in 1926, shows remarkable agreement with their experimental result. This implies that the wave-lengths of the rays entering the atmosphere are not appreciably different from those at altitudes at which we have ourselves taken observations.

Effects of thunder-storms on cosmic rays.—Lake Miguilla, Bolivia, is completely screened from thunder-storm influences and yielded the same value of the cosmic rays as regions not so screened. Also sea-level observations taken in the midst of heavy thunder-storms showed no influence of these on cosmic ray readings.

### I. Introductory

THE experiments reported herewith were performed in the High Andes of Bolivia, and on the ocean voyage thereto in the summer of 1926. They had four principal objectives.

The first was to see whether in high altitude lakes in the southern hemisphere the altitude-ionization curve would coincide with that found in lakes

in the northern hemisphere. This curve was particularly sensitive in the very high altitude lakes obtainable in the High Andes and the spectral distribution found in 1925 could be more accurately tested. If the northern hemisphere and the southern hemisphere curves coincided it would go a long way toward eliminating the possibility that the rays are generated by the incidence of high-speed beta-rays on the very outer layers of our atmosphere—about the only hypothesis which could put the source of these rays in the last tenth of the air above the earth—the approximate height reached in Millikan and Bowen's high-altitude sounding-balloon work (see below). For such beta-rays would be expected to be influenced by the earth's magnetic field so as to generate stronger radiation over the poles than over the equator. In latitude 17, south, we should be completely screened from such pole effects, particularly if we could get into suitable high-altitude pockets in the mountains.

The second was to obtain further crucial tests of the C. T. R. Wilson hypothesis that these rays may be due to the integration of the effects of the impact in the earth's atmosphere of electrons endowed with many millions of volts of energy acquired in thunder-storms. Lakes in suitable pockets in the High Andes would be completely screened from such effects. Also a comparison of the rays found in thunder storm areas with those found in large regions, like California, which are comparatively free from thunder-storms, might furnish check observations upon this point.

The third was by determining through deep under-water observations the zero readings of new electroscopes, to obtain new checks on our value of the ionization due to the cosmic rays at sea-level, a quantity for which as yet there have been wide divergences between the results of different experimenters.<sup>1</sup>

The fourth was to get into suitable pockets or valleys in very high mountains where the rays are three or four times as intense as at sea-level, and there to make more reliable tests on directional effects in cosmic rays—in particular to see whether the Milky Way is more or less effective than other portions of the sky in sending these rays into the earth.

#### II. GEOGRAPHICAL DISTRIBUTION

The South American lake chosen as most suitable for the under-water readings was Lake Miguilla near Caracoles, Bolivia. This is a snow-fed lake 125 feet deep lying at an altitude of 15,000 feet (4570 meters), 3200 feet higher than Muir Lake, in which our highest under-water readings in the northern hemisphere had been taken. It was perhaps 700 feet in width and 2000 feet in length, and surrounded on all sides by mountains, the angular elevation of which as seen from our raft anchored in the middle was about 17 degrees. The cosmic rays coming in at an angle less than this to the surface are so fully absorbed by their long path through air that their total contribution to the cosmic rays coming in from the hemisphere

<sup>&</sup>lt;sup>1</sup> Swann, Phys. Rev. **29**, 372 (1927), finds the ionization due to them on the summit of Pike's Peak 0.75 per cc per sec. per atmosphere, while we found them in the same place to be close to 5 ions.

above is negligible. The lake, on the other hand, was completely screened by the mountains from the hypothetical rays generated in thunder-storms. The zero readings of the two electroscopes used, *i.e.*, the readings corresponding to depths of immersion of more than 20 meters, were 6.8 for electroscope number 1, and 2.4 for electroscope number 4. Of course the readings of significance for our purpose were those taken near the upper part of the depth-ionization curve. The differences between the foregoing zeros and the readings at depths corresponding to the upper part of the curve are given in Table I. Accidents to the electroscopes after these readings were taken prevented the lower part of the curve from being as accurately filled in, but this was of no importance since this portion could be studied much more accurately in lakes nearer home.

Table I

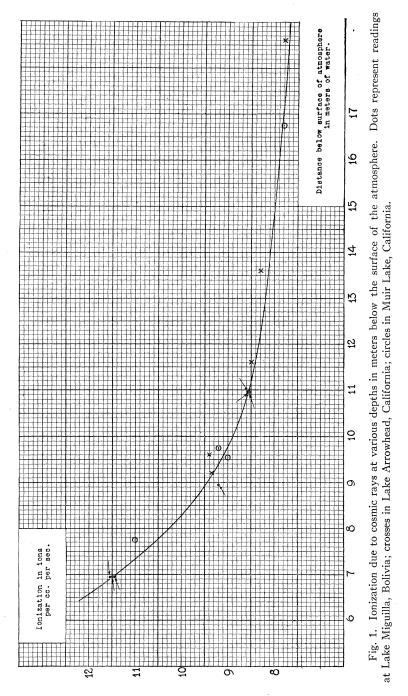
Ionization readings at various depths in Lake Miguilla. The values are expressed in ions per cc per sec.

Depth beneath surface	0	1/2  m.	1 m.	3 m.	5 m.
Electroscope No. 1 (1st run)	7.74	5.26	4.04	1.8	1.14
Electroscope No. 1 (2nd run)			4.1		1.15
Electroscope No. 4			4.14		1.14
Means	7.74	5.26	4.09	1.8	1.14

We have plotted these readings taken in South America on the same sheet with those taken with electroscope No. 1 in 1925 in Muir Lake and in Arrowhead Lake in California,<sup>2</sup> adding the new cosmic ray readings to the old zero, 7.4, instead of to the new zero, 6.8, and have then drawn a smooth curve to fit as nearly as possible the whole set of points taken from both hemispheres. Fig. 1 shows this curve. It furnishes the best of evidence that the cosmic rays are quite independent of geographical position, the points taken in the southern hemisphere fitting this one curve quite as well as do the points taken in the northern hemisphere. The two points that fall farthest from the curve correspond to single readings, and hence should be given little weight in comparison with points which represent the means of three or four readings.

The reading at the surface, viz. 7.74, not indicated in Fig. 1, falls about two ions above the curve, as it should do in conformity with the Muir Lake work, because of the presence at the surface of local radiations from the mountains and the atmosphere. In conformity, too, with the findings at Muir Lake the reading 5.26 at 1/2 meter is slightly above the curve, since this thickness of water is insufficient for the complete absorption of these softer rays from the mountains and the atmosphere. From the absorption coefficient of the hardest of the radioactive radiations it is easy to compute that no such rays can affect the reading at one meter depth. In this work, too, the three different observations taken at one meter are so consistent that this point may be taken as fixing the curve with considerable certainty at this elevation. The readings at five meters are seen to be equally con-

<sup>&</sup>lt;sup>2</sup> Millikan and Cameron, Phys. Rev. 28, 851 (1926).



cordant. The present curve is then the best we have yet presented for studying spectral distribution. (See section V.)

We also took a series of readings on the surface of Lake Titicaca (altitude 12,500 feet, 3820 meters) and found there the mean reading 12.3 ions in electroscope No. 1. It is interesting to compare this result with very careful and very complete sets of readings on shipboard taken all the way from Los Angeles to Mollendo, Peru. The mean of a very large number of consistent readings taken in this way with electroscope No. 1 was 8.29. The difference between these readings at sea and those taken in just the same way on Lake Titicaca is 12.32-8.29=4.03. The increase in cosmic rays shown by the curve between sea-level and 12,500 feet is 12.0-8.8=3.2. The excess of 0.8 ions per cc. per sec. is easily accounted for by the proximity of mountains bordering Titicaca, for these are very low-lying and should have added a little, but only a little, to the surface reading. So that the Titicaca readings speak, though in a less precisely quantitative way than the curve, for the uniformity of the cosmic rays the world over at a given height.

Further, if we take the difference between the zero reading of electroscope No. 1 and the reading on Lake Titicaca we obtain 12.32-6.8=5.52. Now, the cosmic ray ionization shown at 12,500 feet by our original cosmic ray curve for electroscope No. 1 was 12.3-7.4=4.9, and the cosmic ray ionization at 12,500 taken from the new cosmic ray curve is 12.0-7.4=4.6. The two curves are then at this point in fairly close agreement and together they yield an ionization at 12,500 0.77 ions lower than that observed on the surface of Lake Titicaca, in close agreement with the 0.8 ions excess obtained above, so that from every angle from which we can approach it from our data the surface reading at Lake Titicaca is about 0.8 ions above the value of the cosmic rays alone at that elevation. From the old curve, from the new curve, and from the Titicaca surface readings, then, the conclusion that the cosmic ray ionization at 12,500 is not far from 4.6 ions is unavoidable.\*

Sea-level observations taken in the midst of powerful thunder-storms showed no influence of these storms upon the electroscope readings.

#### III. SEA-LEVEL COSMIC RAY IONIZATION

The value of the sea-level cosmic ray ionization which we have heretofore published from our 1925 observations with electroscope No. 1 was 1.4 ions per cc per second. This value can be obtained directly by reading off the ordinate at 10.33 meters of the former cosmic ray curves for electroscope No. 1 and subtracting the zero value, viz., 7.4. The same procedure applied to the cosmic ray curve for electroscope No. 3 gives, however, 1.6 ions per cc per second. Whether the discrepancy was due to different effects of electroscope walls or to imperfections in the determination of electroscope capacities, we were in 1925 uncertain. Electroscope No. 3 suffered an accident which made a redetermination of its capacity impossible, but electroscope No. 1, used again in this work, and the new electroscope No. 4, identical in con-

<sup>\*</sup> Swann's conclusion that the ionization due to cosmic rays on Pike's Peak is less than 1 ion is quite irreconcilable with all three of these findings.

struction with No. 3, had their capacities measured carefully at the initiation of the present work. When placed side by side and subjected to the same change in radiation, they showed, within the limits of observational error, the same change in ionization, thus indicating that though No. 1 had a volume of 1890 cc and a wall thickness about 2 mm, while No. 4 had a volume of 3210 cc and a wall thickness of 3 mm, yet these wall and volume differences had no appreciable effect on the indicated ionization per cc. This fact is brought sharply to light in Table II, in which the maximum divergence from the mean value of I/cc/sec is under three percent, which is quite as close as our claims for precision can be in these particular readings.

TABLE II

Equality of the ionization in two electroscopes of different walls and volumes.

Place of observation	No. 1	No. 4	Mean
Side by side readings, Pasadena Institute campus Side by side readings in small boat Balboa Harbor	$12.36 \\ 8.79$	$8.54 \\ 4.78$	
Total differences in incoming radiation	3.57	3.76	3.66
Side by side readings, Caracoles mountain side Side by side readings, Titicaca Lake surface	$\begin{array}{c} 20.42 \\ 12.32 \end{array}$	16.03 7.86	
Total differences in incoming radiation	8.10	8.17	8.13
Side by side readings, Lake Miguilla, 1 m. immersion Side by side readings, Lake Miguilla, 5 m. immersion	10.9 7.94	$\substack{6.54\\3.50}$	
Total differences in incoming radiation	2.96	3.04	3.00

In the new composite curve, Fig. 1, involving readings in both hemispheres, the ionizations in Lake Miguilla given by both electroscopes No. 1 and No. 4, after the subtraction of their zero readings and the addition of the differences to the old zero 7.4, are included, so that the new curve has a greater generality, not only as to lack of dependence upon geographical position, but also as to freedom from instrumental peculiarities than any we have thus far presented.

It will be seen, however, that the sea-level value of the cosmic rays as read from the new curve is exactly 1.4, the same we have heretofore published for electroscope No. 1 alone. Further, we have very recently built a new steel electroscope of highly increased sensitivity and have found that although its shape, wall material and volume are different from the corresponding elements in No. 1 and No. 4, yet the sea-level ionization yielded by it in an exceedingly satisfactory series of depth-ionization readings (to be presented in a subsequent article) is again 1.4 with, however, an absolute uncertainty not yet accurately determined. We shall therefore fix upon 1.4 as the value of the cosmic ray ionization at sea-level within an ordinary electroscope. This value may need slight modification to conform to the peculiarities of individual electroscopes, but the foregoing agreements in the case of three different electroscopes indicate that, for rays as hard as the cosmic rays, such influences will be but slight. We shall present a later report upon the effect of walls.

# IV. Tests on the Influence of the Milky Way on Cosmic Ray Intensities

The position of the sun having been shown by all observers to have no influence whatever upon the intensity of the cosmic rays, the next important step of the same kind is obviously to make as careful tests as possible upon the effect of the Milky Way. Up to the time at which the following experiments were performed Kolhörster<sup>3</sup> had obtained indications at an altitude of 3550 meters of something like a 15 percent increase in the cosmic radiations coincident with the culmination of the Milky Way, although he could bring to light no such effect at sea-level.

Since unambiguous evidence upon this point would have the most important bearing upon the source of the cosmic rays, we sought a valley at the highest obtainable altitude where the surrounding mountains would cut off entirely any possible influences from the Milky Way during a period of at least four or five consecutive hours. We found such a situation at the mining camp of the Caracoles Tin Company, altitude 15,400 feet, an elevation at which according to the curve of Fig. 1 the ionization due to cosmic rays has the very considerable value of 5.5 ions. On account of the shielding effect of the mountains this value was reduced at the point of observation

TABLE III

Observations on directional effects on cosmic rays.

1st Series 3 days duration	2nd Series	3 days duration
4:22 P. M.— 6:17 P. M. 16.4 min. 6:47 P. M.—10:25 P. M. 16.3 max. 8:55 A. M.—11:14 A. M. 15.8 max. 11:51 A. M.— 3:15 P. M. 16.1 max. 3:49 P. M.— 8:28 P. M. 15.8 max. 8:44 P. M.—11:01 P. M. 15.95 max. 8:52 A. M.—11:05 A. M. 15.7 max. 11:19 A. M.— 1:46 P. M. 15.75 max. 2:10 P. M.— 4:39 P. M. 16.1 min. 5:19 P. M.— 6:56 P. M. 16.4 min.	9:19 P. M.— 2:17 2:40 A. M.— 7:30 7:55 A. M.— 1:05 1:47 P. M.— 6:44 6:58 P. M.—11:20 1:39 A. M.— 7:43 8:10 P. M.— 1:12 2:13 P. M.— 6:45 6:57 P. M.—10:16	P. M. 12.83 min. P. M. 12:92 max. P. M. 12.80 min. P. M. 12.97 max. A. M. 12.90 min. A. M. 13.02 max. A. M. 13.08 min.
Mean = 16.03 Mean max = 15.93; mean min. = 16.17	Mean = Mean max = 12.97; m	12.94 ean min. = 12.90

to 3.6 ions. The Milky Way was practically out of sight between the hours of 2:30 and 7.

In this position we took two sets of continuous readings, observing night and day for a period of three days in each set. The actual readings are given in Table III. Such accuracy as was obtained is revealed in the consistency of the readings themselves. It will be seen that in the first set the "mean max.", which signifies merely the mean of the observations taken when the Milky Way was in culmination, is a trifle less than the "mean min." (Milky Way out of sight). But in the second set these relations

<sup>&</sup>lt;sup>3</sup> Kolhörster, Sitz. Ber. d. Preuss. Akad. 34, 366 (1923).

become inverted. The observational error of the means can scarcely be more than about 0.1 ions, or three percent of the 3.6 ions under observation.

Although the precision and consistency of our electroscope readings still leave much to be desired, we think that a conservative conclusion from these readings is that, if there be any effect whatever of the Milky Way upon the cosmic radiations, the rays coming to us from its direction cannot be six percent greater nor less than are those coming from the portion of the heavens at right angles to the Milky Way. This is in agreement with our preceding less discriminating measurements, and also with recent very careful work at sea-level done subsequently to this by Hoffmann4 and Steinke,5 who can find there no directional effect in cosmic rays at all; but it is at variance with results reported by Büttner<sup>6</sup> and by Kolhörster.<sup>7</sup> The present work, however, was probably done under quite as favorable conditions as to altitude and constancy of temperature as have ever been used. It is very important to obtain unambiguous evidence upon this point. No entirely reliable conclusions about the origin of the rays can be drawn until it is settled. As yet the case for a favored region from which the rays come does not seem to have been established, but more sensitive tests can be made and will be made in the near future. For the purposes, however, of the computations in the next section the foregoing results wholly justify the assumption of a cosmic radiation coming in to the earth equally from all directions and producing an ionization in accordance with the Gold table,8 instead of with the usual linear absorption law used by all other observers. The application of the linear absorption law leads to widely different results at high altitudes. Indeed, with the same value of the absorption coefficient, viz. 0.25 per meter of water, and the same value of the sea-level ionization the Gold table gives an ionization of the "top" of the atmosphere of 75 ions per cc per second, while the linear law yields 19 ions per cc per second.

#### V. Spectral Distribution of Cosmic Rays

From our analysis, with the aid of the Gold table, of our depth-ionization curve from electroscope No. 3 we brought to light sharply in 1926 a definite hardening of the cosmic rays from  $\mu = 0.30$  per meter of water at the depth beneath the surface of the atmosphere corresponding to the altitude of Muir Lake, to  $\mu = 0.18$  at the bottom of the curve, that is, between say 13 meters and 23 meters beneath the surface of the atmosphere. We commented in our 1926 paper on the fact that electroscope No. 1 gave a slightly lower value of  $\mu$  for the top of the curve than did electroscope No. 3. The new work with two new electroscopes, viz. No. 4 and the new sensitive one of steel mentioned above, checks the former results with electroscope No. 1 and indicates that the former curve of No. 3 was the less reliable of the two. The new composite curve, Fig. 1, is considerably more reliable than are either

<sup>4</sup> Hoffmann, Ann. d. Physik 80, 779 (1926).

<sup>&</sup>lt;sup>5</sup> Steinke, Zeits. f. Physik. 42, 570 (1927).

<sup>&</sup>lt;sup>6</sup> Büttner, Zeits. f. Geophys. 2, 187 (1926); 2, 291 (1926).

<sup>&</sup>lt;sup>7</sup> Kolhörster, Naturwiss. 14, 936 (1926).

<sup>8</sup> Gold, Proc. Roy. Soc. A82, 152 (1909).

of the former curves, since it both extends to higher altitudes where the curve is very sensitive, and is a composite of readings with several electroscopes and in widely separated localities.

The upper portion of the new curve, i.e., the portion between the abscissas 6.4 and 9.3 or from about a half meter below the surface of Lake Miguilla to about the same depth beneath the surface of Arrowhead, yields, with the aid of the Gold table, used precisely as in our 1926 paper, the coefficient  $\mu = 0.25$ . Further, the curve yields the same value of  $\mu$  whether tested from 6.3 meters to 7.4 meters, or from 7.4 to 8.4, or from 8.4 to 9.4; but from 9.3 to 10.3 (sea-level) the value of  $\mu$  comes out 0.23. In other words, there is a slight hardening of the rays detectable in the region between Lake Miguilla and sea-level. From 10.3 m to 12.3 m there is still further hardening, the curve yielding the value  $\mu = 0.20$  with a discrimination of one point, or at most two points, in the last place. At the lower end of the new composite curve, however, for example between the depths 12 m and 18 m, the coefficient has dropped definitely to a mean value of 0.15, which is also the mean value yielded by the old curve of electroscope No. 1. The conclusion which we before reached of a spectral distribution of a little less than an octave is reaffirmed by this new work, though the range in  $\mu$  is now from 0.25 to 0.15 instead of from 0.30 to 0.18. The corresponding wave-lengths, if computed by A. H. Compton's formula, are 0.000525A and 0.00032A.

#### VI. THE HARDNESS OF THE COSMIC RAYS COMING INTO THE TOP OF THE ATMOSPHERE AND THE IONIZATION PRODUCED IN THE ATMOSPHERE BY THEM

Having now fixed upon the value of the ionization at sea-level due to cosmic rays at 1.4 I/cc/sec and the absorption coefficients up to a height of 15,000 feet (4570 meters), we can now, with the aid of the Gold table, compute the precise values of the ionization due to cosmic rays at all altitudes up to this limit with much greater confidence than heretofore.

Further, if the rays which enter the top of the atmosphere are of the same wave-length as those found at an altitude of 15,000 feet we can compute the ionization at all high altitudes. The observations made by Millikan and Bowen<sup>9</sup> with sounding balloons furnish excellent means of testing this hypothesis. Before making the tests it is interesting to point out how large a change of the ionization at high altitudes corresponds to a small change in the value of  $\mu$ . Thus using the value of  $\mu$  found in the literature in 1921, viz., 0.57, the ionization at the "top" of the atmosphere (treated as homogeneous) comes out 463 I/cc/sec if the rays are assumed to come in vertically and to produce 1.4 I/cc/sec at sea-level. Lowering the value of  $\mu$  from 0.57 to 0.25 reduces this "top" value of the ionization to 18.5, i.e., to about 1/25th of the first mentioned value. This merely illustrates how sensitive very high altitude flights are for the fixing of mean absorption coefficients. Again, if the rays come in from all directions instead of vertically, high altitude flights show the difference in ionization very easily, for with a value

<sup>&</sup>lt;sup>9</sup> Millikan and Bowen, Phys. Rev. 27, 353 (1926).

of  $\mu = 0.25$  the Gold table gives an ionization at the "top" of the atmosphere of 75 I/cc/sec in place of the 18.5 I/cc/sec computed for vertical rays.

For the sake, then, of gaining information as to the character of the cosmic rays when they enter the atmosphere, we have computed with the aid of the Gold table the number of ions to be expected at a series of altitudes up to the top on the hypothesis that from 15,000 feet up  $\mu = 0.25$ . The results from sea-level up, taking for the sea-level ionization 1.4 and for the region 9.3 m to 10.3 m  $\mu = 0.23$  and for all altitudes above 9.3 m  $\mu = 0.25$ , are contained in Table IV.

Table IV

Ionization of the atmosphere per cc per sec. by cosmic rays.

Altitude (kilometers)	Distance below top of atmosphere (meters of water)	I	Altitude (kilometers)	Distance below top of atmosphere (meters of water)	I
0 0.9 5 6 7 8 9	10.33 9.3 5.3 4.5 3.85 3.25 2.73 2.25	1.4 1.88 7.04 9.58 11.80 14.81 18.20 21.1	11 12 13 14 15 17.5	1.87 1.53 1.30 1.11 .93 .62 .42	25.8 30.3 33.8 36.6 40.4 48.0 54.1 75.0

In the record of Millikan and Bowen's high altitude flight they give the mean discharge rate, in the ascent from 5 km to 15 km and the descent again to 5 km, as 46.2 I/cc/sec. They also give the discharge rate of the same electroscope at the earth's surface as 15.4 ions. The local radiation plus the cosmic rays at the point in Pasadena where the last reading was taken was 5.4 ions, thus making the zero of this electroscope 10 ions, on the assumption that it would agree with the three tested above in the value of the ionization produced by a given external source. This gives the mean value of the ionization due to cosmic rays during the flight from the 5 km altitude up to the 15 km altitude and back to the 5 km altitude as 46.2-10= 36.2 I/cc/sec. This multiplied by the total number of seconds of the duration of the flight from 5 km up to 15 km and back to 5 km, namely 8100, gives 29,300,000 as the observed number of ions formed per cc. during this interval. The theoretical number obtained by dividing the time of flight into 15 minute intervals and multiplying the number of seconds in the interval by the value of the ionization at the mean height during the interval (Table IV) is 29,250,000.

The extreme closeness of the numbers is of course accidental, but the agreement shows that the curve of Fig. 1 extended up to the top of the atmosphere with the aid of the coefficient which holds at its upper end, namely,  $\mu = 0.25$  strikes exactly our observed point at 15 km altitude, which is 0.91 of the way to the "top." This fact lends a new significance to Table IV, which now represents, not hypothetical ionizations, but actual ionizations as determined by an experimental cosmic ray curve well filled in from 4570 m

down, and with one point inserted at the 15 km altitude. From these facts the inference is scarcely avoidable that the cosmic rays represent a spectral region or band which starts quite sharply at  $\mu=0.25$  on the long wave-length side and extends about an octave (possibly more) towards shorter wavelengths. The reduction of these absorption coefficients to wave-lengths is, of course, uncertain, Dirac's formula being probably the most correct, but for the sake of comparison with our former data we give the corresponding Compton wave-lengths viz; 0.00053 and 0.00032A. Dirac's formula gives thirty percent shorter wave-lengths.

Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena. November 7, 1927.