

# **Demography and Economic Development: The Interaction between Institutions and Educational Choices**

Inaugural-Dissertation

zur Erlangung des Grades Doctor oeconomiae publicae (Dr. oec. publ.)

an der Ludwig-Maximilians-Universität München

2018

vorgelegt von

**Gerrit Julius Reinhard Leander Meyerheim**

Referent:	Professor Dr. Uwe Sunde
Korreferent:	Professor Dr. Gerhard Illing
Promotionsabschlussberatung:	11. Juli 2018

# Contents

Acknowledgments . . . . .	viii
<b>Preface</b>	<b>1</b>
<b>1 The Role of the Timing of the Demographic Transition for Economic Development</b>	<b>7</b>
1.1 Introduction . . . . .	7
1.2 Growth Empirics Reconsidered . . . . .	13
1.2.1 Hypotheses and Empirical Strategy . . . . .	13
1.2.2 Data and Sample Construction . . . . .	16
1.2.3 Reduced Form Estimation Results . . . . .	17
1.3 A Simple Unified Growth Model . . . . .	22
1.3.1 Individual Preferences and Constraints . . . . .	22
1.3.2 Production . . . . .	23
1.3.3 The Intragenerational Equilibrium . . . . .	24
1.3.4 Equilibrium Dynamics . . . . .	25
1.3.5 Outline of the Behaviour of the Model . . . . .	26
1.3.6 A Quantitative Analysis . . . . .	29
1.4 Towards Unified Growth Empirics . . . . .	32
1.5 Conclusion . . . . .	40
<b>2 From Stagnation to Growth: Demography, Democracy, and Comparative Development</b>	<b>43</b>
2.1 Introduction . . . . .	43
2.2 Related Literature . . . . .	49
2.3 Empirical Patterns: A Tale of Three Transitions . . . . .	51
2.4 The Model . . . . .	54
2.4.1 Individual Preferences and Budget Constraints . . . . .	55
2.4.2 The Production of Human Capital . . . . .	57
2.4.3 Production of Output . . . . .	57
2.4.4 Public Policies and Political Regimes . . . . .	58

2.4.5	Timing of Events . . . . .	61
2.4.6	Dynamics . . . . .	62
2.5	The Intra-Generational Equilibrium . . . . .	62
2.5.1	Optimal Education, Fertility and Consumption . . . . .	63
2.5.2	Optimal Policies and Political Equilibrium . . . . .	64
2.6	Equilibrium Dynamics . . . . .	67
2.6.1	Democratisation and the Demographic Transition . . . . .	67
2.7	Illustrative Simulation . . . . .	72
2.7.1	Quantitative Implementation . . . . .	72
2.7.2	Baseline Simulation Results . . . . .	74
2.7.3	Comparative Development: A Tale of Three Transitions . . . . .	76
2.8	Conclusion . . . . .	91
<b>3</b>	<b>Income and Democracy Revisited</b>	<b>94</b>
3.1	Introduction . . . . .	94
3.2	The Model . . . . .	96
3.2.1	Individuals . . . . .	97
3.2.2	Human Capital . . . . .	98
3.2.3	Production and Technology . . . . .	98
3.2.4	Political Economy . . . . .	99
3.2.5	Timing . . . . .	100
3.3	Analytical Results . . . . .	101
3.4	Specification and Data . . . . .	104
3.5	Results . . . . .	105
3.6	Conclusion . . . . .	109
<b>4</b>	<b>Long-Run Dynamics of Inequality: Insights from a Unified Growth Theory</b>	<b>111</b>
4.1	Introduction . . . . .	111
4.2	Theoretical Framework . . . . .	120
4.2.1	The Model . . . . .	120
4.2.2	Evolution of the Economy . . . . .	123
4.2.3	Long Run Dynamics and Inequality . . . . .	125
4.3	Dynamics of Inequality . . . . .	128
4.3.1	Quantitative Implementation . . . . .	128
4.3.2	Evolution of Income Distribution: Gini Index . . . . .	129
4.3.3	Income and Inequality . . . . .	132
4.3.4	Evolution of Upper Tail Inequality: Top Income Shares . . . . .	137
4.4	The Dynamics of Global Inequality . . . . .	138
4.4.1	Dynamics of Inequality: A tale of two countries . . . . .	138

4.4.2	Dynamics of Global Inequality . . . . .	142
4.5	Mechanism and Further Research . . . . .	144
4.6	Conclusion . . . . .	148
<b>References</b>		<b>150</b>
<b>Appendix A Appendix to Chapter 1</b>		<b>157</b>
A.1	Simulation Details . . . . .	157
A.2	Additional Results . . . . .	159
<b>Appendix B Appendix to Chapter 2</b>		<b>160</b>
B.1	Analytical Results . . . . .	160
B.1.1	Individual Optimisation Problem . . . . .	160
B.1.2	Determination of Optimal Tax Rates in the Simple Case . . . . .	161
B.1.3	Dropping the Primogeniture Assumption . . . . .	162
B.2	Additional Figures . . . . .	163
<b>Appendix C Appendix to Chapter 3</b>		<b>166</b>
C.1	Extension to the Degree of Democratisation . . . . .	166
<b>Appendix D Appendix to Chapter 4</b>		<b>168</b>
D.1	Returns to Ability and Income Distribution . . . . .	168
D.2	Details of Calibration . . . . .	170
<b>Eidesstattliche Versicherung</b>		<b>174</b>

## List of Tables

1.1	The Determinants of Income Growth: Cross-Section Estimates . . . . .	18
1.2	The Determinants of Income Growth: Within-Country Panel Estimates . . .	19
1.3	Robustness: Nonlinearity and Changing Importance of Growth Determinants	21
1.4	Reduced-Form Estimates Reconsidered: Data vs. Simulation . . . . .	38
1.5	TFP-Proxies: Data vs. Simulation . . . . .	39
3.1	Income and Democracy: Replication and Baseline Results . . . . .	106
3.2	Income and Democracy: Replication and Baseline Results . . . . .	107
3.3	Income and Democracy: Heterogeneity within Groups . . . . .	108
3.4	Income and Democracy: Heterogeneity within Groups . . . . .	109
A.1	Sensitivity of Reduced-Form Estimates: Data vs. Simulation . . . . .	159

## List of Figures

1.1 Implications of a Delayed Onset of the Economic Transition. . . . .	9
1.2 Distribution of the Onset of the Transition in the Estimation Sample . . . . .	16
1.3 Simulated Development Path of the Baseline Economy . . . . .	32
1.4 Simulated Development Path of Two Economies . . . . .	33
1.5 Distribution of the Onset of the Transition in the Data and Simulation Sample	36
1.6 Secular Stagnation . . . . .	41
2.1 Long-Run Development Dynamics: Demographics, Democracy, and Income	52
2.2 The Effect of Democracy on Income Growth . . . . .	53
2.3 The Effect of the Demographic Transition on Income Growth . . . . .	53
2.4 The Timing of the Demographic Transition . . . . .	76
2.5 Long-Run Development Dynamics by the Timing of the Demographic Transition	77
2.6 Long-Run Development Dynamics: Data vs. Simulation . . . . .	78
2.7 Long-Run Development Dynamics for Different Sets of Countries: Data vs. Simulation . . . . .	79
2.8 Long-Run Development Dynamics: Population . . . . .	80
2.9 Long-Run Development Dynamics: Education . . . . .	81
2.10 The Effect of the Transition to Democracy on Income Growth: Data vs. Simulation . . . . .	83
2.11 The Effect of the Demographic Transition on Income Growth: Data vs. Simulation . . . . .	84
2.12 Interactions Between Transitions: Data vs. Simulation . . . . .	85
2.13 Interactions Between Transitions: Data vs. Simulation . . . . .	87
2.14 The Role of Demographic Development: Baseline Simulation vs. Counterfactual	89
2.15 The Role of Democratisation: Baseline Simulation vs. Counterfactual . . . . .	91
2.16 The Role of the Consensual Democratisation Scenario: Baseline Simulation vs. Counterfactual . . . . .	92
4.1 Development and Changing Patterns of the Income-Inequality Nexus . . . . .	113
4.2 Long Run Dynamics: Illustration . . . . .	126
4.3 Long Run Development: Share of Skilled from year 0 to 2000 . . . . .	129

4.4	Income and Skill Distribution from Stagnation to Growth: 1800-2000 . . . . .	130
4.5	Long Run Inequality Dynamics (Gini Index): Simulated Model 1900-2000 . .	131
4.6	Long Run Inequality Dynamics (Gini Index): 1850-2000 Data . . . . .	131
4.7	Inequality Dynamics 1960-2000 (Gini Index): Data . . . . .	132
4.8	Inequality and Development: Simulation . . . . .	133
4.9	Inequality and Development: Empirical "Kuznets' Relationship" . . . . .	134
4.10	Inequality and Development: "Kuznets' Relationship" Data 1960-2000 . . .	135
4.11	Top Income Shares 1960-2000: Simulation . . . . .	138
4.12	Evolution of Top Income Shares 1960-2000: Data US . . . . .	139
4.13	Income Inequality (Gini Index) in US and France: Data 1960-2010 . . . . .	140
4.14	Top Income Shares in US and France: Data 1960-2010 . . . . .	141
4.15	Income Inequality (Gini Index) in US and France: Simulation 1960-2010 . . .	142
4.16	Top Income Shares in US and France: Simulation 1960-2010 . . . . .	143
4.17	Evolution of Global Inequality 1820-2000: Simulation and Data . . . . .	145
4.18	A Human Capital Kuznets' Curve: Simulation and Data . . . . .	146
4.19	Global Inequality in Life Expectancy (Gini Index): Simulation and Data 1820-2000 . . . . .	147
B.1	The Timing of the Demographic Transition . . . . .	163
B.2	The Timing of the Demographic Transition . . . . .	163
B.3	The Role of Demographic Development: Baseline Simulation vs. Counterfactual	164
B.4	The Role of Democratisation: Baseline Simulation vs. Counterfactual . . . . .	164
B.5	The Role of the Consensual Democratisation Scenario: Baseline Simulation vs. Counterfactual . . . . .	165
D.1	Returns to Ability and Income Distribution (balanced growth path) . . . . .	171
D.2	Ability Bias Technological Change and Income Distribution . . . . .	171

## Acknowledgments

First I want to thank my co-author and first supervisor Uwe Sunde for his guidance and belief. He both inspired me to pursue a PhD as well as to investigate the open questions of demography, economic development, and long-run growth that are the basis of this dissertation. Furthermore, I have benefited immensely from discussions with him that have shaped my thinking over the last five years. Additionally, I want to thank Gerhard Illing for thoughtful comments and standing in as my second supervisor. Special thanks go to Carsten Eckel for both completing my dissertation committee as third supervisor as well as allowing me to be part of the research training group GRK 1928 of the German Science Foundation. I am further indebted to Matteo Cervellati, who helped to co-author chapters two and four of this dissertation and who, apart from Uwe, is the person I learnt from the most during the past five years.

I am greatly indebted to the German Science Foundation for providing me with funding and further resources for three years of this dissertation. I additionally want to thank Carsten Eckel, Andreas Haufler and Martin Kocher for their time spent on the organisation of GRK 1928 and the very pleasant working atmosphere created by them.

Furthermore, I want to thank the Department of Economics, especially Susan Fay, Julia Zimmermann, Manuela Beckstein, Silke Englmaier, Caroline Benkert, and Sibel Avsar for their great amount of administrative work, patience, and help on organisational matters.

Further thanks go to the members of the cohort of 2013 and colleagues at the Chair for Population Economics, namely Sebastian, Lukas (Buchheim), Mira, Marie, Lukas (Rosenberger), and Ilka for making this a very enjoyable time. Special thanks go to my two officemates Rainer and Andreas, who started out as colleagues, but became friends over the years.

Finally, I want to thank my parents, Beatrix and Bernhard as well as my sister Caecilia for their unending love and unconditional support. This would have been impossible without you. Thank you.



The power of population is indefinitely greater than the power in the earth to produce subsistence for man.

—Thomas Malthus

# Preface

Economic growth and development in the history of human kind has for the most period been stagnant. Since the evolution of human species to its modern form around 150.000 years ago sustained growth has been an unknown phenomenon until about 150 years ago. The transition from an epoch of stagnation to a regime of sustained economic growth represents both one of human kind's most remarkable achievements as well as greatest puzzles. While comparatively little of the natural circumstances changed over the past millennia, income per capita has undergone a previously unthinkable tenfold increase over the past two centuries with profound effects on education, health and living standards in general across the world (see, among others Maddison (2006)). However, the rise in income per capita has not been distributed equally across societies. In fact, variation in the timing of the take-off from stagnation to growth has led to massive worldwide differences in income per capita. While inequality across countries had been fairly moderate two centuries ago the onset of sustained growth only in some countries has widened the gap between the richest and poorest countries considerably.

This dissertation is part of the ambitious quest to unearth the reasons and consequences of both this exceptional transformation and the simultaneous divergence in incomes. Its contribution originates from the perspective that both institutions and educational choices are malleable along the development path, resulting in questions about their mutual interaction: What are the effects of individual decisions on education? Do these decisions influence future institutions? How do these reshaped institutions then feed back into individuals' decisions? Answering these questions requires a long-term perspective and

careful modelling of the single mechanisms in question. It is the aim of this dissertation to shed new light on these questions and to develop new, theoretically-substantiated contributions and empirical applications. I do this by applying a coherent methodology that combines unified growth theories with quantitative implementations and reduced form estimations.

The first chapter “The Role of the Timing of the Demographic Transition for Economic Development”, which is joint work with Uwe Sunde, investigates the importance of the demographic transition on a country’s development. Starting from the observation that existing empirical work focuses on the estimation of growth models that implicitly or explicitly rely on the assumption of a balanced growth path (see, for example Mankiw *et al.* (1992)) it asks the question on whether this is the correct way of capturing cross-country income differences. Indeed when one considers the time frame laid out above it is safe to say that a balanced growth path is a very recent observation and that transition dynamics towards this path are highly non-linear. Unified growth theory is one strand of the economic growth literature that explicitly focuses on these underlying non-linear dynamics. However, up until today unified growth theory is rarely used as a structural foundation to motivate empirical work. Some of this can be attributed to the fact that unified growth models typically abstract from capital accumulation, a crucial part of growth regressions and a stylised fact of economic development. The paper argues that adopting a long-run perspective of development is both more consistent with observed empirical growth patterns and offers valuable new insights into the mechanics of comparative development differences across countries. Furthermore it can also provide some intuition for future growth paths. As the demographic transition constitutes a one-off, non-recurring change in growth trajectory, relative differences in the timing can have profound effects. The analysis proceeds to show that the empirically measurable effect of the demographic transition on growth performance is not only considerable but has lasting effects well beyond after its onset. Additionally, the transition reshapes the importance of factors of production. While physical capital was the most important factor of production for growth before the

transition its importance diminished afterwards. Human capital, on the other hand, while not irrelevant pre-transition becomes the dominant factor of production afterwards. Building on this preliminary result the developed theoretical model of this chapter tries to capture the undergoing structural transformations in the economy. Via including physical capital in the model it is able to show that an economy can experience a long phase of stagnation, followed by an increase in population size, saving and physical capital until a point is reached where a changing technological environment makes investment in education (and therefore human capital) worthwhile. After this point a demographic transition occurs and the economy experiences a decline in population growth, an increase in human capital and a diminished importance of physical capital. The theoretical model is then simulated numerically to match the data patterns of growth. Variation in starting conditions allows varying the onset of the demographic transition and therefore generates artificial data that can be used for a synthetic panel analysis. The results of the model are in line with the empirical considerations formulated beforehand. Namely, the onset of the transition positively contributes to economic development and changes the importance of factors of production, with physical capital becoming less and human capital becoming more important after the transition, respectively. Lastly, the analysis sheds new light on the secular stagnation debate. As the demographic transition and the associated increase in human capital is a one-off, non-recurring event, a slowdown in growth is neither surprising nor too worrying as the effects of one-off changes slowly wash out over time.

While the demographic transition was undoubtedly a very important event, most of the world, starting with Europe at the end of the 19<sup>th</sup> century, experienced another massive change in its fundamental structure. Most of today's states underwent a process of democratisation which placed the decisions about the allocation and distribution of income, goods and services from the hands of a small elite in the hands of the general public. The second chapter, "From Stagnation to Growth: Demography, Democracy, and Comparative Development", which is joint work with Matteo Cervellati and Uwe Sunde, examines the interactions between these two transformations. Both processes undeniably

had a significant effect on individual decision making. However, evidence presented so far has focused on mono-causal explanations that attribute the secular take-off in growth to either transition. The theoretical model developed within this project gives new insights into both channels and demonstrates that mono-causal explanations focusing only on one particular transition are misleading since they strongly interact with each other. The model studied is a unified growth model based on the prototype of Galor and Weil (2000), extended by an institutional component. The framework of the institutional part is based on the seminal contribution of Acemoglu and Robinson (2000). This institutional element allows studying the influence of demographic factors on institutional changes. Furthermore, it is possible to investigate different scenarios of democratisation and compare their impact on macroeconomic performance. For example, a peaceful democratisation with subsequent investment in public goods entails positive effects for each member of the population, while a forcible seizure of power by the majority of the population will result in redistribution at the expense of the previously ruling class. The theory posits that the timing and types of transitions critically depend on a country's human capital formation. The better the environment of a country for the formation of human capital stock, e.g., through better health care or educational institutions, the more likely it is that both democratisation and the demographic transition occur earlier. The worse the conditions, the more likely it is that both transitions occur later, as the incentive to form human capital is lower and the ruling class has enough resources to defend the status quo. Later democratisation also, for the most part, implies redistribution of resources to the previously marginalized since investment in public goods have low returns due to the comparatively low human capital stock. The structure of the model allows simulation based on observable data moments. The simulation results are both able to replicate the general patterns of development as well as the strong effects of either transition on growth performance, as observed by the empirical work focusing on the effect of either transition.

A further question that is directly prompted by these results is tackled by the third chapter "Income and Democracy Revisited". If economic transformation is also driving

institutional change should increasing income inevitably lead to social and political change? The answer that this chapter delivers is differentiated. With the help of a simple unified growth model, augmented by institutional change, it can be shown that rising income per capita can have both positive and negative effects on the likelihood of democratisation. Crucially, the effect depends on the source of income. If increases in income are human capital driven, then the effect is positive. Otherwise the effect is either non-existent or negative. As in the previous chapters the demographic transition plays a vital role in facilitating this transition since only after it a secular acceleration in the accumulation of human capital takes place. Empirical results from dynamic panel data estimation confirm the theoretical conjecture of heterogeneous effects of income per capita increases. Countries that are demographically advanced display a positive effect, while countries that are still prior to their demographic transition have a negative one. Additionally, the results lend further credibility to the importance of accounting for non-linear dynamics in empirical analysis of growth and development.

Building on this knowledge, another important question arises: how does the return to human capital change over time and what is the resulting effect on the distribution of income? This question has, because of the great influence of Thomas Piketty's book "Capital in the Twenty-First Century" (2014) among other reasons, come under increased scrutiny in recent years. The fourth chapter, "Long-Run Dynamics of Inequality: Insights from a Unified Growth Theory", which is joint work with Matteo Cervellati and Uwe Sunde, provides a possible answer to this question. It uses a variant of the prototype unified Growth theory by Cervellati and Sunde (2015a). As it accounts for an entire development horizon it is able to capture transitory dynamics that would otherwise be lost in steady state oriented growth models. Endogenous changes in the composition of the population in terms of skills and in the returns to individual abilities change both the entire distribution of income as well as upper tail inequality. Differential returns to both skill and ability allow the extraction of effects during different phases of development. More precisely, skill biased technical change, present after an initial push towards industrialisation will initially increase

inequality. However, as virtually the entire population picks up skilled jobs during the process of development the skill premium declines and inequality decreases. Nonetheless, technical change still factors into the return to labour. However, since the entire population works in similarly skilled occupations the effect of technical changes channels into returns at the individual level: technical change becomes strongly ability biased. This results in a subsequent increase in both total and top share inequality, with the amplitude depending on the size of the ability bias.

# Chapter 1

# The Role of the Timing of the Demographic Transition for Economic Development<sup>1</sup>

## 1.1 Introduction

Existing empirical work on growth is generally based on the estimation of growth models, or growth accounting frameworks, that implicitly or explicitly rely on the notion of a balanced growth path or on the convergence to such a balanced growth path. This methodology is typically based on both theoretical and econometric considerations. Yet, in light of long-run dynamics, the notion of a balanced growth path appears counterfactual, and an empirical analysis of long-run growth patterns restricting attention to balanced growth seems overly restrictive.

Unified growth theory models explicitly focus on the non-linear dynamics underlying long-run growth patterns and investigate the mechanisms behind the transition from stagnation to growth. The central insight of unified growth theory is the close interconnection between the economic transition from stagnation to growth and the onset of the demographic

---

<sup>1</sup>This chapter is joint work with Uwe Sunde.



transition. However, unified growth theory is rarely used to motivate or structurally inform empirical work. At most, unified growth models are simulated for illustrative purposes. Moreover, by focusing on demographic dynamics, unified growth theories typically abstract from the accumulation of capital.

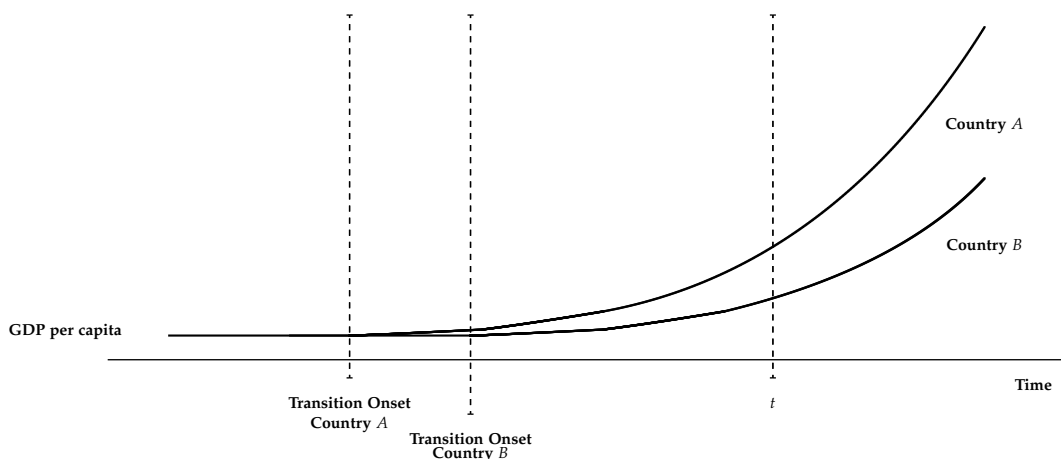
This paper argues that adopting a long-run perspective of growth and development might not only be more consistent with the empirical growth patterns, but it might offer completely new insights into the mechanics behind cross-country comparative development differences as well as insights into likely scenarios of future development dynamics in developed and less developed countries. This argument is motivated by the observation of the secular acceleration in economic well-being over human history as consequence of the economic transition, see, e.g., Galor (2011) and Jones and Romer (2010). Since the seminal contribution by Galor and Weil (2000), the main mechanisms that have been proposed view the demographic transition as a critical turning point for the endogenous take-off from stagnation to sustained growth. However, instead of merely constituting the historical background, it appears possible that this transition, which involves a decline in fertility and an increase in education, has long-lasting implications for economic performance even long after its onset due to the massive, non-recurring and prolonged structural change in the economy that pertains both to sectoral and demographic composition.

Figure 1.1 illustrates this by depicting two countries that follow development paths that only differ in terms of the timing of the economic (and demographic) transition, but that are otherwise entirely identical.<sup>2</sup> The first implication that follows from the figure is a positive correlation between the time that has passed since the onset of the transition and the income that is observed across countries at a given point in time. In other words, at a given point in time, countries that have experienced the transition earlier are, everything else equal, richer than countries that underwent the transition later. The second implication that follows

---

<sup>2</sup>A comparison of long-run development patterns in the data for different countries reveals a picture similar to that in the figure. Assuming that different countries follow identical paths that are only shifted in terms of the timing of the take-off is a working hypothesis that does not restrict the empirical analysis and to which we will return below.

from the figure is an acceleration in income growth after the onset of the transition within countries over time. These implications have been largely neglected in the empirical growth literature.



**Figure 1.1:** *Implications of a Delayed Onset of the Economic Transition.*

The analysis in this paper proceeds in three steps. As a first step, we investigate the empirical relevance of the timing of the onset of the demographic transition for economic development. Following the insight of the prototype unified growth theory that is based on the close connection between demographic and economic development, the empirical analysis exploits information on the time of the fertility transition as proxy for the onset of the transition to sustained growth and tests the relevance of accounting for the time since the onset of the transition on economic development across countries and within countries over time. We estimate empirical specifications that account for country-specific heterogeneity and global growth trends by including country and time fixed effects, and that therefore identify the effect by exploiting within-country variation over time. Consistent with the hypothesis, the timing of the fertility transition indeed seems to have systematic long-lasting effects on economic development and explains income and growth differences across countries.

In a second step, we develop a novel unified growth theory that can serve as basis for empirical work and rationalise the empirical analysis. Following most unified growth

models, the theory is based on the close connection between demographic and economic development. Differently from existing models, we consider a two-sector framework and explicitly account for endogenous population dynamics, education, and accumulation of physical capital. This framework delivers an analytically tractable model that can be simulated without resorting to corner solutions. The simulated growth path exhibits highly non-linear dynamics in the core elements of the model, population, education, capital, and output, with the fertility transition marking the crucial turning point of the long-run development path. The new framework, while purposefully simple, has the same ingredients as the conventional growth and development accounting frameworks. At the same time, it allows illustrating the different implications of non-linear dynamics for empirical work and to derive testable implications.

The third and final step of the analysis explores further implications on the basis of simulated data based on a calibrated version of the model. This analysis is supposed to illustrate the capability of the model to generate data that deliver similar empirical patterns as those observed in the actual data. Moreover, the simulated model can be used to explore likely scenarios of future development. By documenting the model's ability to generate similar empirical patterns as observed in the data, the analysis provides a bridge between reduced form evidence and a structural model, documenting that a simulation based on heterogeneity in a single parameter can generate data patterns that provide qualitatively similar reduced form estimation results to those obtained in the empirical analysis before. Moreover, the analysis provides insights to the secular stagnation debate, by documenting that the long-run transition dynamics are likely to imply systematically lower growth rates as the non-recurring and prolonged transition dynamics fade out.

This paper contributes to several strands of the literature. By extending an otherwise standard empirical growth framework and providing evidence for a systematic role of the timing of the demographic transition for contemporaneous economic performance, the empirical results add to earlier empirical investigations of the cross-country determinants of income growth, like Mankiw *et al.* (1992), Benhabib and Spiegel (1994), Krueger and

Lindahl (2001). Our estimation framework extends the standard framework to the explicit consideration of the time since the onset of the transition and the results document that the time since the onset of the demographic transition is a relevant variable that has not been accounted for in earlier estimates. In this respect the analysis also complements the recent findings on the role of the fertility transition for cross-country income differences by Galor (2012) and Dalgaard and Strulik (2013). The results also relate to the literature investigating the role of human capital and cross-country income differences, see de la Fuente and Domenech (2006), Cohen and Soto (2007), Portela, Alessie and Teulings (2010) and Sunde and Vischer (2015). The empirical results also suggest a successively smaller importance of physical capital for economic development in later phases of development, while human capital appears to become important after the onset of the transition. By broadening the perspective of growth empirics beyond the notion of balanced growth, the evidence contributes to the ongoing debate about the importance of factors that are thought to be relevant for explaining growth, such as physical capital, or human capital, and in particular to the mixed results regarding the role of human capital for growth.<sup>3</sup> By documenting the importance of accounting for structural differences in behaviour across different phases of demographic development, the results of this paper complement recent evidence for the existence of important non-linearities in the growth process depending on the stage of the demographic transition, see Cervellati and Sunde (2011a, 2011b).

The paper also contributes to the unified growth literature by proposing a simple quantifiable model that can be used for empirical applications. By extending a prototype unified growth framework with endogenous fertility and education to the accumulation of physical capital, the model contributes to the literature that has mostly abstracted from physical capital, see, e.g. the seminal contribution by Galor and Weil (2000), and subsequent work by Kogel and Prskawetz (2001), Galor and Moav (2002), Doepke (2004), and Cervellati and Sunde (2005) (see Galor, 2011 for a comprehensive account of the role of the demographic

---

<sup>3</sup>The literature has encountered difficulties in detecting a significant role of human capital within countries overtime. See Cohen and Soto (2007).

transition in the unified growth literature), or focused on the role of capital accumulation while abstracting from population dynamics, such as, e.g., Hansen and Prescott (2002), Galor and Moav (2006), or Galor et al. (2009). The main mechanism leading to the transition is based on the demographic transition as the critical turning point for the endogenous take-off from stagnation to sustained growth and constitutes an extension of the framework proposed by Strulik and Weisdorf (2008).

The model also delivers additional insights regarding the relative importance of capital and human capital along the process of development, thereby providing a coherent framework for hypotheses that have been made in the literature, such as the role of physical capital being associated mostly with the onset of the transition (e.g., Hansen and Prescott, 2002), while human capital has been hypothesised to play a crucial role for the transition and growth after the transition (e.g., Galor and Weil, 2000, and Doepke, 2004). By documenting a sizable heterogeneous effect during the different phases of development, with a diminishing role of physical capital after the transition and an increasingly important role of human capital for economic development after the growth take-off, the results suggest that sample composition and omitted non-linearities in the empirical specification might explain some of the mixed findings regarding the growth effect of human capital in the literature.

Besides documenting the empirical validity of the unified growth approach, a central implication is the fading out of transitional forces as the demographic and economic transitions fade out. This insight has far-reaching consequences for the growth outlook across the world. Lately, the debate about the “new secular stagnation hypothesis” has gained momentum. This debate has largely focused on the prolonged recovery from cyclical fluctuations and the respective challenges for policy.<sup>4</sup> Only few papers have pointed to the possibility of a long-run decline in growth potential, such as Gordon (2012, 2014, 2016) who emphasised unfavorable dynamics in demographic composition, education, and globalisation, for the growth prospects of the US. The results of this paper suggest the

---

<sup>4</sup>See the recent book edited by Teulings and Baldwin (2014) and the session on “The Economics of Secular Stagnation” at the Allied Social Science Associations Conference 2015. See also OECD (2015).

largely neglected role of the fading out of transitional dynamics, thereby providing a formal treatment of arguments made by Cervellati et al. (2017).

The remainder of the paper is structured as follows. Section 2.3 contains the empirical analysis, including a description of the empirical strategy and the data, as well as the empirical results. Section 2.4 presents a unified growth model that rationalises the empirical analysis. Section 1.4 provides further insights on the basis of a calibrated version of the model, and Section 3.6 concludes.

## **1.2 Growth Empirics Reconsidered**

### **1.2.1 Hypotheses and Empirical Strategy**

As suggested by Figure 1.1, the hypothesis of this paper is that the timing of the transition from stagnation to growth might have implications for the growth performance for a prolonged period. This implication is essentially common to all unified growth models, regardless of the precise mechanism that underlies the transition. This section presents reduced form empirical evidence for this hypothesis. The empirical analysis focuses on three aspects of this hypothesis. The first aspect is that countries that have experienced the transition earlier are presumably, everything else equal, richer than countries that underwent the transition later. The second aspect that follows from the figure refers to the development dynamics within the same country overtime and predicts an acceleration in income growth after the onset of the transition. Hence, as long as a country does not reach a balanced growth path (if it exists), the transition dynamics as captured by the time since the onset of the transition should be detectable. However, depending on the long-run growth process, and in particular whether the transitional income dynamics have more or less momentum due to the substantial structural and demographic change associated with the transition than the productivity growth mechanisms underlying the dynamics along the balanced growth

path, the effects of the time since the onset might be non-monotonic.<sup>5</sup> The third aspect refers to the relative importance of the main (proximate) determinants of growth. In particular, the different mechanisms that have been proposed in the unified growth literature to explain the take-off predict that the roles of physical and human capital might change during the different phases of the transition from stagnation to sustained growth. While the role of physical capital has been associated mostly with the onset of the transition, human capital has been predicted to play a crucial role for the transition and as a main determinant of growth after the transition.<sup>6</sup> This implies a potential interaction in the importance of the main growth determinants and the time since the onset of the transition.

To investigate the empirical relevance of the first hypothesis, the empirical analysis exploits cross-country variation as in a conventional empirical growth framework. (see, e.g., Benhabib and Spiegel, 1994, Krueger and Lindahl, 2001, or Cohen and Soto, 2007). The dependent variable is the change in log GDP per capita in a country  $i$  from time  $t - 1$  to time  $t$ , which is regressed on a lagged term  $\ln y_{i,t-1}$ , the change in the (log) capital stock,  $\Delta k_{i,t}$ , the change in (log) human capital,  $\Delta HC_{i,t}$ , as well as time fixed effects,  $\phi$ .<sup>7</sup> We extend a standard linear growth regression by including the time since the onset of the transition,  $TSO_{i,t}$ , as additional regressor,

$$\Delta y_{i,t} = y_{i,t} - y_{i,t-1} = \psi y_{i,t-1} + \beta \Delta k_{i,t} + \gamma \Delta HC_{i,t} + \zeta TSO_{i,t} + \phi_t + u_{i,t}. \quad (1.1)$$

This empirical model can be estimated using data in long differences (as is done in most

---

<sup>5</sup>More complicated dynamics that lead to overtaking or temporarily lower growth around the transition are also conceivable, but the growth effect is a first-order implication of the model and therefore delivers a testable hypothesis.

<sup>6</sup>Models along the line of Hansen and Prescott (2002) predict a crucial role of physical capital accumulation for initiating the take-off, but a diminishing role of physical capital for sustained growth afterwards. Models focusing on human capital, like Galor and Weil (2000) or Doepke (2004), predict an increasing importance of human capital also after the onset of the transition. By being based on a formation process (like schooling) that requires years, the effect of human capital is also likely to be delayed in time (being fully effective after the completion of the fertility transition).

<sup>7</sup>Following the literature, human capital is included in levels, consistent with a macro-Mincer specification, see Krueger and Lindahl (2001) and Cohen and Soto (2007). In addition, this specification does not restrict the coefficient of physical capital and past income to be the same, as in some existing empirical studies.

of the empirical growth literature), or by pooled OLS using panel data (at 10 or 20 years frequency, for instance).<sup>8</sup>

The second hypothesis is tested using within-country variation over time. The dependent variable is log GDP per capita in a country  $i$  at time  $t$ ,  $y_{i,t}$ . The main explanatory variables are the stocks of (log) physical capital,  $k_{i,t}$ , the level of (log) human capital,  $HC_{i,t}$ , and a lagged term  $y_{i,t-1}$  that captures convergence dynamics. Country fixed effects and common time fixed effects account for time invariant cross-country heterogeneity and global growth trends, respectively. The estimation framework is

$$y_{i,t} = \phi y_{i,t-1} + \beta k_{i,t} + \gamma HC_{i,t} + \zeta TSO_{i,t} + \mu_i + \nu_t + \varepsilon_{i,t} \quad (1.2)$$

where  $\mu_i$  is the country fixed effect and  $\nu_t$  is the common time dummy. To test the prediction of an acceleration of growth in a country after the transition, we estimate model (1.2) using a linear or non-linear specification with respect to the time since onset ( $TSO$ ) variable. This involves extending an otherwise standard linear growth regression that exploits within country variation over time, by including the time since the onset of the transition,  $TSO_{i,t}$  (or its second order polynomial), as additional regressor.<sup>9</sup>

The third hypothesis about the changing importance of the different growth determinants during different phases of the transition is tested by extending the estimation framework to the inclusion of the interactions between the years since the onset of growth and physical as well as human capital. This model nests the specifications that are estimated conventionally and deliver identical estimates if the (implicit) restriction that the coefficients  $\zeta$  or  $\zeta$  are equal to zero is justified. This allows for testing the hypotheses that derive from the unified growth framework against a well-defined alternative, namely the conventional empirical growth framework.

---

<sup>8</sup>This specification replicates and extends the standard specification in recent empirical growth studies (see, e.g., Benhabib and Spiegel, 1994, Krueger and Lindahl, 2001, or Cohen and Soto, 2007).

<sup>9</sup>See Cohen and Soto (2007) for a derivation of this specification. The variable  $TSO_{i,t}$  is country-specific but varies over time, which allows us to estimate its coefficient together with country and time fixed effects.

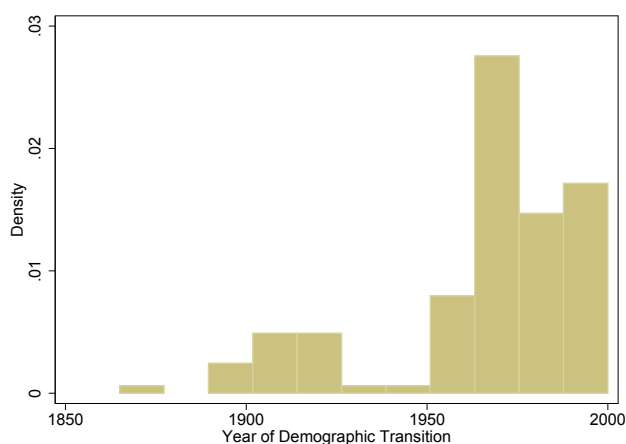


We estimate the effect of the time since the onset of the transition on economic development within countries over time. The empirical specifications account for country-specific heterogeneity and global growth trends by including country and time fixed effects, and therefore identify the effect by exploiting within-country variation over time.

## 1.2.2 Data and Sample Construction

The empirical analysis is based on cross-country panel data for income per capita, physical capital and human capital from the Penn World Tables version 9.0 by Feenstra *et al.* (2015). In these data, physical capital stocks are computed using the perpetual inventory method based on investment data taken from national accounts. Human capital is measured as an index based on average years of schooling from Barro and Lee (2013) and Cohen and Leker (2014). The data on the onset of the fertility transition is from Reher (2004).<sup>10</sup>

The estimation is conducted for 10-year panel data, 20-year panel data and long differences for a sample of 133 countries over the period 1950-2014. Figure 1.2 depicts the distribution of the onset of the transition in the data set.



**Figure 1.2:** *Distribution of the Onset of the Transition in the Estimation Sample*

---

<sup>10</sup>The variable  $TSO_{i,t}$  is computed as the difference between the respective calendar year and the year of the onset of the fertility transition according to Reher (divided by 100). The variable takes value 0 before the transition.

### 1.2.3 Reduced Form Estimation Results

Table 1.1 presents the results obtained from estimating specification (1.1). Column (1) replicates the standard specification in long differences 1960-2010 as estimated in the literature by restricting  $\zeta$  to zero. Physical and human capital affect growth significantly positively, while log income per capita in 1960 exhibits a negative effect, consistent with convergence dynamics.

Column (2) presents the results for the unrestricted model by including the time since the onset of the transition. The results document a significantly positive influence of time since the onset of the demographic transition,  $TSO$ , on income or income growth.<sup>11</sup> The coefficient estimates of physical capital is basically unaffected, whereas the effect of human capital remains positive and significant, but becomes around 20% larger in size. Also the convergence term becomes larger in size once time since onset is added as regressor.

The remaining columns repeat the same analysis by estimating pooled OLS using data at 20-year and 10-year frequencies over the same period. In all estimations, the time since the onset of the transition enters as highly significant determinant of cross-sectional income differences, consistent with the hypothesis derived from the insights of unified growth theory.<sup>12</sup> Again, the inclusion has little consequences for the effects of physical capital, whereas the coefficients for human capital and lagged income levels are increased in absolute terms by the inclusion of the time since onset variable.

Table 1.2 reports the estimation results of the empirical specification (1.2) using panel data with 20-year, 10-year and 5-year frequencies. Due to the inclusion of country fixed effects, these estimates are identified by within-country variation over time, relative to the period mean (as consequence of time fixed effects). The replication of the standard

---

<sup>11</sup>Note that the estimates of all coefficients are identical when using income levels as dependent variable, with the only exception being the coefficient on past income (which is increased by 1 in the levels regression).

<sup>12</sup>This finding is unaffected by the specification of the human capital variable. Table ?? presents the results of a full specification that includes human capital in levels and changes, as suggested by Sunde and Vischer (2015), delivers qualitatively similar results.

**Table 1.1:** *The Determinants of Income Growth: Cross-Section Estimates*

Dependent Variable:	Change in Log GDP per Capita					
Data Frequency:	Long Differences		20 years		10 years	
	(1)	(2)	(3)	(4)	(5)	(6)
$y_{t-1}$	-0.048 [0.043]	-0.136** [0.060]	-0.042** [0.017]	-0.080*** [0.021]	-0.022*** [0.008]	-0.042*** [0.010]
$\Delta k_t$	0.579*** [0.052]	0.554*** [0.048]	0.578*** [0.038]	0.548*** [0.036]	0.559*** [0.037]	0.537*** [0.037]
$\Delta HC_t$	0.699** [0.290]	0.954*** [0.297]	0.372* [0.198]	0.606*** [0.213]	0.341** [0.161]	0.501*** [0.170]
Time since Onset <sub>t</sub>		0.651*** [0.244]		0.283*** [0.083]		0.148*** [0.041]
Year Fixed Effects			Yes	Yes	Yes	Yes
Observations	91	91	270	270	586	586
Countries	91	91	114	114	114	114
Adjusted $R^2$	0.690	0.704	0.592	0.606	0.471	0.482

Results from Pooled OLS regressions. The dependent variable  $y_{t-1}$  is the change in (the log of) real GDP at constant 2011 national prices (in mil. 2011US\$) divided by population size (in millions),  $k_t$  is (the log of) capital stock at constant 2011 national prices (in mil. 2011US\$),  $HC_t$  is (the log of) the human capital index, see data description of the PWT 9.0 for details. “Time since Onset” measures the years since the onset of the demographic transition, divided by 100. Sample period: long differences 1960-2010, 20-year panel {1950, 1970, 1990, 2010}, 10-year panel {1950, 1960, ..., 2010}. Columns (1)-(2): results of long differences. Columns (3)-(4): results at 20-year data frequency. Columns (5)-(6): results at 10-year data frequency. Standard errors are clustered at the country level. \*, \*\*, \*\*\* denote significance at 10%, 5%, and 1% levels.

specification, which restricts the effect of  $TSO$  to zero, reveals no significant effects of physical and human capital on economic development in the 20-year panel in Column (1). Column (2) relaxes the restriction on  $\zeta$  by including  $TSO$  as additional explanatory variable. Again, accounting for the time since the onset of the demographic transition has a significant impact on economic performance, even after controlling for lagged income, physical and human capital. Qualitatively similar results emerge for panel data in 10-year and 5-year frequencies. Throughout, the estimates reject the null hypothesis of  $\zeta$  being zero. Instead, the results show that  $TSO$  is a significant determinant of development, compatible with the hypothesis of an acceleration of growth within countries after the onset of the transition. The coefficients of the lagged income and physical capital variables are fairly similar in the restricted and in the extended specifications, while the effect of human capital is significant and even declines in size when  $TSO$  is controlled for, suggesting that the demographic

dynamics to some extent capture dynamics in human capital.

For robustness, we replicated the estimation using alternative estimators that account for the potential bias due to the dynamic panel specification (Nickell, 1981), using a bias-corrected least squares dummy variable estimator, CLSDVE, along the lines of Bruno (2005a, 2005b). Again, the results are qualitatively and quantitatively very similar, with similar results.<sup>13</sup>

**Table 1.2:** *The Determinants of Income Growth: Within-Country Panel Estimates*

Dependent Variable	Log GDP per Capita					
	20 years		10 years		5 years	
	(1)	(2)	(3)	(4)	(5)	(6)
$y_{t-1}$	-0.013 [0.056]	-0.069 [0.046]	0.344*** [0.065]	0.286*** [0.054]	0.698*** [0.050]	0.671*** [0.048]
$k_t$	0.535*** [0.072]	0.507*** [0.069]	0.383*** [0.059]	0.381*** [0.057]	0.152*** [0.039]	0.152*** [0.039]
$HC_t$	0.457** [0.223]	0.477** [0.227]	0.402** [0.154]	0.404** [0.161]	0.150* [0.088]	0.157* [0.092]
Time since Onset <sub>t</sub>		2.063*** [0.753]		0.903*** [0.315]		0.391** [0.180]
Country Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	272	272	588	588	1304	1304
Countries	114	114	114	114	114	114
Adjusted $R^2$	0.792	0.820	0.823	0.831	0.892	0.893

Results of fixed effects regressions. The dependent variable  $y_{t-1}$  is (the log of) real GDP at constant 2011 national prices (in mil. 2011US\$) divided by population size (in millions),  $k_t$  is (the log of) capital stock at constant 2011 national prices (in mil. 2011US\$),  $HC_t$  is (the log of) the human capital index, see data description of the PWT 9.0 for details. “Time since Onset” measures the years since the onset of the demographic transition, divided by 100. Sample period: 20-year panel {1950,1970,1990,2010}, 10-year panel {1950,1960,...,2010}, 5-year panel {1950,1955,...,2010,2014}. All specifications include a full set of country fixed effects and common time dummies. Standard errors are clustered at the country level. \*, \*\*, \*\*\* denote significance at 10%, 5%, and 1% levels.

Finally, Table 1.3 estimates an extended specification of (1.2) by including either a quadratic specification of the time since the onset of the transition,  $TSO$ , to capture potential non-linearities of the effect, or by including an interaction of time since the onset of the

<sup>13</sup>These estimates are based on panel data with 10-year and 5-year frequency, which retains a sufficiently long time series dimension for alternative estimators. The unreported results deliver qualitatively identical but more noisy results and are available upon request.

transition with physical and human capital to capture changes in the relative importance of these factors at different points during the development process. The results in Columns (1), (3), and (5) suggest that the effect of the time since the onset of the demographic transition is non-linear, exhibiting an increasing and concave effect that peaks around a quarter of a century after the onset of the demographic transition. The results in Columns (2), (4) and (6) suggest that, in line with the prediction of a changing importance of different growth determinants in the different phases of the transition, the importance of physical capital for growth declines with time since the transition. The effect of human capital increases in interaction with the onset of the transition. However, this effect is not statistically significantly different from zero, potentially due to measurement issues in the human capital variable. Overall, the results reject the null hypothesis that *TSO* plays no role and, therefore, suggest the relevance of extending the standard linear estimation framework to accounting for the long-run development dynamics suggested by unified growth theory. The results of bias-corrected least squares dummy estimators confirm these results.<sup>14</sup>

Taken together, the results of this first systematic investigation of the implications of a perspective of long-run growth in line with unified growth theory are consistent with the hypothesis proposed in the introduction. The timing of the fertility transition seems to have long-lasting effects on economic development and explains income and growth differences across countries. In particular, the empirical results are consistent with the hypothesis of a positive correlation between the time that has passed since the onset of the transition and the income that is observed across countries at a particular point in time  $t$ , in the sense that countries that have experienced the transition earlier are, everything else equal, richer than countries that underwent the transition later. The results are also consistent with a temporary acceleration in income growth after the onset of the transition when focusing on within-country dynamics. Finally, the evidence points to non-linear dynamics and the potentially changing importance of the main growth determinants. These findings qualify

---

<sup>14</sup>The findings are qualitatively similar when using bias corrected estimators. Detailed results are available upon request.

**Table 1.3: Robustness: Nonlinearity and Changing Importance of Growth Determinants**

Dependent Variable	Log GDP per Capita					
	20 years		10 years		5 years	
	(1)	(2)	(3)	(4)	(5)	(6)
$y_{t-1}$	-0.069 [0.049]	-0.059 [0.052]	0.302*** [0.054]	0.325*** [0.057]	0.679*** [0.047]	0.682*** [0.048]
$k_t$	0.507*** [0.068]	0.526*** [0.071]	0.377*** [0.057]	0.394*** [0.057]	0.151*** [0.040]	0.171*** [0.039]
$HC_t$	0.478* [0.288]	0.337 [0.280]	0.282 [0.194]	0.211 [0.173]	0.014 [0.104]	0.000 [0.093]
Time since Onset $_t$	2.061** [0.795]	3.242*** [1.122]	1.166*** [0.381]	2.917*** [0.702]	0.729*** [0.218]	2.144*** [0.408]
(Time since Onset $_t$ ) <sup>2</sup>	0.002 [0.160]		-0.235* [0.132]		-0.275*** [0.079]	
Time since Onset $_t \times k_t$		-0.158 [0.108]		-0.196*** [0.074]		-0.163*** [0.041]
Time since Onset $_t \times HC_t$		0.562 [0.460]		0.278 [0.320]		0.210 [0.175]
Country Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	272	272	588	588	1304	1304
Countries	114	114	114	114	114	114
Adjusted $R^2$	0.819	0.821	0.833	0.837	0.895	0.897

Results of fixed effects regressions. The dependent variable  $y_t$  is (the log of) real GDP at constant 2011 national prices (in mil. 2011US\$) divided by population size (in millions),  $k_t$  is (the log of) capital stock at constant 2011 national prices (in mil. 2011US\$),  $HC_t$  is (the log of) the human capital index, see data description of the PWT 9.0 for details. "Time since Onset" measures the years since the onset of the demographic transition, divided by 100. Sample period: 20-year panel {1950,1970,1990,2010}, 10-year panel {1950,1960,...,2010}, 5-year panel {1950,1955,...,2010,2014}. All specifications include a full set of country fixed effects and common time dummies. Standard errors are clustered at the country level. \*, \*\*, \*\*\* denote significance at 10%, 5%, and 1% levels.

the results from earlier empirical investigations of the cross-country determinants of income growth by documenting that the onset of the transition is a relevant variable that has not been accounted for in earlier estimates. The results obtained with an empirical framework that controls for past income and time fixed effects also demonstrates that the role of the time since the onset of the transition is robust to (and different from) the consideration of convergence dynamics.<sup>15</sup>

In the next section, we develop a simple unified growth framework that can rationalise

<sup>15</sup>In this respect the analysis complements the recent findings on the role of the fertility transition for cross-country income differences by Galor (2012) and Dalgaard and Strulik (2013) and extends empirical growth studies (e.g., Krueger and Lindahl, 2001, Cohen and Soto, 2007).

these results and demonstrates the new insights of a unified growth approach in comparison to those obtained with a balanced growth view. Furthermore it can be used as theoretical underpinning of the reduced form growth empirics shown so far.

## 1.3 A Simple Unified Growth Model

### 1.3.1 Individual Preferences and Constraints

The model presented in this section extends the unified growth framework by Strulik and Weisdorf (2008) by including both education and physical capital accumulation. Consider an overlapping generations model. The population of generation  $t$  is of size  $L_t$  and consists of identical individuals. Individuals live for two periods: childhood and adulthood. During childhood, an individual only consumes one unit of agricultural output (“food”). This consumption reflects a biological necessity, or subsistence. During adulthood, individuals receive utility from goods and the number of children that they give birth to,  $n_t$ . Following Strulik and Weisdorf (2008), the utility of individuals is quasi-linear in goods,  $m_t$ , and logarithmic in the size of offspring  $n_t$ ,

$$u_t = m_t + \gamma \ln n_t \quad \gamma > 0. \quad (1.3)$$

The life of an adult consists of two sub-periods. In the first sub-period, an individual supplies labour to production in agriculture. In the second period, the individual supplies human capital, reflected by labour that is potentially augmented by an education investment during the first sub-period of adulthood, and capital, which is accumulated through savings during the first sub-period of adulthood, to the production of “manufactured” goods. Individuals maximise their utility by making fertility, education and savings decisions. Reproduction is asexual, and  $n_t$  represents the desired number of offspring. Education is chosen in terms of the share  $e_t \geq 0$  of time during the first sub-period of adulthood that the individual wants to invest in his own education. This time spent on education reduces the time available for work in the agricultural sector. The wage income from working in the

agricultural sector in the first sub-period of life is used to finance fertility  $n_t$  and savings  $s_t$ . Therefore, both expenditures for children and savings are in terms of agricultural goods. This determines the budget constraints for the two sub-periods of adult life,

$$(1 - e_t)w_t^L = n_t + s_t \quad (1.4)$$

$$s_t r_t + (1 + e_t)w_t^H = m_t, \quad (1.5)$$

where  $w_t^L$  is the wage obtained in the agricultural sector during the first sub-period of adult life,  $w_t^H$  is the return to human capital, and  $r_t$  is the return to savings that serve as productive capital.

### 1.3.2 Production

There are two sectors of production. The agricultural sector uses labour  $L_t$  and agricultural technology  $A_t$  to produce food, which is consumed by the children of the respective generation. Agricultural production exhibits decreasing returns to scale, and follows

$$Y_t^A = \mu A_t^\varepsilon L_t^\alpha \quad \mu > 0, \{\alpha, \varepsilon\} \in (0, 1). \quad (1.6)$$

The manufacturing sector uses capital  $K_t$  (in the form of savings from the first period of life), human capital  $H_t$  (i.e., labour that is augmented by education in the first period of life) and manufacturing technology  $M_t$  in a production process that exhibits constant returns to scale,

$$Y_t^M = \delta M_t^\phi H_t^\psi K_t^{1-\psi} \quad \delta > 0, \{\phi, \psi\} \in (0, 1). \quad (1.7)$$

Total output (income) of a generation is given by

$$Y_t = Y_t^A + Y_t^M. \quad (1.8)$$

Notice that all agricultural output produced by generation  $t$  is consumed as food by that



generation's offspring, or as their savings, so that there are no relative prices between manufactured and agricultural goods. Wages and factor returns are determined on competitive markets and are given by

$$w_t^L = \frac{Y_t^A}{L_t} = \mu \frac{A_t^\varepsilon}{L_t^{1-\alpha}} \quad (1.9)$$

$$w_t^H = \frac{\partial Y_t^M}{\partial H_t} = \psi \delta M_t^\phi \left( \frac{K_t}{H_t} \right)^{1-\psi} \quad (1.10)$$

$$r_t = \frac{\partial Y_t^M}{\partial K_t} = (1-\psi) \delta M_t^\phi \left( \frac{H_t}{K_t} \right)^\psi . \quad (1.11)$$

The aggregate stocks of human and physical capital depend on the education and savings decisions of the respective generations:

$$H_t = L_t(1 + e_t) \quad (1.12)$$

$$K_t = s_t L_t = \left[ (1 - e_t) w_t^L - n_t \right] L_t . \quad (1.13)$$

Hence, without education, the stock of human capital in the second period of life corresponds to the stock of labour.

### 1.3.3 The Intragenerational Equilibrium

The individual decision problem involves maximizing (1.3) subject to the constraints (1.4) and (1.5). Inserting (1.4) into (1.5) and plugging in factor rents gives an expression for  $m_t$  which can be substituted back into (1.3). As a result the objective function becomes an expression of  $n_t$  and  $e_t$ . Taking first order conditions for  $n_t$  as well as  $e_t$  and solving the resulting system of equations for  $e_t$  we obtain

$$e_t = \max \left\{ 0, 2\psi - 1 - \frac{\gamma}{\delta M_t^\phi} \left[ \frac{\psi}{(1-\psi)w_t^L} \right]^{1-\psi} \right\} . \quad (1.14)$$

Solving the first order condition of  $e_t$  for  $n_t$  and combining it with (1.14) yields

$$n_t = (1 - e_t)w_t^L - \frac{1 - \psi}{\psi}(1 + e_t)w_t^L = \begin{cases} \frac{2\psi - 1}{\psi} \cdot w_t^L & \text{if } e_t = 0, \\ \frac{\gamma}{\delta M_t^\phi} \frac{(w_t^L)^\psi}{\psi^\psi (1 - \psi)^{1 - \psi}} & \text{if } e_t > 0. \end{cases} \quad (1.15)$$

Combining (1.15) with (1.4) yields optimal savings as

$$s_t = \frac{1 - \psi}{\psi}(1 + e_t)w_t^L = \begin{cases} \frac{1 - \psi}{\psi} \cdot w_t^L & \text{if } e_t = 0, \\ 2(1 - \psi)w_t^L - \frac{\gamma}{\delta M_t^\phi} \left[ \frac{(1 - \psi)w_t^L}{\psi} \right]^\psi & \text{if } e_t > 0. \end{cases} \quad (1.16)$$

As can be seen the optimal values of fertility and savings change depending on whether education is positive or not.<sup>16</sup>

### 1.3.4 Equilibrium Dynamics

The dynamics of the model are described by the state variables,  $A_t$ ,  $M_t$ , and  $L_t$ . Following Strulik and Weisdorf (2008), we assume that technology in the two sectors evolves according to a learning-by-doing process, with the dynamics of the state variables given as follows,

$$L_{t+1} = L_t n_t \quad (1.17)$$

$$A_{t+1} = A_t + Y_t^A \quad (1.18)$$

$$M_{t+1} = M_t + Y_t^M \quad (1.19)$$

The stocks of  $K_t$  and  $H_t$  are determined endogenously according to (1.13) and (1.12). In terms of growth rates, this setting implies

---

<sup>16</sup>Note that for education to be a viable choice we must restrict  $\psi > \frac{1}{2}$ .

$$g_{t+1}^L = \frac{L_t n_t - L_t}{L_t} = n_t - 1 \quad (1.20)$$

$$g_{t+1}^A = \frac{Y_t^A}{A_t} = \frac{\mu L_t^\alpha}{A_t^{1-\varepsilon}} \quad (1.21)$$

$$g_{t+1}^M = \frac{Y_t^M}{M_t} = \frac{\delta H_t^\psi K_t^{1-\psi}}{M_t^{1-\phi}} \quad (1.22)$$

### 1.3.5 Outline of the Behaviour of the Model

Given an initial state with small population size and low productivity levels, fertility around replacement level, no education and little saving the evolution of the economy is governed by the following dynamical system:

$$\left\{ \begin{array}{l} e_t = 0 \\ n_t = \frac{2\psi-1}{\psi} \cdot w_t^L \\ s_t = \frac{1-\psi}{\psi} \cdot w_t^L \\ L_{t+1} = n_t L_t \\ A_{t+1} = w_t^L L_t + A_t \\ M_{t+1} = \delta M_t^\phi s_t^{1-\psi} L_t + M_t \end{array} \right.$$

Population grows as long as  $n_{t+1} \geq n_t > 1$ , which implies:

$$1 < n_t \leq (1 + g_{t+1}^A)^{\frac{\varepsilon}{1-\alpha}}. \quad (1.23)$$

Note that this holds at either very low levels of agricultural technology ( $A_t \rightarrow 0$ ) or high of labour share in agriculture ( $\alpha \rightarrow 1$ ), which are both characteristics of the Malthusian era. As a result population increases, leading to a differential growth in productivity, favouring the productivity in the agricultural sector. To see this note that the growth rates of the two sectors are given by

$$\begin{aligned} g_{t+1}^A &= \frac{Y_t^A}{A_t} = \frac{\mu L_t^\alpha}{A_t^{1-\varepsilon}} \\ g_{t+1}^M &= \frac{Y_t^M}{M_t} = \frac{\delta s_t^{1-\psi} L_t}{M_t^{1-\phi}}, \end{aligned}$$

with low savings and no human capital initially undermining possible productivity growth in manufacturing. However, as long as fertility is increasing savings per person will also increase. Note first that increasing fertility, given by (1.23) is equivalent to an increase in the agricultural wage if education is zero:

$$n_{t+1} \geq n_t \iff w_{t+1}^L \geq w_t^L \quad \text{if } e_t = 0$$

Therefore, since savings are given by  $s_t = (1 - \psi) \cdot w_t^L / \psi$ , increases in fertility lead to increases in the agricultural wage  $w_t^L$ , and ultimately increasing savings  $s_t$ . Increases in saving in turn lead to higher manufacturing productivity, which eventually makes education viable once the necessary condition

$$2\psi - 1 - \frac{\gamma}{\delta M_t^\phi} \left[ \frac{\psi}{(1-\psi)w_t^L} \right]^{1-\psi} > 0, \quad (1.24)$$

holds. Due to monotonicity, this condition holds at some point in time. As soon as education is positive the dynamical system is given by:

$$\begin{cases} e_t &= 2\psi - 1 - \frac{\gamma}{\delta M_t^\phi} \left[ \frac{\psi}{(1-\psi)w_t^L} \right]^{1-\psi} \\ n_t &= \frac{\gamma}{\delta M_t^\phi} \frac{(w_t^L)^\psi}{\psi^\psi (1-\psi)^{1-\psi}} \\ s_t &= 2(1-\psi)w_t^L - \frac{\gamma}{\delta M_t^\phi} \left[ \frac{(1-\psi)w_t^L}{\psi} \right]^\psi \\ L_{t+1} &= n_t L_t \\ A_{t+1} &= w_t^L L_t + A_t \\ M_{t+1} &= \delta M_t^\phi (1 + e_t)^\psi s_t^{1-\psi} L_t + M_t \end{cases}$$

Compared to the state with no education, this implies higher savings and a reduction in fertility.<sup>17</sup> Intuitively, the agricultural wage is now large enough to sustain a minimum level of children dictated by the preference structure. This implies that resources are shifted to attain the highest possible level of consumption, leading to comparatively lower fertility and higher savings. More formally, for fertility the following must hold,

$$\frac{\gamma}{\delta M_t^\phi} \frac{(w_t^L)^\psi}{\psi^\psi (1-\psi)^{1-\psi}} < \frac{2\psi - 1}{\psi} \cdot w_t^L \Leftrightarrow \frac{\gamma}{\delta M_t^\phi} \left[ \frac{\psi}{(1-\psi)w_t^L} \right]^{1-\psi} < 2\psi - 1,$$

which is precisely the necessary condition for positive education given by (1.24). Analogously for savings it must hold that,

$$\begin{aligned} 2(1-\psi)w_t^L - \frac{\gamma}{\delta M_t^\phi} \left[ \frac{(1-\psi)w_t^L}{\psi} \right]^\psi &> \frac{1-\psi}{\psi} \cdot w_t^L \Leftrightarrow \\ 2\psi - 1 &> \frac{\gamma}{\delta M_t^\phi} \left[ \frac{\psi}{(1-\psi)w_t^L} \right]^{1-\psi}, \end{aligned}$$

which again is the necessary condition for positive education given by (1.24). The increase in education and the increasing level of savings imply a boost in industrial productivity growth to the point that  $n_{t+1} \geq n_t$ , no longer holds and fertility declines. This is the case when

$$n_t \leq \left[ \frac{(1 + g_{t+1}^A)^\varepsilon}{(1 + g_{t+1}^M)^\frac{\phi}{\psi}} \right]^\frac{1}{1-\alpha}. \quad (1.25)$$

For later use, let us denote the first period for which  $n_{t+1} \leq n_t$  as  $t^*$ . This is the turning point in demographic development – the onset of the demographic transition. The subsequent decline in fertility directly induces a decline in the growth rate of agricultural productivity since it primarily depends on increases in population size. Manufacturing productivity continues to grow faster as education rises. Once the economy is in the vicinity of the maximum level of education the growth rate of manufacturing productivity declines as well.

---

<sup>17</sup>Note that fertility may still be increasing overall at this point, but at a slower pace than compared to a state with no education.

In the long run condition (1.25) becomes

$$n_t \approx \left[ \frac{(1 + g_{t+1}^A)^\varepsilon}{(1 + g_{t+1}^M)^{\frac{\phi}{\psi}}} \right]^{\frac{1}{1-\alpha}} \approx 1,$$

and the dynamical system is given by:

$$\left\{ \begin{array}{l} e_t = 2\psi - 1 \\ n_t = 1 \\ s_t = 2(1 - \psi)w_t^L \\ L_{t+1} = L_t \\ A_{t+1} = w_t^L L_t + A_t \\ M_{t+1} = \delta M_t^\phi (1 + e_t)^\psi s_t^{1-\psi} L_t + M_t \end{array} \right.$$

### 1.3.6 A Quantitative Analysis

The model exhibits a long-run development pattern that exhibits a transition from a quasi-Malthusian regime associated with slow growth, high fertility, and no education, towards a balanced growth path with low fertility and a positive level of education. To illustrate these dynamics of long-run development, we quantify and simulate the model. The same simulation will be extended in the next section to provide a link to the reduced form estimation results presented before.

**Parametrisation and Calibration.** The simulation makes use of the fact that, given a set of initial values for  $A_t$ ,  $M_t$ , and  $L_t$ , one can compute the equilibrium  $n_t$ ,  $e_t$ , and  $s_t$ , and thus  $K_t$ , and  $H_t$ . With this, income as well as the values of  $A_t$ ,  $M_t$ , and  $L_t$  for the next generation can be computed, and the entire development path can be simulated forward.

In order to strike a balance between the OLG structure and the need for a sufficiently large number of within-country observations along the development path for a comparison with the empirical results, we set the length of a generation to be 10 years and simulate the

model for a sequence of 150 generations. While this simulation is useful for a comparison with actual data using reduced form estimation, the choice of the particular simulation window from year 1000 to year 2500 is without consequence for the main result and mainly serves the purpose of illustration. Below we will restrict the analysis to a time frame that is comparable to the panel data used in the empirical analysis above. It is important to remember, however, that all simulations are based on this long simulation window.<sup>18</sup>

The parameters of the model are set to match observable data moments for England (United Kingdom) in the Penn World Tables (version 9.0) by Feenstra *et al.* (2015). The labour (human capital) share of manufacturing production is set to  $\psi = 0.62$ , which is also in line with the value used by Hansen and Prescott (2002). The labour share in agriculture is set to the same value  $\alpha = 0.62$ , which is also in line with Doepke (2004).

The parameter  $\mu$  is normalised to one, while the values for  $\varepsilon$ ,  $\delta$ ,  $\phi$  and  $\gamma$  are obtained by matching the balanced growth path moments of the model to observable data (for details see Appendix A.1). This gives values  $\varepsilon = 0.51$ ,  $\delta = 1.27$ ,  $\phi = 0.36$  and  $\gamma = 3.0$ . This calibration satisfies plausible requirements like constant population along the steady state, but mainly serves illustrative purposes.

The initial conditions  $L_0$ ,  $A_0$  and  $M_0$  are set such that  $n_0 = 1$ ,  $e_0 = 0$  and  $s_0 \approx 0$ . Given  $L_0 > 0$  any value of  $A_0$  greater than zero will lead to positive savings. At the same time it will determine how quickly an economy undergoes the demographic transition with a larger value of  $A_0$ , *ceteris paribus*, anticipating the transition.

Solving (1.15), for  $n_0 = 1$  gives

$$L_0 = \left[ \frac{2\psi - 1}{\psi} \cdot \mu A_0^\varepsilon \right]^{\frac{1}{1-\alpha}}.$$

Solving (1.14) for  $e_0 = 0$  gives

---

<sup>18</sup>We also conducted an alternative simulation for generations lasting 20 years. From the perspective of the traditional OLG literature, the duration of 10 years for a generation is relatively short but effectively allows for relatively fast dynamics while maintaining an internally consistent structure. Allowing for interactions between generations would require more restrictive assumptions about internal consistency regarding rational expectations or the possibility for re-optimisation without a clear gain in terms of quantitative behaviour. Since the simulation approach taken here is sufficient to make the point of interest, we leave this extension for future research.

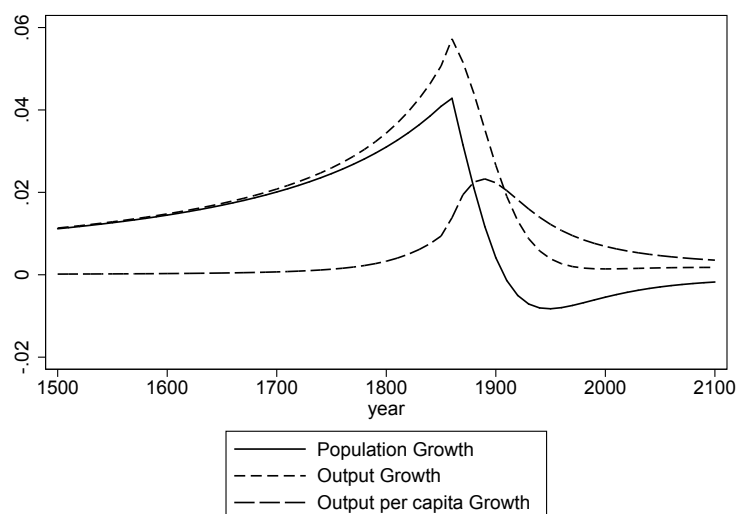
$$M_0 < \left\{ \frac{\gamma}{\delta(2\psi - 1)} \left[ \frac{\psi L_0^{1-\alpha}}{(1-\psi)\mu A_0^\varepsilon} \right]^{1-\psi} \right\}^{\frac{1}{\phi}}$$

$$M_0 < \left\{ \frac{\gamma}{\delta(2\psi - 1)} \left[ \frac{2\psi - 1}{1 - \psi} \right]^{1-\psi} \right\}^{\frac{1}{\phi}},$$

where a lower value of  $M_0$  shifts the time period where education becomes viable further backward. The values used in the baseline simulation are  $A_0 \approx 0.009$ ,  $L_0 \approx 0.00015$  and  $M_0 \approx 52.2430$ . This generates a simultaneous demographic and education transition at 1860.

**Illustrative Simulation.** Figure 1.3 illustrates the working of the model by plotting the long-run evolution of growth rates of aggregate output, population, and output per capita over the period 1500 to 2100. This illustrative simulation pinpoints several characteristic features of the unified growth model. As consequence of the choice of parameters and initial conditions, the model exhibits a demographic transition with an onset of the decline in fertility in 1860. While population grows at an increasing rate before this turning point, the growth rates decline thereafter and converge to a stable population with fertility at replacement. This fertility transition marks a turning point in the model dynamics, as fertility starts declining and gives rise to increased education attainment per capita. Aggregate output grows at an even faster rate before the onset of the fertility transition than population, mainly as consequence of the population-related growth of agricultural productivity, as stipulated by (1.21) in combination with (1.20). The fertility transition leads to an acceleration of education acquisition, which fosters productivity growth in the manufacturing sector as consequence of (1.22) and leads to sustained output growth even after population growth declines. Ultimately, however the increase in education acquisition loses momentum. This and the decline in population growth towards a stable population implies that the growth in aggregate human and physical capital also abates. As consequence, aggregate output growth stabilises at a balanced growth rate that is lower than during the transition when output growth is affected by the one-off dynamics related to the demographic transition.





**Figure 1.3:** *Simulated Development Path of the Baseline Economy*

The combination of the population and output dynamics implies a growth pattern in output (income) per capita that is somewhat delayed compared to the peaks in both population growth and aggregate output growth, with output per capita growth peaking just before 2000.

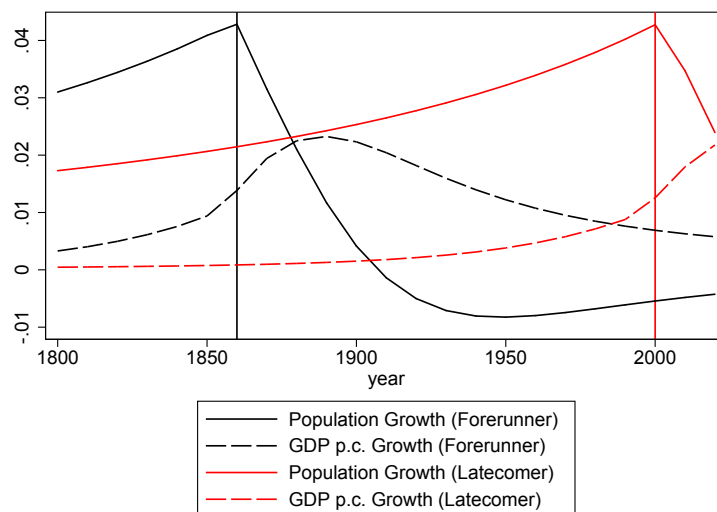
## 1.4 Towards Unified Growth Empirics

We proceed investigating the implications of these patterns for the interpretation of data and the reduced form estimation results of Section 2.3. The intuition behind this analysis is to use the model as data generating process that generates the typical patterns of unified growth theories. The interest is not so much in the precise mechanisms underlying the model, but rather its capability of generating non-linear long-run dynamics in the core factors underlying the usual empirical growth studies, namely output, population, and physical and human capital.

The analysis of the empirical implications proceeds in three steps. First, we analyse the dynamics of the model from the perspective of two model economies that only differ in the timing of the onset of the fertility transition. In a second step, we extend this analysis

by generating a synthetic cross-country panel data set that consists of a number of model economies that only differ in terms of the timing of their (demographic and economic) transition, but that are otherwise identical, and analysing this panel data set through the lens of reduced form regressions. The final step of the analysis uses the model to make inference about potential scenarios of development dynamics in the future.

**The Role of the Transition: Two Economies.** Consider a second economy that is identical in all dimensions to the baseline economy, but differs with respect to the initial productivity in the agricultural sector, for instance due to differences in soil quality or geo-climatological conditions. In particular, the second economy has a slightly lower initial agricultural productivity, which implies a later onset of the demographic transition due to the delayed technological development and the associated slower population dynamics. To be concrete, we simulate this economy to have an onset of the demographic transition in 2000.



**Figure 1.4:** *Simulated Development Path of Two Economies*

Figure 1.4 plots the trajectories for population and output per capita growth for these two economies. Obviously, both economies exhibit the same dynamics, although on a different time scale. The delay in demographic transition entails a delay in income per capita growth.

Observing both economies at a given point in time, or during a limited observation period, such as 1960-2010 as in the reduced form analysis, delivers an incomplete picture of the development path. Moreover, obviously the (implicit or explicit) assumption of a balanced growth path with different growth rates is not warranted. Reduced form estimates are thus likely to be heavily influenced by the observation period for which they are conducted, and the sample on which they are performed. Empirical results obtained with a sample of forerunner countries that are observed during comparable periods on their long-run development trajectory, similar to the baseline economy, are likely to differ from results obtained with a sample of latecomer countries. Notice that this is true even when the maintained assumption about parameter stability across countries is satisfied, since in the example the only difference between the two economies is in initial conditions, not structural parameters.

**A Synthetic Panel Analysis.** In the following, we sharpen this intuition by deriving an appropriate specification for reduced form empirical analysis from the model. This will allow us to reconsider the empirical patterns and compare the results obtained for the data with those obtained with a synthetic panel of simulated economies for which the data generating process is known and given by the framework presented before.

To derive a reduced form estimation framework, denote the inverse of the manufacturing share in total output by

$$\frac{Y_t}{Y_t^M} = \frac{1}{1 - \sigma_t^A} \quad \text{with } \sigma_t^A = \frac{Y_t^A}{Y_t},$$

and rewrite (1.8) in terms of output of the manufacturing sector. Using (1.7), total output is then given by

$$Y_t = \frac{1}{1 - \sigma_t^A} \cdot \delta M_t^\phi H_t^\psi K_t^{1-\psi}.$$

Dividing through by  $L_t$  and taking logs delivers as expression for log output per capita,

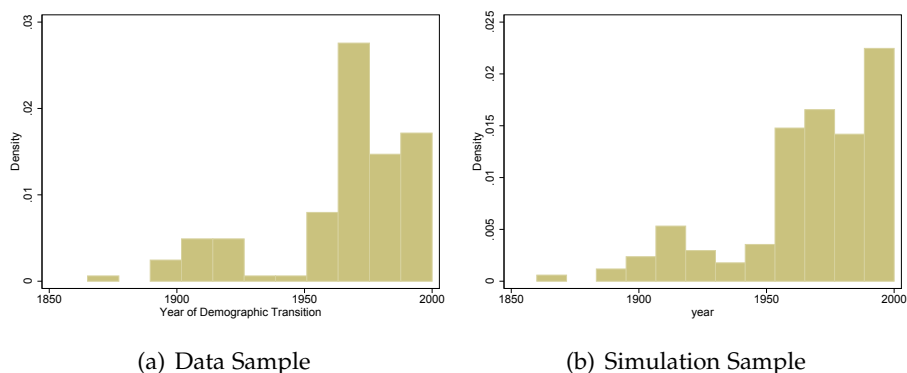
$$\ln y_t = \ln \left( \frac{Y_t}{L_t} \right) = \ln \left( \frac{1}{1 - \sigma_t^A} \right) + \ln \delta + \phi \ln M_t + \psi \ln h_t + (1 - \psi) \ln k_t .$$

Substituting the expressions for  $h_t = \frac{H_t}{L_t}$  and  $k_t = \frac{K_t}{L_t}$  from (1.12) and (1.13) indicates that human and physical capital are determined by  $e_t$ ,  $s_t$  and  $n_t$ . In light of the corresponding analytical expressions (1.14) and (1.16) and the discussion of the dynamical system, it is clear that human and physical capital dynamics are crucially affected by the state of demographic development, i.e., whether  $e_t$  is zero or positive (and whether the associated fertility transition has already begun). We therefore proxy this qualitatively different behaviour before and after the onset of the fertility transition by introducing a binary indicator variable  $post_t$  that takes value 0 before the onset of the fertility transition, and 1 after. To be precise, we set  $post_t = 1$  for all  $t \geq t^*$ , where  $t^*$  corresponds to the time of the onset of the demographic transition as defined in Section 1.3.5. To account for the qualitatively different dynamics in savings and education before and after the demographic transition, we estimate an empirical model with interactions of these variables with  $post_t$ . In order to mimic the construction of the human capital index in the Penn World Tables, we apply a quadratic fit to  $e_t$  in order to generate a conforming human capital index in the simulated data. Moreover, since productivity  $M_t$  is unobserved, we use a proxy variable approach for the data regressions. In particular, noticing from (1.19),  $M_t$  can be proxied by  $Y_{t-1}$  and a random (potentially autocorrelated) component  $\varepsilon_t$ . Combining this, the empirical specification for a given country is expressed by

$$\begin{aligned} \ln y_t = & a \ln Y_{t-1} + b_1 (pre_t \times \ln h_t) + b_2 (post_t \times \ln h_t) \\ & + b_3 (pre_t \times \ln k_t) + b_4 (post_t \times \ln k_t) + C + \varepsilon_t , \end{aligned} \tag{1.26}$$

where  $C$  is a constant that captures  $\delta$  as well as potential other control variables such as country and period effects, and  $a$  and  $b_i$  ( $i = 1, 2, \dots, 4$ ) are coefficients to be estimated.

To investigate the empirical relevance of this specification we use simulated data from a synthetic panel of countries. In particular, following the same logic as Cervellati and



**Figure 1.5:** *Distribution of the Onset of the Transition in the Data and Simulation Sample*

Sunde (2015a), we simulate an artificial cross country panel data set that consists of 114 countries that are identical in terms of parameters and initial conditions. The only exception is  $A_0$ , which is assumed to differ for exogenous reasons, e.g., geo-climatic conditions.<sup>19</sup> A lower  $A_0$  implies a later fertility transition, ceteris paribus. The  $A_0$  of the baseline country generates a fertility transition in 1860. Subsequently, we simulate additional countries while adjusting (decreasing)  $A_0$  for these countries in order to match the observed distribution of dates of the onset of the demographic transition in the data.

Figure 1.5 depicts the respective distributions of onset dates of the transition in the data and compares it to those in the simulated sample. By construction, the simulated distribution matches the empirical distribution, with minor deviations stemming from the fact that in the simulation the dates are forced to correspond to decades.

We use the simulated data, which have been obtained by varying only one country-specific, time-invariant parameter (the initial condition  $A_0$ ), to estimate the empirical model (1.26) and compare the estimates to the corresponding estimates for the same data as in Section 2.3.

Table 1.4 shows the respective estimation results for data and simulation. The estimation results exploit variation within countries over 10 year intervals. The similarity of the

---

<sup>19</sup>Cervellati and Sunde (2015a) provide an analysis based on differences in extrinsic mortality that can be driven by geo-climatological factors.

estimates obtained with simulation data and with actual data is striking. In the simplest specification, physical capital and human capital both exert a significant positive effect that is of comparable size (Columns (1) and (2)). When allowing for heterogeneous effects before and after the demographic transition (when  $pre_t = 1$  and  $post_t = 1$ , respectively), the effect of physical capital is approximately the same. For human capital, the effect is zero by construction in the simulated data, since  $e_t = 0$ . After the demographic transition, the effect is significantly positive. In the actual data, the effect of human capital is positive but not statistically significantly different from zero before the demographic transition. After the transition, human capital has a positive and significant effect on economic performance. The qualitative and quantitative similarity of the estimates obtained for simulated and actual data demonstrates the model's potential as a data generating process that delivers new insights into growth empirics.

Taking the simulation as data generating process allows for further insights regarding the appropriate specification of the empirical framework, however. In particular, the development dynamics are governed by the productivity parameters as crucial state variables. While in the model it is clear by construction that it is  $M$  that is most relevant in this dimension, factor productivity is unobserved in the data, which implies the need for appropriate proxies. To investigate this issue in more detail, Table 1.5 presents the results from estimates of expanded empirical models for data and simulation. Columns (1) and (2) take advantage of the fact that the value of the relevant state variable reflecting technology is known to be  $M$  in the simulation, which allows for an exact specification of the empirical model. Columns (3) and (4) display the corresponding results in the data using aggregate GDP  $Y_{t-1}$  as a proxy for technological level as suggested in (1.26). Columns (5) and (6) report the corresponding results when using GDP per capita  $y_{t-1}$  as alternative proxy. Several findings are noteworthy. First, the reduced form estimation is able to account for all variation in the simulated data, as suggested by the Adjusted  $R^2$  of 1. Second, the estimation results obtained with the simulated data are again very similar to those obtained with actual data, regardless of the specification of the productivity proxy, although the similarity is

**Table 1.4:** *Reduced-Form Estimates Reconsidered: Data vs. Simulation*

Dependent Variable	Log GDP per Capita			
	Simulation		Data	
	(1)	(2)	(3)	(4)
$k_t$	0.533*** [0.009]	0.588*** [0.041]		
$HC_t$	0.477*** [0.013]	0.531*** [0.184]		
Pre_Transition $_t \times k_t$			0.532*** [0.010]	0.575*** [0.044]
Post_Transition $_t \times k_t$			0.530*** [0.010]	0.561*** [0.048]
Pre_Transition $_t \times HC_t$				0.242 [0.246]
Post_Transition $_t \times HC_t$			0.478*** [0.012]	0.585*** [0.184]
Country Fixed Effects	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes
Observations	798	700	798	700
Countries	114	114	114	114
Adjusted $R^2$	0.999	0.812	0.999	0.815

Results of fixed effects regressions. The dependent variable  $y_t$  is (the log of) real GDP at constant 2011 national prices (in mil. 2011US\$) divided by population size (in millions),  $k_t$  is (the log of) capital stock at constant 2011 national prices (in mil. 2011US\$),  $HC_t$  is (the log of) the human capital index, see data description of the PWT 9.0 for details. “Pre/Post” are binary indicators that take value 1 for each observation before the onset (at or after the onset, respectively) of the demographic transition  $t^*$ . Sample period: 10-year panel {1950, 1960, ..., 2010}. All specifications include a full set of country fixed effects and common time dummies. Standard errors are clustered at the country level. \*, \*\*, \*\*\* denote significance at 10%, 5%, and 1% levels.

higher for aggregate GDP as proxy of the state variable.

Additional results reported in the Appendix document the stability of the coefficient estimates for different sample restrictions. In light of the highly non-linear dynamics of long-run development, it is interesting and reassuring to see that a correctly specified empirical framework is able to deliver consistent estimates of the reduced form coefficients regardless of the sample composition. In particular, the estimation results obtained for the simulated data and using the theoretically appropriate specification delivers almost identical coefficient estimates regardless whether the sample is restricted to forerunner countries that are already converging to the balanced growth path during the observation period,

**Table 1.5: TFP-Proxies: Data vs. Simulation**

Dependent Variable	Log GDP per Capita					
	Simulation		Data			
	(1)	(2)	(3)	(4)	(5)	(6)
Log $M_t$	0.282*** [0.001]	0.283*** [0.001]				
Log $Y_{t-1}$			0.226*** [0.066]	0.219*** [0.062]		
$y_{t-1}$					0.344*** [0.065]	0.342*** [0.064]
$k_t$	0.542*** [0.000]		0.499*** [0.052]		0.383*** [0.059]	
$HC_t$	0.063*** [0.002]		0.230 [0.184]		0.402** [0.154]	
Pre_Transition $_t \times k_t$		0.543*** [0.000]		0.493*** [0.053]		0.388*** [0.057]
Post_Transition $_t \times k_t$		0.544*** [0.000]		0.479*** [0.057]		0.384*** [0.059]
Pre_Transition $_t \times HC_t$				-0.025 [0.245]		0.176 [0.203]
Post_Transition $_t \times HC_t$		0.061*** [0.002]		0.282 [0.184]		0.401** [0.158]
Country Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	798	798	588	588	588	588
Countries	114	114	114	114	114	114
Adjusted $R^2$	1.000	1.000	0.805	0.807	0.823	0.825

Results of fixed effects regressions. Data: The dependent variable  $y_t$  is (the log of) real GDP at constant 2011 national prices (in mil. 2011US\$) divided by population size (in millions),  $k_t$  is (the log of) capital stock at constant 2011 national prices (in mil. 2011US\$),  $HC_t$  is (the log of) the human capital index, see data description of the PWT 9.0 for details. "Pre/Post" are binary indicators that take value 1 for each observation before the onset (at or after the onset, respectively) of the demographic transition  $t^*$ . Sample period: 10-year panel {1950, 1960, ..., 2010}. Standard errors are clustered at the country level. \*, \*\*, \*\*\* denote significance at 10%, 5%, and 1% levels.

or to latecomer countries that are pre-transitional for most of the observation period. The same holds when restricting the observation period by dropping observation waves.<sup>20</sup> For the actual data this is only true to a certain extent. Coefficient estimates for the effects of physical and human capital vary substantially across specifications by a factor 2 or more.

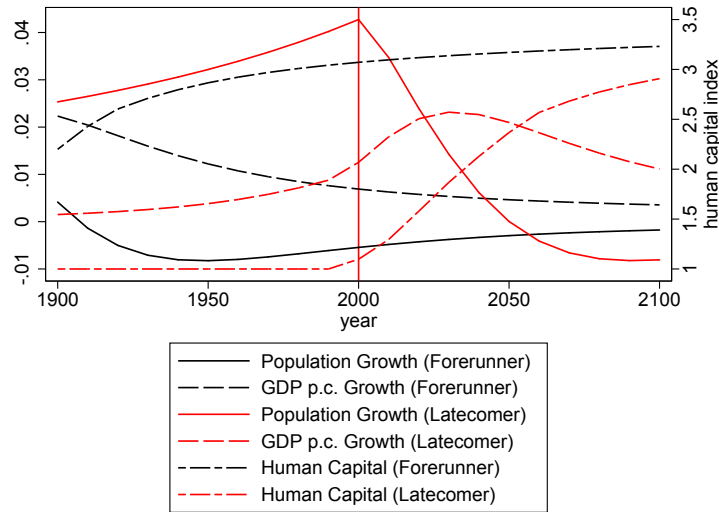
<sup>20</sup>See Table A.1 in the Appendix for details.



**Future Growth and Secular Stagnation.** The dynamics of long-run development illustrated by the model and the empirical results regarding the somewhat more moderate dynamics after the onset of the demographic transition have additional implications for the perspectives of future growth. In particular, the demographic transition and the associated economic take-off imply singular, non-recurrent dynamics. Depending on the observation period, this suggests that it might be natural to expect slower growth in the future for countries that recently underwent the demographic transition and that will converge to a growth path with the momentum of the transition slowly fading out. Figure 1.6 illustrates this by plotting the dynamics for the same two economies as before, a forerunner country that experienced the onset of the demographic transition in 1860 and a latecomer country that experienced the onset in 2000. The trajectory of output per capita growth for the forerunner country indicates that GDP per capita growth is declining. The reason is that both engines of transitional growth – population and human capital – become weaker, as reflected by the fact that population growth is declining ever since the onset of the demographic transition and has almost reached a stable demographic trajectory by 2000, while the human capital index already stabilises at a high level. For the latecomer country, on the other hand, population dynamics peak in 2000, education is just about to take off, and GDP per capita growth experiences a sharp acceleration. Thus, the weakening of transitional dynamics implies a slow-down in growth, providing a formal justification of the arguments made by Cervellati et al. (2017).

## 1.5 Conclusion

The results of this article provide support for the hypothesis that the time since the economic and demographic transition is an important determinant of economic development. Consistent with the implications from the unified growth literature, the results suggest that countries that experience an earlier take-off from stagnation to growth are richer and develop faster. The effect of the time since the transition is quantitatively important. Moreover, the results show a significant interaction of the time since the transition with physical



**Figure 1.6:** *Secular Stagnation*

and human capital. While physical capital might have been important for the take-off, its importance for development does not increase after the transition. On the other hand, human capital gains importance for the economic development process after the transition.

The findings have far-reaching implications for the interpretation of empirical results. For instance, empirical frameworks that implicitly build on a notion of balanced growth might lead to misleading results since its estimates are sensitive to the sampling in terms of observation period or sample countries in terms of forerunners or latecomers. This suggests that the weak and often insignificant average effect of both physical and human capital obtained in linear regressions exploiting within country variability over time might result from an undetected heterogeneity in the importance of these growth determinants in the transition to sustained growth.

Overall, the results suggest that the time since the onset of the transition is an important determinant of economic development that has been omitted in previous empirical studies of economic growth. The results provide support for empirical specifications that account for the level of demographic development and that are consistent with the implications from the unified growth framework, rather than exclusively focusing on dynamics around the

balanced growth path. The results also suggest that empirical estimates of the growth effects of particular factors might be affected by sample composition in terms of the countries and observation periods that are used for estimation.

## Chapter 2

# From Stagnation to Growth: Demography, Democracy, and Comparative Development<sup>1</sup>

### 2.1 Introduction

The reasons for the enormous differences in development across the world that have motivated economists since Adam Smith are still not fully understood today. Intensified research efforts over the past decades have delivered many valuable theoretical insights and increasing data availability has led to a substantial empirical literature about the drivers of growth and the factors that might be responsible for comparative development differences. Improved econometric techniques and identification methods entailed a better understanding and delivered widely cited empirical results about the causal drivers of growth over the last two decades. Most empirical studies on growth and development exploit cross-country panel data and panel econometric techniques, such as the inclusion of country and time fixed effects, and additionally make use of a variety of identification techniques including difference-in-difference and instrumental variable strategies. Among

---

<sup>1</sup>This chapter is joint work with Matteo Cervellati and Uwe Sunde.

the candidate explanations for the development differences across the world emerging from this literature are institutions such as democracy and property rights, public policies and infrastructure, as well as demographic factors, among others.

Despite its undisputed importance, the focus on identifying particular causal determinants of development differences across the world and the use of appropriate identification strategies has intrinsic limitations. First, providing credible causal evidence for the relevance of a particular factor or channel does not necessarily imply the irrelevance of other channels. In spite of an intense debate, there is still no agreement on the dominance of a particular (set of) factors or channels. Instead, despite the method-related focus on the identification of single causal factors, most researchers would probably disagree with a monocausal explanation of comparative development differences. Second, the typical approach of reduced-form estimation strategies crucially relies on the correct specification of the empirical framework and the assumption about stable relationships between the variables of interest. The use of linear specifications prevalent in the literature is typically motivated by approximations around the balanced growth path. These assumption can be problematic in the context of non-linear long run development dynamics which imply changes in the importance of different factors or mechanisms during the transition from stagnation to growth. Typically, reduced form empirical approaches are limited in their ability to unfold the dynamic interactions between multiple drivers of growth.

This paper argues that the focus on monocausal explanations – the quest for “the” factor or mechanism that is responsible for long-run development differences – might be seriously misleading if long-run economic development is the result of major transitions in different dimensions that are characterised by bilateral feedbacks.

As a first step to illustrate the potential relevance of these issues for a better understanding of the process of long-run comparative development we present the empirical patterns of long-run development in three dimensions: income, democracy, and demographic dynamics. From a global perspective, trajectories of economic, political and demographic development are highly non-linear over the past two centuries. The timing of the acceleration in all

dimensions is roughly synchronous, suggesting a reading of long-run development as a “tale of three transitions” that are potentially strongly intertwined. Looking at the same data through the lens of a difference-in-differences framework, it appears that in isolation, both the democratic and the demographic transitions have positive effects on economic development. Moreover, the transition in one dimensions might facilitate subsequent development in other dimensions.

The strong co-movement in the three stylised dimensions of development and the previous arguments raise questions about the interpretation of reduced form evidence and their ability to identify the drivers of long run development. The reason is that it is precisely the main advantage of the most credible identification strategies – the isolation of the effect of a single variable from a mono-causal perspective – that limits the scope and usefulness of this approach for the question of the reasons for long-run development differences. Instead, the data patterns appear to indicate that a better understanding of the mechanisms behind the multi-directional, dynamic interactions between different dimensions and channels of development necessarily requires a more structural approach. Existing work in this direction is still very limited, however.

In this paper, we attempt to take a first step in this direction by proposing a unified growth theory that allows for an analytical characterisation of the dynamics of economic, political and demographic development as well as for a quantitative analysis that has a direct implications for the link to empirical work. The theory represents a “prototype” model that combines previously studied and empirically relevant mechanisms. In particular, the model combines a mechanism that is based on demographic development along the seminal work of (Galor and Weil, 2000) and a textbook version of institutional change in terms of democratisation along the lines of (Acemoglu and Robinson, 2000). These model components naturally complement each other and their combination allows investigating the implications of interactions between democratisation and the demographic transition for the dynamics of the economic development process within a unified framework. For the purposes of this paper, the original models are extended in several important directions

and made suitable for quantitative analysis. In particular, the quantity-quality trade-off that drives education and fertility choices and thus demographic dynamics is extended beyond the representative agent framework by considering different population groups. This allows us to study the differential patterns of education and fertility across different groups, thereby endogenizing a key driver of inequality and democratisation that is typically taken as exogenous in the democratisation literature. The consideration of public good provision along the lines of (Lizzeri and Persico, 2004) as alternative to fiscal redistribution implies the possibility of different scenarios for the transition to democracy, which can be conflictual or peaceful, with implications for policies and subsequent economic development.

The theory delivers novel predictions about the dynamic interactions between economic, demographic and institutional (democratic) development. Technological change in the context of demographic development implies changes in the demand for human capital as well as in inequality, and ultimately leads to transitions in all three dimensions. The model also indicates, however, that the relative timing of the transitions cannot be established a priori. The demographic and democratic transitions (weakly) complement each other since a transition in one dimension tends to increase the likelihood of a transition in the other. To study the fundamental drivers of long run development we use the model to derive predictions about the role of country-specific characteristics. In particular, the theory predicts that countries with characteristics that make investments in human capital more productive exhibit earlier transitions in all dimensions. The model also delivers predictions about asymmetric effects on the timing of the demographic and democratic transition and on the democratisation scenario. Specifically, countries in which the returns to education are relatively low are expected to experience a greater delay in democratisation than in the demographic transition. The reason is related to a change in the relative importance of the drivers of democratisation, making a peaceful (efficiency-enhancing) democratic transition less likely and increasing the incentives for the ruling elite to hold on to non-democratic institutions.

In a next step, we illustrate the analytical predictions by simulating a dynamic version

of the model. The comparison of a simulated economy with an alternative economy that is identical except for the productivity of education investments illustrates the analytical predictions and allows for a fresh look at the empirical patterns. In line with the analytical results, the simulation results show that the timing of the three transitions tends to be closely aligned. The results also show that countries with higher returns to education investments experience earlier transitions in all dimensions, consistent with the observation of forerunner countries that underwent the industrial revolution, the demographic transition and the (typically peaceful) transition to democracy already during the nineteenth century. Countries with lower returns to education experience a delayed growth take-off and a later demographic transition, but an even more delayed transition to democracy, which, on top, tends to be more violent on average. The model therefore delivers new insights into the changing interactions between demographic and political development that have not been explored and documented.

In a last step, we use the simulated data to qualify the interpretation of empirical cross-country panel studies. Using a simulated panel of countries that are all identical apart from one time-invariant parameter that determines the return to education investments, we can produce development patterns that closely resemble the empirically observed patterns of the “tale of three transitions”. By keeping the data-generating process under tight control, the only driver of differences in the cross-country patterns can be the time-invariant country specific characteristic that delivers heterogeneity across countries. We then apply a difference-in-differences approach and control for country and time fixed effects in the simulated data. Similar to the empirical patterns shown before, this exercise documents sizeable accelerations in income after democratisation and after the demographic transition. This analysis illustrates the potential problems with the interpretation of reduced form estimation strategies. In line with the usual interpretation offered in the empirical literature, the effects obtained when regressing on the simulated data are indeed “causal” in the sense that a transition indeed implies a significant increase in output. At the same time, this finding does not warrant further claims about a monocausal driver of long-run development



that are sometimes made in the literature. For instance, the evidence for a causal effect of each transition cannot be interpreted in a mono-causal fashion since, by construction, the estimated reduced-form effect of the demographic transition subsumes also indirect effects on, and of, transitions in other dimensions.

Another important and largely overlooked related point is related to the non-linear dynamics of long run development. By construction, the only driver of differences in comparative development in the simulated model is a time-invariant, country-specific characteristic. Nevertheless, when applying the usual difference-in-differences methodology exploiting within country variability over time by accounting for country and period fixed effects to the simulated data, one can identify the effects of each transition in isolation. The usual interpretation of this as causal effects of a particular transition while accounting for all relevant (observable and unobservable) country-specific characteristics such as geography, past history, culture etc., is not necessarily correct, however, if, as in our model, the differences in country-specific characteristics has non-linear or time-variant effects on the patterns of development. In the specific example, we know that the only fundamental cause of comparative development differences is the country-specific parameter that governs the returns to investments in education. These findings indicate the need for a shift in focus towards non-linear dynamics in different dimensions, which implies a central role for the relative timing of the respective transitions in the different dimensions for a better understanding of comparative development. At the same time, a direct consequence of this change of focus is a unified framework that reconciles the prevailing, and seemingly opposing, views on comparative development.

The remainder of the paper is structured as follows. Section 2.2 puts the paper into the perspective of the existing literature. Section 2.3 presents the stylised empirical patterns motivating the analysis. Section 2.4 lays down the model. Section 2.5 presents the equilibrium at a given moment in time, and Section 2.6 presents the results regarding the dynamics of development in the different dimensions. Section 2.7 illustrates the results by ways of a simulation of the model for different scenarios. Section 3.6 concludes. Proofs of analytical

results and details of the simulation are relegated to the Appendix.

## 2.2 Related Literature

This paper contributes to, and complements, several strands of the existing literature. The insight of dynamic interactions between major mechanisms behind long-run development reconciles different views about the fundamental drivers of development in the literature. One view is that human capital is the key determinant of growth and development, and primarily explains differences in development by differences in the human capital stock (see, e.g., Gennaioli et al., 2013), without necessarily explaining why these differences emerge in the first place. A second view, the unified growth literature, studies the mechanisms behind the transition from stagnation to growth, and provides a long-run perspective on the reasons for differences in development and human capital acquisition across regions and countries. In particular, following the seminal work by Galor and Weil (2000), this literature emphasises the interplay between technical change, the demand for education, and the demographic transition, which ultimately brings about a switch in fertility from quantity to quality and the related economic transition from quasi-stagnation to sustained growth. By focusing on demographic mechanisms, this literature has largely abstracted from institutions whose key role for enabling countries to develop economically has been emphasised by a substantial literature following the work by Acemoglu and Robinson (2000, 2006) and Acemoglu *et al.* (2001). While there is empirical support for each of these views, the relative importance of human capital, a long-run transition mechanism coupled with the demographic transition, or the transition towards inclusive, growth-enhancing institutions for explaining the differences in comparative economic development it is still a matter of intense debate. The results of this paper hopefully have some potential to reconcile these views and re-focus the debate on the interaction between different causal mechanisms.

The paper also contributes to the theoretical literature on the drivers behind the demographic transition, which mainly focuses role of the fertility transition (see, e.g., Galor and Weil, 2000, Dalgaard and Strulik, 2015, 2016), of a mortality decline (see, e.g., Boucekine,

de la Croix, and Licandro, 2003, Cervellati and Sunde, 2005, Dalgaard and Strulik, 2014), or of the transition in fertility and mortality (de la Croix and Licandro, 2013, Cervellati and Sunde, 2015a) for development.<sup>2</sup> None of these contributions considers endogenous changes in institutions as the model developed below.

At the same time, the theory contributes to the literature on democratisation that has mainly focused on exploring mechanisms underlying the extension of the franchise. These mechanisms include the implicit or explicit threat of conflict (see, e.g., Acemoglu and Robinson, 2000, 2001, Bertocchi and Spagat, 2001, and Conley and Temimi, 2001) or efficiency reasons for the adoption of democracy (see, e.g., Bourguignon and Verdier, 2000, Lizzeri and Persico, 2004, Llavador and Oxoby, 2005, Cervellati et al., 2008, or Gradstein, 2007). Only few recent papers have considered the possibility of different transition regimes and studied their implications (Cervellati et al., 2012, 2014). Other contributions focused on the interaction between technology, institutions, and the demand for education (see, e.g., Galor and Moav, 2006, Galor et al., 2009). However, neither of these contributions explicitly accounts for the demographic transition and its implications for redistribution and democratisation. The present paper thereby provides a natural link between these different strands of the literature and shows that a broader perspective delivers valuable insights to a better understanding of the empirical patterns of long-run development.

On the empirical side, there is growing evidence for the role of fertility and the fertility transition for education and thereby development (see, e.g., Becker et al., 2010, Ashraf and Galor, 2011). Likewise, mortality and its decline during the demographic transition has been shown to crucially affect education acquisition, population dynamics, and thereby economic development (see, e.g., Cervellati and Sunde, 2011, 2013, 2015b). There is little and only indirect evidence on the role played by institutions in this context (see, e.g., Strittmatter and Sunde, 2013). The quantitative analysis in the present paper contributes to this literature and delivers new implications that can be tested in future work.

---

<sup>2</sup>Studies on the interplay between mortality and fertility on development in the Malthusian context include Voigtländer and Voth, 2013a, 2013b.

Regarding institutions, there is ample empirical evidence regarding the role of development and education for democratisation (see, e.g., recent work by Acemoglu et al., 2008, Cervellati et al., 2014, and Murtin and Wacziarg, 2014), as well as for the effect of democratisation on development (see, e.g., Papaioannou and Siourounis, 2008, and Cervellati and Sunde, 2014). Also here, however, empirical work has not appreciated the potential influence of the demographic transition for institutional change and its effects on economic development. An exception is recent work by (Dyson, 2013) and (Wilson and Dyson, 2017) that has pointed out that progress in the demographic transition, in terms of changes in mortality, fertility, population growth and the age structure, accelerate the transition to democracy using cross-country panel data for the period 1970-2005. The results presented below shed new light on these issues and demonstrate not only a likely role of demographic development for democratisation and economic development, but also important feedback effects.

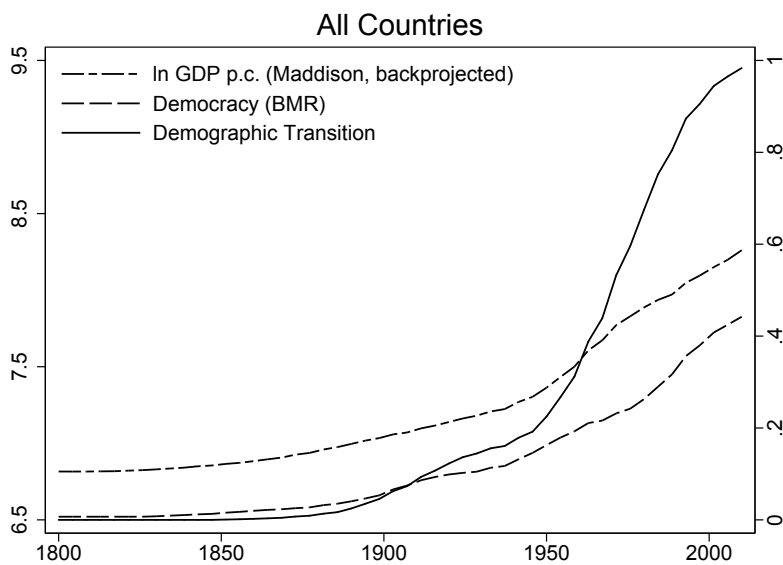
### **2.3 Empirical Patterns: A Tale of Three Transitions**

The motivation for the analysis can be seen by considering the stylised facts of historical development patterns in Europe and the world. Figure 2.1 depicts the average level of income per capita across 145 countries in the world over the period (decades) 1800-2010. The figure also includes the average democratisation, as well as their status regarding the demographic transition.<sup>3</sup> A look at these data suggests that there is essentially a parallel co-movement of GDP per capita, democracy and demographic transition. Across countries, the transition to democracy essentially paralleled the demographic transition until after the

---

<sup>3</sup>Income data are from the New Maddison Project Database, see Bolt and van Zanden (2014). Missing data are obtained by back-projecting the level of the first income observation of a respective country assuming a stagnant trend. Data on democracy are from Boix et al. (2013), and information on whether a country has undergone the demographic transition is from Reher (2004). The demographic transition corresponds to the onset of the fertility transition, which represents the last phase of the demographic transition and which usually begins with a decline in mortality that preceded the fertility decline by up to several decades. In light of earlier work, see, e.g., Cervellati and Sunde (2015b), the onset of the mortality transition triggers the transition in education and ultimately the fertility transition, and therefore appears more relevant for economic development. This implies that the coding of the demographic transition as the onset of the fertility transition is rather conservative in that it makes countries appear as undergoing the transition very late.

second world war demographic development appears to have accelerated and ultimately almost all countries in the sample underwent the demographic transition. Also the transition to democracy seems to have accelerated after the second world war, but not to the same extent, with only half of the countries in the sample exhibiting democratic institutions at the end of the observation period.

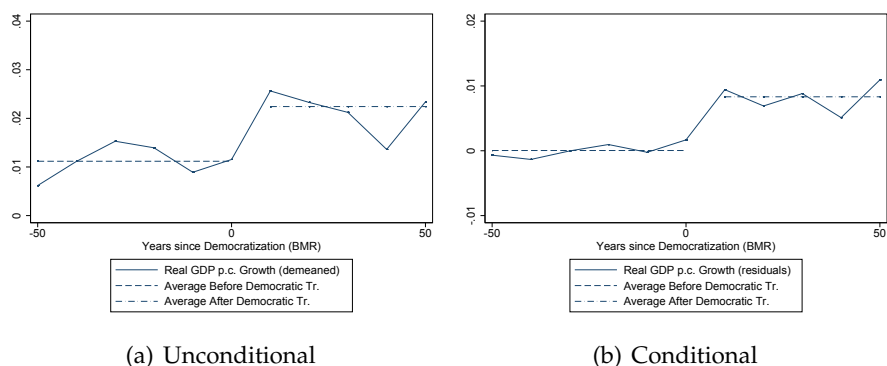


Local polynomial plots of period averages for 145 countries. Sources: New Maddison Project Database, see Bolt and van Zanden (2014), Boix et al., (2013), and Reher (2004).

**Figure 2.1:** Long-Run Development Dynamics: Demographics, Democracy, and Income

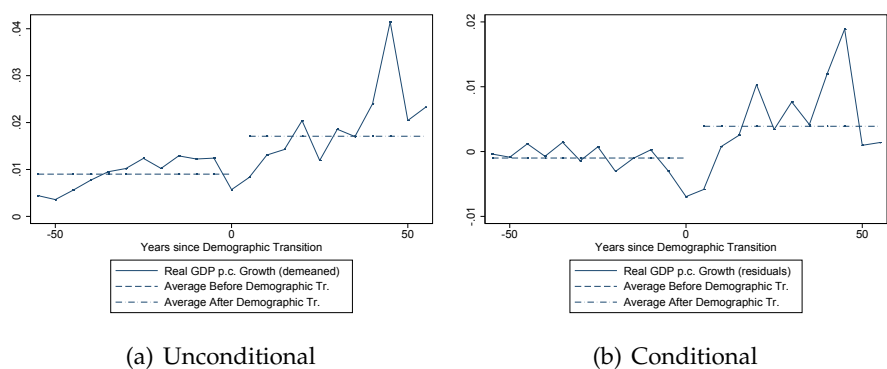
A large body of empirical work has tried to identify the causal effect of improved institutions, in terms of democratisation, on economic development exploiting a differences-in-differences strategy based on panel data and variation in institutions. Following this logic, Figure 2.2 plots the data for log income per capita relative to the timing of the transition to democracy. The left panel of Figure 2.2 plots the average income per capita for five decades before and after the transition to democracy, the right panel plots the average residuals after taking out country fixed effects and common time effects. The plots are indicative of a positive effect of democratisation on economic development, consistent with the findings in the literature (see, e.g., Papaioannou and Siourounis, 2008, Cervellati and Sunde, 2014,

Acemoglu et al., 2017).



**Figure 2.2:** *The Effect of Democracy on Income Growth*

While suggestive of a causal effect of improved institutions, this view of the data neglects the possibility that also the demographic transition had some influence on economic development. In fact, the analogous plot of economic development relative to the timing of the demographic transition seems to suggest a similar effect, see Figure 2.3. While this might be expected in light of the dynamics shown in Figure 2.1, this illustrates the problem with the attempt to identify a single factor to explain economic development dynamics when it is likely that there are different dynamic forces that affect development and that influence each other.



**Figure 2.3:** *The Effect of the Demographic Transition on Income Growth*

Viewed separately, each of these data patterns is suggestive of a causal effect of democrati-

sation or the demographic transition, respectively, on economic development. Taken together, the evidence is consistent with the view that both views, that the demographic transition as well as the transition to democracy mark important turning points for economic development. At the same time, it is not clear how the transition in one dimension affects the transition in the other dimension, or whether one single underlying mechanism drives all these dynamics. Based on this evidence, it seems difficult a priori to disentangle a single dimension as the one of primary importance. Moreover, the identification of such a mono-causal relationship appears a less obvious task than a quasi-experimental perspective might suggest. In fact, taking a purely empirical perspective about about the fundamental drivers behind the dynamics of development appears limited. In the following, we develop a model that illustrates this point in greater clarity.

## **2.4 The Model**

This section presents a stylised model that can be used to investigate the fundamental interactions between democratisation and the demographic transition. The model structure reflects a combination of two models that are known from the literature: a standard unified growth model of the demographic and economic transition and a model of institutional change in terms of endogenous democratisation. In particular, the framework combines a modified version of the seminal unified growth model by Galor and Weil (2000) (see also Galor, 2011) with a model of endogenous democratisation along the lines of the model of Acemoglu and Robinson (2000, 2006). We extend the model of democratisation by accounting for the possibility of a transition that is initiated by the elite, capturing elements from alternative models of democratisation in the literature (see, e.g., Lizzeri and Persico, 2004). This prototype model of long-run development can be used to demonstrate the consequences of going beyond a monocausal view of long-run development by combining fairly established frameworks and mechanisms and studying the value added by considering their interplay. This allows us to focus attention on previously unexplored aspects and implications of the well-understood mechanisms of these models.

### 2.4.1 Individual Preferences and Budget Constraints

The model is based on an overlapping-generations structure where individuals live for two periods. Individuals reproduce asexually. During their childhood in period  $t - 1$ , individuals consume a fraction of their parents' income. As adults in period  $t$  they are endowed with efficiency units of labour according to a human capital production function that is described below. Crucially, human capital depends on education investments made by the parent generation for their children. Additionally, adults are endowed with one unit of time, which they inelastically supply to the labour market.

Heterogeneity between individuals arises with respect to the income potential. While individuals are identical regarding their time and human capital endowment they differ in wealth. In particular, there are property rights over land thereby dividing the population into two groups, landowners (rich) and landless (poor). The group of landowners has size  $L^r$ , so that the group of landowners constitutes a fraction  $\delta_t \equiv \frac{L_t^r}{L_t}$  of the total population. Members of this group derive income from supplying efficiency units of labour as well as from returns to land. In contrast, a fraction  $(1 - \delta_t) \equiv \frac{L_t^p}{L_t}$  of individuals own no land and derive income only from supplying efficiency units of labour. Following Bertocchi (2006), landowners bequeath their land possessions by primogeniture, i.e. the entire land stock of a family is inherited by the first child; additional children remain landless.<sup>4</sup> The income potential available to an individual thus consists of wage income, which arises via the market wage per efficiency unit of labour, as well as of land rents for landowners. Hence, in the following the index  $i$  reflects this heterogeneity, by denoting landowners,  $i = r$ , or landless,  $i = p$ , whose factor incomes are:

$$\begin{aligned}\bar{z}_t^r &= w_t h_t + \frac{p_t N}{\delta_t L_t} \\ \bar{z}_t^p &= w_t h_t,\end{aligned}$$

---

<sup>4</sup>This assumption is not critical for the results. In Appendix B.1.3 we discuss the implications of relaxing this assumption and assuming the alternative extreme of equal bequests to all children.



where  $N$  denotes the total amount of land,  $w_t$  is the wage per efficiency unit of labour  $h_t$ , and  $p_t$  is the land rent, as faced by generation  $t$ .

Preferences of an individual  $i$  are defined over consumption  $c_t^i$  and the quantity  $n_t$  as well as the quality (measured by human capital  $h_{t+1}$ ) of their offspring  $n_t^i h_{t+1}^i$ . This relation is represented by the utility function

$$u_t^i = (1 - \gamma) \ln(c_t^i) + \gamma \left[ \ln(n_t^i) + \beta \ln(h_{t+1}^i) \right], \quad (2.1)$$

where  $\gamma \in (0, 1)$  represents the weight of altruism towards children and  $\beta \in (0, 1)$  is the preference for human capital. Individual decisions involve an optimal choice of consumption, number of children and their education. The utility function is strictly increasing and strictly quasi-concave in its arguments and satisfies the necessary boundary conditions thus guaranteeing an interior solution for the maximisation problem.

Individuals receive a disposable income, which they devote to consumption  $c_t^i$  and raising children. Each child requires a fraction  $r$  of the parental income regardless of quality, spent for nurturing and basic needs. Moreover, educating a child costs an additional fraction of income per unit of education  $e_{t+1}$  to be attained.<sup>5</sup> As in Galor and Moav (2002b), we assume that  $r$  is small enough to guarantee positive rates of population growth,

$$r < \gamma. \quad (A1)$$

Individuals face a subsistence consumption constraint,  $\tilde{c}$ , which must be satisfied at all times. Hence, denoting the disposable income by  $z_t^i$ , the budget constraint faced by individual  $i$  reads

$$z_t^i \geq c_t^i + n_t^i \cdot z_t^i \left[ r + e_{t+1}^i \right] \quad (2.2)$$

$$c_t^i \geq \tilde{c}. \quad (2.3)$$

---

<sup>5</sup>Deviating from Galor and Weil (2000), we assume that child costs accrue in terms of parental income, not time.

As discussed in detail below, disposable income  $z_t^i$ , may differ from the factor income  $\bar{z}_t^i$  from labour and land rents because of taxation and redistribution.

## 2.4.2 The Production of Human Capital

Individual labour income derives from the endowment with efficiency units of labour. More precisely it depends on the human capital that an individual can supply to the production of output during adulthood. This human capital is determined by parental decisions. In particular, the human capital production function is increasing and strictly concave in parental education expenditure,  $e_{t+1}$ , and technological progress,  $g_{t+1}$ . Additionally, individuals can obtain human capital  $h_{t+1}^p$  from the government via means of public goods provision, which will be specified below. As result human capital of an individual in period  $t + 1$  is given by

$$h_{t+1}(e_{t+1}, g_{t+1}) = [\kappa(1 + g_{t+1})e_{t+1} + \rho]^\theta + h_{t+1}^p, \quad (2.4)$$

with  $\kappa > 0$ ,  $\beta \in (0, 1)$  and  $\rho > 1$  being parameters. In particular, the parameter  $\theta$  can be seen as representing the effectiveness of parental investments in the production of human capital.

As in Galor and Weil (2000), this specification implies that technology and education are complementary in the production of human capital. This formulation also implies that human capital endowments do not differ across landowners and landless, since education decisions are fully determined by aggregate variables.

## 2.4.3 Production of Output

In period  $t$ , a total mass of  $L_t$  individuals enters the labour force. This total mass of individuals is endowed with efficiency units of labour  $h_t$  obtained from education investments made by their parents as discussed above. Every period the economy produces a single homogeneous good using land and efficiency units of labour as the inputs within a CES production technology

$$Y_t = A_t \left[ (H_t)^{\frac{\sigma-1}{\sigma}} + (N)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad \sigma > 1, \quad (2.5)$$

where  $N$  denotes the exogenous and fixed stock of land and  $H_t = h_t L_t$  the total amount of efficiency units of labour. Factors are augmented by a total factor productivity  $A_t$ .<sup>6</sup>

Markets are competitive and factors are paid their marginal product:<sup>7</sup>

$$w_t = \frac{\partial Y_t}{\partial H_t} = A_t \left[ (H_t)^{\frac{\sigma-1}{\sigma}} + (N)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} (H_t)^{-\frac{1}{\sigma}} \quad (2.6)$$

$$p_t = \frac{\partial Y_t}{\partial N} = A_t \left[ (H_t)^{\frac{\sigma-1}{\sigma}} + (N)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} (N)^{-\frac{1}{\sigma}} \quad (2.7)$$

#### 2.4.4 Public Policies and Political Regimes

The key model ingredient that creates income inequality and hence a distributional conflict is the assumption of property rights over land paired with unequal land endowments. This inequality and the resulting redistributive conflict are the driving force behind democratisation.<sup>8</sup>

Generally, income sources can be subject to a flat tax rate  $\tau_t^i$  with  $i \in (H; N)$ , respectively. However, taxation is costly (for example due to bureaucracy). Following Okun (1975) and Acemoglu and Robinson (2006), this cost  $C(\tau_t)$  is related to the level of taxation,

$$C'(\tau_t^i) > 0, \quad C''(\tau_t^i) > 0, \quad C(0) = 0 \quad \text{and} \quad C(1) = 1, \quad (A2)$$

giving rise to a Laffer curve.<sup>9</sup> Tax revenues are therefore given by taxation of income sources

---

<sup>6</sup>Given the focus on demographic dynamics, we follow Galor and Weil (2000) and abstract from physical capital.

<sup>7</sup>As will become clear below, the main mechanism and result would remain unchanged if instead one were to assume that production is in the hands of landowners who pay wages below productivity if their political power allows them to do so. In essence, this would be equivalent a case that allows for regressive redistribution.

<sup>8</sup>This is the main departure from the original model by Galor and Weil (2000), who assume that no property rights are defined over land, but who consider an otherwise almost identical economy.

<sup>9</sup>In the simulation, the parametric specification is  $C(\tau_t^i) = (\tau_t^i)^2$ . This formulation ensures interior equilibria with tax rates less than 100%.

net of costs

$$R_t = \left[ \tau_t^H - C(\tau_t^H) \right] w_t H_t + \left[ \tau_t^N - C(\tau_t^N) \right] p_t N. \quad (2.8)$$

Alternatively, the government can collect identical lump-sum transfers  $\omega$  from individuals to finance a public good, namely additional human capital through public schooling efforts. The corresponding production function of human capital generated by public schooling is

$$L_t \cdot h_{t+1}^p = L_t \cdot \alpha \left[ \kappa(1 + g_{t+1})e_{t+1}^p + \rho \right]^\theta, \quad (2.9)$$

where  $e_{t+1}^p$  is the level of education expenditure chosen by the government and  $\alpha \in (0, 1)$  represents the degree of investment translated into actual human capital. Because of scale effects in production the cost for a unit of additional human capital is assumed to be fraction of individual total child cost

$$\phi(r + e_{t+1}^p) \quad \phi \in (0, 1)$$

Aside of the budgetary costs, the public good requires unanimous consensus in the population.<sup>10</sup> That is there must be no preferable alternative for either group. To keep things simple, we assume that either redistribution or public good provision is feasible, but no combination of redistribution and public goods provision, is possible.

Having determined the policy space, it remains to specify how policies are chosen. The institutional framework governing the political economy side of the model is an adaptation of the model by Cervellati *et al.* (2012), which combines the textbook model of democratisation under the shadow of conflict by Acemoglu and Robinson (2000, 2006) and an efficiency-based democratisation process along the lines of Lizzeri and Persico (2004). The key element of the model is redistributive conflict that derives from the unequal endowments of the

---

<sup>10</sup>This assumption resembles the efficiency-enhancing role of consensus in the society as in Cervellati *et al.* (2008, 2014). It is also exemplified by the fact that everybody has to participate in financing the public good.

different population groups, which implies that preferences over policies may differ between landowners and landless.

Following the literature, the political institutions are reflected in the distribution of *de jure* political power to determine political decisions. These decisions refer to the tax rate and the provision of public goods. There are two alternative scenarios to consider in this context: the *de jure* political power can either be in the hands of the landowners, or in the hands of the landless.

The allocation of *de jure* political power of a generation  $t$  is inherited from the previous generation. However, this allocation may be changed depending on the distribution of *de facto* political power. In particular, in a situation with the rich as the ruling elite they can always decide to keep their exclusive political power, or decide to share it with the poor. Likewise, the poor can decide to fight for political power, or accommodate the oligarchic elite. The allocation of *de jure* political power hence depends on the preferences and the dominance in terms of *de facto* political power. Building on a literature on political conflict, the success probability of conflict is increasing in the resources that can be credibly mobilised by a party. We capture this idea by modelling the *de facto* political power of the rich compared to the poor by the following inequality

$$\delta_t L_t \bar{z}_t^r \geq (1 - \delta_t) L_t \bar{z}_t^p, \quad (2.10)$$

where the left-hand side of (3.9) represents the fighting potential in terms of aggregate income of the landowners, while the right-hand side is the fighting potential of the landless. This is a simplified version of a standard conflict success function, see, e.g., Hirshleifer (2001) and captures the elements that are essential for the present analysis of the equilibria of a richer class of conflict models.

If the political elite, which is assumed to be initially represented by the landowners, prevails in terms of political power, a revolution is infeasible and decisions about policies are made by the landowners. Since they constitute a minority of the population, such a system will be called oligarchy in the following. If, however, the landless are able to mobilise

enough *de facto* political power to dominate, it is the landless who can establish a political regime where they have the *de jure* power to determine policies. Since in this case policies can be decided by majority voting, this case will be labelled a democracy.<sup>11</sup>

Notice that this modelling also comprises the case in which democratisation is the commitment device that allows the elite to avoid revolution by giving up *de jure* political power.<sup>12</sup> Alternatively, the elite can offer democratisation voluntarily if this implies economic benefits.<sup>13</sup> Taken together, this discussion implies that the model delivers a simple representation of the different scenarios of the allocation of political power, as well as of transition scenarios. Starting from an oligarchy of the landowners, a transition to democracy can occur under a conflict if it is imposed by the landless as consequence of *de facto* political power prescribed by the condition (3.9). Policies in this democracy can be either redistribution or public good provision depending on the preferences of the landless. Alternative, a transition to democracy can be consensual in the sense of being initiated by the landowners with the aim to implement public goods.

#### 2.4.5 Timing of Events

Generation  $t$  with mass  $L_t$  enters the economy and endowments are realised. The political regime and the policy of the previous generation represent the benchmark for political interactions. The timing of events within a generation  $t$  is as follows:

1. The Political regime is implemented by the group with *de facto* power according to (3.9).

---

<sup>11</sup>Under democracy, the optimal policy is determined by the median voter, who is a member of the group of landless, whereas under oligarchy the optimal policy is that of a member of the landowners.

<sup>12</sup>A straightforward extension to incorporate this idea is introducing an assumption about the costs and consequences of outright revolution. In this sense this modelling approach is a generalisation of the framework of Acemoglu and Robinson (2000, 2006).

<sup>13</sup>Another straightforward extension in this direction would be an unequal distribution of land among the landowners and a resulting conflict of interest about public goods provision among the landed elite, paralleling the setting in Lizzeri and Persico (2004).

2. The policy (taxation with redistribution, public goods provision or neither) is decided upon.
3. Decisions about fertility and resources spent on education of children.
4. Realisation of income, consumption, utility, death.

### 2.4.6 Dynamics

The model is closed by determining the dynamics of the economy. Similar to Galor and Weil (2000) technological progress between periods  $t$  and  $t + 1$  is increasing in education  $e_t$  and population size  $L_t$

$$g_{t+1}(e_t, L_t) \equiv \frac{A_{t+1} - A_t}{A_t} = (e_t + \rho)a(L_t), \quad (2.11)$$

with  $\rho > 1$ , and

$$a(L_t) = \min \{ \Phi L_t, a^* \}$$

which reflects the scale effect that determines the dynamics of the economy in the absence of positive human capital investments.<sup>14</sup>

## 2.5 The Intra-Generational Equilibrium

This section derives the intra-generational equilibrium. The dynamics, in particular the dynamic interactions between the processes of democratisation and demographic transition, will be studied in section 2.6.

---

<sup>14</sup>The functional form is borrowed from Lagerlöf (2006).

### 2.5.1 Optimal Education, Fertility and Consumption

Individuals belonging to generation  $t$  choose the number of children and their amount of education subject to their respective budget constraint.<sup>15</sup>

The optimal education choice depends on technological progress. Optimisation of utility with respect to  $e_{t+1}^i$  gives the following relation between  $e_{t+1}^i$  and  $g_{t+1}$ :

$$Q(e_{t+1}, g_{t+1}) \equiv \beta(r + e_{t+1})h_e(e_{t+1}, g_{t+1}) - h(e_{t+1}, g_{t+1}) \begin{cases} = 0 & \text{if } e_{t+1} > 0, \\ \leq 0 & \text{if } e_{t+1} = 0. \end{cases} \quad (2.12)$$

In order to ensure a positive level of technological progress with individuals choosing an education level of zero, we follow Galor and Weil (2000) and impose the assumption that

$$Q(0, 0) = rh_e(0, 0) - h(0, 0) < 0. \quad (A3)$$

If (A2) is satisfied, there exists a positive threshold level of technological progress  $\hat{g} > 0$  above which individuals of generation  $t$  choose positive levels of education for their children. Solving for  $e_{t+1}$  gives:

$$e_{t+1} = \max \left\{ 0, \frac{1}{1 - \beta\theta} \left[ \beta\theta r - \frac{\rho}{\kappa(1 + g_{t+1})} \right] \right\}. \quad (2.13)$$

with  $e'(g_{t+1}) > 0$  and  $e''(g_{t+1}) < 0$  for all  $g_{t+1} > \hat{g}$ . The expression for  $e_{t+1}$  is positive for a given set of parameters if and only if

$$g_{t+1} > \frac{\rho}{\kappa\beta\theta r} - 1 \equiv \hat{g},$$

meaning that the decision to educate exclusively depends on technological progress.<sup>16</sup>

The solution of the optimisation for fertility and consumption also exhibits a corner

---

<sup>15</sup>See Appendix B.1 for details on the optimisation problems of landowners and landless.

<sup>16</sup>This implies that for positive level of technological progress that enables a positive education level the following parametric version of (A2) has to hold,  $\rho > \kappa\beta\theta r$ , otherwise a zero (or even negative) growth is consistent with a positive optimal education level.



solution as long as potential income is below the income necessary to satisfy the subsistence constraint

$$z_t^i \leq \bar{z} \equiv \frac{\tilde{c}}{1 - \gamma}, \quad i \in \{r; p\}. \quad (2.14)$$

Consequently the solution to the maximisation problem to the two groups is given by

$$n_t^i = \begin{cases} \frac{\gamma}{r+e_{t+1}} & \text{if } z_t^i \geq \bar{z}, \\ \frac{1-\tilde{c}}{r+e_{t+1}z_t^i} & \text{if } z_t^i \leq \bar{z}. \end{cases} \quad (2.15)$$

$$c_t^i = \begin{cases} (1 - \gamma) \cdot z_t^i & \text{if } z_t^i \geq \bar{z}, \\ \tilde{c} & \text{if } z_t^i \leq \bar{z}. \end{cases} \quad (2.16)$$

Given (A2), (A2), and (3.3), technological progress and variations in income affect the optimal education and fertility decisions of the two groups as follows. Technological progress between periods  $t$  and  $t + 1$  (weakly) increases the education investment in children for both groups and decreases fertility correspondingly. An increase in disposable income  $z_t^i$  (due to increased efficiency units of labour, landholding, transfers or public goods provision from the previous period) implies an increase in the quantity of offspring with no effect on the quality for both groups as long as the subsistence constraint binds (that is,  $z_t^i \leq \bar{z}$ ). If the subsistence constraint does not bind (that is  $z_t^i \geq \bar{z}$ ), an increase in disposable income has no effect on education investments or fertility of both groups. Likewise, public goods provision from the previous period or an increase in the tax rate  $\tau_t$  has no effect on either quantity or quality for both groups if the subsistence constraint is not binding, (that is  $z_t^i \geq \bar{z}$ ).

## 2.5.2 Optimal Policies and Political Equilibrium

To be able to solve for the optimal education and fertility decisions, it is necessary to determine the relevant budgets under the different possible scenarios. In particular, the institutional environment determines the individual budgets of landowners and landless

via the implemented policies.

Within the given policy space, preferences for redistribution differ across landowners and landless. Landowners oppose redistribution while the landless prefer a strictly positive tax rate on land incomes compared to a state with no taxes.<sup>17</sup>

To see this, note that the indirect utility of each group is strictly increasing in income with disposable income being determined by the tax rate. Additionally, indirect utilities are concave functions of the tax rate. Since the policy space is one-dimensional, the conditions for applying the Median Voter Theorem are satisfied. The median voter is defined by the political regime in place (the distribution of *de facto* political power). Given that only positive tax rates  $\tau_t$  are allowed, the optimal tax rate preferred by landowners is zero since they would be losing income, while the optimal tax rate for the landless is positive and time invariant.<sup>18</sup>

The public is financed such that the lump-sum transfers collected from every individual exactly pay the production, that is

$$\alpha L_t \cdot \phi(r + e_{t+1}^p) \leq L_t \cdot \omega. \quad (2.17)$$

Maximising (2.9) with respect to (2.17) gives the optimal amount of public education

---

<sup>17</sup>In case of fiscal redistribution, only land income is taxed in equilibrium, i.e.,  $\tau^H = 0$ . To see this, note that taxing wage income would reduce total income of both groups by the same amount even after receiving transfers and is therefore wasteful. Additionally, as taxation is confined to non-negative tax rates and redistribution is lump-sum such that total government revenue is the sum of individual transfers taxation is (weakly) progressive.

<sup>18</sup>Differentiating indirect utility with respect to the tax rate gives

$$\begin{aligned} \frac{\partial V_t^r}{\partial \tau_t} &= A \cdot \left[ 1 - C'(\tau_t) - \frac{1}{\delta_t} \right] \leq 0 \\ \frac{\partial V_t^p}{\partial \tau_t} &= B \cdot [1 - C'(\tau_t)] \leq 0 \end{aligned}$$

which implies that the optimal tax poor is constant over time and implicitly defined by the solution to:

$$1 - C'(\tau^*) = 0.$$

$$e_{t+1}^p = \max \left\{ 0, \frac{1}{1-\theta} \left[ \theta r - \frac{\rho}{\kappa(1+g_{t+1})} \right] \right\}. \quad (2.18)$$

Note that, given it is feasible, public education is provided earlier than its private counterpart since  $\hat{g}^p \equiv \frac{\rho}{\kappa\theta r} - 1 < \hat{g}$ .

The optimal policies for the two groups can then be determined by comparing indirect utilities under the different policy alternatives. Landowners always strictly prefer zero taxation and hence no redistribution. However, they face a trade-off between either no taxation (and thus no redistribution) or the concession of political power and public goods provision. This is only a possibility if the landless prefer public goods provision as well since surrendering power to them while they still prefer redistribution would lead to the latter. This implies that the trade-off for the landless is stricter and therefore means that if the prefer public goods provision the landowner will also not object to it.

Formally, the landless face a trade-off between public good provision but no redistribution, or redistribution but no public good. In particular, the indirect utility of the landless from the public good exceeds the indirect utility from redistribution if and only if

$$\begin{aligned} (1-\gamma) \ln [(1-\gamma)(w_t h_t - \omega)] + \gamma [\ln(n_t^*) + \beta \ln(h_{t+1}^* + h_{t+1}^p)] \\ > \\ (1-\gamma) \ln \left[ (1-\gamma) \left[ w_t h_t + \frac{R_t}{L_t} \right] \right] + \gamma [\ln(n_t^*) + \beta \ln(h_{t+1}^*)], \end{aligned}$$

where  $h_{t+1}^p$  is the additional human capital produced under the public good. Substituting (2.8) and (2.9) shows that the landless prefer the public good over redistribution if and only if

$$\left[ 1 + \alpha \cdot \frac{h_{t+1}^p}{h_{t+1}} \right]^{\beta \frac{\gamma}{1-\gamma}} > \frac{w_t H_t + [\tau_t - C(\tau_t)] p_t N}{w_t H_t - L_t \cdot \omega}. \quad (2.19)$$

The political equilibrium itself is characterised by the desired political regime of the population group that dominates in terms of *de facto* political power under the prevailing

conditions. Hence, depending on condition (3.9), it is the landowners or the landless who decide about the regime in order to implement their desired policy. Given the discussion above, the landowning elite may find it profitable to implement a democracy compared to the status quo of zero taxation in order to allow for the implementation of the public good if indirect utility is higher. Once the landless dominate in terms of *de facto* political power, they either implement a regime with public good provision, or a regime of positive taxation and redistribution. In this sense, the political equilibrium comprises elements of democratisation under a revolutionary threat as in the framework by Acemoglu and Robinson (2000, 2006), as well as the possibility for a voluntary transition to democracy by the landed elite motivated by economic benefits as in Lizzeri and Persico (2004).

Hence, depending on the political equilibrium and the associated levels of redistribution or public good provision (which can be either positive and zero, zero and positive, or both zero), current period budgets are given by

$$z_t^r = \begin{cases} w_t h_t + \frac{p_t N}{\delta_t L_t} & \text{under no policy ,} \\ w_t h_t + \frac{p_t N}{L_t} \left[ \tau^* - C(\tau^*) + \frac{1-\tau^*}{\delta_t} \right] & \text{under redistribution ,} \end{cases} \quad (2.20)$$

$$z_t^p = \begin{cases} w_t h_t & \text{under no policy ,} \\ w_t h_t + \frac{p_t N}{L_t} [\tau^* - C(\tau^*)] & \text{under redistribution .} \end{cases} \quad (2.21)$$

## 2.6 Equilibrium Dynamics

### 2.6.1 Democratisation and the Demographic Transition

The preceding analysis implies that there are two intertwined dynamic processes at work. The dynamics of technological progress ultimately imply that education investments become positive, triggering a demographic transition by a shift of resources from fertility to the education of children. At the same time, the fact that technological progress tends to reduce inequality by making human capital more important in the production process has twofold implications for the political equilibrium. On the one hand, it makes the landless stronger

in terms of *de facto* political power in the sense of condition (3.9). On the other hand, it reduces inequality while making human capital more important, thereby slowly shifting the focus from redistributive conflict to the demand for public goods provision. Moreover, the demographic transition, once it occurs, reinforces this tendency. Hence, the model presented so far implies that two transitions occur along the equilibrium path, a demographic transition and a transition in the political regime that either provides redistribution or public goods provision. This section presents the main contribution of the paper by investigating the interactions between these two transitions along the equilibrium path.

The analysis begins by establishing the existence and uniqueness of a transition to a democratic equilibrium. Given that the economy starts out in a situation of an oligarchy by the landowners, there exists a unique time  $\hat{t}$  at which a permanent transition to democracy occurs. To see this, note that with the landowners exhibiting primogeniture their share in population at each and every point in time is given by

$$\delta_t = \delta_0 \frac{L_0}{L_t}.$$

Substituting factor prices (3.6) and (3.7) into the respective budgets (2.2) for landowners and landless and plugging back into (3.9) allows reformulating the condition for the landless to obtain *de facto* political power to

$$L_t > \frac{N}{h_t} \left[ \frac{1}{1 - 2\delta_0 \frac{L_0}{L_t}} \right]^{\frac{\sigma}{\sigma-1}}. \quad (2.22)$$

From the results about fertility and education decisions, it is clear that  $L_t$  and increases over time. Therefore, for a given parameter configuration, it can be ensured by the choice of initial conditions and applying the intermediate value theorem that there exists a unique time period (generation)  $\hat{t}$  in which the economy will switch to the democratic regime as the landless become the politically more powerful group. This period is also the latest period in which a democratic transition is observed, and the democratic regime equilibrium is absorbing.

In addition to the finding that the economy eventually experiences a transition to

democracy, the framework has additional implications for the scenario under which this transition happens, i.e., whether the transition is initiated by the land-owning elite in pursuing more efficient public goods provision, or by the landless, in order to implement public goods or redistribution. The alternative scenario of a transition to democracy is that the landowners deliberately transfer *de jure* political power to the landless in order to establish a consensus that allows for the implementation of the public good. This scenario requires that the landless strictly prefer the public good to fiscal redistribution, i.e., condition (2.19) must hold. Denote the first period (generation) for which this is the case as  $\check{t}$ .

Notice that it is ultimately demographic development that makes the transition to democracy possible. On the one hand, population dynamics and income dynamics reinforce each other and affect the allocation of political power reflected in (2.22). On the other hand, the demographic transition, in the sense of an incentive to provide education to the children as implied by an interior solution to (3.3), is a central driver for both the landowners to prefer a transition to democracy in order to implement public goods and for the landless to have the same preference. Hence, the first period for which  $e_{t+1} > 0$  as implied by (3.3) is denoted  $\bar{t}$ .

A priori, the sequence  $\hat{t}$ ,  $\check{t}$  and  $\bar{t}$  is not clear. It is not obvious whether the demographic transition occurs before the transition to democracy and under which scenario the transition to democracy occurs. In fact, the democratic transition can occur before the demographic transition in terms of the onset of positive education investments and an associated reduction in fertility has taken place.

At the same time, democratisation can have profound effects on demographic development, by affecting the costs for children through increasing disposable income among the landless either through redistribution or public good provision. Hence, the ultimate effect of democratisation on the demographic transition in the model is closely linked to whether the landless are still subsistence constrained when democratisation occurs.

Summing up, the equilibrium dynamics can be analysed by determining in which sequence  $\hat{t}$ ,  $\check{t}$  and  $\bar{t}$  occur, i.e., in which sequence conditions (2.22), (2.19) and (3.3) are

fulfilled. Closer inspection reveals that there is a key parameter that influences the relative timing of the three transitions: for a given parametric environment, a greater level of  $\theta$  increases human capital and therefore positively affects both transitions.

In order to study the differential effect of  $\theta$  in a pre-transitional environment insert (3.8) into (3.4), accounting for the fact that  $e_t = 0$ . This gives the following three expression for demographic transition, democratic transition and public goods democracy

$$L_t > \frac{1}{\kappa\beta\theta r\Phi} - \frac{1}{\rho\Phi}, \quad (2.23)$$

$$L_t > \frac{N}{h_t} \left[ \frac{1}{1 - 2\delta_0 \frac{L_0}{L_t}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (2.24)$$

$$\left[ 1 + \alpha \cdot \frac{h_{t+1}^p}{h_{t+1}} \right]^{\beta \frac{\gamma}{1-\gamma}} > \frac{w_t H_t + [\tau_t - C(\tau_t)] p_t N}{w_t H_t - L_t \cdot \omega}. \quad (2.25)$$

These conditions allow for an analytical characterisation of the main predictions of the model. We begin by stating a useful preliminary result.

**Lemma 1.** *Before the demographic transition, a higher  $\theta$  has a non-negative effect on population as long as  $\rho > 1$ , ceteris paribus.*

*Proof.* From (2.16), pre-transitional fertility is given by  $n_t^i = \frac{1 - \bar{c}_t}{r z_t^i}$ , and the level of human capital is  $h_{t+1} = \rho^\theta$ . Hence,  $h_{t+1}$  is increasing in  $\theta$  as long as  $\rho > 1$ . Furthermore,  $z_t$  is increasing in  $\theta$  as long as  $\rho > 1$ , and hence  $L_{t+1}$  is also increasing in  $\theta$  as long as  $\rho > 1$ .  $\square$

We are now in the position to characterise the timing of the demographic transition.

**Proposition 1.** *A higher  $\theta$  implies an earlier onset of the demographic transition, i.e., a smaller  $\bar{L}$ , ceteris paribus.*

*Proof.* From (2.23) it follows immediately that the transition takes place if and only if population is past the threshold  $\bar{L}_t$  that determines the onset of the demographic transition as

$$\bar{L}_t > \frac{1}{\kappa\beta\theta r\Phi} - \frac{1}{\rho\Phi},$$

which is strictly decreasing in  $\theta$ . Hence, a greater  $\theta$  implies a lower population size required for the onset of the demographic transition. Moreover, from Lemma 1 it follows that a greater  $\theta$  accelerates population growth.  $\square$

Likewise,  $\theta$  affects the timing of the transition to democracy.

**Proposition 2.** *A higher  $\theta$  implies an earlier onset of the democratic transition, i.e. a smaller  $\hat{t}$ , ceteris paribus.*

*Proof.* Consider (2.24) and notice that  $h_t$  is strictly increasing in  $\theta$  given  $\rho > 1$ . Likewise, from Lemma 1  $L_{t+1}$  is also increasing in  $\theta$ . Hence, ceteris paribus, the left hand side of (2.22) is larger at any time  $t$  if  $\theta$  is higher while the right hand side is smaller for a higher  $\theta$  at any time  $t$ , thus leading to an earlier time  $\hat{t}$  at which the condition binds.  $\square$

Additionally,  $\theta$  also affects the transition scenario.

**Proposition 3.** *Consensual democracy will only arise if there is an incentive to educate. A higher  $\theta$  increases the probability of a consensual democratisation.*

*Proof.* The timing of democratisation is given by the first period at which either the condition for democratisation under conflict, (2.24), or under a consensus, (2.25), binds, i.e. the minimum of  $\hat{t}$  and  $\check{t}$ . If  $e_{t+1}^p = 0$  then condition (2.25) becomes

$$w_t H_t - L_t \cdot \omega > w_t H_t + [\tau_t - C(\tau_t)] p_t N,$$

which can never hold. If  $e_{t+1}^p > 0$  and  $e_{t+1} > 0$  then condition (2.25) becomes

$$\left[ 1 + \alpha \left( \frac{1 - \beta\theta}{1 - \theta} \frac{\theta}{\beta\theta} \right)^\theta \right]^{\beta \frac{\gamma}{1-\gamma}} > \frac{w_t H_t + [\tau_t - C(\tau_t)] p_t N}{w_t H_t - L_t \cdot \omega},$$

where the right hand side decreases with  $\theta$ , while the left hand side increases in  $\theta$  with



$$\lim_{\theta \rightarrow 1} \left[ 1 + \alpha \left( \frac{1 - \beta\theta}{1 - \theta} \frac{\theta}{\beta\theta} \right)^\theta \right]^{\beta \frac{\gamma}{1-\gamma}} = \infty,$$

for any  $\alpha > 0$ .<sup>19</sup> Therefore a higher  $\theta$  leads to an earlier time  $\check{t}$  in which the condition holds; with the possibility of it binding before  $\check{t}$  and even  $\bar{t}$ .  $\square$

A direct corollary of these propositions is that countries that are endowed with a higher  $\theta$  experience an earlier demographic and democratic transition and are more likely to implement public goods during democracy.

**Proposition 4.** *Democratisation implies an accelerated demographic transition if the demographic dynamics are still characterised by Malthusian stagnation, but has no effect on the demographic transition if the demographic dynamics are characterised by a Post-Malthusian equilibrium.*

*Proof.* Democratisation implies higher effective resources for the poor (through fiscal redistribution or the public good). This implies an increase in fertility as consequence of democratisation, but only as long as  $z_t^i \leq \tilde{z}$  as in (2.16). The result follows since  $g'_{t+1}(L_t) > 0$ .  $\square$

## 2.7 Illustrative Simulation

The analytical treatment of the last section has focused on the working of the model and the different transitions as well as their interplay. This section provides a quantitative analysis to study the dynamics of the economy in the demographic and institutional dimensions in more detail and compare them to the actual data.

### 2.7.1 Quantitative Implementation

The simulation of the model covers a horizon of 50 generations of 20 years length each. The quantitative implementation of the model parallels the simulation of the model of Galor and Weil (2000) by Lagerlöf (2006). Differences in parametrisation and initial conditions

---

<sup>19</sup>The same holds for the case in which  $e_{t+1}^p > 0$  and  $e_{t+1} = 0$ .

necessarily arise because of the deviations of the analytical model from the baseline model of Galor and Weil (2000). In addition, the functional form of the human capital production function follows the treatment in Cervellati and Sunde (2015b). All functional form assumptions have been introduced along with the generic model set-up.

The parameter values are identical to the values chosen by Lagerlöf (2006) where possible. The time cost is  $r = 0.15$  and education cost equal to one. Similarly, we assume population growth in the modern growth regime to be balanced,  $\frac{L_{t+1}-L_t}{L_t} = 1$ , and education in the modern growth regime to take  $e^* = 0.075$  (following values taken from de la Croix and Doepke, 2004). The preference parameter for children,  $\gamma = 0.225$ , is set consistent with  $n_t = 1$  in the modern growth regime given  $e^*$ ,  $r$  and  $\tau^e$ . Parameters that play no relevant role once initial conditions are set, are normalised. This refers to the subsistence parameter,  $\tilde{c} = 1$ , land size  $N = 1$ , and the scale parameter  $\Phi = 1$ .

The remaining parameters are set as follows. The CES parameter is set to  $\sigma = 1.3$ . The optimal tax rate that follows from the optimisation of the landless and the functional form assumption about  $C(\tau_t)$  is  $\tau_t^* = 0.5$ . Growth in income per capita in the modern regime is assumed to be 2.4 % p.a., which corresponds to  $g^* \approx 0.6069$ . Inserting  $g^*$  and  $e^*$  into the expressions for education and technological progress allows computing  $\rho$  for a baseline  $\theta = 0.99$  and  $a^*$  (which depends on  $\rho$ ). The preference for human capital is set to  $\beta = 0.7$ . Given these parameters  $\kappa$  is set such that  $\rho > 1$ , which implies  $\kappa = 7.7$ . The parameters governing public good production  $\alpha$  and  $\phi$  are set to  $\alpha = 0.087$  and  $\phi = 0.0115$ , respectively.

Initial conditions for the simulation are set to obtain initially a Malthusian regime in which the *de facto* political power is in the hands of the landowners. This implies education  $e_0 = 0$ , which automatically also determines the levels of  $h_0$  and  $g_0$ . The initial share of the elite in the population is set to  $\delta_0 = 0.05$ . The choice of this parameter is arbitrary and without loss of generality, since the size of the elite does not play a crucial role under the assumption of primogeniture. The initial population size  $L_0$  is set to match the patterns of development. In particular, the absolute size of  $L_0$  depends on the distance to the demographic transition. Since this varies with initial population is adjusted such that all

countries have the same relative distance to the demographic transition. Initial TFP  $A_0$  is set such that, given all other values, population replaces itself in the first period, i.e.  $n_0 = 1$ .

Differences in comparative development patterns are investigated by variation in the parameter  $\theta^i$ .

## 2.7.2 Baseline Simulation Results

We begin the quantitative analysis by simulating the model for a prototype economy. This simulation captures the development of forerunner countries in Europe that experienced the demographic transition during the 19<sup>th</sup> century. As an alternative, we then simulate a latecomer country that undergoes a delayed transition from stagnation to growth. The only difference between the baseline simulation and the alternative simulation is a difference in the human capital parameter  $\theta$ .

The baseline simulation delivers three phases of demographic development, a Malthusian Phase with  $c_t = \bar{c}$  for the landless, a Post-Malthusian phase with  $c_t > \bar{c}$ , fertility increasing in income and no investments in education,  $e_{t+1} = 0$ , and finally a modern growth regime with positive education investments  $e_{t+1} > 0$  and a corresponding decline in fertility. In this simulation, the group of landowners is never at subsistence. This implies differential fertility between the landowners and the landless which only disappears in the long run as the advantage from owning land for disposable income vanishes.

In terms of political regimes, the baseline scenario involves an initial phase of oligarchy of the landowners, who have *de facto* political power. At some point, there is also a transition to democracy in which the landowners offer a transition to democracy and public goods are supplied.

To illustrate the working of the model, we simulate an alternative economy that exhibits some exogenous, country-specific and time-invariant difference that affects  $\theta$  and thus the timing of the transitions. This simulation of an alternative economy is based on the following thought experiment. Suppose this second economy features lower effectiveness of time investments in human capital. This difference has first-order implications for the process

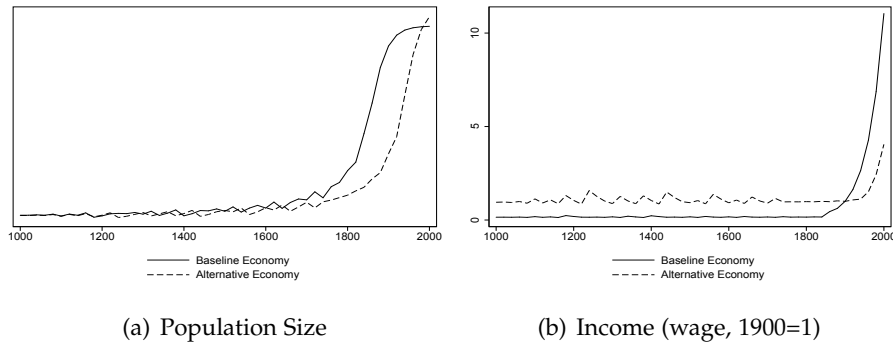
of development as suggested by the analytical results. In particular, in this alternative economy, all transitions (economic, demographic and democratic) are delayed. Moreover, the democratic transition is delayed by more than the demographic transition so that there is an increased asynchrony between the two transitions, and at the same time the transition to democracy is more likely to be conflictual and implement redistribution rather than public goods.

The comparison of the two simulated economies is helpful for understanding the differences in comparative development and the role of the timing of the transition. In order to create comparable scenarios, it is important to remember that we simulate the same model, keeping all parameters and relevant initial values unchanged, except for the effectiveness in human capital production, as proxied by  $\theta$ . This allows us to isolate the role of country-specific factors for the timing of the transition in the different dimensions, and the consequences of a delayed democratisation on the development path alluded to in the last section.

Figure 2.4 displays the long-run trajectories of population size in Panel (a) and income per capita in Panel (b). These plots illustrate the virtually identical development prior to the transition. This phase of development is characterised by pronounced Malthusian cycles in population that also show up in income per capita (which for comparability is depicted in terms of wage income, which is the same for all individuals in the population and indexed to be 1 in 1900 for both countries).<sup>20</sup> The dynamics of the benchmark economy qualitatively matches the stylised pattern of long run development in Western Europe. After a long period of Malthusian stagnation, population growth accelerates with the transition to the Post-Malthusian regime, and eventually a demographic transition, associated with a drop in population growth, occurs. In the simulation this happens in the late 19<sup>th</sup> century. The alternative economy undergoes a very similar transition in the different dimensions, although with a substantial delay.

---

<sup>20</sup>Figure B.2 in the Appendix plots per capita income, which features a declining trend before the demographic transition as result of population growth and relative decline of the importance of land in production.

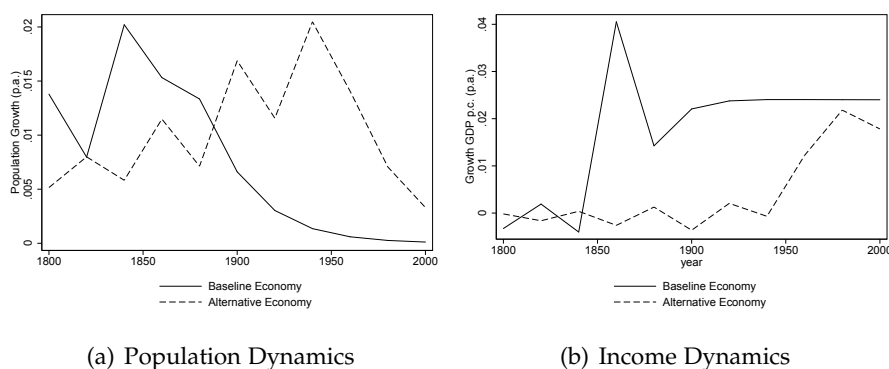


**Figure 2.4:** *The Timing of the Demographic Transition*

Figure 2.5 displays the same dynamics of population growth in Panel (a) and income per capita in Panel (b), zooming in to the period between 1800 and 2000, during which the demographic transition takes place in both countries. The baseline economy reaches the peak in population growth in 1840. This also corresponds to the onset of the demographic transition and of the subsequent decline in fertility as consequence of the increase in education investments. The alternative economy exhibits a peak in 1940, 100 years or five generations later than the baseline country. Thus, the baseline economy already undergoes the transition during the 19<sup>th</sup> century, while the alternative economy enters the demographic transition during the middle of the 20<sup>th</sup> century. Panel (b) illustrates the corresponding dynamics in income per capita. Whereas the baseline economy shows an acceleration of income per capita growth during the second half of the 19<sup>th</sup> Century and enters a balanced growth path in 1900, the alternative economy still exhibits Malthusian dynamics during the 19<sup>th</sup> Century and starts growing only 100 years later. The comparable development patterns before the transition and the earlier take-off in the baseline country imply a pronounced and prolonged divergence in living conditions.

### 2.7.3 Comparative Development: A Tale of Three Transitions

This brings us in the position to reconsider the empirical patterns of historical development patterns in Europe and the world discussed in Section 2.3. To do that, consider a world



**Figure 2.5:** Long-Run Development Dynamics by the Timing of the Demographic Transition

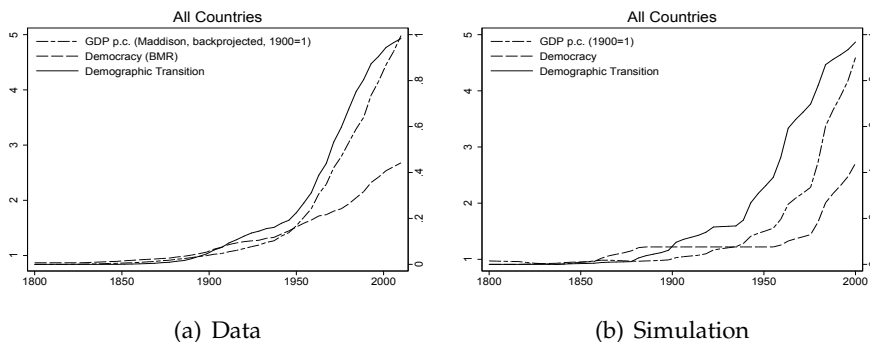
with many economies that only differ in the effectiveness of education investments ( $\theta$ ). In particular, assume that there is a distribution of  $\theta$  for 145 artificial economies that implies a distribution of the timing of the demographic transition in the artificial world that matches that in the data that underlie Figure 2.1.<sup>21</sup>

**Comparative Dynamics.** Figure 2.6(a) replicates the stylised empirical facts of historical development patterns for income, demography and democracy shown in Figure 2.1 and contrasts it to the patterns generated from the simulated world of 145 artificial countries over the period (decades) 1800-2010. Interestingly, the simulation of the stylised model captures the empirical pattern surprisingly closely.<sup>22</sup> In particular, the almost simultaneous development of demographic and democratic development and the acceleration in income growth in response to the transitions in the two dimensions is clearly visible. The same is true for the divergence between the demographic transition, which is ultimately undergone by all countries, and democratic development, which falls behind and is completed in less than half of all countries captured by the model simulation. Hence, overall the simulated data exhibit very similar patterns as the empirical data, which is noteworthy given that they

<sup>21</sup>The demographic transition can be targeted by choosing  $\theta$  as consequence of Proposition 1. Figure B.1 in the Appendix provides a comparison of the timing of the demographic transition in the actual data and in the data set of artificial economies.

<sup>22</sup>Log GDP per capita exhibits more pronounced dynamics in the simulation. For better comparability, the data for GDP per capita can also be normalised to the respective level in 1900, with similar patterns.

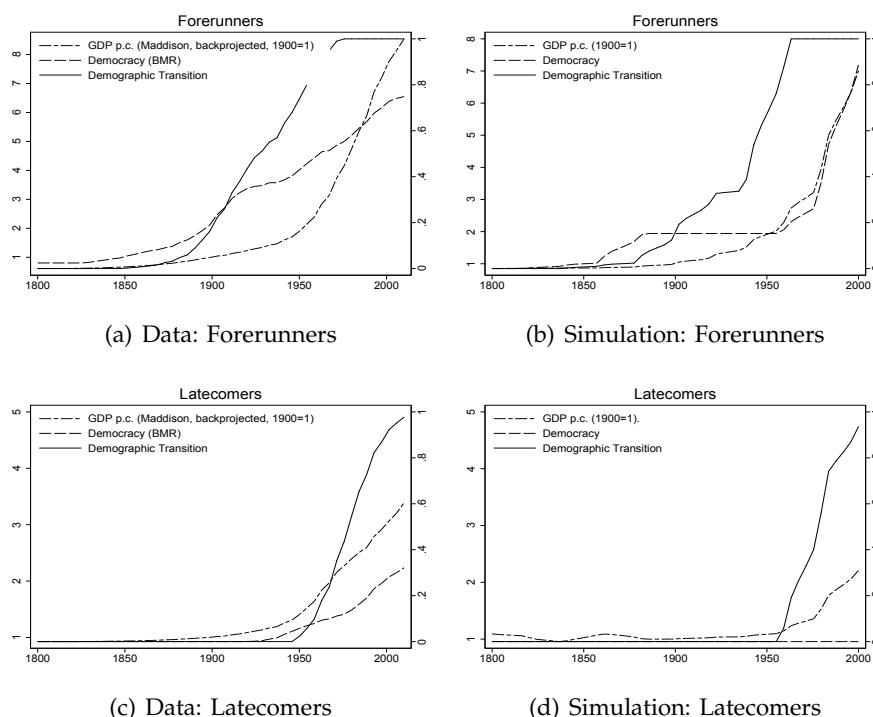
have been generated with the very same data generating process and heterogeneity in only one country-specific, time-invariant parameter, namely  $\theta$ .



**Figure 2.6:** *Long-Run Development Dynamics: Data vs. Simulation*

To investigate whether the model also captures the differences in the dynamics for different subsets of countries, we split the data and simulation samples by the timing of the transition. In the data, we follow the categorisation of Reher (2004) and distinguish forerunner and follower countries that underwent their demographic transition before 1965, and trailer and latecomer countries that underwent their demographic transition only after 1965.<sup>23</sup> The respective data patterns are shown in Figure 2.7(a) and (c). We apply the same categorisation to the simulated data. The respective results are depicted in Figure 2.7(b) and (d). The simulation results not only account for the earlier onset of income growth in the early transitions countries. More importantly, they also replicate the earlier onset in the demographic transition as well as the slow-down in democratisation that is observed in the empirical data for the latecomer countries. Overall, the similarities between the data patterns and the corresponding simulation are striking when considering that the simulation results are based on the identical model that only differs in the effectiveness of education ( $\theta$ ).

<sup>23</sup>The classification of countries follows Reher (2004) who labels countries with an onset of the fertility decline before 1935 and between 1950 and 1964 as forerunners and followers, respectively, and countries where the fertility transition began after 1965 as trailers and latecomers. Very similar patterns emerge when splitting countries by the period of first democratisation following the definition of democratisation waves by Huntington (1993).

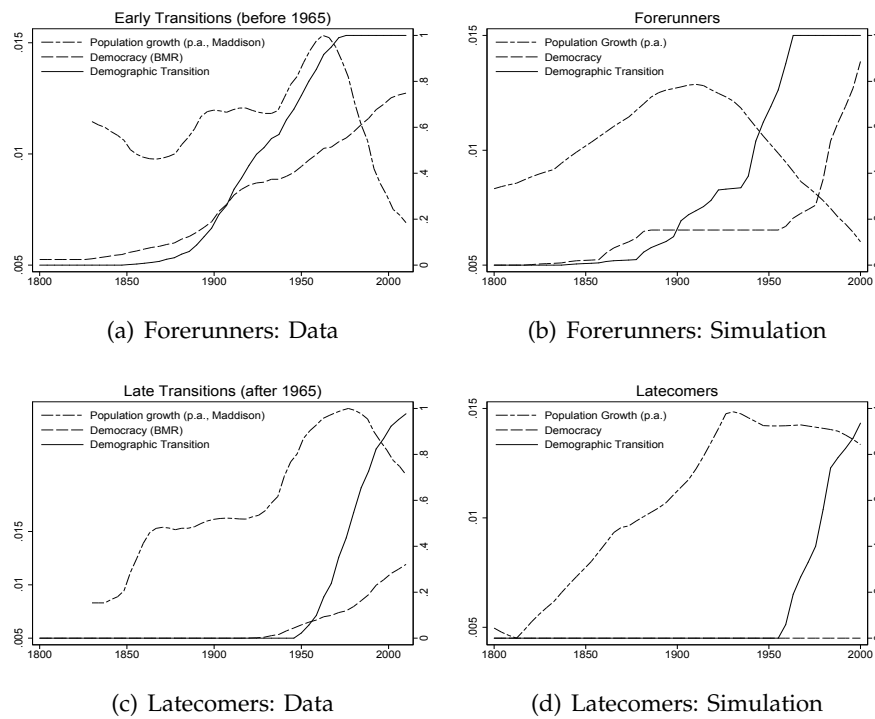


**Figure 2.7:** Long-Run Development Dynamics for Different Sets of Countries: Data vs. Simulation

**Dynamics in Other Dimensions.** To investigate the mechanisms behind the transitions as well as the delays in the transitions, Figure 2.8 displays the population dynamics in the data for forerunners and latecomers, respectively, as well as the corresponding simulation results. The plots indicate that indeed the timing as well as the qualitative and quantitative patterns of population growth are captured quite well by the model, despite the fact that none of the parameters and initial conditions was matched to this moment. The most striking difference between data and simulation is the extent of the decline in population growth in forerunner countries. The simulation implies an earlier and less rapid decline in population growth than the data, where the decline in population growth only commences visibly after the Second World War, whereas the simulation displays an earlier and smoother decline in population growth. Quantitatively, the results are very comparable, however. For latecomer countries, the simulation exhibits much more moderate population growth than the data. Both aspects can be rationalised by mortality patterns, which are missing entirely



in the framework of Galor and Weil (2000).<sup>24</sup> The mismatch between simulation and data might thus be an indication that the neglected positive check of mortality might constitute a relevant omission of the model and that a mechanism based entirely on the interplay of a quantity-quality trade-off and the demand for education misses out an element influencing population dynamics. A clear indication for this is that the epidemiological transition, which happened in the aftermath of the Second World War appears to have led not only to a decline in mortality, but also to a convergence between the simulated and actual population dynamics.<sup>25</sup>



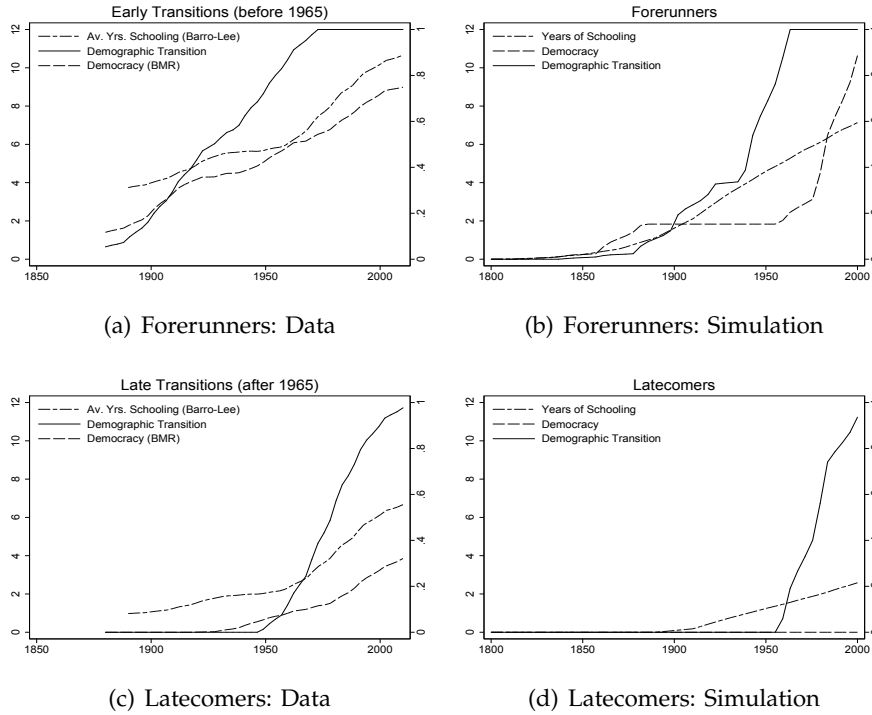
**Figure 2.8:** Long-Run Development Dynamics: Population

The flip side of the population dynamics are the dynamics of education acquisition. Figure 2.9 shows the corresponding patterns in the data and the simulation. Again, the

<sup>24</sup>See Cervellati and Sunde (2015b) for a unified growth model that incorporates mortality dynamics.

<sup>25</sup>See Acemoglu and Johnson (2007) and Cervellati and Sunde (2011c) for a discussion of the epidemiological transition and its implications in the context of the demographic transition.

simulated data capture the data patterns considerably well qualitatively as well as quantitatively, despite the fact that years of schooling need to be imputed from the parental education investments.<sup>26</sup>



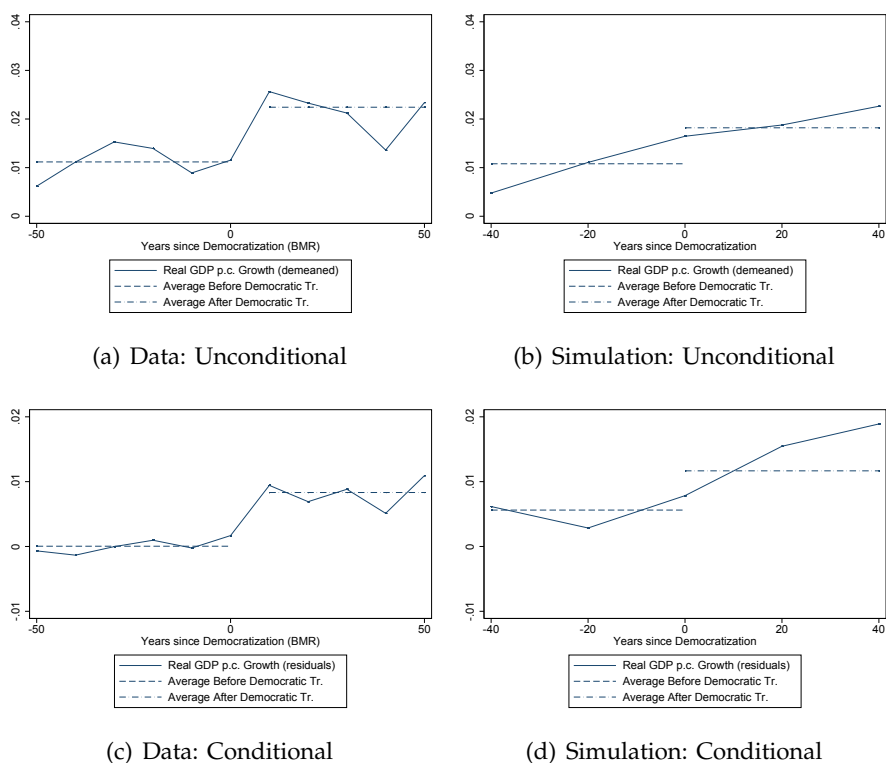
**Figure 2.9:** Long-Run Development Dynamics: Education

As consequence of the divergent population dynamics across forerunner and latecomer countries, comparing countries appears not entirely adequate. Alternatively, one might consider comparing the shares of the total population alive at a certain point in time living under a particular regime, like democracy. Unreported results show that plotting the respective population shares under democracy in the full sample as well as in the subsamples of forerunner and latecomer countries in the data does not greatly affect the results regarding the similarity between empirical patterns and the corresponding simulation results. Again, the simulation captures the data patterns rather well.

<sup>26</sup>Schooling in period  $t$  is calculated via  $years_t = \frac{e_t \times 12}{e^t}$ . This way years of schooling start at zero and become twelve years on the balanced growth path with a smooth transition.

**The Dynamic Effects of Transitions.** Next, we return to the effect of democratisation on economic development and investigate whether the model can generate patterns that are consistent with those in the data also in this dimension. One way to illustrate this is to consider the potential bi-directional feedbacks between the demographic transition and the transition to democracy directly. Panels (a) and (c) of Figure 2.10 replicate the respective patterns of Figure 2.2 in terms of raw average income per capita relative to the transition to democracy, or the residuals of income per capita after taking out country fixed effects and common time effects. These empirical patterns suggest that, on average, the demographic transition accelerated the transition to democracy, as indicated by the fairly low share of democracies among the countries under observation before the demographic transition and the increase of this share thereafter. Panels (b) and (d) of Figure 2.10 plot the corresponding pattern for the simulated data. The simulated income data exhibit a strong dynamic trend, which becomes obvious in the unconditional data. However, when considering the data after conditioning out country and period effects, the simulated data show a pronounced acceleration in economic development after the onset of the transition to democracy. This is consistent with the effect in the data and often reported in empirical studies on the basis of two-way fixed effects panel regressions. However, the simulated data have been obtained with the same model as data generating process, and only heterogeneity in  $\theta$ . This implies that a two-way fixed effects approach does not account for country-specific time-invariant factors when these interfere with the long-run dynamics in non-linear ways. Moreover, the attention on one dimension only, democratisation, is overly restrictive as it neglects potentially important other dynamic processes that interact with the institutional dynamics.

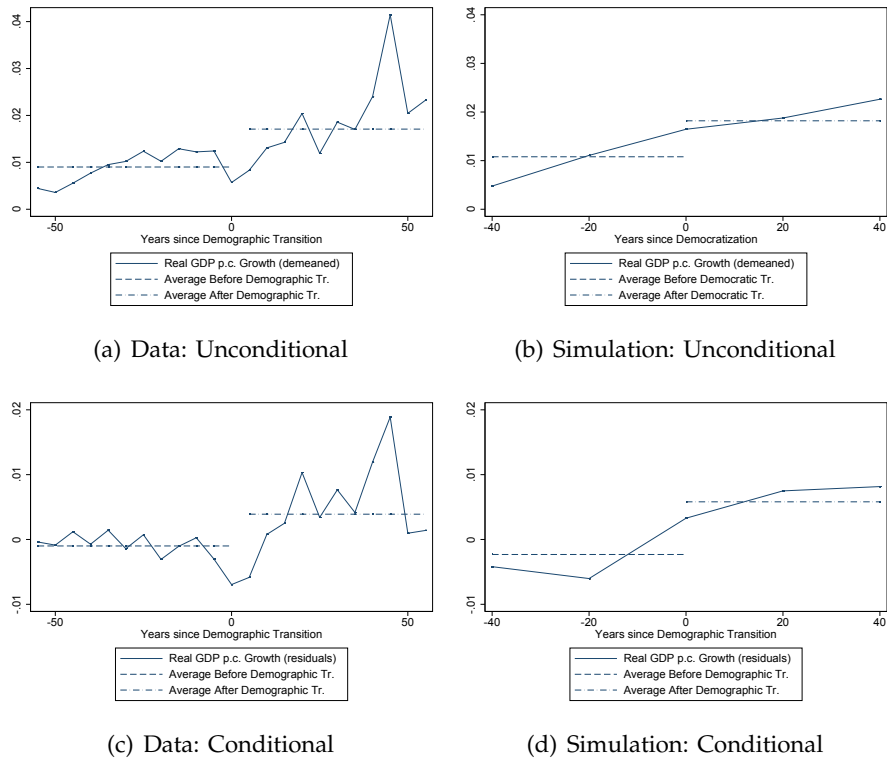
This is illustrated by the analogous plot of economic development relative to the timing of the demographic transition. Panels (a) and (c) of Figure 2.11 replicate the respective patterns of Figure 2.3 and contrast them with the corresponding patterns in the simulated data, shown in panels (b) and (d). Again, the model delivers qualitatively and quantitatively very similar patterns as in the data. In particular, the consideration of the effect after accounting for country and period effects is strikingly similar.



**Figure 2.10:** *The Effect of the Transition to Democracy on Income Growth: Data vs. Simulation*

**Bi-Directional Interactions.** The analysis so far suggests that long-run development patterns are the outcome of interactions between dynamics in different dimensions. In particular, the model as data generating process implies that economic development is affected by the demographic transition as well as by the transition to democracy. These transitions interfere with each other, as has become clear from the analytical results. Hence, we continue with a more detailed investigation of these interactions between the demographic and democratic transitions. To this end, we order the data by the temporal distance to the demographic transition (the onset of the transition in fertility), as well as by the temporal distance to the democratic transition, respectively.

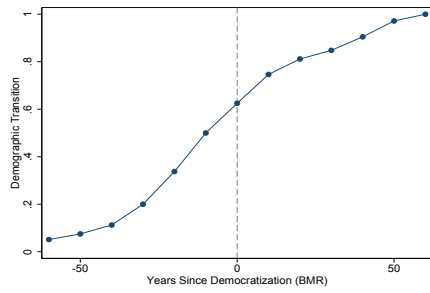
Panels (a) and (c) of Figure 2.12 present binscatter plots of the status of demographic development relative to the year of democratisation in the data. From this perspective the data suggest that most countries had already undergone the demographic transition before



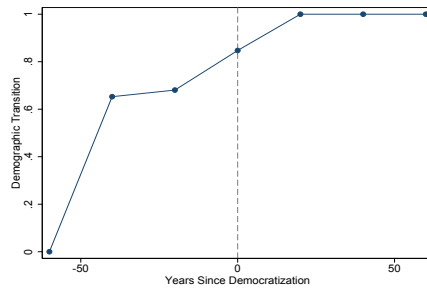
**Figure 2.11:** *The Effect of the Demographic Transition on Income Growth: Data vs. Simulation*

democratisation. Nevertheless, there is a small increase around the year of democratisation. Panels (b) and (d) show the corresponding patterns for the simulated data.

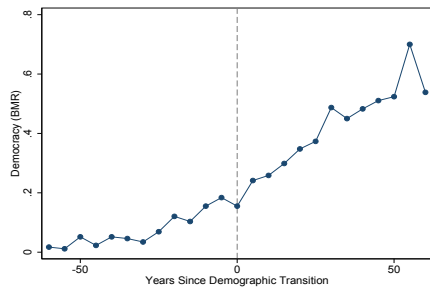
Figure 2.13 revisits this point about the relative timing of the demographic and democratic transition by plotting the data separately for the countries that completed the demographic transition early and late. Interestingly, both in the data as well as in the simulation the emerging patterns for the role of the time to/since democratisation for the demographic transition are almost identical. In contrast, the data reveal that for countries with early demographic transitions (before 1965) the democratic transition is on the way around the time of the demographic transition, whereas for countries with late demographic transition, democratisation occurs mainly in the aftermath of the demographic transition. The data confirm the impression from the time series data showed in Figure 2.7 that the demographic transition had commenced in almost half of the forerunner countries before the transition to



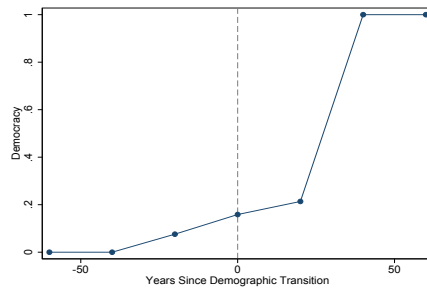
(a) Data: Demographics



(b) Simulation: Demographics



(c) Data: Democracy



(d) Simulation Democracy

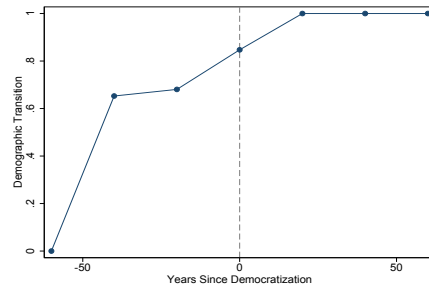
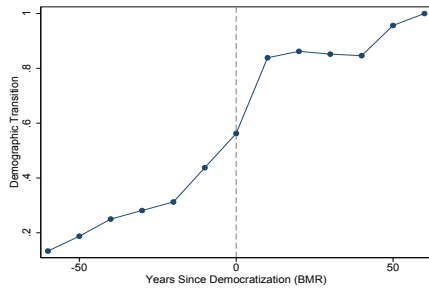
**Figure 2.12:** *Interactions Between Transitions: Data vs. Simulation*

democracy. In contrast, the transition to democracy preceded the demographic transition in roughly 50% of the (European) forerunner countries. For latecomer countries, the patterns are substantially different. Almost all countries of this group had commenced their demographic transition by the time of the transition to democracy. On the other hand, very few countries of this group had experienced a transition to democracy before the demographic transition.

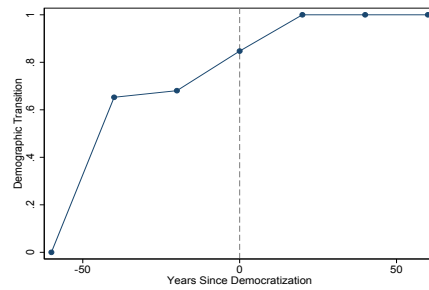
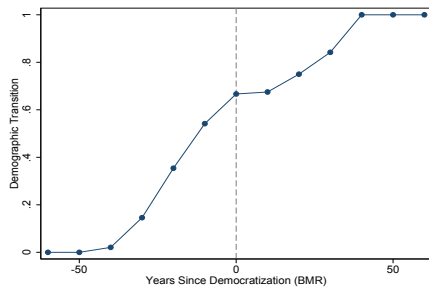
Additional analysis reveals that the take-off in GDP per capita occurred essentially at the same time as the transition to democracy, with the demographic transition being somewhat delayed. Compared to this, the pattern for countries with late demographic transitions (after 1965) reveals not only a later demographic transition, but also a much later and less prevalent transition to democracy, as well as a more moderate increase in incomes (see Figure 2.12).

Taken together, the figures suggest that the transitions in the demographic and institutional domains complement each other in the sense of a discrete improvement in the respective other dimension around the timing of a transition. This pattern reveals two insights. Among the group of the forerunner countries, it is the demographic transition that occurs somewhat earlier and influences institutional development. At the same time, there is a pronounced feedback of democratisation once it occurs, in terms of an acceleration of the demographic development. Among the group of latecomer countries, the democratic transition occurs with an even greater delay compared to the demographic transition. This implies that the effect of democratisation on the demographic transition becomes essentially invisible, whereas the effect of the demographic transition on democratisation becomes more pronounced.

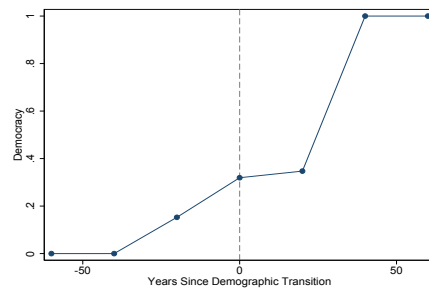
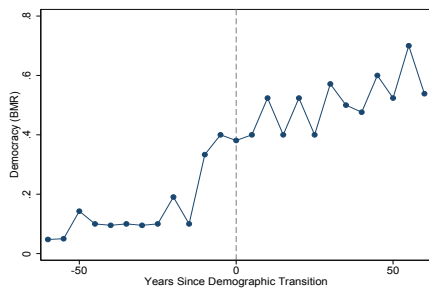
Overall, the comparison of empirical patterns and the corresponding simulations are consistent with the view that both the demographic transition as well as the transition to democracy mark important turning points for economic development. At the same time, the transition in one dimension seems to affect the transition in the other dimension. Based on this evidence, it seems difficult to disentangle a single dimension as the one of primary



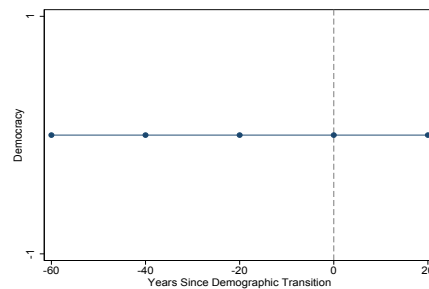
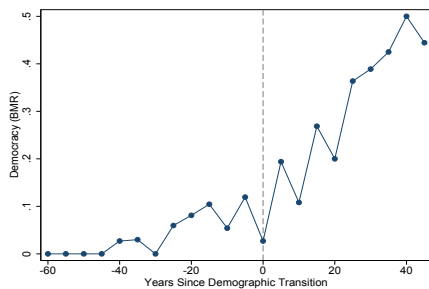
(a) Data: Demographics (early transitions) (b) Simulation: Demographics (early transitions)



(c) Data: Demographics (late transitions) (d) Simulation: Demographics (late transitions)



(e) Data: Democracy (early transitions) (f) Simulation Democracy (early transitions)



(g) Data: Democracy (late transitions) (h) Simulation Democracy (late transitions)

**Figure 2.13:** *Interactions Between Transitions: Data vs. Simulation*

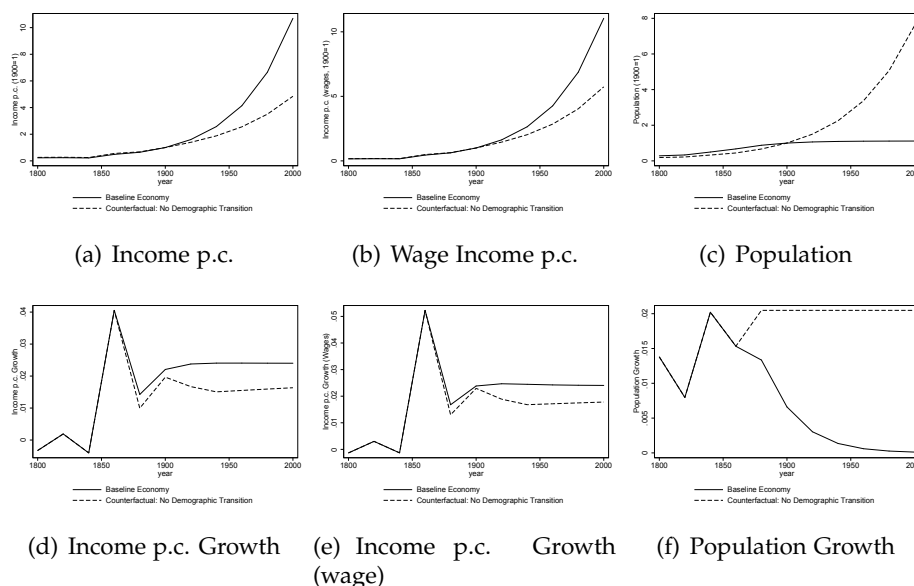


importance. Moreover, the identification of such a mono-causal relationship appears a formidable task. In our perspective, the scope for learning more about the dynamics of development from single-dimensional empirical analyses is therefore limited.

**Counterfactual Experiments.** Due to the interactions of demographic and institutional dynamics, the model generates complex dynamics. To illustrate the interplay between the different dynamic forces, we end by presenting the results of simulations under alternative counterfactual scenarios that illustrate the role of the different elements of the model. In particular, the model naturally lends itself to investigate how the dynamics are influenced by the two components, the demographic and institutional part. We do this by comparing the baseline simulation which incorporates the demographic and democratic transition along the equilibrium path, and different counterfactual scenarios that correspond to a simulation of the model economy with either the possibility of democratisation (and therefore redistribution or public goods provision), or the possibility of the demographic transition (and therefore a transition to education investments in children) being switched off.

The first of these counterfactual simulations compares the baseline economy to an economy that never experiences the demographic transition. This version of the model focuses on the role of institutional change (along the lines of Acemoglu and Robinson (2006) or Lizzeri and Persico (2004)) without allowing for a demographic transition. Concretely, in this economy it is assumed that parents cannot invest in the education of their children, such that  $e_{t+1} = 0$  by assumption. Figure 2.14 plots the trajectories of income per capita, wage income and population in levels (panels (a)-(c)) and growth rates (panels (d)-(e)) for the baseline economy (solid line) and compares it to the counterfactual simulation of an identical country in which the demographic transition is ruled out by constraining education investments to be zero (dashed line). Notice that for this economy the transition to democracy coincides with the (consensual) demographic transition. The growth plots show that income per capita growth (as well as wage growth) experience a peak during the transition to democracy. As consequence of the demographic transition, with its decline in

fertility and increase in education investment, population growth declines in the baseline case, while growth of income per capita stabilises on a balanced growth path with sustained growth.<sup>27</sup> The counterfactual economy without a demographic transition exhibits no decline in fertility and hence sustains high population growth rates. Without education, the development of income per capita evolves along a flatter trajectory. The differences in growth rates manifest in substantial level differences as suggested by the panels in the first row of the figure. This indicates that, despite a positive effect of democratisation on income development, the demographic transition plays an important role for the development dynamics. If, as is the case here, the demographic transition coincides with democratisation, part of the demographic effect might be mistakenly associated with democratisation.



**Figure 2.14:** *The Role of Demographic Development: Baseline Simulation vs. Counterfactual*

Plots in (a), (b) and (c) are for indexed values (1900 = 1).

The second of these counterfactual exercises involves a comparison of a simulation of the full version of the model to a version in which the possibility of democratisation

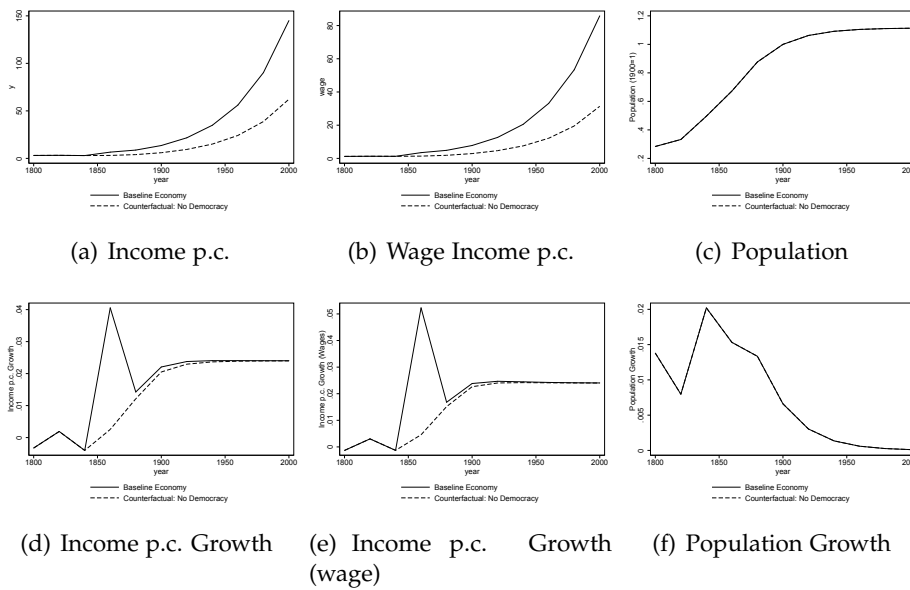
<sup>27</sup>The slight discrepancy in population levels in panel (c) despite identical growth rates in panel (f) are a visual artefact that is due to the normalization of population to a value of 1 in 1900. Figure B.3 in the Appendix presents the corresponding graph for absolute rather than normalized levels.

is switched off (in terms of no possibility to overcome the oligarchic political regime through conflict as reflected by the guns condition (3.9) or by public investments in human capital). Hence, in this counterfactual model economic development is closely related to the demographic transition along the lines of Galor and Weil (2000), while institutional change (democratisation) is absent. Figure 2.15 plots the corresponding levels and growth rates of income per capita, wage income and population for the baseline economy (solid line) and compares it to the counterfactual simulation (dashed line).<sup>28</sup> The results suggest that the peak in income growth that occurs in the baseline simulation as consequence of democratisation (under the consensual scenario entailing public good provision) is not present in the counterfactual simulation. Without the possibility for democratisation, development is plainly driven by demographics in terms of fertility and population growth, as well as the transition to education. Ultimately, in the long run both economies settle on virtually identical balanced growth paths. The figure also shows that the institutional component (democratisation) leaves population dynamics completely unaffected. The reason is that at the moment of the transition, the economy has already entered the phase of Post-Malthusian dynamics.

The third counterfactual exercise investigates the role of the scenario under which democratisation occurs. As indicated before, the transition to democracy occurs under the initiative of the elite in the baseline economy, since  $\hat{t} < \check{t}$ . Switching off the possibility of providing the education-enhancing public good implies restricting the democratisation scenario to be conflictual. Figure 2.16 plots the corresponding paths for income per capita, wage income and population for the full model as well as this counterfactual. In the baseline economy, the delay in conflictual democratisation is 80 years. Hence, in comparison to the baseline economy, the counterfactual economy lacks the initial growth momentum of consensual democratisation in 1860. Nevertheless, due to the demographic transition, there

---

<sup>28</sup>Again, the slight discrepancies in income, wage and population levels in panels (a), (b), and (c) despite identical growth rates in (d), (e), and (f), respectively, during the early time periods are due to the normalization of population to a value of 1 in 1900. Figure B.4 in the Appendix presents the corresponding graph for absolute rather than normalized levels.



**Figure 2.15:** *The Role of Democratisation: Baseline Simulation vs. Counterfactual*

Plots in (a), (b) and (c) are for indexed values (1900 = 1).

is an acceleration of growth. In 1940, however, a conflictual democratisation occurs, which brings with it redistribution and a related resource loss as consequence of the costly taxation. Again, since at the time of democratisation the economy is in a Post-Malthusian equilibrium, the transition scenario has no consequences for population dynamics.<sup>29</sup>

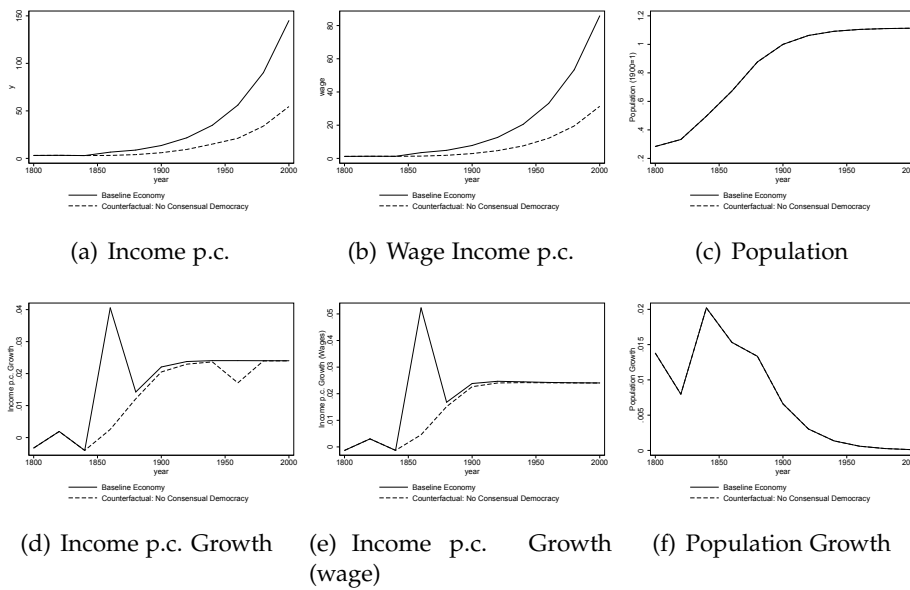
Taken together, these exercises indicate that each transition implies a marked effect on development.<sup>30</sup>

## 2.8 Conclusion

This paper started with the claim that the focus on monocausal explanations – the quest for “the” fundamental determinant of long-run development differences – might be seriously

<sup>29</sup>Compare also Figure B.5 in the Appendix for the corresponding graph for absolute rather than normalized levels.

<sup>30</sup>If democratisation occurred while part of the population is still in a Malthusian regime, democratisation would lead to a more pronounced spike in population growth. In this case, democratisation would also imply a faster transition to lower fertility, and thus a faster decline in population growth as consequence of the relaxation of the budget constraint of individuals that are still subsistence constrained.



**Figure 2.16:** *The Role of the Consensual Democratisation Scenario: Baseline Simulation vs. Counterfactual*  
 Plots in (a), (b) and (c) are for indexed values (1900 = 1).

misleading if long-run economic development is the result of major transitions in different dimensions that are characterised by bilateral feedbacks. To illustrate this point, we presented a comprehensive model of long-run development in which demographic and institutional transitions both affect the dynamics of development. The framework combines the essence of two workhorse models of unified growth theory and democratisation and shows that this comprehensive model exhibits dynamics that are able to replicate the three transitions in the data that relate to the economic, demographic and institutional environment. The findings point towards important, but so far largely neglected, dynamic complementarities of these transitions, suggesting that monocausal explanations and reduced form empirical evidence of causal effects of single factors on the process of development that focus entirely on demographic or institutional aspects are likely to be misleading. The numerical patterns generated by the model are consistent with empirical evidence on the patterns of development and the demographic as well as democratic transition. The analytical and quantitative results thus demonstrate that development ultimately is likely to be the result of different interacting transitions.

The main findings of this paper have important implications for future research on comparative development. First, by demonstrating that democratisation may in large part be the result of demographic dynamics, claims about the unique primacy of institutions appear unwarranted. This does not mean that institutions and democratic transitions do not play an important causal role for long-run development. In fact, the results show that democratisation accelerates the demographic transition. However, attributing to them the entire effect on development while disregarding the demographic dynamics that lead to democratisation provides a slightly distorted interpretation of the role of institutional change. At the same time, institutional change is itself an important determinant not only of the economic but also of the demographic transition.

We also demonstrate the consequences of different country-specific characteristics on the dynamics of development in the three dimensions, in particular on the timing of the demographic and democratic transition. Only variation in a single country-specific and time-invariant factor is shown to be able to generate the qualitatively different patterns of timing in the different transitions that are found in the data. At the same time, differences-in-difference estimates based on fixed effects estimators as in much of the existing literature are shown to be unable to detect the role of the country-specific factor as result of the non-linear and mutually interacting multi-dimensional dynamics.

We hope that the results of this paper are able to shed new light on the recent debate about the fundamental determinants of comparative development and provide a new perspective on the causal drivers of growth and development.

## Chapter 3

# Income and Democracy Revisited

### 3.1 Introduction

Early investigations of the effect of income on democracy have found a strong positive correlation between income and democracy. This result was consistent with Lipset's "Modernisation Hypothesis" that was generally perceived as a stylised fact. However, this observation was challenged by a very influential article by Acemoglu et al. (2008). Herein Acemoglu et al. (2008) argue that previous studies of the effect of income on democracy failed to control for factors that simultaneously affect both variables. They show that the inclusion of country fixed effects removes any statistical correlation between income and democracy. Inclusion of country fixed effects accounts for omitted variable bias due to country-specific time invariant factors that jointly affect income and democracy. The authors then argue that this result coupled with the observed positive correlation between income and democracy of previous studies is evidence for the importance of "critical junctures" for economic development that create divergent development paths.

However, Cervellati et al. (2014) have shown that the result of a mean zero effect of income on democracy has at least been partly driven by sample composition. They document that a mean zero effect of income on democracy is consistent with substantial variation across subsamples. More precisely, they provide evidence that the effect is negative

for former colonies and positive for non-colonies.

This paper contributes to this debate in two aspects. It develops a simple unified growth model in the spirit of Galor and Weil (2000) with an institutional component that accounts for a critical moment in the development path of a country: the demographic transition. The demographic transition and its accompanying take-off in the accumulation of human capital fundamentally changes the composition of income and thereby power. Additionally, the demographic transition is a one-off scenario thereby implying a non-linear effect that might not be captured by the inclusion of country fixed effects in a linear regression framework. The analytical results suggest that increases in income per capita are detrimental to the cause of democracy before the demographic transition as they are bound to benefit those in power. Conversely, after the transition and the accompanying increase in education and human capital rises in income per capita benefit the masses thereby triggering a stronger push towards democracy.

Second, I test the predicted effect of the theory using the same panel approach as Acemoglu et al. (2008) while accounting for the level of demographic development. The results reveal substantial heterogeneity in the effect of income on democracy, depending on demographic development. The effect of income on democracy is positive in countries that are demographically developed but negative in countries that are not. Within demographically developed countries, the effect of income is also heterogeneous and more positive the more advanced the level of demographical development with the opposite holding as well.

The findings have several relevant implications. First, the results document a significantly heterogeneous effect of income on democracy, thereby providing further evidence to the argument of Cervellati et al. (2014) that a zero mean effect of income on democracy might be driven by sample composition. Additionally the results also show sizeable differences within demographically (un-)developed countries, thereby further strengthening the argument of the importance of the demographic transition. Second, the existence of a heterogeneous effect of income suggests that the results from a linear regression framework may be flawed in this context as the model is misspecified. The significantly different effects of income



depend on demographic development, which exhibits highly non-linear dynamics. Not including this important structural change results in the failure to capture a fundamental change in the income distribution dynamics of a country.

Besides Acemoglu et al. (2008) the documented results of income on democracy are mixed. The literature has documented the role of sample composition for explaining mixed results (see for example Cervellati et al. (2014), Benhabib *et al.* (2013) or Moral-Benito and Bartolucci (2012)). The influence of demographic dynamics on democracy has been studied by Dyson (2013) as well as Wilson and Dyson (2017), who find that progress in the demographic transition accelerates the transition to democracy by using cross-country panel data. I complement these findings by providing a simple economic intuition for the observed heterogeneous effects across different samples and demonstrating a possible interaction between demographic development and democracy.

The rest of this paper is structured as follows. Section 3.2 presents a unified growth model that can create heterogeneous effects of income on democracy. Section 3.3 presents the analytic results and derives the testable predictions. Section 3.4 outlines the specification and data used to reproduce the results of Acemoglu et al. (2008) and test the theoretical predictions. Section 3.5 presents the results and Section 3.6 concludes. A small extension to the model from Section 3.2 is presented in the Appendix.

## 3.2 The Model

Consider an overlapping generations model. Every individual lives for two periods and reproduces asexually. During childhood in period  $t - 1$ , individuals consume a fraction of their parents' income. As adults in period  $t$  they are endowed with  $h_t$  efficiency units of labour according to a human capital production function that is described below. Herein human capital depends on the education investments made by the parent generation. Adults are endowed with one unit of time, which they inelastically supply to the labour market. While individuals are identical in their human capital endowment they differ in wealth. Namely, the population is divided into two classes: rich and poor. The rich constitute a

fraction  $\delta_t$  of the population and derive their income from landholding and supplying their labour to the market. The poor on the other hand represent a fraction  $1 - \delta_t > \delta_t$  of the population and only receive labour income.

### 3.2.1 Individuals

Individuals generate utility from consumption  $c_t$ , the number of children  $n_t$  and their human capital  $h_{t+1}$  when they enter the labour force. The preferences are represented by the utility function:

$$u_t = (1 - \gamma) \ln(c_t) + \gamma \ln(n_t h_{t+1}) \quad \gamma \in (0, 1), \quad (3.1)$$

The disposable income of an individual is made up of the market wage per efficiency unit of labour, as well as of land rents if they belong to the class of the rich. Therefore, by denoting rich,  $i = r$ , and poor,  $i = p$ , disposable income is given by:

$$z_t^r = w_t h_t + \frac{p_t N}{\delta_t L_t} \quad (3.2)$$

$$z_t^p = w_t h_t, \quad (3.3)$$

where  $N$  is the total amount of land,  $w_t$  the wage per efficiency unit of labour  $h_t$ , and  $p_t$  the land rent at time  $t$ . Disposable income is devoted to consumption  $c_t^i$  and raising children. Each child costs a fraction  $r$  of the parental income regardless of quality. Moreover, educating a child requires an additional fraction of income per unit of education  $e_{t+1}$ .<sup>1</sup> As in Galor and Moav (2002b), I assume that  $r$  is small enough to guarantee positive rates of population growth

$$r < \gamma. \quad (A1)$$

---

<sup>1</sup>This is different to Galor and Weil (2000), who assume that child costs are in terms of parental time, not income.

### 3.2.2 Human Capital

Individual labour income depends on the human capital that an individual can supply to the production of output during adulthood. This human capital is determined by parental decisions. In particular, the human capital production function is increasing and strictly concave in parental education expenditure,  $e_{t+1}$ , and technological progress,  $g_{t+1}$

$$h_{t+1}(e_{t+1}, g_{t+1}) = [\kappa(1 + g_{t+1})e_{t+1} + 1]^\theta, \quad (3.4)$$

with  $\kappa > 0$  and  $\theta \in (0, 1)$  being parameters.

Like Galor and Weil (2000), this specification implies that technology and education are complements in the production of human capital. Additionally, the formulation also implies that human capital does not differ across classes as education decisions are fully determined by aggregate variables.

### 3.2.3 Production and Technology

In period  $t$ , a total mass of  $L_t$  individuals enters the labour force. This mass of individuals is endowed with efficiency units of labour  $h_t$  obtained from education investments made by their parents as discussed above. Every period the economy produces a single homogeneous good using land and efficiency units of labour as the inputs using the following production technology

$$Y_t = A_t \left[ (H_t)^{\frac{\sigma-1}{\sigma}} + (N)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad \sigma > 1, \quad (3.5)$$

where  $N$  denotes the exogenous and fixed stock of land. Factors are augmented by a total factor productivity  $A_t$ .

Markets are competitive and factors are paid their marginal product:

$$w_t = \frac{\partial Y_t}{\partial H_t} = A_t \left[ (H_t)^{\frac{\sigma-1}{\sigma}} + (N)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} (H_t)^{\frac{-1}{\sigma}} \quad (3.6)$$

$$p_t = \frac{\partial Y_t}{\partial N} = A_t \left[ (H_t)^{\frac{\sigma-1}{\sigma}} + (N)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} (N)^{\frac{-1}{\sigma}} \quad (3.7)$$

The model is closed by determining the dynamics of the economy. Technological progress between periods  $t$  and  $t + 1$  is increasing in education  $e_t$  and the previous level of technology  $g_t$

$$\begin{aligned} g_{t+1}(e_t, g_t) &\equiv \frac{A_{t+1} - A_t}{A_t} \\ &= \min \{ (e_t + \eta)g_t, g^* \} \quad \eta > 1, g_0 > 0 \end{aligned} \quad (3.8)$$

with  $g^*$  denoting steady state technological progress.

### 3.2.4 Political Economy

As in the literature on institutions the decisions are made by the actors with *de jure* political power. Thereby two scenarios are possible: the *de jure* political power being either in the hands of the rich or the poor.<sup>2</sup>

Thereby I assume that the status quo of *de jure* political power is inherited from the previous generation. That is, if either class holds *de jure* political power at the end of period  $t$  they will continue to do so at the beginning of period  $t + 1$ . However, the allocation of *de jure* political power may change depending on the distribution of *de facto* political power. More precisely, in a situation where *de jure* political power is initially held by the rich, the poor can decide to challenge the rich in trying to become the rulers themselves. Therefore, the class that has *de jure* political power must also hold the *de facto* political power. As in the literature on political conflict the success probability of a power struggle depends on the available resources either side can expend. This relationship between available resources is

---

<sup>2</sup>I refrain from defining explicit policies since the focus of the model is on the effect of income on *transitioning* to democracy.

captured by

$$\delta_t L_t z_t^r \stackrel{\geq}{\leq} (1 - \delta_t) L_t z_t^p, \quad (3.9)$$

where the left-hand side of (3.9) represents the available resources for fighting of the rich, while the right-hand side are the available resources of the poor.<sup>3</sup>

This simple version of a standard conflict success function, see, e.g., Hirshleifer (2001) contains the important elements for the analysis of the effect of income on transitioning to democracy.<sup>4</sup> If the rich hold more total resources than the poor a revolution is infeasible meaning they hold both *de facto* and *de jure* political power. As they are a minority, this resembles a state of oligarchy. Conversely, if the poor hold more total resources than the rich they obtain *de facto* and therefore *de jure* political power. Since they are the majority this state will be a democracy.

### 3.2.5 Timing

A generation  $t$  with mass  $L_t$  enters the economy and endowments are realised. The political regime is inherited from the previous generation  $t - 1$ . Within generation  $t$  the following timing holds:

1. The Political regime is implemented by the class with *de facto* power according to (3.9).
2. Individuals decide on consumption fertility and education.
3. Realisation of income, consumption, utility, death.

---

<sup>3</sup>Note that I abstract both from cost of revolution (as present Acemoglu and Robinson (2000)) or differential ability of the classes to mobilise resources as it would unnecessarily complicate the analysis without changing the qualitative results.

<sup>4</sup>The model can be easily extended to account for changes within already partly democratic environments, such as extending the voting franchise. For details see Appendix C.1

### 3.3 Analytical Results

A member of generation  $t$  allocates second period income between consumption and expenditure on children. Maximising (3.1) subject to (3.2) or (3.3) gives:

$$n_t^i = \frac{\gamma}{r + e_{t+1}} \quad (3.10)$$

$$c_t^i = (1 - \gamma) \cdot z_t^i, \quad (3.11)$$

implying that fertility choice is independent of income and the same across both classes.

The choice of optimal education depends on technological progress. Maximisation of utility with respect to  $e_{t+1}^i$  gives the following relation between  $e_{t+1}^i$  and  $g_{t+1}$ :

$$Q(e_{t+1}, g_{t+1}) \equiv (r + e_{t+1})h_e(e_{t+1}, g_{t+1}) - h(e_{t+1}, g_{t+1}) \begin{cases} = 0 & \text{if } e_{t+1} > 0, \\ \leq 0 & \text{if } e_{t+1} = 0. \end{cases} \quad (3.12)$$

In order to ensure the existence of a positive level of technological progress without education I follow Galor and Weil (2000) and impose the assumption that

$$Q(0, 0) = rh_e(0, 0) - h(0, 0) < 0,$$

which implies

$$\frac{1}{\kappa\theta r} - 1 > 0. \quad (A2)$$

The following Lemma establishes the change of the qualitative dynamics within the economy.

**Lemma 2.** *If  $g^* > \frac{1}{\kappa\theta r} - 1$ , there exists a time period  $\bar{t}$  above which the education level becomes positive.*

*Proof.* Solving (3.12) for  $e_{t+1}$  gives:

$$e_{t+1} = \max \left\{ 0, \frac{1}{1-\theta} \left[ \theta r - \frac{1}{\kappa(1+g_{t+1})} \right] \right\} .$$

If (A2) is satisfied, then for any  $\theta$ ,  $r$ , and  $\kappa$

$$\theta r - \frac{1}{\kappa} < 0$$

Subsequently, this implies

$$g_{t+1} < \frac{1}{\kappa\theta r} - 1$$

for some  $t$ . Solving (3.8) recursively gives:

$$g_{t+1} = \eta^{t+1} g_0$$

for all  $t$  with  $e_t = 0$ . Since  $g_{t+1}$  is monotonically increasing over time and  $g^* > \frac{1}{\kappa\theta r} - 1$  the result follows from the Intermediate Value Theorem.<sup>5</sup> The time period is uniquely defined by:

$$\bar{t} = \ln \left[ \frac{1}{g_0} \left( \frac{1}{\kappa\theta r} - 1 \right) \right] - 1$$

□

We are now in position to state the main result of the theory.

---

<sup>5</sup>Assuming  $g^* > \frac{1}{\kappa\theta r} - 1$  is not particularly restrictive as it simply implies that a modern growth regime will have some level of education.

**Proposition 5.** *Increases in income per capita have differential effects on the likelihood of democratisation. Namely, increases in income per capita affect the probability of democratisation either negatively or not at all before the demographic transition while they can have positive effects afterwards.*

*Proof.* Rewriting condition (3.9) gives the following relation for oligarchy

$$\frac{N}{h_t L_t} > (1 - 2\delta_t)^{\frac{\sigma}{\sigma-1}}, \quad (3.13)$$

where the right hand side is a constant since (3.10) is the same for both classes and therefore relative class size remains constant. In a pre-transitional environment (3.13) simplifies to

$$\frac{N}{L_t} > (1 - 2\delta_t)^{\frac{\sigma}{\sigma-1}},$$

with income per capita given by

$$\begin{aligned} \frac{Y_t}{L_t} &= A_t \left[ \left( \frac{H_t}{L_t} \right)^{\frac{\sigma-1}{\sigma}} + \left( \frac{N}{L_t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &= A_t \left[ 1 + \left( \frac{N}{L_t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \end{aligned} \quad (3.14)$$

As can clearly be seen the only ways to increase income per capita in a pre-transitional environment is either via increases in TFP  $A_t$  and land  $N$  or via a reduction in population size. However, increases in TFP have no effect on (3.13), while increases in land or decreases in population size increase the left hand side, thereby making democratisation less likely. Therefore increases in income per capita either have no or negative effects on the likelihood of democratisation in a pre-transitional environment.

As soon as the economy undergoes the demographic transition individuals invest in education. Subsequently (3.13) and (3.14) become:

$$\frac{N}{[\kappa(1 + g_{t+1})e_{t+1} + 1]^\theta L_t} > (1 - 2\delta_t)^{\frac{\sigma}{\sigma-1}},$$

and



$$\frac{Y_t}{L_t} = A_t \left[ \left( [\kappa(1 + g_t)e_t + 1]^\theta \right)^{\frac{\sigma-1}{\sigma}} + \left( \frac{N}{L_t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} .$$

Now, both education level  $e_{t+1}$  and the rate of technological progress  $g_{t+1}$  positively affect (3.14) and negatively the left hand side of (3.13). The effects of TFP, land and population size stay the same. However, note from (3.10) that increases in the rate of technological progress and education level reduce fertility, therefore reducing  $L_{t+1}$ . This second order effect increases future income per capita even further but also increases the left-hand side of (3.13). Nonetheless, the effect of education and technological progress on the likelihood of democratisation is positive if  $h_{t+1}L_{t+1} > h_tL_t$ , i.e.

$$\frac{r\kappa(1 + g_{t+2}) - 1}{r\kappa(1 + g_{t+1}) - 1} \cdot n_t > 1$$

which holds for any  $g_{t+1} < g^*$  if  $n_t \geq 1$ . Consequently, increases in income per capita can have a positive effect on the likelihood of democratisation but the strength of this effect may vary. □

In summary the theory delivers two testable predictions:

1. Increases in income per capita negatively affect the likelihood of democratisation if the economy has not yet undergone the demographic transition.
2. Increases in income per capita positively affect the likelihood of democratisation if the economy has undergone the demographic transition.

### 3.4 Specification and Data

The benchmark empirical put forward by Acemoglu et al. (2008) is

$$d_{i,t} = \alpha d_{i,t-1} + \gamma y_{i,t-1} + \delta_i + \mu_t + u_{i,t} , \tag{3.15}$$

with  $d_{i,t}$  being the democracy score of country  $i$  at time  $t$ . In order to capture persistence and mean-reverting dynamics a lagged dependent variable is included. The coefficient  $\gamma$

captures the effect of lagged log income per capita on democracy and is therefore the focus of the analysis. In order to account for time invariant cross-country heterogeneity and global trends a full set of country ( $\delta_i$ ) and time ( $\mu_t$ ) fixed effects is included. Additional omitted factors are captured in the error term  $u_{i,t}$ .

To account for heterogeneous effects of income on democracy I extend the baseline specification by allowing income per capita to have different effects on democracy for different groups of countries, namely

$$d_{i,t} = \alpha d_{i,t-1} + \gamma_1 (\text{forefollow}_i \times y_{i,t-1}) + \gamma_2 (\text{traillate}_i \times y_{i,t-1}) + \delta_i + \mu_t + u_{i,t}, \quad (3.16)$$

where  $\text{forefollow}_i$  and  $\text{traillate}_i$  indicate whether a country had an early (pre 1960) or late demographic transition. This specification allows isolating the effect of the demographic transition since the possibility of heterogeneous convergence processes and development dynamics between the two groups of countries are accounted for.

The empirical analysis uses the five and ten-year panel data sets over the period 1960-2000 from Acemoglu et al. (2008). The dependent variable is the democracy index by Freedom House normalised to the range from 0 to 1. Higher values indicate higher values of democracy. The data on income per capita come from the Penn World Tables. Information on the demographic transition is obtained from the data compiled by Reher (2004).

### 3.5 Results

Table 3.1 presents the results of estimating models (3.15) and (3.16) over the period 1960 to 2000 for the five-year panel. Column (1) replicates the results of Acemoglu et al. (2008). Results differ slightly since some observations that are not in the database of Reher (2004) are lost. Nonetheless, the result of Acemoglu et al. (2008) stands: once controlling for fixed effects the effect of income on democracy is zero. Columns (2) and (3) report the results for the split sample. The findings support the hypotheses of the theory. Consistent with the theory the results document a positive and highly significant effect of income on

**Table 3.1:** *Income and Democracy: Replication and Baseline Results*

	Democracy (Freedom House)		
	Full Sample	Forefollow	Trailate
	(1)	(2)	(3)
Democracy <sub>t-1</sub>	0.371*** (0.054)	0.582*** (0.095)	0.289*** (0.056)
Income <sub>t-1</sub>	-0.032 (0.038)	0.137*** (0.045)	-0.089* (0.049)
Panel	5-years	5-years	5-years
Observations	718	262	456
Countries	102	34	68
R-Squared	0.139	0.377	0.092

Fixed-effect OLS. Panel regressions with clustered standard errors in brackets. \*, \*\*, \*\*\* denote significance at 10%, 5%, and 1% levels. All regressions include country fixed effects and time fixed effects.

democracy if the country underwent an early demographic transition and a negative albeit less significant effect of income on democracy if the country experienced a late demographic transition. Again, this is in line with the theory as the effect of income on democracy can be zero.

Table 3.2 reports the results of estimating models (3.15) and (3.16) over the period 1960 to 2000 for the ten-year panel. Again, the results for the full sample replicate the results obtained by Acemoglu et al. (2008). In the split samples the qualitative patterns of the results from the 5-year panel remain intact. The effect of income on democracy is positive for countries with an early demographic transition and negative for ones with a late transition. However, the coefficients are insignificant. Nonetheless, it is interesting to note that the coefficient of the lagged dependent variable is positive and highly significant for early transitions and negative and significant for late transitions, indicating a higher degree of political stability in countries with an earlier transition.

**Table 3.2:** *Income and Democracy: Replication and Baseline Results*

Democracy (Freedom House)			
	Full Sample	Forefollow	Trailate
	(1)	(2)	(3)
Democracy <sub>t-1</sub>	-0.038 (0.087)	0.338*** (0.126)	-0.171** (0.083)
Income <sub>t-1</sub>	0.014 (0.069)	0.204 (0.136)	-0.080 (0.080)
Panel	10-years	10-years	10-years
Observations	718	262	456
Countries	102	34	68
R-Squared	0.139	0.377	0.092

Fixed-effect OLS. Panel regressions with clustered standard errors in brackets. \*, \*\*, \*\*\* denote significance at 10%, 5%, and 1% levels. All regressions include country fixed effects and time fixed effects.

Since the regression framework is a dynamic panel with fixed effects and the number of observations is small the results could potentially be biased (see Nickell (1981)). However, the resulting bias is unlikely to be the driving force behind the heterogeneous effects between the groups as there is no reason statistical bias should vary systematically across subsamples.

In order to further refine the heterogeneity of the effect of income on democracy I extend (3.16) to allow for more granularity between the groups. More precisely the new specification is

$$\begin{aligned}
 d_{i,t} = & \alpha d_{i,t-1} + \gamma_1 (fore_i \times y_{i,t-1}) + \gamma_2 (follow_i \times y_{i,t-1}) \\
 & + \gamma_3 (trail_i \times y_{i,t-1}) + \gamma_4 (late_i \times y_{i,t-1}) + \delta_i + \mu_t + u_{i,t},
 \end{aligned}
 \tag{3.17}$$

where  $fore_i$ ,  $follow_i$ ,  $trail_i$  and  $late_i$  indicate varying degrees of development with regards to the demographic transition (from most to least advanced).

**Table 3.3:** *Income and Democracy: Heterogeneity within Groups*

Democracy (Freedom House)				
	Forerunner	Follower	Trailer	Latecomer
	(1)	(2)	(3)	(4)
Democracy <sub>t-1</sub>	0.619*** (0.131)	0.433*** (0.112)	0.331*** (0.054)	0.233** (0.090)
Income <sub>t-1</sub>	0.197*** (0.061)	0.102 (0.081)	-0.059 (0.086)	-0.159*** (0.051)
Panel	5-years	5-years	5-years	5-years
Observations	179	83	199	257
Countries	23	11	30	38
R-Squared	0.415	0.244	0.112	0.076

Fixed-effect OLS. Panel regressions with clustered standard errors in brackets. \*, \*\*, \*\*\* denote significance at 10%, 5%, and 1% levels. All regressions include country fixed effects and time fixed effects.

Table 3.3 presents the results of estimating models (3.17) over the period 1960 to 2000 for the five-year panel. The results document a very strong heterogeneous effect between the groups. Namely column (1) shows that the effect of income and democracy is positive and highly significant for demographically very developed countries. Once the level of demographic development decreases so does the size of the coefficient, as is the case for the group of followers in column (2). The effect of income on democracy is still positive but lower than for the forerunners and insignificant. Columns (3) and (4) describes that once one moves to demographically less developed countries the effect of income on democracy indeed reverses. The effect is negative but insignificant for the group of trailers, while it is strongly negative and highly significant for the group of latecomers.

Table 3.4 shows the results of estimating models (3.15) and (3.16) over the period 1960 to 2000 for the ten-year panel. As with the previous specification the qualitative pattern of results stays intact. The effect of income on democracy is positive for demographically very advanced countries and decreases when demographic development declines to the point of

**Table 3.4:** *Income and Democracy: Heterogeneity within Groups*

	Democracy (Freedom House)			
	Forerunner	Follower	Trailer	Latecomer
	(1)	(2)	(3)	(4)
Democracy <sub>t-1</sub>	0.457*** (0.094)	-0.161 (0.210)	-0.164 (0.115)	-0.234** (0.115)
Income <sub>t-1</sub>	0.246 (0.186)	0.206 (0.168)	-0.116 (0.119)	-0.129 (0.088)
Panel	5-years	5-years	5-years	5-years
Observations	95	38	99	122
Countries	22	11	26	37
R-Squared	0.269	0.066	0.038	0.069

Fixed-effect OLS. Panel regressions with clustered standard errors in brackets. \*, \*\*, \*\*\* denote significance at 10%, 5%, and 1% levels. All regressions include country fixed effects and time fixed effects.

becoming negative for the latecomer countries. Also, as in the previous specification the effect of past democracy also declines with less demographic development.

Altogether the empirical results suggest a strongly heterogeneous effect of income on democracy. Consistent with the developed theory the effect of income strongly depends on the level of demographic development with low levels leading to a negative effect and high levels to a positive one, respectively.

### 3.6 Conclusion

This paper argues that the effect of income on democracy is heterogeneous. The demographic transition fundamentally changed the income composition of a country towards human capital oriented growth. This changed the composition and distribution of income and thereby ultimately also the distribution of power. Enriched masses had an easier push towards democracy than in previously more unequal societies where gains in income per

capita were largely absorbed by those in power. As a result one should expect that the extent of demographic development has a considerable influence on the effect of income on democracy.

This notion is underlined by the theoretical model developed in this paper. Subsequent empirical analysis of this testable implication shows that the non-linear dynamics of demographic development are not sufficiently captured by the fixed effect of a linear regression framework. Namely, the results reveal substantial heterogeneity in the effect of income on democracy both between demographically developed and non-developed countries as well as within both groups.

Overall, the results suggest that there exists sizeable heterogeneity in the effect of democracy on income and that the demographic transition is an important determinant of both the absolute size and the sign of this effect. Additionally, they provide further support for specifications that account for non-linear effects during a countries development path. Lastly, they also suggest that empirical studies finding a zero mean effect of income on democracy might be driven by sample composition.

## Chapter 4

# Long-Run Dynamics of Inequality: Insights from a Unified Growth Theory<sup>1</sup>

### 4.1 Introduction

Economists have always had an interest in a better understanding of the long-run dynamics of inequality, but the recent years have shown a revived interest in inequality research, both in terms of uncovering better data and new theories to explain the empirical patterns. This new work has produced empirical results that contradict the “conventional wisdom” of a hump-shaped relationship between economic development and inequality, and a secular decline in inequality, by documenting a substantial increase in inequality in the recent decades in most developed economies. As consequence, there is an ongoing discussion about the reasons for this recent surge in inequality, particularly at the top end of the income distribution, and considerable confusion about why the “conventional wisdom” might not hold (anymore). Various explanations for the observed dynamics of inequality have been put forward, which include the role of skill-biased technical change, the role of heterogeneity

---

<sup>1</sup>This chapter is joint work with Matteo Cervellati and Uwe Sunde.



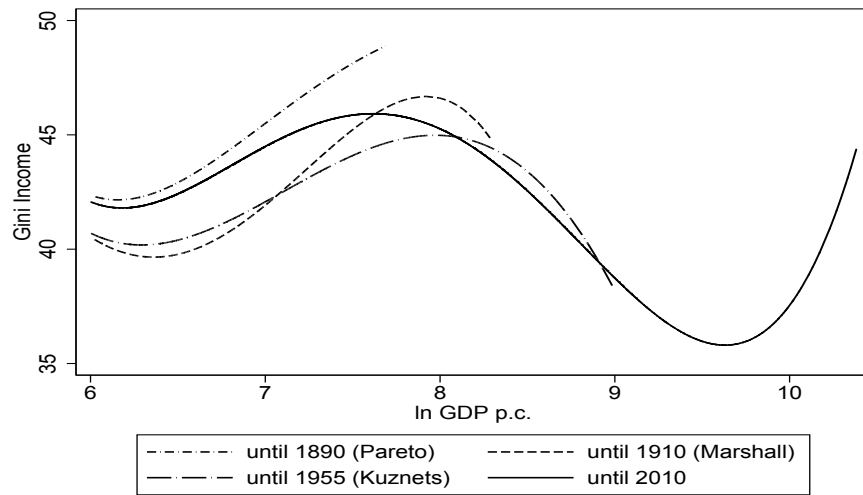
in ability, entrepreneurial success, as well as institutional factors.

This paper proposes a theoretical framework of long-run development that decomposes the dynamics of inequality along the long-run development path into the various forces and explanations that have been put forward in the literature and that is consistent with the “conventional wisdom” as well as the recent surge in inequality.

The analysis is rooted in the first systematic attempts to isolate the determinants of inequality and its dynamics. These attempts are attributed to Pareto (1896), who, by looking at statistical data, uncovered regularities in the shape of the upper part of the income (and wealth) distribution. His work provided arguments about the expected changes of the entire income distribution during the process of transition to an industrial society, instead of focusing on particular statistics of the distribution. Shifting the focus away from the role of social classes that prevailed at the time, Pareto associated inequality with the heterogeneous ability of different individuals to generate income, such as skills and attitudes. This ability changed with a changing structure of opportunities, and hence the respective returns. In particular, he argued that in archaic societies one would expect to observe a mass of individuals at the lowest levels of incomes, close to economic subsistence and he predicted a more dispersed income distribution to emerge as an economy develops. Lack of data prevented Pareto to substantiate his intuitions on the evolution of the full income distribution. Other early thinkers, most notably Marshall (1910), also predicted an increase in inequality during the process of development.

Half a century later, Kuznets (1955) offered a different and more subtle view on the global dynamics of inequality, which he even more explicitly linked to long-run structural change. Empirical observations lead him to conjecture an eventual decline in inequality, which he attributed to the “rise in income share of the lower groups within the non-agricultural sector of the population”. However, Kuznets conceded that, given the lack of reliable data, his conjecture was “perhaps 5 per cent empirical information and 95 per cent speculation, some of it possibly tainted by wishful thinking” (Kuznets, 1955, p.26). Kuznets’ conjecture, which is often interpreted as implying a hump-shaped relationship between income and inequality,

**Figure 4.1:** *Development and Changing Patterns of the Income-Inequality Nexus*



Note: The figure depicts the changing patterns of kernel-weighted local polynomial regression estimates of the relationship between income and inequality by using all available data from van Zanden et. al. (2014) from 1850 to 1890, 1910, 1955 and 2010, respectively.

has sparked an intense and ongoing debate.

The last decades have seen an unprecedented effort in data collection and measurement in order to substantiate or qualify these conjectures. Different strands of the literature have documented changes in inequality in different dimensions and parts of the income and wealth distribution. These changes include, in particular, the dynamic evolution in inequality in the population at large (as measured by, e.g., the Gini index), the increase in upper tail inequality, and the changing patterns of the world income distribution in terms of inequality across countries at the global level. The recent evidence points to an increase in inequality between skilled and unskilled workers, which has been argued to be linked to skill biased technical change, and to an increase in residual inequality, particularly in the upper tail of the income distribution, in developed countries.

These patterns of long-run inequality dynamics are documented in Figure 4.1, which plots the changing relationship between development and inequality using the latest available data. The figure also illustrates that the previous conjectures about this relationship had indeed been informed by the data available at the time of writing. However, the dynamics

of inequality exhibit more complicated patterns than previously thought. Based on the new data and emerging evidence of a recent increase in inequality, Kuznets' conjecture has come under attack.

The empirical patterns of the evolution of inequality have been documented previously based on variation within and across countries. While the empirical patterns in these different dimensions are presumably related to each other, existing explanations for the dynamics of inequality have focused on particular time periods or parts of the income distribution in isolation from each other, contributing to entirely separate strands of the literature. To date, there is no consensus about the mechanisms that lie behind each of the specific empirical patterns. Even more importantly, a coherent theoretical framework that allows studying the evolution of inequality in these different dimensions over the course of development is still missing. As a consequence, it is hard to evaluate the relative importance and interlinkages of the mechanisms that lie behind the empirical patterns of inequality dynamics that have been documented in the different dimensions, and the empirical relevance and mutual consistency of the different hypotheses that have been brought forward to explain them.

This paper tries to make progress by proposing a simple theoretical framework that delivers predictions about the long-run dynamics of the entire income distribution, including the upper tail, both within and across countries. By focusing on the implications of long-run dynamics during a transition from almost stagnant to sustained growth, the approach in fact nests the perspectives originally put forward by Pareto and Kuznets, namely that the dynamics in these different dimensions of inequality are systematically related to each other and are the result of the changing shape of the income distribution in countries that undergo the different stages of the non-linear process of long-run development. In spite of its intuitive appeal, this is, to our knowledge, the first research effort to scrutinise this view and relate it to the recent surge of empirical results.

The framework builds on the unified growth model proposed by Cervellati and Sunde (2005). The main building block of the theory is a general equilibrium OLG occupational

choice model in which agents are initially endowed with different abilities and have to choose whether to stay unskilled or to become skilled. Becoming skilled is more rewarding for individuals with higher ability, but involves a time-consuming education process. Across generations, the individual incentives to become skilled are crucially related to the endogenous change in returns to skills and life expectancy. The engine of long run development that eventually drives a transition from quasi-stagnation to sustained growth is the acquisition of ability-intensive skilled human capital at the aggregate level.

This baseline set-up is extended in two main dimensions. First, we propose a generalised version of a standard Mincer-type representation by allowing for (log) levels of human capital that are increasing and convex in ability. An important implication of the proposed formulation is that even retaining the (standard) assumption of a (time invariant) normal distributions of ability it generates an empirically relevant income distribution. In particular, the income distribution approximates a log normal around the mean and a Pareto on the upper tail. A second new element of the theory is the explicit consideration of both skill-biased technological change (following Nelson and Phelps, 1966) and ability-biased technical changes within the skilled population (following Galor and Moav, 2000). While the former endogenously affects inequality between skilled and unskilled, which shapes the income distribution in the early phases of the transition, the latter is the main driver of inequality within skill groups and shapes the income distribution in later stages of development as the economy approaches balanced growth paths with the great majority of the population being skilled.

The model allows for an analytical characterisation of the evolution of the full income distribution. Inequality dynamics result from endogenous changes in the composition of population in terms of skills and in the returns to individual abilities. By characterising the changes in the relative importance of the two sources of inequality, the model delivers predictions about the change of the entire income distribution during the process of transition from stagnation to growth. In particular, the model delivers predictions about the dynamics of inequality in the population at large (in terms of e.g. Gini Index) and in the upper tail of

the income distribution (in terms of e.g. of top 10, 1 and 0.1 per cent income shares) within a simple and analytically tractable framework.

The model delivers a set of new testable predictions about the evolution of the income distribution within countries over time, about the changing returns to skills and ability, and about the evolution of global inequality during the last two hundred years. Starting from an almost degenerate income distribution with a large majority of the population being unskilled and earning similar incomes, the transitional dynamics of the model predict an increase in income inequality with an income distribution on the balanced growth path that is approximately log normal around the mean but with thicker (Pareto) tails. This shape results from the interplay of a normal ability distribution coupled with endogenously changing convex returns to ability. The theory also predicts non-monotonic dynamics in income inequality in the population at large, reflected in, e.g., the Gini index. Specifically, the model predicts a hump-shaped inequality pattern that is the result of an initial increase and a subsequent decrease in the skill-related heterogeneity within the population. These dynamics in the skill composition are due to long-run development forces and resemble the dynamics conjectured by Kuznets. They are followed by a later phase that is characterised by another increase in inequality, which is driven by the accelerating dispersion of the returns to ability among the group of the skilled. While upper tail inequality and top income shares tend to monotonically increase over the course of development, the rise is more pronounced when the economy converges to the balanced growth path. Finally, by accounting for the differential timing of growth take-offs across countries, the theory also delivers predictions about the evolution of global inequality.

Using a quantitative version of the model, which allows for a simulation of the dynamic evolution of a model economy as well as a multi-country extension, the theoretical predictions are confronted with the empirical data patterns. The comparison of the dynamic evolution of the simulated baseline economy to the data in terms of changes in Gini index, relationship between income and inequality (the Kuznets Curve) and in terms of the evolution in upper tail inequality (in terms of the top 10, 1 and 0.1 percent income shares)

documents the model's ability to generate realistic data patterns. In particular, the dynamics exhibit a hump-shaped evolution of inequality resembling the Kuznets hypothesis, as well as a subsequent increase in inequality. This pattern is consistent with empirical observations of the dynamics of the Gini coefficient over a long time horizon. In addition, the monotonically increasing role of ability for productivity implies an increase in top income shares. This pattern interacts with the transition to mass education and gains momentum in the later phases of the development process. The model thereby generates patterns that resemble those of increasing top income shares documented in the data.

Beyond the dynamics of inequality within a country, a straightforward extension of the model the model can be used to study global inequality dynamics. As a first step to explore this issue, we run a simple counterfactual exercise by considering two economies that only differ in terms of the parameter driving ability-biased technical change. The results illustrate that the two economies exhibit similar dynamics in development and inequality (in terms of Gini) for most of their history but eventually display a markedly different evolution in the top income shares. A comparison with data from the US and France delivers similar patterns, contributing new insights to the current debate on the changing role of time invariant country specific characteristics. The results also exemplify that explaining diverging patterns in top income shares in mature economies does not necessarily require identifying time variation in country specific characteristics (like e.g. institutional or fiscal environments, etc). By highlighting the role of the transitional processes, in particular the economic transition and the simultaneous demographic transition in education attainment, the model also implies that the overall dynamics of development have relevant consequences for global inequality patterns. The simulation of an artificial world that consists of countries that exhibit identical features except for the timing of their transition, the global inequality patterns are shown to also exhibit non-linearities. In fact, the increase in global inequality stipulated by the model predictions related to delays in the transition across countries is consistent with the observed empirical patterns.

Finally, with the inequality dynamics implied by the model being closely related to de-

mographic dynamics, one would expect similar patterns with these demographic dynamics. The exploration of some predictions of the theoretical framework turns out to be informative about the specific mechanisms highlighted by the model. The comparison of the dynamics in education inequality or inequality in life expectancy with the dynamics implied by the model reveals very similar patterns. The results thereby provide a natural link to recent empirical findings of a Kuznets-type pattern in education over the long run, see Morisson and Murtin (2014), or in life expectancy, see Bourguignon and Morrisson (2002), providing further support for the view of a unified model of long-run development being consistent with the long-run dynamics of inequality.

**Related Literature.** This paper contributes to the literature in several ways. The prototype unified growth theory presented here extends that by Cervellati and Sunde (2015b) to study the changing shape of the full income distribution of within and cross-country inequality.<sup>2</sup> Recently, research on establishing stylised facts regarding changes in inequality in the income distribution at large, in the income appropriated by the top income earners and inequality at the global level (worldwide) has experienced a revival. The accumulating evidence for a sharp increase in upper tail inequality (measured in terms of income accruing to top income earners) that followed Piketty (2014) and Piketty and Saez (2014) materialised in an intense debate on the explanations and likely causes, including recent contributions by Acemoglu and Robinson (2015), Blume and Durlauf (2015), Krusell and Smith (2015), Jones (2015), and Ray (2015). Specific theories have been offered to explain income and wealth distributions with fat (Pareto) tails, see, e.g., Benhabib et al. (2011), and Piketty and Zucman (2015), Gabaix et al. (2016), and Jones and Kim (2018). This literature focuses exclusively on steady states or transitional dynamics around the balanced growth path and concentrates on

---

<sup>2</sup>Alternative models of the demographic transition focus on the role of the fertility transition (see, e.g., Galor and Weil, 2000, Dalgaard and Strulik, 2015, 2016), a mortality decline (see, e.g., Boucekine, de la Croix, and Licandro, 2003, Cervellati and Sunde, 2005, Dalgaard and Strulik, 2014), or the transition in fertility and mortality (de la Croix and Licandro, 2013) for development. By illustrating the extent to which inequality dynamics within and across countries can be accounted for by demographic patterns, the paper also sheds new light on the importance of long-run demographic dynamics for economic performance, as emphasised recently by Cervellati *et al.* (2017).

random returns in the tradition of Pareto, who also inquired about the role of randomness, which he called the “set of unknown causes, acting now in one direction, then in another, to which, given our ignorance of their true nature, we give the name of chance”, for generating fat tails in the income distribution. Contrary to the perspective taken by Pareto, the recent literature abstracts, however, from long-run changes in the full distribution of incomes. Differently from the share of income appropriated at the top, the long-run dynamics of the entire income distribution has received much less empirical and theoretical attention in this strand of the literature. Our contribution is to fill this gap by focusing attention on the role of long-run dynamics in the context of the economic and demographic transition.

Some earlier theories have offered insights regarding the long-run drivers of inequality in the transition out of agriculture and their interactions with capital markets and institutions, see Aghion *et al.* (1999), Lindert and Williamson (2003) and Acemoglu and Robinson (2006), respectively. This theoretical research, which is closer to the arguments by Kuznets, was not specifically aimed at characterising the full income distribution and, accordingly, does not deliver explicit predictions on the evolution of top income shares. This perspective has been criticised, among others by Piketty (2014), on the ground that the empirical measures of overall inequality are based on non-comparable and often unreliable data and by pointing out that some of the structural predictions that suggest phases of reductions in inequality do not square with the recent evidence on the top income shares. The theory presented below addresses this point and thereby provides a link between the theoretical literatures focusing on the long-run dynamics of overall inequality and the evolution of top income shares. of inequality have not been at the center stage of the unified growth literature about the transition from stagnation to growth, even though some models have featured inequality as a factor underlying the transition, see, e.g., see, e.g., Galor and Moav (2006) and Galor *et al.* (2009).

Empirical research investigating the conjecture by Kuznets about the effect of structural change on inequality has mainly concentrated on summary statistics of the full distribution of incomes, in particular the Gini index. Empirical evidence based on cross-country panel



data deliver mixed, and sometimes contradictory, findings regarding the relevance of this conjecture. The results often depend on the sample of countries and the time period under consideration (see, e.g., Jha, 1996, Tam, and the references therein). By focusing on the role of non-linear dynamics, our theory provides a straightforward explanation for these mixed results.

Finally, a recent empirical literature has established new stylised facts about the evolution of global inequality, see Bourguignon and Morrisson (2002), van Zanden et al. (2014), Milanovic (2014) and Solt (2016). Our paper provides a coherent framework that is able to produce these stylised facts while accounting for a realistic pattern of inequality dynamics within countries.

The rest of the paper is organised as follows. Section 4.2 presents the model. In Section 4.3 we contrast the model predictions for within-country inequality dynamics with those observed in the data. Section 4.4 looks at global inequality dynamics. Section 4.5 studies some specific predictions of the proposed mechanism and Section 4.6 concludes. Proofs of analytical results and details of the simulation are relegated to the Appendix.

## 4.2 Theoretical Framework

### 4.2.1 The Model

While the analytical results do not hinge on specific functional forms, we report the respective specifications of functional forms used in the quantitative analysis below as we introduce them in the model.

**Set-up.** The model economy features overlapping generations of individuals denoted by  $t \in \mathbb{N}^+$ . Each generation,  $t$  faces a life expectancy  $T_t$  (with maximum life spell  $\bar{T}$ ).

Individuals differ in terms of innate ability  $a \in [0, 1]$  which is drawn from a (truncated) normal distribution  $a \sim N(\mu, \sigma^2)$ . Individuals are price takers and decide to be either unskilled,  $U$  or skilled,  $S$ , with corresponding unit wages  $w_t^u$  and  $w_t^s$ , respectively. Ability is assumed to be more valuable for skilled human capital. For simplicity, assume that ability

does not affect the level of unskilled human capital,  $h_t^u$ , while it increases the returns of skilled workers. The function  $h_t^s(a)$  maps ability into skilled human capital and, given the wage for skilled workers  $w_t^s$  (that will be determined below), into income  $y(a) = w_t^s \cdot h_t^s(a)$ . The main trade-off faced by individuals is between the time spent in acquiring skills,  $\underline{e}^s > 0$ , and the relative lifetime returns to unskilled and skilled occupations. The dynamics of the returns to ability in skilled human capital play an important role for the patterns of inequality in the last decades, as is discussed below.

**Production and Wages.** A unique final consumption good is produced with a constant returns to scale aggregate production function using unskilled and skilled human capital as inputs. The production function we use in the quantitative analysis has CES functional form

$$Y_t = A_t [(1 - x_t) (H_t^u)^\eta + x_t (H_t^s)^\eta]^\frac{1}{\eta}, \quad (4.1)$$

with  $\eta \in (0, 1)$  and  $x_t \in (0, 1) \forall t$ . The model is closed in general equilibrium with wages equalling marginal productivity

$$w_t^s = \frac{\partial Y_t}{\partial H_t^s}, \quad w_t^u = \frac{\partial Y_t}{\partial H_t^u}. \quad (4.2)$$

**Optimal Occupational Choices.** Individuals choose between becoming skilled or staying unskilled in order to maximise their lifetime income taking both life expectancy,  $T_t$ , and wages  $w_t^u$  and  $w_t^s$  as given. If staying unskilled individuals offer a given level of human capital to the market that for simplicity is assumed to be time invariant and unrelated to ability,  $h_t^u$ , earning a (per period income) given by  $y_t^u = w_t^u h_t^u$ .

Investing in skilled human capital is assumed to involve a fix cost in terms of time  $\underline{e}^s > 0$  that can be interpreted as, e.g. time devoted to training, internships or education. The returns to skilled human capital are assumed to be increasing and (weakly) convex in the level of ability so that

$$h_t^s(a) \quad \text{with} \quad h_t^s(a) \geq 0 \quad \text{and} \quad h_t^s(a) \geq 0$$

which implies a level of individual (per period) income given by:  $y_t^s(a) = w_t^s h_t^s(a)$ .

The main trade-off faced by individuals in choosing their occupation is therefore between the investment in  $\underline{e}^s$  and the lifetime returns. Practically agents have to compare the lifetime income gained by staying unskilled,  $T_t h_t^u w_t^u$  or becoming skilled  $(T_t - e) h_t^s(a) w_t^s$ .

**Returns to skills and ability.** As studied below, the full income distribution at each point in time depends on the population share of individuals becoming skilled, on the skill premium and on the returns to ability within the skilled population. The balanced growth path of the model implies that, in the limit, (almost) the entire population acquires skilled human capital.

A frequently used functional formulation that fits the general properties of the production of skilled human capital is the exponential specification  $h(a) = e^a$  that, in the balanced growth path and coupled with the assumption of a normal distribution of ability, would imply a linear (log) income,  $\ln y(a) = \ln w + a$ , which implies that the full income distribution is *Log Normal*.

However, for the issue at hand a serious limitation of the log linear representation is that it produces counterfactual predictions about the distribution at the upper tail, which tends to be approximately *Pareto* distributed, i.e., exhibit a thicker tail. A growing recent literature focuses specifically, and exclusively, on the upper tail to produce income distributions that fit the empirical evidence for top income shares over the last decades both in terms of shape of the distribution (Pareto) and in terms of changes of the Pareto coefficient, see, e.g., Gabaix *et al.* (2016) and Jones and Kim (2018).

Since the goal of the paper is to explore the changes in inequality arising from income differences both between and within skill groups over the full transition from stagnation to growth, we adopt a generalised representation that allows for log returns to be non linear in ability. In particular, in the simulation of the model we use the formulation

$$h(a) = (1 - a)^{-\phi}, \tag{4.3}$$

with  $\phi > 0$ . This formulation has the advantage of allowing an analytical characterisation of the income distribution on the balanced growth path and a direct comparison of its features relative to those of the Log Normal distribution. In Appendix D.1 we show that the specification (4.3) produces an income distribution that is line with the empirical patterns both for the population at large around the mean *and* for the upper tail. Furthermore the thickness of the tail (governing the top income shares) is driven by a single parameter,  $\phi > 0$ . In particular, the density at the top is approximately log normal if  $\phi = 1$  while the tails are increasingly thicker for  $\phi > 1$ . Another advantage of this formulation is that the parameter  $\phi$  drives the returns to ability which provides a tractable characterisation of ability biased technical change. As a result, specification (4.3) is able to produce empirically relevant data moments for both inequality in the population at large (e.g. Gini) and for the high income earners (e.g. top income shares).

#### 4.2.2 Evolution of the Economy

To characterise the dynamic evolution of the model economy, we consider intergenerational externalities in reduced form in three dimensions, representing skill-biased technical change, ability-biased technical change, and a health externality affecting life expectancy.<sup>3</sup>

**Skill Biased Technological Change.** We follow Nelson and Phelps (1966) by assuming that the returns to skilled human capital in production,  $x_t$ , increases with the share of skilled workers in the previous generation,  $\lambda_{t-1}$ , and with the scope for further improvement,  $1 - x_{t-1}$ :

$$\frac{x_t - x_{t-1}}{x_{t-1}} = X(\lambda_{t-1}, x_{t-1}) = \lambda_{t-1}(1 - x_{t-1}). \quad (4.4)$$

**Ability Biased Technological Change.** Following Galor and Moav (2000) we assume that the return to ability in skilled human capital is increasing with the growth rate of

---

<sup>3</sup>This modelling essentially follow Cervellati and Sunde (2005) and the parametrisation of Cervellati and Sunde (2015). The main novelty in terms of dynamics is the consideration of ability-biased technical change.

productivity which, in turns, depends on the share of skilled workers in the previous period  $\lambda_t$

$$h_t^s(a, g_t) = (1 - a)^{-\phi(g_t)} \quad \text{with} \quad \phi'(\cdot) > 0 \quad (4.5)$$

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = G(\lambda_t \bar{a}_t) \quad (4.6)$$

Irrespective of the specific functional forms, which are needed only for the numerical implementation, this modelling of skill and ability biased technical change creates a positive feedback loop between skill acquisition and the intensity of ability within skills that affects the skill premium and the returns to ability for skilled individuals.

**Life Expectancy.** The dynamics of the model are closed by assuming, as in Cervellati and Sunde (2005), that life expectancy increases in the share of skilled individuals in the previous generation. While the precise functional form is of not directly relevant, in the simulation we assume the linear formulation

$$T_t = Y(\lambda_{t-1}) = \underline{T} + \rho \lambda_{t-1}, \quad (4.7)$$

where  $\underline{T}$  is the baseline longevity and  $\rho > 0$  reflects the scope for improvement. Since  $\lambda \in (0, 1)$ , the maximum level of adult longevity is  $\bar{T} = \underline{T} + \rho$ .

**The Share of Skilled Individuals.** The share of individuals that optimally choose to be skilled at each point in time is characterised as a general equilibrium outcome resulting from optimal choices within each generation of individuals  $t$  that, as discussed above, maximise lifetime income via occupational choice and the resulting equilibrium wages given by (4.2). As characterised in Cervellati and Sunde (2005) the equilibrium share of skilled individuals denoted by,

$$\lambda_t = \Lambda(x_t, T_t) \quad (4.8)$$

which is an increasing, S-shaped, function of life expectancy (that for any wage reduces the incidence of the time investment  $e > 0$  and is increasing in the level of skill premium (that relates to  $x_t$ ). The derivation of the equilibrium share of skilled workers can be characterised also in terms of a threshold level of individual ability  $a$  above which individuals find it optimal to be skilled. Consequently, the threshold level of ability is initially (for low levels of  $T$ ) large, but eventually (as  $T$  increases) converges to zero as almost all individuals regardless of their ability find it profitable to become skilled. This has relevant implications for inequality since it affects the ability spread between the most able and the least able individual among the group of skilled individuals.

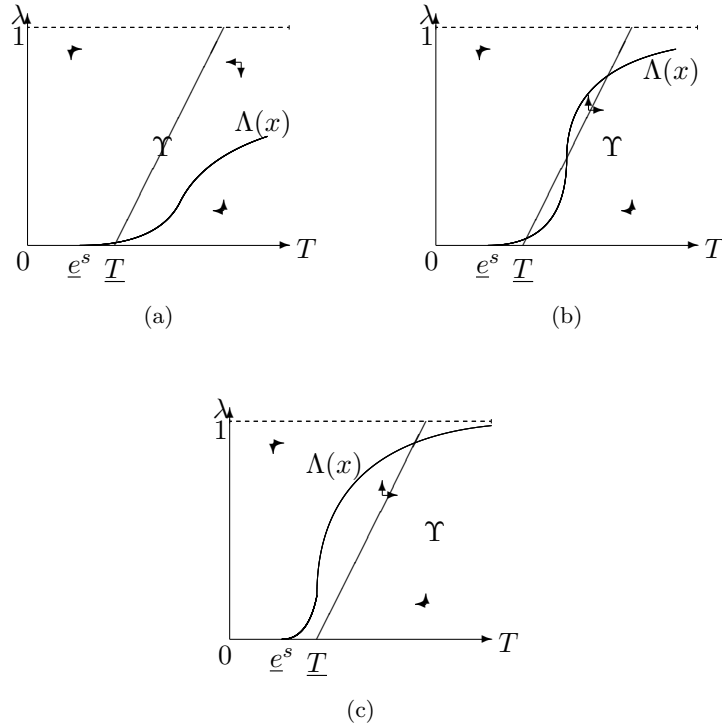
### 4.2.3 Long Run Dynamics and Inequality

This simple occupational choice model endogenously delivers a structural transition from an underdeveloped economy with a majority of unskilled workers to an economy with almost only skilled workers as it approaches the balanced growth path.

For given initial conditions in terms of level of total factor productivity, life expectancy and returns to skills, the dynamic evolution can be characterised by iterating forward the state variables at each point in time as they result from the system of equations given by (4.4) to (4.8). For low enough initial levels of technology,  $A_0$ , returns to skilled human capital,  $x_0$  and life expectancy  $T_0$  the system is characterised, by a very low share of skilled individuals in equilibrium which, in turn, implies slow improvements in technology and life expectancy. The returns to skills,  $x_t$ , and life expectancy are, eventually, sufficiently large to trigger a fast transition to a balanced growth path characterised by a mostly skilled population, large life expectancy and high returns to skills and ability. Figure 4.2 illustrates the evolution of the conditional dynamic system for low, intermediate and high returns to skills ( $x_t$ ), respectively.

The mechanics of the unified theory imply that the change in the distribution of income is the result of the structural process of skill acquisition and skill biased technical change, and of the increase in within skilled inequality to increasing returns to ability. As a result

**Figure 4.2:** Long Run Dynamics: Illustration



Note: Illustration of the dynamic interactions between the share of skilled workers and life expectancy.

the framework delivers the full evolution of the income distribution and hence provides a workhorse model to study the long-run dynamics of inequality.

Consider first the role of the changing composition of the population. From Figure 4.2, in the early (pre-transitional) phase the majority of the population is unskilled, while on the balanced growth path the majority is skilled, respectively. Abstracting from differential returns to ability for skilled human capital, this implies that inequality between skill groups is negligible in the very early phase of development as well as in the late phases of development as the economy settles on the balanced growth path. The reason is that during these phases, the income distribution coincides with degenerate skill distributions where almost all individuals have made the same occupational choice. Inequality begins to increase as the skill composition begins to change in the run-up to the transition. In fact, for

exogenous wages, inequality would peak when the skill distribution is most polarised, with half of the population unskilled and skilled, respectively. At some point during the transition, however, inequality starts declining.<sup>4</sup> This pattern aligns both with the speculations by Pareto on the rising inequality out of a degenerate distribution at subsistence level in the phase of exit from what he called “archaic” to modern societies, and with the arguments by Kuznets about an eventual decline in inequality. This decline is indeed related to the increase in the share of income in the lower segments of the population that move out of agriculture as conjectured by Kuznets. In view of the one-to-one mapping between share of skilled workers and income, this part of the model therefore delivers predictions that are broadly in line with the “Kuznets-Curve”.

The dynamic evolution of the income distribution also depends on the evolution of inequality within skill groups. In particular, this evolution is driven by the share of skilled individuals who experience different returns to ability and ability-biased technical change. The model delivers clear analytical predictions on this dimensions as well. First, the monotonic increase in the share of skilled workers also implies a reduction in the threshold level of ability (of the indifferent worker) and, accordingly, an increase in the ability gap between the most and the least able workers in the skilled population.<sup>5</sup> During the early phase of development, only the few most able individuals decide to become skilled. Correspondingly, the dispersion of ability, and therefore income inequality related to the return to ability, within the skilled population is low. As the share of skilled workers increases, so does the dispersion of ability among the skilled and, accordingly, inequality. Hence, with heterogeneous ability, the distribution of income is not degenerate and inequality on the balanced growth path is unambiguously larger than in the pre-transitional phase. Ability-biased technical change leads to a further dynamic force that

---

<sup>4</sup>While preserving the analytical prediction of a hump-shape, the actual location of the peak in inequality in terms of the share of skilled workers, also depends on the strength of the process of skilled biased technical change that affects the relative wages of skilled and unskilled over time.

<sup>5</sup>Specifically, average ability among the skilled population decreases and while the variance of the ability around the mean increases during the process of development.



increases inequality among the skilled. This component of the model therefore implies a monotonic increase in inequality within skill groups (within the group of skilled individuals, to be precise). This process gains momentum as the share of skilled workers increases and is strongest in the last phase of development.

As consequence of the convex log returns to ability as in (4.3), the model also implies an increasingly thick upper tail of the income distribution with a changing Pareto parameter. In the limit, the income distribution associated on the balanced growth path is approximately log normal around the mean with a Pareto tail.

In summary, the inequality between and within skill groups exhibits the following long-run dynamics:

1. A first phase characterised by hump-shaped inequality dynamics that are mainly the consequence of the transition from a mainly unskilled to a mainly skilled population and the corresponding dynamics of inequality between skill groups along the lines described by Kuznets;
2. and a second phase characterised by an increase in inequality due to ability biased technical change within the mostly skilled population.

Notice that the dynamics of inequality effectively depend on the relative strength of these two processes and hence the strength of the dynamics in the returns to ability, which is largely an empirical question.

## **4.3 Dynamics of Inequality**

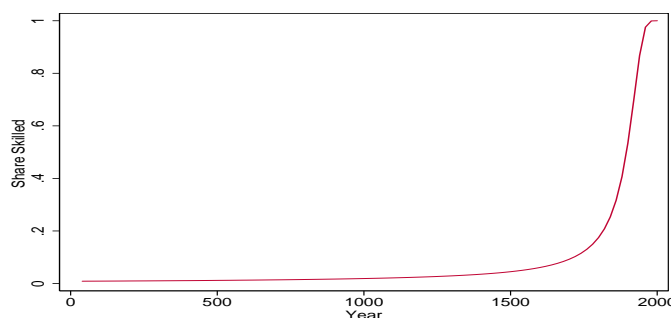
### **4.3.1 Quantitative Implementation**

To illustrate these theoretical predictions and to compare them with actual data we simulate a quantitative version of the model. Following Cervellati and Sunde (2015b), the parametrisation of the model and the calibration of the time invariant parameters is based either on empirical observations where available (e.g., regarding the elasticity of substitution

between skilled and unskilled human capital in the production function), on historical data (e.g., regarding the level of life expectancy), or on the calibration by targeting data moments on the balanced growth path (e.g., regarding the parameters governing the speed of technological progress). A brief discussion of the calibration procedure is reported in Appendix D.2.

The model is simulated by setting initial conditions and computing numerically the evolution of all state variables resulting from equations (4.4) to (4.8). We simulate the endogenous evolution of all variables of interest from year 0 to year 2000.

**Figure 4.3:** *Long Run Development: Share of Skilled from year 0 to 2000*



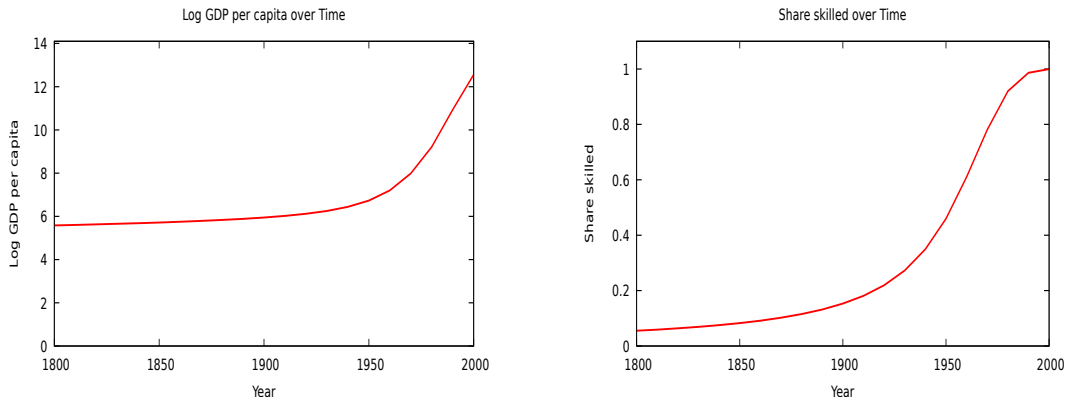
Note: This figure illustrates the simulation of the dynamic evolution of the model from year 0 to year 2000. The simulation is conducted for generations of 10 years. The timing is set by assuming a balanced growth path (a level of share of skills above 0.99) as of year 2000.

### 4.3.2 Evolution of Income Distribution: Gini Index

We begin the evaluation of the empirical relevance of the model dynamics by exploring the long-run dynamics of inequality generated by the model and comparing them to those in the data observed over the last two hundred years. We focus attention on the countries that began the transition to sustained growth during the nineteenth century, the so-called forerunner countries. This allows us to compare the predicted evolution of inequality in the model economy and in the data over time by looking at the Gini Index, which represents the most widely used summary statistic for inequality in the distribution at large.

Figure 4.5 depicts the evolution of the simulated economy over the period 1900-2000 in

**Figure 4.4:** *Income and Skill Distribution from Stagnation to Growth: 1800-2000*

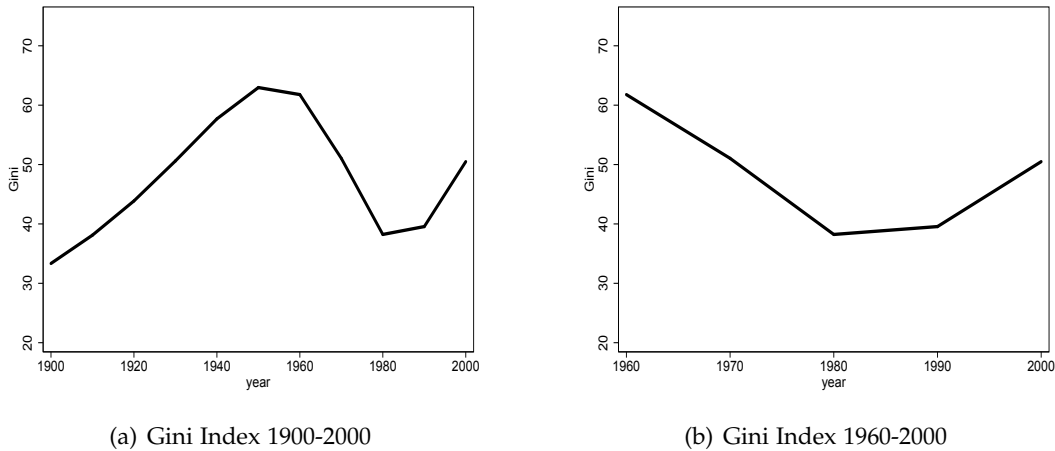


Note: This figure zooms in on the transition period by extracting from the full time series of simulated variables the data for GDP per capita and the share of skilled workers over the period 1800-2000.

Panel (a) and zooms in on the period 1960-2000 in Panel (b). The simulation illustrates the initial increase in inequality associated with the process of skill acquisition. The reduction in the Gini after 1950 is related to a skill composition of the population in which the unskilled workers become a minority. In 1950, the share of unskilled in the model economy is already below 50 percent. By 1970 the share of unskilled individuals is around 20 percent. As the vast majority of the population acquires skilled human capital the role of within skilled inequality, associated with the spread in the distribution of ability and the ability premium, which gets amplified by ability-biased technical change, gain momentum. Eventually, this leads to a reversal in the dynamics of inequality, which begins to increase again as consequence of increasing inequality within the group of skilled individuals.

Figure 4.6 depicts the corresponding evolution of the Gini index in the data for the sample of forerunner countries that underwent the structural transition (related to demography in general and education in particular) during the late nineteenth century. Panel (a) depicts the raw data and the mean from Bourguignon and Morrisson (2002) covering the period 1850-1990. Panel (b) replicates the analysis using data from van Zanden et. al. (2014) (van Zanden *et al.*, 2014), which cover the period 1900-2000. While exhibiting slight differences in terms of the precise timing, both data sets exhibit the same qualitative patterns: inequality

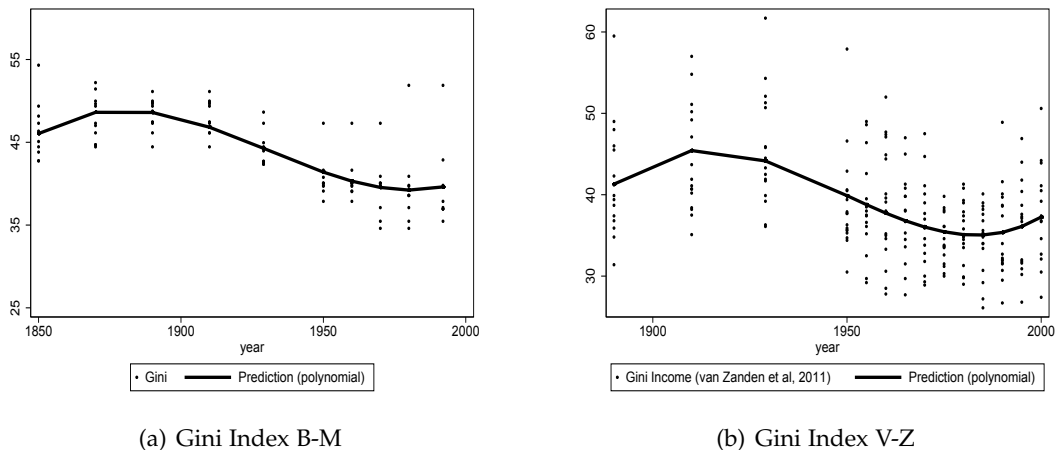
**Figure 4.5:** Long Run Inequality Dynamics (Gini Index): Simulated Model 1900-2000



Note: The Figure depicts the Gini Index computed from the dynamic simulation of the model economy. Panel (a) plots the simulation of the full period while Panel (b) restricts attention to the period 1960-2000. See text for details of the calibration and Section D.2 in the appendix.

dynamics are characterised by a hump-shaped evolution of inequality, until the decline in inequality is reversed and followed by an increase during the last decades of the 20th century.

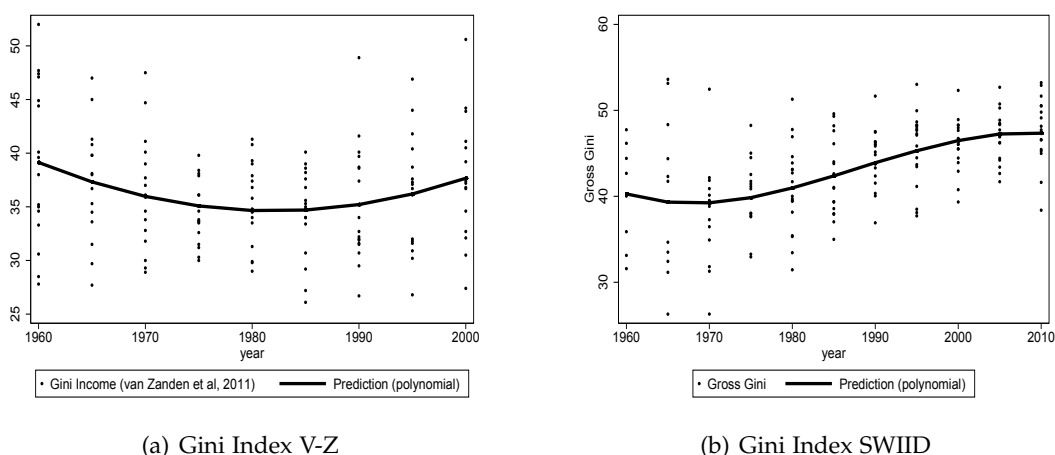
**Figure 4.6:** Long Run Inequality Dynamics (Gini Index): 1850-2000 Data



Note: Panel (a) reports the evolution of the (average) Gini using data from Bourguignon and Morrison (2002) for the sample of forerunners countries (Western Europe and Western Regions and Offshoots). Panel (b) replicates the same analysis using data from van Zanden et al. (2014) (Western Europe and Western Offshoots).

Figure 4.7 reveals the same patterns when restricting attention to the last half century, over the period 1960-2010, where in addition to the data compiled by van Zanden et al. (2014) there are also data available from the alternative source of the Standardised World Income Inequality Database, SWIID. While the different data sources provide a slightly different picture in terms of timing and magnitude, they nevertheless deliver a similar qualitative picture and both point at a reversal in the reduction in income inequality from around the 1970 to 1980.

**Figure 4.7:** *Inequality Dynamics 1960-2000 (Gini Index): Data*



Note: Panel (a) reports the evolution of the (average) Gini using data from van Zanden et. al. (2014) for the sample of forerunner countries (Western Europe and Western Regions and Offshoots). Panel (b) replicates the same analysis using data from the Standardised World Income Inequality Database, SWIID (Western Europe and Western Offshoots).

### 4.3.3 Income and Inequality

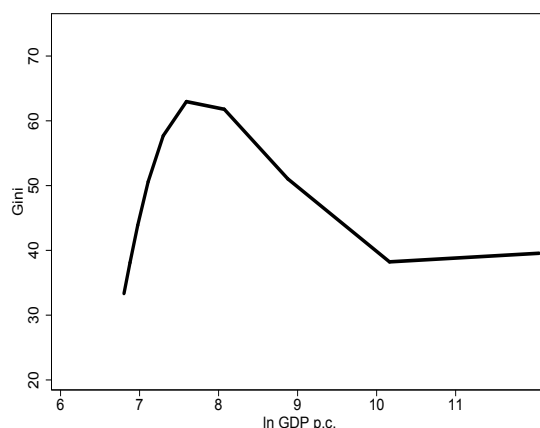
Next, consider the implicit relationship between inequality and income that has become known as the “Kuznets-Curve”. The theoretical framework provides clear predictions about the evolution of inequality and hence this implicit relationship during different phases of development. In particular, the model predicts a hump-shaped (first positive then negative) relationship between inequality and income for low to intermediate levels of development (income), and a reversal to a positive relationship for high levels of development (income).

While, strictly speaking, these predictions are based on the long-run dynamics along

the development path, the model predictions can also be interpreted as reflecting the cross-sectional relationship between income (rather than phases of the development path or time) and inequality. In fact, the common practice of exploring this implicit relationship follows the empirical literature that has tried to test the empirical validity of the conjecture raised by Kuznets (1955). Initially, the empirical evidence of a hump-shape relationship was, indeed, viewed as aligning with Kuznets's conjecture.

To illustrate this model prediction, Figure 4.8, depicts the cross-sectional relationship between income and inequality as implied by the same simulated data contained in Figure 4.5.

**Figure 4.8:** *Inequality and Development: Simulation*

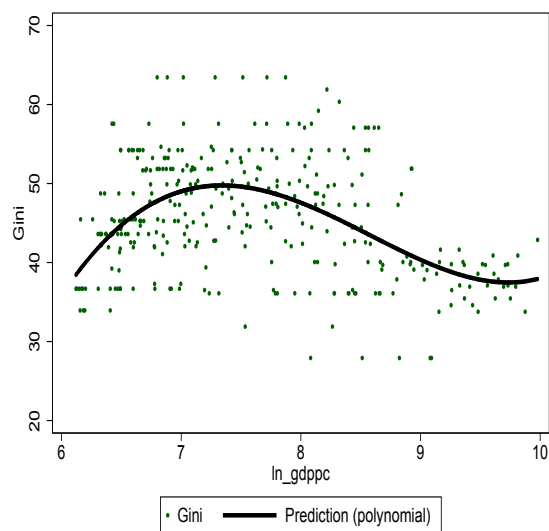


Note: The Figure depicts the relationship between the Gini Index in each year, computed from the dynamic simulation of the baseline model economy and the respective level of income in the same year. See text for details of the calibration and Section D.2 in the appendix.

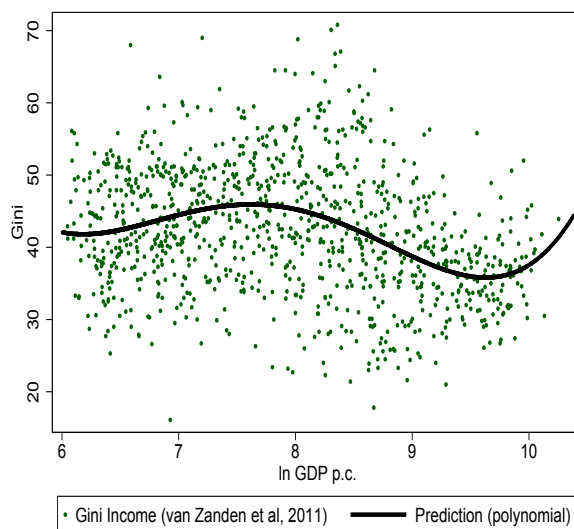
Figure 4.9 plots the same relationship using available country panel data. Panel (a) depicts the corresponding pattern that emerges from the data by Bourguignon and Morrison (2002), which has the advantage of going back to 1820 but covers only the period until 1992. Panel (b) shows the relationship for data from van Zanden et. al. (2014), which cover the period 1850-2000. While again differing in terms of the location of the peak and magnitudes, the two data sources reveal a hump-shaped relationship for low to intermediate income levels along the lines of a Kuznets curve, but an increasing relationship for high income

levels. The same pattern consistently emerges also in Figure 4.10 when using data from the Standardised World Income Inequality Database, SWIID, which covers the period 1960-2000.

**Figure 4.9: Inequality and Development: Empirical "Kuznets' Relationship"**



(a) Data B-M 1820-1992

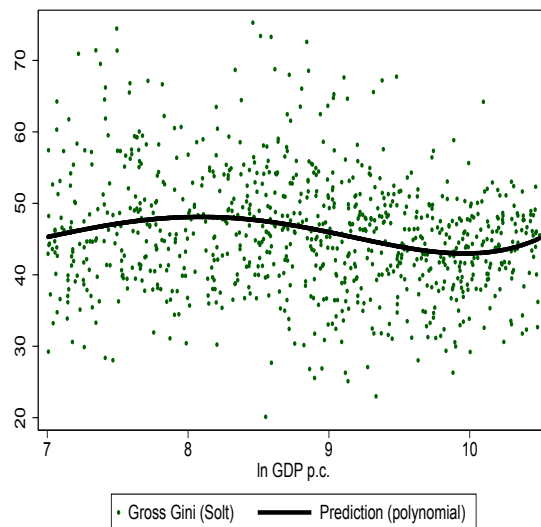


(b) Data V-Z 1850-2000

Note: Panel (a) reports the the relationship between income and inequality (measured by the Gini Index) using all available data from Bourguignon and Morrison (2002) for the period 1820-1992. Panel (b) replicates the same analysis using data from van Zanden et. al. (2014) using data for all countries over the period 1850-2000.

The analysis so far documented the highly non linear pattern of the relationship between

**Figure 4.10:** *Inequality and Development: "Kuznets' Relationship" Data 1960-2000*



Note: The figure depicts the the relationship between income and inequality (measured by the Gini Indxe) using all available data from the Standardised World Income Inequality Database, SWIID for all available countries over the period 1960.

income and inequality. According to the model, this pattern is caused by the interplay of inequality between skill groups related to the dynamics of the skill distribution during the transition and inequality within skill groups related to the return to ability among the population that acquires high education/skills.

The fact that the main drivers of inequality, and as consequence the relationship between income and inequality, change during the different phases of development has important implications for the interpretation of the mixed empirical findings and the changing perception of the relevant stylised facts. In particular, early work, most notably by (Pareto, 1896) and (Marshall, 1910) among others, had predicted an increase in inequality during the process of development. With retrospect, this observation is consistent with the increase in inequality during the early phases of development because of the structural process of skill acquisition that leads to an increase in inequality, mainly between skill groups. By the time (Kuznets, 1955) revisited the relationship, the share of skilled had already substantially increased in the most developed countries, and the contribution of rising inequality started fading out.



Finally, the renewed interest of economists in the last decades coincides with the period in which the most developed economies had reached a mature phase where rising inequality within skill groups is the main driver of overall inequality. This perspective implies that observers of the same empirical relationship at different points in time (or more precisely, during different phases of development) come to different (or even opposite conclusions) regarding the relationship between income and inequality.

To illustrate the empirical relevance of this argument, consider again Figure 4.1 from the introduction, which plots a smooth estimate of the relationship between income and inequality using all the data available today (from van Zanden et al., 2014) from 1850 until 1890, 1910, 1955 and 2010, respectively. Hence, the thought experiment of this figure is showing the empirical evidence available up to the point of writing of the respective contributions. The figure illustrates that, having had the data available to us today, both Pareto and Marshall have (correctly) inferred a positive association from the data. Similarly, by 1955, Kuznets have pointed out (correctly) a reversal in the relationship, implying a hump-shape of the overall relationship. Finally, there is another reversal in the income-inequality nexus when one incorporates the last decades of data.

Figure 4.1 also shows the relevance of sample composition for the interpretation of the mixed findings in the empirical literature. Substantial research effort has been devoted recently to the improvement of the measurement of inequality. Given the highly non-linear relationship between inequality and income documented above, it appears clear restricting attention to different samples (as is often done implicitly because of limited data availability) can lead to entirely different estimates. For instance, considering again Figure 4.1, non-linear estimates based on a sample that is restricted to countries (or periods) with levels of log income per capita between, e.g., 6 and 10, would likely deliver a hump-shaped relationship, whereas a U-shape relationship would emerge when restricting the sample to countries (or periods) with log income per capita between 7 and 11. The same argument applies, a fortiori, for the use of different databases that report information for different countries, periods and levels of income (see e.g. again Figures 4.9 and 4.10).

#### 4.3.4 Evolution of Upper Tail Inequality: Top Income Shares

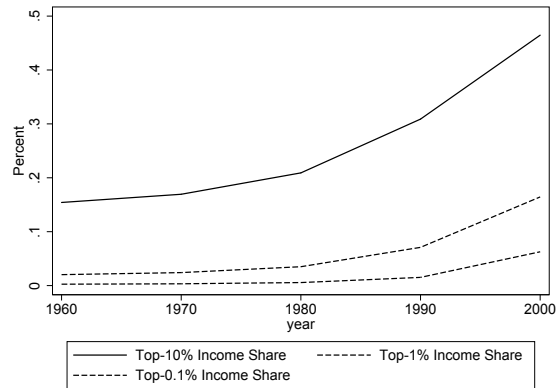
We now consider the implied predictions for tail inequality by looking at the income shares of the top 10, 1 and 0.1 per cent income earners. Sparked by the discussion in Piketty (2014) and Piketty and Saez (2014) and fuelled by the World Wealth and Income Database Project initiated by Alvaredo *et al.* (2016), a burgeoning empirical literature has documented an increase in upper tail inequality for a growing number of countries in recent years. This empirical pattern has stimulated theoretical research on the mechanisms behind the shape and the dynamics of the top tail of the income (and wealth) distribution. Effectively, this literature is based on mechanisms related to power laws in the productivity distribution.

While being stylised, a comparative advantage of our simple occupational choice framework presented in Section 4.2 is that it allows for an analytical characterisation as well as for a quantitative simulation of the evolution of the entire income distribution, rather than restricting attention exclusively to the upper tail of the distribution. In addition, the model allows for the joint consideration of the dynamics of endogenous skill acquisition, of the returns to ability, and of skill-biased and ability-biased technical change, which provides the unique possibility to study the long-term dynamics of inequality in different dimensions, including in the top income shares. In particular, recall that the model implies that the top income earners are skilled workers from the top tail of the ability distribution. Ability-biased technical change increases their income shares during the later phases of development, when the economy converges to the balanced growth path, implying an increase in top income shares.

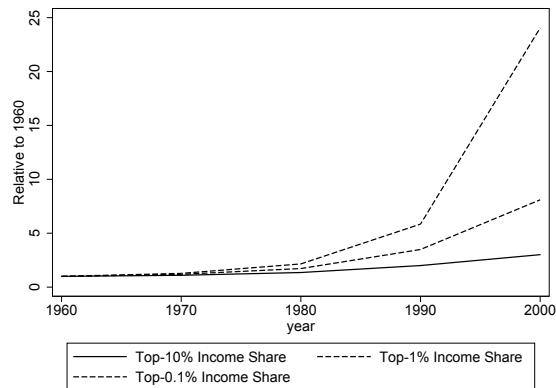
Figure 4.11 illustrates these predictions by plotting the evolution of the top 10, 1 and 0.1 percent top income shares from the same simulation of the model that was used to compute the evolution of the Gini Index in Figure 4.5. Panel (a) depicts the change in absolute top income shares while Panel (b) depicts the evolution of the respective top incomes shares normalised to their level in 1960. The figure documents the monotonic increase in top income shares, which is more pronounced higher up in the tail of the distribution.

Figure 4.12 plots the corresponding patterns using data for the USA (both in absolute

**Figure 4.11: Top Income Shares 1960-2000: Simulation**



(a) Simulation: Top Income Shares Simulation



(b) Simulation: Top Income Shares Normalised

Note: Simulated top income shares from baseline model. See text for details of the calibration and Section D.2 in the appendix.

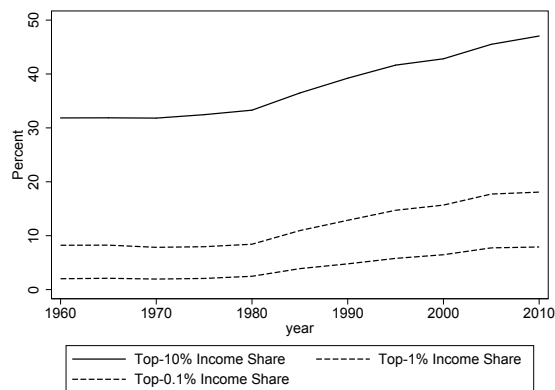
and in relative terms).

## 4.4 The Dynamics of Global Inequality

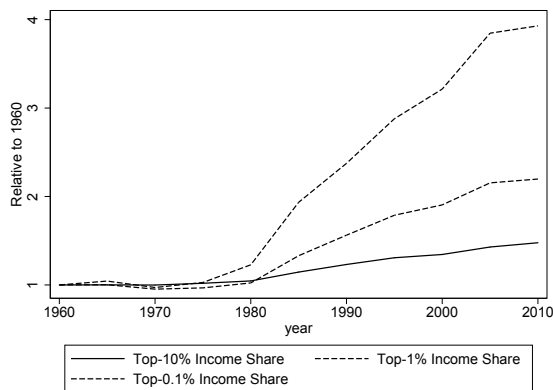
### 4.4.1 Dynamics of Inequality: A tale of two countries

The considerations so far have focused on the dynamics of inequality within a country during the process of long-run development. While this view has the advantage of clarifying the mechanisms behind long-run inequality dynamics and delivered patterns consistent

**Figure 4.12:** *Evolution of Top Income Shares 1960-2000: Data US*



(a) Top Income Shares



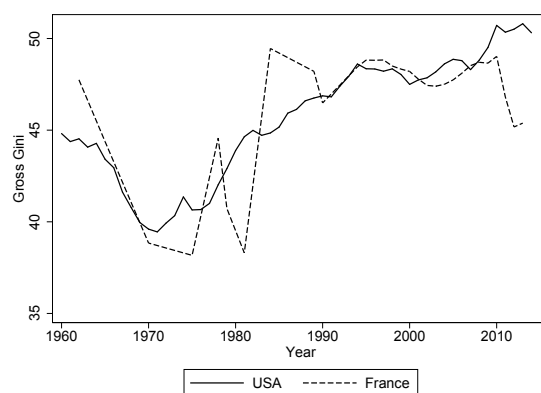
(b) Top Income Shares: Normalised

with those observed in the data, it has largely disregarded cross-sectional differences across countries. In fact, top income shares have increased in most developed countries over the last few decades. However, the extent of this increase differs, sometimes significantly, from one country to another, despite often similar dynamics in the rest of the distribution.

In this respect, the cases of the US and France stand out as prototypical examples of developed countries. Both countries display relatively similar dynamics in the overall income distribution (as reflected by, e.g., the Gini Index). This is illustrated in Figure 4.13, which plots the Gini index for the two countries over the past half century. The figure indicates that inequality in the income distribution at large declined in both countries until

about the early 1970s. This phase was followed by a subsequent increase in inequality to levels of the Gini higher than those in 1960. This increase was roughly similar in both countries.

**Figure 4.13:** *Income Inequality (Gini Index) in US and France: Data 1960-2010*

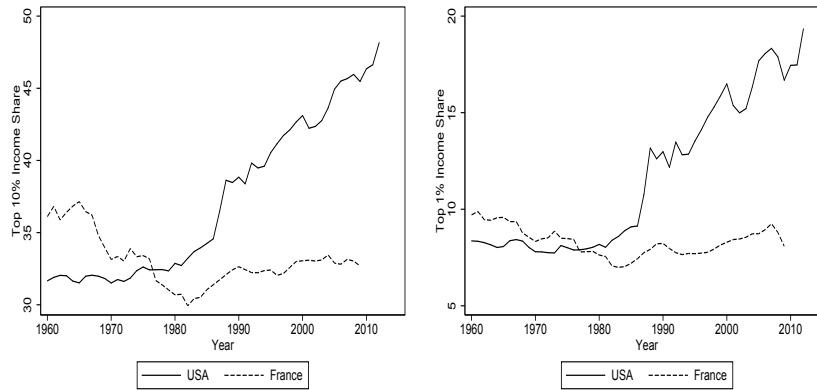


Note: Evolution of income inequality measured in terms of Gini Index in US and France, respectively. Data from the Standardised World Income Inequality Database, SWIID.

During the same period, both countries show markedly different dynamics in top income shares, as documented in Figure 4.14. In fact, the top income shares in the US and France have been roughly comparable until around 1980. After 1980, income shares have increased in both countries, but the increase in France has been substantially smaller than the one observed in the US.

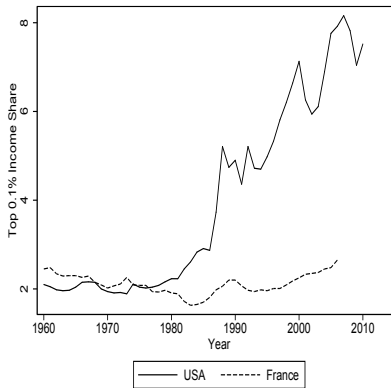
The question about the fundamental causes of the empirical patterns of the top income shares has sparked a lively and ongoing academic debate that is likely to be unsettled for at least a while. The model developed in Section 4.2 delivers a useful tool to explore the relevance of changes in the returns to ability for the observed dynamics of the income distribution in reduced form. In particular, the empirical patterns are compatible with the conjecture that compared to France, the US has been characterised by similar dynamics in the skill composition and in skill-biased technical change, which explains the similar overall dynamics in inequality. At the same time, the difference in the dynamics of top income shares suggests differences in the extent of ability-biased technical change. Recall that,

**Figure 4.14: Top Income Shares in US and France: Data 1960-2010**



(a) Top 10 %

(b) Top 1 %



(c) Top 0.1 %

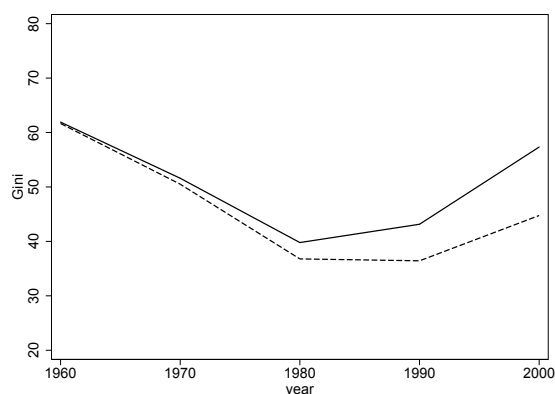
Note: Panel (a), (b) and (c) depict the overtime evolution of the top 10, 1 and 0.1 percent top income shares in US and France, respectively. Data from World Wealth and Income Database, WID.

from an analytical perspective, the returns to ability, and their dynamics, have a first order effect on overall inequality only in the mature phases of development, when the economy converges to the balanced growth path. While affecting inequality within the group of skilled individuals, and therefore also the Gini index, the effect of ability bias should be expected to be largest for the top incomes, which go to the most able individuals among the skilled.

To illustrate these predictions, Figure 4.15 plots the evolution of the Gini index for the baseline simulation together with the simulation of another model economy that is

identical in all dimensions but features a less pronounced ability biased technical change. The dynamics of inequality are roughly comparable, showing a decline in inequality up to about 1980 and then an increase, which is less pronounced in the baseline economy than in the economy with the less pronounced ability-biased technical change. Figure 4.16 shows the corresponding simulations for the top income shares in the two model economies. Consistent with the discussion, the top income shares increase in both economies, but the increase is more pronounced in the baseline economy with stronger ability-biased technical change, and greater higher up in the tail of the income distribution.

**Figure 4.15:** *Income Inequality (Gini Index) in US and France: Simulation 1960-2010*

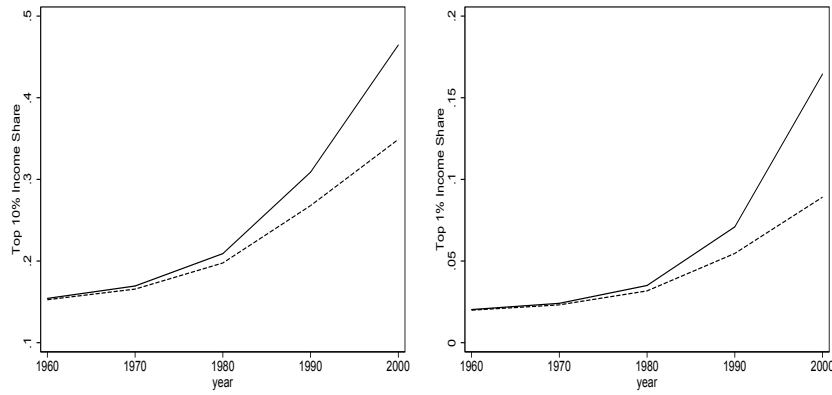


Note: Evolution of Gini Index in two simulated model economies that are identical in all dimensions but the returns to ability (see text for details).

#### 4.4.2 Dynamics of Global Inequality

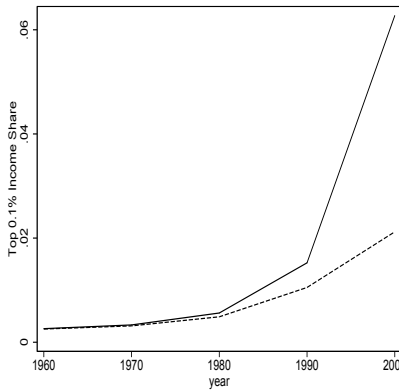
Finally, we turn to the implications for the dynamics of global income inequality. In this respect, it is important to recall that the model features a long period of stagnant development, which is followed by a rapid transition to sustained growth. This transition is driven by a rapid change in the skill composition of the population. These dynamics imply that if, for whatever reason, different countries experience the growth take-off at different points in time then the long-run dynamics of within-country inequality will affect the inequality between countries. Consequently, global inequality dynamics are closely linked

**Figure 4.16: Top Income Shares in US and France: Simulation 1960-2010**



(a) Top 10 %

(b) Top 1 %



(c) Top 0.1 %

Note: Panel (a), (b) and (c) depict the overtime evolution of the top 10, 1 and 0.1 percent top income shares in two simulated model economies that are identical in all dimensions but the returns to ability (see text for details).

to the within-country inequality patterns. In particular, global inequality is expected to be low during the early phases of (global) development, when all countries are pre-transitional and feature mostly unskilled populations. Global inequality progressively increases as some countries make the transition to sustained growth while others remain underdeveloped and pre-transitional. This prediction is based on the usual feature of unified growth frameworks that the dynamics of per-capita income experience an exponential acceleration during the take-off from stagnation to growth and eventually converge to a sustained growth path.

To generate a simulated counterfactual for the exploration of the empirical relevance



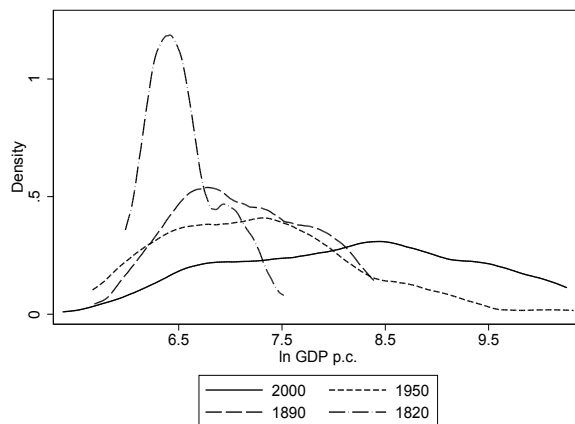
of this prediction, we follow Cervellati and Sunde (2015b) and simulate an artificial world composed of countries that differ only in baseline life expectancy,  $\underline{T}$ . This heterogeneity results from exogenous factors, for instance different geographic environments. As consequence of this assumption, the model features substantial delays in the growth take-off in countries with lower  $\underline{T}$ , but otherwise identical development paths, as documented in detail in Cervellati and Sunde (2015b). This provides an ideal benchmark to study the model implications for global inequality dynamics by isolating the role of the differential timing of the growth take-off across countries as the only source of variation.

In evaluating the empirical validity of the model predictions, we build on a recent stream of the empirical literature, which has documented the evolution of global inequality, see Bourguignon and Morrisson (2002), van Zanden et al. (2014), Milanovic (2014) and Solt (2016). Figure 4.17 plots the resulting patterns of global inequality. Panel (a) shows the pattern emerging from data over the period 1820-2000. Panel (b) plots the corresponding results obtained from simulations of model economy with heterogeneous  $\underline{T}$ . The comparison of the two figures illustrates that the observed increase in global inequality can be replicated by the unified growth framework and correspondingly interpreted as resulting from the (similar) non-linear dynamics of inequality and economic development. In particular, the increase in global inequality is the combination of within-country inequality dynamics that experience an increase during the late phases of development, and the fact that global inequality increases as consequence of countries that experience the transition to sustained growth earlier than others. Ultimately, the model predicts that all countries experience the transition, this just requires sufficient time, but that global inequality does not fully revert as consequence of the divergence of incomes in levels, which is never reversed under the assumption that all countries enter a balanced growth path without spillovers.

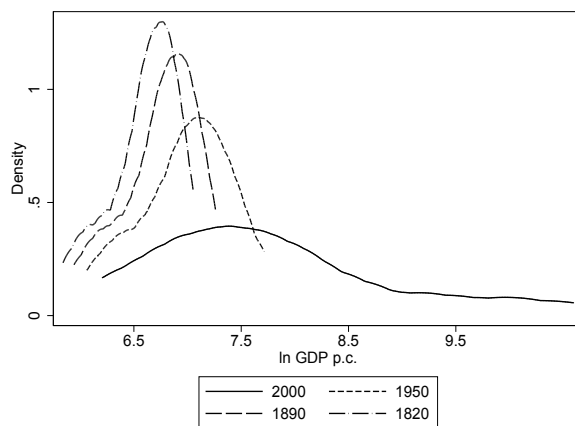
## 4.5 Mechanism and Further Research

To complete the picture, we end by exploring the empirical relevance of some specific predictions of the model. These refer to the dynamics of human capital inequality and

**Figure 4.17:** *Evolution of Global Inequality 1820-2000: Simulation and Data*



(a) Cross-Country Panel Data



(b) Simulated World

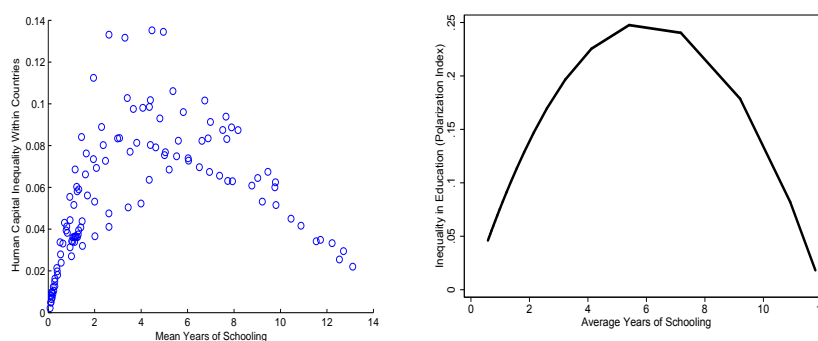
Note: Panel (a) depicts the kernel distributions of income inequality of a simulated world composed by countries that are identical in all parameters but the baseline level of (extrinsic) mortality (see Cervellati and Sunde, 2015 for details). Panel (b) depicts the kernel distribution of income inequality in the world using data from van Zanden et al. (2014). For both simulation and data the kernels are computed for the years 1820, 1890, 1950 and 2000.

global inequality in life expectancy.

**A Human Capital Kuznets Curve.** The main engine of growth in the model is the acquisition of skilled human capital. The model economy passes from an equilibrium characterised by few skilled individuals to a population with almost only skilled individuals. As shown in Section 4.3, this feature implies an intermediate peak in income inequality, which is in

fact driven by a peak in inequality between skill groups. Interpreting this skill acquisition in terms of education acquisition, the model thus implies a “Human Capital Kuznets Curve”. Figure 4.18 depicts the relationship between the average level of education across countries (measured by average years of schooling in each country) and human capital inequality within countries. Panel (a) plots the data pattern, replicating work by Morrison and Murtin (2013). Panel (b) shows the corresponding result of the simulation of the baseline model.<sup>6</sup>

**Figure 4.18:** *A Human Capital Kuznets’ Curve: Simulation and Data*



Note: Panel (a) depicts the relationship between average education and human capital inequality the baseline simulation. Panel (b) reports the relationship between average years of schooling and human capital inequality in the data by Morrison and Murtin (2013, Figure 6) which is based on a sample of 60 countries over the period 1820-2000.

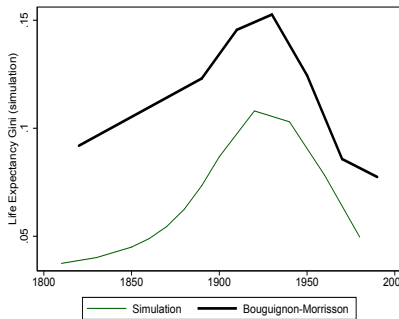
**Dynamics of Inequality in Life Expectancy.** Another specific feature of the theoretical model is the close, bi-directional relationship between the skill composition and life expectancy. In particular, the acquisition of skilled human capital is facilitated by higher life expectancy. At the same time, dynamically, the acquisition of skills in the population has a positive externality on life expectancy. As a consequence, the dynamics of life expectancy within a country are expected to exhibit a pronounced increase during the transition to sustained growth. Contrary to income, which keeps growing on the balanced growth path, the increase in life expectancy eventually experiences a slowdown, and life expectancy con-

---

<sup>6</sup>Here we follow the calibration of Cervellati and Sunde (2015) with the acquisition of skilled human capital being associated with 12 years of education. By construction, this corresponds to the average number of years of education in the model on the balanced growth path.

verges to its (exogenously determined) upper bound.<sup>7</sup> These dynamics have implications for the evolution of global inequality in life expectancy across countries that can be contrasted with the respective empirical patterns. In particular, based on this discussion the inequality in life expectancy across countries is expected to increase over time as consequence of some countries undergoing the take-off to growth earlier than others. Eventually, however, and contrary to the respective dynamics in income, global inequality in life expectancy is predicted to decrease as countries undergo the transition and life expectancy in these countries approaches its upper limit. Figure 4.19 validates this prediction by depicting the simulated data (of the artificial world used for the analysis of global inequality dynamics) and the actual data from Bourguignon and Morrison (2002).

**Figure 4.19:** *Global Inequality in Life Expectancy (Gini Index): Simulation and Data 1820-2000*



Note: The Figure depicts the overtime evolution of the global inequality in life expectancy across countries (measured by a Gini Index) in the simulated (artificial) world economy and in the data by Bourguignon and Morrison, 2002.

**Discussion and Extensions.** The model has specific features that can be used to derive further untested predictions. Also the theoretical framework is sufficiently simple and could be extended to address further questions that have been subject to empirical analysis but for which, at the moment, no systematic theoretical research is available. In particular, the model features endogenous increases in survival rates and ageing as consequence of overlapping age cohorts. This also implies a change in the age composition of the working population

<sup>7</sup>The assumption of the existence of an upper bound is not needed for the dynamics and the corresponding results regarding inequality in life expectancy, see also the corresponding discussion in Cervellati and Sunde (2015b).

and implications for the dynamics of inequality in tenure and experience. Finally, the model can account for life cycle dynamics in income, returns to experience, human capital depreciation, and the respective interactions with technological progress. Extending the model to the explicit consideration of these elements will allow deriving predictions about the evolution of lifetime inequality, as well as the evolution of time and cohort inequality. Such an extended model would be the basis for a first exploration of the role of ageing for the changing patterns of the income distribution and the dynamics of top income shares.

## 4.6 Conclusion

Understanding the evolution of economic inequality is a long-standing topic in economics, which has recently attracted a lot of attention in the view of the perception of growing inequity both within countries and across countries. A massive effort has been made over the last decade in terms of data collection and measurement of the dynamics of several dimensions of inequality. These range from evolution of measures of inequality in the entire population over time (e.g., using Gini indices), to the dynamics of inequality due to the development in the income shares of the richer segments of the population, to the evolution of the inequality across countries and the world population.

In this paper we contribute a first attempt to study the evolution of these different dimensions of inequality through the lens of a structural theoretical model of long-term development. We do so by exploring the analytical predictions derived from a unified growth framework over the very long-run. The model, which is suitable for quantitative analysis, allows for a full characterisation of the evolution of the economy from quasi-stagnant equilibria in which the vast majority of the population is unskilled, to a rapid transition in skill acquisition, and eventually to the convergence to a balanced growth path where the dominant source of inequality is inequality within skill groups, which is due to heterogeneous returns to ability.

The model delivers analytical predictions on the dynamics of the distribution of incomes, from an initially almost degenerate distribution in which the vast majority earns similar

(low) levels of income, to an income distribution that is approximately log normal around the mean but with thicker tails as the economy approaches the balanced growth path. A simulated version of the model is used to assess the empirical validity of these predictions. Theoretical predictions are shown to align with the stylised empirical patterns of the evolution of Gini indexes, the changing income-inequality relationship, and the changes in top income shares over time. The model also delivers predictions about the dynamics of the distribution of income at the global level that are consistent with empirical patterns. Finally, the exploration of specific model predictions about the dynamics of inequality in human capital and life expectancy also delivers empirically consistent patterns.

In summary, this paper provides a new comprehensive perspective on the long-run dynamics of inequality and suggests valuable directions for future research. First, the model is amenable to the consideration of more detailed life cycle elements that could allow to study the link between differences of development and the evolution of life time inequality across different age cohorts for the first time. Second, following the tradition of the unified growth literature, the model considers a reduced form evolution of skill and ability bias technical change. An effort to micro-found the changing returns to ability would allow to nest the insights obtained from structural models with the insights obtained from microeconomic theory more closely. In our framework we have focused attention to a deterministic framework since our goal was to isolate the empirical bite of long term structural changes rather than offer a quantitative fix of the empirical patterns. A third, natural, extension is therefore to study the changing interactions between skills and ability in the presence of random shocks. This extension would allow to nest the recent insights from probabilistic growth models into structural (non balanced growth path) theoretical frameworks.

# References

- ACEMOGLU, D. (2002). Directed technical change. *Review of Economic Studies*, **69** (4), 781–809.
- and JOHNSON, S. (2007). Disease and development: The effect of life expectancy on economic growth. *Journal of Political Economy*, **115** (6), 925–985.
- , — and ROBINSON, J. (2001). The colonial origins of comparative development: An empirical investigation. *American Economic Review*, **91** (5), 1369–1401.
- , —, — and YARED, P. (2008). Income and democracy. *American Economic Review*, **98** (3), 808–842.
- , NAIDU, S., RESTREPO, P. and ROBINSON, J. (2017). Democracy does cause growth. *Journal of Political Economy*, forthcoming.
- and ROBINSON, J. (2000). Why did the west extend the franchise? democracy, inequality and growth in historical perspective. *Quarterly Journal of Economics*, **115** (4), 1167–99.
- and — (2001). A theory of political transitions. *American Economic Review*, **91** (4), 938–63.
- and — (2006). *Economic Origins of Dictatorship and Democracy*. Cambridge: Cambridge University Press.
- and — (2015). The rise and decline of general laws of capitalism. *Journal of Economic Perspectives*, **29** (1), 3–28.
- AGHION, P., CAROLI, E. and GARCIA-PENALOSA, C. (1999). Inequality and economic growth: The perspective of the new growth theories. *Journal of Economic Literature*, **37** (4), 1615–1660.
- ALVAREDO, F., ATKINSON, A. B., PIKETTY, T., SAEZ, E. and ZUCMAN, G. (2016). The world wealth and income database. Website: <http://www.wid.world>.
- ASHRAF, Q. and GALOR, O. (2011). Dynamics and stagnation in the malthusian epoch. *American Economic Review*, **101** (5), 2003–2041.
- BARRO, R. and LEE, J.-W. (2013). A new data set of educational attainment in the world, 1950–2010. *Journal of Development Economics*, **104**, 184–198.
- BECKER, S. O., CINNIRELLA, F. and WOESSMANN, L. (2010). The trade-off between fertility and education: Evidence from before the demographic transition. *Journal of Economic Growth*, **15** (3), 177–204.

- BENHABIB, J., BISIN, A. and ZHU, S. (2011). The distribution of wealth and fiscal policy in economies with finitely lived agents. *Econometrica*, **79** (1), 123–157.
- , CORVALAN, A. and SPIEGEL, M. M. (2013). Income and democracy: Evidence from nonlinear estimations. *Economics Letters*, **118** (3), 489–492.
- and SPIEGEL, M. (1994). The role of human capital in economic development: Evidence from aggregate cross-country data. *Journal of Monetary Economics*, **34** (1), 143–173.
- BERTOCCHI, G. (2006). The law of primogeniture and the transition from landed aristocracy to industrial democracy. *Journal of Economic Growth*, **11** (1), 43–70.
- and SPAGAT, M. (2001). The politics of cooptation. *Journal of Comparative Economics*, **29**, 591–607.
- BLUME, L. and DURLAUF, S. (2015). Capital in the twenty-first century: A review essay. *Journal of Political Economy*, **123** (4), 749–777.
- BOIX, C., MILLER, M. K. and ROSATO, S. (2013). A complete data set of political regimes, 1800-2007. *Comparative Political Studies*, **46** (12), 1523–1554.
- BOLT, J. and VAN ZANDEN, J. (2014). The maddison project: collaborative research on historical national accounts. *Economic History Review*, **67** (3), 627–651.
- BOUCEKKINE, R., DE LA CROIX, D. and LICANDRO, O. (2003). Early mortality declines at the dawn of modern growth. *Scandinavian Journal of Economics*, **105** (3), 401–418.
- BOURGUIGNON, F. and MORRISSON, C. (2002). Inequality among world citizens: 1820-1992. *American Economic Review*, **92** (4), 727–744.
- and VERDIER, T. (2000). Oligarchy, democracy, inequality and growth. *Journal of Development Economics*, **62** (2), 285–313.
- BRUNO, G. S. F. (2005a). Approximating the bias of the lsdv estimator for dynamic unbalanced panel data models. *Economics Letters*, **87**, 361–366.
- (2005b). Estimation and inference in dynamic unbalanced panel-data models with a small number of individuals. *Stata Journal*, **5** (4), 473–500.
- CERVELLATI, M., FORTUNATO, P. and SUNDE, U. (2008). Hobbes to rousseau: Inequality, institutions and development. *Economic Journal*, **118**, 1354—1384.
- , — and — (2012). Consensual and conflictual democratization. *B.E. Journal of Theoretical Economics*, **12** (1-33).
- , — and — (2014a). Violence during democratization and the quality of democratic institutions. *European Economic Review*, **66**, 226–247.
- , JUNG, F., SUNDE, U. and VISCHER, T. (2014b). Income, democracy and critical junctures: Comment. *American Economic Review*, **104** (2), 707–719.



- and SUNDE, U. (2005). Human capital formation, life expectancy, and the process of development. *American Economic Review*, **95** (5), 1653–1672.
- and — (2011a). Democratization, violent social conflicts and growth. *IZA Discussion Paper*, **5643**.
- and — (2011b). Disease and development: The role of life expectancy reconsidered. *Economics Letters*, **113**, 269–272.
- and — (2011c). Life expectancy and economic growth: The role of the demographic transition. *Journal of Economic Growth*, **16**, 99–133.
- and — (2013). Life expectancy, schooling and lifetime labor supply: Theory and evidence revisited. *Econometrica*, **81** (5), 2055–2086.
- and — (2014). Civil conflict, democratization, and growth: Violent democratization as critical juncture. *Scandinavian Journal of Economics*, **116** (2), 482–505.
- and — (2015a). The economic and demographic transition, mortality, and comparative development. *American Economic Journal: Macroeconomics*, **7** (3), 1–39.
- and — (2015b). The economic and demographic transition, mortality and comparative development. *American Economic Journal: Macroeconomics*, **7** (3), 189–225.
- and — (2015c). The effect of life expectancy on education and population dynamics. *Empirical Economics*, **48** (4), 1445–1478.
- , — and ZIMMERMANN, K. F. (2017). Demographic dynamics and long-run development: insights for the secular stagnation debate. *Journal of Population Economics*, **30**, 401–432.
- COHEN, D. and LEKER, L. (2014). Health and education: Another look with the proper data. *mimeo, Paris School of Economics*.
- and SOTO, M. (2007). Growth and human capital: Good data, good results. *Journal of Economic Growth*, **12** (1), 51–76.
- CONLEY, J. P. and TEMIMI, A. (2001). Endogenous enfranchisement when groups' preferences conflict. *Journal of Political Economy*, **109** (1), 79–102.
- DALGAARD, C.-J. and STRULIK, H. (2013). The history-augmented solow model. *European Economic Review*, **63** (1), 134–149.
- and — (2014). Optimal aging and death: Understanding the preston curve. *Journal of the European Economic Association*, **12**, 672–701.
- and — (2015). The physiological foundations of the wealth of nations. *Journal of Economic Growth*, **20** (1), 37–73.
- and — (2016). Physiology and development: Why the west is taller than the rest. *Economic Journal*, **126** (598), 2292–2323.

- DE LA CROIX, D. and DOEPKE, M. (2004). Public versus private education when differential fertility matters. *Journal of Development Economics*, **73** (2), 607–629.
- DE LA CROIX, D. and LICANDRO, O. (2013). The child is father of the man: Implications for the demographic transition. *Economic Journal*, **123** (567), 236–261.
- DE LA FUENTE, A. and DOMENECH, R. (2006). Human capital in growth regressions: How much difference does data quality make? *Journal of the European Economic Association*, **4** (1), 1–36.
- DOEPKE, M. (2004). Accounting for fertility decline during the transition to growth. *Journal of Economic Growth*, **9** (3), 347–383.
- DYSON, T. (2013). On demographic and democratic transitions. *Population and Development Review*, **38** (S1), 83–102.
- FEENSTRA, R. C., INKLAAR, R. and TIMMER, M. P. (2015). The next generation of the penn world table. *American economic review*, **105** (10), 3150–82.
- GABAIX, X., LASRY, J.-M., LIONS, P.-L. and MOLL, B. (2016). The dynamics of inequality. *Econometrica*, **84** (6), 2071–2111.
- GALOR, O. (2011). *Unified Growth Theory*. Princeton, NJ: Princeton University Press.
- (2012). The demographic transition: causes and consequences. *Cliometrica*, **6** (1), 1–28.
- and MOAV, O. (2000). Ability-biased technological transition and economic growth. *Quarterly Journal of Economics*, **CXV**, 469–497.
- and — (2002a). Natural selection and the origin of economic growth. *Quarterly Journal of Economics*, **117** (4), 1133–92.
- and — (2002b). Natural selection and the origin of economic growth. *Quarterly Journal of Economics*, **117** (4), 1133–92.
- and — (2006). Das human kapital: A theory of the demise of the class structure. *Review of Economic Studies*, **73** (1), 85–117.
- , — and VOLLRATH, D. (2009). Inequality in land ownership, the emergence of human capital promoting institutions, and the great divergence. *Review of Economic Studies*, **76** (1), 143–179.
- and WEIL, D. N. (2000). Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond. *American Economic Review*, **90** (4), 807–828.
- GENNAIOLI, N., LA PORTA, R., LOPEZ-DE SILANES, F. and SHLEIFER, A. (2013). Human capital and regional development. *Quarterly Journal of Economics*, **105**, 164.
- GORDON, R. J. (2012). Is u.s. economic growth over? faltering innovations confront the six headwinds. *NBER Working Paper*, **18315**.

- (2014). The demise of u.s. economic growth: Restatement, rebuttal, and reflections. *NBER Working Paper*, **18315**.
- (2016). *The Rise and Fall of American Growth*. Princeton, N.J.: Princeton University Press.
- GRADSTEIN, M. (2007). Inequality, democracy and the protection of property rights. *Economic Journal*, **117** (516), 252–269.
- HANSEN, G. D. and PRESCOTT, E. C. (2002). Malthus to solow. *American Economic Review*, **92** (4), 1205–1217.
- HIRSHLEIFER, J. (2001). *The Dark Side of the Force: Economic Foundations of Conflict Theory*. Cambridge: Cambridge University Press.
- HUNTINGTON, S. P. (1993). *The Third Wave: Democratization in the Late Twentieth Century*. Oklahoma: University of Oklahoma Press.
- JHA, S. K. (1996). The kuznets curve: A reassessment. *World Development*, **24** (4), 773–780.
- JONES, C. I. (2015). Pareto and piketty: The macroeconomics of top income and wealth inequality. *Journal of Economic Perspectives*, **29** (1), 29–46.
- and KIM, J. (2018). A schumpeterian model of top income inequality. *Journal of Political Economy*, forthcoming.
- and ROMER, P. (2010). The new kaldor facts: Ideas, institutions, population and human capital. *American Economic Journal: Macroeconomics*, **2** (1), 224–245.
- KÖGEL, T. and PRZKAWETZ, A. (2001). Agricultural productivity growth and the escape from malthusian trap. *Journal of Economic Growth*, **6**, 337–357.
- KRUEGER, A. B. and LINDAHL (2001). Education for growth: Why and for whom? *Journal of Economic Literature*, **39** (4), 1101–1136.
- KRUSELL, P. and SMITH, A. A. (2015). Is piketty’s “second law of capitalism” fundamental? *Journal of Political Economy*, **123** (4), 725–748.
- KUZNETS, S. (1955). Economic growth and income inequality. *American Economic Review*, **45** (1), 1–28.
- LAGERLÖF, N.-P. (2006). The galor–weil model revisited: A quantitative exercise. *Review of Economic Dynamics*, **9** (1), 116–142.
- LINDERT, P. H. and WILLIAMSON, J. G. (2003). Does globalization make the world more unequal? In M. D. Bordo, A. M. Taylor and J. G. Williamson (eds.), *Globalization in Historical Perspective*, Chicago: University of Chicago Press.
- LIPSET, S. M. (1959). Some social requisites of democracy: economic development and political legitimacy. *American Political Science Review*, **53** (3), 69–105.

- LIZZERI, A. and PERSICO, N. (2004). Why did the elites extend the suffrage? democracy and the scope of government, with an application to Britain's 'age of reform'. *Quarterly Journal of Economics*, **119** (2), 707–765.
- LLAVADOR, H. and OXOBY, R. (2005). Partisan competition, growth and the franchise. *Quarterly Journal of Economics*, **120** (3), 1155–1189.
- MADDISON, A. (2006). *The World Economy, Volume 1: A Millennial Perspective*. OECD Publishing, Paris.
- MANKIW, N., ROMER, D. and WEIL, D. (1992). A contribution to the empirics of economic growth. *Quarterly Journal of Economics*, **107**, 407–437.
- MARSHALL, A. (1910). *Principles of Economics*. London: MacMillan and Company.
- MILANOVIC, B. (2014). All the ginis, 1950–2012 (updated in autumn 2014). *mimeo*, World Bank.
- MORAL-BENITO, E. and BARTOLUCCI, C. (2012). Income and democracy: Revisiting the evidence. *Economics Letters*, **117** (3), 844–847.
- MORISSON, C. and MURTIN, F. (2014). The democratic transition. *Journal of Economic Growth*, **19** (2), 141–181.
- MORISSON, C. and MURTIN, F. (2013). The kuznets curve of human capital inequality: 1870–2010. *The Journal of Economic Inequality*, **11** (3), 283–301.
- MURTIN, F. and WACZIARG, R. (2014). The democratic transition. *Journal of Economic Growth*, **19** (2), 141–181.
- NELSON, R. and PHELPS, E. (1966). Investment in humans, technological diffusion and economic growth. *American Economic Review (Papers and Proceedings)*, **56** (2), 69–75.
- NICKELL, S. (1981). Biases in dynamic models with fixed effects. *Econometrica*, **49** (6), 1417–1426.
- OECD (2015). *Escaping the Stagnation Trap: Policy Options for the Euro Area and Japan*. Paris: Organization for Economic Cooperation and Development.
- OKUN, A. (1975). *Equality and Efficiency: The Big Tradeoff*. Washington: Brookings Institution Press.
- PAPAIOANNOU, E. and SIOUROUTIS, G. (2008). Democratization and growth. *Economic Journal*, **118**, 1520–1551.
- PARETO, V. (1896). *Cours d'Économie Politique*. Paris: Librairie Droz.
- PIKETTY, T. (2014). *Capital in the 21st Century*. Cambridge, Mass.: Harvard University Press.
- and SAEZ, E. (2014). Inequality in the long-run. *Science*, **344** (6168), 838–843.
- and ZUCMAN, G. (2015). Wealth and inheritance in the long run. In A. B. Atkinson and F. Bourguignon (eds.), *Handbook of Income Distribution*, vol. 2B, 15, Amsterdam: Elsevier.

- PORTELA, M., ALESSIE, R. and TEULINGS, C. (2010). Measurement error in education and growth regressions. *Scandinavian Journal of Economics*, **112** (3), 618–639.
- RAY, D. (2015). Nit-piketty: A comment on thomas piketty’s capital in the twenty first century. *CESifo Forum*, **1/2015**, 19–25.
- REHER, D. S. (2004). The demographic transition revisited as a global process. *Population, Space and Place*, **10** (1), 19–41.
- SOLT, F. (2016). The standardized world income inequality database. *Social Science Quarterly*, **97** (5), 1267–1281.
- STRITTMATTER, A. and SUNDE, U. (2011). Health and economic development: Evidence from the introduction of public health care. *Journal of Population Economics*, **26** (4), 1549–1584.
- STRULIK, H. and WEISDORF, J. (2008). Population, food, and knowledge: A simple unified growth theory. *Journal of Economic Growth*, **13** (3), 195–216.
- SUNDE, U. and VISCHER, T. (2015). Human capital and growth: Specification matters. *Economica*, **82** (326), 368–390.
- TAM, H. (2008). An economic or political kuznets curve? *Public Choice*, **134** (3-4), 367–389.
- TEULINGS, C. and BALDWIN, R. (eds.) (2014). *Secular Stagnation: Facts, Causes, and Cures*. London: CEPR Press.
- VAN ZANDEN, J. L., BATEN, J., FOLDVARI, P. and VAN LEEUWEN, B. (2014). The changing shape of global inequality 1820–2000 – exploring a new dataset. *Review of Income and Wealth*, **60** (2), 279–297.
- VOIGTLÄNDER, N. and VOTH, H.-J. (2013a). How the west “invented” fertility restriction. *American Economic Review*, **103** (6), 2227–2264.
- and — (2013b). The three horsemen of riches: Plague, war, and urbanization in early modern europe. *Review of Economic Studies*, **80** (3), 774–811.
- WILSON, B. and DYSON, T. (2017). Democracy and the demographic transitions. *Democratization*, **24** (4), 594–612.

# Appendix A

## Appendix to Chapter 1

### A.1 Simulation Details

Using (1.16) one can derive the growth rate of savings as

$$(1 + g_{t+1}^s) = \frac{(1 + g_{t+1}^{hc})(1 + g_{t+1}^A)^\varepsilon}{(1 + g_{t+1}^L)^{1-\alpha}},$$

with  $g_{t+1}^{hc}$  being the growth rate of human capital. Rearranging this expression gives the solution for  $\varepsilon$

$$\varepsilon = \frac{\ln(1 + g_{t+1}^s) + (1 - \alpha) \ln(1 + g_{t+1}^L) - \ln(1 + g_{t+1}^{hc})}{\ln(1 + g_{t+1}^A)}.$$

Assuming that  $(1 + g_{t+1}^A) \approx (1 + g_{t+1}^M) \approx (1 + g_{t+1}^y)$  this gives a value of  $\varepsilon = 0.51$ .

In order to derive  $\phi$  define the share of agriculture in production as

$$\sigma_t^A = \frac{Y_t^A}{Y_t}.$$

This allows to rewrite the expression for GDP per capita growth, such that

$$\begin{aligned}
(1 + g_{t+1}^y) &= \frac{y_{t+1}^A + y_{t+1}^M}{y_t^A + y_t^M} \\
&= \frac{(1 + g_{t+1}^{y^A})y_t^A + (1 + g_{t+1}^{y^M})y_t^M}{y_t} \\
&= \sigma_t^A(1 + g_{t+1}^{y^A}) + (1 - \sigma_t^A)(1 + g_{t+1}^{y^M}) \\
(1 + g_{t+1}^y) &= \sigma_t^A \frac{(1 + g_{t+1}^A)^\varepsilon}{(1 + g_{t+1}^L)^{1-\alpha}} + (1 - \sigma_t^A)(1 + g_{t+1}^M)^\phi (1 + g_{t+1}^{hc})^\psi (1 + g_{t+1}^s)^{1-\psi}.
\end{aligned}$$

Substituting the growth rate of savings gives

$$(1 + g_{t+1}^y) = \sigma_t^A \frac{(1 + g_{t+1}^s)}{(1 + g_{t+1}^{hc})} + (1 - \sigma_t^A)(1 + g_{t+1}^M)^\phi (1 + g_{t+1}^{hc})^\psi (1 + g_{t+1}^s)^{1-\psi},$$

which can be rearranged to obtain a solution for  $\phi$ , namely

$$\phi = \frac{\ln \left[ (1 + g_{t+1}^y) - \sigma_t^A \frac{(1 + g_{t+1}^s)}{(1 + g_{t+1}^{hc})} \right] - \ln(1 - \sigma_t^A) - \psi \ln(1 + g_{t+1}^{hc}) - (1 - \psi) \ln(1 + g_{t+1}^s)}{\ln(1 + g_{t+1}^M)}.$$

The value for  $\sigma_t^A$  can be obtained from World Bank Data.<sup>1</sup> Again assuming that  $(1 + g_{t+1}^A) \approx (1 + g_{t+1}^M) \approx (1 + g_{t+1}^y)$  this gives a value of  $\phi = 0.36$ . The parameters  $\delta$  and  $\gamma$  can be determined by using the balanced growth value of  $n_t$ . More specifically,

$$\begin{aligned}
n_t \equiv (1 + g_{t+1}^L) &= \frac{\gamma}{\delta M_t^\phi} \frac{(w_t^L)^\psi}{\psi^\psi (1 - \psi)^{1-\psi}} \\
\frac{\gamma}{\delta} &= \psi^\psi (1 - \psi)^{1-\psi} \cdot M_t^\phi \cdot \frac{(1 + g_{t+1}^L)}{\sigma_t^A y_t} \\
\frac{\gamma}{\delta} &= \psi^\psi (1 - \psi)^{1-\psi} \cdot \frac{(1 - \sigma^A) y_t}{h_t^\psi k_t^{1-\psi}} \cdot \frac{(1 + g_{t+1}^L)}{\sigma_t^A y_t}.
\end{aligned}$$

This gives a ratio of  $\frac{\gamma}{\delta} = 2.36$ . We choose  $\gamma = 3$  and  $\delta = 1.27$ , which is similar to the values used by Strulik and Weisdorf (2008).

---

<sup>1</sup>The data for the United Kingdom can be retrieved at:  
<https://data.worldbank.org/indicator/NV.AGR.TOTL.ZS?locations=GB>.

## A.2 Additional Results

**Table A.1:** Sensitivity of Reduced-Form Estimates: Data vs. Simulation

Dependent Variable	Log GDP per Capita			
	Forerunners	Latecomers	after 1950	before 2010
Sample	(1)	(2)	(3)	(4)
Simulation				
Log $M_t$	0.289*** [0.003]	0.286*** [0.000]	0.283*** [0.001]	0.283*** [0.001]
$k_t$	0.526*** [0.005]	0.541*** [0.000]	0.541*** [0.000]	0.541*** [0.000]
$HC_t$	0.054*** [0.004]	0.059*** [0.000]	0.062*** [0.002]	0.062*** [0.001]
Observations	189	609	684	684
Countries	27	87	114	114
Adjusted $R^2$	1.000	1.000	1.000	1.000
Data				
Log $Y_{t-1}$	0.226** [0.086]	0.240*** [0.084]	0.226*** [0.066]	0.155** [0.068]
$k_t$	0.307** [0.113]	0.519*** [0.051]	0.499*** [0.052]	0.543*** [0.059]
$HC_t$	0.554 [0.345]	0.264 [0.191]	0.230 [0.184]	0.392* [0.220]
Observations	163	425	588	474
Countries	29	85	114	114
Adjusted $R^2$	0.923	0.758	0.805	0.755
Data				
$y_{t-1}$	0.414*** [0.098]	0.357*** [0.075]	0.344*** [0.065]	0.302*** [0.072]
$k_t$	0.172* [0.096]	0.430*** [0.052]	0.383*** [0.059]	0.426*** [0.065]
$HC_t$	0.702** [0.267]	0.232 [0.168]	0.402** [0.154]	0.440** [0.188]
Observations	163	425	588	474
Countries	29	85	114	114
Adjusted $R^2$	0.936	0.779	0.823	0.774
Country Fixed Effects	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes

Results of fixed effects regressions. Panel A: Simulated data. Panel B: PWT data, the dependent variable  $y_t$  is (the log of) real GDP at constant 2011 national prices (in mil. 2011US\$) divided by population size (in millions),  $k_t$  is (the log of) capital stock at constant 2011 national prices (in mil. 2011US\$),  $HC_t$  is (the log of) the human capital index, see data description of the PWT 9.0 for details. “Forerunners” are countries with an onset of the demographic transition before 1960, “Latecomers” are countries with an onset after 1960. Sample period: 10-year panel {1950, 1960, ..., 2010}. All specifications include a full set of country fixed effects and common time dummies. Standard errors are clustered at the country level. \*, \*\*, \*\*\* denote significance at 10%, 5%, and 1% levels.



# Appendix B

## Appendix to Chapter 2

### B.1 Analytical Results

#### B.1.1 Individual Optimisation Problem

Substituting (2.2) and (3.4) into (3.1) gives rise to the following maximisation problem for the rich:

$$\max_{n_t^r, e_{t+1}^r} (1 - \gamma) \ln \{z_t^r [1 - n_t^r (r + e_{t+1}^r)]\} + \gamma \ln [n_t^r h(e_{t+1}^r, g_{t+1})]$$

**Subject to:**

$$z_t^r [1 - n_t^r (r + e_{t+1}^r)] \geq \tilde{c}(n_t^r, e_{t+1}^r) \geq 0$$

Vice versa substituting (2.2) and (3.4) into (3.1) gives the following problem for the poor:

$$\max_{n_t^p, e_{t+1}^p} (1 - \gamma) \ln \{z_t^p [1 - n_t^p (r + e_{t+1}^p)]\} + \gamma \ln [n_t^p h(e_{t+1}^p, g_{t+1})]$$

**Subject to:**

$$z_t^p [1 - n_t^p (r + e_{t+1}^p)] \geq \tilde{c}(n_t^p, e_{t+1}^p) \geq 0$$

### B.1.2 Determination of Optimal Tax Rates in the Simple Case

Substituting the optimal consumption, fertility and education decisions back gives the following indirect utility function for the rich landowners:

$$V_t^r = \begin{cases} (1 - \gamma) \ln [(1 - \gamma)z_t^r] + \gamma \ln \left[ \frac{\gamma}{r+e(g_{t+1})} h(e_{t+1}, g_{t+1}) \right] & \text{if } z_t^r \geq \tilde{z}, \\ (1 - \gamma) \ln [\tilde{c}] + \gamma \ln \left[ \frac{1 - \frac{\tilde{c}}{z_t^r}}{r+e(g_{t+1})} h(e_{t+1}, g_{t+1}) \right] & \text{if } z_t^r \leq \tilde{z}. \end{cases}$$

Maximisation of these expressions with regard to the tax rate gives

$$\frac{\partial V_t^r}{\partial \tau_t} = \begin{cases} \frac{(1-\gamma) \frac{p_t^N}{L_t} [1-C'(\tau_t) - \frac{1}{\delta_t}]}{z_t^r} & \text{if } z_t^r \geq \tilde{z}, \\ \frac{\gamma \frac{p_t^N}{L_t} [1-C'(\tau_t) - \frac{1}{\delta_t}]}{z_t^r} & \text{if } z_t^r \leq \tilde{z}, \end{cases}$$

with the second order conditions

$$\frac{\partial^2 V_t^r}{\partial \tau_t^2} = \begin{cases} \frac{-(1-\gamma) \left[ \frac{p_t^N}{L_t} C''(\tau_t) z_t^r + \left[ \frac{p_t^N}{L_t} \right]^2 [1-C'(\tau_t) - \frac{1}{\delta_t}]^2 \right]}{[z_t^r]^2} < 0 & \text{if } z_t^r \geq \tilde{z}, \\ \frac{-\gamma \left[ \frac{p_t^N}{L_t} C''(\tau_t) z_t^r + \left[ \frac{p_t^N}{L_t} \right]^2 [1-C'(\tau_t) - \frac{1}{\delta_t}]^2 \right]}{[z_t^r]^2} < 0 & \text{if } z_t^r \leq \tilde{z}, \end{cases}$$

thus identifying a maximum. Repeating the same analysis for the landless gives the indirect utilities

$$V_t^p = \begin{cases} (1 - \gamma) \ln [(1 - \gamma)z_t^p] + \gamma \ln \left[ \frac{\gamma}{r+e(g_{t+1})} h(e_{t+1}, g_{t+1}) \right] & \text{if } z_t^p \geq \tilde{z}, \\ (1 - \gamma) \ln [\tilde{c}] + \gamma \ln \left[ \frac{1 - \frac{\tilde{c}}{z_t^p}}{r+e(g_{t+1})} h(e_{t+1}, g_{t+1}) \right] & \text{if } z_t^p \leq \tilde{z}, \end{cases}$$

with the first order conditions

$$\frac{\partial V_t^p}{\partial \tau_t} = \begin{cases} \frac{(1-\gamma) \frac{p_t^N}{L_t} [1-C'(\tau_t)]}{z_t^p} & \text{if } z_t^p \geq \tilde{z}, \\ \frac{\gamma \frac{p_t^N}{L_t} [1-C'(\tau_t)]}{z_t^p} & \text{if } z_t^p \leq \tilde{z}, \end{cases}$$

and second order conditions

$$\frac{\partial^2 V_t^p}{\partial \tau_t^2} = \begin{cases} \frac{-(1-\gamma) \left[ \frac{p_t^N}{L_t} C''(\tau_t) z_t^p + \left[ \frac{p_t^N}{L_t} \right]^2 [1-C'(\tau_t)]^2 \right]}{[z_t^p]^2} < 0 & \text{if } z_t^p \geq \tilde{z}, \\ \frac{-\gamma \left[ \frac{p_t^N}{L_t} C''(\tau_t) z_t^p + \left[ \frac{p_t^N}{L_t} \right]^2 [1-C'(\tau_t)]^2 \right]}{[z_t^p]^2} < 0 & \text{if } z_t^p \leq \tilde{z}, \end{cases}$$

again identifying a maximum.

### B.1.3 Dropping the Primogeniture Assumption

Assuming there is no primogeniture will give a bequest rule that divides the land stock among all members of the rich. This will not change the budget constraint of a rich individual, which is still given by (2.2):

$$c_t^r + n_t^r z_t^r [r + e_{t+1}] \leq z_t^r.$$

However, it will change the formula for  $\delta_t$ , which is now given by

$$\delta_t = \delta_0 \frac{L_0}{L_t} \prod_{i=0}^t n_i^r.$$

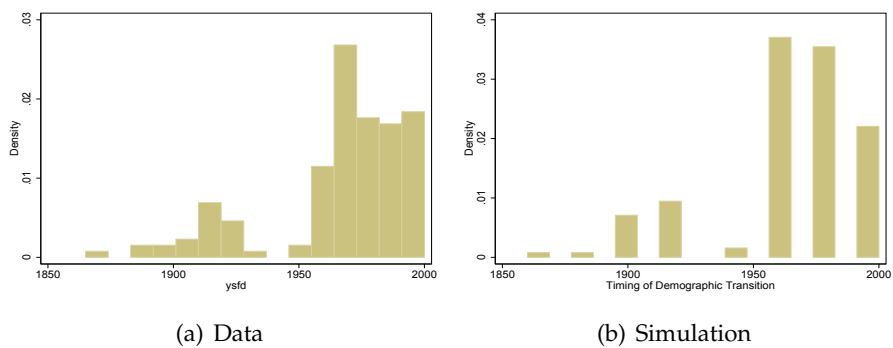
**Lemma 3.** *As long as  $\delta_t < \frac{1}{2} \forall t$  the results regarding the existence of a date at which there is a transition to democracy, as well as the differential effects of  $\theta$  remain intact.*

*Proof.* The necessary condition for democracy, given by (2.22) is now

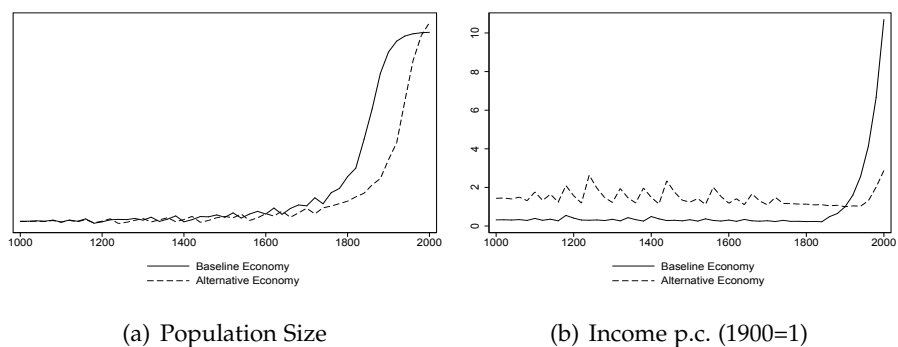
$$L_t > \frac{N}{h_t} \left[ \frac{1}{1 - 2\delta_0 \frac{L_0}{L_t} \prod_{i=0}^t n_i^r} \right]^{\frac{\sigma}{\sigma-1}}.$$

It is straightforward to see that the dynamics established in the text regarding the conditions for democratisation still hold; namely the inevitability of democracy in the (very) long run and the effect of  $\theta$  on the timing. The only possibility for the right hand side of the expression to not be smaller than the left hand side after some  $t$  is for the right hand side to go to infinity. However, if  $\delta_t < \frac{1}{2} \forall t$  holds this possibility is ruled out thus completing the proof.  $\square$

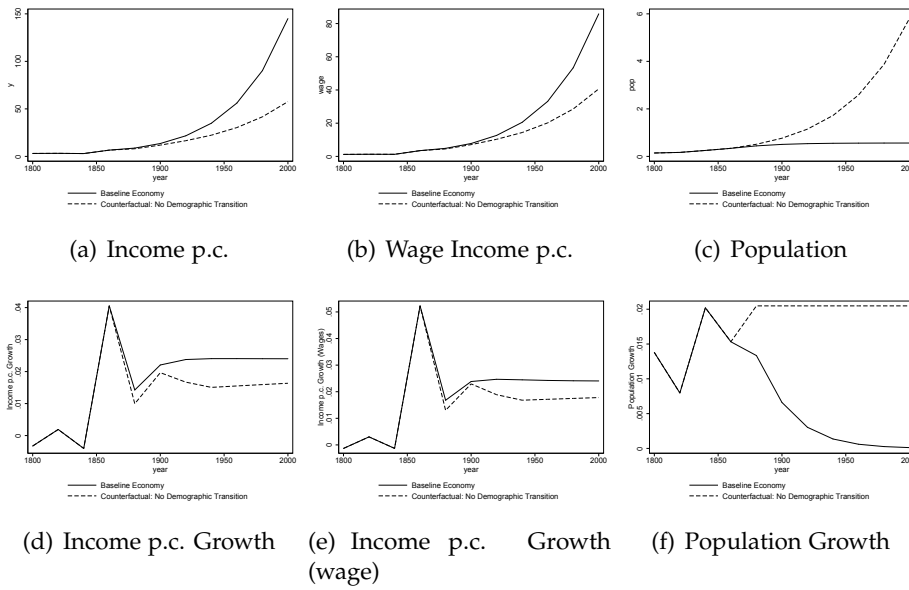
## B.2 Additional Figures



**Figure B.1:** *The Timing of the Demographic Transition*

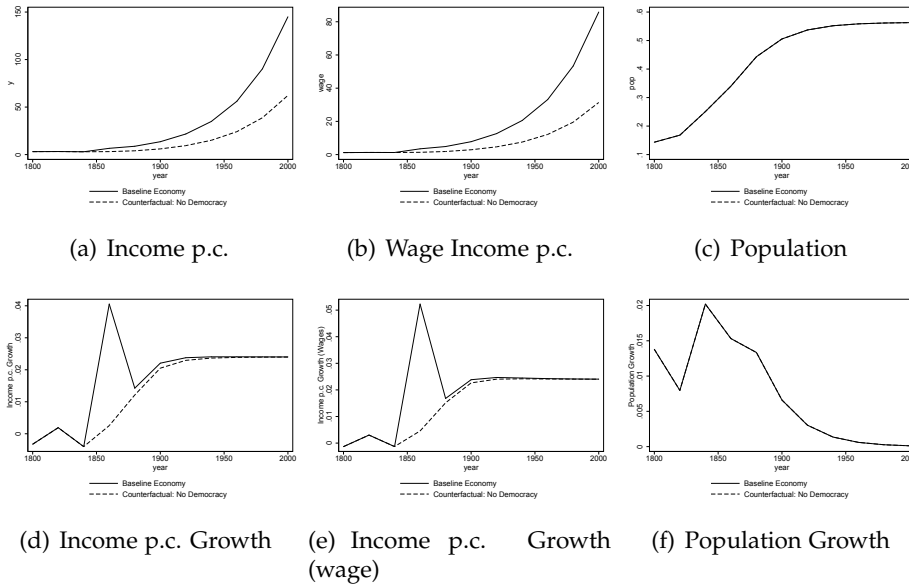


**Figure B.2:** *The Timing of the Demographic Transition*



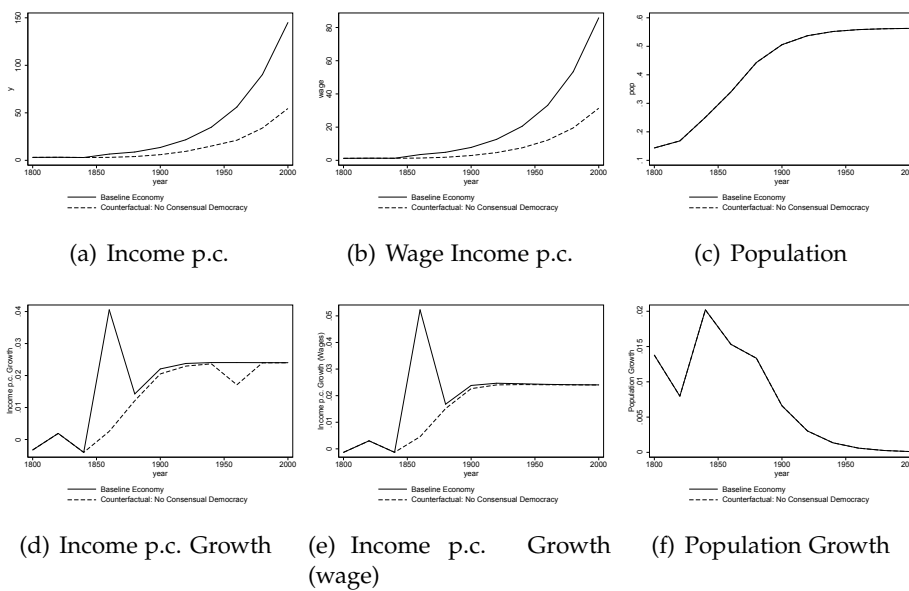
**Figure B.3:** *The Role of Demographic Development: Baseline Simulation vs. Counterfactual*

Plots in (a), (b) and (c) are for absolute values.



**Figure B.4:** *The Role of Democratisation: Baseline Simulation vs. Counterfactual*

Plots in (a), (b) and (c) are for absolute values.



**Figure B.5:** *The Role of the Consensual Democratisation Scenario: Baseline Simulation vs. Counterfactual*

Plots in (a), (b) and (c) are for absolute values.

## Appendix C

# Appendix to Chapter 3

### C.1 Extension to the Degree of Democratisation

Consider again the condition for *de facto* political power

$$\delta_t L_t z_t^r \geq (1 - \delta_t) L_t z_t^p .$$

Now assume that the acquisition of *de facto* political power by the poor does not complete the democratisation process. Instead, because of constraints on the process of acquiring *de jure* from *de facto* power (e.g. because of dependency on bureaucracy that is still in the hands of the rich) only a fraction of the poor is enfranchised. This fraction increases with increasing resources of the poor. In order to achieve full enfranchisement the poor must become powerful enough to completely implement their *de jure* power. Let the level of necessary *de facto* power be given by

$$(1 + \varepsilon) \delta_t L_t z_t^r < (1 - \delta_t) L_t z_t^p ,$$

with  $\varepsilon > 0$ . Now the condition for full democracy becomes

$$\frac{N}{h_t L_t} < \left[ \frac{1 - 2\delta_t}{1 + \varepsilon} - \frac{\varepsilon \delta}{1 + \varepsilon} \right]^{\frac{\sigma}{\sigma-1}} , \tag{C.1}$$

and the degree of enfranchisement is given by

$$\Lambda(h_t L_t) \quad \text{with} \quad \Lambda'(\cdot) > 0 \quad \text{and} \quad \Lambda''(\cdot) > 0.$$

As can be observed the right hand side of (C.1) is independent of income. Therefore introducing a degree of enfranchisement does not alter the main qualitative results, namely the negative or zero effect of income on democracy in a pre-transitional environment and a possible positive effect of income on democracy in a post-transitional setting.



# Appendix D

## Appendix to Chapter 4

### D.1 Returns to Ability and Income Distribution

This Appendix provides an analytical characterisation of the balanced growth path income distribution and a discussion of the comparison with the log normal distribution.

**Analytical Characterisation of the Income Distribution.** Consider a function of returns to ability given by,

$$(1 - a)^{-\phi} \quad \text{with} \quad \phi = \frac{1}{1 - \beta \cdot \lambda_{t-1}} \in [1, \frac{1}{1 - \beta}] \quad (\text{D.1})$$

and ability is normally distributed  $a \sim N(\mu, \sigma^2)$ . To derive the analytical expression of the income distribution use the inverse of (D.1), given by  $a = 1 - y^{-\frac{1}{\phi}}$ , to get

$$\frac{da}{dy} = \frac{1}{\phi} y^{-\frac{(1+\phi)}{\phi}}.$$

Substituting  $y$  into the probability density function of ability ( $N(\mu, \sigma^2)$ ) and multiplying with  $\frac{da}{dy}$  gives

$$\frac{1}{\phi} a^{-\frac{1}{\phi}} e^{-\frac{(1-a)^{-\frac{1}{\phi}} - \mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma a}} \quad (\text{D.2})$$

**Properties of the income distribution in comparison to the log-normal distribution.** The probability density function of a Log Normal Income distribution  $\ln N(\mu, \sigma^2)$  is given by:

$$e^{-\frac{(\ln(a)-\mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma a}} \quad (\text{D.3})$$

Notice first that, for any  $\phi > 0$ , the income distributions (D.2) and (D.3) has the same overall shape with qualitatively similar patterns of both first and second derivatives and both featuring a unique inflexion point. To illustrate the comparison between the two densities let us concentrate on the extreme values of low ability, mean ability and high ability.

For low ability,  $a \rightarrow 0$ , both the Log Normal and the distribution (D.2) converge to the same limit:

$$\lim_{a \rightarrow 0} \left( \frac{1}{\phi} a^{-\frac{1}{\phi}} e^{-\frac{(1-a^{-\frac{1}{\phi}}-\mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma a}} \right) = 0 \quad (\text{D.4})$$

$$\lim_{a \rightarrow 0} \left( e^{-\frac{(\ln(a)-\mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma a}} \right) = 0 \quad (\text{D.5})$$

Let us next evaluate the two income distributions around the mean, that is for  $a = \mu$ :

$$\lim_{a \rightarrow \mu} \left( \frac{1}{\phi} a^{-\frac{1}{\phi}} e^{-\frac{(1-a^{-\frac{1}{\phi}}-\mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma a}} \right) = \frac{1}{\phi} \mu^{-\frac{1}{\phi}} e^{-\frac{(1-\mu^{-\frac{1}{\phi}}-\mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma\mu}} \quad (\text{D.6})$$

$$\lim_{a \rightarrow \mu} \left( e^{-\frac{(\ln(a)-\mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma a}} \right) = \mu^{\frac{\mu}{\sigma^2}} e^{-\frac{(\ln\mu)^2 + \mu^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma\mu}} \quad (\text{D.7})$$

The probability mass at the mean is lower when compared to the log-normal for every  $\beta$ , as the distribution (D.2) has thicker tails. Also notice that the difference in densities around the mean also increases with  $\beta$ .

Finally, for sufficiently high levels of ability,  $a \rightarrow 1$ ,

$$\lim_{a \rightarrow 1} \left( \frac{1}{\phi} a^{-\frac{1}{\phi}} e^{-\frac{(1-a-\frac{1}{\phi}-\mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma a}} \right) = e^{-\frac{(-\mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma}} \cdot \frac{1}{\phi} \quad (\text{D.8})$$

$$\lim_{a \rightarrow 1} \left( e^{-\frac{(\ln(a)-\mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma a}} \right) = e^{-\frac{(-\mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma}} \quad (\text{D.9})$$

which is:

- equal to the density of the log-normal if  $\phi = 1$ , which is the case if  $\beta$  or  $\lambda_{t-1}$  are equal to zero.
- smaller than the density of the log-normal (indicating a longer tail) if  $\phi > 1$ , which it is for any  $\beta \cdot \lambda_{t-1} > 0$ ; also the decrease is convex in decreasing  $\phi$ .
- larger than the density of the log-normal (indicating a shorter tail) if  $\phi < 1$ ;

**Pareto tail.** The probability density function of the Pareto distribution is given by:

$$\frac{\alpha x_m^\alpha}{x^{\alpha+1}} \quad \text{for } x \geq x_m .$$

Denote  $\frac{1+\phi}{\phi} = 1 + \alpha$ , then  $\alpha = \frac{1}{\phi}$  and (D.2) can be rewritten as:

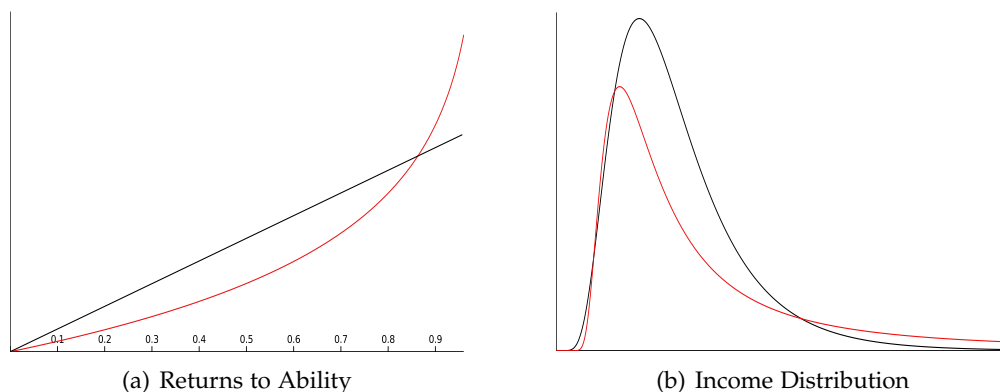
$$\frac{\alpha e^{-\frac{(1-a^{-\alpha}-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma a^{\alpha+1}}} ,$$

which constitutes a transformation of the probability density function of the Pareto distribution. Additionally, note that the larger  $\phi$  becomes the smaller becomes  $\alpha$  giving a fatter tail.

## D.2 Details of Calibration

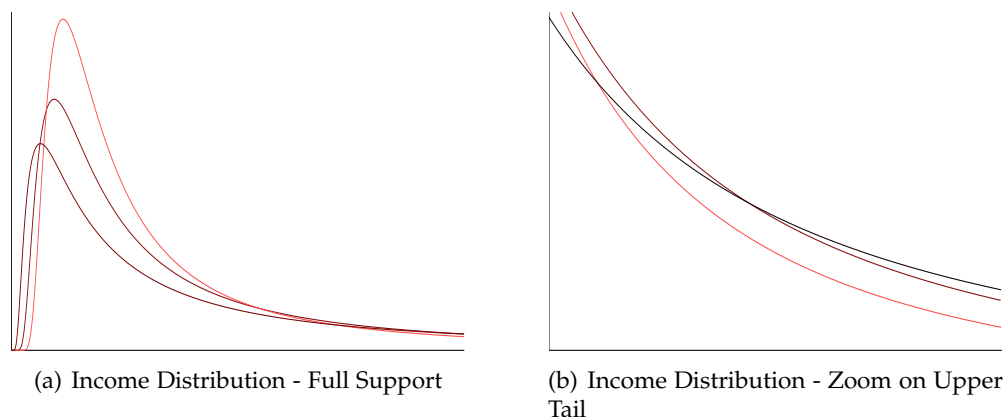
To produce a quantitative version of the model we follow the parametrisation by Cervellati and Sunde (2015) that calibrated the time invariant parameters targeting data moment for developed countries in the balanced growth path. In the following we recall and summarise the procedure.

**Figure D.1:** Returns to Ability and Income Distribution (balanced growth path)



Note: Panel (a) depicts the log returns to ability when assuming linear returns (black line) and for and convex returns (D.1) (red line). Panel (b) depicts the respective income distributions that emerges in the balanced growth path of the model when all individuals are skilled. The black income distribution is exactly log normal while the red income distribution is approximately log normal around the mean but is Pareto on the upper tail.

**Figure D.2:** Ability Bias Technological Change and Income Distribution



Note: Panel (a) depicts the changing shape of the income distribution for higher levels of  $\phi$  (following ability biased technical change). Darker colours indicate higher levels of  $\phi$ . Panel (b) depicts the same distributions in the upper tail. The figure illustrates that the higher the level of  $\phi$  the thicker the tails.

*Length of a generation.* The generation length is set to 10 years.

*Technological Progress.* The parameter of TFP,  $\phi$ , is set to match the average annual growth rate of income per capita on the balanced growth path (which equals the growth rate of technological change), see main text. Data sources: ERS Dataset ([www.ers.usda.gov](http://www.ers.usda.gov)) or historical statistics from the Bank of Sweden ([www.historicalstatistics.org](http://www.historicalstatistics.org)).

*Production Function.* The elasticity of substitution between skilled and unskilled workers is set following the literature. See for instance Acemoglu (2002).

*Human Capital.* The parameter  $\beta$  is set such that the top 1 per cent income share corresponds to approximately 9 % in the baseline simulation. This gives a parameter value of  $\beta = 0.84$ .

*Ability Distribution.* We estimate the income distribution for Sweden in 2000 using micro data from the ECHP dataset for individual incomes of full-time employees aged 25 to 45, which corresponds to the two last cohorts in the dynamic simulation, and equivalently to the two first generations with  $\lambda = 1$  in the data. The income used to estimate the parameters of the ability distribution are converted in US-\$ using an average exchange rate of 9 Kroner for one US-\$ in 2000. The income distribution is approximately log-normal between the 5th and 95th percentile of the data, with slightly thicker tails. The distribution of log incomes has mean 9.7, standard deviation 0.4, and the lowest and highest observed log-incomes are 6.7 and 12.8, respectively, which implies a maximum spread of 6.1. The moments of the income distribution for the age cohort 25-65 are essentially the same, with the lowest, mean, and highest levels of log income being 6.7, 9.7, and 12.8, respectively, and with a standard deviation of 0.41. The ECHP data are based on surveys and refer to total net income from work, which might explain the small differences between the log income per capita from macro data, which is approximately 10 in 2000, and the mean log income from the micro data that is about 9.7. The relevant data moments extracted from this data set are broadly consistent with other data sources based on register data and alternative surveys for gross earnings.

*Adult longevity.* The average of life expectancy at age five in the period 1760-1840 was 48.38, in the period 1790-1810 it was 48.06. Data from the Human Mortality Data Base available at <http://www.mortality.org/>. Similar figures are documented for England, France and

Italy.

*Initial Conditions.* The time axis is set with reference to the convergence to the post-transitional balanced growth path (in terms of  $\lambda$  converging to 1) in 2000. This implies that the choice of  $x_0 = 0.04$  determines the beginning of time in the calibration in the stagnation period. This parametrisation also implies that the income share of unskilled human capital in total production is larger than 99.9% at the beginning of the simulation, and still above 95% in 1800 just before the transition. The initial level of technology is set targeting the level of GDP per capita in Sweden in 2000 equal to 10.03. Data are from [www.historicalstatistics.org](http://www.historicalstatistics.org).

*Cross-country differences in life expectancy.* For background evidence on the role of a higher exposure to diseases in leading to a faster deficit accumulation and earlier death. As alternative scenario, we target a life expectancy at age five of 45 years (compared to 48 years reflecting Sweden around 1800 just before the transition). The data source is UN Population Statistics available at [www.unstats.un.org](http://www.unstats.un.org). Data on life expectancy at five for earlier periods are missing for many countries, including most Sub-Saharan Africa countries in 1960. Alternatively, the available information on child mortality and life expectancy at birth in 1960 can be used to derive an estimate of life expectancy at age five. This delivers a very similar target for the highest mortality countries. In 1960 life expectancy at birth was as low as 33 years in some countries like Afghanistan, and child mortality one third. Assuming a constant death rate below the age of 5, these numbers imply a life expectancy at age five between 44 and 45 years. In some countries, like Swaziland life expectancy at birth is just above 30 years still today (data from the CIA World Factbook).

## Eidesstattliche Versicherung

Ich versichere hiermit eidesstattlich, dass ich die vorliegende Arbeit selbständig und ohne fremde Hilfe verfasst habe. Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sowie mir gegebene Anregungen sind als solche kenntlich gemacht.

Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht. Sofern ein Teil der Arbeit aus bereits veröffentlichten Papers besteht, habe ich dies ausdrücklich angegeben.

München, 19.10.2018

Gerrit Meyerheim

---

Datum

---

Unterschrift