Measurement of the linewidth enhancement factor α of semiconductor lasers

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(Received 15 October 1982; accepted for publication 23 November 1982)

A theory of the amplitude and phase modulation characteristic of a single mode semiconductor laser is presented. In this model the amplitude modulation couples through the complex susceptibility of the gain medium to the phase. We show that this coupling constant can be obtained by a high-frequency modulation experiment. This measured coupling constant is used to infer the linewidth enhancement factor α as discussed by Henry, and Vahala and Yariv. Experiments confirmed the model and we measured a linewidth enhancement factor $|\alpha| = 4.6 \pm 1.0$ for a GaAlAs buried optical guide laser.

PACS numbers: 42.55.Px, 42.60.Fc, 42.60.He, 42.80.Sa

The spectral purity of semiconductor lasers is of utmost importance in applications that utilize the coherence properties of the laser, such as heterodyne detection of optical signals. The extremely high gain in injection lasers as well as strong amplitude-phase coupling causes their spectral properties to differ markedly from other laser systems. Recently, Henry[1] presented a model of the linewidth of single mode semiconductor lasers, which takes line broadening due to the coupling of phase and intensity into account. Independently, Vahala and Yariv[2] developed a semiconductor laser noise theory which treats the carrier density as a dynamic variable and which includes the carrier density dependence of the index of refraction. Both treatments predict a spectral linewidth Δf of the modified Schawlow–Townes form:

$$\Delta f = \frac{v_{gr}^2 h\nu g n_{sp} \alpha_{mir}}{8\pi P_0}(1+\alpha^2), \tag{1}$$

where v_{gr} is the group velocity of the light, $h\nu$ the lasing energy, g the gain, n_{sp} is the spontaneous emission factor,[3] α_{mir} the mirror loss, P_0 the output power per facet, and α the linewidth enhancement factor.[1,2] Until now no direct measurement of this important device parameter was available and its approximate value had to be estimated from frequency shift measurements,[4,5] gain measurements,[4] or gain calculations[6,7] and a subsequent Kramers–Kronig analysis.

The purpose of this letter is twofold: first, we derive the relationship between the small-signal modulation characteristic of a semiconductor laser and the linewidth enhancement factor α, and second, we apply this result to measure α through a high-frequency modulation experiment.

This small-signal analysis is based on the electromagnetic field equation of a laser as derived by Lamb[8] and is applied to the case of index guided single mode lasers with a spatially inhomogeneous carrier density. The polarization $\mathbf{P}(\mathbf{r},t)$ due to the carriers injected into the semiconductor material is given as a function of the electric field $\mathbf{E}(\mathbf{r},t)$ by

$$\mathbf{P}(\mathbf{r},t) = \epsilon_0 \chi(n(\mathbf{r},t),\mathbf{r})\mathbf{E}(\mathbf{r},t), \tag{2}$$

where $\chi = \chi_r + i\chi_i$ is the complex susceptibility which is an explicit function of the carrier density and of the position (in the case where the material is inhomogeneous).

The equation for the time evolution of the cavity mode $E_m(t)$ is obtained by substituting the polarization $\mathbf{P}$, as given by Eq. (2), into the electromagnetic field equation[8]

$$\frac{d^2}{dt^2}\left[\left(1+\frac{\chi_m(t)}{N^2}\right)E_m(t)\right] + \frac{1}{\tau_p}\frac{d}{dt}E_m(t) + \omega_m^2 E_m(t) = -\frac{1}{\epsilon_0 N^2}\frac{d^2 P_{sp,m}}{dt^2}, \tag{3a}$$

$$\mathbf{E}(\mathbf{r},t) \equiv E_m(t)\mathbf{e}_m(\mathbf{r}), \tag{3b}$$

$$\chi_m = \int_V \chi[n(\mathbf{r},t),\mathbf{r}]\,\mathbf{e}_m(\mathbf{r})\cdot\mathbf{e}_m(\mathbf{r})dV, \tag{3c}$$

$$\tau_p \equiv \frac{1}{v_{gr}[\alpha_{sc} + (1/L)\ln(1/R)]}, \tag{3d}$$

$$N^2 \equiv \epsilon/\epsilon_0, \tag{3e}$$

$$\omega_m \equiv 2\pi(c/2LN)m. \tag{3f}$$

The optical mode profile is given by $e_m^2(\mathbf{r})$ and χ_m is the effective susceptibility for the mode m. τ_p, N, and ω_m are the photon lifetime, index of refraction, and resonance frequency, respectively, of the cold cavity, that is, of the cavity when the effects of the lasing transition are removed. The spontaneous emission can be taken into account by adding a driving term $P_{sp,m}$.[2] The explicit form can be obtained from a Langevin noise source treatment which is rather tedious. Fortunately, there is a simpler way to obtain the effects of $P_{sp,m}$ since we are only interested in its average contribution. It is a basic result of quantum mechanics that the ratio of stimulated emission to average spontaneous emission into the lasing mode is equal to the number of photons in the mode, P_m. The average effect of $P_{sp,m}$ can therefore be accounted for in a small-signal analysis by changing the second term (the loss term) in the field equation (3a) from $(1/\tau_p)(dE/dt)$ to

$$\frac{1}{\tau_p}\left(1-\frac{n_{sp}}{P_m}\right)\frac{dE}{dt},$$

where n_{sp} is equal to the ratio of the total to the net stimulated emission rate.

For the small-signal analysis the mode $E_m(t)$ and the mode susceptibility $\chi_m(t)$ are assumed to be of the following form:

$$E_m(t) = E_0[1+\delta(t)]\cos[\omega_1 t + \varphi(t)], \tag{4a}$$

$$\chi_m(t) = \chi_{m,0} + \chi_{m,1}(t) = \chi_{m,0,r} + \chi_{m,1,r}(t) + i[\chi_{m,0,i} + \chi_{m,1,i}(t)]. \tag{4b}$$

The solution for the operating point shows that the mode gain $\chi_{m,0,i}$ is clamped and the lasing frequency ω_1 is shifted slightly from the cold cavity resonance ω_m.

$$\chi_{m,0,i} = \frac{N^2}{\omega_1 \tau_p}\left(1 - \frac{n_{sp}}{P_m}\right), \tag{5a}$$

$$\omega_1^2 = \frac{\omega_m^2}{(1 + \chi_{m,0,r}/N^2)}. \tag{5b}$$

The small-signal behavior is given by the solution of

$$\dot{\varphi}(t) + \frac{1}{2}\frac{\chi_{m,1,r}(t)}{N^2}\omega_1 = 0, \tag{6a}$$

$$\dot{\delta}(t) + \frac{1}{2\tau_p}\frac{n_{sp}}{P_m}\delta(t) - \frac{1}{2}\frac{\chi_{m,1,i}(t)}{N^2}\omega_1 = 0. \tag{6b}$$

In the experiments described below the semiconductor laser is biased above threshold and a small sinusoidally varying current at the frequency Ω is superimposed. Therefore, the steady state sinusoidally modulated solution of the field $E_m(t)$ is of interest. To obtain it we substitute

$$E_m(t) = E_0[1 + (m/2)\cos(\Omega t)]\cos[\omega_1 t + \beta\cos(\Omega t + \Phi)] \tag{7}$$

into Eqs. (6a) and (6b), where m and β are the intensity and phase modulation indices, respectively. For large modulation frequencies Ω $[\Omega \gg (n_{sp}/P_m)(1/\tau_p)]$ the solution is

$$\beta = -\tfrac{1}{2}\alpha_m m, \tag{8a}$$

$$\Phi = 0, \tag{8b}$$

$$\alpha_m(\Omega) \equiv \frac{\chi_{m,1,r}}{\chi_{m,1,i}} = \int_V \frac{\partial\chi_r(n)}{\partial n}\bigg|_{n=n_0(\mathbf{r})} N_1(\mathbf{r},\Omega)e_m^2(\mathbf{r})dV \Bigg/ \int_V \frac{\partial\chi_i(n)}{\partial n}\bigg|_{n=n_0(\mathbf{r})} N_1(\mathbf{r},\Omega)e_m^2(\mathbf{r})dV. \tag{8c}$$

As can be seen from Eq. (8a), the phase and intensity modulation indices are proportional to each other, independent of bias point and optical confinement factor. This result suggests a simple way to obtain the exact value of α_m through a measurement of the ratio of intensity and phase modulation depths, two relatively straightforward measurements, as will be shown below. The factor $\alpha_m(\Omega)$ is defined in Eq. (8c), where $n_0(\mathbf{r})$is the quiescent carrier density at location $\mathbf{r}$, $N_2(\mathbf{r},\Omega)$ the small-signal frequency dependent carrier density fluctuation at position $\mathbf{r}$, and $e_m^2(\mathbf{r})$ the normalized photon density. If the susceptibility $\chi(n,\mathbf{r})$ is linear in n and does not depend on $\mathbf{r}$, then it can be seen from Eq. (8c) that α_m is independent of Ω. Using the relation between the index of refraction N_{ind} and the susceptibility χ, $\epsilon_0 N_{ind}^2 \equiv 1 + \chi$, one obtains that in this case α_m is equal to the linewidth enhancement factor α as defined by Henry.[1]

$$\alpha_m = \frac{\partial\chi_r/\partial n}{\partial\chi_i/\partial n} = \frac{\Delta N_{ind,r}}{\Delta N_{ind,i}} = \alpha. \tag{8d}$$

Please note that in this linear case α_m is totally independent of the spatial distribution of the carrier density modulation (e.g., the incomplete carrier clamping in the wings of the optical mode, or diffusion effects do not enhance the phase modulation as suggested in Ref. 9). However, if the susceptibility $\chi(n,\mathbf{r})$ is not linear in the carrier density or depends on the location (e.g., the optical field penetrates into a material with a different band gap[4]), then the factor α_m is dependent on the modulation frequency and its value measured at high frequencies cannot be used to infer the linewidth broadening factor α.

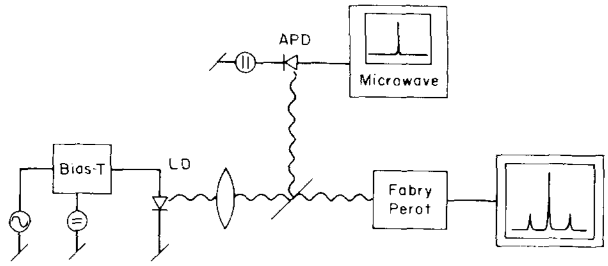

FIG. 1. Setup for measuring the intensity and phase modulation index. The laser diode (LD) is biased well above threshold and a small-signal current at frequency Ω is superimposed. The intensity modulation depth is measured with an avalanche photodiode (APD) and the spectral density with a scanning Fabry–Perot.

The experimental arrangement for measuring the intensity and phase modulation index is shown in Fig. 1. The semiconductor laser is biased above threshold and a small sinusoidally varying current at frequency Ω is superimposed. The intensity and spectral density of the radiation field are given by

$$\text{Intensity} \sim E_0^2[1 + m\cos(\Omega t)], \tag{9a}$$

$$\text{Spectrum: center line at } \omega_1{:}E_0^2[J_0^2(\beta) + m^2 J_1^2(\beta)] \tag{9b}$$

$$\text{First sidebands at } \omega_1 \pm \Omega{:}E_0^2\{J_1^2(\beta) + 1[(m/2)(J_2(\beta) - J_0(\beta))]^2\}, \tag{9c}$$

where $J_n(\beta)$ are nth order Bessel functions. Note that the calculated spectrum is symmetric.

The intensity modulation index m was measured with an avalanche photodiode (S171P Telefunken) calibrated in the measurement setup from dc to 3.5 GHz to an accuracy of $\pm$ 1 dB. The optical spectrum was measured with a confocal scanning Fabry–Perot (Tropel 240). Care was taken to avoid any back reflection into the laser. In a typical measurement, as shown in Fig. 2, the modulation current at frequency Ω was adjusted to produce a desired intensity modulation depth m, which was measured with the photodiode. The phase modulation index β can be found by measuring the relative sideband strength and using Eqs. (9b) and (9c). The factor α_m is then obtained as $\alpha_m = -2(\beta/m)$. Since only the absolute value of β can be measured, the sign of α_m must be obtained by other means. A measurement of the α_m of a buried optical guide laser (BOG Hitachi 3400, $\lambda = 816$ nm) at $\Omega = 2$ GHz, a bias level of 1.3 times threshold, and an intensity modulation depth of 10%, gave $|\alpha_m| = 4.5$. Repetition of this measurement for about 50 different conditions (Ω varied between 1 and 3.5 GHz, bias level varied between 1.3 and 1.9 times threshold, and modulation depth varied between 10% and 30%) gave $|\alpha_m| = 4.6 \pm 0.5$. This value has an additional uncertainty of $\pm$ 10% due to the inaccuracy of the photodiode calibration. A simple structure such as the BOG laser is expected to have the required simple form of the susceptibility so as to render α_m as given by Eq. (8c)

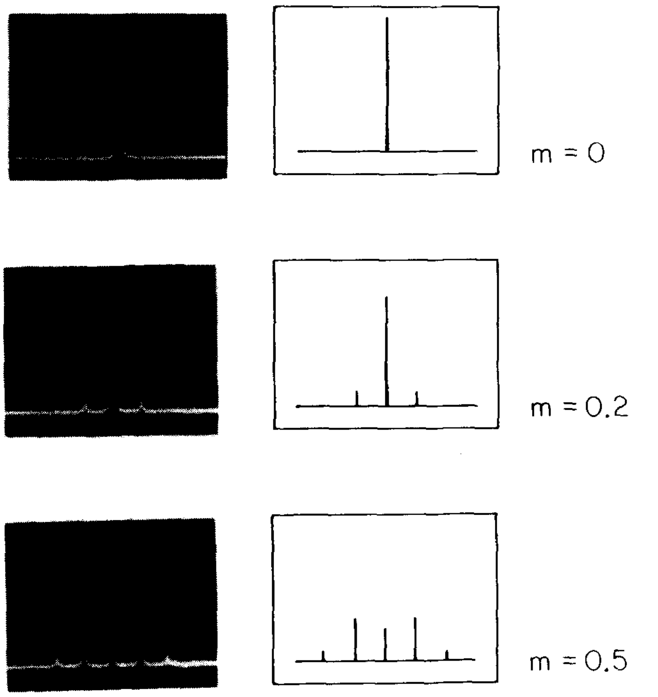

FIG. 2. Measured and calculated optical spectra of a laser diode with an intensity modulation $m = 0$, 20% and 50%. The modulation frequency is $\Omega = 1$ GHz$\times 2\pi$.

frequency independent. Indeed, our measurements of α_m could not detect any dependence on Ω in the range 1–3.5 GHz. Therefore, we conclude that for this BOG laser $|\alpha| = 4.6 \pm 1.0$.

Attempts to measure α for a channeled substrate planar laser (CSP) failed because the measured α_m was strongly dependent on the modulation frequency Ω. This is expected since the optical field penetrates into the GaAs substrate (which has a smaller band gap than the active region) in order to stabilize the transverse mode. As discussed above the susceptibility in such a case is no longer of the required simple form to render Eq. (8c) frequency independent.

In conclusion, we have developed a model for calculating the amplitude and phase modulation of the radiation field of the family of index guided, single mode semiconductor lasers with an inhomogeneous carrier density. In the model amplitude and phase are coupled through the complex susceptibility of the gain medium. We have shown that this coupling constant can be obtained by measuring the ratio of amplitude to phase modulation at high frequencies. Since the same coupling mechanism causes the spectral linewidth broadening, this method enables us to measure directly the linewidth enhancement factor α. Our measurements confirmed the model and we measured a linewidth enhancement factor $|\alpha| = 4.6 \pm 1.0$ for a buried optical guide laser.

The authors thank Dr. M. Nakamura of Hitachi Central Research Laboratories for providing the BOG and CSP lasers used in this experiment. This research was supported by the Office of Naval Research, the National Science Foundation under the Optical Communication Program and by the Army Research Office.

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