The Utilization of Specially Tailored Air Bubbles as Static Pressure Sensors in a Jet

K. K. Ooi
Able Corporation,
Anaheim, Calif.

A. J. Acosta
California Institute of Technology,
Pasadena, Calif.

It is shown that air bubbles of a certain size may be used to measure the fluctuating pressure in a liquid jet. The conditions under which these bubbles accurately reflect the local static pressures are described in detail; the volume shape of the bubbles was determined by holography for a 3.17 mm jet and the change in volume is interpreted as a result of the fluctuating pressure. The experimental results revealed that at any one instant, a wide spectrum of static pressure fluctuation intensities exist in the jet. It was also found that the probability distribution of these intensities has a slightly skewed bell shape distribution and that the fluctuating static pressure peaked at a higher positive value than a negative one.

Introduction

A complete knowledge of the local static pressure is required for a clear understanding of the many phenomenon associated with fluid flows, for example, aerodynamic noise and cavitation. While such pressure measurements on the surface of a body are relatively easy to achieve, the same cannot be said for similar measurements in a free shear layer, such as a jet. This is because one would require a probe whose reading is independent of yaw in any direction and of the dynamic pressure. It is also necessary to use a probe that will not create a substantial flow disturbance when it is in the flow. Although fast response condenser microphones and piezoelectric pressure transducers have been widely used in the past to measure the fluctuating pressure spectrum, as reported in [1], [8], and [10], the measurements obtained were not accurate reflections of the local static pressures. The inaccuracies were due in part to the induced flow disturbance by the probe and partly as a result of spatial averaging over the probes relatively large sensing area. In view of these considerations, it is fairly obvious that the measurement of the local static pressure in a jet is not trivial and continues to present a problem to investigators.

The primary purpose of this paper is to introduce a new concept in the measurement of the instantaneous local static pressure. This concept, as will be demonstrated, will lead to a measurement technique that does not have the many setbacks of the conventional methods described earlier. Some experimental results are also presented as evidences that this new technique is feasible. The current method relied on specially tailored air bubbles as instantaneous pressure indicators. The conditions under which these bubbles could be employed is considered next.

Contributed by the Fluids Engineering Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS and presented at the Fluids Engineering Conference, Houston, Texas, June 20–22, 1983. Manuscript received by the Fluids Engineering Division, August 24, 1983. Paper No. 83-FE-34.

Theoretical Analysis

The motion of a spherical bubble in a fluid is governed by the familiar Rayleigh-Plesset equation viz:

$$\rho(R\ddot{R}+3\dot{R}^2/2)=P_g(T)+P_v(T)-P_\infty(t)-\frac{4\nu\rho\dot{R}}{R}-\frac{2\sigma}{R} \qquad (1)$$

where the symbols are as defined in the nomenclature. In theory, if a complete history of the motion of the bubble is known, then the local static pressure at any instant in time as seen by the bubble can be calculated from the equation above. This is undeniably an involved procedure which will require a continuous monitoring of the bubbles trajectory. It is indeed a difficult task since the bubble is often very small in size, is moving at a high speed and its path is not usually known a priori. Fortunately, this whole analysis can be greatly simplified, as is shown below.

Order of Magnitude Analysis. If the dominant pressure fluctuation in a free shear flow is at a circular frequency of ω, then it is to be expected that the radius of a bubble in the flow field will change at the same frequency. Consequently, the instantaneous bubble radius, $R(t)$, is described by the equation:

$$R(t)=R_0+\epsilon e^{j\omega t} \qquad (2)$$

where R_0 is the reference bubble radius and ϵ is the amplitude of the oscillation about the mean radius. In general, the amplitude of this oscillation is not small and could be of the same order as R_0. By referring all linear dimensions to R_0 and the circular frequency of fluctuation to the resonant frequency of the bubble, ω_0, it follows that:

$$\dot{R}^2 \sim \left(\frac{\epsilon}{R_0}\right)^2\left(\frac{\omega}{\omega_0}\right)^2$$

$$R\ddot{R} \sim \left(\frac{\epsilon}{R_0}\right)\left(\frac{R}{R_0}\right)\left(\frac{\omega}{\omega_0}\right)^2$$

$$\text{pressure term} \sim \frac{1}{\rho u_0^2}\ (\text{where } u_0=R_0\omega_0) \qquad (3)$$

$$\text{viscous term} \sim \left(\frac{\epsilon}{R_0}\right)\left(\frac{\omega}{\omega_0}\right)\Big/\left(\frac{R}{R_0}\right)$$

$$\text{surface tension term} \sim \frac{1}{\text{We}}$$

Furthermore the following ratios are defined as:

$$\frac{R}{R_0} = 0(1)$$

$$\frac{\epsilon}{R_0} = 0(1) \tag{4}$$

and

$$\frac{\omega}{\omega_0} = 0(\delta) \text{ where } \delta << 1$$

From equations (3) and (4) it can be shown that the left hand terms of equation (1) are of the order δ^2 and the viscous term is of the order δ. Therefore these terms can be dropped from equation (1). As a consequence, the equation simplifies to:

$$P_\infty(t) = P_g(T) + P_v(T) - \frac{2\sigma}{R} \tag{5}$$

Since attention will be focused on the local static pressure, it will be more apropriate to rewrite equation (5) with a subscript x yielding:

$$P_{\infty_x}(t) = P_{g_x}(T) + P_{v_x}(T) - \frac{2\sigma}{R_x} \tag{6}$$

The terms with the subscript refer to the corresponding local values. In addition, the partial pressure of the gas is given by:

$$P_{g_x}(T) = P_{g_0}(T)\left(\frac{R_0}{R_x}\right)^{3\gamma} \tag{7}$$

By substituting equation (7) into (6), the following is obtained:

$$P_{\infty_x}(t) = P_{g_0}(T)\left(\frac{R_0}{R_x}\right)^{3\gamma} + P_{v_x}(T) - \frac{2\sigma}{R_x} \tag{8}$$

It is now necessary to devise an experimental technique whereby equation (8) above can be used to compute the local static pressure as sensed by the bubble.

Practical Considerations

It is apparent from equation (8) that the radii of the bubble at position x and at some reference position should be measured before the local pressure could be calculated. An immediate tendency would be to inject a single bubble into the jet, then photographing it at the location of injection and again at position x, a short time later. Although this is a realizable method, it will involve a great deal of picture taking before the entire pressure field in the jet is mapped out. Instead of doing this, a continuous stream of bubbles could be introduced into the jet at a reference point. The instantaneous bubble sizes at the various positions in the jet could then be revealed in a single photograph. Furthermore if it is ensured that the size of the bubble at the point of injection is always the same, then any observed change in the size of the bubble from this reference position could be regarded as a direct result of the pressure fluctuations in the flow. Here again, it should be stressed that the above interpretations are only true provided the conditions stipulated by equation (4) are satisfied. It will be demonstrated that by careful design, the selected bubbles can be made to meet the stated requirements.

Experimental Verification

An axisymmetric jet exiting from a 3.17mm nozzle was chosen as the subject for investigation. Since the jet was to discharge into a stationary medium at a static pressure of only 1.1 atmospheres, it was necessary to restrict the exit velocity of the jet to 12.2 m/s, to avoid cavitation. It has been reported in [1], [8], and [10] that the normalized power spectrum of pressure fluctuations in a jet peaked at a Strouhal number ($=fD/V_j$) of 0.45 and on either side of this value, the spectrum dropped rapidly to insignificant values. Therefore under the present flow conditions, the dominant frequency of the pressure fluctuations would be approximately 1.7 kHz, or a time scale of the order of a millisecond. It follows then the bubble to be used in this jet has to have a response time of at least a tenth of a millisecond if it is to be a good instantaneous pressure indicator. Theoretically, as can be seen from equation (9) below, an infinitely small bubble would easily meet this condition. However the need to be able to measure any change in the size of the bubble easily and accurately require a bubble of finite size. Another requirement is that the bubble has to be sufficiently small to sense the local "point" pressures. It is known from bubble dynamics that the resonant circular frequency of a spherical bubble in an incompressible medium is governed in general by:

$$\omega_0^2 = \frac{3}{\rho R_0^2}\left[\gamma(P_0 - P_v) + 2\frac{(3\gamma - 1)}{3R_0}\sigma\right] \tag{9}$$

and

$$\omega_0^2 = \frac{3}{\rho R_0^2}\left[(P_0 - P_v) + \frac{4\sigma}{3R_0}\right] \text{ for } \gamma = 1$$

It can easily be shown from equation (9) that the response

Nomenclature

D = diameter of the jet at exit
f = frequency of the pressure fluctuation
f_0 = resonant frequency of the bubble
m = number of data points
$P_{g_0}(T)$ = partial pressure of the gas in the bubble at the reference location.
$P_g(T)$ = partial pressure of the gas in the bubble
$P'(t)$ = fluctuating pressure at any instant ($=P_\infty(t)$-P_0)
$P_\infty(t)$ = instantaneous static pressure
P_0 = mean ambient static pressure
$R(t)$ = instantaneous bubble radius
$\dot{R}(t)$ = instantaneous radial velocity of the bubble
$\ddot{R}(t)$ = instantaneous radial acceleration
R_m' = geometric mean radius of the injected bubble
R_0 = reference bubble radius
T = temperature
t = time
u_0 = peripheral velocity of the bubble
V_j = velocity of the jet at exit
We = Weber number
β = damping constant
γ = polytropic index
δ = a quantity of the order much less than unity
ϵ = amplitude of oscillation about the mean bubble radius
ν = kinematic viscosity of water
ρ = density of water
σ = surface tension
ω = circular frequency of the pressure fluctuation
ω_0 = resonant circular frequency of the bubble

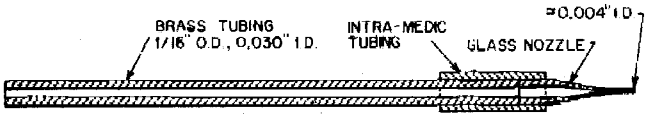

Fig. 1 Diagram showing the geometrical details of the bubble injector

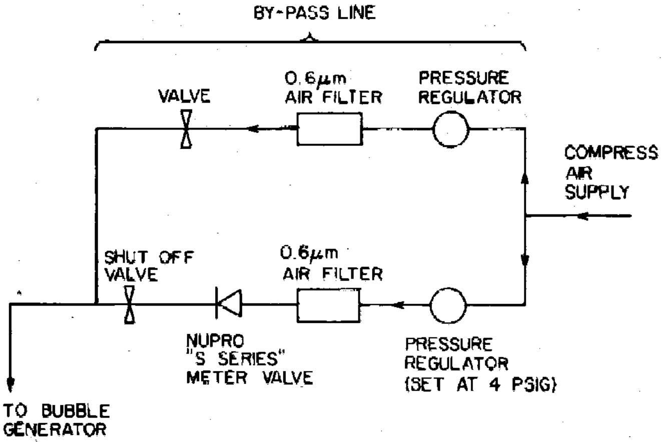

Fig. 2 Diagram of the air bubble injection system

time of a bubble in the size range of 50μm to 100μm is at least an order and a half faster than the time scale of the pressure fluctuation. Any change in volume in a bubble this size due to the fluctuation of pressure could also be easily detectable. Consequently it became the choice for the present work.

Description of Test Apparatus. The required bubble size was generated by a bubble injector which was made up of a glass nozzle drawn from a 1.6 mm outer diameter tubing. The inner diameter of the nozzle at exit was approximately 100μm. This nozzle was attached to a short length of brass tubing as shown in Fig. 1. Bubbles were generated by passing dry compressed air, filtered and regulated to 1.27 atmospheres, through the glass nozzle. The whole injection system is depicted diagramatically in Fig. 2. A meter valve located downstream of the air filter allowed the very small air flow rate (approximately 3×10^{-8} m^3/s) to be finely adjusted.

The primary component of the flow apparatus was the flow assembly which comprised of several sections, as shown in Fig. 3. The jet was created by discharging a column of water from a pressurized vessel through the 3.17 mm nozzle into a stationary water tunnel, the jet being submerged. Flow visualization was accomplished with a holocamera, a description of which is found in [9] and [11]. A general layout of the test apparatus is shown in Fig. 4

Test Procedures. The bubble injector was introduced into the nozzle assembly through a hole in the vaned elbow. The tip of injector was positioned on the jet centerline and at four diameters upstream of the nozzle lip. With the water jet running, the air flow through the injection system was carefully adjusted until a steady stream of individual bubbles was seen exiting from the nozzle lip under stroboscopic lighting. A hologram of the flow field was then taken. A whole series of holograms was obtained this way. In all the tests, the water in the jet and that of the surroundings was kept at a dissolved air content of 11 p.p.m. (approximately 75 percent saturated with air).

Data Reduction. The bubble size as a function of jet position was obtained from the reconstructed images of the holograms. Prior to calculating the static pressure in the jet from equation (8), it was necessary to establish that the rate at which the injected bubbles was dissolving into the jet water was small and that these bubbles acquired steady state soon

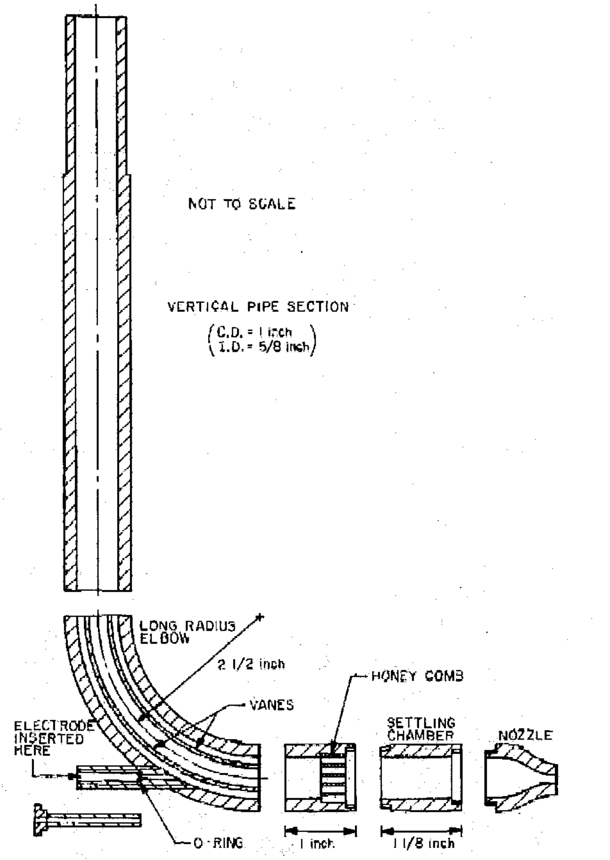

Fig. 3 Diagram showing the geometrical details of the nozzle assembly

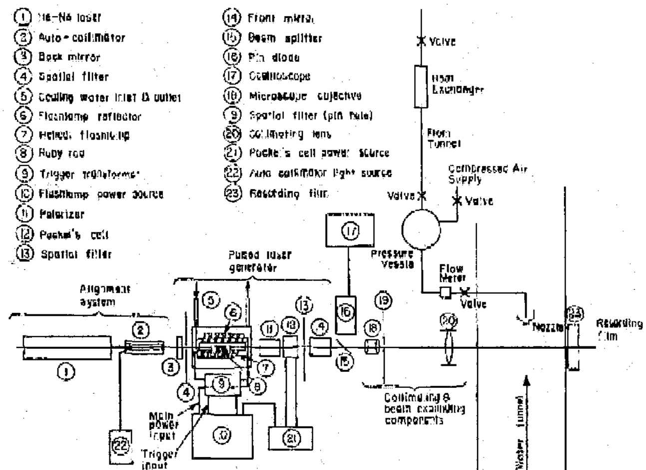

Fig. 4 Diagram showing the layout of the experimental apparatus

after exiting from the flow nozzle. These conditions have to be satisfied if the recorded sizes of the bubble downstream of the jet are to be indicative of the local instantaneous static pressures. Both these effects were found to be negligible under the present test conditions as is demonstrated below.

As the bubbles were accelerated through the flow nozzle, they were set into oscillations. The amplitude of the oscillation with time was reported in [7] as:

$$\epsilon(t) = \epsilon_0 e^{-\beta f_0 t} \cos(2\pi f_0) t \qquad (10)$$

where $f_0 = \omega_0 / 2\pi$. The damping constant for a 65μm bubble (the size of the bubble that was used in the present work) was estimated to be 0.11 by [4] and [5]. By defining steady state as the instant the amplitude of the oscillation was 5 percent or

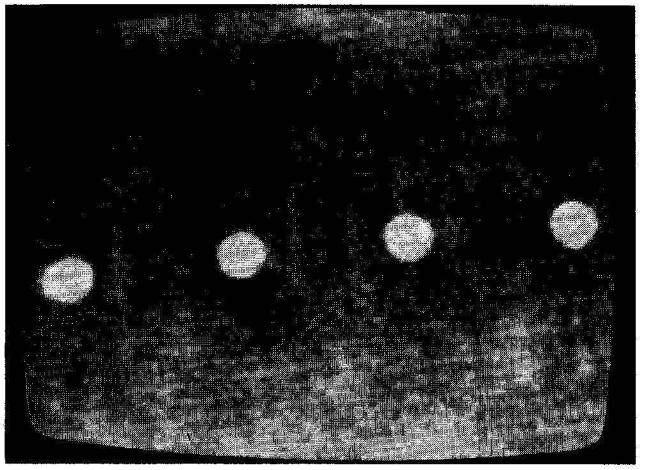

Fig. 5 Photograph of the reconstructed image of a string of injected bubbles in the potential core of the jet, near the nozzle exit. The bubbles are 128μm in diameter.

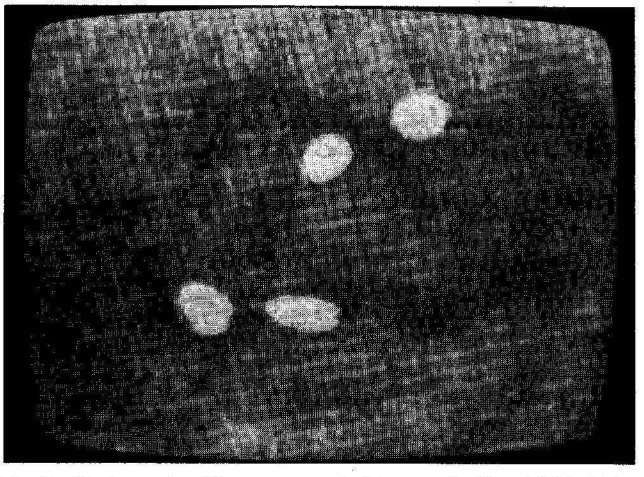

Fig. 6 Photograph of the reconstructed image of ellipsoidal bubbles in the transition region of the jet.

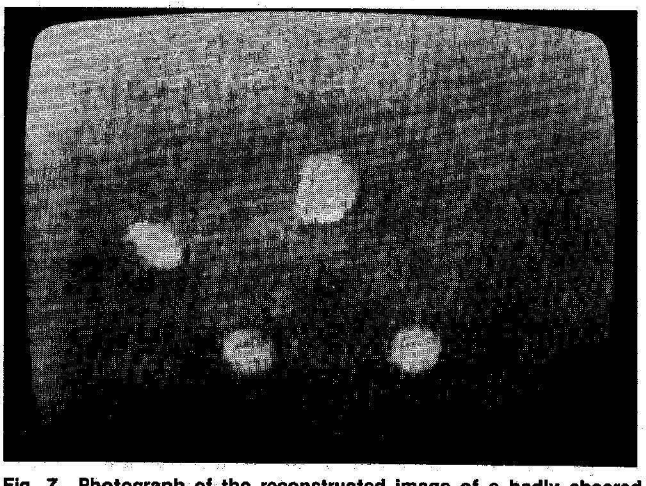

Fig. 7 Photograph of the reconstructed image of a badly sheared bubble in the transition region of the jet

less of the initial amplitude, it followed from equation (10) that the elapsed time was given by:

$$e^{-\beta f_0 t} = 0.05 \tag{11}$$

or $t = 6.2 \times 10^{-4}$s (for a 65μm bubble). This implied that at a jet velocity of 12.2 m/s, the injected bubble acquired steady state at 7.5 mm beyond the nozzle lip. In fact, flow visualization has revealed that equilibrium was reached as near as 5 mm downstream of the nozzle exit. A photographic evidence of this is shown in Fig. 5. As to the rate at which the injected bubbles were going into solution, this problem has been studied in some detail and reported in [6]. Based on this work, it has been estimated that it will take approximately 400 seconds for the present bubbles to completely dissolve in water that is 75 percent saturated. Since the traverse time of the bubbles in the current investigation was of the order of milliseconds, it was inferred that the dissolved volumes of the bubbles were negligibly small. Hence it was concluded that the recorded bubble sizes on the holograms were true reflections of the local instantaneous static pressures.

Equation (8) was further simplified by dropping the vapor pressure term. The justification for doing this lies in the fact that dry air was used in the injector and no significant evaporation was expected to occur in the extremely short traverse time through the region under scrutiny. It could also be readily shown that the initial gas pressure, P_{g_0}, could be obtained from the reduced Rayleigh-Plesset equation, that is

$$P_{g_0} = P_0 + \frac{2\sigma}{R_0} \tag{12}$$

On substitution of P_{g_0}, equation (8) becomes

$$P_{\infty_x}(t) = \left(P_0 + \frac{2\sigma}{R_0}\right)\left(\frac{R_0}{R_x}\right)^{3\gamma} - \frac{2\sigma}{R_x} \tag{13}$$

Observations of the reconstructed images of the holograms showed that in the transition region of the jet, some of the injected bubbles did not retain their spherical form but were ellipsoidal in shape. An example of this situation is shown in Fig. 6. The immediate concern then was whether these ellipsoidal bubbles could still be regarded as good pressure sensors. In reference [13], it was reported that the natural frequency of an ellipsoidal bubble could be very accurately approximated by the response time of a spherical bubble with the same volume. The report also pointed out that the frequency of an ellipsoidal bubble was only slightly dependent on the ratio of the major to the minor axis of the bubble if that ratio was less than four. In the present measurements, the largest change in volume detected was about 3.5 times the original. This translated into an increase of about 50 percent in the radius of the bubble. It follows from equation (9) that a bubble of this size still has a relatively short response time. Hence, all the bubbles in the flow field remained as good indicators of the instantaneous static pressure.

There was however a problem associated with these ellipsoidal bubbles. This problem was the correct determination of the respective volumes. It arose because out of the three minor and major axes of the ellipsoid required for the computation of the volume, only two could be measured from the reconstructed image. It was hoped that the third dimension could somehow be theoretically determined but it has been pointed out in [3] that such a problem is far too complex to solve. Therefore to circumvent this problem, it was assumed that the third axes was given by the geometric mean of the other two measured radii. This meant that the volume of the ellipsoidal bubble was proportional to the cube of the geometric mean radius. Hence an equivalent expression of equation (13) for the ellipsoidal bubbles is:

$$P_{\infty_x}(t) = \left(P_0 + \frac{2\sigma}{R_m}\right)\left(\frac{R_0}{R_m}\right)^{3\gamma} - \frac{2\sigma}{R_m} \tag{14}$$

It is worth pointing out that at present the radius of the bubble could be measured to within 2μm.

One interesting observation is that in all the holograms that were analyzied (17 in all), no fragmentation of the bubbles was ever detected. However, in a number of instances, a few badly sheared bubbles were seen. Such a case is shown in Fig. 7. The volumes of these bubbles were difficult to determine accurately and consequently these bubbles were omitted in the

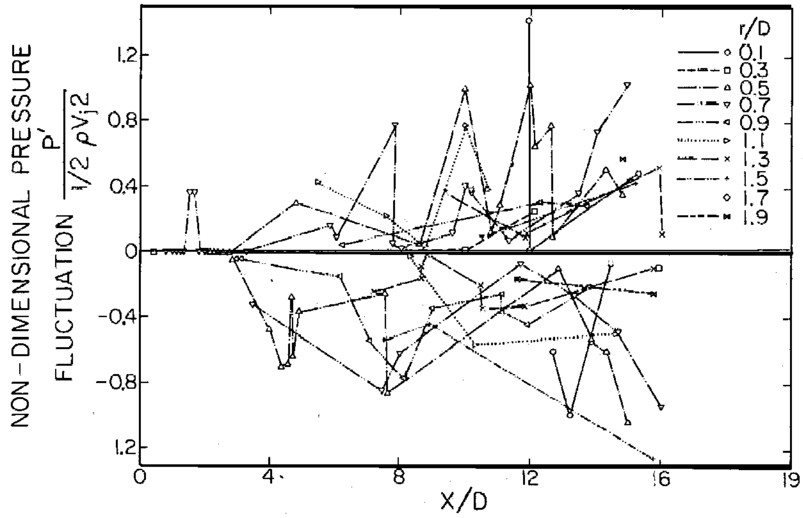

Fig. 8 Graphs of the instantaneous local pressure fluctuations as a function of the dimensionless axial position in the 3.17 mm jet

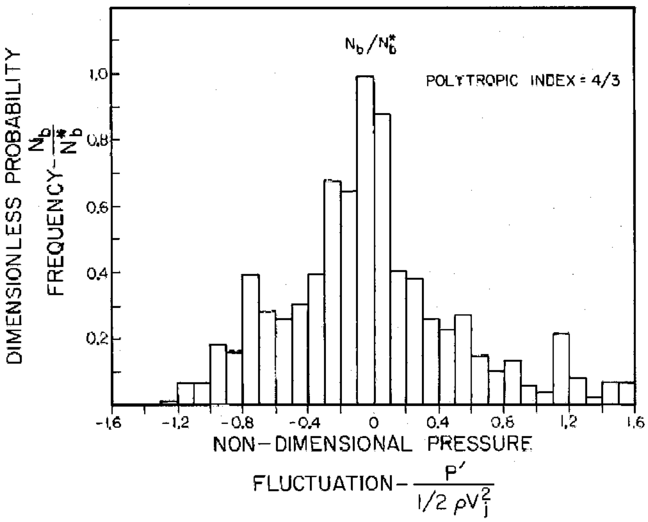

Fig. 9 Probability histogram showing the distribution of the pressure fluctuation intensities in jet. (Polytropic index of 4/3)

analysis. It has also been found that to some degree the tendency of the bubble to distort from its spherical form increases with the bubble size at the point of injection.

Discussion and Conclusions

There is at present no known criteria of establishing the correct value of the polytropic index in equations (13) and (14). Consequently, in the present work, an arbitrary value of 4/3 was used. Figure 8 is a typical example of the measured static pressure fluctuations in the present jet. The pressure fluctuation is expressed as a ratio of the dynamic head and plotted against the non-dimensional axial position. The various lines on the figure join data points at constant radial positions in the jet. It can be clearly seen that a wide spectrum of pressure fluctuations is present at any one instant. It should also be noted that peak fluctuations as intense as 160% of the dynamic head were also recorded although they are not shown in this plot.

A probability histogram which shows the distribution of the intensities of the pressure fluctuations is shown in Fig. 9. This curve was obtained by incorporating all the data from 17 holograms of the jet, under identical flow conditions. It can be inferred from this plot that the pressure fluctuation intensities have a Gaussian-like distribution, but is skewed somewhat. Notice also that the fluctuations peak at a higher positive value than a negative one. A similar histogram of the peak pressure fluctuations has been used to some degree of success in the scaling of cavitation inception data in jets. The details are found in [11].

To have an indication of the rms of the pressure fluctuations, the jet was divided into various regimes as shown in Table 1. The calculation was then performed using the expression:

$$\left\langle\left(\frac{P'}{\frac{1}{2}\rho V_j^2}\right)\right\rangle = \left[\frac{1}{(m-1)}\sum_{m=1}^{m}\left(\frac{P'}{\frac{1}{2}\rho V_j}\right)^2\right]^{1/2} \tag{15}$$

where $\langle(P'/\frac{1}{2}\rho V_j^2)\rangle$ denotes the rms of the dimensionless pressure fluctuation in a particular regime of the jet and m is the number of data points in that region. The computed results are tabulated in Table 1. It can be seen from this table that there are two entries in each region of the jet. The first entry is the calculated rms fluctuation when all the measured data were used. The second entry corresponds to the calculated rms value when a few of the data points were not included in the computation because they exhibited values that were significantly different from the majority of the data in that group. A quick comparison of these two entries clearly shows that in many instances, there is a sizeable difference in the two rms values. This clearly implies that the results are not very accurate due to the limited number of data points. It should be pointed out that the number in parentheses represents the sampling size.

In the rms pressure measurements made to date by conventional means, it has been reported in [2] and [12] that the magnitude of the fluctuation decreases with increasing distance from the jet exit. However, this trend is not apparent in the present case. The cause of this discrepancy is not known. In addition, the currently calculated rms level is considerably higher than those reported by the other authors. It is believed that the lower rms values of the other investigators is due to spatial averaging, since the pressure transducers that they used were much larger by comparison.

It should be apparent by now that the present technique does permit the static pressure in a jet, or any shear flow for that matter, to be measured. Although the pressure measurements in this work were carried out at a jet velocity of only 12.2 m/s, higher velocities could be used if the surrounding ambient pressure was higher. This technique could also be utilized to measure the static pressure in a Lagrangian frame. As a final remark, it should be mentioned that the present method does have a set back which is the time

Table 1 A tabulation of the rms pressure fluctuations in various regions of the present jet (polytropic index = 4/3)

	$0 \leq r/D \leq 0.5$	$0.5 \leq r/D \leq 1.0$	$1.0 \leq r/D \leq 1.5$	$1.5 \leq r/D \leq 2.0$	$2.0 \leq r/D \leq 2.5$	$2.5 \leq r/D \leq 3.0$
$4.0 \leq X/D < 6.0$	0.25(95)	0.43(38)	0.64(13)	–	–	–
	0.20(92)	0.32(35)	0.45(11)			
$6.0 \leq X/D < 8.0$	0.39(43)	0.56(45)	0.76(25)	0.48(3)	–	–
	0.31(41)	0.39(42)	0.65(22)	–		
$8.0 \leq X/D < 10.0$	0.56(24)	0.53(35)	0.56(33)	0.63(14)	–	–
	0.40(20)	0.40(32)	0.43(30)	0.56(13)		
$10.0 \leq X/D < 12.0$	0.66(13)	0.69(38)	0.73(31)	0.83(20)	0.94(3)	–
	0.40(11)	0.54(35)	0.61(28)	0.68(17)	0.77(2)	
$12.0 \leq X/D < 14.0$	0.64(12)	0.65(28)	0.78(18)	0.59(14)	0.69(1)	0.65(7)
	–	0.51(24)	0.49(14)	–	0.58(10)	–
$14.0 \leq X/D < 16.0$	0.60(6)	0.69(10)	0.46(25)	0.68(25)	0.65(9)	0.76(13)
	0.43(5)	–	0.50(22)	–	0.56(8)	0.57(10)

required to analyze the holograms. As an example, it takes approximately ten hours to record all the bubble sizes in a single hologram. This puts a limit to the number of holograms that could be realistically analyzed and consequently affects the accuracy of the statistical results. It is true that an automated, computer aided system could be used to scan the hologram and record the bubble sizes simultaneously as a function of jet position. Unfortunately, such a system is at present unavailable.

Acknowledgments

The authors would like to take this opportunity to thank Professor Sabersky and Dr. Oldenziel for providing many useful suggestions and comments. Dr. J. Katz is deserving of special mention for his valuable assistance on the holographic work. The authors are grateful to Susan Berkley for her help in organizing the manuscript. This work was supported by Naval Sea Systems Command General Hydromechanics Research Program administered by the David Taylor Naval Research and Development Center under Contract No. N00014-75-C-0378, which was greatly appreciated.

References

1 Arndt, R. E. A., and George, W. K., "Pressure Fields and Cavitation in Turbulent Shear Flows," 12th Symp., Naval Hydrodynamics, Washington, D.C., 1978, pp. 327–39.

2 Barefoot, G. L., "Fluctuating Pressure Characteristics in the Mixing Region of a Perturbed and Unperturbed Round Free Jet," Appl. Res. Lab., Tech. Memo., File No. TM 72-165, Penn State Uni., Aug. 1972.

3 Barthes-Biesel, D., and Acrivos, A., "Deformation and Burst of a Liquid Droplet Freely Suspended in a Linear Shear Field," *J. Fluid Mech.*, Vol. 61, Part 1, 1973, pp. 1–21.

4 Clay, C. S., and Medwin, H., *Acoustical Oceanography: Principles and Applications*, Wiley, p. 191.

5 Devin, C., Jr., "Survey of Thermal, Radiation and Viscous Damping of Pulsating Air Bubbles in Water," *J. Acous. Soc. Am.*, Vol. 31, No. 12, Dec. 1959, pp. 1654–1677.

6 Epstein, P. S., and Plesset, M. E., "On the Stability of Gas Bubbles in Liquid-Gas Solutions," *J. Chem. Phys.*, Vol. 18, No. 11, Nov. 1950, pp. 1505–1509.

7 Franklin, R. E., and McMillan, J., "Noise Generation in Cavitating Flows. The Submerged Water Jet I.," Oxford Uni. Engrg. Lab. Report No. 1312/80.

8 Fuchs, H. V., "Space Correlations of the Fluctuating Pressure in Subsonic Turbulent Jets," *J. Sound and Vib.*, Vol. 23, No. 1, 1972, pp. 77–99.

9 Katz, J., "Construction and Calibration of an Holograph Camera Designed for Micro Bubbles Observation in Cavitation Research," CIT Rept. No. Eng. 183-4, Oct. 1979.

10 Lau, J. C., Fisher, M. J., and Fuchs, H. V., "The Intrinsic Structure of Turbulent Jets," *J. Sound and Vibration*, Vol. 22, 1972, pp. 379–406.

11 Ooi, K. K., "Scale Effects on Cavitation Inception in Submerged Jets," CIT Engineering Report, 183-6, Dec. 1981.

12 Sami, S., Carmody, T., and Rouse, H., "Jet Diffusion in the Region of Flow Establishment," *J. F. Mech.*, Vol. 27, Part 2, 1967, pp. 231–252.

13 Strasberg, M., "The Pulsation Frequency of Nonspherical Gas Bubbles in Liquids," *J. Acous. Soc. Am.*, Vol. 25, No. 3, May 1953, pp. 536–537.

DISCUSSION

Reza Taghavi and R. E. A. Arndt[1]

We would like to congratulate the authors for providing a significant piece of information on pressure fluctuations in turbulent jets. However, we do take issue on the authors argument that the discrepancy between their value of the pressure fluctuations R.M.S. and that obtained by other investigators such as Jones et at. [1] and Michalke et al. [2] are merely due to a probe size resolution effect.

Indeed, in a paper by George et al. (3) soon to be published in the *Journal of Fluid Mechanics*, a thorough investigation of the available information obtained from various sources indicates a common value of $\langle p'/\frac{1}{2}\rho U^2 \rangle$ of 0.06. Therefore, something other than probe size is responsible for this discrepancy.

As stated by Tennekes and Lumley [4], in order for a Lagrangian measurement to be stationary, it is necessary that the flow be homogeneous and steady. These conditions are not fulfilled in a turbulent jet flow.

In the experiments reported by the authors, the pressure measurement probes, that is the air bubbles, follow the organized structures of the turbulent jet. Indeed, within a few diameters travel time, uniformly distributed air bubbles tend to migrate toward the core of vortices under the effect of radial pressure gradients [5]. Simple dimensional considerations, as well as photographic data, also tend to prove this argument. When a coordinate system moving with a vortex is considered, it is readily seen that fluid is continuously entering from one side and leaving from the other side of the vortex. Therefore, the pressure measurements reported by the authors are in fact Lagrangian measurements done along immaterial paths (those of vortices), along which the pressure is more likely to deviate from its mean value. That explains the apparent discrepancy between the results of these measurements and (Eulerian) measurements reported by others [1, 2].

[1] St. Anthony Falls Hydraulic Laboratory, University of Minnesota, Minneapolis, Minn. 55414.

The technique described by the authors seems to lead to a de Facto conditional sampling in which pressure is only measured when the probe is in the vicinity of vortex cores. The further the bubbles are from the nozzle lip, the closer they are to the core and the larger the measured pressure R.M.S. is. This could explain the increase $\langle p'/\frac{1}{2}\rho U^2 \rangle$ with downstream distance reported by the authors.

Fortunately, pressure measurements obtained with this technique are most relevant to the study of turbulence governed cavitation since the actual sites of cavitation are measurement probes themselves. The resulting value of the non-dimensional pressure R.M.S. should therefore be used for such studies [6].

Additional References

1 Jones et al., "Spectra of Turbulent Static Pressure Fluctuations in Jet Mixing Layers," *AIAA Journal*, Vol. 17, No. 5, 1979.

2 Michalke, A., and Fuchs, H., "On Turbulence and Noise of an Axisymmetric Shear Flow," *Journal of Fluid Mechanics*, Vol. 70, Part 1, 1975.

3 George, W. K., et al., "Pressure Spectra In Turbulent Shear Flow," AIAA Paper No. 80-0985, 1980.

4 Tennekes, H., and Lumley, J. L., *A First Course in Turbulence*, MIT Press, 1972.

5 Cook, T., and Harlow, F., "Vortex Behaviour in Two Phase Bubbly Flow," *Bulletin of the American Physical Society*, Vol. 28, No. 9, 1983.

6 Arndt, R. E. A., and Taghavi, Reza, "Cavitation Inception in Various Types of Shear Flow," To be presented at the ASCE Speciality Conference in Coeur d'Alene, Idaho, 1984.

Authors' Closure

We welcome Professor Arndt's interest in our jet-flow work, a subject which he himself has contributed to so greatly. We did not mean to put too much weight on the comments with which he takes issue. However, in numerous publications to date, for example, Bradshaw [14] and Knapp et al. [15], it has been clearly stated that the measured values of the probes that are similar to those used by the investigators cited by Professor Arndt are caused by two effects, namely, the effects of the fluctuating pressure and velocity effects caused by variations in the local angle of attack at the probe. Since these probes are unable to differentiate between these two effects, the fluctuating pressure measurements would therefore depend on the probe size relative to the scale of turbulence. In addition, Knapp et al. [15] stressed that "the relation between the fluctuating turbulent pressure and the magnitude sensed by a fixed probe includes a factor that depends on the local deceleration of the fluid element approaching the flow. This unsteady quantity is unknown." As a consequence, errors are introduced into the pressure measurements. It was discussed by Strasberg [16] that the probe errors stated above are probably of the *same order of magnitude* as the pressure fluctuations. If this is the case, then any probe size resolution effects could easily have been masked.

Certainly we would agree that the small air bubbles of our experiment do not follow the fluid trajectories—as is well known. It is quite interesting though that both positive as well as negative pressure excursions are observed showing that the bubbles are not simply entrained in the vortices of the flow. Admittedly, we do not know the actual history of this motion and consequently we do not have the true fluid Lagrangian pressure time history. But that it is "conditioned" to report "core" pressures predominantly seems unlikely for the above reasons. We would, of course, welcome quantitative estimates of this effect but the turbulent mixing structure of such flows is still in a developmental state and so this way prove to be difficult to carry out. We would look forward to the opportunity to review Professor Arndt's own recent work in this field and hope that it may soon become generally available.

Additional References

14 Bradshaw, P., *Experimental Fluid Mechanics*, Pergamon Press, Oxford, 1970.

15 Knapp, R. T., Daily, J. W., and Hammit, F. G., *Cavitation*, McGraw-Hill, New York, 1970.

16 Strasberg, M., "Measurements of Fluctuating Static and Total-head Pressure Gradient in a Turbulent Fluid," DTMB Rept. 1779, 1963.