Large field inflation and moduli stabilisation in type IIB string theory

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Zusammenfassung

Diese Doktorarbeit widmet sich der Realisierung von Axion Inflation in Typ IIB String Theorie. Der Fokus hierbei liegt auf der Stabilisierung der Moduli Felder so, dass diese auf einer hohen Masse stabilisiert sind während ein leichtes, axionisches Feld das alleinige Inflaton ist. Die Herausforderung ist, eine Massenhierarchie zwischen dem Inflaton und den restlichen Moduli Feldern, sowie den Kaluza-Klein- und String-Zuständen, zu schaffen.

Zuerst widmen wir uns nicht-supersymmetrischen Minkowski Vakua im Bereich großer Komplexer Struktur Moduli. Kähler Moduli werden hierbei ignoriert. Wir finden Restriktionen an die Geometrie des internen Raumes, welche masselose Axion erlauben und zudem die restlichen Felder stabilisieren. Desweitern kommen in diesen Modellen nur axionische Komplexe Strukturen als Inflaton Kandidaten in Frage. Danach untersuchen wir Vakua mit nicht-geometrischen Flüssen. Dabei fokusieren wir uns auf eine besonders einfach Klasse von Vakua, die immer mindestens ein masseloses Axion besitzen. Sowohl deren Phenomenologie, als auch Mechanismen um die negative Vakuumsenergie auf null bzw. de Sitter anzuheben, werden diskutiert. Wir stellen ein Model für Axion Monodromie Inflation vor und besprechen dessen Probleme. Der letzte Teil dieser Doktorarbeit beschäftigt sich mit Moduli Stabilisierung in der Nähe der Konifold Singularität. In der Umgebung der Singularität, wenn die Krümmung vernachlässigbar wird, versteht man die effektive Theorie und die logarithmische Struktur der Perioden ist dennoch erhalten. Die Form dieser Perioden führt zu exponentiellen Massehierarchien. Das macht diese Region zu einem guten Kandidaten um dort Moduli Stabilisierung für periodische Inflation zu untersuchen. Wir diskutieren ein solches Modell für Inflation, unter anderem im Hinblick auf die Weak Gravity Conjecture.

Abstract

The emphasis of this thesis is to investigate mechanisms for moduli stabilisation with the aim to realise large field inflation in type IIB string theory, focusing on single field inflation. Large field inflation in string theory is usually driven by an axionic inflaton. String theory compactifications with their plethora of scalar fields have to ensure that large field inflation is purely driven by this axionic field. The challenge is to get a mass hierarchy which justifies axionic single field inflation. The non-inflatonic moduli have to be stabilised at a mass larger than the Hubble scale during inflation, else single field axion inflation is spoiled. Furthermore, a consistent truncation of string theory requires to justify the absence of stringy states and Kaluza Klein fields by stabilising the fields according to the mass hierarchy which was assumed for the effective theory.

One of the applications in this thesis is moduli stabilisation with light axions in the large complex structure limit. We first consider non-supersymmetric no-scale Minkowski vacua without volume moduli. The light axions hereby have to be complex structure moduli. Furthermore, we discuss the constraints on the geometries of the internal space which allow flat axionic directions while the other fields are massive. Then, AdS vacua with non-geometric fluxes are investigated. We consider a simple class of vacua with massless axions and discuss their phenomenology. For applications to inflation, we add uplift terms, either an $\overline{D}3$-brane or a D-term potential, to the F-term scalar potential. We show that Minkowski and de Sitter vacua are possible in these scenarios. Finally, we discuss an F-term axion monodromy model and its challenges. The last part of this thesis is dedicated to moduli stabilisation near the conifold singularity. Exponential mass hierachies appear in this region in moduli space and make it a perfect candidate for realising aligned inflation. Such an aligned inflation model is discussed, also considering the weak gravity conjecture.

Part I

Introduction

Chapter 1

Introduction

In the last century theoretical physics was quite successful in understanding the universe on large as well as small scales. There are two theories, which together describe all four interactions observed in nature. Quantum field theory, particularly the standard model, explains electro-magnetic, weak and strong interactions, while gravity is explained by general relativity. However, some questions of theoretical physics remain unresolved by these theories.

The standard model of particle physics reproduces almost all observations regarding particle interactions to high accuracy. Some exceptions like for instance the non-vanishing neutrino mass and the existence of dark matter, i.e. matter which seems to only interact gravitationally, are not included in the standard model. Also the hierarchy problem, the lightness of the Higgs mass compared to the Planck mass, remains unexplained in the standard model.

General relativity describes the only force which is not included in the standard model, namely gravitation. In contrast to standard particle physics, a quantisation of gravity is still missing. Nevertheless, general relativity is well tested by experiments and an effectively true theory.

One goal of theoretical physics is to have a theory which reproduces general relativity as well as the standard model of particle physics at low energies but also explains what happens at high energies, when quantum effects of gravity become important. The most successful candidate for such a unifying theory is string theory. The concept of string theory is based on a stringy nature of elementary particles. Unlike quantum field theory with its point-like elementary particles, the fundamental building blocks in string theory are one dimensional objects, hence the name *strings*.

These fundamental objects can be open or closed, i.e. both ends are connected.

Furthermore, strings can be excited. Depending on these properties, strings correspond to different particles. For instance, the massless closed string excitation is related to the graviton. String theory contains infinitely many excited states, corresponding to particles with high masses and spins. In the regime where the string length goes to zero, we only need to consider the massless spectrum and can ignore the heavy particles. In terms of mass scales, we work in an effective theory which is valid on energy scales smaller than the string scale. The string scale depends on the string coupling g_s and the internal volume $\mathcal{V}$

$$M_{\mathrm{s}} = \frac{\sqrt{\pi} M_{\mathrm{Pl}}}{g_s^{\frac{-1}{4}}\, \mathcal{V}^{\frac{1}{2}}}\,, \tag{1.0.1}$$

with M_{Pl} the Planck scale. Also higher dimensional objects called *D-branes* are contained in string theory. A Dp-brane is an extended object with $p+1$ spacetime dimensions on which open strings can end.

From several consistency checks it follows that superstring theory is ten dimensional. For instance, Poincare invariance or the absence of a conformal anomaly force the number of spacetime dimensions to 26 for the bosonic and ten for the supersymmetric string. This observation contradicts our observation of four spacetime dimensions. Nevertheless, it is possible for our universe to be indeed ten dimensional. If the six extra dimensions are very small and compact, it is reasonable that we do not (yet) observe them. This is realised by *compactifying* six dimensions on a compact space. The process to derive our four dimensional world from string theory is called dimensional reduction.

One important aspect of string theory is the fact that there is not one string theory, but instead there are five superstring theories, differing in their field content and their amount of supersymmetry. The type II theories, type IIA and type IIB contain only closed strings, are $\mathcal{N}=2$ supersymmetric and differ by the kind of D-branes in the theory. Heterotic theories have $\mathcal{N}=1$ supersymmetry, also only contain closed strings and are not left right symmetric. The left moving excitations correspond to the bosonic string while the right moving are supersymmetric. There are two types of heterotic string theories, differing in their gauge groups, namely $E_8 \times E_8$ and $SO(32)$. The last type of string theory, type I, does not only contain closed but also open strings. All these string theories are believed to be different limits of an eleven dimensional M-theory, which is so far not understood. Furthermore, they are connected by a web of *dualities*. A duality connects two different theories, which produce the same physics.

The stringy dualities are so-called S- and T-duality. The first relates models with strong coupling to those with weak coupling. A theory at strong coupling can be described by a dual theory, which is weakly coupled and therefore better understood. For example, type IIB is S-dual to itself and type I is S-dual to the $SO(32)$ heterotic string. T-duality is a geometric duality which relates different compactifications. For instance, type IIA compactified on a circle with radius R leads to the same physics as type IIB compactified at radius proportional to $1/R$.

String theory is meant to be a theory of everything, therefore it should explain all open questions. To mention two, string theory should solve the cosmological constant problem and explain inflation. We will focus on the latter in this thesis but also include some aspects of how to get a de Sitter vacuum in string theory. Cosmic inflation is a period of very fast expansion in the early universe. So far, its origin is not clarified but there is lots of theoretical and experimental evidence for cosmic inflation.

- Flatness Problem
  The universe is observed to be flat, i.e. the spatial curvature is vanishing. Small deviations from flatness in the beginning of the universe would have become huge after a short time, leading to the need of a highly fine-tuned starting value of the curvature. A period of inflation explains the flatness dynamically. The almost constant energy density of the field that drives inflation, the so-called inflaton, naturally tunes the curvature term flat.

- Horizon problem
  The cosmic microwave background (CMB) has a relatively homogeneous temperature of 2.728 ± 0.004 Kelvin. This implies that there was an interaction between the different regions of the sky. These regions were never causally connected unless there was a phase of superfast expansion. Cosmic inflation therefore naturally explains the homogeneous structure of the CMB by stating that all regions were connected at a certain time.

- Magnetic monopoles
  A third reason is the production of magnetic monopoles, which is a problem without inflation if Grand Unified Theories, which unify electro-magnetic, weak and strong interactions, are realised in nature. Grand Unified Theories predict the existence of a large number of magnetic monopoles and other exotic particles which were produced in the hot beginning of the universe. None of these have been observed so far. A period of inflation would have lowered the density of magnetic monopoles drastically.

Not only the theoretical indications for inflation are compelling, but also the observational data, particularly the CMB. We already mentioned the observation that there must have been an interaction between regions which seemed to not have been causally connected, so-called super-horizon fluctuations. Assuming the information being exchanged with the speed of light, regions in the sky which are separated at angles larger than two degrees, were disconnected without inflation. Other predictions of inflation are a Gaussian and adiabatic CMB spectrum which agrees with observations. Furthermore, inflation explains the large scale structure of the universe. Small density fluctuations during inflation gave rise to the current large scale structure. There is one more prediction of cosmic inflation, which has not yet been observed: the production of primordial gravitational waves.

The produced amount of gravitational waves is related to the tensor-to-scalar ratio. If we observe a tensor-to-scalar ratio r, which is of order 10^{-2} or bigger, we know that the field range which the inflaton ϕ travels during inflation via the so-called Lyth bound

$$\frac{\Delta\phi}{M_{pl}} = \mathcal{O}(1)\sqrt{\frac{r}{0.01}}\,, \tag{1.0.2}$$

is super-Planckian. Inflation models with such a large field range are called *large field inflation*. They are challenging because Planck-suppressed terms in the potential become important and spoil inflation unless there exists a mechanism to protect the potential. Such a mechanism is for instance a shift symmetry of the inflaton.

Obviously, the evidence for inflation is convincing. From a physics perspective, the exponentially fast expansion is driven by one or several scalar fields. We focus on the first explanation in this thesis. In general multi field inflation models predict non-Gaussianities which were not observed so far. If string theory is correct, the scalar field which drives inflation has to be a part of it, which is in principle not difficult to realise.

String theory contains hundreds of scalar fields, none of which was observed. Having one of these scalar fields to drive inflation might be a natural idea. A challenge though is to control the other fields in string theory. As mentioned before, the infinitely excited string states can be integrated out if we do physics below the string scale. Still, there are many not observed particles which are related to the compactification. The dimensional reduction produces an infinite tower of Kaluza Klein states. An effective theory below the Kaluza Klein (KK) scale

$$M_{\rm KK} = \frac{M_{\rm Pl}}{\sqrt{4\pi}\,\mathcal{V}^{\frac{2}{3}}} \tag{1.0.3}$$

does not contain these fields and is therefore desirable for considering inflation. Still, there are other fields in an effective theory below the string and KK scale left, so-called moduli. These are fields parametrising the compactification space. The existence of massless scalar fields would lead to long range fifth forces. Too light moduli fields also lead to the cosmological moduli problem. The energy density of the universe would be dominated by the moduli. These behave like radiation, resulting in an energy density which is too low for nucleosynthesis to occur.

This thesis is dedicated to the realisation of large field inflation in string theory. The symmetry protecting the potential from Planck-suppressed corrections is an axionic shift symmetry

$$\theta \to \theta + c\,. \tag{1.0.4}$$

Axions were first discussed in the context of the QCD axion, which was invented to resolve the strong CP problem. Technically, the standard model allows interactions which violate CP, the combined symmetry of charge conjugation and parity. The QCD axion is a natural mechanism to explain the observed conservation of CP symmetry. In string theory, axions are pseudo scalars with a shift symmetry, and in general not related to the QCD axion. They are components of the complex moduli, for example related to p-forms in the compactified space whose shift symmetry is a remnant of gauge symmetry. The goal of this thesis in one line is to achieve the hierarchy

$$M_{\mathrm{Pl}} > M_{\mathrm{s}} > M_{\mathrm{KK}} > M_{\mathrm{inf}} \sim M_{\mathrm{mod}} > H_{\mathrm{inf}} > M_{\theta}\,, \tag{1.0.5}$$

having an axionic inflaton potential and a self consistent stabilisation. For instance, we have to stick to large volume and stay in the perturbative regime. The right hierarchy of masses is crucial to justify the effective theory. This does not contain stringy fields and Kaluza Klein states. Furthermore, the moduli fields should be heavier than the Hubble scale during inflation H_{inf} to not disturb single field inflation.

In Chapter 2, we derive the behaviour of an inflaton from the Friedmann equations describing the dynamics of the universe. We discuss large field inflation and give a short introduction to axion monodromy and periodic axion inflation. Chapter 3 gives an introduction to compactification of type IIB on Calabi Yau three-folds with fluxes. The two popular moduli stabilisation scenarios, KKLT and the LARGE volume scenario are discussed, too.

The second part of this thesis contains the applications published in [1–4]. Chapter 4 investigates the state of an hierarchically light axion for axion monodromy inflation in Minkowski vacua with the axio-dilaton and complex structure moduli in the large complex structure regime (LCS). We give the conditions for the axion to be light and discuss the constraints on the Calabi Yau geometry. At the end, we show an example with an hierarchically light inflaton in this setting, while Kähler moduli are ignored.

Chapter 5 includes Kähler moduli in its discussion, which leads to the introduction of non-geometric fluxes. First, we shortly motivate these fluxes and introduce their description within double field theory. Then we show examples for special kind of vacua which naturally contain massless axions, so-called *flux-scaling vacua*. These are vacua with a simple, general behaviour. Phenomenological aspects of those vacua as well as their application to axion monodromy inflation are discussed.

Finally, we leave the large complex structure regime and investigate the vicinity of the conifold. Here, exponential mass hierarchies naturally occur even in regions with negligible warping. This region in moduli space is predestined to realise aligned inflation. We discuss the periods for two examples, as well as moduli stabilisation near the conifold. Then, we discuss a toy model on aligned inflation.

Chapter 2

Basics on String Inflation

First we discuss the basics of inflation, where we focus on single field inflation derived from the Friedmann equations. Afterwards we also give a short introduction to axion inflation in string theory. Reviews on string inflation are for instance [5,6].

2.1 Conceptual ideas of inflation

Cosmic inflation was a phase in which the universe expanded exponentially fast. The expansion was much faster than the exponential expansion of our universe at present times. We give a short derivation on the parameters for cosmic inflation, starting with the Friedmann equations. These are the equations which describe a homogeneous and isotropic universe. They can be easily derived by plugging an isotropic homogeneous ansatz into the Einstein equations and are given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\,\pi}{3M_{Pl}^2}\,\rho - \frac{k}{a^2} + \frac{\Lambda}{3}\,, \tag{2.1.1}$$

which is a differential equation for the scale factor $a(t)$ of the universe. The scale factor is a dimensionless parameter for the size of the universe and normalised to one at present time $a(t_0) = 1$. The parameter Λ is the cosmological constant and ρ is the density of the matter and radiation content of the universe. The Hubble parameter H is a measure for the expansion of the universe. It is defined as the time derivative of the scale factor divided by the scale factor

$$H = \frac{\dot{a}}{a}\,. \tag{2.1.2}$$

The factor k denotes the spatial curvature of the universe, vanishing k corresponds to a flat universe. The observed value of k is extremely close to zero and therefore motivates inflation to explain this flatness. Furthermore, we have the second Friedmann equation, the so-called acceleration equation which reads

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\,\pi}{3M_{Pl}^2}(\rho + 3\,p) + \frac{\Lambda}{3}\,. \tag{2.1.3}$$

The third equation describing the dynamics of the universe is the fluid equation

$$\dot{\rho} + 3\,H\,(p + \rho) = 0\,. \tag{2.1.4}$$

The values of the pressure p and the density ρ in the equations above depend on the kind of material. For matter, they read

$$p = 0, \qquad \rho \propto a^{-3}, \qquad a(t) \propto t^{2/3}\,, \tag{2.1.5}$$

whereas for radiation domination the following relations hold

$$p = \rho/3, \qquad \rho \propto a^{-4}, \qquad a(t) \propto t^{1/2}\,. \tag{2.1.6}$$

From the equations (2.1.1), (2.1.3) and (2.1.4), the dynamics of the universe can be calculated, depending on the densities of matter, radiation and the cosmological constant. Since we are interested in inflation, the dominating term in the Friedmann equations should drive an exponential expansion. This is produced by a cosmological constant like term. In terms of pressure and density, this corresponds to $p = -\rho$. Then, the scale of the universe expands exponentially fast with

$$a(t) \propto e^{H\,t}\,. \tag{2.1.7}$$

In general, the dynamics of the universe are described by a mixture of the cosmological constant (or a field which behaves similar to a cosmological constant), radiation and matter.

To see the effect of inflation on the curvature, we rewrite equation (2.1.1) as

$$|\Omega + \Omega_\Lambda - 1| = \frac{|k|}{a^2\,H^2}\,, \tag{2.1.8}$$

with the density parameter Ω defined in terms of the critical density as

$$\Omega = \frac{\rho}{\rho_c}\,, \qquad \rho_c = \frac{3M_{Pl}^2\,H^2}{8\,\pi} \tag{2.1.9}$$

for a given H, Ω_Λ is the density parameter of the cosmological constant (or dark energy) $\Omega_\Lambda = \rho_\Lambda/\rho$. The critical density ρ_c defines the density of a flat universe. In the absence of inflation, this term would deviate from one as

$$|\Omega - 1| \sim t^{2/3} \tag{2.1.10}$$

for matter and for radiation as

$$|\Omega - 1| \sim t\,. \tag{2.1.11}$$

Inflation avoids this problem and drives the sum $\Omega + \Omega_\Lambda$ close to one, which is naturally occurring if the starting value was close enough to one and inflation lasts sufficiently long. Since we want this behaviour in the inflationary case to be generated by a scalar field ϕ, we introduce the density

$$\rho_\phi = \frac{1}{2}\dot{\phi} + V(\phi) \tag{2.1.12}$$

and the pressure

$$p_\phi = \frac{1}{2}\dot{\phi} - V(\phi) \tag{2.1.13}$$

of the scalar field. The first term corresponds to a kinetic energy, while the second is the potential for the scalar. Note that when $\dot{\phi} \ll V$, we obtain a behaviour which has an equation of state $w = \rho/p \approx -1$ and mimics a cosmological constant. Substituting (2.1.12) and (2.1.13) into the Friedmann and fluid equation, we obtain

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\left[V(\phi) + \frac{1}{2}\dot{\phi}^2\right] \tag{2.1.14}$$

and

$$\ddot{\phi} + 3\,H\,\dot{\phi} = -V'(\phi)\,. \tag{2.1.15}$$

2.1.1 Slow roll conditions

The universe expands very fast during inflation, i.e. the scale factor accelerates $\ddot{a} > 0$. This gives the following conditions

$$\ddot{a} > 0 \Leftrightarrow p < -\frac{\rho}{3} \Leftrightarrow \dot{\phi}^2 < V(\phi)\,. \tag{2.1.16}$$

To simplify the equations (2.1.14) and (2.1.15), we assume that the kinetic term is negligible compared to the potential. This is called the *slow-roll approximation*, meaning that inflation is prolonged, the inflaton is rolling very slowly down the potential. Then the equations are given by

$$H^2 \cong \frac{8\pi}{3\,M_{Pl}} V \tag{2.1.17}$$

and

$$3\,H\,\dot{\phi} \cong -V'\,. \tag{2.1.18}$$

The slow-rolling can be captured by defining the parameters η and ϵ. The latter measures the slope of the potential and is defined by

$$\epsilon = \frac{M_{Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2\,. \tag{2.1.19}$$

Note that ϵ has to be positive. The parameter η is a measure for the curvature and is given by

$$\eta = \frac{M_{Pl}^2}{8\pi}\frac{V''}{V}\,. \tag{2.1.20}$$

Those two parameters should fulfill

$$\epsilon \ll 1, \qquad |\eta| \ll 1\,. \tag{2.1.21}$$

These conditions are necessary for slow-roll inflation but they are not sufficient. Inflation ends when the potential becomes steep and reheating occurs. During reheating, the inflaton decays into particles including radiation, and the universe becomes radiation dominated. The minimum duration of inflation necessary to satisfy the observations is measured in terms of *e-foldings* N

$$N = \log\frac{a(t_{end})}{a(t_{initial})}\,. \tag{2.1.22}$$

The minimal number of e-foldings is $\sim 50 - 60$, i.e. the universe expands by a factor $10^{21} - 10^{26}$ during inflation.

We gave a short introduction to single field inflation. In general, inflation could also be driven by multiple fields, so-called *multi field inflation*. There everything is more complex, for instance one has to deal with non-Gaussianities, which were not yet observed. Since there is no evidence for multi field inflation so far, we focus on single field inflation.

2.1.2 The tensor-to-scalar ratio

The motivation for this thesis is large field inflation. For understanding that, we have to introduce the *tensor-to-scalar ratio* r. It is defined as the ratio of tensor fluctuations to scalar fluctuations

$$r = \frac{\delta_t^2(k)}{\delta_s^2(k)} \,. \tag{2.1.23}$$

Tensor and scalar fluctuations are primordial fluctuations, i.e. variations of the density of the universe, which where generated by inflation. These fluctuations triggered structure formation in the universe. The deviation of the density ρ to the average density $\overline{\rho}$ is defined as

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x})}{\overline{\rho}} - 1 = \int \mathrm{dk}\, \delta_{\mathrm{k}}\, \mathrm{e}^{\mathbf{ikx}} \,, \tag{2.1.24}$$

with k the wavenumber. The two types of fluctuations, scalar and tensor fluctuations are defined by their power spectrum $P(k)$, which is deduced from the two point function of the Fourier components of the fluctuations

$$\langle \delta_k \delta_{k'} \rangle = \frac{2\pi^2}{k^3} \delta(k - k')\, P(k) \,. \tag{2.1.25}$$

The scalar fluctuations are fluctuations of the inflaton and behave as

$$\delta_s^2(k) = \frac{k^3}{2\pi^2} P_s(k) = \frac{2\,H^2}{\pi^2 M_{Pl}^2} \,, \tag{2.1.26}$$

whereas tensor fluctuations are fluctuations of the metric and can be measured in the form of primordial gravitational waves. Their Power spectrum is

$$\delta_t^2(k) = \frac{k^3}{2\pi^2} P_t(k) = \frac{H^4}{8\pi^2 M_{Pl}^2\, |\dot{H}| c_s} \,, \tag{2.1.27}$$

where c_s is the speed of sound. The tensor-to-scalar ratio can also be given in terms of the slow-roll parameter ϵ as

$$r = 16\epsilon \,. \tag{2.1.28}$$

Primordial gravitational waves have not yet been measured but an upper bound for the tensor-to-scalar ratio $r < 0.09$ was set by PLANCK [7].

2.2 Large field inflation

Via the so-called Lyth bound, a measurement of the tensor-to-scalar ratio informs us about the field range of the inflaton during inflation. The Lyth bound

$$\frac{\Delta\phi}{M_{Pl}} = \mathcal{O}(1)\sqrt{\frac{r}{0.01}} \tag{2.2.1}$$

claims that when the tensor-to-scalar ratio is too large (i.e. $r \geq 0.01$), the inflaton rolls over super-Planckian field ranges during inflation. This type of inflation, i.e. the inflaton rolls over super-Planckian distances, is called *large field inflation*. Recall that the upper bound for the tensor-to-scalar ratio was given by $r < 0.09$ [7]. Therefore, from an observational point of view large field inflation is still a possibly realistic scenario. Compared to small field inflation, we have to deal with a challenge. Then, suppressed terms in the potential

$$V = -\frac{1}{2}m^2\,\phi^2 + \sum_{i=1}^{\infty} c_i\,\phi^{2i}\,\Lambda^{4-2i}\,, \tag{2.2.2}$$

with the cut-off scale Λ, which can be for instance the Planck scale, are no longer negligible. These chaotic terms would spoil inflation. As can be seen in figure 2.2, Planck-suppressed operators destroy the smoothness of the potential. For large field inflation, the inflaton has to roll over super-Planckian field ranges without any bumps. The challenge for large field inflation is therefore to keep the corrections under control. A symmetry which protects the potential is needed. In string theory, in most cases, this leads to axion inflation. The inflaton is axionic and hence inflation is protected from Planck-suppressed corrections by the axionic shift symmetry. For a review on large field inflation in string theory see for example [8].

2.2.1 Periodic inflation

For periodic inflation, the continuous shift symmetry of an axion is broken to a discrete symmetry and the inflaton potential is periodic. The simplest example is *natural inflation* [9]. Periodic potentials can be generated for example by instantons. The natural inflation potential has a simple cosine structure of the form

$$V(\phi) = \Lambda^4\Big(a - \cos\frac{\phi}{2\pi f_{ax}}\Big)\,, \tag{2.2.3}$$

where f_{ax} is the axion decay constant. Since we want the inflaton to roll over super-Planckian field ranges while keeping the potential flat, the axion decay constant

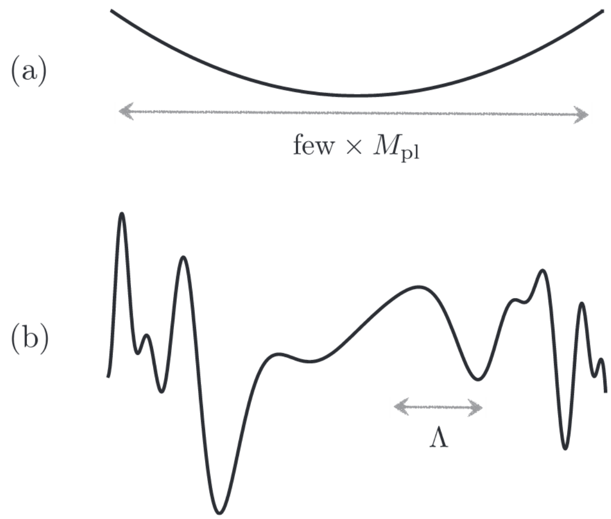

Figure 2.1: Large field inflation [from [5]]
a) A smooth potential is need over super-Planckian field ranges
b) Planck-suppressed operators destroy smoothness

has to be super-Planckian as well. It was shown [10] that a super-Planckian axion decay constant is not allowed for this simple type of inflation due to instantons or light states which lead to bumps in the inflaton potential. Later it was also shown that a super-Planckian axion decay constant does not coincide with a perturbative regime [11].

Aligned inflation is an inflationary scenario which is based on two axions aligning to generate a flat potential for the inflaton [12], see also [13–16]. The basic idea is that the inflaton is a linear combination of axions such that the effective axion decay constant is super-Planckian. Let us look on an example. Take the potential

$$V(\rho,\eta) = \Lambda^4\left[2 - \cos\left(\frac{\rho}{f_1} + \frac{\eta}{g_1}\right) - \cos\left(\frac{\rho}{f_2} + \frac{\eta}{g_2}\right)\right], \tag{2.2.4}$$

where for compactness we use the notation $f_i = 2\pi f_{i,ax}$ and $g_i = 2\pi g_{i,ax}$. The two axions are perfectly aligned if

$$\frac{f_1}{g_1} = \frac{f_2}{g_2}, \tag{2.2.5}$$

whereas a small misalignment $\alpha \neq 0$ leads to a potential which is very flat

$$\alpha = g_2 - \frac{f_2}{f_1} g_1 \,. \tag{2.2.6}$$

With this misalignment we get two axionic combinations ψ and ζ. The axionic combination ψ is heavy and can be integrated out and we are left with a single field potential for the orthogonal axion ζ of the form

$$V(\zeta) = \Lambda^4 \big[1 - \cos\big(\frac{\zeta}{f_{eff}}\big)\big] \tag{2.2.7}$$

with the effective axion decay constant

$$f_{eff} = \frac{f_2\, g_1 \sqrt{(f_1^2 + f_2^2)(f_1^2 + g_1^2)}}{f_1^2\, \alpha} \,. \tag{2.2.8}$$

Each single axion decay constant f_1, f_2, g_1, g_2 remains sub-Planckian though the effective axion decay constant f_{eff} can be super-Planckian.

Let us mention that a super-Planckian axion decay constant can also be obtained by a multitude of axions, such a scenario is called *N-flation* [17–20].

Weak gravity conjecture

In the context of periodic inflation, the weak gravity conjecture [21] has to be considered. In the case of natural inflation, it was shown [10] that a super-Planckian axion decay constant is forbidden while it was assumed that in principle an effective super-Planckian axion decay constant can be allowed like e.g. in aligned inflation. Recently, it was investigated if a large effective axion decay constant is constrained. It turned out that also effective axion decay constants have to obey certain conditions. The starting point for deducing these conditions is the weak gravity conjecture. There exist two versions of this conjecture. The mild form

Any consistent theory with a $U(1)$ gauge field admitting a UV completion with gravity must contain a state with charge to mass ratio greater than that of an extremal black hole

$$\frac{q}{m} \geq \frac{Q_{extrBH}}{M_{extrBH}} \tag{2.2.9}$$

and the strong form, which states that the weak gravity conjecture has to be fulfilled for the lightest charged particle. Physically, the weak gravity conjecture states that extremal black holes have to decay to fundamental particles. Else, infinitely many black hole remnants would be observed. Though being a conjecture, it holds in every string example investigated so far.

In the axionic case, this conjecture can be transformed [22–28] into a statement on the axion decay constant and the instanton action. The instanton action and the axion decay constant have to fulfill the inequality

$$S_{\text{inst}}\, f_{\text{inst}} \leq 1\,. \tag{2.2.10}$$

Again, the weak form implies that the weak gravity conjecture does not need to be fulfilled for the dominant term in the potential, i.e. the inflaton, but for some axion. There is still plenty of research going on concerning the weak gravity conjecture and and concrete realisations of large axion decay constants [29–31]. In recent times, the weak gravity conjecture was also applied to AdS flux vacua and D-branes [32, 33] with the conclusion that non-supersymmetric AdS vacua are metastable.

2.2.2 Axion monodromy inflation

The second example of axion inflation in string theory is axion monodromy inflation. The basic idea is that the axionic shift symmetry is softly broken e.g. by branes or fluxes and the inflaton potential is polynomial

$$V(\phi) = \mu^{4-p}\, \phi^p \tag{2.2.11}$$

Axion monodromy inflation was first discussed in [34, 35] for a scenario with D-branes. There are many senarios in the literature, for instance with a D7-brane deformation modulus as inflaton [36–38] and an inflatonic Higgs-like open string modulus [39]. In [36, 40, 41], *F-term axion monodromy* was discussed, which uses the F-term scalar potential to realise axion monodromy inflation.

Kaloper and Sorbo showed [42] that the slightly broken shift symmetry still protects the potential from corrections which become dominant when the inflaton rolls over super-Planckian field ranges. The idea is that the shift symmetry of the axion can be recovered by a shift in the fluxes and the potential remains protected from corrections. This happens due to the form of the potential

$$V = (f + m\,\phi)^2\,, \tag{2.2.12}$$

which only depends on the axion in combination with a flux shift. Schematically the shift symmetry is recovered by the shifts

$$\phi \to \phi - c/m, \qquad f \to f + c\,, \tag{2.2.13}$$

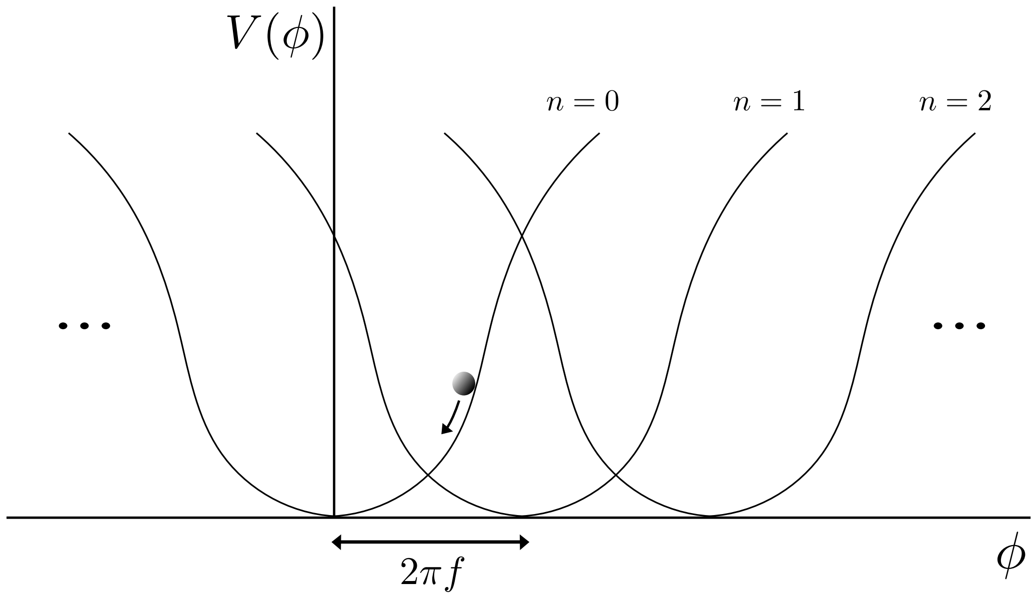

Figure 2.2: Branches [by Irene Valenzuela]
The symmetry of the potential can be recovered by jumping to another flux branch. Choosing a fixed background value for the fluxes breaks the symmetry.

for an axion ϕ and a flux f. Choosing fixed background values for the fluxes breaks the shift symmetry. Figure 2.2 illustrates this. The potential consists of shift symmetric branches until a fixed choice of fluxes chooses one branch and breaks the symmetry. In [43, 44], equation (2.2.13) was explicitly generalised to the type II flux induced geometric scalar potentials. The shift symmetry protects the potential in the sense that it forces the correction terms to still be suppressed

$$\delta V = c_n \frac{\phi^n}{\Lambda^{n-4}} \qquad \rightarrow \qquad \delta V = V_0 \left(\frac{V_0}{\Lambda^4}\right)^n . \tag{2.2.14}$$

The value of the potential in the minimum, i.e. the scale of inflation V_0 has to be smaller than the cut off scale Λ, then this leaves the inflaton potential smooth even when the inflaton travels over super-Planckian field ranges.

Chapter 3

Moduli Stabilisation

Superstring theory is only consistent in ten dimensions while on the other hand we observe a four dimensional world. This discrepancy can be explained by forcing six space dimensions to be very small. Then they are (still) absent in measurements. But the compact space is not only required to be sufficiently small, it also comes with massless scalar fields which have to be stabilised. Stabilisation means that the fields get a mass sufficiently large to coincide with observations. These scalar fields are called *moduli* and the process which makes those fields massive is called *moduli stabilisation*. The case we will discuss in the following are compactifications in type IIB string theory with fluxes. First, we introduce the basic language of Calabi Yau three-folds and the scalar potential generated by dimensional reduction thereon. Then the two most famous moduli stabilisation scenarios on the market, KKLT and LVS are described. The last part of this chapter shortly discusses an alternative moduli stabilisation approach to achieve a potential for a hierarchically light axion.

3.1 Type IIB flux compactifications

Type IIB string theory is $\mathcal{N} = 2$ supersymmetric and contains even dimensional D-branes sourced by p-form RR fields with even p. For a type IIB theory to be realistic, it has to be compactified down to four dimensions. This is usually done on Calabi Yau three-folds due to their preservation of $\mathcal{N} = 2$ supersymmetry in four dimensions which is broken to $\mathcal{N} = 1$ by introducing orientifold planes. We will give a short introduction to Calabi Yau manifolds and their phenomena as well as to the field content and the four dimensional description in terms of $\mathcal{N} = 1$ supergravity. For reviews on flux compactifications see [45–47].

3.1.1 Calabi Yau three-folds

A Calabi Yau N-fold is a Kähler manifold with a nowhere vanishing (N,0)-form Ω_N, it has a Ricci-flat metric and a vanishing first Chern class. Calabi Yau manifolds are not understood in terms of a metric but rather in terms of topological properties. In the case of Calabi Yau three-folds, the defining two properties are the Kähler forms $J_{i\bar{j}}$ and the holomorphic three-form Ω_3. We give a short introduction to those and their relation to moduli fields.

Moduli

There exist two kind of deformations on the Ricci-flat metric, which deform a Calabi Yau manifold into another Calabi Yau manifold

$$\delta g = \delta g_{i\bar{j}}\, dz_i dz^{\bar{j}} + \delta g_{ij}\, dz^i dz^j + \text{h.c.}\,. \tag{3.1.1}$$

The moduli associated with the former deformations are called Kähler moduli, while the latter correspond to complex structure moduli. Kähler moduli describe the volume of a Calabi Yau manifold, while its shape is parametrised by complex structure moduli. The moduli space splits into a product of the moduli spaces of the complex structure and the Kähler sector

$$\mathcal{M}_{\text{c.s}} \times \mathcal{M}_{\text{K}}\,. \tag{3.1.2}$$

The Hodge numbers of the Calabi Yau manifold determine the number of moduli fields. Hodge numbers are defined as the dimensions of the cohomology groups

$$h^{(p,q)} = \dim H^{(p,q)}\,, \tag{3.1.3}$$

where $H^{p,q}$ are forms with p holomorphic and q antiholomorphic indices. In a Calabi Yau three-fold, there are two independent Hodge numbers $h^{2,1} = h^{1,2}$ and $h^{1,1} = h^{2,2}$ as well as the no-where vanishing holomorphic three-from Ω_3, i.e. $h^{3,0} = h^{0,3} = 1$. There are no one-forms and $(2,0)$-forms on Calabi Yau three-folds.

Kähler moduli

The $h^{1,1}$ (1,1)-forms are the Kähler forms $J_{i\bar{j}}$. These fields correspond to two-cycle volumina t^α and are related to the Kähler moduli of the Calabi Yau manifold. For the $(1,1)$- and $(2,2)$-cohomology of $\mathcal{M}$ we introduce bases of the form

$$\begin{aligned} \{\omega_{\mathsf{A}}\} &\in H^{1,1}(\mathcal{M})\,, \\ \{\tilde{\omega}^{\mathsf{A}}\} &\in H^{2,2}(\mathcal{M})\,, \end{aligned} \qquad \mathsf{A} = 1, \ldots, h^{1,1}\,, \tag{3.1.4}$$

and for later convenience we also define $\{\omega_A\} = \{1, \omega_{\mathsf{A}}\}$ and $\{\tilde{\omega}^A\} = \{d\mathrm{vol}_6, \tilde{\omega}^{\mathsf{A}}\}$, with $A = 0, \dots, h^{1,1}$. The latter two bases are chosen as

$$\int_{\mathcal{M}} \omega_A \wedge \tilde{\omega}^B = \delta_A{}^B \,. \tag{3.1.5}$$

The Kähler forms are related to two-cycle volumina via

$$e^{-\phi/2} J = t^\alpha \omega_\alpha \,, \tag{3.1.6}$$

with the dilaton ϕ being related to the string coupling. The real part of Kähler moduli in type IIB orientifolds are given in terms of four-cycles τ_α, which are related to the two-cycles via $\tau_\alpha = \frac{1}{2}\kappa_{\alpha\beta\gamma} t^\beta t^\gamma$ with $\kappa_{\alpha\beta\gamma}$ the triple intersection number defined as $\kappa_{\mathsf{ABC}} = \int_{\mathcal{M}} \omega_{\mathsf{A}} \wedge \omega_{\mathsf{B}} \wedge \omega_{\mathsf{C}}$. The volume of the Calabi Yau three-fold $\mathcal{M}$ is given by $\mathcal{V} = \frac{1}{6}\kappa_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma$.

Periods and complex structure moduli

The (1,1)-forms J are related to volume moduli, whereas the holomorphic three-form Ω_3 depends on complex structure moduli. We choose a symplectic basis for the third cohomology of the Calabi Yau three-fold $\mathcal{M}$ by

$$\{\alpha_\Lambda, \beta^\Lambda\} \in H^3(\mathcal{M}) \,, \qquad \Lambda = 0, \dots, h^{2,1} \,, \tag{3.1.7}$$

which satisfies

$$\int \alpha_\Lambda \wedge \beta^\Sigma = \delta_\Lambda{}^\Sigma \,. \tag{3.1.8}$$

The holomorphic three-form reads in terms of a symplectic basis

$$\Omega_3 = X^\Lambda \, \alpha_\Lambda - F_\Lambda \, \beta^\Lambda, \tag{3.1.9}$$

where the periods X^Λ and F_Λ are functions of the complex structure moduli $\mathcal{U}^i$, with $i = 1, \dots, h^{2,1}$. In terms of the holomorphic three-form Ω_3, the periods are defined as follows

$$X^\Lambda = \int_{A^\Lambda} \Omega_3 \,, \qquad F_\Lambda = \int_{B_\Lambda} \Omega_3 \,, \tag{3.1.10}$$

with A^Λ, B_Λ being the Poincare duals of $\alpha_\Lambda, \beta^\Lambda$. These periods depend holomorphically on the complex structure deformations $\mathcal{U}^i$. The X^Λ can be considered as homogeneous coordinates of the complex structure moduli space. Inhomogeneous coordinates are e.g. defined via $\mathcal{U}^i \sim X^i/X^0$ with $i = 1, \dots, h^{2,1}$. To explicitly calculate the periods, they are expanded around certain points in moduli space. Some of these points are special, for instance the large complex structure point. There, the periods are polynomials up to cubic order. Furthermore we mention another special region, namely the conifold singularity. Here one of the periods contains a logarithmic term. We discuss this in Section 6.

Orientifolds

An orientifold is a manifold with a projection involving the world sheet parity operator Ω, which acts on closed and open strings as

$$\begin{aligned} \text{Closed :} \qquad & \Omega : (\sigma_1, \sigma_2) \to (2\pi - \sigma_1, \sigma_2)\,, \\ \text{Open :} \qquad & \Omega : (\tau, \sigma) \to (\tau, \pi - \sigma)\,. \end{aligned} \tag{3.1.11}$$

Orientifold projections can also contain other discrete symmetries and are induced by adding orientifold planes, so-called O-planes to the theory. Orientifolds have several motivations. For instance, they can cancel the tadpole contribution which is induced by D-branes. Furthermore they break half of the supersymmetry. For instance, in a type IIB string theory compactified on a Calabi Yau three-fold with O-planes we get $\mathcal{N} = 1$ supersymmetry. We use the notation $h^{p,q}_+$ ($h^{p,q}_-$) for fields which are even (odd) under the orientifold projection

$$H^{p,q}(\mathcal{M}) = H^{p,q}_+(\mathcal{M}) \oplus H^{p,q}_-(\mathcal{M})\,, \qquad\qquad h^{p,q} = h^{p,q}_+ + h^{p,q}_-\,. \tag{3.1.12}$$

3.1.2 The scalar potential

In string theory, the massless closed string excitation splits into the metric G, the anti-symmetric B_2 and a trace-part ϕ. Furthermore, type IIB contains so-called Ramond-Ramond p-forms, with p being even. This is the bosonic field content of type IIB and it is summarised in table 3.2. The effective supergravity action for

sector	10d field
NS-NS	ϕ, B_{MN}, G_{MN}
Ramond-Ramond	$\mathrm{C}_0, C_{MN}, C_{MNPQ}$

Table 3.1: Bosonic field content in type IIB.

type IIB string theory is given by

$$S_{\text{IIB}} = \frac{1}{2\tilde{\kappa}_{10}^2}\int d^{10}x\,\sqrt{-G}\Big[e^{-2\Phi}\Big(R+4(\nabla\Phi)^2-\frac{1}{2}|H_3|^2\Big)-\frac{1}{2}|F_1|^2-\frac{1}{2}|\tilde{F}_3|^2 -\frac{1}{4}|\tilde{F}_5|^2\Big]-\frac{1}{4\tilde{\kappa}_{10}^2}\int C_4\wedge H_3\wedge F_3 \tag{3.1.13}$$

with the Ramond-Ramond fluxes

$$\tilde{F}_3 = F_3 - C_0\,H_3\,,\qquad \tilde{F}_5 = F_5-\frac{1}{2}C_2\wedge H_3+\frac{1}{2}B\wedge F_3\,. \tag{3.1.14}$$

The NSNS flux H_3 is the field strength of the Kalb-Ramond two-form B_2. The fluxes are given by

$$H = dB_2\,,\qquad F_1 = dC_0\,,\qquad F_3 = dC_2\,,\qquad F_5 = dC_4\,. \tag{3.1.15}$$

Furthermore, G is the ten dimensional spacetime metric and R is the Ricci scalar. The dilaton ϕ is a scalar field, which is related to the string coupling via $g_s = e^{\phi}$. The factor $\tilde{\kappa}_{10}$ is given by $\tilde{\kappa}_{10}^2 = \frac{1}{2}(2\pi)^7(\alpha')^4$ with α' being related to the string length l_s by $\alpha' = l_s^2$. The action (3.1.13) can be reduced to four dimension

$$\mathbb{R}^{1,3}\times\mathcal{M}_6\,. \tag{3.1.16}$$

After dimensional reduction to four dimensions, these fluxes generate a scalar potential which couples to the moduli fields due to their appearance in the internal metric. On a Calabi Yau three-fold, one- and five-forms are not supported. Hence we only have three-form flux F_3 and H_3, which lead to the potential

$$S = -\frac{1}{4\tilde{\kappa}_{10}^2\,\text{Re}(S)}\int_{\mathbb{R}^{3,1}\times\mathcal{X}} G_3\wedge\star_{10}\overline{G}_3 \sim \int_{\mathbb{R}^{3,1}} d^4x\sqrt{-g_4}\int_{\mathcal{X}} G_3\wedge\star_6\overline{G}_3\,, \tag{3.1.17}$$

where we defined the three-form flux

$$G_3 = F_3 + i\,S\,H_3\,. \tag{3.1.18}$$

The Hodge-star is defined in terms of the metric, which itself depends on the moduli. In this case with three-form fluxes, the Hodge-star only depends on complex structure moduli.

In string theory, fluxes, as well as orientifold planes and D-brane charges contribute to the tadpole condition

$$N_{\text{D}p}^{\text{flux}} + \sum_{\substack{\text{D-branes}\\ \text{O-planes}}\ i} Q_{\text{D}p}^{(i)} = 0\,. \tag{3.1.19}$$

The contribution of (3.1.13) to the D3-tadpole is given by

$$N_{\text{flux}} = \frac{1}{(2\pi)^4(\alpha')^2}\int_{\mathcal{X}} H_3 \wedge F_3\,. \tag{3.1.20}$$

Symplectic basis and prepotential

The metric on a Calabi Yau manifold is in general unknown. However, we can use special geometry [48] for calculations on Calabi Yau three-folds. The integer quantised three-form fluxes are expanded in the symplectic basis as

$$\begin{aligned} \tfrac{1}{(2\pi)^2\alpha'}\,H_3 &= h_\Lambda\,\beta^\Lambda + \overline{h}^\Lambda\,\alpha_\Lambda\,, \qquad & h_\Lambda, \overline{h}^\Lambda &\in \mathbb{Z}\,,\\ \tfrac{1}{(2\pi)^2\alpha'}\,F_3 &= f_\Lambda\,\beta^\Lambda + \overline{f}^\Lambda\,\alpha_\Lambda\,, \qquad & f_\Lambda, \overline{f}^\Lambda &\in \mathbb{Z}\,. \end{aligned} \tag{3.1.21}$$

Then we can expand G_3 as

$$\tfrac{1}{(2\pi)^2\alpha'}\,G_3 = e_\Lambda\beta^\Lambda + m^\Lambda\alpha_\Lambda\,, \qquad \begin{aligned} e_\Lambda &= -i\,S\,h_\Lambda + f_\Lambda\,,\\ m^\Lambda &= -i\,S\,\overline{h}^\Lambda + \overline{f}^\Lambda\,. \end{aligned} \tag{3.1.22}$$

To evaluate the potential in terms of moduli, we have to know how the Hodge star acts on the three-form fluxes. The Hodge star acts on the symplectic basis (3.1.8) as

$$\begin{aligned} \star\alpha &= A\,\alpha + B\,\beta\\ \star\beta &= C\,\alpha + D\,\beta \end{aligned} \tag{3.1.23}$$

with

$$\begin{aligned} A &= -D^T = (\mathrm{Re}\,\mathcal{N})(\mathrm{Im}\,\mathcal{N})^{-1}\\ B &= -\mathrm{Im}\,\mathcal{N} - (\mathrm{Re}\,\mathcal{N})(\mathrm{Im}\,\mathcal{N})^{-1}(\mathrm{Re}\,\mathcal{N})\\ C &= (\mathrm{Im}\,\mathcal{N})^{-1}\,, \end{aligned} \tag{3.1.24}$$

and $\mathcal{N}$ being the period matrix. Then the potential (3.1.17) can be calculated in terms of the period matrix which is defined in terms of the prepotential as [48]

$$\mathcal{N}_{\Lambda\Sigma} = \overline{F}_{\Lambda\Sigma} + 2i\,\frac{\mathrm{Im}(F_{\Lambda\Gamma})X^\Gamma\,\mathrm{Im}(F_{\Sigma\Delta})X^\Delta}{X^\Gamma\,\mathrm{Im}(F_{\Gamma\Delta})X^\Delta}\,. \tag{3.1.25}$$

where

$$F_{\Lambda\Sigma} = \frac{\partial^2 F}{\partial X_\Lambda \partial X_\Sigma} \tag{3.1.26}$$

are the second derivatives of the prepotential with respect to the periods X^Λ and the indices run from $\Lambda, \Sigma = 0, ..., h^{2,1}$. The $\mathcal{N} = 2$ prepotential F is a homogeneous function of degree two of the periods X^Λ. We will use it in our computations at the large complex structure point, where it is known to have the form

$$\widetilde{F} = \frac{\kappa_{ijk} X^i X^j X^k}{X^0} + \frac{1}{2} a_{ij} X^i X^j + b_i X^i X^0 + \frac{1}{2} c \big(X^0\big)^2 + F_{\text{inst.}} \,. \tag{3.1.27}$$

The factors a_{ij}, b_i and c are topological factors while F_{inst} are the instanton contributions. Hence, in terms of a prepotential, the scalar potential (3.1.17) is given by

$$V_F = -\frac{M_{\text{pl}}^4}{4\pi} \frac{1}{\mathcal{V}^2 \, \mathrm{Im}\, \tau} \, (e + m\overline{\mathcal{N}})(\mathrm{Im}\mathcal{N})^{-1}(\overline{e} + \mathcal{N}\,\overline{m}) \,. \tag{3.1.28}$$

Note that this is a positive semi-definite potential, the global minima are Minkowski. This structure is present due to the absence of volume moduli in the scalar potential (3.1.17). The tadpole contribution (3.1.20) in this notation reads

$$N_{\text{flux}} = m \times e = \overline{h}^\Lambda f_\Lambda - \overline{f}^\Lambda h_\Lambda \,. \tag{3.1.29}$$

Field content

Here, we introduce the notation for the closed string complex moduli fields in (3.1.13). The C_p-forms, the dilaton and the moduli of the internal space are scalar fields in the external space, while the field strengths F_p and H_3 can have background values and correspond to fluxes. The scalar fields consist of a real part, which we call saxionic. The imaginary parts have an axionic shift symmetry, which is a remnant of the gauge symmetry of the C_p-forms. These will be the candidates for the inflaton when we realise large field inflation. We expand the RR forms and the

number	modulus	name
1	$S \;= e^{-\phi} - i\, C_0$	axio-dilaton
$h^{2,1}_-$	$U^i = v^i + i\, u^i$	complex structure
$h^{1,1}_+$	$T_\alpha = \tau_\alpha + i\, \rho_\alpha$	Kähler
$h^{1,1}_-$	$G^a = S b^a + i\, c^a$	axionic odd

Table 3.2: Moduli in type IIB orientifold compactifications.

Kalb-Ramond field in terms of (1,1)-forms ω_α and (2,2)-forms $\tilde{\omega}^\alpha$ as

$$B_2 = b^a \omega_a\,, \qquad C_2 = c^a \omega_a\,, \qquad C_4 = \rho_\alpha \tilde{\omega}^\alpha\,. \tag{3.1.30}$$

In type IIB orientifolds, the physical fields are not the two-cycle volumina t_α but four-cycle volumina $\tau_\alpha = \frac{1}{2}\kappa_{\alpha\beta\gamma}t^\beta t^\gamma$. Complex Kähler moduli are then defined by

$$T_\alpha = \frac{1}{2}\kappa_{\alpha\beta\gamma}t^\beta t^\gamma + i\left(\rho_\alpha - \frac{1}{2}\kappa_{\alpha ab}c^a b^b\right) - \frac{1}{4}e^\phi \kappa_{\alpha ab}G^a(G+\overline{G})^b\,, \tag{3.1.31}$$

with G defined in table 3.2, where the moduli we consider are summarised.

3.1.3 Four dimensional supergravity description

After dimensional reduction to four dimensions, the physics of the effective type IIB action (3.1.13) can also be described in terms of an $\mathcal{N}=1$ supergravity. $\mathcal{N}=1$ supergravity can be described by a Kähler potential K, a superpotential W, a Fayet-Illiopolous (FI) term ζ and the gauge kinetic function. The FI term contributes to the D-term potential, which will be discussed in Chapter 5 for examples with abelian gauge fields or a stack of D7-branes. Here, we focus on the F-term scalar potential calculated via Kähler potential and the superpotential for stabilising the fields.

Superpotential

The superpotential on type IIB orientifolds is given by [49]

$$W = \int_{\mathcal{M}} \left[\mathfrak{F} + d_H \Phi_{\rm c}^{\rm ev}\right]_3 \wedge \Omega_3\,, \tag{3.1.32}$$

with

$$\Phi_{\rm c}^{\rm ev} = iS - iG^a\omega_a - iT_\alpha \tilde{\omega}^\alpha \tag{3.1.33}$$

and the holomorphic three-form Ω_3. The subscript in (3.1.32) means that the three-form part of a multi-form is selected, and the operator d_H is defined as $d_H = d - H\wedge$. We use this notation for later convenience when introducing non-geometric fluxes. Equation (3.1.32) reads in a simple setting with H_3 flux and RR three-form flux F_3 and in the absence of geometric flux

$$W = \int_{\mathcal{M}} \left[F_3 + i\,S\,H_3\right]_3 \wedge \Omega_3\,. \tag{3.1.34}$$

This flux superpotential leads to the same potential as (3.1.17), which was shown in [50]. We will refer to (3.1.34) as the *Gukov-Vafa-Witten superpotential* (GVW).

Kähler potential

The kinetic terms of the moduli can be deduced from the Kähler potential

$$K = -\log(S+\overline{S}) - \log\int(\Omega\wedge\overline{\Omega}) - 2\log\mathcal{V}\,. \tag{3.1.35}$$

Recall that the volume $\mathcal{V}$ of the internal space is a function of the Kähler moduli and Ω is related to the complex structure moduli. This potential appears in the kinetic terms of the scalar fields

$$\mathcal{L}_{kin} \sim K_{i\bar{j}}\partial_\mu\phi^i\partial^\mu\overline{\phi}^{\bar{j}}\,, \tag{3.1.36}$$

with the Kähler metric defined by $K_{i\bar{j}} = \frac{\partial^2 K}{\partial\phi^i\partial\overline{\phi}^{\bar{j}}}$. Note that the Kähler metric is used to canonically normalise the moduli fields. It is furthermore the metric on the moduli space of the fields, in the physical regime it hence has to be positive definite.

F-term scalar potential

The F-terms, auxiliary fields which receive a non-zero vev when supersymmetry is broken, are defined as

$$F^i = -e^{\frac{K}{2}}K^{i\bar{j}}D_{\bar{j}}W\,, \tag{3.1.37}$$

with the covariant derivative $D_i = \partial_i + \partial_i K\,W$. The $\mathcal{N}=1$ supergravity F-term scalar potential is then given in terms of the Kähler potential and the superpotential as

$$V_F = \frac{M_{\mathrm{Pl}}^4}{4\pi}\,e^K\Big(K^{I\bar{J}}D_IWD_{\bar{J}}\overline{W} - 3\big|W\big|^2\Big)\,. \tag{3.1.38}$$

The indices run over all moduli. When the superpotential is independent of the Kähler moduli, the last term cancels and we are left with a no-scale potential

$$V_F = \frac{M_{\mathrm{Pl}}^4}{4\pi}\,e^K\Big(K^{I\bar{J}}D_IWD_{\bar{J}}\overline{W}\Big)\,, \tag{3.1.39}$$

where the indices now do not include Kähler moduli. This happens due to the no scale relation $K^{i\bar{j}}K_iK_{\bar{j}} = (\partial_i\partial_{\bar{j}}K)^{-1}\partial_iK\partial_{\bar{j}}K = 3$, with the indices $i,j = 1,...,h^{1,1}$. The global minima of the no-scale potential are Minkowski and the Kähler moduli do not receive a mass. This is the case for the reduced potential term of the action (3.1.13). To stabilise Kähler moduli, other effects have to be included, for instance non-perturbative corrections or non-geometric fluxes.

Shift symmetries in the Kähler potential

In string theory some moduli fields are axionic, i.e. they admit a shift symmetry in the Kähler potential. The shift symmetry has its origin in the gauge symmetry of the fields. Shift symmetric are therefore the Ramond-Ramond p-forms C_p and the Kalb-Ramond field B_2. They are e.g. incorporated as the imaginary parts of the complexified Kähler moduli T^α or the axio-dilaton S. Hence, the Kähler potential is of the form

$$K = -\log\left(S + \overline{S}\right) - 2\log\mathcal{V}(\tau)\,, \tag{3.1.40}$$

where the volume $\mathcal{V}$ purely depends on the four-cycle volumina $\tau \sim T + \overline{T}$. In the complex structure case, the complexified moduli have no origin in RR-forms but are purely geometric. Nevertheless, there are regions in moduli space where a shift symmetry of complex structure moduli exists. In [38] such special points were discussed. Here, we mainly mention two regions with shift symmetry: the large complex structure limit (LCS) and the conifold.

3.2 Moduli stabilisation scenarios

Having seen the scalar potential for stabilising the axio-dilaton and complex structure moduli, we now take a short look at the two most important scenarios for moduli stabilisation, the KKLT scenario [51] and the LARGE volume scenario (LVS) [52]. Both stabilise the axio-dilaton and the complex structure moduli in type IIB string theory via the flux-induced GWV-superpotential at a scale $M_{pl}^2/\mathcal{V}^2$. Whereas the Kähler moduli are stabilised by corrections to the superpotential induced for instance by instantons coming from Euclidean D3-branes or gluino condensation. In the following, we give a short introduction to those stabilisation schemes, assuming that the moduli coupling to fluxes in the GWV-potential are stabilised appropriatly.

3.2.1 KKLT

The scenario of Kachru, Kallosh, Linde and Trivedi (KKLT) [51] is one of the most famous models for moduli stabilisation. It was applied to inflation in KKLMMT [53]. The three-form fluxes stabilise the axio-dilaton and the complex structure fields supersymmetrically. Then, non-perturbative corrections stabilise the volume modulus T as

$$W = W_0 + A_s\, e^{-a_s T}\,. \tag{3.2.1}$$

The pfaffian A_s can depend on complex structure moduli and is in general unknown. In KKLT, it is a number since complex structure moduli were already integrated out. The parameter a_s depends on the origin of the non-perturbative term. The values of A_s, a_s and W_0 are assumed to be real, and the latter is furthermore negative and arises from the flux tree-level superpotential. The axionic part of T is stabilised to zero and we can solve (3.2.1) in terms of the saxion σ as

$$DW = 0 \qquad \to \qquad W_0 \sim -A_s e^{-a_s \sigma_0}\left(1 + \frac{2}{3} a_s\, \sigma_0\right) . \tag{3.2.2}$$

The superpotential W_0 is exponentially small and the resulting minimum is AdS

$$V_{AdS} = -3e^K W^2 = -\frac{a_s^2 A_s^2 e^{-2a_s\sigma_0}}{6\sigma_0} . \tag{3.2.3}$$

This is found by solving the supersymmetry conditions. For uplifting the negative minimum to de Sitter, an $\overline{D}3$-brane is introduced, which leads to a positive-definite contribution to the potential

$$V_{\text{up}} = \frac{\varepsilon}{\mathcal{V}^\alpha} , \tag{3.2.4}$$

where $\alpha = 2$ for a $\overline{\text{D3}}$-brane in the bulk and $\alpha = 4/3$ for a brane located in a warped throat. Figure 3.2.1 shows the AdS minimum and the uplifted de Sitter

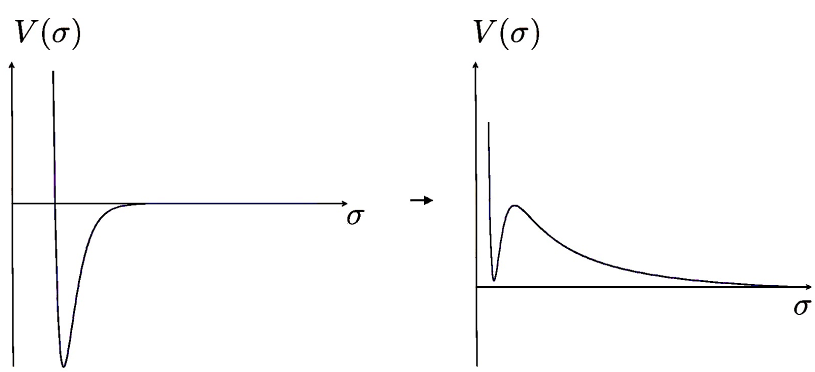

Figure 3.1: KKLT AdS and uplifted dS minima
KKLT has a minimum with negative vacuum energy. After inclusion of an $\overline{D}3$-brane, the vacuum energy becomes positive

vacuum.

3.2.2 LARGE volume scenario

The LARGE volume scenario (LVS) includes corrections in the Kähler potential. The volume is stabilised at an exponentially large value hence the name. For a simple Calabi Yau with two Kähler moduli $\mathbb{P}_{1,1,1,6,9}$[18], the Kähler potential is given by

$$K = -2\log\left(\tau_b^{\frac{3}{2}} - \tau_s^{\frac{3}{2}} + \frac{\xi}{2}\mathrm{Re}(S)^{\frac{3}{2}}\right) , \tag{3.2.5}$$

with $\xi = -\frac{\chi(M)\zeta(3)}{2(2\pi)^3}$ and $\chi(M)$ the Euler number of the internal manifold. For having a minimum, it is necessary that $\chi(M) < 0$. Due to the relation $\chi = 2(h^{11} - h^{21})$, this means that the LVS is valid on manifolds with more complex structure moduli than Kähler moduli $h^{2,1}(M) > h^{1,1}(M)$. Note that the Calabi Yau has a swiss cheese structure with a big four-cycle τ_b and a small cycle τ_s. The LVS assumes that the axio-dilaton and the complex structure moduli are stabilised beforehand by the GVW superpotential similar to KKLT with W_0 its value at the minimum. Furthermore, the superpotential contains a non-perturbative term

$$W_{\mathrm{LVS}}(T) = W_0 + A_s\, e^{-a_s T_s} . \tag{3.2.6}$$

The dominant terms in the scalar potential are

$$V_{\mathrm{LVS}}(T) = e^{K_{cs}}\frac{g_s}{2}\left(\frac{|a_s A_s|^2\sqrt{\tau_s}\, e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{W_0\,|a_s A_s|\,\tau_s\, e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\xi\, W_0^2}{g_s^{\frac{3}{2}}\,\mathcal{V}^3}\right) . \tag{3.2.7}$$

Here K_{cs} denotes the Kähler potential for the complex structure moduli. Even though we are in a large volume regime, all three terms are of the same order of magnitude at the LVS minimum where the Kähler fields are stabilised at

$$\tau_s = \frac{(4\xi)^{\frac{2}{3}}}{g_s} , \qquad \mathcal{V} = \frac{W_0\,\xi^{\frac{1}{3}}}{2^{\frac{1}{3}}\,g_s^{\frac{1}{2}}\,|a_s A_s|}e^{a_s\tau_s} . \tag{3.2.8}$$

Indeed, inserting this solution into (3.2.7), all terms are of order $\mathcal{O}(\mathcal{V}^{-3})$. The volume is stabilised at exponentially large volume, therefore the name LARGE volume scenario. The components of the inverse Kähler metric are at leading order

$$\begin{aligned} K^{\tau_b\bar{\tau}_b} &= \frac{4}{3}\mathcal{V}^{\frac{4}{3}} , \qquad K^{\tau_s\bar{\tau}_s} = \frac{8}{3}\sqrt{\tau_s}\,\mathcal{V} \\ K^{\tau_b\bar{\tau}_s} &= K^{\tau_s\bar{\tau}_b} = 4\tau_s\,\mathcal{V}^{\frac{2}{3}} . \end{aligned} \tag{3.2.9}$$

Using this approximation, the Kähler moduli masses can be easily calculated. The results for the masses of the Kähler moduli are given by

$$
\begin{aligned}
M_{\tau_b}^2 &\sim O(1)\frac{W_0^2\,\xi}{g_s^{\frac{1}{2}}\,\mathcal{V}^3}M_{\rm pl}^2\,, \qquad M_{\rho_b}^2 \sim 0\,, \\
M_{\tau_s}^2 &\sim M_{\rho_s}^2 \sim O(1)\frac{a_s^2\,W_0^2\,\xi^{\frac{4}{3}}}{g_s\,\mathcal{V}^2}M_{\rm pl}^2\,.
\end{aligned}
\tag{3.2.10}
$$

The large volume scenario leads to a massless axion as well as to a light big Kähler modulus τ_b, which is exponentially lighter than the other Kähler moduli, as well as complex structure moduli and the axio-dilaton which also get stabilised at $\mathcal{V}^{-2}$. Uplifting to de Sitter is analogous to KKLT by introducing $\overline{D}3$-branes.

3.3 Moduli stabilisation and single field inflation

A moduli stabilisation scheme for single field inflation requires a hierarchically light inflaton. The main challenge for moduli stabilisation is therefore to keep the inflaton light while all other moduli have to acquire a sufficiently large mass. Actually, not all moduli have to be stabilised, flat axionic directions contribute to dark radiation and do not necessarily contradict observations. Saxions on the other hand must be always heavy.

3.3.1 Controlling the backreaction

In [54] it was shown that the inflaton potential can backreact on the heavy moduli which leads to a flattening of the inflaton potential. The inflaton θ shifts the minimum of the heavy moduli ψ while it rolls down the potential

$$
\psi \to \psi + \delta\psi(\theta)\,. \tag{3.3.1}
$$

The shift of the old minimum scales with the value of the inflaton and also depends on the mass gap to the inflaton. A large mass hierarchy between the inflaton and the other moduli leaves the backreaction on the minimum and the potential small. The shifted moduli minima appear in the inflaton potential and lead to a flattening thereof, picture 3.2 depicts this flattening for the simple flux-scaling example we will consider later. For small values of the inflaton, the backreaction on the polynomial potential is small. For large inflaton values, the potential varies

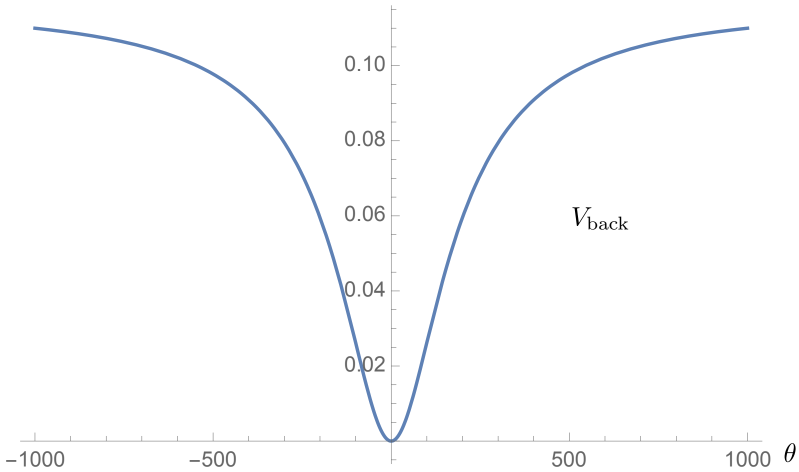

Figure 3.2: An axion monodromy inflation potential considering backreaction. For small values of the inflaton θ (in units of $M_{pl}^4/4\pi$), the potential behaves quadraticly, then linearly and for large θ, the backreaction flattens the potential to a plateau shape.

tremendously from the original polynomial potential and is flattened to a plateau-like shape.

In the following chapters, we do not explicitly consider the effect of backreaction on the potential, but try to realise a sufficiently large mass hierarchy between the inflaton and the heavy fields. For the axion monodromy inflation model with non-geometric fluxes, the effect of backreaction from interactions with heavy moduli on the axion potential was considered in [55]. Our approach is to turn on a potential for a light axion, which stabilises the other moduli and is schematically of the form

$$W_{\text{inf}} = \lambda W + f_{\text{ax}}\, \Delta W\,. \tag{3.3.2}$$

Here, λ is a flux parameter which supposed to be large compared to f_{ax} to ensure the largeness of the first term compared to the second one. Note, that the second term can not be tuned small since f_{ax} is a quantised flux parameter and at least of $\mathcal{O}(1)$. Let us take a closer look at this procedure.

No-scale case

For no-scale vacua which are fixed by vanishing F-terms it is rather simple. The first term fixes all the saxions at

$$V = \lambda^2 V_{\text{mass}}(\sigma_\alpha) + f_{\text{ax}}^2 V_{\text{ax}}(\theta, \sigma_\alpha)\,, \tag{3.3.3}$$

whereas the mixed terms scaling as $\lambda\, f_{\text{ax}}$ vanish due to the minimum condition $D_I W_{\text{mass}} = 0$. After integrating out the heavy moduli by setting $\sigma_\alpha = \overline{\sigma}_\alpha$, the second term is an effective polynomial potential for θ. It is clear from (3.3.3) that for $\lambda \gg f_{\text{ax}}^2$, we get a mass hierarchy between the inflaton and the remaining moduli

$$\frac{m_\theta^2}{m_{\sigma_\alpha}^2} \sim \left(\frac{f_{\text{ax}}}{\lambda}\right)^2 . \tag{3.3.4}$$

The general case

In the general case, the potential is not no-scale and the mixed terms do not vanish in general. In general, the terms λW and $f_{\text{ax}} \Delta W$ have the same scaling, which leads to a destabilisation of the old vacuum. This effect is related to the presence of a linear term in the axion θ in the inflaton potential, whose relative prefactor is generically of order λW. The existence of a linear term depends on the choice of ΔW. An appropriate choice leads to a vanishing linear axion term in the scalar potential and an hierarchically light axion. We conclude that the backreaction is also under control in the AdS case.

3.3.2 Flux-scaling vacua

In the second part of this thesis, we will consider mainly *flux-scaling* vacua. These are vacua which can be computed easily and leave at least one axionic direction massless. Hence, flux-scaling vacua are a good starting point (as λW, i.e. the potential before turning on a term for the axion) for realising axion inflation. The basic properties of these vacua are the following:

- To stabilise N complex moduli, $N+1$ fluxes are turned on, such that all saxions are stabilised but there are axionic flat directions.

- The terms in the superpotential in general have the same overall scaling after plugging in the vacuum expectation values of the fields in the minimum.

- The vevs of the saxions can be controlled by an appropriate choice of fluxes and all masses scale have the same flux dependence, only differing by numerical factors. These masses are therefore not tunable.

As an example, assume a toy potential

$$W = i\,f + i\,h\,A + i\,m\,B\,. \tag{3.3.5}$$

To stabilise the two complex moduli A and B with their imaginary part being axions, three fluxes are turned on. Already at first sight it is clear that one axionic direction remains flat since the axionic field orthogonal to $\phi = h\,\mathrm{Im}(A) + m\,\mathrm{Im}(B)$ does not appear in the potential. For two complex moduli we turned on three fluxes, the saxions are stabilised such that the minimum of the superpotential scales like $W_0 \sim i\,f$, i.e. the real parts of A and B scale as f/h and f/m respectively. Those correspond for instance to the vevs of the volume moduli and the inverse string coupling and can be tuned large for a large flux f, which is the desirable regime. All non flat fields have the same flux-scaling of the mass, only differing by numbers.

Usually, these flux-scaling vacua have a stable non-supersymmetric minimum. Though we will see in Section 5 that a higher number of fields appearing in the potential can lead to tachyonic instabilities. Minima and masses are easily calculated without using numerics. Though they do not allow for a tuning of the stabilised fields, we have flat axionic directions which we can couple to small fluxes. In other words, we use flux-scaling vacua as the term $\lambda\,W$ in (3.3.2), as the first step in our procedure for moduli stabilisation for a light axion.

Part II

Applications

Chapter 4

No-scale Minkowski vacua in LCS

We take the simple flux dependent potential (3.1.17) as a starting point for realising axion monodromy inflation in type IIB. We focus on building models purely from this axio-dilaton and complex structure dependent potential, neglecting Kähler moduli at this point. The vacua we investigate are Minkowski and supersymmetry is broken.
After introducing the notation, we find several No Go theorems and constraints on the geometry if we leave an axionic direction flat and all saxions are stabilised. In the last part of this chapter, we discuss an example with a light axion which avoids the No Go and fulfills the requirements on the Calabi Yau geometry. This chapter is based on the publication [1].

4.1 Notation

We investigate standard type IIB supergravity (3.1.13) with NSNS and RR three-form fluxes. Recall the standard type IIB scalar potential which we use for moduli stabilisation

$$V = -\frac{1}{8\kappa_{10}^2 \mathrm{Re}(S)} \int_{\mathcal{X}} G_3 \wedge \star_6 \overline{G}_3 \tag{4.1.1}$$

with the three-form flux

$$G_3 = F_3 + i\, S\, H_3\,. \tag{4.1.2}$$

Note that this potential in the end, after integrating out the non-inflatonic fields, should lead to an inflaton potential which realises axion monodromy inflation, hence the name *F-term axion monodromy inflation.* Practically, we want to stabilise moduli such that we have an effective theory where this potential depends on one axionic inflaton and the remaining $(1+2\,h^{(2,1)})$ moduli fields are stabilised hierarchically heavy. For explicitly calculating the potential, recall its dependence on the period matrix (3.1.28)

$$V_F = -\frac{M_{\rm pl}^4}{4\pi}\,\frac{1}{\mathcal{V}^2\,{\rm Im}\,\tau}\,(e+m\overline{\mathcal{N}})({\rm Im}\mathcal{N})^{-1}(\overline{e}+\mathcal{N}\,\overline{m})\,, \tag{4.1.3}$$

and the fluxes (3.1.22)

$$\tfrac{1}{(2\pi)^2\alpha'}\,G_3 = e_\Lambda\beta^\Lambda + m^\Lambda\alpha_\Lambda\,, \qquad \begin{aligned} e_\Lambda &= -i\,S\,h_\Lambda + f_\Lambda\,,\\ m^\Lambda &= -i\,S\,\overline{h}^\Lambda + \overline{f}^\Lambda\,. \end{aligned} \tag{4.1.4}$$

The global minima of this potential are Minkowski and given by the equations

$$e_\Lambda + m^\Sigma\,\overline{\mathcal{N}}_{\Sigma\Lambda} = 0\,. \tag{4.1.5}$$

Note that we are interested in non-supersymmetric minima. The equations (4.1.5) correspond to a vanishing of the F-terms of the axio-dilaton F_S and the complex structure moduli $F_{\mathcal{U}^i}$. A supersymmetric minimum furthermore requires the condition $F_T = 0$ to be fulfilled. In the no-scale case, that leads to a vanishing of the superpotential in the minimum

$$F_T = \partial_T W + \partial_T K\,W = \partial_T K\,W = 0 \Rightarrow W_0 = 0\,. \tag{4.1.6}$$

Hence we focus on minima with the equations (4.1.5) being fulfilled and the superpotential at the minimum being non-zero. Now we can calculate the minima of the scalar potential in terms of the period matrix $\mathcal{N}$ or the prepotential F respectively. In the following, we will focus on the large complex structure regime, where the prepotential has a simple cubic form *

$$F = \frac{\kappa_{ijk}X^iX^jX^k}{X^0}\,, \tag{4.1.7}$$

where the triple intersection numbers of the mirror Calabi Yau manifold is κ_{ijk} with $i,j,k = 1,\ldots,h^{2,1}$. The complex structure moduli $\mathcal{U}^i \equiv v^i + i\,u^i$ are defined in terms of the periods via

$$\begin{aligned} X^0 &= 1\,, & F_0 &= -i\,\kappa_{ijk}\,\mathcal{U}^i\,\mathcal{U}^j\,\mathcal{U}^k\,,\\ X^i &= -i\,\mathcal{U}^i\,, & F_i &= -3\,\kappa_{ijk}\,\mathcal{U}^j\,\mathcal{U}^k\,. \end{aligned} \tag{4.1.8}$$

*For this to hold, all complex structure moduli have to be large. On the next page, we will shortly discuss why the quadratic and linear terms in X are neglected even in the simple large complex structure case.

The scalar potential can now be calculated explicitly in terms of the complex structure moduli $\mathcal{U}^i$ and the axio-dilaton S. In our analysis of vacua with massless axions, we assume the general form of (4.1.7), that means we do computations without specifying the triple intersection number κ_{ijk}.

Kähler metric

To check if solutions are in a physical regime, we recall the tree-level Kähler potential for the complex structure moduli, which reads

$$K_{\text{cs}} = -\log\left(-i\int_{\mathcal{X}} \Omega_3 \wedge \overline{\Omega}_3\right) = -\log\left(\kappa_{ijk} v^i\, v^j\, v^k\right). \tag{4.1.9}$$

The imaginary part of $\mathcal{U}^i$ does not appear in the Kähler potential and therefore obeys a continuous shift symmetry $u^i \to u^i + c^i$. These contribute to the axionic content in our scalar potential together with the universal axion $C_0 = \operatorname{Im} S$. The period matrix for a cubic prepotential (4.1.7) takes the following form in terms of the Kähler metric $G_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$

$$\begin{aligned} \operatorname{Im}\mathcal{N}_{ij} &= 4\,\kappa\, G_{i\bar{j}}\,, & \operatorname{Re}\mathcal{N}_{ij} &= 6\,\kappa_{ijk}\,u^k\,, \\ \operatorname{Im}\mathcal{N}_{i0} &= -4\,\kappa\, G_{i\bar{j}}\, u^j\,, & \operatorname{Re}\mathcal{N}_{i0} &= -3\,\kappa_{ijk}\,u^j u^k\,, \\ \operatorname{Im}\mathcal{N}_{00} &= \kappa\left(1 + 4\, G_{i\bar{j}}\, u^i u^j\right), & \operatorname{Re}\mathcal{N}_{00} &= 2\,\kappa_{ijk}\,u^i u^j u^k\,, \end{aligned} \tag{4.1.10}$$

where the Kähler metric computed from (4.1.9) reads

$$G_{i\bar{j}} = -\frac{3}{2}\,\frac{\kappa_{ij}}{\kappa} + \frac{9}{4}\,\frac{\kappa_i \kappa_j}{\kappa^2}\,, \tag{4.1.11}$$

and we have defined

$$\kappa = \kappa_{ijk}\, v^i v^j v^k\,, \qquad \kappa_i = \kappa_{ijk}\, v^j v^k\,, \qquad \kappa_{ij} = \kappa_{ijk}\, v^k\,. \tag{4.1.12}$$

Note that in the physical domain, besides the requirement $s > 0$ for the dilaton, the Kähler metric $G_{i\bar{j}}$ on the complex structure moduli space has to be positive definite.

Remark on the large complex structure limit

The prepotential in the large complex structure limit is given by

$$\widetilde{F} = F + \frac{1}{2} a_{ij} X^i X^j + b_i X^i X^0 + \frac{1}{2} c\left(X^0\right)^2 + F_{\text{inst.}}\,. \tag{4.1.13}$$

This is the general prepotential which is a harmonic function of the homogeneous coordinates of degree two. The first term is the cubic term we introduced previously. The last term are instanton contributions, which are generically small and hence negligible. The factors in front of the quadratic terms are given by

$$a_{ij} = -\frac{1}{2}\int_M h_i \wedge h_j \wedge h_j \,, \tag{4.1.14}$$

with h_i the harmonic $(1,1)$-forms. Furthermore we have

$$b_i = \frac{1}{24}\int_M c_2(M) \wedge h_i \tag{4.1.15}$$

and

$$c = i\,\gamma = \frac{1}{(2\pi i)^3}\chi(M)\zeta(3) \tag{4.1.16}$$

with $c_2(M)$ the second Chern-class and the Euler number $\chi(M)$ of the internal manifold. These quadratic terms can be incorporated into a redefinition of the fluxes.

$$\begin{aligned} \widetilde{h}_0 &= h_0 + b_i \overline{h}^i \,, & \widetilde{h}_i &= h_i + a_{ij}\overline{h}^j + b_i \overline{h}^0 \,, \\ \widetilde{f}_0 &= f_0 + b_i \overline{f}^i \,, & \widetilde{f}_i &= f_i + a_{ij}\overline{f}^j + b_i \overline{f}^0 \,. \end{aligned} \tag{4.1.17}$$

With these redefinitions, the cubic term is sufficient to describe the large complex structure limit as long as

$$\kappa_{ijk} v^i v^j v^k \gg \mathrm{Im}\, c \,. \tag{4.1.18}$$

4.2 No Go's for an axionic flat direction

We can better localise the regions in the landscape with a massless axion and all saxions stabilised by finding No Go theorems for some kind of axions and some kind of geometries respectively. First, we isolate some properties of global vacua with unstabilised axions of the no-scale scalar potential

$$V = M_{\rm pl}^4\, e^K \left[G^{i\bar{j}}\, D_i W D_{\bar{j}} \overline{W} + G^{\tau\bar{\tau}}\, D_\tau W D_{\bar{\tau}} \overline{W} \right] . \tag{4.2.1}$$

The indices of the first term run from $i, j = 1, \ldots, N$, with $N = h^{(2,1)}$. To shorten the notation we combine both terms in (4.2.1) and define $\mathcal{U}^0 = S$ with indices now running $I = 0, \ldots, N$. For later use, we define one axionic direction as

$$\theta = \sum_{I=0}^{N} a_I \, u^I \,, \qquad\qquad a_I = \text{const.}\,. \tag{4.2.2}$$

The axionic shift symmetry implies that the Kähler potential is independent of the axion and purely saxion dependent. The global Minkowski minimum equations then read

$$\partial_I W(\mathcal{U}) = -\partial_I K(v)\, W(\mathcal{U})\,. \tag{4.2.3}$$

The superpotential is independent of the axion since we want this direction to be flat. The holomorphicity of the superpotential then requires it to be also independent of the saxionic partner. Defining $\Theta = \rho + i\,\theta$, this condition reads

$$\partial_\Theta W \equiv 0\,. \tag{4.2.4}$$

Since we are interested in non-supersymmetric minima, which are given by solutions of (4.2.3) with $W|_{min} \neq 0$, this leads to the constraint

$$\partial_\rho K = 0\,. \tag{4.2.5}$$

The conditions (4.2.4) and (4.2.5) are used later to restrict the possible models.

4.2.1 Axion is the universal axion

In the following, we show that the proposal of [41], in which the universal axion $C_0 = c$ is the inflaton, is not possible in this setting. We demand that the minimum conditions (4.1.5) are independent of the universal axion $c = C_0$. These equations can be rewritten as

$$\begin{aligned} c\left(h_\Lambda + \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}\,\overline{h}^{\Sigma}\right) + s\left(\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}\,\overline{h}^{\Sigma}\right) + \left(f_\Lambda + \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}\,\overline{f}^{\Sigma}\right) &= 0\,, \\ -c\left(\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}\,\overline{h}^{\Sigma}\right) + s\left(h_\Lambda + \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}\,\overline{h}^{\Sigma}\right) - \left(\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}\,\overline{f}^{\Sigma}\right) &= 0\,. \end{aligned} \tag{4.2.6}$$

Keeping c a flat direction implies also that the dilaton is unconstrained because both fields couple to the same terms. This then leads to the equations

$$\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}\,\overline{h}^{\Sigma} = 0\,, \qquad\qquad \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}\,\overline{f}^{\Sigma} = 0\,. \tag{4.2.7}$$

These constraints can only be fulfilled for vanishing fluxes. Hence, the universal axion cannot be a flat direction for no-scale Minkowski vacua with three-form flux in type IIB if any other fields are stabilised.

4.2.2 Axion involves the universal axion

Now we consider the case that the universal axion is a part of the axionic linear combination which we want to keep flat. The axion takes the form $\theta = c + u^N$. Applying the condition (4.2.4) on a general superpotential which is given by

$$W = (f_0 + i\,S h_0) + (f_i + i\,S h_i)\mathcal{U}^i + 3(\overline{f}^i + i\,S\overline{h}^i)\,\kappa_{ijk}\,\mathcal{U}^j\mathcal{U}^k - (\overline{f}^0 + i\,S\overline{h}^0)\kappa_{ijk}\,\mathcal{U}^i\mathcal{U}^j\mathcal{U}^k\,, \tag{4.2.8}$$

gives the flux constraints

$$\begin{aligned} 0 &= h_N\,, & 0 &= h_0 + f_N\,, \\ 0 &= \overline{h}^0\,, & 0 &= \kappa_{Nij}\,\overline{h}^j\,, \\ 0 &= h_i + 6\kappa_{Nij}\,\overline{f}^j\,, & 0 &= \kappa_{ijk}\overline{h}^k - \kappa_{Nij}\overline{f}^0\,, \end{aligned} \tag{4.2.9}$$

where $i,j \in \{1,\dots,N\}$ and κ_{ijk} the triple intersection number. Inserting these flux conditions into the minimum equations (4.1.5), we get

$$\begin{aligned} \mathcal{P}_0 &= (f_0 + ch_0) - \tfrac{1}{2}u^i\,\mathrm{Re}\mathcal{N}_{ij}\,(\overline{f}^j + c\overline{h}^j) - u^i\,\mathrm{Im}\mathcal{N}_{ij}\,s\overline{h}^j + \tfrac{1}{3}u^iu^j\mathrm{Re}\mathcal{N}_{ij}\overline{f}^0 = 0\,, \\ \mathcal{Q}_0 &= sh_0 - \tfrac{1}{2}u^i\,\mathrm{Re}\mathcal{N}_{ij}\,s\overline{h}^j + u^i\,\mathrm{Im}\mathcal{N}_{ij}\,(\overline{f}^j + c\overline{h}^j) - (\kappa + u^iu^j\mathrm{Im}\mathcal{N}_{ij})\overline{f}^0 = 0\,, \\ \mathcal{P}_i &= (f_i + ch_i) + \mathrm{Re}\mathcal{N}_{ij}\,(\overline{f}^j + c\overline{h}^j) + \mathrm{Im}\mathcal{N}_{ij}\,s\overline{h}^j - \tfrac{1}{2}u^j\,\mathrm{Re}\mathcal{N}_{ij}\overline{f}^0 = 0\,, \\ \mathcal{Q}_i &= sh_i + \mathrm{Re}\mathcal{N}_{ij}\,s\overline{h}^j - \mathrm{Im}\mathcal{N}_{ij}\,(\overline{f}^j + c\overline{h}^j) + u^j\,\mathrm{Im}\mathcal{N}_{ij}\overline{f}^0 = 0\,. \end{aligned} \tag{4.2.10}$$

We discuss these equations for various forms of the triple intersection number κ_{ijk}.

Case 1: $\kappa_{NNi} \neq 0$ for at least one $i \in \{1,\dots,N\}$

Then the constraints (4.2.9) imply $\overline{f}^0 = 0$ and $\kappa_{ijk}\overline{h}^k = 0$. The equations (4.2.10) then simplify to

$$\begin{aligned} \mathcal{P}_0 &= (f_0 + ch_0) - \tfrac{1}{2}u^i\,\mathrm{Re}\mathcal{N}_{ij}\,\overline{f}^j = 0\,, \\ \mathcal{Q}_0 &= sh_0 + u^i\,\mathrm{Im}\mathcal{N}_{ij}\,\overline{f}^j = 0\,, \\ \mathcal{P}_i &= (f_i + ch_i) + \mathrm{Re}\mathcal{N}_{ij}\,\overline{f}^j = 0\,, \\ \mathcal{Q}_i &= sh_i - \mathrm{Im}\mathcal{N}_{ij}\,\overline{f}^j = 0\,. \end{aligned} \tag{4.2.11}$$

Recall that $\mathrm{Re}\mathcal{N}_{ij}$ only depends on the axions, hence $\mathcal{P}_0$ and $\mathcal{P}_i$ do not stabilise any saxion. Of the remaining $N+1$ equations, we can form the combination

$$\mathcal{Q}_0 + \sum_i u^i\mathcal{Q}_i = s(h_0 + u^i\,h_i) = 0\,. \tag{4.2.12}$$

Since the dilaton is non-zero in the physical regime, there are only N saxion dependent equations left. Therefore, at least one of the $N+1$ saxionic directions remains unconstrained.

Case 2: $\kappa_{NNi} = 0$ **for all** $i \in \{1, \dots, N\}$

We consider $\overline{f}^0 \neq 0$, this corresponds to a prepotential linear in $\mathcal{U}^N$. The situation when $\overline{f}^0 = 0$ is already covered by the discussion in case 1. Now, using (4.2.9), the constraints $\mathcal{Q}_i$ can be rewritten as

$$\begin{aligned}\mathcal{Q}_i = -6\, s\, \kappa_{Nij}\big(\overline{f}^j - u^j \overline{f}^0\big) - \sum_{j=1}^{N-1} \mathrm{Im}\mathcal{N}_{ij}\big(\overline{f}^j - u^j \overline{f}^0\big) \\ - \mathrm{Im}\mathcal{N}_{iN}\big(\overline{f}^N - (u^N - c)\overline{f}^0\big) = 0\,.\end{aligned} \tag{4.2.13}$$

These N conditions fix the axions u^i and c as

$$u^N - c = \frac{\overline{f}^N}{\overline{f}^0}\,, \qquad\qquad u^i = \frac{\overline{f}^i}{\overline{f}^0} \quad \forall\, i \in \{1, \dots, N-1\}\,, \tag{4.2.14}$$

while, by construction, the axionic combination $\theta = c + u^N$ is unstabilised. Recall (4.2.5), i.e. $\partial_\Theta K = 0$. This constrains the dilaton to $s = -\kappa/\kappa_N$, with $\kappa_N = \kappa_{Nij} v^i v^j$ being independent of the saxion v^N. For the other conditions we get

$$\mathcal{P}_i = \frac{1}{\overline{f}^0}\left(\overline{f}^0 f_i + 3\kappa_{ijk}\overline{f}^j\overline{f}^k + s\left(\overline{f}^0\right)^2 \mathrm{Im}\mathcal{N}_{iN}\right) = 0\,, \tag{4.2.15}$$

where in this case the factor $s\,\mathrm{Im}\mathcal{N}_{iN}$ is independent of v^N. Furthermore, also the sum $\mathcal{P}_0 + u^i \mathcal{P}_i = 0$ does not depend on saxions and only gives a constraint for the fluxes. Finally, for the linear combination $\mathcal{Q}_0 + u^i \mathcal{Q}_i = 0$ we obtain after some rewriting

$$\frac{s}{\overline{f}^0}\left(h_0 \overline{f}^0 + h_i \overline{f}^i + 3\kappa_{Nij}\overline{f}^i\overline{f}^j + \left(\overline{f}^0\right)^2 \kappa_N\right) = 0\,. \tag{4.2.16}$$

The term in the brackets has to give zero, since the dilaton is constrained to $s > 0$ in the physical regime. But the terms does not dependent on the saxion v^N, which therefore remains a flat direction.

4.2.3 Axion is purely complex structure

In this case we will see that there is no general No Go theorem but rather a restriction on the form of the prepotential. Indeed, an axion which is massless and purely a combination of complex structure moduli is possible. The simplest cases of geometries admit No Go theorems. The flat axion can be brought to the form $\theta = u^N$. For the superpotential (4.2.8), the condition $\partial_\Theta W \equiv 0$ leads to the constraints

$$\begin{aligned} 0 &= f_N = h_N = 0\,, \qquad & 0 &= \overline{f}^0 = \overline{h}^0\,,\\ 0 &= \kappa_{Nij}\,\overline{f}^j\,, \qquad & 0 &= \kappa_{Nij}\,\overline{h}^j\,, \end{aligned} \tag{4.2.17}$$

with $i, j \in \{1, \ldots, N\}$. Then, the minimum conditions (4.1.5) can be written as

$$\begin{aligned} \mathcal{P}_0 &= (f_0 + ch_0) - \tfrac{1}{2}u^i\,\mathrm{Re}\mathcal{N}_{ij}\,(\overline{f}^j + c\overline{h}^j) - u^i\,\mathrm{Im}\mathcal{N}_{ij}\,s\overline{h}^j = 0\,,\\ \mathcal{Q}_0 &= sh_0 - \tfrac{1}{2}u^i\,\mathrm{Re}\mathcal{N}_{ij}\,s\overline{h}^j + u^i\,\mathrm{Im}\mathcal{N}_{ij}\,(\overline{f}^j + c\overline{h}^j) = 0\,,\\ \mathcal{P}_i &= (f_i + ch_i) + \mathrm{Re}\mathcal{N}_{ij}\,(\overline{f}^j + c\overline{h}^j) + \mathrm{Im}\mathcal{N}_{ij}\,s\overline{h}^j = 0\,,\\ \mathcal{Q}_i &= sh_i + \mathrm{Re}\mathcal{N}_{ij}\,s\overline{h}^j - \mathrm{Im}\mathcal{N}_{ij}\,(\overline{f}^j + c\overline{h}^j) = 0\,. \end{aligned} \tag{4.2.18}$$

The two latter equations (4.2.18) for $i = N$ are solved by fixing one saxionic direction as $\partial_\Theta K \sim \kappa_N/\kappa = 0$. We define the matrix

$$\mathcal{A}_{ij} = \kappa_{Nij}\,. \tag{4.2.19}$$

Then we can make statements depending on the rank of this matrix.

Rank of $\mathcal{A} = 0$ and rank of $\mathcal{A} = 1$

We diagonalise and relabel the matrix $\mathcal{A}$. Then, there exists at most one non vanishing eigenvalue $\mathcal{A}_{11} = \kappa_{N11} \neq 0$. Applying the condition $\partial_\Theta K = 0$ leads to a vanishing of the Kähler metric $G_{Ni} = 0$ for all $i = 1, \ldots N$. This means the saxions are stabilised on the boundary of the physical regime.

Rank of $\mathcal{A} = N - 1$

In this case, the kernel of $\mathcal{A}$ is one dimensional. The flux constraints in the second line of (4.2.17) then lead to fluxes of the form $\overline{f}^i = a^i\overline{f}$ and $\overline{h}^i = a^i\overline{h}$ for the null vector a^i. We can rearrange the equations (4.2.18), to the linear combinations

$$s\overline{h}\,\mathcal{Q}_i + (\overline{f} + c\overline{h})\,\mathcal{P}_i = 0\,, \tag{4.2.20}$$

for each $i = \{1, \dots, N\}$. Explicitly writing this equation in terms of fields and fluxes, we obtain

$$0 = \kappa_{ijk} u^j a^k \Big[(\overline{f} + c\overline{h})^2 + s^2\overline{h}^2\Big] + \Big[(f_i + ch_i)(\overline{f} + c\overline{h}) + s^2 h_i \,\overline{h}\Big]\,. \tag{4.2.21}$$

These equations are trivially fulfilled for $i = N$ and are independent of the saxions v^i. We define the matrix

$$\mathcal{B}_{\mu\nu} = \kappa_{\mu\nu k} a^k\,, \qquad\qquad \mu, \nu \in \{1, \dots, N-1\}\,, \tag{4.2.22}$$

which we assume to be regular. Then (4.2.21) stabilises the $N-1$ axions u^μ as

$$u^\mu = -\big(\mathcal{B}^{-1}\big)^{\mu\nu} \frac{(f_\nu + ch_\nu)\big(\overline{f} + c\overline{h}\big) + s^2 h_\nu \overline{h}}{\big(\overline{f} + c\overline{h}\big)^2 + s^2\overline{h}^2}\,. \tag{4.2.23}$$

We build the linear combinations

$$\begin{aligned} \mathcal{P}_0 + \mathcal{P}_i u^i \; &= f_0 + f_\mu u^\mu + \mathcal{B}_{\mu\nu} u^\mu u^\nu \,\overline{f} \; = 0\,, \\ \mathcal{Q}_0 + \mathcal{Q}_i u^i \; &= h_0 + h_\mu u^\mu + \mathcal{B}_{\mu\nu} u^\mu u^\nu \,\overline{h} = 0\,, \end{aligned} \tag{4.2.24}$$

which we combine into

$$0 = \big(h_0\overline{f} - f_0\overline{h}\big) + \big(h_\mu\overline{f} - f_\mu\overline{h}\big)u^\mu\,, \tag{4.2.25}$$

$$0 = h_0 + h_\mu u^\mu + \mathcal{B}_{\mu\nu} u^\mu u^\nu \,\overline{h}\,. \tag{4.2.26}$$

We then have $N+1$ equations (4.2.23), (4.2.25) and (4.2.26), for $N+1$ fields $\{u^1, \dots, u^{N-1}, c, s\}$. We define

$$\mathsf{H} = h_\mu \big(\mathcal{B}^{-1}\big)^{\mu\nu} h_\nu\,, \qquad \mathsf{G} = h_\mu \big(\mathcal{B}^{-1}\big)^{\mu\nu} f_\nu\,, \qquad \mathsf{F} = f_\mu \big(\mathcal{B}^{-1}\big)^{\mu\nu} f_\nu\,, \tag{4.2.27}$$

which are independent of the moduli. Using these definitions in (4.2.25) and applying (4.2.23) we obtain for the dilaton vev

$$s^2 = \big(\overline{f} + c\overline{h}\big) \frac{\overline{f}\,(\mathsf{G} + c\mathsf{H}) - \overline{h}\,(\mathsf{F} + c\mathsf{H}) - \big(\overline{f} + c\overline{h}\big)\big(h_0\overline{f} - f_0\overline{h}\big)}{\big(h_0\overline{f} - f_0\overline{h}\big)\overline{h}^2 - \overline{h}\,\overline{f}\,\mathsf{H} + \overline{h}^2\,\mathsf{G}}\,. \tag{4.2.28}$$

Inserting (4.2.23) into (4.2.26) results in an expression only depending on s^2. Using also (4.2.28), we get the moduli independent expression

$$0 = \overline{h}\, \frac{\big(h_0\overline{f} - f_0\overline{h}\big)^2 + 2h_0\big(\overline{h}\mathsf{F} - \overline{f}\mathsf{G}\big) + 2f_0\big(\overline{f}\mathsf{H} - \overline{h}\mathsf{G}\big) + \mathsf{G}^2 - \mathsf{F}\mathsf{H}}{\overline{f}^2\mathsf{H} + \overline{h}^2\mathsf{F} - 2\overline{f}\,\overline{h}\,\mathsf{G}}\,. \tag{4.2.29}$$

So we end up with N equations for $N+1$ moduli $\{u^1, \dots, u^{N-1}, c, s\}$. Hence, one modulus remains unstabilised.

4.2.4 Summary of No Gos

Here, we shortly summarise the results of the previous section. Our analysis leads us to the following theorem.

> **Theorem**: The type IIB flux-induced no-scale scalar potential does not admit non-supersymmetric Minkowski minima, where a single linear combination of complex structure axions involving the universal axion c is unfixed while all remaining complex structure moduli and the axio-dilaton are stabilised inside the physical domain.

Models with a flat axion which is purely complex structure are restricted in the form of the prepotential.

> **Constraints on the geometry**: A pure unstabilised complex structure axion for a cubic prepotential is only possible if the rank of the triple intersection number in the axionic direction fulfills the inequality
>
> $$2 \leq \mathrm{rk}(\kappa_{Nij}) \leq h^{2,1} - 2\,. \tag{4.2.30}$$

This means we need to consider Calabi Yau manifolds with at least four complex structure moduli $h^{2,1} \geq 4$, to be able to have one axionic modulus unstabilised while the remaining fields receive a mass.

4.3 Example

The constraints on the prepotential are quite restrictive, nevertheless geometries with a single massless axion can be found. The simplest cases have $h^{(2,1)} = 4$. A prepotential which fulfills the requirements is for instance given by

$$F(X_0, X_1, X_2, X_3, X_4) = \frac{X_3^3 + X_1X_2X_3 + X_3X_4^2}{X_0}\,. \tag{4.3.1}$$

The flat axion is u_4 and the matrix $\mathcal{A}_{ij} = \kappa_{4ij}$ has rank two. We choose the following fluxes to be zero

$$f_4 = \overline{f}^0 = \overline{f}^3 = \overline{f}^4 = 0\,, \qquad h_4 = \overline{h}^0 = \overline{h}^3 = \overline{h}^4 = 0\,, \tag{4.3.2}$$

i.e. the superpotential is independent of the axion u_4. While for simplicity we set fixed values for the remaining fluxes. These are given by

$$\begin{aligned} h_0 &= 1\,, & h_3 &= 2\,, & f_0 &= 1\,, & f_3 &= 1\,,\\ h_1 &= -1\,, & \overline{h}^1 &= 1\,, & f_1 &= 1\,, & \overline{f}^1 &= 1\,,\\ h_2 &= 1\,, & \overline{h}^2 &= 1\,, & f_2 &= 4\,, & \overline{f}^2 &= -1\,. \end{aligned} \tag{4.3.3}$$

The condition $\partial_{\mathcal{U}^4} K = 0$ can be solved by $v^4 = 0$. The equations (4.2.18) can be solved iteratively for $\{u^1, u^2, u^3, c, s, v^1\}$ in terms of $\{v^2, v^3\}$ via the following relations

$$\begin{aligned} u_1 &= \frac{2s^2(v_1+v_2)v_3^2 + 2(1+c)((-1+c)v_1 + (1+c)v_2)v_3^2 - s(v_1v_2+v_3^2)}{2s(v_1v_2+v_3^2)}\,,\\ u_2 &= -\frac{2s^2(v_1+v_2)v_3^2 + 2(-1+c)((-1+c)v_1 + (1+c)v_2)v_3^2 + 3s(v_1v_2+v_3^2)}{2s(v_1v_2+v_3^2)}\,,\\ u_3 &= -\frac{sv_1v_2 + v_1^2v_3 - cv_1^2v_3 + sv_3^2 + v_3^3 + cv_3^3}{sv_1v_2 + sv_3^2}\,,\\ s &= \frac{8(-v_1v_2v_3^3 + v_3^5)}{3v_2^4 + v_2^2v_3^2 - 8v_3^4 + v_1^2(-3v_2^2 + 7v_3^2)}\,,\\ c &= \frac{2s(v_1v_2+v_3^2) + v_3(v_1^2 + v_2^2 + 2v_3^2)}{(v_1^2 - v_2^2)v_3}\,,\\ v_1 &= \frac{-9v_2^4v_3^2 + 49v_3^6 + 16v_3^8}{v_2(9v_2^4 - 49v_3^4 + 16v_3^6)}\,. \end{aligned} \tag{4.3.4}$$

Now we are left with the two relations for v^2 and v^3 which are higher order polynomials given by

$$\begin{aligned} f(v_2, v_3) = {}& 27v_2^8 - 72v_2^6v_3^2 + 294v_2^4v_3^4 - 784v_2^2v_3^6 + 48v_2^4v_3^6 + 343v_3^8\\ & - 32v_2^2v_3^8 + 112v_3^{10} = 0 \end{aligned} \tag{4.3.5}$$

and

$$\begin{aligned} g(v_2, v_3) = {}& -729v_2^{15} + 2430v_2^{13}v_3^2 - 891v_2^{12}v3^3 + 6237v_2^{11}v_3^4 + 1782v_2^{10}v_3^5\\ & - 26460v_2^9v_3^6 - 3240v_2^{11}v_3^6 + 8811v_2^8v_3^7 - 3087v_2^7v_3^8 + 7992v_2^9v_3^8\\ & - 19404v_2^6v_3^9 - 3168v_2^8v_3^9 + 72030v_2^5v_3^{10} + 15120v_2^7v_3^{10}\\ & - 16709v_2^4v_3^{11} + 6336v_2^6v_3^{11} - 50421v_2^3v_3^{12} - 39984v_2^5v_3^{12}\\ & - 4608v_2^7v_3^{12} + 52822v_2^2v_3^{13} + 7744v_2^4v_3^{13} + 13720v_2^3v_3^{14}\\ & + 7680v_2^5v_3^{14} - 26411v_3^{15} - 4928v_2^2v_3^{15} - 2816v_2^4v_3^{15}\\ & - 19208v_2v_3^{16} + 7168v_2^3v_3^{16} - 17248v_3^{17} + 5632v_2^2v_3^{17}\\ & - 2048v_2^3v_3^{18} - 2816v_3^{19} + 2048v_2v_3^{20} = 0\,. \end{aligned} \tag{4.3.6}$$

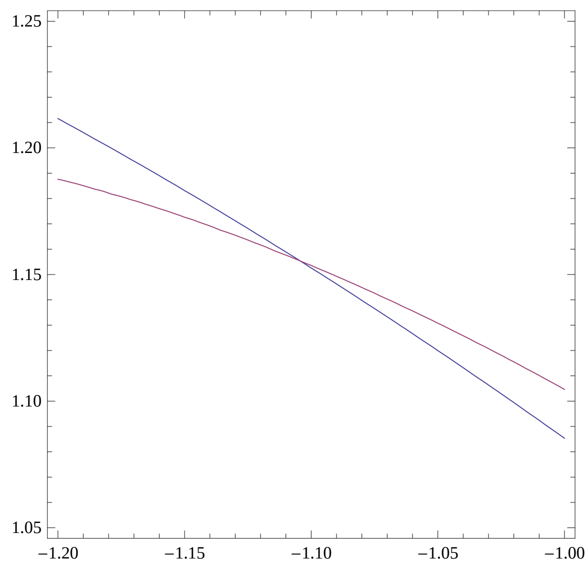

Figure 4.1: Contour plot of the vanishing locus of $f(v_2, v_3) = 0$ (in blue) and $g(v_2, v_3) = 0$ (in red), showing a common zero at $(v_2, v_3) = (-1.104, 1.155)$.

The solution of these two equations are depicted by the plot 4.1, where at the intersection point both equations are fulfilled. Inserting this solution into (4.3.4), the vevs of the fields are given by

$$\begin{aligned} v^1 &= 3.775\,, & u^1 &= 0.492\,, & s &= 0.932\,, \\ v^2 &= -1.104\,, & u^2 &= -0.371\,, & c &= 1.041\,, \\ v^3 &= 1.155\,, & u^3 &= -0.065\,, \\ v^4 &= 0\,, \end{aligned} \tag{4.3.7}$$

and lie in the physical regime with a positive definite Kähler metric. All fields, except for the unstabilised axion, have a mass. It is more involved to check if this solution really is in the large complex structure regime and we leave this task open here.

An axion potential

An axion potential is generated by turning on a flux which couples to the axion, for instance let us choose $f_4 \neq 0$. The inflaton potential we get then is quadratic

$$V_{\text{eff}}(\theta) = \frac{M_{\text{pl}}^4}{4\pi\mathcal{V}^2}\left(f_4\right)^2\left(a + \frac{b}{c}\theta^2\right), \tag{4.3.8}$$

with $a = 0.46$ and $b = 0.33$. After performing a canonical normalisation, we get $c = 1.41$. Flux quantisation forbids to have $|f_4| < 1$, so we have the minimal axion mass for $f_4 = 1$. This example leaves us with an axion mass which is two orders of magnitude too heavy for a large volume $\mathcal{V} \sim 500$. Nevertheless, the example is a proof of principle that a hierarchically light axion with a potential suitable for inflation can be generated in no-scale flux vacua with three-form fluxes in type IIB string theory.

Kähler moduli stabilisation

Investigating the conditions for massless axions, we saw that indeed there are geometries where we can leave an axionic direction flat and later turn on small fluxes to stabilise the axion and give it a small mass compared to the axio-dilaton and the remaining complex structure moduli. Realistic Calabi Yau manifold necessarily have a non-zero number of Kähler moduli. Unfortunately, Kähler moduli do not appear in the Gukov-Vafa-Witten superpotential and therefore are not stabilised at tree-level. In string theory, Kähler moduli are usually stabilised by corrections, e.g. string loop and α'-corrections to the Kähler potential [56, 57] or/and non-perturbative corrections to the superpotential.
The nature of correction terms is their smallness compared to the tree-level potential. This smallness also shows up in the masses of the volume moduli, which are small compared to the fields stabilised by the tree-level potential. This turns out to be problematic since the light axionic inflaton potential is produced by the tree-level superpotential. As an example, the masses of the volume moduli in the LARGE volume scenario are given by

$$\begin{aligned} M_{\tau_b}^2 &\sim O(1)\frac{M_{\text{pl}}^2}{\mathcal{V}^3}\,, \\ M_{\tau_s}^2 &\sim O(1)\frac{M_{\text{pl}}^2}{\mathcal{V}^2}\,, \end{aligned} \tag{4.3.9}$$

whereas the axion has mass $M_\theta \sim O(1)M_{\text{pl}}^2/\mathcal{V}^2$. Since the volume has to be large, one Kähler modulus will spoil single field axion inflation.
The hierarchy between the inflaton and the remaining fields is generated by choosing a large value for the flux terms of the latter moduli, whereas the term stabilising the inflaton can not be chosen arbitrarily small. We conclude that due to the quantisation of fluxes, Kähler moduli which are not stabilised at tree-level will spoil single field inflation due to their lightness.

A note on a different approach to control backreaction

Throughout this thesis, the mass hierarchy is generated by turning on a comparably large flux for the non-inflatonic fields while the axion couples to a small flux of order one. A slightly different approach for realising axion monodromy inflation in type IIB string theory no-scale vacua was considered in [58]. There, the mass hierarchy was generated by tuning the inflaton dependent term small. This is possible since the factors in front of the axion ϕ also dependent on the other moduli z. Schematically the superpotential can be written as

$$W(z,\phi) = W_0(z) + a(z)\,\phi + b(z)\,\phi^2 + \dots\,, \tag{4.3.10}$$

where z are the non-inflatonic complex structure moduli. The terms $a(z)$, $b(z)$ and the higher order terms are tuned to be very small. It was investigated if such a tuning is possible and we get a light axion. The conditions hereby were

1. The coefficients in front of the axion dependent term are small
2. The vev of the axio-dilaton is in the perturbative regime
3. All saxions are stabilised

For Calabi Yau three-folds in the large complex structure regime, the result was negative. A No Go was found, saying that it is not possible at the same time to stabilise the saxions, be in the perturbative regime and tune the coefficients in front of the inflaton ϕ small at the same time. That this is possible for Calabi Yau four-folds was not excluded but is still poorly understood.

4.4 Summary

This approach for realising the moduli stabilisation scheme behind F-term axion monodromy inflation looks at the no-scale scalar potential in type IIB supergravity. The goal was to keep one axionic direction flat while stabilising the remaining fields. There exist No Go theorems for an inflaton which contains the universal axion C_0 Hence the axion has to be a linear combination of only complex structure moduli. It furthermore turned out that there exist restrictions to the form of the prepotential of the Calabi Yau manifold, which constrain the minimal number $h^{(2,1)}$ of complex structure moduli to four. This makes it more difficult to calculate the minimum, nevertheless we proved that such a minimum with a light axion and an axionic inflaton potential in principle exist. So from a pure GVW potential point of view, we can have a potential for an hierarchically light axion realising F-term axion monodromy inflation with the type IIB no-scale scalar potential.

Unfortunately, Calabi Yau manifolds always contain volume moduli. The inclusion of Kähler moduli spoils single field inflation. Avoiding this problem requires tree-level Kähler moduli stabilisation, which in type IIB is only possible in the presence of non-geometric fluxes, which are not on stable string theoretic ground. We discuss tree-level Kähler moduli stabilisation with non-geometric fluxes in the next chapter.

Chapter 5

Non-geometric AdS/dS vacua in LCS

In this chapter we consider axion monodromy inflation with non-geometric fluxes. Unlike in the previous no-scale Minkowski case, we stabilise Kähler moduli at tree-level. Then their masses are of the same order as the remaining moduli masses.
We start this chapter by giving an introduction to non-geometric fluxes and double field theory, which is the framework to capture their physics. Then, moduli stabilisation examples are discussed which leave an axionic direction flat and are starting points for building models of axion monodromy inflation. These vacua are of flux-scaling type. We discuss their phenomenological implications. Then, a term is added to the F-term scalar potential which uplifts the models to de Sitter. For one of this cases we discuss an axion monodromy inflation model. The results of this chapter were published in [2] and [3] and summarised in the proceedings [59].

5.1 Non-geometric fluxes

5.1.1 Motivation

One of the main features of string theory is T-duality. A theory compactified on a circle with radius R can lead to the same physics as another theory compactified on a circle with radius proportional to $\frac{1}{R}$. There are two pairs of T-dual string theories. The two heterotic string theories transform into the eachother by T-duality. Furthermore, type IIA is T-dual to type IIB string theory. One of the effects of T-duality (which is closely related to mirror symmetry) is an interchange of Kähler and complex structure moduli. This duality should become obvious for instance in the scalar potential of type II theories. We consider here the example of an isotropic six-torus, i.e. the three Kähler and complex structure moduli are identical $T_1 = T_2 = T_3$ and $U_1 = U_2 = U_3$. Furthermore, we choose isotropic fluxes $f_1 = f_2 = f_3, h_1 = h_2 = h_3$. In type IIB, the GVW superpotential on such an isotropic six-torus is given by

$$\begin{aligned} W_{IIB} &= \int_{T^6} (F_3 - i\, S\, H_3) \wedge \Omega \\ &= f_0 + 3\, i\, f\, U - 3\, \tilde{f}\, U^2 + i\, f^0\, U^3 \\ &+ S(i\, h_0 + 3\, i\, h\, U - 3\, \tilde{h}\, U^2 + i\, h^0\, U^3)\,. \end{aligned} \tag{5.1.1}$$

T-duality on a torus interchanges complex structure moduli with Kähler moduli as well as odd RR fluxes with even RR fluxes, i.e. type IIB with type IIA. For comparison, let us take a look at the standard type IIA potential on a six-torus. It is given by the RR-part

$$W_{IIA,RR} = \int_{T^6} e^{J_c} \wedge F \tag{5.1.2}$$

with F the RR fluxes F_0, F_2, F_4 anf F_6 and the complex Kähler-form J_c. The NSNS part of the superpotential reads

$$W_{IIA,NSNS} = \int_{T^6} \Omega \wedge (H_3 + \omega\, J_c)\,, \tag{5.1.3}$$

with ω being a geometric NSNS-flux defined in terms of the vielbeins as $\omega_{ab}{}^c = e_i{}^c\Big(\partial_a e_b{}^i - \partial_b e_a{}^i\Big)$. When we add up these potentials, we get the type IIA superpotential on the isotropic torus

$$W_{IIA} = S\,(i\, h_0 - 3\, i\, h\, T) - U\,(3\, i\, h' + \omega'\, T) + f_0 + 3\, i\, f\, T - 3\, \tilde{h}\, T^2 + i\, f^0\, T^3\,. \tag{5.1.4}$$

We named the even RR fluxes and geometric fluxes like in the type IIB example to make comparison easier. Obviously, W_{IIA} and W_{IIB} are not identical under the exchange of Kähler and complex structure moduli $T \leftrightarrow U$ as it was expected from T-duality. To have fully dual potentials, extra fluxes are introduced, so-called non-geometric Q and R-fluxes [60, 61]. They arise by performing T-duality transformations on the geometric NSNS-fluxes:

$$H_{ijk} \xleftrightarrow{T_k} F_{ij}{}^k \xleftrightarrow{T_j} Q_i{}^{jk} \xleftrightarrow{T_i} R^{ijk}\,. \tag{5.1.5}$$

In the presence of all these fluxes, we get scalar potentials for the NSNS-part of type II theories which are dual under exchange of h^{11} and h^{21} moduli. T-duality transformations mix the metric with the Kalb-Ramond field, which leads to a change of geometry. For instance, H-flux on a torus is T-dual to a twisted torus without H-flux. The twisting is captured by *geometric* flux. The new Q- and R-fluxes are called *non-geometric* because they do not live on geometrically understood objects.

5.1.2 Double field theory

The physics behind non-geometric fluxes is not clear from (5.1.5). The best way to capture their physical nature is *double field theory* [62–66], for a pedagogical review see [67]. Double field theory consist not only of the usual ten dimensions but has a double tangent space and doubled coordinates. The T-duality of type II theories is a manifest symmetry in double field theory. The number of compact space dimensions is doubled. The extra coordinates are *winding coordinates* and symbolise the T-duality between a string with momentum p and a string winded around an extra dimension. T-duality transformations are incorporated in the symmetry group $O(D, D)$ of double field theory. The NSNS-part of the action leads to a non-geometric flux-dependent potential and looks as follows in the flux formulation of the DFT action, which is motivated by the scalar potential of gauged supergravity [66, 68, 69]

$$\begin{aligned} S_{\text{DFT}} = \frac{1}{2} \int d^{20}X \, e^{-2d} \Big[& \mathcal{F}_{ABC}\mathcal{F}_{A'B'C'} \Big(\frac{1}{4} S^{AA'} \eta^{BB'} \eta^{CC'} \\ & - \frac{1}{12} S^{AA'} S^{BB'} S^{CC'} - \frac{1}{6} \eta^{AA'} \eta^{BB'} \eta^{CC'} \Big) \\ & + \mathcal{F}_A \mathcal{F}_{A'} \Big(\eta^{AA'} - S^{AA'} \Big) \Big] . \end{aligned} \tag{5.1.6}$$

Here η_{IJ} is the $O(D, D)$ invariant metric

$$\eta_{IJ} = \begin{pmatrix} 0 & \delta^i{}_j \\ \delta_i{}^j & 0 \end{pmatrix} . \tag{5.1.7}$$

and the metric S_{AB} and the doubled vielbeins $E_A{}^I$ are given by

$$S_{AB} = \begin{pmatrix} s^{ab} & 0 \\ 0 & s_{ab} \end{pmatrix} , \qquad E_A{}^I = \begin{pmatrix} e_a{}^i & e_a{}^j b_{ji} \\ 0 & e^a{}_i \end{pmatrix} , \tag{5.1.8}$$

with s_{ab} the flat D-dimensional Minkowski metric. Capital indices should be read like $X^A = (X^a, X_a)$, i.e. having an ordinary and a winding component. $\mathcal{F}_A$ is a flux-component defined as

$$\mathcal{F}_A = \Omega^B{}_{BA} + 2E_A{}^I \partial_I d \tag{5.1.9}$$

with the generalised Weitzenböck connection defined as

$$\Omega_{ABC} = E_A{}^I \partial_I E_B{}^J E_{CJ}\,, \tag{5.1.10}$$

The flux F_{ABC} reads in components

$$\begin{aligned} \mathcal{F}_{abc} &= e_a{}^i\, e_b{}^j\, e_c{}^k\, \mathfrak{H}_{ijk}\,, & \mathcal{F}^a{}_{bc} &= e^a{}_i\, e_b{}^j\, e_c{}^k\, \mathfrak{F}^i{}_{jk}\,, \\ \mathcal{F}_c{}^{ab} &= e^a{}_i\, e^b{}_j\, e_c{}^k\, \mathfrak{Q}_k{}^{ij}\,, & \mathcal{F}^{abc} &= e^a{}_i\, e^b{}_j\, e^c{}_k\, \mathfrak{R}^{ijk}\,, \end{aligned} \tag{5.1.11}$$

i.e. contains the H-flux, geometric flux and the two types of non-geometric fluxes. The curly fluxes correspond to flux orbits given by (here the bivector β^{ab} dual to B_{ab} is vanishing).

$$\begin{aligned} \mathfrak{H}_{ijk} &= \overline{H}_{ijk} + 3\,\overline{F}^m{}_{[\underline{ij}}\, B_{m\underline{k}]} + 3\,\overline{Q}_{[\underline{i}}{}^{mn} B_{m\underline{j}}\, B_{n\underline{k}]} + \overline{R}^{mnp} B_{m[\underline{i}} B_{n\underline{j}} B_{p\underline{k}]} \\ \mathfrak{F}^i{}_{jk} &= \overline{F}^i{}_{jk} + 2\,\overline{Q}_{[\underline{j}}{}^{mi} B_{m\underline{k}]} + \overline{R}^{mni} B_{m[\underline{j}} B_{n\underline{k}]} \\ \mathfrak{Q}_k{}^{ij} &= \overline{Q}_k{}^{ij} + \overline{R}^{mij} B_{mk} \\ \mathfrak{R}^{ijk} &= \overline{R}^{ijk}\,. \end{aligned} \tag{5.1.12}$$

The background fluxes, which are overlined above, can be described in terms of the fields in double field theory, namely the vielbeins, the Kalb-Ramond field B_{ab} and its dual bivector β^{ab}. Schematically, they read

$$\begin{aligned} \overline{H}_{ijk} &\sim \partial_i B_{jk} \\ \overline{F}^i{}_{jk} &\sim e_j{}^i \partial_m e_k{}^m + \tilde{\partial}^i B_{jk} \\ \overline{Q}_k{}^{ij} &\sim \tilde{e}^j{}_i \partial_k \tilde{e}^k{}_m + \partial_k \beta^{ij} \\ \overline{R}^{ijk} &\sim \tilde{\partial}^i \beta^{jk}\,. \end{aligned} \tag{5.1.13}$$

Hence, the action of double field theory contains the full chain of fluxes (5.1.5).

Strong constraint

Double field theory contains more degrees of freedom than allowed by string theory. For being a physically trustworthy theory (and a solution to the string equations of motion), a constraint has to reduce the number of degrees of freedom to the usual one in supergravity. The strongest version of such a constraint is given by

$$\partial_i A\, \tilde{\partial}^i B + \tilde{\partial}^i A\, \partial_i B = 0\,. \tag{5.1.14}$$

and called *strong constraint*. It states that all the fields depend either on a normal coordinate or its dual. Weaker forms of this constraint exist but they still reduce the possibility to turn on all the fluxes (5.1.5) at the same time tremendously. The potential on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ suggested in [60] can be recovered from (5.1.6) by first assuming constant background values for all types of fluxes are turned on and then taking the strong constraint such that the winding dependence vanishes. Background values for fluxes (5.1.13) have to be generated by vielbeins, dual vielbeins, the Kalb-Ramond field and its dual bivector β. The strong constraint kills for instance the dependence on the dual vielbeins, whereas the condition that either B_{ab} or β^{ab} are turned on kills further fluxes.

Status of non-geometric fluxes

The conditions which need to be satisfied to make double field theory a physical theory usually destroy all the possibilities to get all kinds of geometric as well as non-geometric fluxes at the same time. So, from a stringy point of view it is not justified to turn on a potential of the general form we will do in this chapter. It is an open question how to implement such a potential in a physical theory. Simple cases were already investigated [70] but a fully understood and convincing model with different types of fluxes is still lacking. Nevertheless, to avoid the problem of light Kähler moduli, non-geometric fluxes are the only possibility we know. The geometrical nature of non-geometric compactifications is badly understood, we will treat them as perturbations on a standard Calabi Yau.

5.2 Notation

To describe the contribution of non-geometric fluxes to the superpotential, we stay in the four dimensional supergravity language. The theory we consider is a type IIB with O3- and O7-planes. The orientifold projection $\Omega_{\mathrm{P}}(-1)^{F_{\mathrm{L}}}\sigma$ contains, besides the world-sheet parity operator Ω_{P} and the left-moving fermion number

$F_{\rm L}$, a holomorphic involution $\sigma : \mathcal{M} \to \mathcal{M}$. We choose the latter to act on the Kähler form J and the holomorphic $(3,0)$-form Ω_3 of the Calabi-Yau three-fold $\mathcal{M}$ as

$$\sigma^* : J \to +J\,, \qquad \sigma^* : \Omega_3 \to -\Omega_3 \tag{5.2.1}$$

whereas the combined world-sheet parity and left-moving fermion number operator $\Omega_{\rm P}(-1)^{F_{\rm L}}$ acts on the ten-dimensional bosonic fields as

$$\Omega_{\rm P}(-1)^{F_{\rm L}} = \begin{cases} g,\, \phi,\, C_0,\, C_4 & \text{even}\,, \\ B_2,\, C_2 & \text{odd}\,, \end{cases} \tag{5.2.2}$$

For generalising the superpotential to Calabi Yau three-folds with non-geometric fluxes, we introduce the differential operator

$$\mathcal{D} = d - H\wedge\, - F\circ\, - Q\bullet\, - R\llcorner\,, \tag{5.2.3}$$

which has the following action on p-forms:

$$\begin{aligned} H\wedge \;&:\; p\text{-form} \to (p+3)\text{-form}\,, \\ F\circ \;&:\; p\text{-form} \to (p+1)\text{-form}\,, \\ Q\bullet \;&:\; p\text{-form} \to (p-1)\text{-form}\,, \\ R\llcorner \;&:\; p\text{-form} \to (p-3)\text{-form}\,. \end{aligned} \tag{5.2.4}$$

The operator (5.2.3) can be used to define geometric and non-geometric fluxes on Calabi Yau three-folds by its action on the symplectic basis via

$$\begin{aligned} \mathcal{D}\alpha_\Lambda &= q_\Lambda{}^A\omega_A + f_{\Lambda A}\tilde{\omega}^A\,, & \mathcal{D}\beta^\Lambda &= \tilde{q}^{\Lambda A}\omega_A + \tilde{f}^\Lambda{}_A\tilde{\omega}^A\,, \\ \mathcal{D}\omega_A &= \tilde{f}^\Lambda{}_A\alpha_\Lambda - f_{\Lambda A}\beta^\Lambda\,, & \mathcal{D}\tilde{\omega}^A &= -\tilde{q}^{\Lambda A}\alpha_\Lambda + q_\Lambda{}^A\beta^\Lambda\,. \end{aligned} \tag{5.2.5}$$

For a simplified notation, we define

$$\begin{aligned} f_{\Lambda 0} &= h_\Lambda\,, & \tilde{f}^\Lambda{}_0 &= \tilde{h}^\Lambda\,, \\ q_\Lambda{}^0 &= r_\Lambda\,, & \tilde{q}^{\Lambda 0} &= \tilde{r}^\Lambda\,. \end{aligned} \tag{5.2.6}$$

Using this definition, the Bianchi identities for the fluxes [60] are given by

$$\begin{aligned} 0 &= \tilde{q}^{\Lambda A}\tilde{f}^\Sigma{}_A - \tilde{f}^\Lambda{}_A\tilde{q}^{\Sigma A}\,, & 0 &= q_\Lambda{}^A f_{\Sigma A} - f_{\Lambda A}q_\Sigma{}^A\,, \\ 0 &= q_\Lambda{}^A\tilde{f}^\Sigma{}_A - f_{\Lambda A}\tilde{q}^{\Sigma A}\,, & 0 &= \tilde{f}^\Lambda{}_A q_\Lambda{}^B - f_{\Lambda A}\tilde{q}^{\Lambda B}\,. \\ 0 &= \tilde{f}^\Lambda{}_A f_{\Lambda B} - f_{\Lambda A}\tilde{f}^\Lambda{}_B\,, & 0 &= \tilde{q}^{\Lambda A}q_\Lambda{}^B - q_\Lambda{}^A\tilde{q}^{\Lambda B}\,. \end{aligned} \tag{5.2.7}$$

We note that the combined world-sheet parity and left-moving fermion-number transformation act on the fluxes as

$$\Omega_{\rm P}(-1)^{F_{\rm L}} : \begin{cases} \mathfrak{F} \to -\mathfrak{F} \,, \\ H \to -H \,, \\ F \to F \,, \\ Q \to -Q \,, \\ R \to R \,, \end{cases} \tag{5.2.8}$$

which leaves the following fluxes invariant under the orientifold projection:

$$\begin{array}{llllll} \mathfrak{F} & : & & & \mathfrak{f}_\lambda \,, & \tilde{\mathfrak{f}}^\lambda \,, \\ H & : & & & h_\lambda \,, & \tilde{h}^\lambda \,, \\ F & : & f_{\hat\lambda\alpha} \,, & \tilde{f}^{\hat\lambda}{}_\alpha \,, & f_{\lambda a} \,, & \tilde{f}^\lambda{}_a \,, \\ Q & : & q_{\hat\lambda}{}^a \,, & \tilde{q}^{\hat\lambda a} \,, & q_\lambda{}^\alpha \,, & \tilde{q}^{\lambda\alpha} \,, \\ R & : & r_{\hat\lambda} \,, & \tilde{r}^{\hat\lambda} \,. & & \end{array} \tag{5.2.9}$$

Now recall the superpotential

$$W = \int_{\mathcal{M}} \Big[\mathfrak{F} + d_H \Phi_{\rm c}^{\rm ev} \Big]_3 \wedge \Omega_3 \,, \tag{5.2.10}$$

with

$$\Phi_{\rm c}^{\rm ev} = i S - i G^a \omega_a - i T_\alpha \tilde{\omega}^\alpha \,. \tag{5.2.11}$$

For the inclusion of non-geometric fluxes we replace [60]

$$d_H \to \mathcal{D} \,. \tag{5.2.12}$$

Then, one can easily show that the superpotential for geometric and non-geometric fluxes in type IIB is given by

$$\begin{aligned} W = & - \big(\, \mathfrak{f}_\lambda \, X^\lambda - \, \tilde{\mathfrak{f}}^\lambda \, F_\lambda \big) \\ & + i S \, \big(\, h_\lambda \, X^\lambda - \, \tilde{h}^\lambda \, F_\lambda \big) \\ & - i G^a \big(f_{\lambda a} \, X^\lambda - \tilde{f}^\lambda{}_a \, F_\lambda \big) \\ & + i T_\alpha \big(q_\lambda{}^\alpha \, X^\lambda - \tilde{q}^{\lambda\alpha} \, F_\lambda \big) \,. \end{aligned} \tag{5.2.13}$$

Using this superpotential, we consider moduli stabilisation with non-geometric fluxes.

5.2.1 S-dual fluxes

Type IIB string theory is S-dual to itsself. This means it is invariant under $SL(2,\mathbb{Z})$ transformations. The axio-dilaton and the Ramond-Ramond two-form as well as the Kalb-Ramond two-form transform as

$$S \to \frac{aS - ib}{icS + d}\,, \qquad \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} C_2 \\ B_2 \end{pmatrix}, \tag{5.2.14}$$

under $SL(2,\mathbb{Z})$ with $ad - bc = 1$. This leads to Kähler and superpotential transformations of the form

$$K \to K + \log\left(\left|icS + d\right|^2\right) \qquad \Longrightarrow \qquad W \to \frac{1}{icS + d} W\,. \tag{5.2.15}$$

The orientifold odd G-moduli and the complex Kähler moduli transform as

$$G^a \to \frac{1}{icS + d} G^a\,, \qquad T_\alpha \to T_\alpha + \frac{i}{2} \frac{c}{icS + d} \kappa_{\alpha bc} G^b G^c\,. \tag{5.2.16}$$

We want the scalar potential of type IIB to be invariant under S-duality transformations after turning on non-geometric fluxes. To get a scalar potential which includes Q-flux and is S-duality invariant, we have to introduce a new type of fluxes, so-called S-dual P-fluxes. They act on a p-form similar to a Q-flux

$$P\bullet \;:\; \; p\text{-form} \to (p-1)\text{-form}\,, \tag{5.2.17}$$

Q- and P-fluxes transform in an $SL(2,\mathbb{Z})$ doublet

$$\begin{pmatrix} Q \\ P \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} Q \\ P \end{pmatrix}. \tag{5.2.18}$$

In terms of the basis, S-dual fluxes are given by

$$\begin{aligned} -P\bullet \alpha_\Lambda &= p_\Lambda{}^A \omega_A\,, & -P\bullet \beta^\Lambda &= \tilde{p}^{\Lambda A}\omega_A\,, \\ -P\bullet \omega_A &= 0\,, & -P\bullet \tilde{\omega}^A &= -\tilde{p}^{\Lambda A}\alpha_\Lambda + p_\Lambda{}^A \beta^\Lambda\,. \end{aligned} \tag{5.2.19}$$

Using (5.2.15), we see that the superpotential has to be given by

$$\begin{aligned} W^{(3)} = \int_{\mathcal{M}} \Big[\quad & \mathfrak{F} - iSH \\ & + iG^a\left(F \circ \omega_a\right) \\ & + iT_\alpha\left(\left[Q - iSP\right]\bullet \tilde{\omega}^\alpha\right) + \frac{1}{2}\kappa_{\alpha bc} G^b G^c \left(P\bullet \tilde{\omega}^\alpha\right) \quad \Big]_3 \wedge \Omega_3\,. \end{aligned} \tag{5.2.20}$$

In components, this reads

$$W^{(3)} = W^{(2)} + \Big(S\,T_\alpha + \frac{1}{2}\kappa_{\alpha bc}G^bG^c\Big)\,(\,p_\lambda{}^\alpha X^\lambda - \tilde{p}^{\lambda\alpha}F_\lambda)\,. \tag{5.2.21}$$

Later, we will also turn on such an S-dual flux component for realising axion monodromy inflation.

5.2.2 Tadpoles and Freed-Witten anomalies

The fluxes do not only have to fulfill the Bianchi identities (5.2.7) but also tadpole and Freed-Witten anomaly cancellation conditions.

Tadpole contributions

Recall the Dp-brane charge contribution to the tadpole condition (3.1.19)

$$N^{\text{flux}}_{\text{D}p} + \sum_{\substack{\text{D-branes}\\ \text{O-planes}}\ i} Q^{(i)}_{\text{D}p} = 0\,, \tag{5.2.22}$$

which has to be cancelled. A variation of the action with respect to the Ramond-Ramond forms gives us the flux contribution to the tadpoles (in the democratic formulation [71]). We get [72]

$$\delta_{C_p}\,\mathcal{S}_{\text{IIB}} = \frac{1}{2\kappa_{10}^2}\int_{\mathbb{R}^{3,1}\times\mathcal{M}} \frac{(-1)^{\frac{p}{2}}}{2}\,\delta C_p \wedge \Big[\,d_H\,\widetilde{\mathfrak{F}}\,\Big]_{10-p}\,, \tag{5.2.23}$$

with the generalised Ramond-Ramond field strength $\widetilde{\mathfrak{F}}_p = d\,C_{p-1} - H_3 \wedge C_{p-3}$. This is generalised to non-geometric fluxes by replacing the operator $d_H \to \mathcal{D}$. The corresponding Chern Simons terms are schematically given by

$$S_{CS} \sim -\int C^{(4)} \wedge F_3 \wedge H_3 + \int C^{(8)} \wedge Q\cdot F_3 + \int C^{(6)} \wedge F \circ F_3\,. \tag{5.2.24}$$

As a result, the tadpole contributions of three-form fluxes to D3-branes, of geometric fluxes to D5-branes and of non-geometric Q-fluxes to D7-brane tadpoles are

$$\begin{aligned} N^{\text{flux}}_{\text{D3}} &= -\,\mathfrak{f}_\lambda\tilde{h}^\lambda \;+ \tilde{\mathfrak{f}}^\lambda h_\lambda\,,\\ \big[N^{\text{flux}}_{\text{D5}}\big]_a &= +\,\mathfrak{f}_\lambda\tilde{f}^\lambda{}_a - \tilde{\mathfrak{f}}^\lambda f_{\lambda a}\,,\\ \big[N^{\text{flux}}_{\text{D7}}\big]^\alpha &= -\,\mathfrak{f}_\lambda\tilde{q}^{\lambda\alpha} + \tilde{\mathfrak{f}}^\lambda q_\lambda{}^\alpha\,. \end{aligned} \tag{5.2.25}$$

When we consider moduli stabilisation, we always have to take care of fulfilling the tadpole cancellation conditions.

Freed-Witten anomalies

The Freed-Witten anomaly cancellation condition [73] ensures that a cycle supporting flux on which a D-brane is wrapped is free of anomalies. The flux $d\mathcal{F} = H$ for the two-form gauge field

$$\mathcal{F} = F_2 + B_2 \tag{5.2.26}$$

on the D-brane has to vanish $\int_{\Gamma_3} H = 0$ for every three-cycle Γ_3 in the D-brane world-volume. For a cycle with geometric flux wrapped by a D7-brane $\Sigma = \Sigma_+ + \Sigma_-$ the Freed Witten anomaly cancellation condition reads

$$\mathcal{D}[\Sigma] = m^a\Big(\tilde{f}^\lambda{}_a \alpha_\lambda - f_{\lambda a}\beta^\lambda\Big) + m^\alpha\Big(\tilde{f}^{\hat\lambda}{}_\alpha \alpha_{\hat\lambda} - f_{\hat\lambda\alpha}\beta^{\hat\lambda}\Big) = 0\,. \tag{5.2.27}$$

In components, this relation is given by

$$\begin{aligned} 0 &= m^a\,\tilde{f}^\lambda{}_a\,, & 0 &= m^\alpha\,\tilde{f}^{\hat\lambda}{}_\alpha\,, \\ 0 &= m^a\,f_{\lambda a}\,, & 0 &= m^\alpha\,f_{\hat\lambda\alpha}\,. \end{aligned} \tag{5.2.28}$$

In the presence of non-geometric fluxes, these conditions need to be generalised. Note that a $U(1)$ brane can result in a gauging of axionic shift symmetries. This leads to the so-called generalised Green-Schwarz mechanism, which plays an important role for canceling possible chiral gauge anomalies in four dimensions. The Chern-Simons terms of a D7-brane relevant for the Green-Schwarz mechanism are given by

$$\begin{aligned} \mathcal{S}_{\rm CS} \sim \;& \int_{\mathbb{R}^{3,1}\times\Sigma} C_6\wedge F_2 - \int_{\mathbb{R}^{3,1}\times\Sigma'} C_6\wedge F_2 \\ &+ \int_{\mathbb{R}^{3,1}\times\Sigma} C_4\wedge\mathcal{E}\wedge F_2 - \int_{\mathbb{R}^{3,1}\times\Sigma'} C_4\wedge\mathcal{E}'\wedge F_2 + \ldots\,, \end{aligned} \tag{5.2.29}$$

with the four-dimensional abelian gauge field F_2 and the internal background gauge field supported on a D7-brane ζ. The ellipsis indicate that there are additional terms in the Chern-Simons action, which are not important here. The RR-form C_6 can be expanded in terms of even and odd four-forms

$$C_6 = C_{2,\alpha}\tilde\omega^\alpha + C_{2,a}\tilde\omega^a + \ldots\,. \tag{5.2.30}$$

Performing a dimensional reduction of the first line in (5.2.29) to four dimensions, we obtain the Stückelberg mass terms

$$\mathcal{S}^{(1)}_{\rm CS} \sim \int_{\mathbb{R}^{3,1}} 2m^a\,C_{2,a}\wedge F_2\,. \tag{5.2.31}$$

Such a term implies a gauging of shift symmetries of the zero-forms dual to $C_{2,a}$ in four dimensions. Here, we have the relation

$$C_{2,a} \longleftrightarrow -c^a\,, \tag{5.2.32}$$

with the four-dimensional scalars c^a defined in (3.1.30). The gauging of the shift symmetry implies that the scalars transform under a $U(1)$ gauge transformation $A \to A + d\lambda$ as

$$c^a \to c^a + m^a\lambda\,. \tag{5.2.33}$$

A is an open string gauge field on the D-brane with corresponding field strength F_2. The flux induced superpotential also has to be gauge invariant. This leads to constraints on the fluxes. More concretely, in order for the superpotential (5.2.20) to be invariant under (infinitesimal) transformations (5.2.33), we have to impose

$$\begin{aligned} 0 &= m^a \tilde{f}^\lambda{}_a\,, & \qquad 0 &= \kappa_{\alpha bc}\, m^b\, q_\lambda{}^\alpha\,,\\ 0 &= m^a f_{\lambda a}\,, & \qquad 0 &= \kappa_{\alpha bc}\, m^b\, \tilde{q}^{\lambda\alpha}\,. \end{aligned} \tag{5.2.34}$$

Now we need to take a look at the second line in (5.2.29). For this, the RR-four-form is expanded as

$$C_4 = C_2^\alpha\,\omega_\alpha + C_2^a\,\omega_a + \ldots\,, \tag{5.2.35}$$

and the corresponding Stückelberg mass term is given by

$$\mathcal{S}^{(2)}_{\rm CS} \sim \int_{\mathbb{R}^{3,1}} 2\left(\kappa_{\alpha\beta\gamma} m^\beta e^\gamma + \kappa_{\alpha bc} m^b e^c\right) C_2^\alpha \wedge F_2\,. \tag{5.2.36}$$

In four dimensions, the two-forms C_2^α are dual to the scalars ρ_α, the axionic parts of the Kähler moduli T_α. Under open string $U(1)$ gauge transformatiosn $A + d\lambda$, the scalars read

$$\rho_\alpha \to \rho_\alpha + \left(\kappa_{\alpha\beta\gamma} m^\beta e^\gamma + \kappa_{\alpha bc} m^b e^c\right)\lambda\,. \tag{5.2.37}$$

Applying (5.2.34) and the gauge invariance of the general superpotential (5.2.13) leads to the Freed Witten anomaly cancellation conditions

$$0 = \kappa_{\alpha\beta\gamma}\, m^\beta\, e^\gamma\, q_\lambda{}^\alpha\,, \qquad 0 = \kappa_{\alpha\beta\gamma}\, m^\beta\, e^\gamma\, \tilde{q}^{\lambda\alpha}\,. \tag{5.2.38}$$

A D7-brane with vanishing gauge flux sitting on an orientifold even four-cycle automatically satisfies the Freed Witten conditions. Deforming the brane geometrically or turning on gauge flux gives non-trivial constraints. Then, chiral matter exists on the brane and the Kähler modulus corresponding to the wrapped four-cycle cannot be stabilised by non-geometric fluxes.

5.3 Flux-scaling AdS vacua

We now investigate moduli stabilisation with non-geometric fluxes. The vacua we consider are flux-scaling vacua, which were introduced in Chapter 3. First, we look at a simple toy example, subsequently we discuss more involved models, their supersymmetry breaking behaviour and a possible tachyon uplift by a D-term induced by a stack of D7-branes. Furthermore, we take a look at the dilute flux limit in the presence of non-geometric fluxes.

5.3.1 A simple flux-scaling model

We consider a toy manifold with $h^{2,1}_{-} = 0$ and $h^{1,1}_{+} = 1$, i.e. the only fields are the axio-dilaton and one volume modulus. Then the Kähler potential reads

$$K = -3\log(T+\overline{T}) - \log(S+\overline{S})\,. \tag{5.3.1}$$

We turn on some fluxes for the periods $X^0 = 1$ and $F_0 = i$ and get the superpotential

$$W = i\tilde{\mathfrak{f}} + ihS + iqT\,. \tag{5.3.2}$$

Already at this stage we see that only one axionic linear combination appears in the superpotential. The scalar potential arising from this superpotential is rather short:

$$V = \frac{M_{\rm Pl}^4}{4\pi\cdot 2^4}\left[\frac{(hs-\tilde{\mathfrak{f}})^2}{s\tau^3} - \frac{6hqs+2q\tilde{\mathfrak{f}}}{s\tau^2} - \frac{5q^2}{3s\tau} + \frac{1}{s\tau^3}\left(hc+q\rho\right)^2\right], \tag{5.3.3}$$

The axionic direction appearing in the scalar potential is defined as

$$\theta = hc + q\rho\,. \tag{5.3.4}$$

The axion θ receives a mass, whereas the orthogonal axionic direction stays flat. Table 5.3.1 shows the minima of the scalar potential (5.3.3). One of the nicest properties of what we call *flux-scaling vacua* is visible here. The vacuum expectation values of the saxions are controlled by fluxes. In this simple example $s \sim \frac{\tilde{\mathfrak{f}}}{h}$ and $\tau \sim \frac{\tilde{\mathfrak{f}}}{q}$, This means by choosing $\tilde{\mathfrak{f}} > h$ and $\tilde{\mathfrak{f}} > q$, we are in a perturbative large volume regime and gain control over corrections. The first vacuum is supersymmetric. Due to the No-go theorem [74] for massless axions, we have a tachyonic saxion satisfying the Breitenlohner-Freedman bound [75]. The theorem states that

solution	(s,τ,θ)	susy	tachyons	Λ
1	$(-\frac{\tilde{\mathfrak{f}}}{2h},-\frac{3\tilde{\mathfrak{f}}}{2q},0)$	yes	yes	AdS
2	$(\frac{\tilde{\mathfrak{f}}}{8h},\frac{3\tilde{\mathfrak{f}}}{8q},0)$	no	yes	AdS
3	$(-\frac{\tilde{\mathfrak{f}}}{h},-\frac{6\tilde{\mathfrak{f}}}{5q},0)$	no	no	AdS

Table 5.1: Extrema of the scalar potential (5.3.3) for the simplest model.

supersymmetric AdS vacua with a flat axion, have a tachyonic saxion when $W_0 \neq 0$ or a massless saxion when $W_0 = 0$. In all three vacua, the massive moduli have the same scaling with the fluxes. For the fully stable non-supersymmetric AdS vacuum the masses are

$$M^2_{\text{mod},i} = \mu_i \frac{hq^3}{\tilde{\mathfrak{f}}^2} \frac{M^2_{\text{Pl}}}{4\pi \cdot 2^4}\,, \tag{5.3.5}$$

with the numerical values

$$\mu_i \approx (6.2, 1.7\,; 3.4, 0)\,. \tag{5.3.6}$$

The latter two are the axionic directions. Therefore, we do not get a mass gap between axion and saxions by tuning fluxes. Instead, for realising axion inflation, we have to make sure that the so far flat axion gains a light mass.

The value of the cosmological constant in the minimum is negative $V \sim e^K|W|^2 \sim -\frac{hq^3}{\tilde{\mathfrak{f}}^2}$. This simple example leaves not enough freedom to turn on fluxes to stabilise the flat axion. We consider this in more evolved examples.

5.3.2 Inclusion of more moduli

The example we saw in the last section was clearly for a toy Calabi Yau manifold with only one volume modulus. Real Calabi Yau three-folds contain hundreds of moduli, which need to gain mass. In the following, we investigate toy manifolds with

- $h^{1,1}_+ = 1$, $h^{1,1}_- = 0$, $h^{2,1}_- = 1$
  One Kähler and one complex structure modulus. We find stable non-supersymmetric AdS minima.

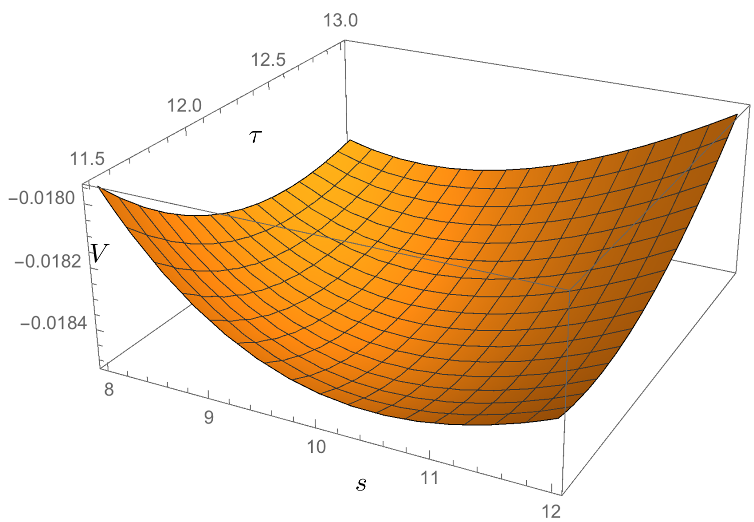

Figure 5.1: The scalar potential V in units of $\frac{M_{\rm Pl}^4}{4\pi\cdot 2^4}$ for $h=q=1$, $\tilde{\mathfrak{f}}=10$, showing the expected stable minimum at $s_0=10$ and $\tau_0=12$.

- $h^{1,1}_+=1$, $h^{1,1}_-=1$, $h^{2,1}_-=0$
  One Kähler and one G-modulus. We find stable non-supersymmetric AdS minima with a massless saxion.

- $h^{1,1}_+=2$, $h^{1,1}_-=0$, $h^{2,1}_-=0$
  Two Kähler moduli. We find non-supersymmetric AdS minima with a tachyon. The example we discuss is a K3-fibration. The results are analogous to a two Kähler moduli model with a swiss cheese which we investigated in [2]. There, the tachyon uplift is also analogous.

In this section, we consider always $h^{2,1}_+=0$. Later, we go on toy orientifolds with $h^{2,1}_+\neq 0$, this produces a D-term and allows for flux-scaling de Sitter vacua. To summarise the results of turning on more fields: the more scalar fields are present, the more likely it is to get tachyons or massless saxions. We shortly discuss three examples. These models include a complex structure modulus, an orientifold odd G-modulus and two Kähler moduli, respectively.

A common feature of the models is the existence of RR tadpoles due to the fluxes. Interestingly, in most examples of AdS vacua we find that $N_{\rm D3}^{\rm flux}$ and $N_{\rm D7}^{\rm flux}$ are negative, as it happens in related T-dual type IIA models [76, 77]. Thus, the flux tadpoles can be compensated by introducing D3- and D7-branes instead of O3- and O7-planes. Magnetised D7-branes that induce D3-charge are in principle allowed but they are constrained by cancellation of Freed-Witten anomalies.

A model with a complex structure modulus

As a first extension of the simplest model, we consider a manifold with $h^{2,1}_{-} = 1$ and $h^{1,1}_{+} = 1$. The Kähler potential is given by

$$K = -3\log(T+\overline{T}) - \log(S+\overline{S}) - 3\log\left(U+\overline{U}\right). \tag{5.3.7}$$

This model corresponds to the isotropic torus $T^6/\mathbb{Z}_2\times\mathbb{Z}_2$. We choose a superpotential of the form

$$W = -\mathfrak{f}_0 - 3\tilde{\mathfrak{f}}^1 U^2 - hUS - qUT\,, \tag{5.3.8}$$

where we used the notation $h := h_1$ and $q := q_1$. This flux-scaling superpotential only depends on the axion $(hc + q\rho)$, while the orthogonal direction remains flat. The scalar potential has two minima. The first one is the supersymmetric AdS minimum with the moduli stabilised as

$$\begin{aligned} &\tau = -18v\,\frac{\tilde{\mathfrak{f}}^1}{q}\,, && s = -6v\,\frac{\tilde{\mathfrak{f}}^1}{h}\,, && v^2 = \frac{1}{9}\,\frac{\mathfrak{f}_0}{\tilde{\mathfrak{f}}^1}\,, \\ &0 = hc + q\rho\,, && u = 0\,. \end{aligned} \tag{5.3.9}$$

As expected from [74], the flat axionic direction leads to a tachyonic saxion above the Breitenlohner-Freedman bound. The second extremum is a non-supersymmetric stable AdS minimum with the vevs for moduli given by

$$\begin{aligned} &\tau = -15v\,\frac{\tilde{\mathfrak{f}}^1}{q}\,, && s = -12v\,\frac{\tilde{\mathfrak{f}}^1}{h}\,, && v^2 = \frac{1}{3\cdot 10^{\frac{1}{2}}}\,\frac{\mathfrak{f}_0}{\tilde{\mathfrak{f}}^1}\,, \\ &0 = hc + q\rho\,, && u = 0\,. \end{aligned} \tag{5.3.10}$$

For the saxion to be physical, the fluxes have to be chosen to be $h, q < 0 < \mathfrak{f}_0, \tilde{\mathfrak{f}}^1$. The superpotential W scales as $\mathfrak{f}_0$. The contribution of the fluxes to the tadpoles are

$$N_{\mathrm{D3}}^{\mathrm{flux}} = \tilde{\mathfrak{f}}^1 h\,, \qquad N_{\mathrm{D5}}^{\mathrm{flux}} = 0\,, \qquad N_{\mathrm{D7}}^{\mathrm{flux}} = \tilde{\mathfrak{f}}^1 q\,. \tag{5.3.11}$$

Note that the flux $\mathfrak{f}_0$ does not contribute to any of the tadpoles. Therefore, by scaling $\mathfrak{f}_0 \gg \tilde{\mathfrak{f}}^1, h, q \sim O(1)$, we can ensure that all moduli are fixed in the perturbative regime. The canonically normalised moduli masses are

$$M^2_{\mathrm{mod},i} = \mu_i\,\frac{hq^3}{(\mathfrak{f}_0)^{\frac{3}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}\,\frac{M^2_{\mathrm{Pl}}}{4\pi\cdot 2^7}\,, \tag{5.3.12}$$

with numerical values

$$\mu \approx \big(2.1,\, 0.37,\, 0.25\,;\, 1.3,\, 0.013,\, 0\big)\,. \tag{5.3.13}$$

The first three states are saxions and the last three are axions. The axionic combination $(q\,c - h\,\rho)$ is massless. Note that the lightest massive mode is axionic, and although not parametrically light, its mass is numerically light. In fact, it is by a factor of 1/5 smaller than the second-lightest massive state, which is purely saxionic. For the gravitino mass the flux dependence is the same as for the moduli masses, with the numerical prefactor given by $\mu_{\frac{3}{2}} \approx 0.152$. For this example, we will later investigate supersymmetry breaking.

A model with orientifold odd G-moduli

Now we consider a Calabi Yau manifold with Hodge numbers $h^{2,1}_{-} = 0$ and $h^{1,1}_{+} = h^{1,1}_{-} = 1$. Adding also geometric flux, we have a potential which contains a G-modulus. Using (3.1.31), the Kähler potential is

$$K = -3\log\Big((T+\overline{T}) + \frac{\kappa}{4(S+\overline{S})}\,(G+\overline{G})^2\Big) - \log(S+\overline{S})\,, \tag{5.3.14}$$

where for later convenience we have set $\kappa := 2\kappa_{\alpha ab}$ for $\alpha = a = b = 1$. We turn on fluxes such that the superpotential (5.2.13) becomes

$$W = i\tilde{\mathfrak{f}} + i\,hS + i\,qT - i\,f\,G\,, \tag{5.3.15}$$

with $\tilde{\mathfrak{f}} = \tilde{\mathfrak{f}}^0$, $h = h_0$, $q = {q_0}^1$, and $f = f_{01}$. For this set of fluxes, the contribution to the tadpoles (5.2.25) is given by

$$N^{\rm flux}_{\rm D3} = \tilde{\mathfrak{f}}h\,, \qquad N^{\rm flux}_{\rm D5} = -\tilde{\mathfrak{f}}f\,, \qquad N^{\rm flux}_{\rm D7} = \tilde{\mathfrak{f}}q\,. \tag{5.3.16}$$

The signs of these tadpoles depend on the signs of the fluxes, which are fixed by setting the vevs of the saxions to a positive value. In the most interesting vacuum discussed below we must demand $q, h < 0 < f, \tilde{\mathfrak{f}}$ for which all tadpole contributions in (5.3.16) are negative.

The scalar potential can be computed from (5.3.14) and (5.3.15), for which we find three extrema which are AdS, a supersymmetric and tachyonic extremum, a non-supersymmetric and tachyonic extremum and the most interesting one is a non-supersymmetric and non-tachyonic minimum. The latter vacuum is characterised by

$$\tau = -\frac{(6+x)}{5(1+x)}\,\frac{\tilde{\mathfrak{f}}}{q}\,, \qquad s = -\frac{1}{x+1}\,\frac{\tilde{\mathfrak{f}}}{h}\,, \qquad \psi = \frac{2x}{x+1}\,\frac{\tilde{\mathfrak{f}}}{f}\,, \qquad 0 = q\rho - f\eta + hc\,, \tag{5.3.17}$$

where for the modulus G we use the notation

$$G^a := \psi^a + i\eta^a\,. \tag{5.3.18}$$

In (5.3.17), we have simplified the formulas by introducing the parameter

$$x = \frac{f^2}{\kappa h q}\,. \tag{5.3.19}$$

As expected, the superpotential (5.3.15) fixes only one linear combination of axions. Again, all the saxion vevs in (5.3.17) can be controlled to be in the perturbative regime since they scale with $\tilde{\mathfrak{f}}$, which has to be chosen large. In the minimum specified by (5.3.17), the superpotential becomes x-independent and we are left with

$$W_0 = -\frac{6i}{5}\,\tilde{\mathfrak{f}}\,. \tag{5.3.20}$$

The other extrema in this model have the same flux-scaling $W_0 \sim \tilde{\mathfrak{f}}$. The scalar potential of the stable non-supersymmetric AdS vacuum reads

$$V_0 = -\frac{2^2\cdot M_{\rm Pl}^4}{4\pi}\,\frac{25}{864}\,\frac{hq^3}{\tilde{\mathfrak{f}}^2}\,(1+x)\,, \tag{5.3.21}$$

where the dependence on x comes from the e^K factor. The masses read

$$M^2_{{\rm mod},i} = \mu_i\,\frac{2^2\cdot M_{\rm Pl}^2}{4\pi}\,\frac{hq^3}{\tilde{\mathfrak{f}}^2}\,(1+x)\,, \tag{5.3.22}$$

with the numerical coefficients

$$\mu_i \approx \big(0.097,\, 0.026,\, 0\,;\, 0.054,\, 0,\, 0\big)\,. \tag{5.3.23}$$

The first three entries correspond to saxionic moduli, while the last three entries are axionic combinations. The minimum not only contains two massless axion but also a massless saxion $(f\tau + q\psi)$. The gravitino mass $M^2_{\frac{3}{2}} = e^{K_0}\,|W_0|^2$ has the same flux dependence (5.3.22), with the numerical factor given by $\mu_{\frac{3}{2}} = \frac{5}{384} \approx 0.013$. We conclude that stable flux-scaling vacua also exist in the presence of orientifold odd G-moduli, though unfortunately we did not find a stable example with all saxions massive.

solution	$(s, \tau_1, \tau_2, \theta)$	susy	tachyons	Λ
1	$(-\frac{\tilde{\mathfrak{f}}}{2h}, -\frac{\tilde{\mathfrak{f}}}{q_1}, -\frac{\tilde{\mathfrak{f}}}{2q_2}, 0)$	yes	2	AdS
2	$(\frac{\tilde{\mathfrak{f}}}{8h}, \frac{\tilde{\mathfrak{f}}}{4q_1}, \frac{\tilde{\mathfrak{f}}}{8q_2}, 0)$	no	2	AdS
3	$(-\frac{\tilde{\mathfrak{f}}}{h}, -\frac{4\tilde{\mathfrak{f}}}{5q_1}, -\frac{2\tilde{\mathfrak{f}}}{5q_2}, 0)$	no	1	AdS
4	$(-\frac{2\tilde{\mathfrak{f}}}{5h}, -\frac{4\tilde{\mathfrak{f}}}{5q_1}, -\frac{\tilde{\mathfrak{f}}}{q_2}, 0)$	no	1	AdS

Table 5.2: Extrema of the scalar potential in the K3-fibration model.

K3-fibration

In the Kähler sector of $\mathbb{P}_{1,1,2,2,2}[8]$ the intersection numbers are such that the Kähler potential splits into sums and is given by

$$K = -2\log(T_1 + \overline{T}_1) - \log(T_2 + \overline{T}_2) - \log(S + \overline{S})\,, \tag{5.3.24}$$

where for simplicity we have set $h^{2,1}_{-} = 0$. Fluxes are chosen such that the superpotential (5.2.13) takes the form

$$W = i\tilde{\mathfrak{f}} + ihS + iq_1T_1 + iq_2T_2\,, \tag{5.3.25}$$

with $\tilde{\mathfrak{f}} = \tilde{\mathfrak{f}}^0$, $h = h_0$ and – for ease of notation – with $q_i = q_0{}^i$. The resulting scalar potential has four AdS vacua summarised in table 5.2, three of which are generalisations of those in table 5.1. The stabilised axion is $\theta = q_1\rho_1 + q_2\rho_2 + hc$, and the potential does not depend on the two orthogonal axion combinations which thus remain unstabilised.

The physical masses of the fields scale with the fluxes in the following way

$$M^2_{\text{mod},i} = \mu_i \frac{hq_1^2q_2}{\tilde{\mathfrak{f}}^2}\frac{M^2_{pl}}{4\pi\cdot 2^4}\,, \tag{5.3.26}$$

where the numerical factors μ_i depend on the specific solution. The cosmological constant is negative and has the same relation to the fluxes as the physical masses. The supersymmetric case contains, as expected, two tachyons above the Breitenlohner-Freedman bound; for the non-supersymmetric vacua, tachyons are below the bound. In vacua 1, 2 and 3 there is a tachyon given by the combination of saxions $\tau_{\text{tac}} = q_2\tau_1 - q_1\tau_2$. In section 5.3.3 we will see that this tachyon can be lifted by adding a D-term to the F-term potential.

Turning to the tadpole conditions, according to (5.2.25) in this model the flux contributions are given by

$$N_{\rm D3}^{\rm flux} = \tilde{\mathfrak{f}} h\,, \qquad \left[N_{\rm D7}^{\rm flux}\right]^1 = \tilde{\mathfrak{f}} q_1\,, \qquad \left[N_{\rm D7}^{\rm flux}\right]^2 = \tilde{\mathfrak{f}} q_2\,. \tag{5.3.27}$$

For the vacua 1, 2 and 3 to have positive vevs for the saxions, we take for concreteness $\tilde{\mathfrak{f}} < 0$ and the remaining fluxes positive. The contributions (5.3.27) to the flux tadpoles are then all negative.

5.3.3 Tachyon uplift by D-brane induced D-Term

To uplift tachyons one could think that taking perturbative and non-perturbative corrections to K and W into account might help. However, since we have taken care of freezing the moduli in the perturbative regime, these corrections are generically suppressed against the tree-level values. Of course, this also holds for the tachyonic mass. The second and more natural option is to have an additional positive-definite contribution such as a D-term potential. Thus, in the following we study how a D-term of a stack of D7-branes contributes to moduli stabilisation and the mass terms. An analogous mechanism to uplift tachyons via D-terms from D-branes was proposed in [78].

To show how the D-term uplift works, we perform our analysis in a concrete model. In particular, we consider the K3-fibration with $h^{1,1}_+ = 2$ and $h^{2,1}_- = 0$ Recall the supersymmetric AdS minimum at

$$\tau_1 = -\frac{\tilde{\mathfrak{f}}}{q_1}\,, \quad \tau_2 = -\frac{\tilde{\mathfrak{f}}}{2q_2}\,, \quad s = -\frac{\tilde{\mathfrak{f}}}{2h}\,, \quad hc + q_1\rho_1 + q_2\rho_2 = 0\,, \tag{5.3.28}$$

and that there also exists a non-supersymmetric AdS minimum at

$$\tau_1 = -\frac{4\tilde{\mathfrak{f}}}{5q_1}\,, \quad \tau_2 = -\frac{2\tilde{\mathfrak{f}}}{5q_2}\,, \quad s = -\frac{\tilde{\mathfrak{f}}}{h}\,, \quad hc + q_1\rho_1 + q_2\rho_2 = 0\,, \tag{5.3.29}$$

which has mass eigenvalues

$$M^2_{{\rm mod},i} = \mu_i \frac{h\, q_1^2\, q_2}{\tilde{\mathfrak{f}}^2} \frac{M_{\rm Pl}^2}{4\pi \cdot 2^4}\,, \tag{5.3.30}$$

with $\mu_i = (-15,\, 11,\, 42\,; 23,\, 0,\, 0)$. The tachyonic mode corresponds to a linear combination of Kähler saxions given by $\tau_{\rm tac} = q_2\tau_1 - q_1\tau_2$. To obtain positive vevs for the saxions we take $\tilde{\mathfrak{f}} < 0$, $h > 0$, $q_1 > 0$ and $q_2 > 0$.

We now introduce a stack of N D7-branes equipped with a $U(1)$ gauge flux with

$$[c_1(L)] = [\mathcal{E}] = l_1 D_1 + l_2 D_2 \,, \tag{5.3.31}$$

where $D_{1,2}$ are two (effective) divisors in $\mathbb{P}_{1,1,2,2,2}[8]$ and $l_{1,2} \in \mathbb{Z}$. The D7-branes are wrapping a four-cycle defined by

$$\Sigma = m_1 D_1 + m_2 D_2 \,, \tag{5.3.32}$$

with $m_{1,2} \in \mathbb{Z}$, which leads to a D-term potential of the form

$$V_D = \frac{M_{\rm Pl}^4}{2{\rm Re}(f)} \xi^2 \,. \tag{5.3.33}$$

Here ξ is the Fayet-Iliopoulos (FI) term of the $U(1) \subset U(N)$ carried by the branes, which is given by

$$\xi = \frac{1}{\mathcal{V}} \int_\Sigma J \wedge c_1(L) \,, \tag{5.3.34}$$

and in (5.3.33) we have assumed that all charged fields have vanishing vevs. The holomorphic gauge kinetic function for the D7-branes is $f = T + \chi S$, where $\chi = \frac{1}{4\pi^2} \int F \wedge F$ denotes the instanton number of the gauge flux on the D7-branes. In the example at hand, the volume is $\mathcal{V} = (t^1)^2 t^2$.

The wrapping numbers (m_1, m_2) and the gauge fluxes (l_1, l_2) are constrained by the generalised Freed-Witten anomaly cancellation conditions (5.2.38), which in the present case lead to

$$m_1 l_1 q_2 + \big(l_1 m_2 + l_2 m_1\big) q_1 = 0 \,. \tag{5.3.35}$$

Using this condition, we find that the FI-parameter can be expressed as

$$\xi = \frac{m_1 l_1}{q_1 \sqrt{\tau_2}\, \mathcal{V}} \big(q_1 \tau_1 - 2 q_2 \tau_2\big) \,. \tag{5.3.36}$$

Note that for a supersymmetric minimum, a vanishing F-term implies a vanishing D-term. And indeed, the values (5.3.28) give a vanishing FI-term. Moreover, ξ also vanishes for the non-supersymmetric minimum in (5.3.29). Therefore, adding the D-term will not change the position of either extremum, but due to its positive-definiteness it is expected to add positive contributions to the squares of the saxion masses.

We now study in more detail the effect of adding a D-term to the former F-term scalar potential. Concretely, we add

$$V_D = \frac{k}{\tau_1^2\tau_2}\frac{\big(q_1\tau_1 - 2q_2\tau_2\big)^2}{\big(m_1\tau_1 + m_2\tau_2\big)\tau_2}\,, \tag{5.3.37}$$

which is obtained by substituting the various ingredients in (5.3.33). Here k is a positive numerical prefactor and for the gauge kinetic function we only included the string tree-level part $\mathrm{Re}(f) = m_1\tau_1 + m_2\tau_2$. As expected, the position of both the supersymmetric (5.3.28) and the non-supersymmetric (5.3.29) extrema do not change. Moreover, from the resulting mass matrix it follows that only the mass eigenvalue corresponding to the tachyonic saxion $\tau_{\rm tac}$ receives corrections and can become positive. In the supersymmetric case a tachyonic state will remain, although above the Breitenlohner-Freedman bound. In the non-supersymmetric extremum there is only one negative mass eigenvalue that receives corrections, which is given by (in units of $M_{\rm Pl}^4/(4\pi)$) *

$$m_{\rm tac}^2 = -\frac{15\,h\,q_1^2\,q_2}{16\,\tilde{\mathfrak{f}}^2} - \frac{375\,q_1^3\,q_2^3\,k}{4\,\tilde{\mathfrak{f}}^3\,(m_1q_1 + 2m_2q_1)}\,. \tag{5.3.38}$$

We observe that the mass can become positive because $\tilde{\mathfrak{f}} < 0$. For instance, choosing $h = 2$, and $q_1 = q_2 = m_1 = m_2 = 1$, implies that $m_{\rm tac}^2$ will turn positive provided $k > -3\tilde{\mathfrak{f}}/50$. We could take for instance $\tilde{\mathfrak{f}} = -10$ and $k = 1$. Thus, the tachyonic mode can be uplifted while the masses of the other moduli do not change. Moreover, as the D-term vanishes in the minimum, the cosmological constant V_0 does not change either.

5.3.4 The dilute flux limit

The dilute flux limit ensures that the fluxes become diluted and their backreaction on the geometry is negligible. We have to check if the fluxes are still diluted in the presence of non-geometric. For investigating this, we look at the action for the fluxes given by [79]

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\Big(\mathcal{L}^{HH} + \mathcal{L}_1^{QQ} + \mathcal{L}_2^{QQ} + \mathcal{L}^{HQ} + \mathcal{L}^{\rm RR}\Big)\,, \tag{5.3.39}$$

*Note that in the following, we have omitted the factor $M_{\rm Pl}^4/(4\pi)$ for ease of notation. It can be re-installed easily by dimensional analysis.

with

$$\begin{aligned}
\mathcal{L}^{HH} &= -\frac{e^{-\phi}}{12} H_{ijk}\, H_{i'j'k'}\, g^{ii'} g^{jj'} g^{kk'}\,, & \mathcal{L}^{HQ} &= \frac{1}{2} H_{mni}\, Q_{i'}{}^{mn}\, g^{ii'}\,, \\
\mathcal{L}_1^{QQ} &= -\frac{e^{\phi}}{4} Q_k{}^{ij}\, Q_{k'}{}^{i'j'}\, g_{ii'} g_{jj'} g^{kk'}\,, & \mathcal{L}_2^{QQ} &= -\frac{e^{\phi}}{2} Q_m{}^{ni}\, Q_n{}^{mi'}\, g_{ii'}\,, \\
\mathcal{L}^{\mathrm{RR}} &= -\frac{e^{\phi}}{12} \mathfrak{F}_{ijk}\, \mathfrak{F}_{i'j'k'}\, g^{ii'} g^{jj'} g^{kk'}\,,
\end{aligned} \tag{5.3.40}$$

where g_{ij} is the internal metric. We consider the simple model with one Kähler modulus, where the fields at the minimum are given by

$$e^{-\phi} \sim s \sim \frac{\hat{\mathfrak{f}}}{h}\,, \qquad g \sim \sqrt{\tau} \sim \frac{\hat{\mathfrak{f}}^{\frac{1}{2}}}{q^{\frac{1}{2}}}\,, \qquad g^{-1} \sim \frac{q^{\frac{1}{2}}}{\hat{\mathfrak{f}}^{\frac{1}{2}}}\,, \tag{5.3.41}$$

where $\hat{\mathfrak{f}} = -\tilde{\mathfrak{f}}$. Substituting these vevs into (5.3.39), we obtain

$$\mathcal{L}^{HH} \sim \mathcal{L}_1^{QQ} \sim \mathcal{L}_2^{QQ} \sim \mathcal{L}^{HQ} \sim \mathcal{L}^{\mathrm{RR}} \sim \frac{h q^{\frac{3}{2}}}{\hat{\mathfrak{f}}^{\frac{1}{2}}}\,. \tag{5.3.42}$$

Remember that we required $\hat{\mathfrak{f}} \gg 1$, and this term is small. Unfortunately, the relevant variable is not the Lagrangian, but the energy-momentum tensor $T_{ij} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g^{ij}}$, appearing on the right-hand-side of the Einstein equation. All contributions to the energy momentum tensor T_{ij} scale in the same way, namely

$$T_{ij}^{HH} \sim T_{1\,ij}^{QQ} \sim T_{2\,ij}^{QQ} \sim T_{ij}^{HQ} \sim T_{ij}^{\mathrm{RR}} \sim hq\,. \tag{5.3.43}$$

Due to flux quantisation, this term is not small but at least one. Therefore, we conclude that the backreaction of the fluxes is of order $\mathcal{O}(1)$.

5.3.5 Soft masses

We now study the supersymmetry breaking behaviour of flux-scaling vacua. The gravity-mediated soft-masses on stacks of D7-branes for a bulk and a sequestered set-up. For the sequestered scenario, anomaly-mediation is the dominant contribution. Such a sequestered scenario is important for lowering the supersymmetry breaking scale to a low or intermediate energy scale. The gravitino mass is given in terms of the Kähler and superpotential as

$$M_{\frac{3}{2}}^2 = e^{K_0} |W_0|^2 \frac{M_{\mathrm{Pl}}^2}{4\pi}\,. \tag{5.3.44}$$

In the flux-scaling scenarios, the gravitino mass is of order of the moduli masses. Soft gaugino and sfermion masses for magnetised D7-branes wrapping an additional four-cycle and supporting the MSSM are added.

Supersymmetry breaking in the bulk

We consider the model with one complex structure modulus in section 5.3.2, where due to Freed Witten anomalies, an extra four-cycle has to be turned on. We do this by introducing a del Pezzo surface of the swiss cheese type of volume form

$$\mathcal{V} = \tau^{\frac{3}{2}} - \tau_s^{\frac{3}{2}} \tag{5.3.45}$$

to avoid deformation moduli. The analysis is analogous to [80]. The superpotential is independent of T_s, whose axionic part receives a mass through a Stückelberg mechanism. The four-cycle modulus τ_s is stabilised by the Fayet-Illiopoulos term which forces the del Pezzo surface to zero size. The F-term vanishes $F^{T^s} = 0$ due to $K^{\overline{T}_s,i}\partial_i K = -2\tau_s \sim 0$. The gaugino masses are then given by the formula

$$M_a = \frac{1}{2}(\mathrm{Re} f_a)^{-1} F^i \partial_i f_a \,, \tag{5.3.46}$$

with

$$F^i = e^{\frac{K}{2}} K^{i\bar{j}} D_{\bar{j}} \overline{W} \,, \tag{5.3.47}$$

and $f_a = T_s + \chi_a S$ is the holomorphic gauge kinetic function for the D7-brane with $\chi_a = \frac{1}{4\pi^2}\int F_a \wedge F_a$ being the instanton number of the gauge flux on the D7-branes. In our example with one complex structure modulus, the gaugino mass is evaluated as

$$M_a^2 = \mu_a \, \frac{h_1(q_1)^3}{(\mathfrak{f}_0)^{\frac{3}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}} \, \frac{M_{\mathrm{Pl}}^2}{4\pi \cdot 2^4} \sim M_{\frac{3}{2}}^2 \,, \tag{5.3.48}$$

with $\mu_a = 12$. The sfermion masses read

$$M_\alpha^2 = M_{\frac{3}{2}}^2 + V_0 - F^{\bar{i}} F^j \partial_{\bar{i}} \partial_j \log Z_\alpha \,, \tag{5.3.49}$$

with Z_α the Kähler metric for the matter field. For magnetised branes on a small shrinkable cycle, one gets at tree-level $Z_\alpha = k_\alpha/\tau$ as argued in [81]. After assuming that we uplifted the AdS minimum to $V_0 = 0$, the sfermion masses read

$$M_\alpha^2 = M_{\frac{3}{2}}^2 - \frac{(F^T)^2}{4\tau^2} = \mu_\alpha \, \frac{h_1(q_1)^3}{(\mathfrak{f}_0)^{\frac{3}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}} \, \frac{M_{\mathrm{Pl}}^2}{4\pi \cdot 2^4} \sim M_{\frac{3}{2}}^2 \,, \tag{5.3.50}$$

with $\mu_\alpha = 28$. This is of the same order as the gravitino mass and therefore the gaugino masses, the gravitino mass and the moduli masses have the same order of magnitude and sequestering is not given. The reason for this is the non-vanishing F-term for the dilaton $F^S \neq 0$.

Sequestered scenario

For sequestering, the F-term $F^S = 0$ at tree-level and is only non-zero after adding corrections. Our flux-scaling examples did not have a vanishing F^S. Nevertheless, we use the model with one G-modulus 5.3.2 as a toy model since there F^T can be tuned to zero.

Gravity mediated gaugino masses

The Kähler potential now also contains $(\alpha')^3$-corrections and is given by

$$K = -2\log\left[\left((T+\overline{T}) + \frac{\kappa}{4(S+\overline{S})}(G+\overline{G})^2\right)^{\frac{3}{2}} + \frac{\xi_p}{2}s^{\frac{3}{2}}\right] - \log(S+\overline{S})\,, \quad (5.3.51)$$

with $\xi_p = -\frac{\chi(\mathcal{M})\zeta(3)}{2\,(2\pi)^3}$. The superpotential (5.3.15) remains the same. The F-terms are given by $F^i = e^{\frac{K}{2}}K^{i\overline{j}}F_{\overline{j}}$ at the minimum are

$$\begin{aligned} F^T &= e^{\frac{K}{2}}\frac{8i}{25}\frac{\tilde{\mathfrak{f}}^2}{q}\frac{8x+3}{1+x}\,, \qquad & F^S &= -e^{\frac{K}{2}}\frac{8i}{5}\frac{\mathfrak{f}^2}{h}\frac{1}{(1+x)}\,, \\ F^G &= e^{\frac{K}{2}}\frac{16i}{5}\frac{\mathfrak{f}^2}{f}\frac{x}{x+1}\,, \end{aligned} \quad (5.3.52)$$

with $x = \frac{f^2}{\kappa h q}$. Having $8x+3=0$, the F-term for the Kähler modulus vanishes $F^T = 0$. Note that κ has to be positive in the physical regime $s_0, \tau_0 > 0$.
Using the definitions of the mass scales (1.0.1), (1.0.3) and the volume $\mathcal{V} \propto \left(2\tau + \frac{\kappa}{2s}\psi^2\right)^{\frac{3}{2}}$, we get

$$\frac{M_{\rm KK}}{M_{\rm s}} \sim \left(\frac{q}{h}\right)^{\frac{1}{4}}\,, \quad (5.3.53)$$

where $x = -\frac{3}{8}$ was applied. We get a small α'-correction $\xi_p(q/h)^{3/2} \ll 1$ for a flux choice $h \gg q$. The minimum is slightly shifted by the addition of this term. Nevertheless, we can see numerically that this shift can be covered by using the corrected form for F^T, which is given by

$$\frac{F^T_\xi}{F^T_0} \sim \xi_p\left(\frac{q}{h}\right)^{\frac{3}{2}} \sim \xi_p\left(\frac{M_{\rm KK}}{M_{\rm s}}\right)^6\,, \quad (5.3.54)$$

at linear order in ζ_p. F^T_0 is the minimal F-term at tree-level for $x \neq -\frac{3}{8}$. The gravity mediated gaugino mass is suppressed as

$$M_a \sim \left(\frac{M_{\rm KK}}{M_{\rm s}}\right)^6 M_{\frac{3}{2}}\,, \quad (5.3.55)$$

compared to the gravitino mass.

Anomaly-mediated gaugino masses

With the tree-level gravity-mediated gaugino masses vanishing at leading order, the one-loop generated anomaly-mediated gaugino masses are expected to be generically larger than the next-to-leading order tree-level masses. In the sequestered LVS scenario, it turned out that even the leading-order anomaly-mediated contribution vanishes due to an extended no-scale structure. Let us estimate this contribution in our model. The anomaly-mediated gaugino masses are given by [82]

$$M_a^{\rm anom} = -\frac{g^2}{16\pi^2}\Big((3T_G - T_R)M_{\frac{3}{2}} - (T_G - T_R)(\partial_i K)F^i \\ - \frac{2T_R}{d_R}F^i \partial_i \log\det Z_{\alpha\beta}\Big), \qquad (5.3.56)$$

where $T_G = N$ is the Dynkin index of the adjoint representation of $U(N)$ and T_R is the Dynkin index of some matter representation R of dimension d_R. In our simple case of unmagnetised D7-branes, there is no charged matter so that the above formula simplifies. Indeed, there is no cancellation between the first and second term and we obtain

$$M_a^{\rm anom} = \frac{1}{16\pi^2 {\rm Re}(f_a)}\frac{8}{3}\,N M_{\frac{3}{2}} = \frac{1}{(4\pi)^{\frac{3}{2}}}\frac{16\,N}{9}\frac{M_{\rm KK}\,M_{\frac{3}{2}}}{M_{\rm Pl}}\,. \qquad (5.3.57)$$

Therefore, we still get a suppression, which generically will be weaker than the next-to-leading order gravity-mediated one (5.3.55). For instance, for $M_{\rm s} \sim 10^{16}\,{\rm GeV}$, $M_{\rm KK} \sim 10^{15}\,{\rm GeV}$ and $M_{\frac{3}{2}} \sim 10^{14}\,{\rm GeV}$, we find $M_a \sim 10^{8}\,{\rm GeV}$ and $M_a^{\rm anom} \sim 10^{11}\,{\rm GeV}$. Therefore, one can get gaugino masses in the intermediate regime.

As argued in [80], the computation of other soft terms is sensitive to higher-order corrections to the matter-field metric and to the uplift, so that we are not pursuing this question here further. Of course, what we have presented is just a toy model, as the brane wrapping the four-cycle is non-chiral and presumably will carry extra massless deformation modes (that also have to be stabilised). The purpose of our analysis was to show how one can arrange for a situation where the gaugino masses are induced by higher-order corrections, and can therefore be parametrically smaller than the gravitino mass scale. This is important for string model building, if one wants to have the supersymmetry breaking scale for the MSSM smaller than the GUT or inflation scale.

5.4 Uplift to de Sitter

An inflationary expanding universe needs a positive cosmological constant. This was not achieved for non-geometric flux-scaling vacua. We need an extra term, i.e. an extra degree of freedom, to lift the vacuum energy of AdS vacua. De Sitter vacua in string theory have been studied from different perspectives [51, 83–92]. Both analytical and numerical approaches have been followed to construct metastable dS vacua. Moreover, as a useful guide, no-go theorems have been derived in the context of the type II [93–101] and heterotic [102–104] superstrings. We focus here on the inclusion of an $\overline{D}3$-brane and a D-term induced by abelian vector fields on certain orientifolds.

5.4.1 D-term

On Calabi Yau orientifolds with $h^{21}_{+} \neq 0$ abelian gauge fields are present which are obtained by reduction of the RR four-form C_4. Then, a D-term potential is generated in the presence of geometric F-flux and non-geometric Q- and R-fluxes [105, 106]. The full scalar potential is the given by

$$V = V_F + V_D + V_{\rm tad}^{\rm NS}\,. \tag{5.4.1}$$

In general, the D-term potential is of the form

$$V_D = -\frac{M_{\rm Pl}^4}{2}\Big[(D_{\hat\lambda} + {\rm Re}\,\mathcal{N}_{\hat\lambda\hat\kappa}\tilde{D}^{\hat\kappa})\,({\rm Im}\,\mathcal{N}^{-1})^{\hat\lambda\hat\sigma}\,(D_{\hat\sigma} + {\rm Re}\,\mathcal{N}_{\hat\lambda\hat\sigma}\tilde{D}^{\hat\sigma}) + \tilde{D}^{\hat\lambda}\,{\rm Im}\,\mathcal{N}_{\hat\lambda\hat\sigma}\,\tilde{D}^{\hat\sigma}\Big] \tag{5.4.2}$$

with

$$\tilde{D}^{\hat\lambda} = \frac{1}{\mathcal{V}}\Big[r^{\hat\lambda}\left(e^{\phi}\mathcal{V} - \tfrac{1}{2}\kappa_{\alpha ab}t^\alpha b^a b^b\right) + q^{\hat\lambda a}\,\kappa_{a\alpha b}t^\alpha b^b - f^{\hat\lambda\alpha}\,t^\alpha\Big] \tag{5.4.3}$$

and

$$D_{\hat\lambda} = \frac{1}{\mathcal{V}}\Big[-r_{\hat\lambda}\left(e^{\phi}\mathcal{V} - \tfrac{1}{2}\kappa_{\alpha ab}t^\alpha b^a b^b\right) - q_{\hat\lambda}{}^{a}\,\kappa_{a\alpha b}t^\alpha b^b + f_{\hat\lambda\alpha}\,t^\alpha\Big]\,. \tag{5.4.4}$$

In the following, we consider the case with $\tilde{r}^{\hat\lambda} = \tilde{q}^{\hat\lambda a} = \tilde{f}^{\hat\lambda}{}_{\alpha} = 0$, then the D-term potential is given by

$$V_D = -\frac{M_{\rm Pl}^4}{2}\Big[({\rm Im}\,\mathcal{N})^{-1}\Big]^{\hat\lambda\hat\sigma} D_{\hat\lambda}\,D_{\hat\sigma}\,. \tag{5.4.5}$$

Naively, this D-term could be generated purely by turning on geometric flux. Unfortunately, the Bianchi identities which these fluxes have to fulfill are so strong, that whenever there is geometric flux turned on, also non-geometric fluxes are present. In this simplified case, the Bianchi identities are schematically of the form

$$r\,\tilde{h} + f\,\tilde{q} = 0. \tag{5.4.6}$$

5.4.2 $\overline{D}3$-branes

Uplifting with an $\overline{D}3$-brane was first discussed in KKLT. The idea in the original KKLT scenario is to uplift an old AdS vacuum to de Sitter in the presence of an $\overline{D}3$-brane. Such an anti-brane generates a term of the form

$$V_{\rm up} = \frac{\varepsilon}{\mathcal{V}^{\alpha}}\,, \tag{5.4.7}$$

with $\alpha = 2$ for an anti-brane in the bulk and $\alpha = 4/3$ for an $\overline{D}3$-brane in the throat. We will consider the latter case. The factor ε has to be very small. When we try to uplift an AdS flux-scaling vacuum to de Sitter with a term like in (5.4.7), we see that the factor α has to be too small to be generated by an anti-brane. Therefore we conclude that an uplift a la KKLT destabilises the vacuum. Nevertheless, it turned out that adding a term (5.4.7) to the scalar potential and then looking for a minimum indeed results in a non-negative cosmological constant.

Example

We consider such a vacuum construction in an example with one Kähler and one complex structure modulus. The tree-level Kähler potential is that of an isotropic torus and reads

$$K = -\log(S+\overline{S}) - 3\log(T+\overline{T}) - 3\log(U+\overline{U})\,, \tag{5.4.8}$$

and the superpotential is chosen to be

$$W = -ifU + ih_0 S - 3ihSU^2 - iqT\,, \tag{5.4.9}$$

with the fluxes $\mathfrak{f}_1 = f$, $\tilde{h}^1 = -h$ and $q_0{}^1 = q$. We look for a Minkowski and a de Sitter minimum of the scalar potential and an uplift term

$$V_{\rm up} = \frac{A}{\mathcal{V}^{\frac{4}{3}}}\,\frac{M_{\rm Pl}^4}{4\pi}\,, \tag{5.4.10}$$

where the parameter A is determined by requirements on the value of the vacuum energy.

Minkowski uplift

First, we minimise the potential

$$V = V_F + V_{\text{up}} \tag{5.4.11}$$

at vanishing vacuum energy. The vacuum expectation values of the saxions are

$$s = \frac{1}{3^{3/4}} \frac{f}{(hh_0)^{1/2}}\,, \quad v = \frac{1}{3^{1/4}} \left(\frac{h_0}{h}\right)^{1/2}, \quad \tau = \frac{f}{3^{1/4} q} \left(\frac{h_0}{h}\right)^{1/2}. \tag{5.4.12}$$

We are in the perturbative large volume regime which we can control if we choose f large enough. Note that this leads to a large contribution to the D3-brane tadpole $N_{\text{D3}} = fh$. The warp dependent parameter A is determined by the condition $V_0 = 0$ to be

$$A = \frac{3^{1/4}}{2} \frac{qh^{3/2}}{h_0^{1/2}}\,. \tag{5.4.13}$$

This term has to be small since its physical origin lies in warping. An appropriate flux choice allows to tune this term small. To ensure positivity of the saxion vevs, our choice for the sign of the fluxes is

$$f > 0\,, \qquad h_0 > 0\,, \qquad h > 0\,, \qquad q > 0\,, \tag{5.4.14}$$

The masses of the moduli have the following flux scaling

$$M^2_{\text{mod}} = \mu_i \frac{q^3 h^{5/2}}{f^2 h_0^{3/2}} \frac{M^2_{\text{Pl}}}{4\pi}\,, \tag{5.4.15}$$

with

$$\mu_i = \{0.8034, 0.4868, 0.03942; 1.5559, 0.2116, 0.0811\}\,. \tag{5.4.16}$$

The first three entries are saxionic while the last three are axionic. The lightest state is a linear combination of saxions.

$$M^2_{\text{s}} = \frac{3^{3/4}\pi}{2^{3/2}} \frac{q^{3/2} h}{f^2 h_0^{1/2}} M^2_{\text{Pl}}, \qquad M^2_{\text{KK}} = \frac{3^{1/2}}{16\pi} \frac{q^2 h}{f^2 h_0} M^2_{\text{Pl}} \tag{5.4.17}$$

so that the relevant ratios are

$$\frac{M^2_{\text{KK}}}{M^2_{\text{s}}} = \frac{1}{2^{5/2} 3^{1/4} \pi^2} \left(\frac{q}{h_0}\right)^{1/2}, \qquad \frac{M^2_{\text{mod},i}}{M^2_{\text{KK}}} = \frac{2^2 \mu_i}{3^{1/2}} \frac{q h^{3/2}}{h_0^{1/2}}. \tag{5.4.18}$$

de Sitter uplift

The analysis of a de Sitter vacuum is a bit more involved. We consider it therefore in an expansion in the vacuum energy $\Lambda = V_0$. The axion vevs stay the same while the vacuum expectation values of the saxions are shifted to

$$\begin{aligned} s &= \frac{1}{3^{3/4}} \frac{f}{(h h_0)^{1/2}} + \frac{2^4 \cdot 7}{3^{5/2}} \frac{f^3 h_0}{q^3 h^3} \Lambda + \mathcal{O}(\Lambda^2)\,, \\ v &= \frac{1}{3^{1/4}} \left(\frac{h_0}{h}\right)^{1/2} - \frac{2^4}{3^2} \frac{f^2 h_0^2}{q^3 h^3} \Lambda + \mathcal{O}(\Lambda^2)\,, \\ \tau &= \frac{f}{3^{1/4} q} \left(\frac{h_0}{h}\right)^{1/2} + \frac{2^4 \cdot 13}{3^2} \frac{f^3 h_0^2}{q^4 h^3} \Lambda + \mathcal{O}(\Lambda^2)\,. \end{aligned} \tag{5.4.19}$$

The parameter A is given by

$$A = \frac{3^{1/4}}{2} \frac{q h^{3/2}}{h_0^{1/2}} + \frac{2^2}{3^{1/2}} \frac{f^2 h_0}{q^2 h} \Lambda + \mathcal{O}(\Lambda^2)\,. \tag{5.4.20}$$

The masses of the moduli are given in an expansion in Λ as

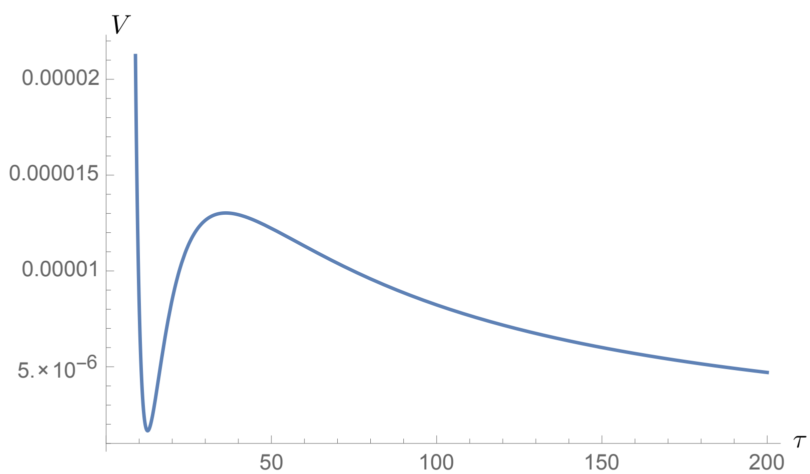

Figure 5.2: The scalar potential $V(\tau)$ in units of $\frac{M_{\rm Pl}^4}{4\pi}$ for $\{s, v\}$ and the axions in their minimum. The fluxes are $h_0 = 10$, $h = q = 1$, $f = 5$ and A is chosen to give a de Sitter minimum.

$$M^2_{\rm mod} = \left(\mu_i \frac{q^3 h^{5/2}}{f^2 h_0^{3/2}} - \tilde{\mu}_i \Lambda + \mathcal{O}(\Lambda^2) \right) \frac{M^2_{\rm Pl}}{4\pi}\,, \tag{5.4.21}$$

with coefficients

$$\mu_i = \{0.8034, 0.4868, 0.03942; 1.5559, 0.2116, 0.0811\}\,, \tag{5.4.22}$$

and

$$\tilde{\mu}_i = \{46.5221, 34.4038, 6.1852; 125.614, 6.5749, 3.6748\}\,. \tag{5.4.23}$$

The moduli masses get lowered by the positive contribution from the cosmological constant. Λ should be not too large to ensure that the moduli are heavy enough and do not become tachyonic. The KK scale and the string scale can also be expanded in terms of the vacuum energy as

$$\begin{aligned}\frac{M_{\rm KK}^2}{M_{\rm s}^2} &= \frac{1}{2^{5/2}3^{1/4}\pi^2}\left(\frac{q}{h_0}\right)^{1/2} - \frac{2^{3/2}}{3\pi^2}\frac{f^2 h_0}{q^{5/2}h^{5/2}}\Lambda + \mathcal{O}(\Lambda^2)\,,\\ \frac{M_{\rm mod,i}^2}{M_{\rm KK}^2} &= \frac{2^2}{3^{1/2}}\mu_i\frac{qh^{3/2}}{h_0^{1/2}} + \frac{2^2}{3^3}\left(2^5\cdot 13\cdot 3^{3/4}\mu_i + 3^{5/2}\tilde{\mu}_i\right)\frac{f^2 h_0}{q^2 h}\Lambda + \mathcal{O}(\Lambda^2).\end{aligned} \tag{5.4.24}$$

A note on $\overline{D}3$-branes and a nilpotent chiral superfield

The scenario above as well as KKLT contain an uplift term sourced by an $\overline{D}3$-brane which was put in by hand. Indeed, configurations with a large number of $\overline{D}3$-branes were shown to be unstable (see for example [107–111]). The argument for instability does not hold in the presence of a single $\overline{D}3$-brane. Recently, configurations with a single $\overline{D}3$-brane gained a better understanding by describing it via a nilpotent chiral superfield [112–114] (for a review see [115]). In an effective theory with purely Kähler moduli, the term (5.4.7) can also be found by adding a nilpotent chiral superfield $S = s + \sqrt{2}\theta\chi + \theta^2 F$ to the Kähler potential. The nilpotency holds for $s = \frac{\overline{\chi}P_L\chi}{2F}$ with χ the goldstino. The uplift term in the bulk as well as the warped uplift term can be recovered. For the latter, the Kähler potential for a simple toy one volume modulus example gets modified as

$$K = -3\,\log\left(T + \overline{T} - s\,\overline{s}\right) \tag{5.4.25}$$

The superpotential gets modified via

$$W = W_0 + \mu^2\, s\,. \tag{5.4.26}$$

Adding this nilpotent field yields an uplift term in the throat

$$V = \frac{\mu^4}{3(T+\overline{T})^2}\,. \tag{5.4.27}$$

Compared to the LVS and KKLT moduli stabilisation scenarios, we consider Kähler moduli stabilisation at tree-level. Then, not only the chiral nilpotent superfield has to be considered but also worldvolume fields of the anti-brane which can not be integrated out as in a purely GVW model. In [116], the uplift term was derived also considering these fields. This is a first step in understanding the supersymmetry breaking and the physics of an $\overline{D}3$-brane.

5.5 Example for axion monodromy inflation

After investigating the examples for moduli stabilisation with a flat axionic direction, we now try to realise our actual purpose, building a model for axion monodromy inflation with a working moduli stabilisation scheme behind it. For this, we start with an example which includes a D-term for uplifting and then turn on a potential for inflation by adding a comparatively small flux, in our example P-flux couples to the inflaton. In the end, we will discuss the implications of this model, including its drawback, which is achieving the correct mass hierarchy.

5.5.1 Step 1

We consider a toy manifold with Hodge numbers $h^{2,1}_+ = 1$, $h^{2,1}_- = 1$, $h^{1,1}_+ = 1$ and $h^{1,1}_- = 0$, i.e. we have one volume modulus T, one complex structure modulus U, one non-abelian gauge field responsible for the D-term contribution and as always, the axio-dilaton S. The superpotential is chosen to be

$$W = i\mathfrak{f}U + i\tilde{\mathfrak{f}}U^3 - ihS + iqT\,, \tag{5.5.1}$$

where for a simplified notation we redefined $\mathfrak{f}_1 = -\mathfrak{f}$, $\tilde{\mathfrak{f}}^0 = \tilde{\mathfrak{f}}$, $h_0 = -h$ and ${q_0}^1 = -q$. For uplifting to Minkowski, we add a D-term potential of the form

$$V_D = \frac{\delta}{v\tau^2}\left(g - \frac{r\,\tau}{3\,s}\right)^2\,, \tag{5.5.2}$$

where $r = f_{\hat{1}\,0}$, $g = f_{\hat{1}\,1}$, and δ is an unphysical positive constant which can be absorbed in a redefinition of the fluxes. By using Bianchi identities, we are left with only one free flux parameter in the D-term, which can be rewritten as

$$V_D = \frac{\delta g^2}{\tau^2 v}\left(1 + \frac{q}{3h}\frac{\tau}{s}\right)^2\,. \tag{5.5.3}$$

Minimising the full potential $V = V_F + V_D$, we see that there exists a tachyon-free Minkowski minimum with two axions stabilised at

$$Re : \Theta = q\rho - hc = 0, \quad u = 0\,, \tag{5.5.4}$$

and the saxions at

$$s = \gamma_1 \frac{\mathfrak{f}^{3/2}}{h\tilde{\mathfrak{f}}^{1/2}}, \quad \tau = \gamma_2 \frac{\mathfrak{f}^{3/2}}{q\tilde{\mathfrak{f}}^{1/2}}, \quad v = \gamma_3 \left(\frac{\mathfrak{f}}{\tilde{\tilde{\mathfrak{f}}}}\right)^{1/2}. \tag{5.5.5}$$

One axionic direction remains massless and will be used as inflaton in step 2. The constant δ is given by

$$\delta g^2 = \gamma_4 \frac{hq\tilde{\tilde{\mathfrak{f}}}}{\mathfrak{f}}\,. \tag{5.5.6}$$

The numerical coefficients in the terms above take the values

$$\gamma_i = \{0.1545, 1.5761, 1.0318, 0.0044\}\,. \tag{5.5.7}$$

We can stay in the physical region, and have $\delta > 0$, by choosing the fluxes $\mathfrak{f}, \tilde{\tilde{\mathfrak{f}}}, h, q > 0$. The saxions are fixed in their perturbative regime for $\mathfrak{f} \gg \tilde{\mathfrak{f}}$ and $\tilde{\tilde{\mathfrak{f}}}, h, q$ of order one. The normalised masses are given by

$$M^2_{\text{mod,i}} = \mu_i \frac{hq^3\,\tilde{\mathfrak{f}}^{5/2}}{\mathfrak{f}^{9/2}} \frac{M^2_{\text{Pl}}}{4\pi}\,, \tag{5.5.8}$$

with prefactors

$$\mu_i = \{0.6986, 0.0152, 0.1318; 0.2594, 0.0524, 0\}\,. \tag{5.5.9}$$

The first three values are saxionic, while the latter are the axions.

5.5.2 Step 2: stabilise the axion

For obtaining an axion potential, we turn on a small perturbation on the superpotential. In our case, this can be done by switching on S-dual P-flux. The inflation generating superpotential reads

$$W = \lambda W_0 - ip\,S\,T\,U\,. \tag{5.5.10}$$

We have to correct the D-term potential by a small perturbation $\Delta\delta$ to ensure a Minkowski minimum after perturbing with W_{ax}. It is given by

$$V_D = \lambda^2 \frac{(\delta_0 + \Delta\delta) g^2}{\tau^2 v} \left(1 + \frac{q}{3h} \frac{\tau}{s}\right)^2 . \tag{5.5.11}$$

with the correction term

$$\Delta\delta \sim -\frac{p\,\mathfrak{f}}{\lambda\, g^2} . \tag{5.5.12}$$

The parameter λ has to be very large to ensure that the stabilised fields can be integrated out. As the axionic direction orthogonal to Θ, we choose $\theta = c$, since $\Theta = 0$. The potential we get after integrating out the stabilised fields is of the form

$$V_{\text{eff}} = B_1\, \theta^2 + B_2\, \theta^4 \tag{5.5.13}$$

with

$$B_1 \sim \frac{\lambda\, p\, h^2\, q^2\, \tilde{\mathfrak{f}}^{5/2}}{\mathfrak{f}^{11/2}} , \qquad B_2 \sim \frac{p^2\, h^3\, q\, \tilde{\mathfrak{f}}^{5/2}}{\mathfrak{f}^{13/2}} . \tag{5.5.14}$$

The quadratic term is dominating over the quartic term for sufficiently large values of the parameter λ assuming the inflaton travels $\theta \sim \mathcal{O}(10)$.

Mass hierarchy

To have a consistent theory, the hierarchy of masses has to be the such that string and Kaluza Klein states are negligible. Let us therefore take a look at the scales for our axion monodromy inflation model. The mass ratios between the moduli masses and the Kaluza Klein scale and the axion mass are given by

$$\frac{M_{\text{KK}}^2}{M_{\text{mod}}^2} \sim \frac{\mathfrak{f}^{3/2}}{\lambda^2\, h\, q\, \tilde{\mathfrak{f}}^{3/2}} , \qquad \frac{M_{\text{mod}}^2}{M_\theta^2} \sim \frac{\lambda\, h\, q\, \tilde{\mathfrak{f}}}{p\, \mathfrak{f}^2} . \tag{5.5.15}$$

Combining these, we get the relation for the KK scale and the axion mass in terms of the fluxes:

$$\frac{M_{\text{KK}}^2}{M_{\text{mod}}^2} \frac{M_{\text{mod}}^2}{M_\theta^2} \sim \frac{1}{\lambda\, p\, \mathfrak{f}^{1/2}\, \tilde{\mathfrak{f}}^{1/2}} . \tag{5.5.16}$$

This ratio has to be clearly larger than one. Unfortunately, fluxes are restricted to integer values, and the ratio will be always smaller than one. Nevertheless, playing around with flux values smaller than one we can get the right mass hierarchy. E.g. take the values $h = 1/220$, $\tilde{\mathfrak{f}} = 1/1810$, $\mathfrak{f} = 6/49$, $q = 1/8$, $g = 1/10$, $p = 1/10000$ and $\lambda = 10$, then table 5.5.2 shows that all mass ratio can be fulfilled.

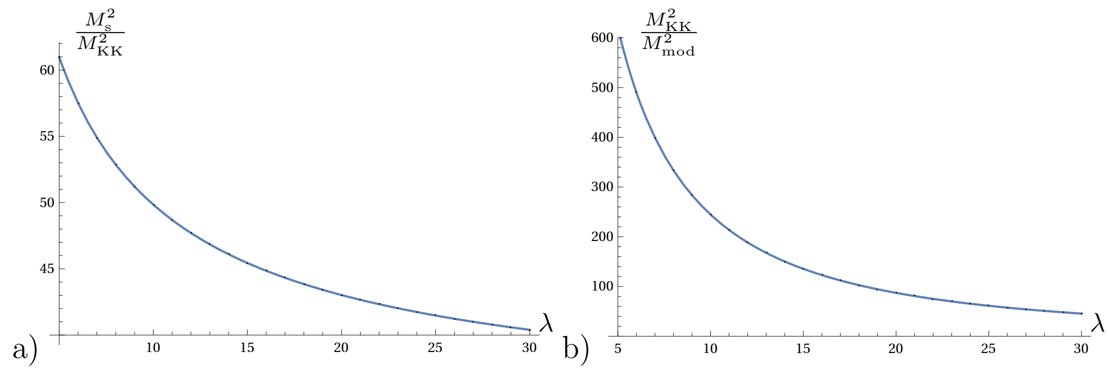

Figure 5.3: Ratio of relevant mass scales for a) string scale over Kaluza-Klein scale and b) the Kaluza-Klein scale over the heaviest modulus. Fluxes are chosen rational with values $h = 1/220$, $\tilde{\mathfrak{f}} = 1/1810$, $\mathfrak{f} = 6/49$, $q = 1/8$, $g = 1/10$ and $p = 1/10000$.

5.5.3 Discussion

We saw that a fully working model was only possible by giving up on integer fluxes. We already saw previously (4.1.17) that the integer fluxes can be redefined to

$$\begin{aligned} \widetilde{h}_0 &= h_0 + b_i \overline{h}^i\,, & \widetilde{h}_i &= h_i + a_{ij}\overline{h}^j + b_i \overline{h}^0\,, \\ \widetilde{q}_0 &= q_0 + b_i \overline{q}^i\,, & \widetilde{q}_i &= q_i + a_{ij}\overline{q}^j + b_i \overline{q}^0 \\ \widetilde{f}_0 &= f_0 + b_i \overline{f}^i\,, & \widetilde{f}_i &= f_i + a_{ij}\overline{f}^j + b_i \overline{f}^0\,. \end{aligned} \tag{5.5.17}$$

The tilded fluxes therefore can be non-integer since we have

$$a_{ij} = -\frac{1}{2}\int_M h_i \wedge h_j \wedge h_j, \qquad b_i = \frac{1}{24}\int_M c_2(M) \wedge h_i\,. \tag{5.5.18}$$

Note that the redefined fluxes can indeed be tuned to be smaller than one, depending on the geometry of the Calabi Yau manifold. The minimal possible value in general can still not be randomly small but only of value 1/24.
Hence, our example would still not be realisable but in principle one has more freedom to choose fluxes such that other models might be realistic.

5.6 Summary

Moduli stabilisation with non-geometric fluxes offers rich possibilities for realising axion monodromy inflation. Despite their unclear status in string theory, non-geometric fluxes allow for stabilising the volume moduli already at tree-level and

Parameter	Value
Δc	$93\,M_{\mathrm{Pl}}$
N_e	61
r	0.0980
n_s	0.9667
$\mathcal{P}$	$2.14 \cdot 10^{-9}$
M_{s}	$1.04 \cdot 10^{17}\,\mathrm{GeV}$
M_{KK}	$1.49 \cdot 10^{16}\,\mathrm{GeV}$
M_{inf}	$4.89 \cdot 10^{15}\,\mathrm{GeV}$
M_{mod}	$\{11.99, 4.81, 2.38, 6.81, 2.47\} \cdot 10^{14}\,\mathrm{GeV}$
H_{inf}	$7.82 \cdot 10^{13}\,\mathrm{GeV}$
M_{θ}	$1.70 \cdot 10^{13}\,\mathrm{GeV}$

Table 5.3: Summary of inflationary parameters for $\lambda = 10$.

thereby circumventing the problem of too light Kähler moduli which would spoil single field inflation. Nevertheless, some issues remain. The backreaction of the fluxes is of order one, i.e. the theory is not in the dilute flux limit. The recurring problem is clearly the difficulty of getting a correct hierarchy of mass scales. If there is a physical reason, for instance an analogon to the weak gravity conjecture for axion monodromy, is still unclear.

Chapter 6

Moduli stabilisation in the vicinity of the conifold

In the past chapters, we focused on a moduli stabilsation scheme for axion monodromy inflation. We considered the regime in moduli space where the complex structure is large. But moduli spaces are rich and offer other interesting points. One of these points is the conifold singularity. There, a three-cycle shrinks to zero size. Near the singularity, exponential mass hierarchies naturally appear. Furthermore, the complex structure moduli admit a shift symmetry. These two properties make it tempting to try moduli stabilisation for axion inflation also in this region in moduli space. The form of the conifold induced terms are periodic and are seducing to consider aligned inflation.
In this chapter, we look at a simple toy model for moduli stabilisation near the conifold and also discuss a more evolved toy model and its application to aligned inflation. The mass hierarchies are in the correct order except for one light Kähler modulus. This chapter is based on [4].

6.1 The conifold

The conifold region has some interesting physical aspects. For instance, at the singularity the warping is infinite. To describe this, the standard supergravity approach is not sufficient. Hence, we will not consider the strongly warped regime but rather stay in the vicinity of the singularity.

6.1.1 Periods near the conifold

In the vicinity of the conifold, the form of the periods has an interesting form, and the effective supergravity approach is still valid. We saw before that a choice of complex structure moduli is described by the periods

$$X^i = \int_{B_i} \Omega \tag{6.1.1}$$

and

$$F_i = \int_{A_i} \Omega \tag{6.1.2}$$

of the holomorphic three-form Ω. In the moduli space of the Calabi Yau three-fold, these periods can correspond to projective coordinates. A complex rescaling of the holomorphic three-form leaves the complex structure invariant. In such a moduli space $\mathcal{M}$ of a Calabi Yau manifold there may be a region in which one of the periods vanishes. Let us call this period/coordinate $F_c = Z$. The point in moduli space where this coordinate shrinks to zero is a singularity called *conifold*. Moreover, for a closed loop around the conifold singularity, the symplectic dual period undergoes a monodromy $X^c \to X^c + F_c$. The remaining periods should stay finite at the conifold locus. Near $F_c \sim 0$, this is captured by the period over the dual cycle containing a logarithmic term

$$X^c \sim \text{const.} + \frac{1}{2\pi i} Z \log Z\,. \tag{6.1.3}$$

Plugging the logarithmic form of the periods into the Kähler metric $G_{i\bar{j}}$, the singularity becomes visible. The metric

$$G_{i\bar{j}} \sim \log Z\overline{Z} \tag{6.1.4}$$

is divergent near $Z = 0$. For applications to axion inflation, note that the Kähler potential is invariant under $Z \to e^{i\theta} Z$, which implies a shift symmetry for the phase of the conic modulus.

The physical meaning of this singularity [117] is a break down of the effective theory at this point of moduli space. Former massive states become massless. But our effective theory is only valid if these states are massive and can be integrated out, which happens to be the case in the non-singular points in moduli space. In the vicinity of the conifold, the effective theory is valid and the periods are still of the form (6.1.3). The unusual logarithmic form of a period nevertheless leads to interesting new physics and hierarchies in the well-understood domain.

6.1.2 The conifold and warping

It was shown [118] that a conifold corresponds to a warped geometry. The simplest ansatz for such a geometry is the following

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}dy^m dy^n \,. \tag{6.1.5}$$

The external four dimensional coordinates are x^μ, while y^μ are the coordinates on the internal space. The warp factor $A(y)$ depends on these internal coordinates and can be expressed in terms of the volume $\mathcal{V}$ and the conic modulus Z [119,120] as

$$e^{-4A(y)} \sim 1 + \frac{1}{(\mathcal{V}|Z|^2)^{\frac{2}{3}}} \,. \tag{6.1.6}$$

The warp factor obviously vanishes for

$$\mathcal{V}|Z|^2 \gg 1 \,, \tag{6.1.7}$$

and the metric is unwarped. This is a regime where the standard supergravity regime is valid and we can compute physics without considering extra massless states in the strongly warped region. Physically, this condition means that the physical size of the three-cycle is large

$$\mathrm{Vol}(A) = \mathcal{V}^{\frac{1}{2}} \left| \textstyle\int_A \Omega_3 \right| = (\mathcal{V}|Z|^2)^{\frac{1}{2}} \,, \tag{6.1.8}$$

and we are not at the singularity.

6.2 Example for moduli stabilisation: Mirror of the quintic

We consider as a simple example for moduli stabilisation at the conifold the mirror dual of the quintic $\mathbb{P}_4[5]^{(101,1)}$. The Hodge numbers of this manifold are given by $h^{1,1} = 101$ and $h^{2,1} = 1$. It is predestined to use its complex structure sector as a one parameter example. The complex structure regime is given by the parameter ψ and the hypersurface constraint

$$P = \sum_{i=1}^{5} Z_i^5 - 5\psi \prod_{i=1}^{5} Z_i = 0 \,. \tag{6.2.1}$$

This hypersurface constraint becomes singular, i.e. $P = \partial_i P = 0$ for $i = 1, \dots, 5$ for the co-dimension one locus $\psi = 1$. The second derivatives do not vanish so that one has a conifold singularity. At the singularity $u = 5(\psi - 1) = 0$, a three-cycle B^1 shrinks to zero size, i.e. the corresponding period has to vanish like $F_1 = \int_{B^1} \Omega_3 \sim u + O(u^2)$. The explicit form of the periods can be deduced by solving the Picard-Fuchs equations, fourth order differential equations depending on the periods. We use here the solution of the Picard-Fuchs equations in the regime $|\psi| < 1$ [121], which is given by the periods

$$\varpi_f(\psi) = -\frac{1}{5} \sum_{n=1}^{\infty} \frac{\lambda^{2n}\, \Gamma\left(\frac{n}{5}\right) (5\,\psi)^n}{\Gamma(n)\, \Gamma^4\left(1 - \frac{n}{5}\right)}\,. \tag{6.2.2}$$

via

$$\varpi_i(\psi) = -\left(\frac{2\pi i}{5}\right)^3 \varpi_f(\lambda^i \psi) \tag{6.2.3}$$

with $i = 0, 1, 2, 4$ and $\lambda = \exp(2\pi i/5)$.

6.2.1 Periods of the mirror ot the quintic

After expansion around the conifold locus $u \sim 0$ and application of an $\mathrm{Sp}(4;\mathbb{Z})$ transformation, the periods read [122, 123]

$$\begin{aligned} F_0 &= \tilde{a}_0 + \tilde{b}_0\, u + \dots\,, \\ F_1 &= a u + \dots \\ X^0 &= a^0 + b^0\, u + \dots\,, \\ X^1 &= -\frac{1}{2\pi i} F_1 \log u + c + d\, u + \dots \end{aligned} \tag{6.2.4}$$

with the numerical values of the parameters

$$\begin{aligned} a &= \frac{\sqrt{5}}{2\pi i}\,, \quad c = 1.07072586843016\,, \quad d = -0.0247076138044847 \\ a^0 &= 12.3900325542991\,, \quad b^0 = 2.033209433405164 \\ \tilde{a}_0 &= 6.19501627714957 - 0.64678699225205\, i \\ \tilde{b}_0 &= 1.016604716702582 - 0.075383347561773\, i\,. \end{aligned} \tag{6.2.5}$$

For readability we introduce inhomogeneous coordinates by dividing by X^0 and define the conic modulus via

$$Z = \frac{F_1}{X^0} = \frac{a}{a^0} u + O(u^2)\,. \tag{6.2.6}$$

The period vector $\Pi^T = (F_0, F_1, X^0, X^1)$ is then given by

$$\Pi = X^0 \begin{pmatrix} \tilde{A}_0 - \tilde{B}_0 Z + O(Z^2) \\ Z \\ 1 \\ -\frac{1}{2\pi i} Z \log Z + C + DZ + O(Z^2) \end{pmatrix} \tag{6.2.7}$$

with parameters

$$\begin{aligned} \tilde{A}_0 &= \frac{\tilde{a}_0}{a^0} = \tfrac{1}{2} - 0.05220220282659\, i \\ \tilde{B}_0 &= -\frac{a^0 \tilde{b}_0 - b^0 \tilde{a}_0}{a\, a^0} = 0.08641932600114 \\ C &= \frac{c}{a^0} = 0.08641932600114 = \tilde{B}_0 \\ D &= \frac{1}{a}\left(d - \frac{b^0 c}{a^0} + \frac{a}{2\pi i} \log\left(\frac{a}{a^0}\right)\right) = -\frac{1}{4} + 0.001859112592390\, i\,. \end{aligned} \tag{6.2.8}$$

In this patch the Kähler potential admits a shift symmetry $Z \to e^{i\theta} Z$, furthermore the linear term vanishes. The Kähler potential is of the form

$$\begin{aligned} K_{\text{cs}} &= -\log\left[-i\Pi^\dagger \Sigma\, \Pi\right] \\ &= -\log\left[\frac{1}{2\pi}|Z|^2 \log(|Z|^2) + A + O(|Z|^2)\right] \end{aligned} \tag{6.2.9}$$

with $A = 0.10440$ and the symplectic pairing

$$\Sigma = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}\,. \tag{6.2.10}$$

Now we want to use these periods to stabilise moduli close to the conifold singularity. For small Z, the periods are expanded as

$$\begin{aligned} X^0 &= 1, & X^1 &= -\frac{1}{2\pi i} Z \log Z + \tilde{B}_0 + DZ + \ldots \\ F_0 &= \tilde{A}_0 - \tilde{B}_0\, Z + \ldots\,, & F_1 &= Z\,. \end{aligned} \tag{6.2.11}$$

6.2.2 Stabilising the conic modulus Z

We turn on fluxes such that the superpotential is given by

$$\begin{aligned} W &= f\,X^1 + ihSF_1 + ih'SF_0 \\ &= f\left(-\frac{1}{2\pi i} Z\log Z + \tilde{B}_0 + D\,Z + \dots\right) + ihS\,Z + ih'S(\tilde{A}_0 - \tilde{B}_0\,Z + ...)\,. \end{aligned} \tag{6.2.12}$$

We assume $h \gg h'\,\tilde{B}_0$. The F-term condition $D_Z = 0$ is used to stabilise the conic modulus. In the expansion around $Z = 0$ it reads

$$\underbrace{\frac{f}{2\pi i}\log Z - ih\,S}_{\text{order }\log Z} + \underbrace{\frac{f}{2\pi i} - D\,f}_{\text{order }O(1)} + \dots = 0\,. \tag{6.2.13}$$

Solving this equation, the vacuum expectation value of the conic modulus is indeed exponentially small as required for consistency

$$Z \sim \hat{C}\,e^{-\frac{2\pi h}{f}S}\,, \qquad \text{with} \quad \hat{C} = \exp\left(-1 + 2\pi i\,D\right)\,. \tag{6.2.14}$$

The mass of the conic modulus can be computed via the second derivative of the scalar potential in the minimum

$$V_{Z\overline{Z}} = \partial_Z \partial_{\overline{Z}} V = e^K\,G^{Z\overline{Z}} \partial_Z(D_Z W)\,\partial_{\overline{Z}}(D_{\overline{Z}}\overline{W})\Big|_{D_Z W = 0}\,, \tag{6.2.15}$$

with the Kähler metric component

$$G_{Z\overline{Z}} \sim -\frac{1}{2\pi A}\log(|Z|^2)\,. \tag{6.2.16}$$

When we insert this we get

$$V_{Z\overline{Z}} \sim -\frac{1}{2\text{Re(S)}\mathcal{V}^2|Z|^2}\frac{f^2}{2\pi\log(|Z|^2)}\,. \tag{6.2.17}$$

After normalising $M_Z^2 = \frac{1}{2}G^{Z\overline{Z}}V_{Z\overline{Z}}$, the mass of the canonically normalised complex structure modulus becomes

$$M_Z^2 \sim \frac{M_{\text{pl}}^2}{4\text{Re(S)}\mathcal{V}^2|Z|^2}\frac{Af^2}{\log^2(|Z|^2)}\,. \tag{6.2.18}$$

Using the expression for the string scale, $M_{\rm s}^2 = \frac{M_{\rm pl}^2 g_s^{\frac{1}{2}}}{\mathcal{V}}$, one can write

$$M_Z^2 \sim \frac{M_{\rm s}^2}{\mathcal{V}|Z|^2} \frac{A f^4 g_s^{\frac{5}{2}}}{16\pi^2 h^2} \,. \tag{6.2.19}$$

Obviously the condition for negligible warping $\mathcal{V}|Z|^2 \gg 1$ has to be fulfilled to ensure that the conic modulus is significantly lighter than the string scale. That is another way to show that this is the condition for our effective theory to be valid. The F-term for the dilaton is given by

$$D_S W = \left(i\, h\hat{C} e^{-\frac{2\pi h}{f} S} + ih'\, \tilde{A}_0\right) - \frac{1}{S+\overline{S}} \left(\tilde{B}_0\, f + \frac{f}{2\pi i} \hat{C}\, e^{-\frac{2\pi h}{f} S} + ih' S \tilde{A}_0\right) \tag{6.2.20}$$

and indentical to what one gets after inserting the solution for the conic modulus in the superpotential, which reads then

$$W_{\rm eff} = \tilde{B}_0\, f + \frac{f}{2\pi i} \hat{C}\, e^{-\frac{2\pi h}{f} S} + ih' S \tilde{A}_0 + \ldots \,. \tag{6.2.21}$$

The exponential term generated through inserting the solution for Z is mimicking a $D(-1)$-instanton term.

6.2.3 Stabilisation of the axio-dilaton

Now we want to stabilise the axio-dilaton. We consider the cases with $h' \neq 0$ and $h = 0$.

<u>*Case A:*</u> $h' \neq 0$

For non-zero h', the effective superpotential after integrating out the conic modulus reads

$$W_{\rm eff} = \tilde{B}_0\, f + ih' S \tilde{A}_0 \tag{6.2.22}$$

and we get for the F-term F_S the equation

$$D_S W_{\rm eff} = -\frac{1}{S+\overline{S}} \left(-ih' \overline{S} \tilde{A}_0 + \tilde{B}_0\, f\right) \,. \tag{6.2.23}$$

Setting this F-term to zero, one gets the following values for the real and imaginary parts of the axio-dilaton

$$\frac{1}{g_s} = \frac{f}{h'}\,\mathrm{Im}\Big(\frac{\tilde{B}_0}{\tilde{A}_0}\Big)\,, \qquad C_0 = -\frac{f}{h'}\,\mathrm{Re}\Big(\frac{\tilde{B}_0}{\tilde{A}_0}\Big)\,. \tag{6.2.24}$$

Inserting these vevs into the solution for the conic modulus, its flux- and geometry-dependence reads

$$Z \sim \hat{C}\,\exp\left(2\pi i\,\frac{\tilde{B}_0}{\tilde{A}_0}\frac{h}{h'}\right)\,. \tag{6.2.25}$$

Taking care of being in the perturbative regime and the conic modulus being close to the conifold, the fluxes have to fulfill the inequalities $|h| > |h'|$ and $|f| > |h'|$, while the relative signs of the fluxes depend on the sign of the geometrical parameters $\mathrm{Im}(\frac{\tilde{B}_0}{\tilde{A}_0})$. The mass of the axio-dilaton is given by

$$M_S^2 = \frac{M_{\mathrm{pl}}^2}{\mathcal{V}^2\mathrm{Re}(S)}\frac{|\tilde{B}_0 f|^2}{A}\,. \tag{6.2.26}$$

The mass of the conic modulus is exponentially larger than the axio-dilaton mass. Therefore, we can integrate it out.

$$\frac{M_S^2}{M_Z^2} \sim |Z|^2 \sim \exp\left(-4\pi\,\mathrm{Im}\Big(\frac{\tilde{B}_0}{\tilde{A}_0}\Big)\frac{h}{h'}\right)\,, \tag{6.2.27}$$

Case B: $h' = 0$

In this case the effective superpotential after inserting the vev for the conic modulus has a KKLT-like exponential term and is of the form

$$W_{\mathrm{eff}} = \tilde{B}_0\,f + \frac{f}{2\pi i}\hat{C}\,e^{-\frac{2\pi h}{f}S}\,. \tag{6.2.28}$$

The equation for the F-term $D_S W = 0$ reads

$$ih\hat{C}e^{-\frac{2\pi h}{f}S} - \frac{f\tilde{B}_0}{2\mathrm{Re(S)}} + \ldots = 0\,. \tag{6.2.29}$$

Defining $\tilde{s} := \frac{2\pi h}{f}\mathrm{Re}(S)$ and taking the values of the coefficients for the quintic (6.2.5) one gets for the axion $C_0 = -\frac{f}{h}\left(\mathrm{Re}(D) + \frac{1}{4}\right)$ and a transcendental relation for the saxion

$$e^{\tilde{s}} = \frac{|\hat{C}|}{\pi\tilde{B}_0}\tilde{s}\,, \qquad \text{with} \quad \hat{C} = \exp\left(-1 + 2\pi i D\right) \tag{6.2.30}$$

For the mirror of the quintic we get $\lambda = \frac{i\hat{C}}{\pi \tilde{B}_0} \sim 1.37$. In that case, the transcendental equation does not admit a solution. Nevertheless, we consider a toy potential for inflation assuming there is a solution for the dilaton in the perturbative regime. Calculating the potential and inserting (6.2.30) gives

$$V(\theta) = \frac{1}{A\,\mathcal{V}^2(S+\overline{S})}\, 2|f\tilde{B}_0|^2 \left[1 - \cos\left(\tfrac{2\pi h}{f}\theta\right)\right] . \tag{6.2.31}$$

Then, we can compute the potential for the canonically normalised field $\tilde{\theta} = \theta/(\sqrt{2}\mathrm{Re}(S))$

$$V(\tilde{\theta}) = \frac{1}{A\,\mathcal{V}^2\mathrm{Re}(S)}\, |f\tilde{B}_0|^2 \left[1 - \cos\left(\sqrt{2}\tilde{s}\,\tilde{\theta}\right)\right] \tag{6.2.32}$$

so that we can read off the axion decay constant $f_{\tilde{\theta}} = 1/(\sqrt{2}\tilde{s}) < 1$, which is sub-Planckian. For the masses of the conic modulus and the axion we get

$$M_{\tilde{\theta}}^2 \sim \frac{M_{\mathrm{pl}}^2 hf}{\mathcal{V}^2} \frac{4\pi|\tilde{B}_0|^2\tilde{s}}{A} \tag{6.2.33}$$

and

$$M_Z^2 \sim \frac{M_{\mathrm{pl}}^2 hf}{\mathcal{V}^2} \frac{A}{8\pi\,|\tilde{B}_0|^2\tilde{s}} . \tag{6.2.34}$$

Obviously, both fields are of the same scale up to geometry dependent terms and there is no mass hierarchy.

6.3 The conic LVS

The condition $\mathcal{V}|Z|^2 \gg 1$ together with the fact that the conic field is stabilised at exponentially small values $Z \sim e^{-\mathrm{sth}}$ makes the LARGE volume scenario the natural candidate for Kähler moduli stabilisation close to the conifold, since the volume is exponentially large.

Let us take a look if this moduli stabilisation procedure is indeed fulfilling the constraint. We consider a swiss-cheese Calabi Yau three-fold with the corrected Kähler potential

$$K = -2\log\left(\tau_b^{\frac{3}{2}} - \tau_s^{\frac{3}{2}} + \frac{\xi}{2}\mathrm{Re}(S)^{\frac{3}{2}}\right) . \tag{6.3.1}$$

The conic LARGE volume scenario then is based on the superpotential

$$W_{\rm inst}(T_s) = W_0 + A_s\, Z^N\, e^{-a_s T_s}\,. \tag{6.3.2}$$

The pfaffian A_s is in general an unknown function of complex structure moduli. We are interested in a LARGE volume scenario close to the conifold point and therefore parametrise the unknown pfaffian $A_s(Z,U)$ as $A_s(U) * Z^N$ to extract the dependence on the conic modulus. We do this to see under which conditions the relation $\mathcal{V}|Z| \gg 1$ can be satisfied.
The only difference to the standard LVS is the general unknown pfaffian $A_s \to A_s\, Z^N$. Up to CY-geometry dependent coefficients of order one, after freezing the axion ρ_s, the dominant terms in the scalar potential read

$$V_{\rm LVS}(T) = e^{K_{cs}}\frac{g_s}{2}\left(\frac{|a_s A_s Z^N|^2\sqrt{\tau_s}\, e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{W_0\,|a_s A_s Z^N|\,\tau_s\, e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\xi\, W_0^2}{g_s^{\frac{3}{2}}\,\mathcal{V}^3}\right). \tag{6.3.3}$$

Recall that the masses of the fields are

$$\begin{aligned} M^2_{\tau_b} &\sim O(1)\frac{W_0^2\,\xi}{g_s^{\frac{1}{2}}\,\mathcal{V}^3}M^2_{\rm pl}\,, \qquad M^2_{\rho_b} \sim 0\,, \\ M^2_{\tau_s} &\sim M^2_{\rho_s} \sim O(1)\frac{a_s^2\,W_0^2\,\xi^{\frac{4}{3}}}{g_s\,\mathcal{V}^2}M^2_{\rm pl}\,, \end{aligned} \tag{6.3.4}$$

and do not depend on the parameter A_s. Hence, they do not depend on the conic modulus Z. Note also the lightness of the big Kähler modulus τ_b. Later on, this lightness becomes problematic for our inflationary scenario.
The small Kähler modulus and the volume at the minimum are stabilised at

$$\tau_s = \frac{(4\xi)^{\frac{2}{3}}}{g_s}\,, \qquad \mathcal{V} = \frac{W_0\,\xi^{\frac{1}{3}}}{2^{\frac{1}{3}}\,g_s^{\frac{1}{2}}\,|a_s A_s Z^N|}e^{a_s\tau_s}\,. \tag{6.3.5}$$

Now, the constraint $\mathcal{V}|Z|^2 \gg 1$ can be explicitly calculated in the considered example and becomes

$$\mathcal{V}|Z|^2 \sim \exp\left[\frac{a_s}{g_s}\left(\frac{h(N-2)}{f} + (4\xi)^{\frac{2}{3}}\right)\right]. \tag{6.3.6}$$

This term is large for $N > 1$. Even when this is not satisfied, an appropriate tuning of the fluxes allows to fulfill the requirement of negligible warping. We conclude that the LARGE volume scenario is indeed the natural Kähler moduli stabilisation procedure near the conifold.

6.4 Example for aligned inflation

For realising aligned inflation, we make use of the shift symmetry of the complex structure moduli at the conifold. Since we need an exponential term which we get by integrating out the exponentially heavy conic modulus, we need to consider an example with one additional complex structure modulus.
The inflationary scenario will be based on a toy model which is similar to the projective space $\mathbb{P}_{11226}[12]$ but assumes simplified periods. For later discussion, we also calculate the periods of $\mathbb{P}_{11226}[12]$.

6.4.1 Periods of $\mathbb{P}_{11226}[12]$

The mirror of the this manifold has two complex structure moduli that appear as deformations of the hypersurface constraint

$$P = z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 - 12\psi\, z_1\, z_2\, z_3\, z_4\, z_5 - 2\phi\, z_1^6\, z_2^6\,. \tag{6.4.1}$$

The conifold is defined by the equation $864\psi^6 + \phi = 1$. For the parameter ψ and ϕ small, the fundamental period reads [124, 125]

$$\varpi_f(\psi,\phi) = -\frac{1}{6}\sum_{n=1}^{\infty} \frac{\Gamma\left(\frac{n}{6}\right)(-12\,\psi)^n\, u_{-\frac{n}{6}}(\phi)}{\Gamma(n)\,\Gamma^2\left(1-\frac{n}{6}\right)\,\Gamma\left(1-\frac{n}{2}\right)} \tag{6.4.2}$$

with

$$u_{-\frac{n}{6}}(\phi) = \frac{e^{-i\pi\frac{n}{12}}}{2\,\Gamma\left(\frac{n}{6}\right)} \sum_{m=0}^{\infty} \frac{e^{i\pi\frac{m}{2}}\,\Gamma\left(\frac{m}{2}+\frac{n}{12}\right)\,(2\phi)^m}{m!\,\Gamma\left(1-\frac{m}{2}-\frac{n}{12}\right)}\,. \tag{6.4.3}$$

This holds for $|\phi| < 1$ and $\left|\frac{864\psi^6}{\phi\pm1}\right| < 1$. The periods which are solutions to the Picard-Fuchs equations can be deduced from (6.4.2) and read

$$\varpi_i(\psi,\phi) = -\frac{(2\pi i)^3}{\psi}\varpi_f(\lambda^i\psi, \lambda^{6i}\phi)\,, \tag{6.4.4}$$

with $i = 0,\ldots,5$ and $\lambda = \exp(\pi i/6)$. The symplectic basis [126, 127] can be computed by multiplication with the transition matrix

$$\begin{pmatrix} F_0 \\ F_1 \\ F_2 \\ X^0 \\ X^1 \\ X^2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \varpi_0 \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \\ \varpi_4 \\ \varpi_5 \end{pmatrix}, \tag{6.4.5}$$

and reads

$$\begin{aligned} F_0 &= 4323.04i - 1548.4i\,\xi + 107.7i\,\phi - 3893.22i\,\xi^2 - 278.46i\,\xi\phi + 27.78i\,\phi^2\,, \\ F_1 &= 3191\,\xi + 172.29\,\phi + 7583.59\,\xi^2 - 533.36\,\xi\phi + 65.2\,\phi^2\,, \\ F_2 &= (-492.72 + 1976.76i) + (372.45 - 302.3i)\xi - (258.97 + 58.87i)\phi \\ &\quad -(436.95 - 262.39i)\xi^2 - (6.5 + 14.41i)\xi\phi - (3.09 - 24.14i)\phi^2 \end{aligned} \tag{6.4.6}$$

and

$$\begin{aligned} X^0 &= -(994.58 + 184.76i) + (859.48 + 471.9i)\xi + (10.04 - 112.7i)\phi \\ &\quad +(1831.84 + 2209i)\xi^2 - (136.09 - 124.82i)\xi\phi + (3.13 + 10.25i)\phi^2\,, \\ X^1 &= -\frac{1}{2\pi i}F_1 \log F_1 + 784.36i - 4997.53i\,\xi - 185.8i\,\phi + \dots\,, \\ X^2 &= 369.52i - 943.81i\,\xi + 225.4i\,\phi - 4418i\,\xi^2 - 249.64i\,\xi\phi - 20.5i\,\phi^2\,. \end{aligned} \tag{6.4.7}$$

For computing these periods, we stick to the region around the point $\psi = \psi_0 = 864^{-\frac{1}{6}}$ and $\phi = 0$. Defining $\psi = \psi_0 + \xi$ the periods up to quadratic order in (ξ, ϕ) are computed numerically up to $n \sim 20000$ in (6.4.2). For simplicity, we want to switch to inhomogeneous coordinates $F_0 = 1$. Therefore we substitute

$$\phi \to -18.52\,\xi + 25.09i\,Z - 231.17\,\xi^2 + 408.85i\,\xi Z + 222.58\,Z^2 \tag{6.4.8}$$

and

$$\xi \to 1.97\,Y + 1.13i\,Z - 62.84\,Y^2 - 8.7i\,ZY + 4.07\,Z^2\,. \tag{6.4.9}$$

Then the periods take the simple form

$$\begin{aligned} F_0 &= 1\,, \\ F_1 &= Z\,, \\ F_2 &= (0.46 + 0.11i) + (1.10 - 2.17i)Y - 0.19\,Z \\ &\quad - (7.34 - 14.73i)\,Y^2 + (2.71 + 1.42i)\,YZ + (0.11 - 1.69i)\,Z^2 \end{aligned} \tag{6.4.10}$$

and

$$\begin{aligned} X^0 &= (-0.04 + 0.23i) + (1.10 + 0.06i)Y + 0.17\,Z \\ &\quad - (7.34 + 1.83i)\,Y^2 + (0.55 + 1.42i)\,YZ + (0.11 - 0.17i)\,Z^2\,, \\ X^1 &= -\frac{1}{2\pi i}Z\log Z + 0.18 - 0.42\,Y - 1.43i\,Z + \dots\,, \\ X^2 &= 0.09 - 2.19\,Y + 14.67\,Y^2 - 2.84i\,YZ - 0.22\,Z^2\,. \end{aligned} \tag{6.4.11}$$

In terms of the new variables Z and Y, the Kähler potential takes a simple form and reads

$$\begin{aligned} K_{\rm cs} &= -\log\left[-i\Pi^\dagger\Sigma\,\Pi\right] \\ &= -\log\left[\frac{1}{2\pi}|Z|^2\log\left(|Z|^2\right)+A+{\rm Re}Y+B\,({\rm Re}Y)^2+C\,|Z|^2\ldots\right]. \end{aligned} \tag{6.4.12}$$

with $A = 0.44$ and $B = -19.05$ and $C = -2.86$. The Kähler potential exhibits a shift symmetry for both complex structure fields. The conic modulus has an axionic phase $Z \to e^{i\theta}Z$, while the imaginary part of Y exhibits a shift symmetry $\mathrm{Im}(Y) \to \mathrm{Im}(Y) + \theta$. In the later realisation of aligned inflation, we will use the shift symmetry of Y for building an axion inflation model.

6.4.2 Moduli stabilisation

We now want to create a mass hierarchy by using the procedure we introduced earlier

$$W = W_0 + \Delta W_{ax}\,. \tag{6.4.13}$$

The basic idea is that ΔW_{ax} is generated by terms of order Z. First, we take a look at the tree-level term to give a mass to the non-inflatonic fields.

Stabilising saxions and one axion

After out integration of the Z-modulus, we are left with an effective Kähler potential

$$K_{\rm eff} = -2\log\mathcal{V} - \log(S+\overline{S}) - \log\left(A + \tfrac{1}{2}(Y+\overline{Y})\right) \tag{6.4.14}$$

and an effective superpotential

$$W_{\rm eff} = f\,\alpha + h'\,\beta S + \hat{f}'\,\gamma Y \tag{6.4.15}$$

with general $\alpha, \beta, \gamma \in \mathbb{C}$ and $A \in \mathbb{R}$. Since in that case the minimum equations $D_S W = 0$ and $D_Y W = 0$ stabilise the dilaton at zero, we include one more order into the Kähler potential and take

$$\begin{aligned} K_{\rm eff} &= -2\log\mathcal{V} - \log(S+\overline{S}) - \log\Big(A + \kappa\,{\rm Re}Y - ({\rm Re}Y)^2\Big) \\ W_{\rm eff}^{(0)} &= i\alpha\big(f + h'\,S + \hat{f}'\,Y\big) \end{aligned} \tag{6.4.16}$$

and for the superpotential we choose $A, \alpha, \kappa \in \mathbb{R}$, and for simplicity $\alpha = \beta = \gamma$. At the minimum, we get the vacuum expectation values

$$\begin{aligned} \Sigma &= \hat{f}'\,\zeta_0 + h'\,c_0 = 0 \\ s = s_0 &= \frac{1}{h'}\sqrt{f^2 - A\,\hat{f}'^2 + \kappa f \hat{f}'} \\ \mathrm{Re}(Y) = y_0 &= \frac{1}{\hat{f}'}\Big(- f + \sqrt{f^2 - A\,\hat{f}'^2 + \kappa f \hat{f}'}\Big) . \end{aligned} \tag{6.4.17}$$

This simplifies for $f/\hat{f}' \gg 1$ and $\kappa = 0$ to

$$\Sigma = 0\,, \qquad s_0 = \frac{f}{h'}\,, \qquad y_0 = -\frac{A\hat{f}'}{2f} + O\Big((\hat{f}'/f)^2\Big)\,. \tag{6.4.18}$$

6.4.3 Aligned Inflation

Aligned inflation is now realised by including the exponential term coming from integrating out the conic modulus. The superpotential then reads

$$\begin{aligned} W_{\mathrm{eff}} &= W_{\mathrm{eff}}^{(0)} + W_{\mathrm{eff}}^{(1)} \\ &= i\alpha\big(f + h'\,S + \hat{f}'\,Y\big) + \frac{f\hat{C}}{2\pi i}\,\exp\Big(-\tfrac{2\pi}{f}\big(hS + \hat{f}Y\big)\Big)\,. \end{aligned} \tag{6.4.19}$$

To generate such a superpotential, we have started with

$$W_{\mathrm{eff}} = i\alpha\big(f + h'\,S + \hat{f}'\,Y\big) + Z\,\Big(f + h\,S + \hat{f}\,Y\Big) + ... \tag{6.4.20}$$

and inserted the vev for the conic modulus Z. The exponential term creates a potential for the axionic inflaton orthogonal to Σ. Naivly integrating out the non-inflatonic fields leads to

$$\begin{aligned} D_S W_{\mathrm{eff}}|_{S_0,Y_0} &= c_S\,|Z|\exp\Big(-\tfrac{2\pi i}{f}\Theta\Big)\,, \\ D_Y W_{\mathrm{eff}}|_{S_0,Y_0} &= c_Y\,|Z|\exp\Big(-\tfrac{2\pi i}{f}\Theta\Big) \end{aligned} \tag{6.4.21}$$

with $\Theta = \big(hc + \hat{f}\zeta\big)$ and $c_S, c_Y \neq 0$. It turns out that in this simple approach $V_{\mathrm{eff}}|_{S_0,Y_0}$ is independent of the inflaton and therefore does not generate an inflationary potential and is non-vanishing at order $O(|Z|^2)$. In the true vacuum (at order $O(|Z|^2)$) the vacuum energy should be zero and the remaining axion Θ should be stabilised at $\Theta = 0$. This happens only after considering the backreaction of the exponential term on the saxions vevs. Perturbing around the leading order values y_0, s_0 by $\Delta y_0 \sim O(|Z|)$ and $\Delta s_0 \sim O(|Z|)$, we fix Δy_0 and Δs_0 by requiring $D_S W_{\rm eff}|_{S_0,Y_0,\Theta=0} = D_Y W_{\rm eff}|_{S_0,Y_0,\Theta=0} = 0 + O(|Z|^2)$. For $\kappa = 0$, $f/\hat{f}' \gg 1$ and at leading non-vanishing order in $|Z|$ we find

$$\Delta s_0 \sim -\frac{f}{2\pi\alpha h'}\Big(1+\frac{4\pi h}{h'}\Big)\,|Z|\,, \qquad \Delta y_0 \sim -\frac{A\hat{f}}{2\alpha f}\,|Z|\;. \tag{6.4.22}$$

Then we get an axion dependent potential consisting of the terms

$$\begin{aligned} V_1 &= G^{S\overline{S}}\, D_S W\, D_{\overline{S}}\overline{W} = \frac{|Z|^2}{2\pi^2}\; f^2\Big(1+\frac{4\pi h}{h'}\Big)^2\,\Big(1-\cos\big(\tfrac{2\pi}{f}\Theta\big)\Big) \\ V_2 &= G^{Y\overline{Y}}\, D_Y W\, D_{\overline{Y}}\overline{W} = \frac{|Z|^2\,A}{4\pi^2}\,\hat{f}'^2\,\Big(1+\frac{4\,\pi\hat{f}}{\hat{f}'}\Big)^2\Big(1-\cos\big(\tfrac{2\pi}{f}\Theta\big)\Big)\,. \end{aligned} \tag{6.4.23}$$

For $f/\hat{f}' \gg 1$ and $h/h' \gg 1$ we get the effective inflaton potential

$$V_{\rm eff} = e^{K_{\rm eff}}\,(V_1+V_2) \sim \frac{4|Z|^2}{A\mathcal{V}^2}\;\frac{fh^2}{h'}\;\Big(1-\cos\big(\tfrac{2\pi}{f}\Theta\big)\Big)\,, \tag{6.4.24}$$

which is indeed vanishing at $\Theta = 0$. The canonically normalised axion for $f/f' \gg 1$ is given by

$$\tilde{\Theta} = \frac{h'}{\sqrt{A}\hat{f}'}\Theta\,, \tag{6.4.25}$$

and results in the potential

$$V_{\rm eff} \;=\; \frac{4|Z|^2}{A\mathcal{V}^2}\;\frac{fh^2}{h'}\,\Big(1-\cos\big(\tfrac{2\pi\sqrt{A}(h\hat{f}'-h'\hat{f})}{fh'}\tilde{\Theta}\big)\Big)\;\equiv\;V_0\Big(1-\cos\big(\tfrac{\tilde{\Theta}}{f_{\tilde{\Theta}}}\big)\Big)\,. \tag{6.4.26}$$

The axion decay constant can be read off and is given by the flux ratio

$$f_{\tilde{\Theta}} = \frac{f}{2\pi\sqrt{A}}\,\frac{h'}{h\hat{f}'-h'\hat{f}}\,. \tag{6.4.27}$$

This flux dependent axion decay constant can be tuned large by choosing appropriate fluxes and lead to a smooth inflaton potential.

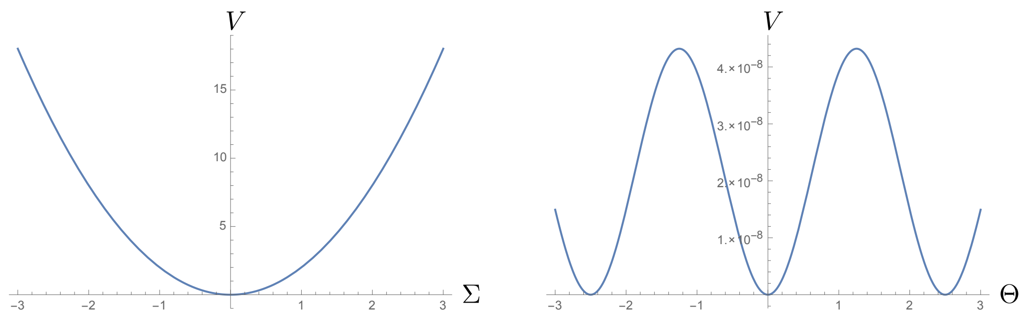

Figure 6.1: Scalar potential for the two axions Σ and Θ for $f = 10$, $h' = \hat{f}' = 1$, $h = -\hat{f} = 2$, $\hat{C} = 1$ and $A = 0.1$.

Mass scales

In the previous chapters on moduli stabilisation for large field inflation the models suffered from an incorrect hierarchy of masses unless we give up on integer fluxes. So let us look at the mass hierarchy in the aligned inflation scenario near the conifold. In our example, the canonically normalised moduli masses are all scaling like

$$M^2_{\mathrm{mod}} = \frac{f^2 g_s}{A\mathcal{V}^2}\,. \tag{6.4.28}$$

While the canonically normalised axion $\tilde{\Theta}$ has a mass which is exponentially suppressed by a factor $|Z|^2$ compared to the other fields

$$M^2_{\tilde{\Theta}} = \frac{V_0}{f^2_{\tilde{\Theta}}}\, M^2_{\mathrm{Pl}} \sim \frac{|Z|^2}{f\,\mathcal{V}^2}\, M^2_{\mathrm{Pl}}\,. \tag{6.4.29}$$

Since the conic modulus was scaling like $M_Z \sim \frac{M_{mod}}{|Z|^2}$, we obtain a mass hierarchy for the moduli of the form

$$M_{\tilde{\Theta}} < M_{\mathrm{mod}} < M_Z\,. \tag{6.4.30}$$

Obviously, our procedure is justified regarding the moduli mass and we have an hierarchically light axion. The scale of inflation is given by

$$M^2_{\mathrm{inf}} \sim V_0^{\frac{1}{2}} \sim \frac{f^{\frac{1}{2}}|Z|}{\mathcal{V}}\,, \tag{6.4.31}$$

and exponentially heavier than the moduli masses. Using the scaling $g_s \sim 1/f$, we get for the ratios of the mass scales

$$\frac{M^2_{\mathrm{inf}}}{M^2_{\mathrm{mod}}} \sim \frac{(\mathcal{V}|Z|^2)}{f^{\frac{1}{2}}|Z|} \qquad \text{and} \qquad \frac{M^2_{\mathrm{inf}}}{M^2_Z} \sim (\mathcal{V}|Z|^2)\frac{|Z|}{f^{\frac{1}{2}}}\,. \tag{6.4.32}$$

Due to the constraint $\mathcal{V}|Z|^2 \gg 1$ the inflationary scale is larger than the moduli masses, but for sufficiently small $|Z|$ can be lower than the mass of the conic complex structure modulus. For a complete consideration we also need to look at the Kähler moduli sector. We use the masses derived in the conic LARGE volume scenario above and see that the inflaton mass is hierarchically heavier than the mass of the lightest Kähler modulus

$$\frac{M_{\tilde{\Theta}}^2}{M_{\tau_b}^2} \sim \frac{\mathcal{V}|Z|^2}{f^{7/2}} \tag{6.4.33}$$

In terms of the effective theory this means, that as long as we can ignore stringy states and our ansatz is valid, the light four-cycle modulus is also lighter than the inflaton

$$\frac{M_{\tilde{\Theta}}^2}{M_{\tau_b}^2} \sim \frac{M_{\mathrm{s}}^2}{f^2 M_Z^2}\,, \tag{6.4.34}$$

which is unfortunate for single field inflation. The mass hierarchy is given by

$$M_{\tau_b} < M_{\tilde{\Theta}} < M_{\mathrm{mod}} < M_{\mathrm{inf}} \sim M_Z < M_{\mathrm{KK}} < M_{\mathrm{s}} < M_{\mathrm{Pl}}\,, \tag{6.4.35}$$

so for single field inflation, everything fits except for the lightness of the small Kähler modulus. In table 6.1, the concrete flux, Z and volume dependence of the mass scales are summarised.

Weak gravity conjecture

Let us discuss if the axion decay constant in this model can be tuned to be super-Planckian and satisfy the weak gravity conjecture (2.2.10)

$$S_{\mathrm{inst}}\, f_{\mathrm{inst}} \leq 1\,. \tag{6.4.36}$$

We did not not induce aligned inflation via an instanton term but by considering a special region in moduli space with exponential terms. Nevertheless, we analyse here if the weak gravity conjecture holds in our example.
The action for the instanton is

$$S_{\mathrm{inst}} = \frac{2\pi}{f}(hs_0 + \hat{f}y_0) \sim \frac{2\pi}{f}hs_0 \sim \frac{2\pi h}{h'}\,, \tag{6.4.37}$$

which leads to a clear violation of the weak gravity conjecture

$$S_{\mathrm{inst}}\, f_{\tilde{\Theta}} \sim \frac{fh}{\sqrt{A}(h\hat{f}' - h'\hat{f})} > 1\,. \tag{6.4.38}$$

Scale	(Mass)^2 in $M_{\rm Pl}^2$
string scale $M_{\rm s}^2$	$\frac{1}{f^{1/2}\,\mathcal{V}}$
Kaluza-Klein scale $M_{\rm KK}^2$	$\frac{1}{\mathcal{V}^{4/3}}$
conic c.s. modulus M_Z^2	$\frac{f}{\mathcal{V}^2\|Z\|^2}$
inflationary mass scale $M_{\rm inf}^2$	$\frac{f^{1/2}\|Z\|}{\mathcal{V}}$
other moduli $M_{\rm mod}^2$	$\frac{f}{\mathcal{V}^2}$
gravitino mass $M_{3/2}^2$	$\frac{f}{\mathcal{V}^2}$
large Kähler modulus $M_{\tau_b}^2$	$\frac{f^{5/2}}{\mathcal{V}^3}$
inflaton $M_{\tilde\Theta}^2$	$\frac{\|Z\|^2}{f\,\mathcal{V}^2}$

Table 6.1: Moduli masses and scales with $g_s \sim 1/f$.

The weak form can still be satisfied if there is some instanton fulfilling the inequality. We assume a potential of the form

$$V \sim e^{-2\,S_{\rm inst}}\left(1-\cos\left(\tfrac{\tilde\Theta}{f_{\tilde\Theta}}\right)\right)+e^{-2\,S^{(2)}_{\rm inst}}\left(1-\cos\left(\tfrac{k\,\tilde\Theta}{f_{\tilde\Theta}}\right)\right), \tag{6.4.39}$$

with $k\in\mathbb{Z}$. If the factor k is large enough, the axion decay constant $f_{\tilde\Theta}/k$ is small. For $S_{\rm inst} < S^{(2)}_{\rm inst}$, the dominating term is the first one.

$$S_{D(-1)} = 2\pi s_0 = \frac{2\pi f}{h'}\,, \qquad f_{D(-1)} = \frac{h'}{2\pi\sqrt{A}\hat f'} = \frac{f_{\tilde\Theta}}{k} \tag{6.4.40}$$

with $k=(f\,\hat f')/(h\hat f' - h'\hat f)$. This axion decay constant can fulfill the weak gravity conjecture for large k. Furthermore, for $f/h' > 1$, this is the sub-leading term in the potential and inflation is driven by the potential with the super-Planckian axion decay constant.

6.4.4 Polynomial terms in the periods

The choice of superpotential was simplified. We neglected higher order polynomial terms which appear in the periods. We take a look at the effects of these higher order terms by considering a superpotential with a quadratic term

$$W_{\text{eff}} = i\alpha\big(f + h'\, S + \hat{f}'\, Y\big) + iBY^2 + \frac{f\hat{C}}{2\pi i}\,\exp\Big(-\tfrac{2\pi}{f}\big(hS + \hat{f}Y\big)\Big)\,. \tag{6.4.41}$$

If the quadratic term dominates over the exponential term depends on the value of B, which is geometry dependent. Figure 6.2 shows the effective potential for Θ (by dashed lines) for two different values of the parameter B. We see that

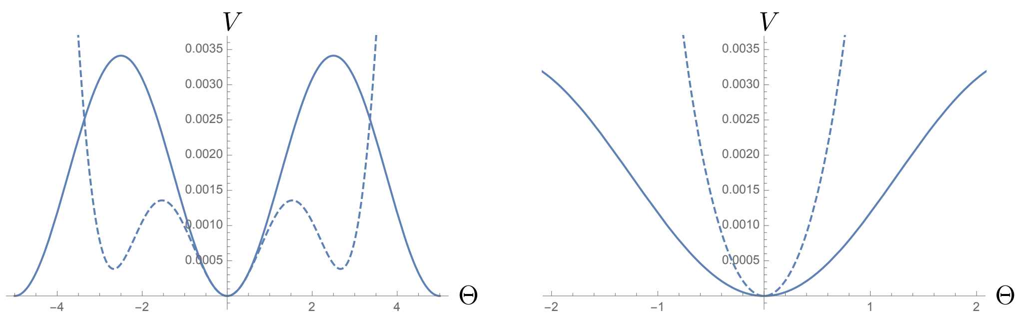

Figure 6.2: Scalar potential (dashed lines) for the axion Θ for $f = 10$, $h' = \hat{f}' = 1$, $h = -\hat{f} = 1$, $\hat{C} = 1$, $A = 0.1$ and $B = 0.01$ in the left-handed plot and $B = 0.1$ in the right-handed plot. For comparison, the solid lines show the potential for $B = 0$.

for small enough values of B, the potential approximately overlaps with the pure exponential potential for small Θ. But even if this term was sufficiently small, there are infinitely many more polynomial terms in the periods. These terms are in general not vanishing and also not converging. They would stabilise the massless axion dominantly compared to the exponential term.

6.5 Summary

Moduli stabilisation near the conifold allows for hierarchies which are not present in the large complex structure case. Even though we did not consider moduli stabilisation at the conifold singularity but rather in its vicinity with negligible warping, the mass hierarchies were still present. In our discussion, the property which leads to the interesting effect is indeed the logarithmic term in one of the

periods and not the warping.
From a pure moduli stabilisation perspective, the conifold region is quite interesting. The applications to aligned inflation turned out to be a bit more tricky. Even though the exponential term, which arises after integrating out the conic modulus, in principle can realise aligned inflation, it only worked in a toy example. When we considered a real Calabi Yau manifold, namely $\mathbb{P}_{11226}[12]$ we faced a problem, the appearance of higher order polynomial terms in the periods which dominate over the exponential term.
Let us mention that the suitability of the conifold region to realise large field inflation was also investigated in [128]. There, the periods were calculated up to quite high orders in the polynomial term for the mirror quintic. The resulting potential was tested if it was able to drive axion monodromy inflation. The result was negative.

Part III

Conclusions

Chapter 7

Conclusions

We summarise the results of our work on moduli stabilisation in the context of large field inflation in flux compactifications of type IIB string theory and discuss the status of large field inflation in string theory in general.

7.1 Summary of the results

In this thesis, we investigated moduli stabilisation in type IIB string theory such that large field inflation is realised. This becomes important if the tensor-to-scalar ratio r is larger than $r \geq 10^{-2}$. In the near future, experiments can measure such small values of r. Independently of this motivation, we considered several different moduli stabilisation schemes with flat and light axionic directions with geometric as well as non-geometric fluxes in different regions in the complex structure moduli space.

As a first result, we found that purely geometric no-scale Minkowski vacua with flat axions in type IIB are highly constrained. We found a No Go theorem claiming that any axion involving the universal axion cannot be a flat direction if the remaining fields are massive. Furthermore, the internal geometry must have at least four complex structure moduli and only specific forms of the prepotential are allowed. We showed an example of a vacuum with a massless axion and turned on a potential for the axion as a toy model for axion monodromy inflation. We achieved a polynomial mass hierarchy between the axion mass and the other moduli fields. The absence of Kähler moduli stabilisation makes this scenario unrealistic.

In the next step we included the stabilisation of volume moduli. Since the axion should be the lightest state, Kähler moduli need to be stabilised at tree level. This corresponds to moduli stabilisation with non-geometric fluxes. We turned on a scalar potential with geometric and non-geometric fluxes at the same time. From a stringy perspective it is not clear that this is an allowed configuration since there might be too many degrees of freedom. Also the physical nature of these non-geometric compactifiactions is badly understood. We treated non-geometric fluxes as small perturbations on the Calabi Yau. We considered the simplest class of vacua with all saxions stabilised, so-called flux-scaling vacua. These vacua contain at least one massless axion, while we control the saxions such that we stay in a perturbative large volume regime. Several aspects of these vacua are discussed, for instance soft supersymmetry breaking, the dilute flux limit and the uplift of tachyon masses. The masses of the stabilised fields have the same scaling with the fluxes, therefore flux-scaling vacua contain no hierarchically light axion. Nevertheless, non-supersymmetric flux-scaling vacua are a good starting point to realise F-term axion monodromy inflation. The inflaton potential is generated by turning on a small flux for a previously flat axion. In general, these vacua are AdS. We discussed uplifting to Minkowski and de Sitter by adding an $\overline{D}3$-brane and a D-term generated by abelian vector fields and non-geometric fluxes. Finally, we turned on a potential for a previously flat axion and considered axion monodromy inflation. One crucial consistency check is the hierarchy of masses. In our case, the string and KK scale has to be larger than the moduli scale to justify our moduli stabilisation starting point. Since we were interested in single field inflation, we also have to make sure that the mass of the moduli which are not axionic is larger than the Hubble scale during inflation. However, the hierarchy of mass scales is not satisfied as long as fluxes are integer quantised. In the large complex structure limit, a redefinition of fluxes can lead to effectively non-integer fluxes. Nevertheless, they are not sufficiently small to control the mass hierarchy.

In the last part of this thesis, we considered moduli stabilisation close to the conifold singularity. At the singularity, the geometry is strongly warped and the effective supergravity description breaks down. Nevertheless, in the vicinity of the conifold the warping is negligible and we can control the theory. Here, the logarithmic structure of the conic period leads to interesting new physics. The natural mechanism to stabilise volume moduli at the conifold is the LARGE volume scenario. There the condition for negligible warping can be easily satisfied. The logarithmic period structure gives rise to exponential mass hierarchies. A light axion mass and a potential for aligned inflation were considered in a toy manifold. For this aligned inflation model, we discussed the weak gravity conjecture. The big Kähler modulus in the LARGE volume scenario is very light, and turned out to be lighter than the inflaton. In the toy model, we ignored quadratic and higher order terms in the periods. In general, periods contain polynomial terms of all orders, which do not converge. Hence it is not clear that the exponential term gives the dominant contribution to the axion potential.

7.2 Outlook on large field inflation in string theory

It remains a challenge in general to realise large field inflation in string theory. Other ansätze and models for moduli stabilisation also face serious problems. The fine tuning ansatz of [58], as well as models with open string moduli [129,130] were not successful in realising axion monodromy inflation. Particularly the missing control over the hierarchy of scales was problematic in the considered moduli stabilisation scenarios. If there is an underlying mechanism responsible, comparable to the weak gravity conjecture, which forbids axion monodromy models in string theory is not yet clear.

But not only the stringy realisation of axion monodromy inflation remains problematic, also the missing control over the instanton terms conjectured by the weak gravity conjecture, which destroys periodic axion inflation models if the strong version holds. Work in the past, also including the lattice weak gravity conjecture [26] states that the strong form always holds in string theory [29]. Then quantum gravity forbids this kind of large field inflation. The lattice weak gravity conjecture claims that in any gauge theory coupled to gravity, any spot on the charge lattice consistent with Dirac quantisation should contain a (possibly unstable) superextremal particle. Recently, it was discussed that in an effective theory with discrete symmetries, for instance an orientifold, the sublattice weak gravity conjecture does not (yet?) restrict all periodic axion inflation models [131].

A fully fledged model which realises large field inflation in string theory still remains an open problem. All kinds of investigated models so far suffer from problems. If large field inflation is at all possible in string theory is therefore an interesting theoretical question, which will become highly relevant if primordial gravitational waves will be measured.

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