Spacetime Geometry from Graviton Condensation: A new Perspective on Black Holes

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Zusammenfassung

In dieser Dissertation stellen wir eine neuartige Sichtweise auf Raumzeitgeometrien vor, welche selbige als Produkt der Kondensation einer hohen Anzahl von Quanten auf einer ausgezeichneten, flachen Hintergrundmetrik betrachtet. Wir untermauern diesen Ansatz im Hinblick auf die Rolle von Raumzeitsingularitäten innerhalb der Quantenfeldtheorie sowie mit semiklassischen Betrachtungen schwarzer Löcher.

Abgesehen von der allgemeinen Relativitätstheorie ist in allen anderen Theorien die Raumzeit eine feste gegebene Größe, so dass etwaige Singularitäten nur im Zusammenhang mit Observablen auftauchen und dann durch Renormierungstechniken entfernt oder eben als schlicht unphysikalisch eingestuft werden. Das singuläre Verhalten von Raumzeiten an sich hingegen ist pathologisch. Die Vorhersage solcher Singularitäten im Sinne geodätischer Unvollständigkeit fand ihren Höhepunkt in den berühmten Singularitätentheoremen von Hawking und Penrose. Obwohl diese Theoreme nur sehr grundlegende Annahmen voraussetzen, stellen wir ihre physikalische Relevanz in Frage. Am Beispiel eines schwarzen Lochs zeigen wir, dass jegliche klassische Detektortheorie zusammenbricht weit bevor geodätische Unvollständigkeit einsetzen kann. Davon abgesehen verdeutlichen wir, dass das Heranziehen von Punktteilchen zur Diagnose von Raumzeitanomalien in Bereichen starker Krümmung eine zu starke Vereinfachung darstellt.

Angesichts dieser Ergebnisse drängt sich die Frage auf, inwieweit Quantenobjekte von Raumzeitsingularitäten betroffen sind. Basierend auf der für quantenmechanische Testteilchen zugeschnittenen Definition geodätischer Unvollständigkeit sammeln wir Ideen für eine Vollständigkeitsanalyse dynamischer Systeme. Eine Weiterentwicklung unserer Ansätze zeigt, dass insbesondere die Schwarzschildgeometrie keinerlei pathologisches Verhalten bezüglich der Betrachtung von Quantenobjekten hervorruft.

Dieses Resultat soll aber nicht darüber hinwegtäuschen, dass die hier verwandte semiklassische Behandlung weiterhin mit vielen ungelösten Paradoxa behaftet ist. Es ist für uns deshalb unabdingbar, auch den geometrischen Hintergrund quantentheoretisch zu beschreiben.

Unsere ersten Schritte in diese Richtung machen wir mit Hilfe eines nicht-relativistischen Modells, das unter Verwendung skalarer Felder nur die wichtigsten Eigenschaften der allgemeinen Relativitätstheorie umfasst. Hierbei fassen wir schwarze Löcher als gebundene Quantenzustände auf, die aus einer Vielzahl N von langwelligen Gravitonen bestehen und einem kollektiven Potential unterworfen sind. Wir modellieren unser System so, dass es sich an einem Phasenübergang befindet. Dies hat zwangsläufig zur Folge, dass keine Störungstheorie mehr anwendbar ist und wir gezwungen sind auch eine numerische Analyse unseres Modells vorzunehmen. Dabei erhalten wir starke Anzeichen dafür, dass

unsere Modellbeschreibung Korrekturen aufweist, die mit $1/N$ skalieren. Dies entspricht genau dem zugrundeliegenden so genannten Quanten-N-Portrait schwarzer Löcher und vermag das entscheidende Merkmal hinsichtlich einer Auflösung des schon lange bestehenden Informationsparadoxons zu sein. Trotz dieser ermutigenden Ergebnisse müssen wir feststellen, dass solch ein nicht-relativistisches Modell schlichtweg keine ausreichend befriedigende Beschreibung schwarzer Löcher liefern kann.
Wir wenden uns im Folgenden deshalb einem relativistischen Verfahren zu, um Raumzeitgeometrien mit gebundenen Quantensystemen großer Konstituentenzahl in Einklang zu bringen. Basierend auf einer nicht-trivialen Vakuumstruktur, welche eine Kondensation von Gravitonen zulässt, stellen wir eine Verbindung zwischen In-Mediums Modifikationen und einem kollektiven Potential her. Dabei betrachten wir die Minkowski-Raumzeit als fundamental und verstehen andere Raumzeitgeometrien als gebundene Zustände auf Minkowski, welche sich im Grenzwert unendlicher Konstituentenzahlen ergeben. Diese Konstruktion gebundener Zustände ist analog zur Beschreibung von Hadronen in der Quantenchromodynamik - es werden die gleichen nicht-perturbativen Methoden verwendet, wie z.B die Hilfsstrombeschreibung oder die Operatorprodukt-Entwicklung. Da wir vor allem an der Beschreibung schwarzer Löcher interessiert sind, entwickeln wir obige Konstruktion im Hinblick auf die Schwarzschildgeometrie. Darauf basierend wiederholen wir, wie deren Konstituenten- sowie Energiedichte auf Partonebene hergeleitet werden kann und wie sich zeigen lässt, dass sich die Masse eines schwarzen Loches proportional zur Anzahl seiner Konstituenten verhält. Wir nutzen die genannten Ergebnisse sodann, um über die Partonebene hinaus Vorhersagen zu treffen. Wir betrachten insbesondere die Streuung eines skalaren Teilchens am schwarzen Loch und zeigen explizit, dass die Konstituentenverteilung eine Observable darstellt, welche prinzipiell in einem Experiment gemessen und damit unsere Theorie stützen könnte. Darüber hinaus vermitteln wir eine Methodik, die auch zur Beschreibung der Formation schwarzer Löcher sowie der Hawking-Strahlung eingesetzt werden kann.
All diese Einsichten ermöglichen es nicht zuletzt, das Mysterium schwarzer Löcher aufzulösen. Gleichsam heben sie hervor, wie fragwürdig eine semiklassische Behandlung schwarzer Löcher im Allgemeinen ist. Da unser Formalismus nicht nur auf die Geometrie schwarzer Löcher beschränkt ist, sondern eine viel weitläufigere Anwendung zulässt, wagen wir auch die Beschreibung des de-Sitter-Raumes. Hier stellen wir eine Prognose zur Errechnung der Vakuumsenergie auf, um eine natürliche Erklärung für die kosmologische Konstante zu finden. Zusätzlich erwähnen wir, dass selbst die Quantenchromodynamik und, viel allgemeiner, jegliche Theorie gebundener Zustände von unserem Formalismus profitieren kann.

Losgelöst hiervon erläutern wir eine alternative Beschreibung, welche klassische Lösungen als kohärente Zustände im Limes hoher Besetzungszahlen von sogenannten Korpuskeln interpretiert. Hier werden wir uns auf die Beschreibung des Anti-de-Sitter-Raumes konzentrieren. Da wir uns im Laufe dieser Dissertation hauptsächlich mit der Notwendigkeit einer Konstituentenbeschreibung schwarzer Löcher befassen, verweisen wir für diese korpuskuläre Beschreibung auf den Anhang.

Abstract

In this thesis we introduce a novel approach viewing spacetime geometry as an emergent phenomenon based on the condensation of a large number of quanta on a distinguished flat background. We advertise this idea with regard to investigations of spacetime singularities within a quantum field theoretical framework and semiclassical considerations of black holes.

Given that in any physical theory apart from General Relativity the metric background is determined in advance, singularities are only associated with observables and can either be removed by renormalization techniques or are otherwise regarded as unphysical. The appearance of singularities in the spacetime structure itself, however, is pathological. The prediction of said singularities in the sense of geodesic incompleteness culminated in the famous singularity theorems established by Hawking and Penrose. Though these theorems are based on rather general assumptions we argue their physical relevance. Using the example of a black hole we show that any classical detector theory breaks down far before geodesic incompleteness can set in. Apart from that, we point out that the employment of point particles as diagnostic tools for spacetime anomalies is an oversimplification that is no longer valid in high curvature regimes.

In view of these results the question arises to what extent quantum objects are affected by spacetime singularities. Based on the definition of geodesic incompleteness customized for quantum mechanical test particles we collect ideas for completeness concepts in dynamical spacetimes. As it turns out, a further development of these ideas has shown that Schwarzschild black holes, in particular, allow for a evolution of quantum probes that is well-defined all over.

This fact, however, must not distract from such semiclassical considerations being accompanied by many so far unresolved paradoxes. We are therefore compelled to take steps towards a full quantum resolution of geometrical backgrounds.
First steps towards such a microscopic description are made by means of a non-relativistic scalar toy model mimicking properties of General Relativity. In particular, we model black holes as quantum bound states of a large number N of soft quanta subject to a strong collective potential. Operating at the verge of a quantum phase transition perturbation theory naturally breaks down and a numerical analysis of the model becomes inevitable. Though indicating $1/N$ corrections as advertised in the underlying so-called Quantum-N portrait relevant for a possible purification of Hawking radiation and henceforth a resolution of the long-standing information paradox we recognize that such a non-relativistic model is simply not capable of capturing all relevant requirements of a proper black hole treatment.
We therefore seek a relativistic framework mapping spacetime geometry to large-N

quantum bound states. Given a non-trivial vacuum structure supporting graviton condensation this is achieved via in-medium modifications that can be linked to a collective binding potential. Viewing Minkowski spacetime as fundamental, the classical notion of any other spacetime geometry is recovered in the limit of an infinite constituent number of the corresponding bound state living on Minkowski. This construction works in analogy to the description of hadrons in quantum chromodynamics and, in particular, also uses non-perturbative methods like the auxiliary current description and the operator product expansion. Concentrating on black holes we develop a bound state description in accordance with the isometries of Schwarzschild spacetime. Subsequently, expressions for the constituent number density and the energy density are reviewed. With their help, it can be concluded that the mass of a black hole at parton level is proportional to its constituent number. Going beyond this level we then consider the scattering of a massless scalar particle off a black hole. Using previous results we can explicitly show that the constituent distribution represents an observable and therefore might ultimately be measured in experiments to confirm our approach. We furthermore suggest how the formation of black holes or Hawking radiation can be understood within this framework. After all, the gained insights, capable of depriving their mysteries, highlights the dubiety of treating black holes by means of classical tools. Since our setup allows to view other, non-black-hole geometries, as bound states as well, we point out that our formalism could also shed light on the cosmological constant problem by computing the vacuum energy in a de Sitter state. In addition, we point our that even quantum chromodynamics, and, in fact, any theory comprising bound states, can profit from our formalism.

Apart from this, we discuss an alternative proposal describing classical solutions in terms of coherent states in the limit of an infinite occupation number of so-called corpuscles. Here, we will focus on the coherent state description of Anti-de Sitter spacetime. Since most parts of this thesis are directed to find a constituent description of black holes we will exclude this corpuscular description from the main part and introduce it in the appendix.

Introduction

The Standard Model of physics (SM) and General Relativity (GR) are deemed as the most rigorously and extensively confirmed theories describing nature. While to date the Standard Model is the most successful theory of particle physics uniting the electromagnetic, the weak and the strong forces, General Relativity explains physics on cosmological scales with gravity representing a manifestation of spacetime curvature.

The Large Hadron Collider (LHC) has tested the Standard Model with unprecedented precision. In 2013, even the Higgs Boson, responsible for the masses of fundamental particles, was tentatively confirmed to exist [1].
Concerning GR, being the basis of current cosmological models, the Planck space mission, following COBE and WMAP, is dedicated to measure the cosmic microwave background (CMB) anisotropies. The results of the mission strongly support the ΛCDM (Λ Cold Dark Matter) model, i.e. the standard model of cosmology equipped with a cosmological constant. This model is based on the assumption that the large scale structure of our universe is the result of primordial density perturbations caused by quantum fluctuations. The perturbations on the surface of last scattering are today's observed temperature anisotropies in the CMB. The measurements fit the prediction that the primordial inhomogeneity is quasi scale invariant and obeys Gaussian statistics.

Although both the SM and GR are in excellent agreement with experiments and there indeed have been no signals yet that point to extensions beyond[1], there still remain many questions. Compiling a list, it would include the identity of dark matter and dark energy as well as the hierarchy problem, the strong CP problem and the cosmological constant problem - the last three of which can be grouped to the so-called naturalness problems. We will introduce them in the next section. Afterwards we will turn to open issues concerning solely gravity - a theory that has gained high popularity also among non-physicists due to the rich selection of fascinating effects and puzzles it offers.

Naturalness problems

The notion of naturalness addresses the free parameters of a given effective theory. It implies that these parameters should be of order one in natural units. Specializing to technical naturalness we assume that a parameter is naturally small if setting it to zero enhances the symmetry of the theory [2].

[1]For example, the LHC has provided no signs of supersymmetry, nor has Planck revealed any non-gaussianities or isocurvature perturbations.

The hierarchy problem

The Standard Model Lagrangian comprises 19 free parameters, all of which have to be determined by experiment. While fermion and gauge field masses, protected by chiral and gauge symmetries, only receive logarithmic (cut-off dependent) radiative corrections, elementary scalar fields such as the Higgs are not protected by any symmetry and have quadratically divergent corrections. Thus, the Higgs mass should be of the order of the cut-off scale of the Standard Model viewed as an effective theory. The natural value of the Higgs mass would therefore amount to the Planck mass M_p. Experimental data, however, has shown that its mass is of the order of the electroweak scale [1]. This implies that there is a variation by many orders of magnitude requiring extreme fine-tuning.

Possible solutions to this hierarchy problem are offered by supersymmetry, which introduces new degrees of freedom in order to cancel the divergences. In a similar way, this problem is avoided by composite Higgs models (e.g. technicolor). On the other hand, it is tempting to lower the cut-off scale itself. This might be done via extra dimensions (see e.g. ADD model [3], Randall-Sundrum model [4])

The strong CP problem

Another problem related to naturalness is the strong CP (charge parity) problem. Taking the chiral limit, i.e. setting the masses of the quarks to zero, the QCD Lagrangian exhibits a global vector and axial vector symmetry $U_V \times U_A$. While the vector symmetry part, corresponding to baryon number and isospin symmetry, is in good agreement with experiment, the axial symmetries are spontaneously broken by the formation of quark-antiquark condensates ($\langle \bar{q}q \rangle \neq 0$). Consequently, Goldstone bosons are expected to appear. Admittedly, we rather deal with pseudo Goldstone bosons since the three lightest quarks (the u, d and s quarks) are only approximately zero. Instead of the expected nine pseudo Goldstone bosons, however, only eight such bosons have been observed by experiment - an issue known as the $U_A(1)$ problem. The problem is actually solved by the chiral anomaly of the current j_5^μ corresponding to the symmetry $U_A(1)$ with the anomaly inducing a non-zero divergence of the current. Hence, due to Noether's theorem, there exists a $U_A(1)$-symmetry transformation changing the QCD Lagrangian. Providing possible instanton contributions, this changing term,

$$\mathcal{L}_\theta = \theta \frac{g^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu}, \tag{1}$$

must be included in the Lagrangian according to 't Hooft [5]. Here, $G_{\mu\nu}$ is the field strength tensor, $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$ its dual and g is the quark-gluon coupling constant. With that the $U_A(1)$ problem is solved. As can be seen, the additional term in the Lagrangian of QCD incorporates a parameter θ, $0 \leq \theta \leq 2\pi$. In order to agree with experimental measurements this vacuum angle is constrained to be smaller than 10^{-9}. In the light of the comparably wide range θ can take theoretically, it does not seem natural, that CP violation is measured to be so small.

A well-known solution to the strong CP problem is offered by Peccei-Quinn theory [6], which rests upon the insight that an additional global $U(1)$ chiral symmetry might effectively rotate the gained θ-vacuum term away. Here, a (quasi) $U_{PQ}(1)$ invariant Lagrangian is constructed whose symmetry gets spontaneously broken. As a consequence, a new pseudo Goldstone scalar field, called axion, arises. The axion term of the constructed Lagrangian is constructed such that the vacuum angle θ gets effectively canceled.

Though providing an elegant solution to the strong CP problem, it has to be mentioned that the axion has not been detected yet[2].

The cosmological constant problem

Concerning the ΛCDM model naturalness issues arise in terms of the cosmological constant Λ, the value of the energy density of vacuum, which is associated with dark energy and cold dark matter.

Prior to the discovery of the accelerated expansion of the universe the cosmological constant problem was constituted by the question why the observed cosmological constant was equal to zero. Today, the question is reformulated to why the observed value of Λ only amounts to $10^{-120}M_p^4$, where M_p denotes the Planck mass. From a theoretical point of view, quantum fluctuations should be responsible for the vacuum energy. Summing up their zero-point energies up to some cut-off M_{UV}, the result will be quartically divergent after renormalization, i.e. proportional to m^4, where m represents the highest mass of the theory considered. Already the electron, having a mass of $\sim 10^{-23}M_p$, would require a fine-tuning of about 30 orders of magnitude in order to match with measurements. Even worse, if the universe is described by an effective quantum field theory down to the Planck scale, there would be a difference of 120 orders of magnitude between the theoretical and the observed vacuum energy density. Therefore, the cosmological constant problem is not solely given by these huge differences, but is also by the enormous sensitivity to the effective theory chosen.

Since the cosmological constant problem already occurs at very low energy scales and has visible effects only on large scales, the problem is best addressed if gravity is modified in the infrared (IR) region. The aim of such modifications is to weaken the strength of gravity on these scales. This could happen by means of screening mechanisms, respectively degravitation [7, 8, 9]. In that case graviton condensates can be formed whose energy density compensates the cosmological constant. Therefore, the cosmological constant itself could be large but gravitate weakly so that the cosmic acceleration would still be small [10]. Alternatively, Newton's constant could be promoted to a high-pass filter in order to modify the effect of long wavelength sources such as the cosmological constant [11]. Another possibility is given by self-acceleration as, for example, in the DGP model [12]. This model, however, has to struggle with ghosts and has been ruled out by observations [13].

[2]This is due to the fact that the axion hardly interacts with any other Standard Model particle making it a good candidate for dark matter, by the way.

Problems of gravity ...

... in the UV

Apart from these naturalness problems present in both the Standard Model and General Relativity one of the most outstanding problems in contemporary physics is the following: While the Standard Model successfully combines the strong, the weak and electromagnetic interactions within quantum field theory, gravity has so far resisted to be formulated by a fundamental quantum theory.

At the classical level, gravity is described by Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}, \tag{2}$$

which connect the presence of matter, described by the energy momentum tensor $T_{\mu\nu}$, to the curvature of spacetime, encoded in the metric $g_{\mu\nu}$, respectively the Ricci tensor $R_{\mu\nu}$. G_N denotes Newton's constant. The corresponding Einstein-Hilbert action

$$S_{EH} = \int \mathrm{d}^4 x \sqrt{-g} \left(\frac{1}{16\pi G_N} (R - 2\Lambda) + \mathcal{L}_M \right), \tag{3}$$

where $g = \det g_{\mu\nu}$ and $\mathcal{L}_M$ describes the matter content of the theory, turns out to be perturbatively non-renormalizable as simple power counting shows: Performing a perturbative expansion around a flat background $\eta_{\mu\nu}$ of the metric one will encounter more and more powers of G_N in each step. Given that G_N is dimensionful, this entails more and more powers of momenta at each loop level. Hence, the higher the order of the expansion, the more ultraviolet (UV) divergences arise. Eventually, an infinite number of counter terms to cancel the divergences is needed in the renormalization procedure. Therefore, gravity is perturbatively non-renormalizable and has predictive power only in the sense of an effective theory which is valid up to the Planck scale $M_p = \sqrt{1/(8\pi G_N)}$. That is, any prediction made comes with corrections in powers of E/M_p. Beyond the Planck scale gravitational amplitudes violate perturbative unitarity and gravity requires a UV completion.

Before the rise of the Standard Model, physicists were confronted with the same situation for the other three forces of nature [14]: The Schrödinger equation, for example, can be treated perturbatively only up to the electron mass scale and must be replaced by Quantum Electrodynamics if higher energies wish to be considered. The same goes for Fermi's theory which is UV completed by a theory involving massive vector bosons and the Higgs boson. Concerning the strong force, the chiral Lagrangian, suitable at low energies, has found its UV theory in Quantum Chromodynamics (QCD).

An ultimate theory embedding gravity at low energies, i.e. a generally accepted quantum theory of gravity, however, has not been found yet. A possible UV completion is offered by string theory introducing a new length scale at which particles are no longer pointlike.

Asymptotic safety [15, 16], originally developed by Weinberg, aims to find a quantum theory of gravity as well. This concept rests upon a UV completion by means of an interacting fixed point of the renormalization group flow of gravity, i.e. a point where the running of the gravitational coupling $\alpha_g = p^2/M_p^2$ with the four momentum p

stops in order to evade unitarity violations. If asymptotic safety proved valid, this would equally mean that gravity is non-perturbatively renormalizable and insofar a fundamental rather than an effective theory. Theories possessing a non-interacting fixed point are perturbatively renormalizable via the integration of new, weakly coupled degrees of freedom. The integration of massive vector bosons in Fermi's theory as mentioned above can be invoked as a good example. Methods of UV completion relying on the existence of an RG fixed point are summarized as Wilsonian UV completion [17]. Albeit no definite verdict can be drawn on Wilsonian UV completion with respect to gravity, there are indications of failure [18].

However, as Dvali et al. have pointed out, there is still the possibility for gravity to be self-complete, see e.g. [19, 20]. Indeed, considering trans-Planckian collisions the effective theory of gravity predicts the formation of black holes [21, 22, 23]. According to [19, 24] black hole formation is even unavoidable in Einstein gravity scattering experiments provided the energy is above the Planck mass, $E \geq M_p$, and the impact parameter is smaller than the Planck length, $l_p = \sqrt{\hbar G_N}$. The argumentation rests upon the so-called generalized uncertainty principle, see e.g. [25, 26, 27, 28, 29], stating that the Planck length represents the absolute lower bound on any distance that can be probed. So, the created black hole will have a mass $M = E$ and a Schwarzschild radius $r_s = 2G_N M$ such that for energies way above the Planck scale it will be a well-behaved classical object. In this way, unitarity violations in high energy scattering processes are prevented via the formation of classical black holes. Self-completeness therefore represents an efficient mapping between deep UV and IR physics opening the possibility to dispense with a Wilsonian UV completion. Notice that this approach is strongly related to the idea of classicalization [30, 31, 32, 33] with black holes serving as classicalizing tools.

... in the IR and in general

The plethora of reflections on gravity in the UV shall not distract from the fact there are still puzzles in the low-energy regime as well.

Indeed, we just encountered such a puzzle in the previous section in form of the cosmological constant which can be viewed as a source of infinite extent. Therefore, as mentioned, IR modifications of gravity might have the power to overcome this problem.

Apart from that we have the general problem that GR offers the possibility to have spacetime singularities. As we will explain later these are far different from singularities occurring in all other theories. According to the singularity theorems formulated by Hawking and Penrose the occurence of such singularities implies the existence of geodesic incompleteness [34]. That is, objects might come from nowhere or disappear correspondingly. In fact, the problem is best known from black hole solutions of the Einstein equations (2).

But, sticking to black holes, this is not the only feature that causes so much fascination concerning these objects. There are far more mysteries surrounding them. To mention here are the origin of black hole entropy [35, 36, 37] and the famous information paradox, see e.g. [38, 39].

Following Bekenstein's argumentation black holes ought to carry entropy to be in accordance with the second law of thermodynamics [35]. At first, this insight seemed

to be in conflict with the no-hair theorem [40, 41, 42, 43] stating that all stationary black holes can be completely characterized by just their mass, charge and angular momentum. Based on considerations of Hawking [44], Bekenstein argued that the problem would be solved if the entropy was proportional to the horizon area of the black hole [36].
So far, however, no microscopic explanation, i.e. an explanation in terms of statistical mechanics, for this entropy has been found, although Strominger and Vafa were able to derive the entropy of five-dimensional extremal black holes in string theory [45].

The information paradox, on the other hand, stems from the semiclassical treatment of black hole thermodynamics. Using quantum field theory methods on curved spacetime Hawking showed that black holes evaporate particles in such a way as if they were black bodies [46, 47]. This implies that the radiation does not capture any information. Hence, while black holes unceasingly absorb information in the course of soaking in material, they are not able to release this information. Clearly, this fact is in conflict with the quantum mechanical principle of unitary time evolution. The severity of this problem becomes clear if one is made aware that the distinguished physicists Hawking, Thorne and Preskill made a bet on the outcome of the paradox in 1997. While Hawking and Thorne were in favor of GR, Preskill argued the opposite. In 2004, Hawking conceded the bet, convinced that black holes leak information [48]. Nevertheless, there is no real consensus yet and there have been several other proposals to solve the paradox. One of these is the possibility of remnants staying over after a period of informationless radiation release [49]. Another proposal is black hole complementarity [50, 51, 52] assuming that information is both absorbed and reflected by the black hole. A violation of the no-cloning theorem [53, 54] is precluded by the fact that any observer can only witness the course of action on his own side of the horizon, but never on both sides at the same time. This ansatz, however, seems to bring about another paradox, the firewall paradox. According to Almheiri et al. it is inconsistent to hold on to all postulates used for complementarity [55]. In particular, preserving unitarity implies a violation of the equivalence principle at the horizon. The authors argue that at this point a firewall, i.e. a domain of highly energetic quanta, must be formed burning up any observer before he can enter the black hole.
However, in our opinion most attempts are flawed by the fact that they try to solve the problem within the semiclassical approximation - an approximation which is the reason for the paradox arising in the first place. After all, according to the AdS/CFT correspondence [56, 57] black holes in Anti-de Sitter space (AdS) are dual to a conformal field theory (CFT) and as the field theory side is unitary for sure, so should be the gravity side. In string theory, for example, the fuzzball idea, positing that black holes can be described as balls of strings, has been put forward [58, 59, 60].

Nevertheless, finding a full-fledged quantum field theoretical treatment to resolve all the outlined problems is equally important. This thesis is an attempt in doing so.

Outline

Based on the issues mentioned above we establish a quantum description for black holes in this thesis. Keeping in mind other unresolved problems concerning gravity we try to model our approach in a quite general manner.

There are two main parts in which this thesis can be divided: A part involving classical methods, more precisely a classical and a semiclassical part, and a purely quantum theoretical part.

We will start with the purely classical treatment of black holes and its consequences in the next chapter. In particular, we will deal with the singularity theorems of Hawking and Penrose [34, 44, 61, 62, 63]. We demonstrate that, though such considerations have mathematical legitimacy, they must be considered as irrelevant if quantum theory is believed to lay the foundations from which all classical physics arises.
We substantiate this result in chapter 2 within a semiclassical treatment. Using the example of a Schwarzschild geometry we investigate in how far quantum test particles are affected by classical singularities.
While usually the semiclassical approximation is assumed to hold whenever big objects are considered we note that within the so-called N portrait [64] presented in chapter 4 black holes represent an exception. We will address this issue particularly in chapter 3.

Having gained enough motivation all subsequent chapters will then concentrate on the main objective of this thesis, namely to deliver a well-conceived quantum theoretical description for black holes and spacetime geometries in general.
Reviewing the N portrait we recognize that deviations from semiclassicality can be achieved by means of strong collective effects. We will capture these ideas subsequently within a toy model. Thereupon, chapters 6 and 7, reflecting the heart of this work, will establish a more profound constituent treatment of spacetime. Here, we will develop novel ideas inspired by methods from quantum chromodynamics which we apply in accordance with the guidelines of GR. Concentrating on the Schwarzschild solution we show how the structure of black holes can be probed within our framework. In particular, we consider the scattering of a massless scalar particle off a black hole and show that the constituent number represents a directly observable quantity - a result inherently refused by the (semi)classical approximation and, at least in principle, verifiable.

Finally, we will recapitulate our findings and provide prospects for future developments in the field.

Detached from the black hole theme we introduce an alternative constituent description of spacetime. Here, we concentrate on Anti-de Sitter spacetime to uncover corpuscular corrections. This part can be found in the appendix.

The metric signature used in this thesis is $(-,+,+,+)$. We will stick to natural units $\hbar = c = k_B = 1$ in the purely classical treatment of Part I while $\hbar$ will be restored for semiclassical considerations and in the second part.

Part I

The classical description of spacetime geometry

Chapter 1

The problem of singularities

General Relativity offers a rich selection of fascinating phenomena - spacetime singularities being undoubted one of the most interesting ones. After all, spacetime singularities play a very special role when compared to singularities arising in other physical fields. The reason behind will be discussed in this chapter. Presenting the singularity theorems we will furthermore show that such singularities are indeed an inevitable feature of Einstein's theory. Simple estimations, however, indicate that the actual presence of singularities cannot be confirmed within such a classical theory.

1.1 Singularities in GR vs. other gauge theories

A few months after the publication of Einstein's field equations (2) in 1915, Schwarzschild found one of the first exact solutions [65]. It describes the exterior gravitational field of a spherical mass. That could either simply be a planet, but also, in case that the so called Schwarzschild radius r_s is equal or bigger than its physical radius, a non-rotating, non-charged black hole. In Schwarzschild coordinates the line element is of the form

$$\mathrm{d}s^2 = -\left(1 - \frac{r_s}{r}\right)\mathrm{d}t^2 + \frac{1}{\left(1 - \frac{r_s}{r}\right)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2 \tag{1.1}$$

where $\mathrm{d}\Omega_2^2 = \mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\phi^2$ denotes the metric on a two-dimensional sphere of unit radius. As it appears, the metric has singularities at $r = r_s$ and $r = 0$. While the former turns out to be a mere coordinate singularity as can be confirmed for example when switching to Kruskal-Szekeres coordinates the latter turns out be coordinate independent. Invoking the Kretschmann scalar, $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ with $R_{\alpha\beta\gamma\delta}$ representing the Riemann curvature tensor[1], it can be shown that for $r \to 0$ the curvature of spacetime turns to infinity. Therefore, at this point, spacetime itself is no longer well-defined and a true gravitational singularity occurs.

Similarly, in Friedmann-Robertson-Walker (FRW) spacetime, describing a homogeneous, isotropic expanding universe, the Einstein equations imply that, if $\rho + 3p > 0$ for all times t, with ρ being the total energy density and p the pressure, there is a singularity at the origin of the universe, i.e. for $t \to 0$.

[1]Using Christoffel symbols $\Gamma_{\alpha\beta\gamma} = \frac{1}{2}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\beta\gamma}\right)$ the Riemann curvature tensor is given by $R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho{}_{\nu\sigma} - \partial_\nu \Gamma^\rho{}_{\mu\sigma} + \Gamma^\rho{}_{\mu\lambda}\Gamma^\lambda{}_{\nu\sigma} - \Gamma^\rho{}_{\nu\lambda}\Gamma^\lambda{}_{\mu\sigma}$.

There have been found many other solutions involving such singularities, but they were all exhibiting exact symmetries. It was therefore believed by Einstein himself [66, 67] and many other physicists at that time [68, 69, 70] that either these singularities were pathological artifacts caused by the imposed symmetry or that they were utterly untenable effects which could not be admitted to occur in generic spacetimes.

In the following decades little progress was made concerning the prediction of singularities. Only in 1955, the year of Einstein's death, Raychaudhuri published an equation, the Raychaudhuri equation [71], which proved to be a fundamental lemma for the singularity theorems developed by Hawking and Penrose [34, 44, 61, 62, 63]. These singularity theorems, established from 1965 on, changed things dramatically: They proved the occurrence of singular behavior in generic scenarios, thus notably not depending on exact symmetries.

The reason why these theorems represented a major cut in the development of General Relativity can be understood as follows:
In all other physical theories the spacetime manifold and metric structure are given in advance, such that we know time and place of all spacetime events. The values of mathematical objects (i.e. fields, scalars,...) of interest are then determined at these events. If a physical quantity turns out to be infinite or undefined in another way, renormalization methods offer a way out. Otherwise the singularity is regarded as unphysical in the sense that it stems from an incomplete theory.

However, if spacetime, represented by a smooth manifold and non-degenerate metric, is plagued by singularities itself we have no means of localizing these singularities in time and space, because at the points where these are supposed to occur the notion of spacetime is simply not defined anymore.

1.2 Defining spacetime singularities

Although there is no entirely satisfactory definition of gravitational singularities, it is widely accepted to define a singularity through geodesic incompleteness. The holes in spacetime occurring due to a singularity should cause geodesics to have a finite affine length, i.e. geodesics which are inextendible but have a finite range of affine parameter. A spacetime is called geodesically incomplete if it possesses at least one incomplete geodesic. Vividly speaking, geodesic incompleteness implies that freely moving observers or particles have histories that do not exist before or after a finite interval of proper time. Now, Geroch constructed an example of a spacetime which is timelike, null and spacelike geodesically complete, but still contains a timelike inextendible curve of bounded acceleration and finite affine length [72]. Nevertheless, at least timelike and null geodesically incomplete spacetimes, i.e. those where freely falling particles can appear out of nowhere or disappear should be considered pathological.

Consequently, the above mentioned singularity theorems are mathematical statements about conditions for spacetimes to be timelike or null geodesically incomplete. As such they only indicate the presence of singularities. They can neither make predictions on their location nor their physical nature.

We will now introduce these theorems.

1.3 On the singularity theorems

The assumptions about spacetime the singularity theorems are based on are rather general. They can be summarized as the positivity of energy, a reasonable causality assumption stating, for example, that there be no closed timelike curves, and the existence of strong gravitational fields such that trapped surfaces may be formed, i.e. regions of spacetime which nothing can escape from.
Following these assumptions all singularity theorems share a distinctive formulation [73]:

If a given spacetime satisfies
(a) an energy condition,
(b) a causality condition, and
(c) a boundary or initial condition
then it contains at least one incomplete causal geodesic.

The main concept behind the proof of these theorems is as follows:
Given a causal structure it is shown that for a pair of certain events, there must be causal curves of maximal length connecting them. Furthermore, if the spacetime satisfies the so-called generic condition plus an energy condition it can be proven that a complete causal geodesic must contain pairs of conjugate points[2]. Using this information one can then construct a contradiction rendering the spacetime non-spacelike geodesically incomplete.

The most general singularity theorem applicable to both collapse and cosmological scenarios is the Hawking-Penrose theorem [34]:

Theorem 1.3.1. No spacetime M can satisfy all of the following three requirements together:

$$M \text{ contains no closed timelike curves,} \tag{1.2}$$

$$\text{every inextendible causal geodesic in } M \text{ contains a pair of conjugate points,} \tag{1.3}$$

$$\text{there exists a future- (or past-)trapped set } S \subset M. \tag{1.4}$$

In order to thoroughly understand this theorem and other singularity theorems it is important to be familiar with some definitions and basic results of the causal structure of spacetime. Being aware that such technicalities might distract from the actual content and implications of these theorems we will refer the interested reader to the standard litertaure (see e.g. [34, 74]). Offering a better physical understanding we will present the conditions of these theorems now in detail.

1.3.1 Energy conditions and the Raychaudhuri equation

We will start by presenting the energy conditions as they also play a vital role in Raychaudhuri's equation (1.12). In essence, these conditions aim to capture the requirement

[2]By conjugate points we mean, roughly speaking, that neighboring geodesics meet each other twice, namely at so-called conjugate points.

of having a (physically sensible) positive energy. To see this, let us assume that the energy momentum tensor can be decomposed as

$$T^{\mu\nu} = \rho e_0^\mu e_0^\nu + p_1 e_1^\mu e_1^\nu + p_2 e_2^\mu e_2^\nu + p_3 e_3^\mu e_3^\nu \tag{1.5}$$

with ρ denoting the energy density and p_i the pressure and the vectors e_α^μ forming an orthonormal basis. Let v^μ be a normalized, future-pointing, timelike vector representing the four-velocity of an observer in spacetime and k^μ a future directed null vector. An observer in spacetime with four-velocity v^μ measures the energy density to be $T_{\mu\nu}v^\mu v^\nu$. If the *weak energy condition* holds this energy density is non-negative, i.e.

$$T_{\mu\nu}v^\mu v^\nu \geq 0, \tag{1.6}$$

which also implies that $\rho \geq 0$ and $\rho + p_i > 0$. The *null energy condition* is fulfilled if

$$T_{\mu\nu}k^\mu k^\nu \geq 0 \tag{1.7}$$

whereas the *strong energy condition* stipulates that

$$\left(T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu}\right) v^\mu v^\nu \geq 0. \tag{1.8}$$

Recalling Einstein's field equations,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} \;\;=\;\; 8\pi G_N T_{\mu\nu}, \tag{1.9}$$

relating spacetime curvature (represented by the Ricci tensor $R_{\mu\nu}$) and the presence of matter (represented by the energy momentum tensor $T_{\mu\nu}$), we can equivalently understand these energy conditions as geometrical statements. Neglecting the cosmological constant Λ, for example, we can express the strong energy condition as

$$R_{\mu\nu}v^\mu v^\nu \geq 0. \tag{1.10}$$

As we will see in a moment, the strong energy condition implies that gravitation is an attractive force.
Let us therefore assume to have a congruence of timelike geodesics $x^\mu(s,\tau)$, i.e. a family of timelike curves such that for any point in an open set of a given spacetime manifold there passes exactly one curve of this family [75]. Here, the parameters s and τ label which geodesic we mean and the affine parameter along this very geodesic, respectively. In order to assess how these timelike geodesics behave as time evolves we need to have a look at the deviation vector $\xi^\mu = \frac{\partial x^\mu}{\partial s}$ between two neighboring geodesics. As it turns out we find

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} = -R^\mu_{\nu\alpha\beta}v^\nu \xi^\alpha v^\beta \tag{1.11}$$

for the acceleration between two neighboring geodesics. Clearly, (1.11) reveals that spacetime curvature causes geodesics to deviate. The question now is what happens if the strong energy condition holds. This will be answered by Raychaudhuri's equation

(1.12). As can be shown $\xi^\mu \nabla_\mu v^\nu = v^\mu \nabla_\mu \xi^\nu$, where ∇_μ denotes the covariant derivative [75]. In general, a tensor field is said to be parallelly transported along a curve if its covariant derivative along that curve vanishes. Therefore, introducing the tensor field $B^\nu_\mu = \nabla_\mu v^\nu$ we recognize that B^ν_μ measures the failure of ξ^μ to be parallelly transported along the given geodesic.
The reason for such a failure can be traced back to three different reasons influencing the volume of the congruence. First, and most interesting for our considerations, there can be an expansion or contraction of the congruence volume which is given by the divergence θ of v^μ, $\theta = \nabla_\mu v^\mu$. That is, for $\theta > 0$ the geodesics of the congruence are drifting apart while for $\theta < 0$ they are bent towards each other which could possibly lead to caustics, i.e. a crossings of geodesics, or conjugate points[3]. Secondly, we can also have a distortion of the shape (without changing the volume). This is given by a symmetric trace-free tensor $\sigma_{\mu\nu} = B_{(\mu\nu)} - \theta\delta_{\mu\nu}$. Besides, there also may be a rotation of the volume expressed by an antisymmetric part $\omega_{\mu\nu} = B_{[\mu\nu]}$ so that in general[4] $B_{\mu\nu} = 1/3\theta\delta_{\mu\nu} + \sigma_{\mu\nu} + \omega_{\mu\nu}$.
Far more interesting now is the evolution of the parameter θ with proper time. Since $\theta = B^\mu_\mu$ and $\frac{d\theta}{d\tau} = (\nabla_\mu \theta) v^\mu$ we end with

$$\frac{\mathrm{d}\theta}{\mathrm{d}\tau} = -\frac{1}{3}\theta^2 - \sigma^{\mu\nu}\sigma_{\mu\nu} + \omega^{\mu\nu}\omega_{\mu\nu} - R_{\mu\nu}v^\mu v^\nu, \tag{1.12}$$

which is known as the Raychaudhuri equation. The importance of this equation becomes clear if we consider a congruence of timelike geodesics being hypersurface orthogonal ($\omega_{\mu\nu} = 0$, see [75]). If furthermore the strong energy condition is fulfilled, then (1.12) implies that the expansion must decrease in time, $\frac{\mathrm{d}\theta}{\mathrm{d}\tau} \leq -\theta^2$, because $\sigma_{\mu\nu}$ is a purely spatial tensor. Consequently, gravitation is an attractive force focussing timelike geodesics in the progress of time. Furthermore, in that case a simple integration shows that the geodesics must converge to a single point, a caustic, if the congruence has a negative initial expansion.
A similar argumentation for null congruences also leads to caustics [75]. Note that such a caustic only signals a breakdown of the evolution equation (1.12), i.e. a singular behavior of the congruence. A priori, their presence does not imply a singularity of spacetime in the sense of geodesic incompleteness. However, it can be regarded as a precursor of the singularity theorems which also contain causality conditions as we have seen.

1.3.2 Causality conditions

Concerning the causality conditions there exists a whole hierarchy. The strongest causality condition is global hyperbolicity meaning that for given initial data on some spacelike hypersurface both past and future developments must be uniquely predictable

[3]A proper definition of conjugate points can be given by means of so-called Jacobi fields which are solutions of the geodesic deviation equation (1.11). Given two points along a timelike geodesic, these are said to be conjugate if there exists a Jacobi field along that geodesic which is non-zero except at these two points.

[4]Note that we are operating in three spatial dimensions here.

by means of Einstein's equations. The Hawking-Penrose theorem, however, merely uses the so-called strong causality condition:

Definition 1.3.1. At a point $p \in M$, M denoting a spacetime manifold, the *strong causality condition* holds if, given a neighborhood U_p of p, there is another neighborhood $V_p \subset U_p$ of p such that every causal segment with endpoints in V_p lies entirely in U_p. Hence, every neighborhood of p contains a neighborhood of p, which no causal curve intersects more than once.

The simplest of all causality conditions is the chronological condition which simply rules out closed timelike curves.

The last conditions occurring in the singularity theorems are initial and / or boundary conditions.

1.3.3 Initial and boundary conditions

In most of the theorems such conditions are provided by the existence of closed trapped surfaces or reconverging lightcones. This implies that there are regions in spacetime where not only ingoing but also outgoing null geodesics are converging. Such regions develop in spherical gravitational collapse scenarios, for example [76].

1.3.4 Outlining the proof of the Hawking-Penrose theorem

A priori the singularity theorems do not indicate that there is a singularity in the first place. However, having a look at the Hawking-Penrose theorem, it seems to be the most acceptable way out. Indeed the proof of theorem (1.3.1) can be sketched as follows [34]: The starting point is the assumption of having both a future-trapped set S and strong causality holding throughout. If condition (1.3) is assumed to hold as well, this gives rise to a contradiction. In fact, (1.3) is a consequence of the following corollary:

Corollary 1.3.1. Let γ be an inextendible causal geodesic. If the strong energy condition holds and the generic condition[5] holds along γ, then either γ is incomplete or γ contains a pair of conjugate points.

Therefore, the second requirement of the singularity theorem is a consequence of the strong energy condition, the generic condition and causal geodesic completeness. The physical statement behind, just like in Raychaudhuri's equation (1.12), is that gravity is an attractive force leading to the focussing of geodesics.

Not willing to give up any of the other assumptions, it must be concluded that the Hawking-Penrose theorem implies geodesic incompleteness and therefore singularities. Since a thorough proof of the theorem is quite lengthy and not relevant for this thesis we refer the reader to the original literature [79]. See also [74].
Let us now rather have a closer look at the theorem's requirements.

[5]This condition asserts that causal geodesics contain at least one point at which some quantity constructed from the Riemann tensor and the tangent to the curve is non-zero. In other words, at some point a freely falling observer must be subject to tidal acceleration [77]. By means of the generic condition and employing Raychaudhuri's equation (1.12) corollary (1.3.1) can be proven [78, 79].

1.3.5 Discussion on the Hawking-Penrose theorem

The first requirement of theorem (1.3.1), i.e. that there be no closed timelike curves, seems plausible for the existence of a spacetime making time travel possible would entail many temporal paradoxes. In fact, in 1949 Gödel found such a solution to Einstein's field equations [80]. His solution represents a dust model with non-zero cosmological constant. However, it does not allow for Hubble expansion and is therefore ruled out by current observations [81]. Still, this is not the only solution found containing closed timelike curves (see e.g. [82, 83]). Of course, observers might be protected from detecting any chronological violation if closed curves were crossing event horizons. This possibility is known as cosmic censorship [84, 85]. Still, at least for the big bang singularity, there must be a caveat. Also, if the Novikov self-consistency conjecture [86] held the probability of events giving rise to paradoxes would be set to zero.
We will, however, not engage ourselves in such constructions. After all, the chronology assumption is of global nature and therefore has no chance to be falsified (or verified) by any local physical measurement.

As we have seen, the second requirement of the singularity theorem is a consequence of the strong energy condition, the generic condition and causal geodesic completeness. The strong energy condition needed to insure the second requirement is actually somewhat debatable. In fact, there exist physically reasonable matter configurations violating the strong energy condition. The energy momentum tensor of massive scalar fields such as the Higgs field, for example, does not satisfy this condition [44, 73, 87]. Besides, there are inflationary scenarios violating the strong energy condition [88] and today's cosmological constant ($\Lambda > 0$) is not compatible either [89]. Nevertheless, while it might be that the Hawking-Penrose theorem is ruled out by experiment, this does not rule out the whole plethora of singularity theorems. For instance, there is a singularity theorem by Penrose using the null energy condition [61] which is far less restrictive. The generic condition, on the other hand, is supposed to be satisfied by all sufficiently generic spacetimes. It states that at some point along each causal curve an effective non-zero curvature is encountered. Highly symmetric spacetimes, however, might not obey the generic condition [73, 78]. Also, the weak energy condition can be violated. Here, quantum effects, such as the Casimir effect can give rise to negative local energy densities [90]. Still, averaged energy conditions, permitting localized violations of the energy conditions along null or timelike geodesics, are expected to hold in any case [88]. Geodesic completeness, of course, represents an indispensable prerequisite for non-singular spacetimes by definition.

A trapped set S as suggested in the third condition (1.4) of the theorem is assumed to arise in gravitational collapse, for example [61, 91, 92, 93, 94]. Such collapse is expected for stars with masses bigger than the Chandrasekhar limit, i.e. about 1.4 solar masses [95, 96].

1.4 Limitations of singularity forecasts

Though being part and parcel of General Relativity, the singularity theorems do not necessarily imply that there exist real singularities in our universe, be it in the past, present or future.

As pointed out above, the anchor of all singularity problems can be traced back to the fact that gravity, as opposed to all other fundamental forces, is purely attractive. One way out can be found in the modification of GR in such a way as to make gravity repulsive in some situations. In the Einstein-Cartan theory, for instance, spin-spin interactions can be repulsive and therefore prevent singularites [97]. There are also theories with matter exhibiting negative energy densities.

In any way, GR can only reliably describe gravitational interactions up to the Planck scale where quantum fluctuations of the spacetime metric become important and cannot be ignored any longer. Note furthermore, that the Planck length only sets the strong coupling scale of gravity if the background geometry is flat. Otherwise this scale will also depend on the curvature of the considered spacetime.

After all, spacetime singularities can only pose a physical threat if they can be probed by a detector, i.e. if there is a sensible observable. We therefore might wonder if there is any detector theory capable of doing so. As we demonstrate in appendix (A) we have to expect, depending on the initial conditions chosen, a (classical) particle detector to break down far before the Planck distance. The breakdown of the detector theory can then be interpreted in two different ways: Either, the test-mass limit breaks down and back reaction becomes so strong that the particle moves as if it were on a geodesic of a now perturbed, new spacetime geometry, or, sticking to the old background, the particle's motion is accelerated and therefore no longer geodesic. Indeed, for a mass slightly above the test-mass limit the particle's motion can be described by the so-called MiSaTaQuWa equations [98, 99]. The latter interpretation goes under the name gravitational self-force [100, 101]. Besides, in a realistic treatment, a finite mass also implies extended objects. The dynamics of these was extensively studied in [102, 103, 104, 105, 106]. In essence, we can summarize, that such bodies no longer follow geodesics - not even their center of mass.

Hence it seems that singularities in the sense of geodesic incompleteness have to be considered as irrelevant as far as realistic classical detectors are concerned. This necessarily leads to the question in how far quantum detectors are affected by classical singularities. Hints, that these might be unconcerned by the presence of such singularities already exist [107].

Chapter 2

Towards a quantum theoretical probing of classical black holes

In the previous chapter we have learnt about the singularity theorems by Hawking and Penrose. These are an essential component of the (classical) theory of General Relativity and seem to be set in stone. We have collected several indications that are motivating to doubt their power of predictability in a physical sense. Especially, with regard to black holes which are described by a singular spacetime manifold a further analysis of the status of the singularity theorems is of paramount importance. After all, it might be that the theorems are nothing but a differential geometry's statement about geodesics and classical physics.

In order to shed light on the dark, we have to ask whether classically singular spacetimes must also be quantum theoretically singular. Clearly, spacetimes such as the Schwarzschild spacetime are singular when probed with classical test particles. Bound to geodesic motion these particles must inevitably disappear once falling into the black hole as they reach the singularity within finite proper time. Testing a spacetime with quantum particles or quantum fields, however, the outcome is not clear a priori. Not least because we have not introduced a definition of quantum singularities, yet. We will expand on such a definition in the following - first for quantum particles and afterwards for quantum fields. Note that the latter definition does not merely represent a supplement. It rather is an essential prerequisite if dynamical spacetimes shall also be examined for singularities. Subsequently we will draw a conclusion about the singularity status of black holes within this framework.

2.1 Defining quantum mechanical singularities

In the following we seek a definition of spacetime singularities that is appropriate for quantum test particles. So far, there is no general agreement on the definition of singularities in the quantum realm. In their pioneering paper [107] based on [108] Horowitz and Marolf argue that a quantum system is singular if the evolution of a state is not uniquely defined for all time. This idea is closely tied to its counterpart in the classical theory. If for a classical test particle in some spacetime its geodesic is incomplete the predictability of its evolution breaks down. The uniqueness of evolution, i.e. a unitary time evolution, for all time is therefore not guaranteed. This is as strict as one can be when transferring the requirements of the (classical) singularity theorems to the quantum realm since a fully predictable time evolution implies global hyperbolicity - the strictest of all causality conditions (1.3.2). Besides, such a causality condition already ensures by itself that closed timelike curves cannot occur [109]. In this sense, the singularity definition of Horowitz and Marolf resembles an analogy to a quite strict classical counterpart.

The implementation of the quantum singularity idea works as follows:
The evolution of a quantum mechanical state is governed by the Schrödinger equation and therefore by the Hamiltonian of the system. This operator is a symmetric partial differential operator on an L^2 space with a dense domain. It is proportional to the Laplacian. Considering a freely moving particle in a geodesically complete spacetime, it is known that the Laplacian has a unique self-adjoint extension [110]. Therefore, concentrating on static spacetimes, Horowitz and Marolf argue that a spacetime is quantum mechanically incomplete if the Laplacian has no unique self-adjoint extension. In particular, they consider a static, globally hyperbolic spacetime with a scalar particle of mass $m > 0$ being governed by the Klein-Gordon equation $(\nabla^\mu \nabla_\mu + m^2)\psi = 0$. This wave equation can be rewritten as

$$\frac{\partial^2 \psi}{\partial t^2} = -A\psi \tag{2.1}$$

such that a spacetime can be called quantum mechanically non-singular if the spatial portion A of the Klein-Gordon wave operator has a single self-adjoint extension A_E, i.e. if it is essentially self-adjoint.
Due to the following theorem and corollary, a sufficient condition for this to be the case is to consider L^2 solutions to $A\psi \pm \mathrm{i}\psi = 0$ and show that there is only one square integrable solution.

Theorem 2.1.1. Let T be a symmetric operator on a Hilbert space $\mathcal{H}$. Then the following three statements are equivalent:

(a) T is self-adjoint

(b) T is closed and $\mathrm{Ker}(T^* \pm \mathrm{i}) = \{0\}$

(c) $\mathrm{Ran}(T \pm \mathrm{i}) = \mathcal{H}$

Here, "Ker" and "Ran" denote the kernel and range of the given operators, respectively. Based on this we also get the following corollary:

Corollary 2.1.1. Let T be a symmetric operator on a Hilbert space $\mathcal{H}$. Then the following three statements are equivalent:

(a) T is essentially self-adjoint

(b) $\mathrm{Ker}(T^* \pm \mathrm{i}) = \{0\}$

(c) $\mathrm{Ran}(T \pm \mathrm{i})$ is dense

A proof of Theorem (2.1.1) and Corollary (2.1.1) may be found in [111]. For a better context consult [112]. Speaking about the "basic criterion" or "von-Neumann criterion" for self-adjointness in the following we implicitly refer to Theorem (2.1.1) or Corollary (2.1.1), respectively.
Provided that the operator A is essentially self adjoint, the Klein-Gordon equation reduces to

$$\mathrm{i}\frac{\partial\psi}{\partial t} = \sqrt{A_E}\psi \tag{2.2}$$

with $\psi(t) = \exp(-\mathrm{i}t\sqrt{A_E})\psi(0)$. In that case the wave equation will not contain any divergence at any time and unitarity is preserved.

However, in order to actually analyze whether the spatial part A of the Klein-Gordon operator has a single self-adjoint extension A_E, we must solve the pair of equations

$$(A^* \pm \mathrm{i})\psi = 0 \tag{2.3}$$

and count the number of independent solutions in $\mathcal{H}$, i.e. the dimension of $\mathrm{Ker}(A^* \pm \mathrm{i})$. Let us illustrate this approach by an example.

2.1.1 Checking for quantum mechanical singularities in an example spacetime

For a simple application[1] let us consider a static, spherically symmetric background

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + r^{2p}\mathrm{d}\Omega_n \tag{2.4}$$

where $\mathrm{d}\Omega_n$ denotes the standard metric on an n-dimensional sphere. Unless $p = 1$, which corresponds to Minkowski spacetime in spherical coordinates, a classical singularity occurs either for $r \to 0$ or $r \to \infty$. If a quantum singularity appears as well can be decided by analyzing the equation

$$(\triangle \pm \mathrm{i})\psi = 0 \tag{2.5}$$

where $\triangle$ denotes the Laplacian operator. Separating variables we have $\psi \propto f(r)Y(\Omega_n)$ for the wave function of our test particle. The Laplacian in curved spacetime is given by

$$\triangle\psi = \frac{1}{\sqrt{|q|}}\partial_i(\sqrt{|q|}q^{ij}\partial_j\psi) \tag{2.6}$$

where q denotes the determinant of the spatial part of the metric, which in our case is R^{2n}. (2.5) can then be evaluated to be

$$\partial_r^2 f + n\frac{p}{r}\partial_r f - \frac{\alpha}{r^{2p}}f \pm \mathrm{i}f = 0 \tag{2.7}$$

[1] Note, that this example is a close adaption of [107].

with α being the eigenvalue of the angular part of the Laplacian. For our purposes the value of α is irrelevant which is why we will set it to zero. Near the origin (2.7) reads

$$\partial_r^2 f + n\frac{p}{r}\partial_r f = 0. \tag{2.8}$$

There are two solutions to this equation: $f_1(r) = \text{const.}$ and $f_2(r) = r^{1-np}$. The former solution clearly is square integrable with respect to the proper volume element $r^{np}\mathrm{d}r\mathrm{d}V_n$ where $\mathrm{d}V_n$ stands for the infinitesimal volume element of the n-sphere. f_2, however, fails to be in L^2 if $p \geq 3/n$. Consequently, in order to have a unique solution, i.e. to have a quantum mechanically non-singular geometry, $p \overset{!}{\geq} 3/n$. All other geometries are neither classically nor quantum mechanically regular.

Using this method a wide range of classically singular spacetimes has been uncovered to be wave-regular when tested with quantum particles, e.g. dilatonic black holes [107] and some cylindrical spacetimes [113]. Still, there exist other spacetimes where the singularity remains under quantum mechanical considerations, e.g. BTZ spacetime [112] and the negative mass Schwarzschild spacetime [107].
It should be mentioned that Ishibashi and Hosoya [114] performed the same analysis choosing the Hilbert space to be a Sobolev space rather than L^2. Concerning the negative mass Schwarzschild spacetime they also find a quantum singularity. In any case, this outcome should not bother us. Actually, said singularity prevents us from having no stable ground state [115].

Of course, this analysis can be performed the other way round. From [110] we know that if a spacetime is geodesically complete, then it is non-singular quantum mechanically as well.

2.2 Extending the singularity analysis to quantum field theory

The method of Horowitz and Marolf [107], being applicable with respect to quantum mechanics, can only judge whether static spacetimes are singularity free in a quantum sense. To get a deeper insight into the singularity problems an extension of the quantum mechanical analysis to a quantum field theory description is due. Especially, if we want to consider general time-dependent backgrounds we have to employ quantum field theory.

One of the most burning questions in GR is the status of the singularity in Schwarzschild spacetime. With a positive mass this spacetime is no longer static as in the above mentioned analysis where a negative mass was considered. Rather, there is a horizon and the roles of space and time coordinates are switched in the interior region. Therefore, we have to deal with a dynamical spacetime and an application of quantum field theory instead of quantum mechanics is inevitable.

As a start, we will ascertain whether the negative mass Schwarzschild spacetime is also singular in the quantum field theoretical sense. For our analysis we use the

Schrödinger representation [116] of quantum field theory as this formulation proves to be the most appropriate for investigating the quantum completeness of generic spacetimes. In analogy to the quantum mechanical case, we deduce the functional multiparticle Hamilton operator for a massive scalar field in this geometry. The Hamiltonian is then examined in order to decide whether there is a unique time evolution for the quantum fields.

Before going into details let us shortly introduce the most important features of the Schrödinger representation.

2.2.1 Schrödinger representation of quantum field theory

The Schrödinger representation of quantum field theory provides the framework needed for the application of the mathematical theorems concerning symmetric Schrödinger operators on Hilbert spaces.
Within this framework the usual quantum mechanical quantities like the wave function and the Hamilton operator are no longer functions of the (spatial) coordinates. We are rather dealing with functionals depending on the quantum fields. The wave functional $\Psi[\varphi]$ has the interpretation of a multiparticle state and the functional Hamilton operator $H[\varphi]$ constitutes its energy operator.
The fields in quantum field theory live in Fock spaces $\mathcal{F}$ made out of Hilbert spaces $\mathcal{H}$ representing zero-, one-, two-...particle states. More precisely, Fock space is a direct sum of tensor products of single-particle Hilbert spaces. If the fields considered are representing free bosons we have the following symmetrized Fock space [117],

$$\mathcal{F}_\nu(\mathcal{H}) = \bigoplus_{n=0}^{\infty} S_\nu \mathcal{H}^{\otimes n}, \tag{2.9}$$

where projection operator S_ν acts symmetrizing [2]. Furthermore, this operator is self-adjoint and commutes with the single-particle Hamiltonians $H(x_j)$. It projects into the subspace of states which are invariant under permutations of the variables. The multiparticle Hilbert space $\mathcal{H}^{\otimes n}$ is given by

$$\mathcal{H}^{\otimes n} \equiv \bigotimes_{k=1}^{n} \mathcal{H} \tag{2.10}$$

with $\mathcal{H}^0 \equiv \mathbb{C}$. Note that if the Hilbert space itself is separable, so will be the Fock space [118]. The general form of the multiparticle Hamiltonian will be a direct sum of one-particle Hamilton operators [119],

$$H = \sum_{j\in\mathbb{N}} \mathbb{1} \otimes \ldots \otimes \mathbb{1} \otimes H(x_j) \otimes \mathbb{1} \otimes \ldots, \tag{2.11}$$

where the single particle operator $H(x_j)$ acts on the j^{th} particle. In general it is not true that a sum or a product of self-adjoint operators is self adjoint as well. Therefore, one has to be sure, that the basic criterion for self-adjointness can be used for Fockspace Schrödinger operators as well. At least for a free scalar theory in a static spacetime the

[2] For free fermions we would have an antisymmetrizing operator instead.

validity of the basic criterion can be shown to hold. Note that in the interacting case there might appear difficulties [120, 121].

Let us therefore concentrate on a free scalar field of mass m in the following. The action for such a field on a curved background is given by [122, 116]

$$S = -\int \mathrm{d}^4x \frac{\sqrt{-g}}{2} \left\{ g^{\mu\nu}\partial_\mu \Phi(x) \partial_\nu \Phi(x) + \left(m^2 + \zeta R\right) \Phi(x)^2 \right\}, \tag{2.12}$$

where ζ is a constant standing for a non-minimal coupling to gravity represented by the Ricci scalar R. By calculating the canonical conjugate momentum, $\pi(\boldsymbol{x}) = \frac{\delta \mathcal{L}(x)}{\delta(\partial_0 \Phi(x))} = \sqrt{-g} g^{00} \partial_0 \Phi(x)$ we get the Hamilton operator

$$\begin{aligned} H[\Phi] &= \int \mathrm{d}^3x \left(\pi(x)\partial_0\Phi(x) - \mathcal{L}(x)\right) \\ &= \int \mathrm{d}^3x \frac{\sqrt{-g}}{2} \left\{ \frac{g_{00}}{-g} \pi^2(x) + \left(|\nabla\Phi(x)|^2 + \left(m^2 + \zeta R\right)\Phi(x)^2\right) \right\} \end{aligned} \tag{2.13}$$

where the spatial metric factors are encoded in the $|\nabla\Phi(x)|^2$-term and d^3x is the spatial volume element neglecting the corresponding Jacobian. The equal-time commutation relation between the field $\Phi(x)$ and the conjugate momentum $\pi(x)$ is

$$[\Phi(t, \boldsymbol{x}), \pi(t, \boldsymbol{y})] = i\delta(\boldsymbol{x} - \boldsymbol{y}). \tag{2.14}$$

Switching to a coordinate Schrödinger representation let us work in a Fock space whose basis is constructed from the time-independent field operator $\Phi(\boldsymbol{x})$. $|\phi\rangle$ shall be an eigenstate of $\Phi(\boldsymbol{x})$ with eigenvalue $\phi(\boldsymbol{x})$, i.e. the spectrum of $\Phi(\boldsymbol{x})$ contains the fields $\phi(\boldsymbol{x})$ as eigenvalues. These fields are just scalar functions of the coordinates. They are classical fields and not operators acting on the state of field content. The coordinate, i.e. ϕ-representation, of an arbitrary state $|\Psi\rangle$ in Fock space is the wave functional $\Psi[\phi] = \langle\phi|\Psi\rangle$. Note that $|\Psi\rangle$ is time dependent. Since $\left[\frac{\delta}{\delta\phi(\boldsymbol{x})}, \phi(\boldsymbol{y})\right] = \delta(\boldsymbol{x} - \boldsymbol{y})$ we can deduce the conjugate momentum in a functional representation,

$$\pi(\boldsymbol{x}) = -\mathrm{i}\frac{\delta}{\delta\phi(\boldsymbol{x})}. \tag{2.15}$$

By means of this expression the Schrödinger equation can now be turned into a functional differential equation. When separating off the time dependence of the state[3], i.e. $\Psi[t, \phi] = \Psi[\phi]\exp(-\mathrm{i}Et)$, we end up with a stationary functional Schrödinger equation:

$$\begin{aligned} H[\phi]\Psi[\phi] &= -\frac{1}{2}\int \mathrm{d}^3x \sqrt{-g} \left\{ \frac{g_{00}}{-g} \frac{\delta^2\Psi}{\delta\phi(\boldsymbol{x})^2}[\phi] + \left(|\nabla\phi(\boldsymbol{x})|^2\Psi[\phi] + \left(m^2 + \zeta R\right)\phi^2(\boldsymbol{x})\Psi[\phi]\right) \right\} \\ &= E\Psi[\phi]. \end{aligned} \tag{2.16}$$

Here, E denotes the energy eigenvalue corresponding to $H[\phi]$.

In analogy to the quantum mechanical case above, the Hamilton operator $H[\phi]$ can then be tested for essential self-adjointness using the basic criterion (2.1.1).

[3] This can be done because the Hamiltonian has no explicit time dependence.

2.2.2 Quantum field probes of black hole singularities

After collecting the mathematical prerequisites we can proceed with an application of the formalism. We will consider a negative-mass Schwarzschild type black hole and deduce that its geometry is quantum field theoretically complete by using the basic criterion.

Quantum probes of static curved spacetimes

In the last section we derived the form of the functional Hamilton operator of free scalar fields on generically curved spacetimes. Focussing on the basic criterion (Theorem (2.1.1)) we will have to check whether the equation

$$H[\phi]\Psi[\phi] \pm \mathrm{i}\Psi[\phi] = 0 \tag{2.17}$$

has a unique solution different from $\Psi = 0$. Plugging in the Hamilton operator for a free massive scalar field we get

$$-\frac{1}{2}\int \mathrm{d}^3x\sqrt{-g}\left\{\frac{g_{00}}{-g}\frac{\delta^2\Psi}{\delta\phi(\boldsymbol{x})^2}[\phi] + \left(|\nabla\phi(\boldsymbol{x})|^2\Psi[\phi] + \left(m^2+\zeta R\right)\phi^2(\boldsymbol{x})\Psi[\phi]\right)\right\} \pm \mathrm{i}\Psi[\phi] = 0 \tag{2.18}$$

It will be sufficient to consider the ground state wave functional $\Psi_0[\phi]$ only. Assuming that this functional has no nodes and is positive everywhere our ansatz for the vacuum state is [116]

$$\Psi_0[\phi] = \eta e^{-F[\phi]}, \tag{2.19}$$

where η is a normalization constant. A motivation for this Gaussian ansatz can be found following the example of the harmonic oscillator in quantum mechanics. The a priori unknown functional $F[\phi]$ must then obey the following equation,

$$\begin{aligned} -\frac{1}{2}\int \mathrm{d}^3x\sqrt{-g}\left\{\frac{g_{00}}{-g}\left(-\frac{\delta^2F[\phi]}{\delta\phi^2} + \left(\frac{\delta F[\phi]}{\delta\phi}\right)^2\right)\right\} \quad &= \\ \mp\mathrm{i} + \frac{1}{2}\int \mathrm{d}^3x\sqrt{-g}\left(|\nabla\phi(\boldsymbol{x})|^2 + \left(m^2+\zeta R\right)\phi(\boldsymbol{x})\right). \end{aligned} \tag{2.20}$$

As it stands, $F[\phi]$ will be a general quadratic functional of $\phi(\boldsymbol{x})$,

$$F[\phi] = \int \mathrm{d}^3x\mathrm{d}^3y\phi(\boldsymbol{x})f(\boldsymbol{x},\boldsymbol{y})\phi(\boldsymbol{y}). \tag{2.21}$$

Using $F[\phi]$ we plug $\Psi_0[\phi]$ into (2.18). For convenience, $f(\boldsymbol{x},\boldsymbol{y})$ shall be symmetric in $\boldsymbol{x}$ and $\boldsymbol{y}$. The second functional derivative can then be identified with the constant and the squared first functional derivative with the term containing the Klein-Gordon operator acting on the field operator eigenvalues:

$$\int \mathrm{d}^3x\left[\frac{g_{00}}{\sqrt{-g}}f(\boldsymbol{x},\boldsymbol{x})\right] = \pm\mathrm{i}, \tag{2.22}$$

$$2\int \mathrm{d}^3x \left[\frac{g_{00}}{\sqrt{-g}} \int d^3y \int d^3z \phi(\boldsymbol{z}) f(\boldsymbol{z},\boldsymbol{x}) f(\boldsymbol{x},\boldsymbol{y}) \phi(\boldsymbol{y})\right] = \qquad (2.23)$$
$$\frac{1}{2}\int \mathrm{d}^3x \sqrt{-g}\phi(\boldsymbol{x}) \left(-\Delta + \left(m^2 + \zeta R\right)\right) \phi(\boldsymbol{x}).$$

The formalism is now complete and we are ready to analyze the negative-mass Schwarzschild black hole metric which is given by (1.1). Following the aforementioned steps we get

$$\int \mathrm{d}^3x \left[\frac{1+\frac{r_s}{r}}{r^2\sin(\theta)} f(\boldsymbol{x},\boldsymbol{x})\right] = \pm\mathrm{i}, \qquad (2.24)$$

$$2\int \mathrm{d}^3x \left[\frac{1+\frac{r_s}{r}}{r^2\sin(\theta)} \int \mathrm{d}^3y \mathrm{d}^3z \phi(\boldsymbol{z}) f(\boldsymbol{z},\boldsymbol{x}) f(\boldsymbol{x},\boldsymbol{y}) \phi(\boldsymbol{y})\right] =$$
$$\frac{1}{2}\int \mathrm{d}^3x r^2 \sin(\theta) \phi(\boldsymbol{x}) \left[-\Delta + m^2\right] \phi(\boldsymbol{x}). \qquad (2.25)$$

Note that we have omitted the Ricci scalar R as the Schwarzschild solution represents a vacuum solution of the Einstein equations and thus $R = 0$. Of course, we can use the symmetries and eliminate the angular dependence by going to the equatorial plane.
This makes clear that Equation (2.22) can only be satisfied if $f(\boldsymbol{x},\boldsymbol{x})$ and therefore $f(\boldsymbol{x},\boldsymbol{y})$ is a complex function with imaginary values. Taking (2.25) and assuming that $f(\boldsymbol{x},\boldsymbol{y})$ is indeed a function with imaginary values we end up with a contradiction because the Laplace-Beltrami operator Δ is positively defined. Hence, we must conclude that (2.17) has no solution different from $\Psi \equiv 0$ for a Schwarzschild black hole with negative mass. This constitutes that the Hamilton operator is indeed essentially self-adjoint and that the state $\Psi[\phi]$ has a unique time evolution for all times.

Summarizing, the classical singularity of the negative mass Schwarzschild spacetime remains when probed with quantum fields. This result is in accordance with the quantum mechanical case considered in [107].

Note, that there is another criterion for essential self-adjointness apart from the basic criterion, namely Weyl's limit point-limit circle criterion [123]. Contrary to the basic criterion, however, it has the disadvantage that it is only applicable to symmetric Schrödinger operators on a half-line[124]. This makes the Schrödinger representation not a mere convenient representation for our purposes but rather a necessity. We will not apply this criterion here although we could since the negative-mass Schwarzschild spacetime has the desired properties. The interested reader is referred to [111, 124].

Of course, it was not our initial intention to check whether the negative-mass Schwarzschild singularity remains when tested with quantum fields. The main purpose of the formalism developed is to draw a verdict on the common (positive-mass) Schwarzschild singularity. In static spacetimes, the evolution of quantum fields is unitary which, in particular, preserves state normalization.
However, we must be careful when treating a dynamical spacetime such as the (interior) Schwarzschild spacetime. The reason is that the quantum theory of dynamical spacetimes treated as external backgrounds does not require a unitary evolution any longer [125]. Hence, the notion of quantum mechanical completeness used for static spacetimes must now be advanced.

2.2.3 Extending the framework to dynamical spacetimes

In order to extend the definition of quantum singularities to time-dependent spacetimes we take over a definition developed by Stefan Hofmann and Marc Schneider [126]. We call a general, i.e. possibly time-dependent, globally hyperbolic spacetime quantum complete (to the left) if the Schrödinger wave functional of a free test field can be normalized at the initial time t_0 with the value of the normalization being an upper bound for all times $t \in (0, t_0)$.

Using this definition as a guideline, one can prove that the interior of a Schwarzschild black hole is quantum complete, despite being geodesically incomplete [126]:

According to Geroch [127], a globally hyperbolic spacetime is diffeomorphic to $\mathbb{R}\times\Sigma_t$ and foliates into Cauchy hypersurfaces Σ_t, $t \in \mathbb{R}$. For the interior of a Schwarzschild black hole, the spacelike Cauchy hypersurfaces are given by $\{t_0\} \times \mathbb{R} \times S^2$, where $t_0 \in (0, 2M)$.

Given these hypersurfaces we can employ a (1+3)-decomposition being familiar from the ADM-formulation of General Relativity[4]. Describing a globally hyperbolic spacetime we can define a purely spatial metric

$$q_{ij} = g_{ij} + n_i n_j \tag{2.26}$$

where n_a denotes the unit (timelike) normal vector to Σ_t for which $n^2 = -1$. For an infinitesimal distance $\sqrt{q_{ab}\mathrm{d}x^a\mathrm{d}x^b}$ on the hypersurface, the proper time τ of a co-moving observer is differing from the coordinate time t by the lapse function $N_\perp$, i.e. $\mathrm{d}\tau = N_\perp \mathrm{d}t$. Physically, this function represents the rate of flow of proper time with respect to t, $N_\perp = -t^\mu n_\mu$. The distance between two infinitesimally separated points on the hypersurface is given by

$$\mathrm{d}x^i = N^i_\parallel(t, x^i)\mathrm{d}t \tag{2.27}$$

with $N_\parallel$ representing the movement tangential to Σ_t. $N_\parallel$ is called shift vector. The line element of a given spacetime, $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$, can now be expressed in terms of the variables q_{ij}, $N_\perp$ and $N_\parallel$:

$$\mathrm{d}s^2 = (nN_\perp \mathrm{d}t)^2 + q_{ij}\left(\mathrm{d}x^i + N^i_\parallel \mathrm{d}t\right)\left(\mathrm{d}x^j + N^i_\parallel \mathrm{d}t\right). \tag{2.28}$$

Following [126] the same goes for the action and hence the Hamiltonian of a free scalar field which is given by

$$H = \int_{\Sigma_t} \mathrm{d}\mu(\boldsymbol{x})\, h = \int_{\Sigma_t} \mathrm{d}\mu(\boldsymbol{x})\left(N_\perp h^\perp + N_\parallel{}^a h^\parallel{}_a\right), \tag{2.29}$$

where we definded $\mathrm{d}\mu(\boldsymbol{x}) \equiv \mathrm{d}^3x\sqrt{q}$ and

$$h^\parallel{}_a = \pi\partial_a\Phi/\sqrt{q} \tag{2.30}$$

$$h^\perp = \tfrac{1}{2}\left[\tfrac{1}{q}\pi^2 + q^{ij}\partial_i\Phi\partial_j\Phi + \left(m^2 + \zeta R\right)\Phi^2\right]. \tag{2.31}$$

[4]The ADM formulation is an approach to GR (and more generally to gauge theories) that emphasizes its field theoretic, rather than geometric, character. In particular, the dynamics of the gravitational field are viewed in terms of a Hamiltonian system. For more details see [128].

Adapting the spacetime coordinates to the slicing[5], $N_{\parallel}^{a} = 0$ and $N_{\perp} = \sqrt{-g_{00}}$. Again, each hypersurface Σ_t is equipped with a Fock space and the coordinate representation of a state $|\Psi\rangle$ on that Fock space is the wave functional $\Psi[\phi](t)$. It satisfies the functional generalization of the (now) time-dependent Schrödinger equation,

$$\mathrm{i}\partial_t \Psi[\phi](t) \;=\; H[\Phi](t)\;\Psi[\phi](t). \tag{2.32}$$

Here, $H[\Phi](t)$ denotes a functional constructed from the Hamilton density

$$h \;=\; \tfrac{1}{2}\left[\frac{\sqrt{-g_{00}}}{q}\frac{\delta^2}{\delta\phi^2} + q^{ij}\partial_i\Phi\partial_j\Phi + \left(m^2 + \zeta R\right)\Phi^2\right] \tag{2.33}$$

with any explicit dependence on t being due to the given geometry (see (2.13)). As usual, the normalization of the wave functional Ψ is given by

$$\|\Psi\|^2(t) = \int \mathrm{D}\phi\;\Psi^*[\phi](t)\Psi[\phi](t), \tag{2.34}$$

where $\mathrm{D}\phi$ is the measure over all field configurations in each hypersurface Σ_t. Static spacetimes require this norm of the wave functional to be time independent guaranteeing unitary time evolution. On a dynamical spacetime considered as an external background, however, there is no obligation for a unitary time evolution because probability with respect to the the space of field configurations might be lost to the background. In this respect, $H[\Phi](t)$ does not have to be a self-adjoint operator on the space of wave functionals any longer. For the probability density $\|\Psi[\phi]\|^2(t)$ we rather require in a dynamical spacetime that

$$\|\Psi[\phi]\|^2(t) \leq \|\Psi[\phi]\|^2(t_0) \quad \forall t \in (0, t_0) \tag{2.35}$$

is fulfilled [126]. In other words, probability must not be gained from the background since the background is not resolved in terms of dynamical degrees of freedom. A violation of the initial normalization implies that back reaction has to be taken into account. As explained above, a singularity analysis in that case becomes redundant because the spacetime to be analyzed gets destroyed.
As in the static case it will be sufficient to study the ground state wave functional only [126]. Note, that this is not clear a priori. We will pursue this issue more detailed in the next section. Using the generalized Gaussian ansatz

$$\begin{aligned} \Psi^{(0)}[\phi](t) &= N^{(0)}(t)\,\mathcal{G}^{(0)}[\phi](t), \\ \mathcal{G}^{(0)}[\phi](t) &= \exp\left[-\tfrac{1}{2}\int_{\Sigma_t}\mathrm{d}\mu(\boldsymbol{x})\mathrm{d}\mu(\boldsymbol{y})\phi(\boldsymbol{x})K(\boldsymbol{x},\boldsymbol{y},t)\phi(\boldsymbol{y})\right] \end{aligned} \tag{2.36}$$

and plugging it into the functional Schrödinger equation (2.32) results in an evolution equation for the Φ-independent factor $N^{(0)}(t)$,

$$N^{(0)}(t) \;=\; N_0 \exp\left[-\tfrac{\mathrm{i}}{2}\int_{t_0}^{t}\mathrm{d}t'\int_{\Sigma_{t'}}\sqrt{-g_{00}}\mathrm{d}\mu(\boldsymbol{z})\;K(\boldsymbol{z},\boldsymbol{z},t')\right], \tag{2.37}$$

[5] Note that static spacetimes, as treated before, generically possess such a split with vanishing shift vector [44].

while the evolution for the kernel $K(\boldsymbol{x},\boldsymbol{y},t)$ is described by a Φ-dependent nonlinear integro-differential equation [126].

Specializing to Schwarzschild coordinates both equations can be simplified by exploiting the given symmetries. Still, they are not soluble in a straight forward way. We will not go into details of the calculation here and refer the interested reader to [126]. The important outcome is that

$$\begin{aligned} \|\Psi^{(0)}\|^2(\tau) &\rightarrow |\ln(\tau)|^{-v(\Sigma)\Lambda}\left(\tau^{3/4}|\ln(\tau)|\right)^{N(\Lambda)} \\ &\rightarrow 0 \end{aligned} \tag{2.38}$$

for $\tau \to 0$. Here $\tau \equiv t/r_s$ and $N(\Lambda)$ stands for the number of momentum modes with $|k| \in [0, \Lambda^{1/3}]$. Besides, it was shown that already $\Psi^{(0)}[\Phi](\tau) \to 0$ as the black hole singularity is approached, meaning the wave functional itself has vanishing support towards the singularity [126].

Therefore, we can safely conclude that a Schwarzschild black hole is indeed quantum complete with respect to free scalar fields in the ground state.

Within this framework it can be furthermore shown that for Friedmann-Robertson-Walker spacetime[6] the probability density $\|\Psi^{(0)}\|^2(\tau)$ is time independent [126]. Hence, also these types of geodesically incomplete spacetimes are not affected by quantum singularites.

2.2.4 The role of excitations...

As we have seen in the last section Schwarzschild spacetime can be proven to be quantum complete if the ground state of the Schrödinger functional field states is considered. For static spacetimes a ground state analysis suffices as the criterion for quantum completeness is given by a unitary time evolution such that excited modes can be excluded to destroy quantum completeness. For dynamical spacetimes, however, the influence of excitations cannot be neglected because excited modes might get more populated than the initial ground state mode due to the enhancement of curvature with time and the background field providing more energy as a consequence. In order to be sure that the Schwarzschild geometry is quantum complete we therefore need to check whether the wave functional for free fields in excited modes is bounded from above accordingly.

Indeed, it can be shown that the norm of excited states is not only bounded in a Schwarzschild geometry, but approaches zero in the progress of time just like the ground state [129]:
Following [129], excitation states $\Psi^{(n)}[\phi](t)$ can be constructed in the Schrödinger representation iteratively by means of functional creation operators $(a[f])^*(t)$, where f are classical on-shell fields:

$$\Psi^{(n)}[\phi](t) = ((a[f])^*)^n\,\Psi^{(0)}[\phi](t). \tag{2.39}$$

[6]The FRW metric describes a homogenous, isotropic universe. It is described by $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}(\Sigma)^2$ with $a(t)$ denoting the scale factor according to which the universe either expands or contracts.

Then,

$$\|\Psi^{(n)}\|^2(\tau) = \kappa_n(\langle f, f\rangle)^n \|\Psi^{(0)}\|^2(\tau), \tag{2.40}$$

where κ_n is some combinatoric factor. Since $(\langle f, f\rangle)^n$ does not grow faster than $\|\Psi^{(0)}\|^2(\tau)$ approaches zero as $\tau \to 0$ for all n we can conclude that

$$\|\Psi^{(n)}\|^2(\tau) \xrightarrow{\tau\to 0} 0. \tag{2.41}$$

In fact, the authors of [129] find that $(\langle f, f\rangle)^n$ is a constant factor.

That is, Schwarzschild spacetime is quantum complete for excited states as well. What remains to be seen is whether this is also the case if interacting fields are considered.

2.2.5 ...and interacting fields

Concerning interacting fields we choose their coupling to be weak initially such that they can be treated within perturbation theory. Introducing a Φ^4-interaction into the Hamiltonian (2.33) and carrying out a programm analogous to the free-field case, it can be shown that [129]

$$\|\Psi_{int}\|^2(\tau) \xrightarrow{\tau\to 0} \|\Psi_{free}\|^2(\tau). \tag{2.42}$$

This implies that the interactions among the fields diminish whilst the singularity is approached. It might be suspected, that this behavior is caused by a decreasing particle density around the singularity. In any case (2.42) implies that quantum completeness is guaranteed for such fields as well. If, on the other hand, this was not the case, we could conclude that the fields entered a strong-coupling regime. A singularity analysis would then become obsolete.

2.3 A short bottom line

In this chapter we have presented a framework in which geometrically singular spacetimes can be probed by quantum objects. In particular, we observed that Schwarzschild black holes are quantum complete whilst being geodesically incomplete. Hence, we must draw the conclusion that near the would-be singularity measurement devices and observables can no longer be described classically. Rather, they must represent inherent quantum objects.

Though evading the classical singularities, this semiclassical point of view might not be sufficient to describe the nature of black holes or other singular configurations. As we will see in the next chapter, black holes, albeit no longer being plagued by singularities, are still surrounded by many paradoxes in this approach.

Chapter 3

The failure of a semiclassical black hole treatment

Aspects calling for a new black hole description

Introducing the notion of singularities in the quantum realm we were able to conclude in the last chapter that Schwarzschild spacetime is quantum regular. Convinced that classical physics is emergent from an underlying quantum theory the classical black hole singularity no longer is of any physical relevance in this respect. While this is true for Schwarzschild spacetime we have also seen that there exist other spacetimes still suffering from singular behavior in a semiclassical treatment. This implies that quantum field theory in curved spacetime cannot evade all classical irregularities. After all, though solving the singularity problem, even quantum probes on a classically left Schwarzschild background do not seem to be satisfactory for quite many reasons.

In the following we will present several aspects that call for a full quantum description of black holes - the most prominent being the famous information paradox. We will start with black hole entropy in a purely classical regime.

3.1 Black hole entropy and the information paradox

The concept of black hole entropy is based on two theorems of General Relativity [130].

1) Hawking's area theorem

This theorem states that the horizon area A of a black hole cannot decrease [44],

$$\frac{\mathrm{d}A}{\mathrm{d}t} \geq 0. \tag{3.1}$$

Clearly, in a classical situation, one cannot expect anything else for there is no particle emission possible in the presence of a horizon.

2) No hair theorem

According to Israel all stationary black hole solutions of the Einstein equations, including charged and rotating ones, can be completely characterized by three parameters only [40, 41, 42, 43]: mass M, charge Q and angular momentum J. An external observer is ignorant to any other information, i.e. hair, for example the kind of material the black hole was initially built from. This is because all other information is enclosed behind the event horizon.

Recollecting the second law of thermodynamics, given a thermodynamic system, the sum of the entropies of the participating subsystems is increasing. Holding on to the no hair theorem Bekenstein consistently realized that this would imply a violation of the second law of thermodynamics [35]. Any object carrying entropy will lead to a decrease of entropy in the exterior of the black hole when crossing its horizon. The black hole entropy should therefore increase. This however, a priori, seems to be excluded by the no hair theorem. Being reminiscent of the second law of thermodynamics this observation has led Bekenstein to link black hole entropy S and horizon area A. The result is the Bekenstein-Hawking formula [36, 37, 46, 47]:

$$S = \frac{A}{4l_p^2}. \tag{3.2}$$

In compliance with the first law of thermodynamics,

$$\mathrm{d}M = T\mathrm{d}S, \tag{3.3}$$

black holes must be associated with some finite temperature T if Bekenstein entropy is taken seriously. Indeed, Bardeen, Carter and Hawking found in 1973 that Einstein's equations suggest an analogous law with the entropy being identified with the horizon area and the temperature T being captured by the surface gravity κ of the black hole[1] [131],

$$\mathrm{d}M = \kappa\mathrm{d}A. \tag{3.4}$$

Now, for a Schwarzschild black hole $\kappa = \hbar/(G_N M)$, i.e. non-zero [132]. At that time, this was a paradoxical situation since black holes were believed to have zero temperature by default.

3.1.1 Hawking radiation

In 1975 Hawking employed a semiclassical reasoning to resolve this paradox. In such an approach the background geometry is left purely classical while the particles and fields acting on that geometry are treated quantum theoretically. Hawking showed that a distant observer will detect a thermal spectrum of particles emitted from the black hole having a temperature $T = \kappa$ [46, 47], which in the Schwarzschild case corresponds to

$$T = \frac{\hbar}{G_N M}. \tag{3.5}$$

This observation renders Bekenstein's idea of a finite black hole entropy proportional to the horizon area consistent. Accordingly, the classical law (3.1) must now be superseded as the black hole's mass and the area of its horizon is caused to decrease over time due

[1] Note that in this formula changes of charge and angular momentum have been left out.

to Hawking radiation. As we will see this is not the end of the story. Even in this semiclassical line of reasoning we will encounter problems concerning the entropy of the black hole.

3.1.2 Negative heat capacity

As can be seen from (3.5) black holes are attributed with a temperature that is inversely proportional to their mass. Therefore, while heat flows out of the black hole during its evaporation, peculiarly its temperature increases further. This negative heat capacity implies an untypical behavior for thermodynamical systems.

f

3.1.3 Information paradox

Hawking's calculations furthermore indicate that black hole evaporation by means of thermal radiation does not preserve information. In short, this is because the information of a state falling into a black hole cannot be regained for Hawking radiation is thermal and therefore no carrier of information.

To be more precise consider a collapsing mass shell described by the Schwarzschild spacetime (1.1). Albeit reflecting a time-independent geometry let us take this simple metric for the description of an evaporating black hole. We can do so as long as the evaporation process is slow [39]. Following the thoughts of [39, 133] imagine two spacelike slices S_1 and S_2 at different times t_1 and $t_2 > t_1$, respectively. While in the exterior region such slices may be described by $t_1 =$ const. and $t_2 =$ const., they are given by $r_1 =$ const. and $r_2 =$ const. in the interior with $r_2 > r_1$. Both regions shall be joined by a smooth interpolating segment across the horizon. Considering the evolution from the first to the second slice the connecting segment obviously has to stretch. Albeit the Schwarzschild geometry is time-independent, the connecting segment, covering both parts of the interior and exterior region, is time dependent. This is possible since the Schwarzschild coordinates cannot be used in the near-horizon region. As a result, this uncovered time dependence and the stretching between successive spacelike slices leads to an increase in the wavelength of present fields. Consequently, particles, or more precisely Hawking pairs, are created out of the vacuum. Another way to see this is that the vacua of both slices are different from each other.

The particles created are in an entangled state,

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle_{in} |0\rangle_{out} + |1\rangle_{in} |1\rangle_{out} \right), \tag{3.6}$$

where $|0\rangle$, $|1\rangle$ represent the occupation numbers 0 or 1 for a given mode, respectively. While one particle is emitted and contributes to Hawking radiation the other one stays located inside the Schwarzschild surface. The entanglement entropy associated with such a configuration, i.e. the von Neumann entropy, is

$$S = -\mathrm{tr}\,(\rho \ln \rho) \tag{3.7}$$

where the density matrix $\rho = |\psi\rangle\langle\psi|$. Of course, for the whole system, which is in a pure state, the entanglement entropy vanishes. However, tracing over the "*out*"-subsystem we get ρ_{in} and

$$S_{in} = \ln(2). \tag{3.8}$$

During the collapse of the mass shell we can pick many more such spacelike slices and for each pair created the entropy increases by ln(2). In the end there are two possible outcomes. Either the black hole evaporates entirely or evaporation stops at some point and there is a remnant of Planck-size order left. Since there is nothing left from the black hole, the first choice means that the final state cannot be given by a wave function. There is nothing left the radiation can be entangled with and so information is lost irretrievably. Rather, the final state is given by a density matrix. But having started from a pure initial state and ending in a mixed state indicates the violation of unitary evolution - a truly fatal conflict with one of the most profound principles of quantum mechanics. Still, the second choice does not offer a satisfactory solution either. Indeed, there is no evident violation of quantum mechanics. However, the remaining lump of matter corresponds to an entanglement entropy $S = N\ln(2)$. As

$$S = (\#\text{ of internal states}) \tag{3.9}$$

we have a degeneracy amounting to[2] 2^N. Therefore, the degeneracy of the remnant is unbounded despite being of finite size and energy [39].
Either way, we are left with a paradox.

3.2 ...really a paradox? Solution proposals

At this point, we must analyze the underlying reasons for these puzzles. First of all, Hawking performed his calculations in a limit where the background spacetime is completely decoupled,

$$G_N \to 0, \qquad M \to \infty, \qquad r_s = 2G_N M \text{ fixed}. \tag{3.10}$$

This semiclassical approach neglects any back reaction from the emitted particles on the classical background. Besides, infinitely heavy black holes can never resemble realistic models. The limits (3.10) are only an approximation. Also, once a large number of quanta has been emitted, we might no longer have a classical evolution.

Therefore, we cannot conclude that there really exists a paradox in the first place. The crucial question to ask is whether in a full quantum treatment there is still a clash between quantum mechanics and gravity.

Of course, so far a theory of quantum gravity has not been found and it would be otiose to ponder about a resolution. In any case we know that there is no information paradox in string theory. According to the AdS/CFT correspondence [56, 57], the most rigorous realization of the holographic principle [134], black holes in AdS are dual to a conformal field theory. Since the field theory side is unitary for sure, so should be the gravity side.

[2]Note that we have neglected the Boltzmann constant in this formula.

Black hole complementarity

Making use of the holographic principle a solution to the information paradox might be the concept of black hole complementarity [50, 51, 52]. This idea by Susskind based on earlier work of 't Hooft preserves both the equivalence principle and unitary evolution at the semiclassical level. However, this approach is on the expense that locality must be given up. There is both a reflection and a crossing of information at the event horizon in some sense. Concerns that this reasoning would violate the no-cloning theorem are invalidated by the fact that no observer can confirm both courses of action simultaneously. Introducing a stretched horizon, a surface slightly above the event horizon, an infalling observer will see himself and his bit of information just passing through the event horizon while an asymptotic observer would see the stretched horizon in a dissipative way for being heated up by the infalling bit of information and reradiating it in the form of Hawking radiation.

Firewalls

In [55] Marolf and Polchinski et. al. point out, however, that there are mutual inconsistencies in the complementarity assumptions[3]. In short, these are 1. unitary time evolution, 2. the validity of the semi-classical approach outside the black hole and 3. the equivalence principle ("no drama"). If information escapes from a black hole and shall be described by a pure state, the radiation emitted at late times must be entangled with the past Hawking radiation. At the same time the outgoing particles must be entangled with the ingoing ones as well. That is, preserving unitarity, there is a contradiction with the quantum mechanical principle of monogamous entanglement. The authors of [55] therefore suggest a breaking of the interior and exterior entanglement by means of high-energy quanta at the horizon. This conjecture trying to solve the apparent inconsistency in black hole complementarity goes under the name "firewall". Giving up the equivalence principle the firewall proposal has led to controversial debates among physicists. While some are in favor of it (e.g. [135, 136]) others raise their sceptisism (e.g. [137, 138, 139]).

ER=EPR

Further, inspired by [140], Maldacena and Susskind proposed the existence of non-traversable wormholes between entangled states [141]. This feature, the so-called ER=EPR correspondence[4], could circumvent the need for a firewall.

[3]Their arguments are partly based on Page's work [144].

[4]ER stands for Einstein and Rosen who discovered wormhole solutions (Einstein-Rosen bridges) [142] of the Einstein equations while EPR stands for Einstein, Podolsky and Rosen, famous for their paper on quantum correlations [143].

3.3 A complete quantum treatment as the ultimate cure

As for now, no final verdict on the solution of the information paradox and other related problems can be drawn.

Page's approach and black holes as the fastest scramblers in nature

Following conjectures by Page in the end all confusion might be caused by the semiclassical treatment of black holes itself [144]. The reasoning of Page is exclusively based on quantum information theory. His calculations indicate that the entanglement entropy of the black hole's radiation increases until half the black hole has been evaporated and subsequently decreases. At the end of the evaporation process the entropy drops to zero. The amount of information I released is defined by

$$I \equiv \ln(\dim\mathcal{H}) - S \tag{3.11}$$

where $\dim\mathcal{H}$ denotes the dimension of the Hilbert space under consideration. Assuming a big black hole radiating a large number of quanta, information will effectively be released only at the halfway point of evaporation, i.e. at the so-called Page time[5]. Until the black hole has been evaporated entirely, the full information is regained and the radiation is purified. Furthermore, inspired by string theory considerations suggesting that the information is encoded in the black hole's internal degrees of freedom [45, 56], Hayden and Preskill showed that after the Page time information is released very rapidly [145]. In order to preserve complementarity, the time scale for this release is then given by the scrambling time

$$t_{scr} \geq r_s \log\left(\frac{r_s}{l_p}\right), \tag{3.12}$$

which is the time it takes to scramble, i.e. thermalize, information. As it turns out, black holes saturate the lower bound for the scrambling time making them the fastest scramblers in nature [146].

String theory solutions

As announced, (super)string theory might be able to resolve the mentioned problems connected to black holes. Indeed, Mathur and Lunin proposed that black holes should be considered as "fuzzballs", i.e. loosely speaking, bound states of strings [59, 60]. Also matrix models have been suggested [147, 148, 149]. Besides, Strominger and Vafa succeeded in deriving the microscopic origin for the entropy of a five-dimensional extremal black hole [45]. Whether their results can be transferred to Schwarzschild black holes remains to be seen.
Apart from that, all these solutions rely on a certain UV completion of gravity at short distances. This, however, is not a mandatory requirement for a solution.

[5]Strictly speaking, Page time is defined as the time when the black hole's entropy has halved.

Following a new path

In the next part we will present an approach that works independently of any UV completion. It assumes that black holes (of sizes $R \gg l_p$) are made up of long-wavelength gravitons which are subject to a collective potential. From this perspective, the collective effects of IR physics are responsible for the many puzzles occurring in the semiclassical treatment.
In a sense, this approach is analogous to continuum mechanics, where classical observables like temperature or pressure arise due to underlying microscopic degrees of freedom. The ideal gas law $PV = Nk_BT$, for example, is a well known manifestation of this phenomenon. Similarly, we expect that the classical concept of a spacetime emerges from a long-wavelength (coarse-grained) limit of microscopic gravitational degrees of freedom.
Clearly, such an approach inherently prevents the formation of singularities, horizons or other peculiarities present in General Relativity. These should therefore be considered as an oversimplification of the underlying quantum theory. In this respect, the interior Schwarzschild solution, for example, should only be taken seriously in a geometric sense.

Before proceeding we want to mention that our arguments calling for a full quantum treatment of black holes shall not be misunderstood as an invalidation of QFT in curved spacetime. Rather, we intend to clarify that a semiclassical reasoning might ignore important quantum effects which could have the power to solve the mentioned prevailing paradoxes.

Part II

Towards a quantum description of spacetime

Chapter 4

Black holes as Bose-Einstein condensates of gravitons

4.1 Introducing black hole's Quantum-N portrait

In a recent series of papers by Dvali and Gomez [64, 150, 151, 152, 153] a new conceptual framework concerning black hole physics was proposed. In this work black holes are treated as bound states, respectively condensates, of $N \gg 1$ weakly interacting (i.e. long-wavelength $\lambda = \sqrt{N} l_p$) gravitons.
Within the Schwarzschild radius, the classical gravitational theory is strongly coupled. This necessitates to sum up an infinite number of equally important terms in the perturbation series. Therefore, in order to represent the black hole interior the bound state itself must be a non-perturbative object. The individual interactions between gravitons can be described by means of the perturbative vertices in Einstein theory.
Building on the work of 't Hooft and Witten [154, 155], Dvali and Gomez used the generic large-N logic to argue that the dimensionless gravitational coupling α is proportional to $1/N$. This guarantees that the effective quantum theory of gravitons in Minkowski spacetime is weakly coupled even if non-perturbative objects are considered. The reason for this lies in the possibility that gravitating objects like black holes can be treated non-perturbatively due to large collective effects.
Besides, it was suggested that the black hole condensate is at the critical point of a quantum phase transition. Since quantum phase transitions necessarily imply large quantum correlations any semiclassical description is rendered invalid. A good example for this can be found in [156]. Therefore, this novel framework is able to exhibit black hole properties that cannot be captured in a semiclassical treatment of gravity.
Most importantly, the underlying quantum physics might be able to resolve the many mysteries and paradoxes surrounding classical black holes. The semiclassical description is only restored by sending the number of quanta N in the gravitational field to infinity. At finite N there are deviations from classicality. Usually, quantum processes, such as the decay into a two particle state, are expected to be exponentially suppressed, i.e. $\langle \text{out}| \exp(-S/\hbar) |\text{in}\rangle \sim \exp(-N)$ where S denotes the action of the system under consideration. Bose-Einstein condensates, however, receive $1/N$ corrections [64, 153]. These corrections can accumulate over the black hole's lifetime to a $\mathcal{O}(1)$-effect [153].

Unitarization through black hole formation is by definition only possible if the evaporation of the black hole itself is a unitary process. Therefore, a UV completion of gravity through black hole production crucially relies on a microscopic understanding of black holes and their radiation.
The motivation for a description of black holes in terms of long-wavelength constituents can be seen as follows. Consider an unstable, homogenous, spherical body of radius R and mass M subject to gravitational collapse. As predicted by classical physics the collapse will unavoidably lead to the formation of a black hole. As long as $R \gg r_s = 2G_N M$ linearized gravity is a good approximation to use. The gravitational field is essentially Newtonian and is given by a potential $\phi(r) \sim 1/r$ in the exterior, respectively $\phi(r) \sim r^2$ in the interior region. The occupation number N of quanta in the gravitational field can be obtained by comparing the gravitational part of the energy

$$E_{grav} \sim \frac{M r_s}{R} \sim \frac{M^2 G_N}{R} \tag{4.1}$$

with the sum of the energies of the individual gravitons having some wavelength λ and occupation number N_λ

$$\sum_\lambda \frac{\hbar N_\lambda}{\lambda} \simeq \frac{\hbar N}{R}. \tag{4.2}$$

It follows that

$$N \simeq \frac{M^2}{M_p^2}, \tag{4.3}$$

where we used that the peak of the wavelength distribution is at $\lambda = R$ and introduced the Planck mass[1] $M_p = \sqrt{\hbar/G_N}$. Of course, this can only hold, when the interaction of the individual gravitons both among each other and with the collective potential is negligible. Having this, we can also conclude that for $R \gg r_s$ there is no gravitational self-sourcing. The condensate cannot be self-sustained. This also shows that all objects except black holes have a substantial part of their energy carried by other constituents than gravitons. Such objects therefore can only exist with the help of an external source.
However, once the radius of the sphere crosses its Schwarzschild horizon, the energy gets dominated by gravitons with wavelength $\lambda = r_s$. Since by default an object of size R can at most be compressed to have a radius r_s we infer from (4.3) that black holes described in terms of N constituents comprise maximally packed states of gravitons. Any increase of the number of quanta would inevitably lead to an increase of the black hole's mass and equally its radius. Furthermore, at this point the gravitational energy becomes of the order of the energy of the source. That is, the self-sourcing by the collective gravitational energy becomes important. The black hole condensate becomes self-sustained. Notice that the interactions among the individual gravitons remain weak unless $r_s \sim l_p$.

4.1.1 The Universality of N

Having a self-sustained system at hand it suffices to know the occupation number N which has now become a universal quantity in the description of black holes - hence the

[1] Note that we neglect any numerical factors here as we are interested in basic scaling relations only.

notion "Quantum-N portrait":
The number N can be translated into a measure of energy or mass as we have seen. Likewise it represents the Schwarzschild radius and therefore the wavelength of the individual gravitons. The dimensionless interaction strength between individual gravitons can also solely be given in terms of N,

$$\alpha \equiv \hbar \frac{G_N}{\lambda^2} = \frac{1}{N}. \tag{4.4}$$

Therefore, we have a close resemblance to the no-hair theorem [157] found in the classical black hole description.
From equation (4.4) we also see that due to the description of black holes as large-N objects, the individual coupling between the gravitons is extremely weak. On the other hand, the collective coupling αN is of order unity indicating a strong collective binding potential. An important consequence of this picture is that black holes always balance on the verge of self-sustainability, since the kinetic energy $\hbar/r_s$ of a single graviton is just as large as the collective binding potential $-\alpha N\hbar/r_s$ produced by the remaining $(N-1)$ gravitons. If we give a graviton inside the condensate just a slight amount of extra energy, its kinetic energy will be above the escape energy of the bound state. In [64] it was therefore concluded that black holes are leaky condensates.

4.1.2 Leakiness and emergent thermality

The collapse and leakage process can be parametrized as a self-similar decrease of N. Let us consider a $2 \to 2$ scattering of two constituent gravitons in which one of them gains above-threshold energy and as a consequence can escape the bound state. Since the escape wavelength is $\lambda_{esc} = \sqrt{N} l_p$, the decay rate is given by

$$\Gamma_{esc} \simeq \frac{1}{N^2} N^2 \frac{\hbar}{l_p \sqrt{N}}. \tag{4.5}$$

The first $1/N^2$ factor here comes from the squared amplitude of the scattering amplitude while the second factor is purely combinatoric rooting in the $\binom{N}{2}$ possibilities to choose two out of N gravitons that shall scatter. Since these factors cancel, the decay rate is determined by the characteristic energy of the process. Of course, due to phase space arguments the $2 \to 2$ scattering is the most probable. Scattering processes including more gravitons are suppressed by higher powers of N. The overall decay rate is given by

$$\Gamma_{esc} = \frac{\hbar}{l_p \sqrt{N}} + \mathcal{O}\left(N^{-3/2}\right). \tag{4.6}$$

The characteristic time scale during which one graviton of wavelength $l_p\sqrt{N}$ leaves the continuum can be read off to be $\hbar\Gamma^{-1}$, such that the leakage of particles

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{1}{l_p \sqrt{N}} + \mathcal{O}\left(N^{-3/2}\right). \tag{4.7}$$

In terms of the black hole mass this formula can be recast in the form

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{\hbar}{l_p^2 N} + \mathcal{O}\left(N^{-2}\right). \tag{4.8}$$

Now, we might wonder how a quantum effect like Hawking radiation can be understood in this picture of highly occupied graviton states.
First of all, defining a temperature $T = \hbar/(\sqrt{N} l_p)$ the thermal of a black hole with mass M, $T = \hbar/(G_N M)$, i.e. Hawking temperature [46, 47], is reproduced.
Secondly, we can establish a connection to Hawking radiation. To this end, one has to keep in mind that Hawking's calculations were performed in a semiclassical situation. Transferred to the N portrait this regime corresponds to the double-scaling limit

$$N \to \infty, \qquad l_p \to 0, \qquad \sqrt{N} l_p \text{ fixed}, \qquad \hbar \text{ fixed}. \tag{4.9}$$

In this case, the exact thermal spectrum of Hawking radiation is recovered:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{T^2}{\hbar}. \tag{4.10}$$

Summarizing, Dvali and Gomez were able to show that for large N their framework correctly reproduces the thermal evaporation of black holes. This is remarkable because at no point they fall back to the geometric concepts used in the standard derivation. On the contrary, Hawking temperature and radiation emerge from a quantum effect, namely the depletion, respectively evaporation, of quanta out of the leaky black hole condensate. Similarly, the odd negative heat capacity associated to black holes can be simply ascribed to the fact that the number of black hole constituents N decreases as a result of the quantum depletion.

4.1.3 Black holes as classicalons

Now, clearly, the occupation number N can be understood as the parameter measuring the classicality of a given object composed out of gravitons, in this case black holes. The number of gravitons produced in the gravitational field of any elementary particle is negligibly small, for example for an electron we get $N \sim G_N m_e^2/\hbar \approx 10^{-44}$. This shows why elementary particles cannot be considered as classical gravitating objects on scales of the order of their would-be Schwarzschild radius even though they contribute to a standard Newton law at large distances. Black holes, on the other hand, are the most classical quantum objects one can imagine. Therefore, they can be called classicalons [19, 24, 30, 31, 32].

4.1.4 Quantum corrections

It furthermore has to be noted that the leading corrections to the above results, e.g. (4.6), are given by $1/N$. In the semiclassical picture the only non-perturbative quantum corrections are of the form $\exp(-S/\hbar)$, i.e $\exp(-N)$, where S denotes the action describing the black hole. Of course, taking into account only the classical black hole action, these corrections are not sensible to any microscopic features. In the quantum portrait,

however, black holes are modeled as condensates at the verge of a quantum phase transition. Therefore, the leading corrections to the above (semi)classical ($N \to \infty$) picture are not exponentially but only $1/N$ suppressed.
This deviation is of paramount importance: As mentioned, above semiclassical picture has to struggle with the famous information paradox. Even by consideration of the exponentially suppressed quantum corrections, there is no way to get back all the information that was once put into a black hole. However, having $1/N$ corrections at our disposal, it is possible to retrieve all information over the black hole's lifetime. The reason is that these corrections are present for each step of emission resulting in an accumulation that ultimately leads to a $\mathcal{O}(1)$-effect.
In conclusion, from the corpuscular point of view the famous information paradox is just an artefact of working in the strict (semi)classical $N \to \infty$ limit. To the same extent, it is only in the $N \to \infty$ limit that the black hole loses its hair. In fact, black holes reveal $1/N$-hair [153].
At this point we want to mention once more that this framework suggested by Dvali and Gomez has nothing to do with a completion of gravity at short distances, such as string theory (see (3.2)). Rather, it aims at resolving classical backgrounds in terms of quantum constituents. Typical length scales involved are macroscopic distances such as the Schwarzschild radius in the case of black holes. Moreover, the black hole is modeled as a bound state of long-wavelength gravitons $\sim r_s$. Hence, the physics of sufficiently large black holes can be regarded as independent from any ultraviolet physics.
In that sense, all seemingly mysterious properties must be due to quantum collective effects of the infrared constituents.

4.2 Toy models for the N portrait

Having no quantum theory of gravity at hand it is hard to draw a serious verdict on the Quantum-N portrait along the lines of [64, 150, 151, 152, 153]. To at least get a glimpse several toy models of Bose-Einstein condensates on the verge of a quantum phase transition have been investigated (see e.g. [158, 159]).
In [158] a system of attractive bosons in one spatial dimension was studied. For one thing, it was shown that quantum effects are important at the critical point. It was pointed out that this holds true even if the number N of particles becomes large - totally against the intuition that large-N systems should be treatable classically. Secondly, the entanglement entropy of different modes was demonstrated to peak around the critical point and to be mainly supported by long-wavelength modes.
Using a $(d+1)$-dimensional Bose-Einstein condensate it was shown in [159] that the scrambling time, or more precisely the quantum break time[2], scales as $r_s \log(N)$. Transferring this result to black holes, the authors argue that their findings are in full agreement with the predictions made by [145, 146] which would support the fast-scrambling conjecture (see (3.3)).
While the papers [158] and [159] reverted to ordinary Bose-Einstein condensates we

[2] Quantum break time describes the time scale required to depart from the mean-field approximation by $\mathcal{O}(1)$. Around instabilities of the Gross-Pitaevskii equation, i.e. for black holes described within the N portrait, it can be shown to coincide with the scrambling time [159].

propose a derivatively coupled toy model [160]. We do so because in gravity the interaction terms of the metric fluctuation field with itself are coupled derivatively. Since this is also reflected in the N-portrait by the momentum dependence of the graviton interaction strength (4.4) we hope to be able to study further aspects of black holes within the given picture. In particular, our construction is aimed to stay at the point of a quantum phase transition irrespective of the occupation number N of the classicalon state. In order to check whether these assumptions are justified we consider quantum perturbations around a highly occupied classical state (Bogoliubov approximation). Our results indicate that the perturbative approach is not applicable, which is exactly what we expect to see if the system indeed manages to stay at the verge of a quantum phase transition. Therefore, we have indications for the claims of [151, 152], even though only a subsequent numerical and non-linear analysis can finally decide about the status of our model.

4.2.1 Bose-Einstein condensates at the verge of quantum phase transition

Let us begin by reviewing the well-known non-relativistic model of weakly interacting massive bosons as it was used in [158] and [159]. This allows us to get familiar with the properties of a quantum phase transition. For a thorough introduction to fundamental concepts used therein, we refer the reader to the relevant literature, e.g. [161, 162]. The discussion closely follows [156] where a condensate of N bosons of mass m with an attractive interaction in one dimension of size V at zero temperature is described. The second quantized Hamiltonian of such a system is given by

$$\hat{H} = \frac{\hbar^2}{2m} \int_0^V \mathrm{d}x \, (\partial_x \hat{\Psi})^\dagger (\partial_x \hat{\Psi}) - \frac{U}{2} \int_0^V \mathrm{d}x \, \hat{\Psi}^\dagger \hat{\Psi}^\dagger \hat{\Psi} \hat{\Psi}, \tag{4.11}$$

where $\hat{\Psi}(x,t)$ and $\hat{\Psi}^\dagger(x,t)$ are the boson field operators that annihilate and create a particle at the position x, respectively. The operator $\hat{\Psi}^\dagger\hat{\Psi}$ is the particle density operator. The first term in (4.11) represents the kinetic energy and the second one describes the particle interactions. The explicit form of the interaction term can be derived from the assumption of having a dilute gas of bosons, compare for example to [161, 162]. The range of inter-particle forces is much smaller than the average distance between the particles. Therefore, only two-body interactions have to be considered while interactions of three or more particles can be neglected. Furthermore, at sufficiently low temperature the scattering between two particles is dominated by the s-wave contribution to the wave function which is characterized by a single parameter, the scattering length. This leads to the concept of an effective interaction described by a contact potential $V_{\text{int}} = U\delta(x - x')$ (which has already been integrated above), where U is a parameter of dimension [energy] $\times$ [length] controlling both the strength and the sign of the interaction. Since we are considering attractive interactions, U will be positive in the following.

The dynamics of $\hat{\Psi}(x,t)$ are given by the Heisenberg equation

$$\mathrm{i}\hbar\,\partial_t\hat{\Psi} = \left[\hat{H},\hat{\Psi}\right] \tag{4.12}$$

$$= \left[-\frac{\hbar^2}{2m}\partial_x^2 + U(\hat{\Psi}^\dagger\hat{\Psi})\right]\hat{\Psi}, \tag{4.13}$$

where the equal time commutation relations

$$\left[\hat{\Psi}(x,t),\hat{\Psi}^\dagger(x',t)\right] = \delta(x-x') \qquad \left[\hat{\Psi}(x,t),\hat{\Psi}(x',t)\right] = 0 \tag{4.14}$$

have been used. Note that $\hat{\Psi}^\dagger$ is related to the operator $\hat{\Pi}$, which is the canonical conjugate of $\hat{\Psi}$, by $\hat{\Pi} = \mathrm{i}\hbar\hat{\Psi}^\dagger$.

Applying the mean-field approximation amounts to replacing the operator $\hat{\Psi}(x,t)$ by a function $\Psi_0(x,t)$ which constitutes a classical field having the meaning of an order parameter. This replacement is justified when the quantum ground state is highly occupied. In this case the non-commutativity of the field operator is a negligible effect. Since $N \gg 1$, the states $|N\rangle$ and $|N+1\rangle$ can be regarded as physically equivalent. This motivates the following definition: $\Psi_0(x,t) = \langle N-1|\hat{\Psi}(x,t)|N\rangle$. For identical bra- and ket-states the expectation value would be zero. Taking the average over stationary states, whose time dependence is separated in the usual way, we see that the time dependence of Ψ_0 is given by

$$\Psi_0(x,t) = \langle N-1|e^{\frac{\mathrm{i}E_{N-1}t}{\hbar}}\hat{\Psi}(x)e^{-\frac{\mathrm{i}E_N t}{\hbar}}|N\rangle$$

$$= \Psi_0(x)\exp\left(-\frac{\mathrm{i}\mu t}{\hbar}\right), \tag{4.15}$$

where $\mu = \frac{\partial E}{\partial N} \approx E_N - E_{N-1}$ is the chemical potential representing the energy needed to add one more particle to the system. Inserting this ansatz in (4.13) yields the stationary Gross-Pitaevskii equation[3] :

$$\left(-\frac{\hbar^2}{2m}\partial_x^2 - \mu + U|\Psi_0(x)|^2\right)\Psi_0(x) = 0. \tag{4.16}$$

A trivial solution that fulfills the periodic boundary conditions $\Psi_0(0) = \Psi_0(V)$ is given by

$$\Psi_0^{(\mathrm{BE})}(x) = \sqrt{\frac{N}{V}} \quad \text{fixed.} \tag{4.17}$$

[3]Note that equation (4.16) may, after separating the time dependence, also be derived from the action principle $\delta S = \delta\int\mathcal{L}\,\mathrm{d}t = 0$ with the Lagrangian $\mathcal{L}$ given by

$$\mathcal{L} = \frac{\mathrm{i}\hbar}{2}\int_0^V \mathrm{d}x(\Psi_0^*\partial_t\Psi_0 - \Psi_0\partial_t\Psi_0^*) - \frac{\hbar^2}{2m}\int_0^V \mathrm{d}x\,(\partial_x\Psi_0)^*\,(\partial_x\Psi_0) + \frac{U}{2}\int_0^V \mathrm{d}x(\Psi_0^*\Psi_0)^2.$$

Quantizing the resulting conjugate momentum is consistent with the commutation relations (4.14) and the Hamiltonian (4.11).

This solution corresponds to the homogenous Bose-Einstein condensate. However, this solution represents the minimal energy configuration only for $U < U_c$. The critical value has been derived in [156] to be $U_c = \hbar^2\pi^2/(mVN)$. For $U > U_c$ the ground state is given by an inhomogenous solution $\Psi_0^{(\mathrm{sol})}(x)$ describing a soliton. By increasing the parameter U, i.e. the interaction strength, the ground state of the system undergoes a phase transition from the Bose-Einstein phase to the soliton phase once the critical point U_c is reached. As the authors in [156] have shown, this point of phase transition is characterized by a cusp in the chemical potential $\mu(U)$, the kinetic energy $\epsilon_{kin}(U)$ and the interaction energy $\epsilon_{int}(U)$ per particle as functions of U.

The main result of [156] was to show that at the point of a phase transition quantum corrections to Ψ_0 become important such that a purely classical description is no longer possible, therefrom the name 'quantum phase transition'. A suitable way to investigate this effect is provided by the Bogoliubov approximation in which the classical field Ψ_0 is furnished with small quantum corrections $\delta\hat{\Psi}$. A proper quantum mechanical treatment, of which the details are given in the next section, allows to derive the famous energy spectrum of the Bogoliubov excitations

$$\begin{aligned}\epsilon(k) &= \left(\left(\frac{\hbar^2\delta k^2}{2m}\right)^2 - \frac{\hbar^2 UN}{mV}\delta k^2\right)^{1/2} \\ &= \left(\left(\frac{\pi\hbar^2}{mV}\right)^2 \delta k^2 \left[\left(\frac{V}{2\pi}\right)^2 \delta k^2 - \frac{U}{U_c}\right]\right)^{1/2} .\end{aligned} \tag{4.18}$$

Due to the periodic boundary conditions, the momentum δk of the Bogoliubov modes is quantized in steps of $2\pi/V$. From (4.18) it is clear that once the interaction strength approaches the value U_c, the energy of the first Bogoliubov mode ($\delta k = 2\pi/V$) vanishes. Consequently, the excitation of the first mode becomes energetically favorable and the condensate is depleting very efficiently. This is the characteristic property of a quantum phase transition. This picture is further substantiated by calculating the occupation number of excited Bogoliubov modes

$$n(\delta k) = \frac{\hbar^2\delta k^2/2m - UN/V}{2\epsilon(\delta k)} - \frac{1}{2} , \tag{4.19}$$

which shows that the vanishing of $\epsilon(\delta k)$ is accompanied by an extensive occupation of the corresponding quantum states. This means that the Bogoliubov approximation is no longer applicable and quantum corrections are significant. For values $U > U_c$ the energy becomes imaginary, which signals the formation of a new ground state that is given by the soliton solution $\Psi_0^{(\mathrm{sol})}(x)$, compare to the discussion in [156]. Moreover, the work of [158, 163] shows that the system becomes drastically quantum entangled at the critical point, which is yet another characterization of quantum phase transition.

By making the N dependence of U_c explicit and introducing the new dimensionless coupling parameter $\alpha = UmV/(\hbar^2\pi^2)$, the condition for the breakdown of the Bogoliubov approximation becomes

$$\alpha = \frac{1}{N} . \tag{4.20}$$

This is exactly the condition for self-sustainability in the case of a black hole (see chapter 5). These considerations closely follow [151], where the authors wanted to illustrate the relation between black hole physics and Bose-Einstein condensation at the critical point. Of course, in this toy model the relation (4.20) is not generically realized, but has to be imposed by adjusting the model parameters by hand, i.e. for a given value of N, the interaction strength U has to be chosen appropriately. In the case of GR the left hand side of equation (4.20) is k-dependent which in principal could allow for a generic cancelation between the two terms in the squared bracket in the last line of (4.18). This cancelation is assumed to take place up to $1/N$-corrections.

The aim of our work is to present a non-relativistic scalar model that is in principle able to account for this cancelation and thus generically stays at the point of a quantum phase transition independent of the chosen parameters. It is not possible to derive this result within the Bogoliubov approximation since a high occupation of quantum states is the defining property of a quantum phase transition. However, the breakdown of the perturbative approach is a necessary condition and therefore provides an important indication.

4.2.2 Using derivative couplings instead

In the following, we will consider a special non-relativistic, classicalizing theory mimicking General Relativity. As in [156], we choose to confine our theory in a 1-dimensional box of size V. To be concrete, we consider the following Hamiltonian for the second quantized field $\hat{\Psi}(x)$ measuring the particle density at position x,

$$\hat{H} = \frac{\hbar^2}{2m}\int_0^V \mathrm{d}x : (\partial_x\hat{\Psi})^\dagger(\partial_x\hat{\Psi}) : +\lambda\int_0^V \mathrm{d}x : \Big((\partial_x\hat{\Psi})^\dagger(\partial_x\hat{\Psi})\Big)^2 : \tag{4.21}$$

$$+\kappa\int_0^V \mathrm{d}x : \Big((\partial_x\hat{\Psi})^\dagger(\partial_x\hat{\Psi})\Big)^3 : , \tag{4.22}$$

where : : denotes the normal ordering. Note that the purpose of this Hamiltonian is only to provide a toy model which describes bosons which are derivatively coupled, since this is the most important aspect of GR that leads to classicalization. In particular, we do not have in mind some fundamental underlying system, which could effectively be described by this Hamiltonian, as it was the case for the model presented in section (4.2.1).

We are now looking for solutions of the Heisenberg equation

$$\mathrm{i}\hbar\,\partial_t\hat{\Psi} = \Big[\hat{\Psi}, \hat{H}\Big] \tag{4.23}$$

$$= -\frac{\hbar^2}{2m}\partial_x^2\hat{\Psi} - 2\lambda\,\partial_x\Big[(\partial_x\hat{\Psi}^\dagger)(\partial_x\hat{\Psi})^2\Big] - 3\kappa\,\partial_x\Big[(\partial_x\hat{\Psi}^\dagger)^2(\partial_x\hat{\Psi})^3\Big], \tag{4.24}$$

in which the field operator is again replaced by a classical field $\Psi_0(x)$. Note, that we will drop the subscript 0 in the following. We try to generalize the known homogeneous BEC solution (4.17). We can separate the time dependence as in (4.15), leading to the following stationary Gross-Pitaevskii equation for the classical field:

$$-\frac{\hbar^2}{2m}\partial_x^2\Psi - 2\lambda\,\partial_x\Big[(\partial_x\Psi^*)(\partial_x\Psi)^2\Big] - 3\kappa\,\partial_x\Big[(\partial_x\Psi^*)^2(\partial_x\Psi)^3\Big] = \mu\Psi(x). \tag{4.25}$$

Since $\Psi(x)$ is a complex field, this equation has in general the following class of solutions

$$\Psi_k(x) = \sqrt{\frac{N}{V}} \exp(\mathrm{i}kx), \tag{4.26}$$

where the momentum k is quantized in steps of $2\pi/V$ by implementing periodic boundary conditions. The number of particles is denoted by N. Even though this solution is not x-independent like (4.17), it still describes a homogeneous density of particles. The phase factor simply means that this configuration has a non-vanishing total (angular) momentum. Inserting (4.26) in the Hamiltonian (4.21) results in the polynomial

$$\frac{H^{(0)}}{V} = \frac{\hbar^2}{2m} z + \lambda z^2 + \kappa z^3 \tag{4.27}$$

where

$$z = \frac{N}{V} k^2. \tag{4.28}$$

However, not every solution (4.26) is a local minimum of the energy (4.27). For sure, one minimum is given by $k = 0$ (since the kinetic energy contributes positively), which would exactly correspond to the Minkowski vacuum in the case of General Relativity given that this is the global energetic minimum of the theory (4.21). Moreover, by appropriately choosing the coefficients[4] λ and κ, we can construct a second minimum of (4.27) at $z_0 = Nk_0^2/V$ with positive energy, denoted with Ψ_{k_0}, where $k_0 > 0$. It is easy to show that the corresponding solution not only minimizes (4.27) (that is, minimizing the energy within the sub-class of solutions (4.26)) but is also given as a minimum in complete field space (that is, it is a minimum for general fluctuations $\Psi = \Psi_{k_0} + \delta\Psi$). It is this solution that will turn into the classicalon which corresponds to the black hole solution of General Relativity. Furthermore, it should be noted that the chemical potential is zero due to the relation $\mu = \partial E/\partial N|_{k_0} \propto \partial H^{(0)}/\partial z|_{z_0}$, which follows directly from (4.28).

Bogoliubov theory

We will study the leading quantum perturbations $\delta\hat{\Psi}(x)$ about the classical condensate $\Psi_{k_0}(x)$. To this end, we write

$$\hat{\Psi}(x) = \frac{1}{\sqrt{V}} \sum_k \hat{a}(k) e^{\mathrm{i}kx} = \frac{1}{\sqrt{V}} \hat{a}(k_0) e^{\mathrm{i}k_0 x} + \frac{1}{\sqrt{V}} \sum_{k \neq k_0} \hat{a}(k) e^{\mathrm{i}kx}, \tag{4.29}$$

where $\hat{a}(k)$ is the annihilation operator of the momentum mode k. The Bogoliubov approximation consists in treating the first term in (4.29) classically due to the large

[4] Note that this choice implies $\lambda < 0$ and $\kappa > 0$. If we considered a system with only two particles that occupy high momentum states, the Hamiltonian (4.21) would not be bounded from below for this particular parameter choice. This is due to the fact that the expectation value of the third term would vanish for a two-particle state. However, this problem could be easily solved by adding for example the term $\left((\partial_x^2\hat{\Psi})^\dagger(\partial_x^2\hat{\Psi})\right)^2$ with an arbitrarily small positive coefficient. Since we are only interested in describing states with a high occupation number, this problem does not arise and we stick to the original version (4.21).

occupation of the state with momentum k_0. The second term presents a small quantum correction. On account of this, the replacement

$$\hat{a}(k_0) \to \sqrt{N_0} \tag{4.30}$$

is introduced, which allows to identify $\Psi_{k_0}(x)$ with the first term in (4.29). The second term is simply the Fourier representation of the quantum perturbation $\delta\hat{\Psi}(x)$. We want to calculate the perturbation series up to second order in $\delta\hat{\Psi}(x)$ or $\hat{a}(k \neq k_0)$. Note that once we allow for an occupation of the momentum states with $k \neq k_0$, we have to distinguish between N_0, the number of particles in the ground state, and N, the total number of particles. Since we want to express everything in terms of N, the normalization condition

$$\hat{a}^\dagger(k_0)\hat{a}(k_0) = N - \sum_{k\neq k_0} \hat{a}^\dagger(k)\hat{a}(k) \tag{4.31}$$

has to be employed. This means that the zeroth order $H^{(0)}$ terms contribute to the second order $H^{(2)}$ when we express N_0 in terms of N. Inserting (4.29) and (4.31) into the Hamiltonian (4.21), results in the following quadratic order expression:

$$H^{(2)} = \sum_{\delta k\neq 0} \left[\epsilon_0^{(1)}\hat{a}^\dagger\hat{a} + \epsilon_0^{(2)}\hat{b}^\dagger\hat{b} + \epsilon_1(\hat{a}^\dagger\hat{b}^\dagger + \hat{b}\hat{a})\right], \tag{4.32}$$

where the decomposition $k = k_0 + \delta k$ has been used and the (re-)definitions

$$\hat{a}(\delta k) \equiv \hat{a}(k_0 + \delta k) \tag{4.33}$$

$$\hat{b}(\delta k) \equiv \hat{a}(k_0 - \delta k) \tag{4.34}$$

as well as

$$\epsilon_0^{(1)} = (k_0 + \delta k)^2 P_0 + \Lambda_0 \tag{4.35}$$

$$\epsilon_0^{(2)} = (k_0 - \delta k)^2 P_0 + \Lambda_0 \tag{4.36}$$

$$\epsilon_1 = (k_0^2 - \delta k^2) P_1 \tag{4.37}$$

apply. Here, the polynomials P_0, P_1 and Λ_0 are functions of the combination z_0 and the coefficients m, λ and κ:

$$P_0 = \frac{\hbar^2}{4m} + 2\lambda z_0 + \frac{9}{2}\kappa z_0^2 \tag{4.38}$$

$$\Lambda_0 = -k_0^2\left(\frac{\hbar}{4m} + \lambda z_0 + \frac{3}{2}\kappa z_0^2\right) \tag{4.39}$$

$$P_1 = \lambda z_0 + 3\kappa z_0^2 \tag{4.40}$$

Note that when using the extremal energy condition $\partial H^{(0)}/\partial z|_{z_0} = 0$, see equation (4.27), we obtain

$$P_0 = P_1 \quad \text{and} \quad \Lambda_0 = 0 \tag{4.41}$$

due to the relations $2V(P_0 - P_1) = \partial H^{(0)}/\partial z|_{z_0}$ and $2V\Lambda_0/k_0^2 = -\partial H^{(0)}/\partial z|_{z_0}$, respectively. Furthermore, it can be checked that

$$P_0 > 0 \tag{4.42}$$

if z_0 corresponds to the minimum of (4.27) because $2VP_1 = z_0\, \partial^2 H^{(0)}/\partial z^2|_{z_0}$. In this case, also $\epsilon_0^{(1)}$ and $\epsilon_0^{(2)}$ are strictly positive. The Hamiltonian (4.32) is almost of the Bogoliubov form and can be diagonalized by means of the transformation

$$\hat{\alpha} = u\hat{a} + v\hat{b}^\dagger \quad \text{and} \quad \hat{\beta} = u\hat{b} + v\hat{a}^\dagger, \tag{4.43}$$

where $u, v \in \mathbb{R}$. Setting the off-diagonal terms to zero and requiring standard commutation relations for $\hat{\alpha}$ and $\hat{\beta}$ implies

$$\epsilon_1 \left(u^2 + v^2\right) - 2u\, v \frac{\epsilon_0^{(1)} + \epsilon_0^{(2)}}{2} = 0 \tag{4.44}$$

as well as

$$u^2 - v^2 = 1\ . \tag{4.45}$$

These two equations are solved by

$$u = \pm \frac{1}{\sqrt{2}} \left(\frac{1}{2} \frac{\epsilon_0^{(1)} + \epsilon_0^{(2)}}{\epsilon} + 1 \right)^{1/2}, \quad v = \pm \frac{1}{\sqrt{2}} \left(\frac{1}{2} \frac{\epsilon_0^{(1)} + \epsilon_0^{(2)}}{\epsilon} - 1 \right)^{1/2}, \tag{4.46}$$

where

$$\epsilon = \sqrt{\frac{1}{4} \left(\epsilon_0^{(1)} + \epsilon_0^{(2)} \right)^2 - \epsilon_1^2}. \tag{4.47}$$

Note that, as mentioned above, $\epsilon_0^{(1)}$ and $\epsilon_0^{(2)}$ are strictly positive, whereas the sign of ϵ_1 depends on the value of δk. Thus in order to fulfill (4.44), we have to choose u and v in (4.46) both positive when $\delta k < k_0$ and one of both has to be chosen negative when $\delta k > k_0$. In both cases the diagonalized version of (4.32) reads

$$H^{(2)} = \sum_{\delta k \neq 0} \left[\left(\epsilon + \frac{1}{2}(\epsilon_0^{(1)} - \epsilon_0^{(2)}) \right) \hat{\alpha}^\dagger \hat{\alpha} + \left(\epsilon - \frac{1}{2}(\epsilon_0^{(1)} - \epsilon_0^{(2)}) \right) \hat{\beta}^\dagger \hat{\beta} + \epsilon - \frac{1}{2}(\epsilon_0^{(1)} + \epsilon_0^{(2)}) \right]. \tag{4.48}$$

Using the definitions (4.35), (4.36) and (4.37), we find $\epsilon = 2P_0 k_0 |\delta k|$ and $(\epsilon_0^{(1)} - \epsilon_0^{(2)})/2 = 2P_0 k_0 \delta k$. Note that ϵ is strictly positive. By employing the relation $\hat{\alpha}(\delta k) = \hat{\beta}(-\delta k)$ we find

$$H^{(2)} = \sum_{\delta k \neq 0} \left[2 \left(\epsilon + \frac{1}{2}(\epsilon_0^{(1)} - \epsilon_0^{(2)}) \right) \hat{\alpha}^\dagger \hat{\alpha} + \epsilon - \frac{1}{2}(\epsilon_0^{(1)} + \epsilon_0^{(2)}) \right]. \tag{4.49}$$

Accordingly, the vacuum $|0\rangle$ of the Fock space is defined as

$$\hat{\alpha}|0\rangle \ = \ 0. \tag{4.50}$$

It follows from the Hamiltonian (4.49) that the combination

$$e(\delta k) \equiv 2\left(\epsilon + \frac{1}{2}(\epsilon_0^{(1)} - \epsilon_0^{(2)})\right) \tag{4.51}$$

is the energy of the quasi particles created by $\hat{\alpha}^\dagger(\delta k)$ with momentum $k_0 + \delta k$. Since the vacuum of our theory is defined with respect to $\hat{\alpha}$, it contains a non-vanishing amount of excited real particles associated with $\hat{a}$ (and $\hat{b}$ equivalently). This effect goes under the name quantum depletion and occurs physically due to the interactions amongst the particles which necessarily pushes some of them to excited states. Their precise number is given by

$$\langle 0|\hat{a}^\dagger(\delta k)\hat{a}(\delta k)|0\rangle = v^2(\delta k) \ . \tag{4.52}$$

This allows to rewrite the energy of the quasi particles associated with $\hat{\alpha}$ as

$$e(\delta k) = \begin{cases} 8P_0 k_0\, \delta k & \text{for } \delta k > 0 \\ 0 & \text{for } \delta k \leq 0 \end{cases} \tag{4.53}$$

and the number of depleted real particles with momentum $k_0 + \delta k$ as

$$v^2(\delta k) = \frac{1}{2}\left(\frac{k_0^2 + \delta k^2}{2k_0|\delta k|} - 1\right) . \tag{4.54}$$

The above results can easily be generalized to a derivatively coupled theory with an arbitrary number of higher order terms

$$H = \sum_{r=1}^{r_{\max}} c_r \int_0^V \mathrm{d}x \ :(\partial_x \Psi^\dagger \partial_x \Psi)^r: \ . \tag{4.55}$$

Note that the coefficients c_r have dimension [energy][length]$^{3r-1}$. The standard kinetic term corresponds to $r = 1$ for which the coefficient is $c_1 = \hbar^2/(2m)$. The energy of the quasi particles and the number of depleted particles are given by (4.53) and (4.54) where P_0 now is given by the generalized expression

$$P_0 = \sum_{r=1}^{r_{\max}} c_r \frac{r^2}{2}\left(\frac{N}{V}\right)^{r-1}\left(k_0^2\right)^{r-1} , \tag{4.56}$$

and k_0 is determined as a minimum of the generalized version of (4.27)

$$\frac{H^{(0)}}{V} = \sum_{r=1}^{r_{\max}} c_r \left(k^2\right)^r \left(\frac{N}{V}\right)^r . \tag{4.57}$$

The coefficients c_r have again to be chosen such that there is a non-trivial minimum.

Discussion

Our results incorporate the vanishing of the energy gap for $\delta k < 0$. This (at least partly) vanishing energy gap can be considered as an indication for the occurrence of a quantum phase transition, as we discussed in section 4.2.1. Moreover, we see that the Bogoliubov modes become highly occupied for $\delta k \gg k_0$. This in fact signals a breakdown of the Bogoliubov theory anyway, as two succeeding terms in the quantum perturbation theory compare as

$$N_0 \left(k_0 + \delta k\right)^2 k_0^2 \delta N \sim N_0^{1/2} \left(k_0 + \delta k\right)^3 k_0 \delta N^{3/2} \,, \tag{4.58}$$

where δN denotes the number of excited particles in the momentum state $k_0 + \delta k$. Equation (4.58) clearly shows that the number of excited particles should at least be suppressed as $\delta N \sim N_0 k_0^2/\delta k^2$. The result for the number of depleted particles (4.54) is, however, completely the opposite, as it is not suppressed but enhanced for large δk. Therefore, we can safely conclude that the perturbative approximation has broken down. Again, this is in accordance with the expectation of being at the quantum critical point because at this point the system behaves purely quantum and cannot even approximately be described classically. Hence, the breakdown of the Bogoliubov theory was expected, since it amounts to calculate the perturbative quantum corrections around a classical ground state.

Note that the breakdown is also intuitive from the viewpoint of a vanishing energy gap for the quasi particles with $\delta k < 0$. Of course, neither $\hat{a}$ nor $\hat{b}$ particles can directly be related to the direction of $\hat{\alpha}$ or $\hat{\beta}$ particles in phase space. But the vanishing of the energy gap should somehow be transferred into the sector of physical $\hat{a}$ and $\hat{b}$ particles. Since a vanishing energy gap means that it is indefinitely easy to excite the quasi particles, we seem to recover this behavior in the high momentum sector of $\hat{a}$ and $\hat{b}$ particles.

We can also perform the Bogoliubov approximation around the global minimum of (4.27) at $k = 0$. Due to the derivatively coupled nature of the interaction terms, the higher order terms in (4.21) do not contribute, which in turn implies that the Hamiltonian (4.32) is already diagonal. Therefore, there is no depletion of the vacuum which allows us to further extend the GR analogy: This state would simply correspond to the Minkowski vacuum in the case of GR.

Contrary to model (4.11), where the critical point is actually reached and crossed by sufficiently increasing the interaction strength U, in our model there is some indication that the system stays at the point of quantum phase transition and does not organize itself in a new classical ground state. However, this indication is only inferred from the observation of the breakdown of the Bogoliubov theory. To get some solid measures, we need to go beyond the Bogoliubov approximation in the next step [160]. This can be achieved by a full quantum mechanical treatment of the theory (4.21). The diagonalization of the Hamiltonian can be performed under the assumption that only the lowest k momentum eigenstates are significantly occupied (given that we are supposed to sit in a local minimum, this seems to be a good assumption). Therefore, it suffices to diagonalize the Hamiltonian within a Hilbert subspace containing only a finite number of states describing N bosons occupying k different momentum eigenstates. For k chosen appropriately small the calculation is numerically feasible and has been performed in the case of the non-derivatively coupled model in [156]. By means of this calculation we would be able to address quantitative questions, such as the size of the energy gap, the number and spectrum of depleted particles or the amount of quantum entanglement in the system.

Besides, the generalization of our results to a relativistic theory offers another promising prospect of future research. This necessitates to apply the ideas of the Bogoliubov approach to a relativistic theory and would be a significant step towards a more quantitative treatment of the black hole condensate in General Relativity. Of course, such computations have the disadvantage that they involve a resummation of infinitely many equally important terms. The reasoning of the quantum-N portrait circumvents this issue by modeling collective binding effects with an effective binding potential. For an ultimate proof of the microscopic picture proposed, however, it will be necessary to go beyond this approximation. Hence, a confrontation with the problems concerning the relativistic nature of GR is unavoidable.

4.2.3 Numerical treatment of our model

As we have seen in the last section there is a momentum region δk for which the energy of the excitation modes vanishes. This implies, that it would not cost any energy for the bosons to pass over to an excited state. Depletion would then be very efficient and the occupation number of excited Bogoliubov modes would become extensive. On the other hand, this behavior signals a breakdown of the Bogoliubov approximation used. Quantum effects cannot be neglected any longer. Therefore, in order to gather more information about the condensate behavior it is inevitable to perform a numerical analysis.

The starting point for this analysis is the Hamiltonian (4.21) of our system which can be cast in a more convenient form by means of the decomposition (4.29) into creation annihilation operators. We furthermore set $\hbar^2/(2m) = 1$, $\lambda = -2$ and $\kappa = 1$. Having a non-relativistic, i.e. number conserving, system at hand the Hamiltonian can be diagonalized exactly for a chosen fixed number N of particles. We perform this diagonalization with respect to the following momentum eigenstate basis $|n_{k_0}, n_{k_0+\delta k}, n_{k_0+2\delta k}, ..., n_{k_0-\delta k} ....\rangle$ with $n = k/V$. As mentioned before, we have to truncate the range of the angular momenta to keep our analysis numerically feasible. The first thing we are interested in is the status of the energy gap.

Energy gap analysis

For analyzing the energy gap we only take $k \in [k_0, k_{0+\delta k}, k_{0-\delta k}]$. As can be seen from plots (4.1) and (4.2) there is already a closing of the energy gap if the number of particles is increased from only 50 to 100. Concerning the N-portrait, this fits the expectation of a energy-gap closing with increasing N. Indeed, the closing turns out to be proportional to $1/N$ rather than following an exponential law (see (4.3)).

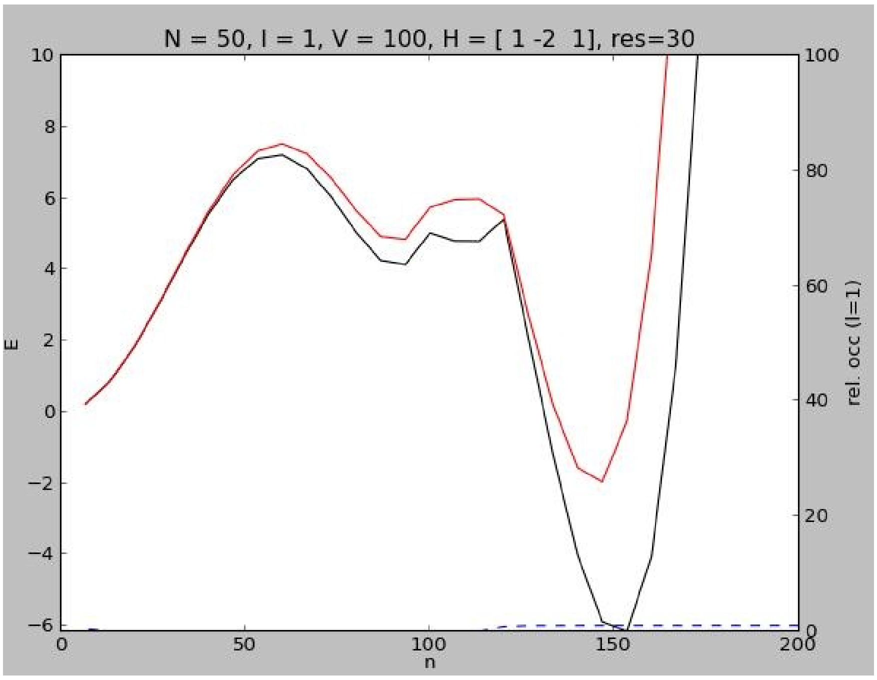

Figure 4.1: Illustration of the energy gap between the ground state mode and the first excited mode. The number of particles considered is $N = 50$ in a volume $V = 100$. E denotes the energy, $n = k/V$. Further parameters are: $\hbar^2/(2m) = 1$, $\lambda = -2$, $\kappa = 1$.

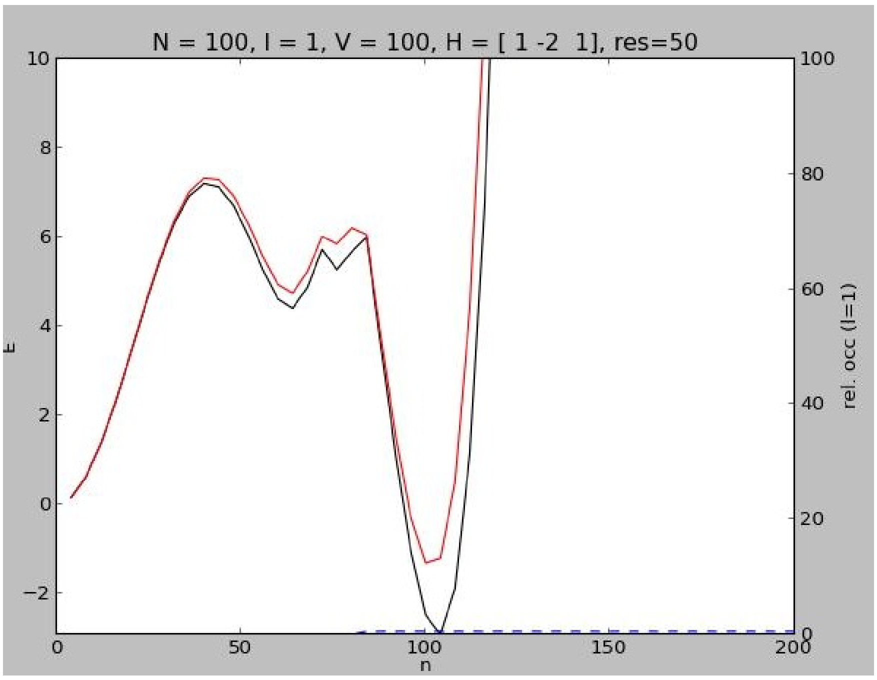

Figure 4.2: Energy gap between the ground state mode and the first excited mode. The number of particles considered is $N = 100$. All other parameters are as in figure (4.1).

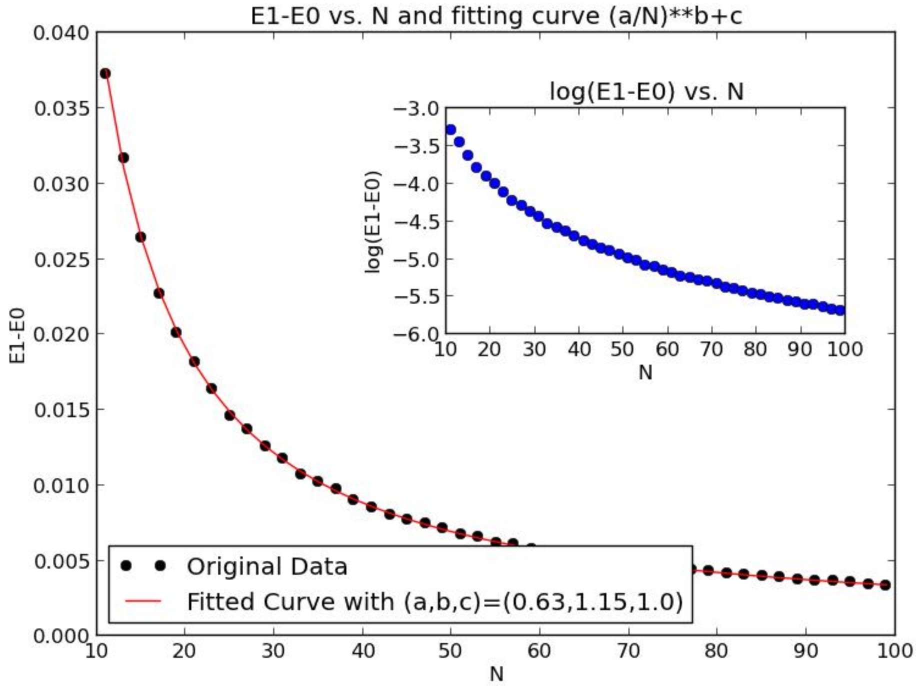

Figure 4.3: Closing of the energy gap between the ground state mode and the first excited mode ($E_1 - E_0$) for an increasing number of quanta N.

First problems occuring

Though obtaining such promising results we cannot trust the previous analysis. This becomes obvious when illustrating the ground-state energy E_0 depending on the angular momentum k for various numbers of particles. For each fixed number of particles considered the ground-state energy seems to decrease infinitely when taking more and more momenta into account (see (4.4)).

Clearly, for $N < 3$ it was expected that our model requires a modification since the λ-term of the Hamiltonian can no longer insure that the energy is bounded from below. However, for $N \geq 3$ we did not expect such unbounded behavior when considering the classical Hamiltonian. From the Bogoliubov analysis, on the other hand, we could have expected that a numerical treatment of our model is not capable to capture the quantum mechanical behavior when perturbing the system only slightly around the black hole minimum k_0.

In fact, taking only a small amount of higher excitation modes into account we totally ignore the impact contributed by the (global) Minkowski minimum. Although we are confident that a wider range of angular momenta will resolve the problem of an ever dropping ground state energy we are not able to show this by means of the used code. At this point, we want to remark that the problem occurring is characteristic for the Hamiltonian's derivative structure. It does not occur in non-derivatively coupled systems by construction which is why a numerical analysis in these cases works smoothly [156, 158].

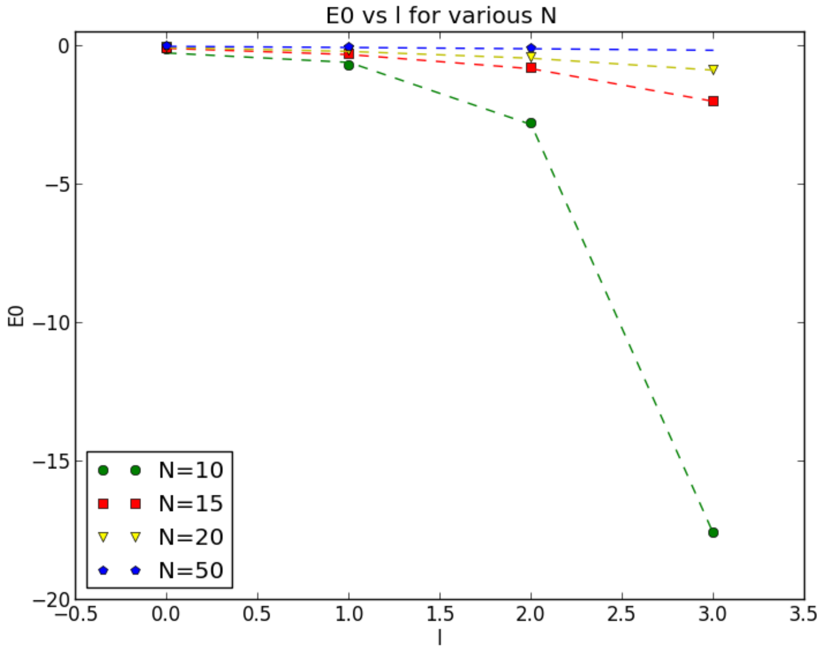

Figure 4.4: Dropping of the ground-state energy with increasing momentum (here denoted by l) for various numbers N of particles.

More problems in sight

After all, we might have the problem that our ansatz to model a black hole within such a system was wrong in the first place, although we expect black holes to be quasi-stationary states.
Indeed, we cannot hope to model black hole characteristics by means of a stationary solution. Of course, questions for the entanglement entropy can be investigated in this system in accordance with [158]. However, if we are interested in depletion effects we must have look the system's time evolution. The population of the k_0-mode should decrease in time such that in the end all particles are populating the Minkowski vacuum and much higher excitation modes, resembling Hawking radiation, in order to not violate energy momentum conservation.

This analysis of our model and the related problems pointed out make it obvious that the ideas behind the Quantum-N portrait of black holes cannot be illustrated to full satisfaction by means of simple toy models.
For this reason we now want to seek a more profound quantum theoretical treatment of black holes. In order to do so we present a constituent description of spacetime based on quantum chromodynamics in the next chapter.

Chapter 5

Finding the building blocks of spacetime

In the course of this thesis we have seen that a consistent description of black holes cannot be formulated in a classical way without the appearance of singularities. We have furthermore learnt that not even a semiclassical framework comes without paradoxes. Therefore, more fundamental concepts have to be sought.

The idea that black holes may be described as bound states of gravitons made a major contribution to recent developments in this area.
Though several models (e.g. [158, 159, 160]) have been established to support this idea, the picture itself remained rather qualitative. So far, there is no solid underlying quantum theory. Besides, the N-portrait [64] itself cannot explain how such black holes are built in the first place. To this end, the idea has to be promoted with more fundamental principles. Taking it seriously that black holes can be viewed as bound states of gravitons it seems reasonable to draw parallels with quantum chromodynamics (QCD) where hadrons are described as bound states of constituent quarks.
Of course, a black hole is not simply a hadron. Following [64] they are not built from just a few constituents but are rather large-N objects. Besides, they are ignorant of QCD concepts such as asymptotic freedom and color confinement.
Consequently, there are many obstructions that might prevent us from an analogous characterization of black holes. We will, nevertheless, establish a quantitative framework for realizing a relativistic description of a quantum bound state that mimicks the bound state description developed for QCD.

Let us motivate our quantum field theoretical approach in the following.

5.1 The motivation behind

Based on the asymptotic flatness of the Schwarzschild solution, a black hole is fully characterized by its total mass. This allows to interpret the Schwarzschild metric in terms of the exterior gravitational field of an isolated body. Duff showed that the Schwarzschild solution can be obtained by resumming infinitely many tree-level scattering processes involving weakly coupled gravitons and the black hole as an external source on Minkowski spacetime [164]. Therefore, the exterior of a Schwarzschild black

hole admits both a geometrical and a quantum mechanical description based on the S-matrix.
As for the interior region we have seen that there is a clash of both these descriptions. To be more specific, we already encountered problems at the semiclassical level.

Consequently, while GR is capable of providing us with perturbative deformations of spacetime, strong light-cone deformations implying non-perturbative effects are not included in its scope.

As we have mentioned before, we do not claim that QFT on curved spacetime is not valid. Rather, we want to explore the possibility that QFT on flat spacetime, being capable of describing non-perturbative effects, is fundamental - even for the description of black hole interiors. We take the point of view that the Minkowski light cone is a distinguished light cone in accordance with locality principles. Other spacetime geometries, such as the Schwarzschild spacetime, should appear as derived concepts.
Concerning black holes, the geometrical description can be reinterpreted as an emergent phenomenon in the exterior region[1] while for the inside region it merely represents an artifact void of any Hilbert space connection.

A quantum bound state description of spacetime might shed light in the dark. Before going into details of the construction of such bound states let us first review how such states enter the Hamiltonian spectrum. After all, we are usually dealing only with one-particle states.

5.2 Quantum bound states in the Hamiltonian spectrum

Having a look at the Hamiltonian spectrum of an interacting theory we will find that bound states enter the spectrum just like one-particle states and multi-particle states do. Let us illustrate this for simplicity by means of a scalar theory following [170]: Considering the entire Hilbert space comprising free and interacting scalar particles the completeness relation is given by

$$\mathbb{1} = |\Omega\rangle\langle\Omega| + \sum_{\Lambda} \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_{\boldsymbol{p}}(\Lambda)} |\Lambda_{\boldsymbol{p}}\rangle\langle\Lambda_{\boldsymbol{p}}|, \tag{5.1}$$

where $E_{\boldsymbol{p}}(\Lambda) \equiv \sqrt{|\boldsymbol{p}|^2 + m_\Lambda^2}$ is the energy of the eigenstates[2] $|\Lambda_{\boldsymbol{p}}\rangle$ with momentum $\boldsymbol{p}$ and mass m_Λ. As before, $|\Omega\rangle$ denotes the non-perturbative vacuum. By means of this expression we can rewrite the two-point function $\langle\Omega|\phi(x)\phi(y)|\Omega\rangle$ as

$$\langle\Omega|\phi(x)\phi(y)|\Omega\rangle = \sum_{\Lambda} \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_{\boldsymbol{p}}(\Lambda)} \langle\Omega|\phi(x)|\Lambda_{\boldsymbol{p}}\rangle\langle\Lambda_{\boldsymbol{p}}|\phi(y)|\Omega\rangle, \tag{5.2}$$

[1]This is similar to geometric optics which can be viewed as long distance effective description derived from quantum electrodynamics.

[2]Note that such a state $|\Lambda_{\boldsymbol{p}}\rangle$ can be obtained by employing Lorentz invariance of the theory and boosting an eigenstate $|\Lambda_{\boldsymbol{0}}\rangle$.

where we assumed that $x^0 > y^0$. Note that $\langle\Omega|\phi(x)|\Omega\rangle\langle\Omega|\phi(y)|\Omega\rangle$ is usually zero by symmetry or Lorentz invariance, respectively [170]. Using

$$\langle\Omega|\phi(x)|\Lambda_{\boldsymbol{p}}\rangle \;=\; \langle\Omega|\phi(0)|\Lambda_0\rangle e^{-\mathrm{i}px}|_{p^0=E_{\boldsymbol{p}}} \tag{5.3}$$

and lifting the three-dimensional integration occurring in (5.1) to a four-dimensional one, we get

$$\langle\Omega|\phi(x)\phi(y)|\Omega\rangle \;=\; \sum_\lambda \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{i}}{p^2 - m_\Lambda^2 + \mathrm{i}\varepsilon}|\langle\Omega|\phi(0)|\Lambda_0\rangle|^2 e^{-\mathrm{i}p(x-y)}. \tag{5.4}$$

The case $y^0 > x^0$ works analogously. Both cases can be summarized in the so-called Källén-Lehmann spectral representation [170]:

$$\langle\Omega|\mathrm{T}\phi(x)\phi(y)|\Omega\rangle \;=\; \int_0^\infty \frac{\mathrm{d}m^2}{2\pi}\underbrace{\sum_\Lambda 2\pi\delta(m^2 - m_\Lambda^2)|\langle\Omega|\phi(0)|\Lambda_0\rangle|^2}_{\rho(m^2)}\triangle(x-y,\, m^2). \tag{5.5}$$

Here, $\triangle(x-y,\, m^2)$ denotes the Feynman propagator. Furthermore, we introduced the spectral function $\rho(m^2)$. As the overlap matrix element $\langle\Omega|\phi(0)|\Lambda_0\rangle$, usually parametrized by the so-called decay constant Γ, indicates, the spectral function contains information about the full spectrum of the Hamiltonian. In fact, one-particle- and bound states contribute delta peaks while multi-particle states are represented by a continuous spectrum.

Of course, certain threshold energies have to be reached until bound states or multi-particle states can be created. Above such energies, however, these states can enter the S-matrix as well [171, 172]. In this case the bound states are prepared as asymptotic in- and out-states just like one-particle states. In particular, they can be represented by well separated wave packets which, being evaluated at infinity, can be reduced to simple plane waves. From a physical point of view it is then also clear that bound states loose their spatially extended character in this limit and are effectively reduced to point particles.

We will not go into details at this point. For a thorough derivation the interested reader is referred to [169].

It should be mentioned nevertheless that there is no loss of information concerning the non-trivial structure of the bound state in such a treatment. This is due to the fact that the interaction itself takes place locally anyways and S-matrix theory can only achieve the resolution of the bound state's interior indirectly by measuring scattering angles, for example.

Let us now see how bound states can be constructed in practice. Historically, this problem was first addressed within the framework of QCD.

5.3 Quantum bound states in QCD

Going back to the roots of QCD it was not clear how to describe hadronic bound states such as protons, for example.

The reason can be seen when having a look at the QCD Lagrangian:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_f \bar{q}_f \left(\mathrm{i} \not{D} - m_f \right) q_f. \tag{5.6}$$

Here $G^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g f^{abc} \mathcal{A}^b_\mu \mathcal{A}^c_\nu$ is the gluonic field strength tensor with $\mathcal{A}^a_\mu$ representing the gluon fields and f^{abc} the structure constants of $SU(3)$. The index $a = 1, ..., 8$ counts the number of gluon fields. q_f denote the quark fields with different flavors f and $D_\mu = \partial_\mu - \mathrm{i} g \mathcal{A}^a_\mu T^a$ stands for the gauge covariant derivative with g representing the bare coupling constant and T^a the eight generators of the Lie algebra of $SU(3)$.

Experimentally, the mass, spin and electrical charge of hadrons were known and effective model building allowed to study hadron reactions at energies sufficiently low to neglect their internal structure.

However, protons, and hadrons in general, are not part of the QCD Hamiltonian itself, albeit being part of its spectrum. The reason is that they are no elementary degrees of freedom. They just appear as color singlet bound states of quarks and gluons which, in turn, are no asymptotic states and thus do not show off in the physically observed spectrum. Therefore, understanding the internal structure of hadrons in terms of their constituents presented a tremendous problem.
From an experimental point of view access to the proton's interior can be gained using scattering experiments. The charge radius of the proton sets the average length scale for confining color within protons. Hadrons in general cannot leak color, and chromodynamics can only be studied if probes are employed that can resolve length scales smaller than the charge radius. This can be achieved in deep inelastic scattering processes. Here, high-energetic leptons are emitting virtual photons which probe the composite structure of the target bound state. The only information that can be observed is the recoil of the emitter. This information, however, suffices to draw conclusions on the proton's interior.

A major breakthrough in the description of hadronic bound states was made by Shifman et al. [165, 166]. They were the first to postulate the existence of a non-perturbative ground state supporting the creation of bound states by means of auxiliary currents. The main difference to the perturbative vacuum is that it allows for quarks and gluons to condense. In turn, condensates of quarks and gluons, i.e. normal-ordered contributions in correlation functions, parametrize the a priori unknown ground state. This way, non-perturbative effects are mapped to the details of this ground state such that generic observables factorize in perturbative, i.e. computable, and non-perturbative, i.e. parametrized, pieces.

Concerning the cross section of deep inelastic scattering processes as mentioned above there is a division into so called hard and soft contributions.
The hard part (Wilson coefficient) can be computed perturbatively in terms of the elementary quark and gluon degrees of freedom. A vital prerequisite for the application of perturbation theory is that QCD features asymptotic freedom for individual interactions at sufficiently small distances.
The soft part, on the other hand, encodes non-trivial information about the internal structure of the bound state. This information is given by structure functions, or more

precisely parton distribution functions. Unlike the hard part of the cross section confinement, salient for non-abelian gauge theories, causes the soft part to be subject to non-perturbative effects. Given that the distribution functions often cannot be determined from first principles matching to experiments at a given scale is required. Predictivity then follows from the renormalization group evolution of these distribution functions.

At this point it is important to mention that confinement is a priori not due to collective effects, where all constituents create an average potential for each single constituent. Nevertheless, Shifman et al. [165, 166] showed that questions pertaining to the internal structure of hadronic bound states can be formulated in a mean-field language. In a sense, the non-perturbative vacuum effectively acts as a mean-field source with respect to which quarks and gluons can condense while perturbative particles propagate in the background created by these condensates.

A note on nomenclature

As just mentioned, the possibility of condensation processes plays an important role in the characterization of bound states. In order to describe asymptotically free objects we will denote the corresponding Minkowski vacuum by $|0\rangle$. Condensed particles, on the other hand, will be connected to a "vacuum medium" $|\Omega\rangle$ of Minkowski. We will use this nomenclature only as a bookkeeping device helping us to distinguish which expressions can be treated perturbatively and non-perturbatively, respectively. In this respect, the designation $|\Omega\rangle$ describes a "non-perturbative ground state" on which particles can be injected such that non-trivial composite structures are supported by enabling condensation processes. This injection is achieved by an auxiliary current. The "perturbative vacuum" $|0\rangle$, on the other hand, is ignorant towards such currents. Again, we want to stress that this approach does by no means imply that Minkowski spacetime consists of two different vacuum sectors. In fact, there is only one unique vacuum[3] and condensation is a non-perturbative effect generated by an effective (mean-field) potential due to particle interactions.

The main question we wish to pursue now is whether the interior of Schwarzschild black holes admit a similar quantum bound state description. To this end let us explain how the auxiliary current construction works. As we will see this construction is not tied to the QCD framework but rather is a general tool for representing bound states.

5.3.1 Auxiliary current description

The task of auxiliary currents $\mathcal{J}$ is to encode information that identifies the quantum bound state $\mathcal{B}$ in the non-perturbative vacuum $|\Omega\rangle$. In this respect the auxiliary current description (ACD) generalizes the construction of free n-particle states where creation operators acting on the perturbative vacuum $|0\rangle$ store information about momenta, spins, gauge quantum numbers etc. of these particles. In fact, being interested in their microscopic composition, we can expand a given bound state in terms of a Fock basis

[3] According to the Wightman axioms of QFT. See e.g. [167]

of multiparticle states. Sticking to QCD, for example, hadrons might be expanded in a basis of quark- and gluon states. If these states are in accordance with the quantum numbers and isometries of the sought bound state, there will be a non-zero overlap.

Within the perturbative framework, suitable for describing scattering processes, there is no dynamical constituent representation of bound states based on elementary degrees of freedom. At the kinematical level, however, we can identify the state $\mathcal{B}$ by a list $\mathcal{L}$ comprising all quantum numbers that characterize the bound state. These are given by the eigenvalues of the corresponding Casimir operators of Minkowski, such as mass and spin, but also by gauge quantum numbers, for example charge, isospin and color in the case of QCD. Note that the precise composition of the currents $\mathcal{J}$ acting on $|\Omega\rangle$ in terms of elementary degrees of freedom is irrelevant apart from the fact that it has to implement the correct quantum numbers of the list $\mathcal{L}$.

The current for the ρ meson, for example, is given by $\mathcal{J}_\rho^\mu = 1/2(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d)$ [165, 166], where u and d are up and down quark fields, respectively. This current has the correct isospin, charge and color quantum numbers to represent a ρ meson. For the description of a proton the current should contain two up quarks and one down quark with total spin 1/2. If additionally we included a gauge-invariant combination of the non-abelian field strength tensor or any other gluonic content the quantum number of the state $\mathcal{J}|\Omega\rangle$ would be still the same. Accordingly, there is a vast variety of auxiliary currents possible.

Now, suppose $|\mathcal{L}\rangle = |K, Q\rangle$ is a momentum eigenstate corresponding to momentum K and quantum numbers Q. Expanding $|\mathcal{B}\rangle$ in this basis,

$$\langle\mathcal{B}|\mathcal{J}(x)|\Omega\rangle = \sum\!\!\!\!\!\!\int_{\mathcal{L}(\mathcal{B})} \mathcal{B}^*(\mathcal{L})\langle\mathcal{L}|\mathcal{J}(x)|\Omega\rangle. \tag{5.7}$$

Here, the integral is over the energy-momentum, the sum runs over the quantum numbers collected in the list $\mathcal{L}$, and $\mathcal{B}(\mathcal{L})$ denotes the bound state wave function. For an appropriately chosen auxiliary current this overlap matrix element does not vanish. Using furthermore translational invariance of the vacuum state we get

$$\begin{aligned} \langle\mathcal{B}|J(x)|\Omega\rangle &= \sum\!\!\!\!\!\!\int_{\mathcal{L}(\mathcal{B})} \mathcal{B}^*(\mathcal{L})\, e^{-\mathrm{i}Kx}\langle\mathcal{L}|J(0)|\Omega\rangle & (5.8)\\ &= \Gamma_{\mathcal{B}} \sum\!\!\!\!\!\!\int_{\mathcal{L}(\mathcal{B})} \mathcal{B}^*(\mathcal{L})\, e^{-\mathrm{i}Kx} & (5.9) \end{aligned}$$

where we have introduced the parameter $\Gamma_{\mathcal{B}} = \langle\mathcal{L}|\mathcal{J}(0)|\Omega\rangle$. Notice that this parameter is non-perturbative and encodes structural information about the bound state. In QCD it is called decay constant. Dealing with $J(x)|\Omega\rangle$ analogously results in

$$J(x)|\Omega\rangle = \Gamma_{\mathcal{B}} \int \frac{\mathrm{d}^4K}{(2\pi)^4}\, e^{-\mathrm{i}Kx}|\mathcal{L}\rangle \tag{5.10}$$

such that

$$\int d^4x \exp(iPx)J(x)|\Omega\rangle = \Gamma_{\mathcal{B}} \int \frac{d^4K}{(2\pi)^4} \int d^4x \, e^{-i(P-K)x}|K,Q\rangle \quad (5.11)$$

$$= \Gamma_{\mathcal{B}}|P,Q\rangle. \quad (5.12)$$

Solving for $|P,Q\rangle$ and plugging the result into $|\mathcal{B}\rangle = \int \frac{d^4P}{(2\pi)^4}\mathcal{B}(P,Q)|P,Q\rangle$ finally yields

$$|\mathcal{B}\rangle = \Gamma_{\mathcal{B}}^{-1} \int \frac{d^4P}{(2\pi)^4}\mathcal{B}(P,Q) \int d^4x \, e^{iPx}\mathcal{J}(x)|\Omega\rangle. \quad (5.13)$$

Note that the wave function $\mathcal{B}(P,Q)$ localizes the information carried by the auxiliary current in $|\mathcal{B}\rangle$. Just like $\Gamma_{\mathcal{B}}$, the wave function therefore is intrinsically non-perturbative. As we will see later, it can be related to the constituent distribution of the bound state. Furthermore, on-shell we have $P^2 = -M_{\mathcal{B}}^2$ where $M_{\mathcal{B}}$ denotes the mass of the bound state. So, four-dimensional momentum integration is only chosen for convenience. In particular, $\mathcal{B}(P,Q) = \delta^{(1)}(P^2+M_{\mathcal{B}}^2)\hat{\mathcal{B}}(P,Q)$ with $\hat{\mathcal{B}}(P,Q)$ representing the on-shell wave function of the state $|\mathcal{B}\rangle$. In this respect, (5.13) contains information about the mass scale of the bound state as well.

The auxiliary current description in connection with the LSZ reduction formalism

Let us mention at this point that the action of the current on the non-perturbative vacuum is reminiscent of the role of creation operators in a free theory. Therefore, it suggests itself that the auxiliary current description can be related to the Lehmann-Symanzik-Zimmermann (LSZ) reduction formalism [168] which is a tool to calculate S-matrix elements. At this point, it is worth appreciating its simplicity when applied to free states $|\chi\rangle = |\boldsymbol{k}, \mathcal{Q}\rangle$:

$$a^\dagger(\boldsymbol{k}, \mathcal{Q})|0\rangle = \Gamma_\chi^{-1} \int d^3x \, e^{ikx}\mathcal{J}(x)|\Omega\rangle \, , \quad (5.14)$$

where k is on–shell and $\boldsymbol{k}$ is the particle's three momentum.

In the auxiliary current description, excitations of the perturbative vacuum are generated by acting with the auxiliary current on the perturbative vacuum on a spatial slice at an arbitrary time. The current then simply reduces to the field operator creating a scattering state from the vacuum. For example, $\mathcal{J}(x) = \chi(x)$ for a single particle scalar scattering state $|\chi\rangle$. Since k is on-shell, an ingoing scattering state would be given by

$$|\boldsymbol{k}, \mathcal{Q}, \text{in}\rangle = -i\frac{2\pi}{k^0(\boldsymbol{k})}{\Gamma_\chi}^{-1} \int d^4x \, e^{ik\cdot x}D(x)\mathcal{J}(x)|\Omega\rangle, \quad (5.15)$$

with $D(x)$ denoting the equation of motion operator associated with $|\chi\rangle$[4].

Hence, the auxiliary current description reduces to the famous Lehmann-Symanzik-Zimmermann reduction formula when applied to scattering states. This implies that the auxiliary current description allows for a unified framework for S-matrix processes involving perturbative as well as non-perturbative states.

[4] Note that in (5.15) we have omitted the boundary term and any disconnected contributions.

5.3.2 Calculating observables in QCD - sum rules

Having constructed bound states by means of the auxiliary current description we can now go on and try to evaluate observables concerning the non-perturbative hadronic structure within this framework.

The method of QCD sum rules [165, 166], developed by Shifman, Vainshtein and Zakharov, has become a widely accepted tool to do so. Originally used for the determination of simple static hadronic characteristics like masses and coupling constants, they were also employed to calculate hadronic wave functions, form factors and decay widths (see e.g. [173, 174, 175, 176]).
In this treatment, making use of the auxiliary current description introduced in the last section, hadrons are represented by their interpolating quark currents. To make predictions of an observable related to a hadronic state the correlation function

$$\Pi_{\mu\nu} = \mathrm{i} \int \mathrm{d}^4x \; \mathrm{e}^{\mathrm{i}qx} \langle \Omega | \mathrm{T} \mathcal{J}_\mu(x) \mathcal{J}_\nu(0) | \Omega \rangle \tag{5.16}$$

of the corresponding currents $\mathcal{J}$ is introduced. Taking vector currents, which are color currents in QCD and hence are conserved, a transverse tensor structure can be factored out of this two-point function,

$$\Pi_{\mu\nu} = \left(q_\mu q_\nu - q^2 \eta_{\mu\nu} \right) \Pi(q^2). \tag{5.17}$$

For currents taken at large virtualities, i.e. in a region of large spacelike momentum transfers $Q^2 \equiv -q^2 \gg \Lambda_{QCD}$ in hadron scattering, the effective quark-gluon coupling at one-loop level[5] $\alpha_s(Q^2) \propto \frac{1}{\beta_0 \ln(Q^2/\Lambda_{QCD}^2)}$ becomes small and the quarks predominantly propagate at short distances [177].
On the other hand, if q^2 is transferred from large negative to positive values, the average distance between the points x and 0, where the currents are inserted in the correlation function, grows. The long-distance quark-gluon interactions become important and, eventually, hadrons are formed. The hadronic states then also contribute to $\Pi_{\mu\nu}$[178].

QCD sum rules allow to link both momentum regimes by virtue of a dispersion relation.
At spacelike momentum transfers the operator product expansion (OPE) (see [179, 17, 180]) can be applied to $\Pi_{\mu\nu}$. This procedure allows to separate the short- and long-distance contributions in a systematical manner. The time-ordered product of two currents, which can be shown to be singular if evaluated at the same points in spacetime (see [181]), is expressed as a sum of local (composite) operators O_d of increasing mass dimension d,

$$\Pi(q^2) = \sum_{d=0} C_d(q^2, \mu) \langle \Omega | O_d(0, \mu) | \Omega \rangle, \tag{5.18}$$

with the corresponding c-number coefficient functions $C_d(q^2, \mu)$ being ordered in their singularity in inverse powers of $x^2 = 0$. They are known as Wilson coefficients. While these can be described perturbatively as an expansion in $\alpha_s(Q^2)$, the operators $O_d(0, \mu)$ are parametrized in terms of vacuum condensates. Contrary to the Wilson coefficients

[5] In this formula the parameter β_0 encodes the number of colors and quark flavors while Λ_{QCD} is the asymptotic scale parameter of QCD.

the condensates do not depend on the specific currents and can therefore be considered as universal. The lowest-dimension operator ($d = 0$) is the unit operator associated with the leading perturbative contribution. Gluon radiative corrections to this diagram are suppressed by the small coupling.
At momenta below some scale μ, a normalization point, long-distance effects become important and non-trivial quark- and gluon condensates, i.e. power corrections, start contributing to the correlation function. Note that the introduction of such a scale is important in order to allow the OPE for theories such as QCD, where also non-perturbative effects have to be accounted for [180]. In fact, the expansion (5.18) has only been proven in pure perturbation theory [179]. It is merely an expectation, that there exists a window for which μ is large enough such that non-perturbative corrections to the Wilson coefficients can be neglected and equally small enough that the condensates are μ-independent.
Note also, that only operators with compatible quantum numbers have to be taken into account. This leads to the fact that the $d = 0$ operator is not followed until the quark condensate $\langle \bar{q}q \rangle$ of dimension three. While such quark condensates are directly incorporated in the OPE, gluon condensates are accounted for in terms of the external field method. Here, gluons are treated as background sources for the current quarks, i.e. quark propagation is considered in a gauge-field background. Employing the Fock-Schwinger gauge condition $x^\mu A_\mu(x) = 0$ then allows to express the gluon fields in terms of the gluon field strength tensor. This offers the advantage that the operator product expansion only runs over gauge covariant terms.
So far, the condensates cannot be computed from first principles and are subject to experimental input.

Let us illustrate now the sum rule method with an example:
The amplitude for the emission and absorption of a $b\overline{u}$ quark pair in the vacuum by means of the external current $\overline{u}\gamma_\mu\gamma_5 b$, corresponding to a B-meson, is given by

$$\Pi_{\mu\nu} = \mathrm{i} \int \mathrm{d}^4x \; \mathrm{e}^{\mathrm{i}qx} \langle \Omega | \mathrm{T}\overline{u}(x)\gamma_\mu\gamma_5 b(x)\overline{b}(0)\gamma_\nu\gamma_5 u(0)|\Omega\rangle. \tag{5.19}$$

For $q^2 \ll m_b^2$, where m_b represents the bottom quark mass, the leading perturbative contribution is determined by a quark-loop diagram as shown in figure (5.1).

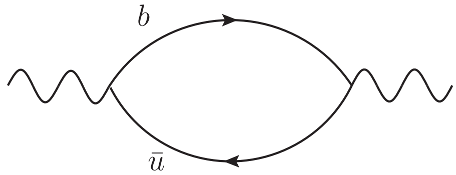

Figure 5.1: Leading perturbative contribution to the emission and absorption of a $b\overline{u}$ quark pair in the vacuum.

The suppressed radiative corrections to this diagram are illustrated in figure (5.2 a)). The other diagrams (figures (5.2 b) and c))) belong to higher orders in perturbation theory and take interactions with the vacuum quark- and gluon fields into account.

These are treated as external fields and form the condensate, i.e. the non-perturbative contributions.

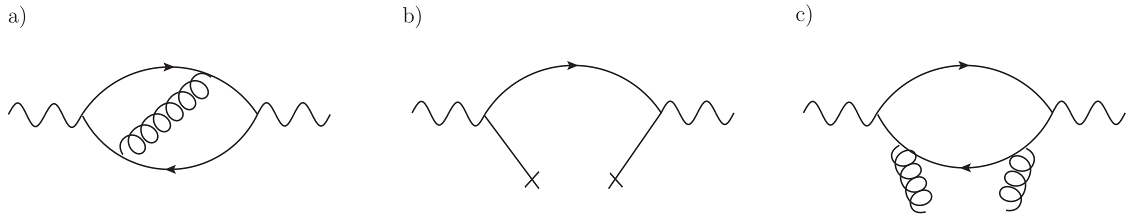

Figure 5.2: Corrections to the emission and absorption of a $b\overline{u}$ quark pair in the vacuum: a) one-gluon exchange, b) quark condensate, c) gluon condensate. Figure adapted from [178].

Note that convergence of the OPE is by no means obvious. Still, it is supposed to be a good approximation if truncated after a few terms [165].

A determination of the hadronic content of the correlation function in the timelike region is granted by the unitarity condition, which relates the imaginary part of the correlation function, i.e. the observable spectral density ρ, to the sum over all intermediate hadronic states compatible with the quantum numbers of the quark currents. The unitarity condition is obtained by inserting a complete set of intermediate hadronic states $|b\rangle$ into (5.16) and exploiting translational invariance,

$$\rho \equiv \mathrm{Im}\Pi_{\mu\nu}(q^2) = \sum_b \langle\Omega|\mathcal{J}_\mu|b\rangle\langle b|\mathcal{J}_\nu|\Omega\rangle(2\pi)^4\delta^{(4)}(q - p_b + p_\Omega). \tag{5.20}$$

Isolating the ground state contribution and introducing a shorthand notation for the spectral density of excited (higher resonance) and continuum (multihadron) states,

$$\mathrm{Im}\Pi_{\mu\nu}(q^2) = \Gamma_{b0}^2\delta^{(4)}(q - p_{b_0} + p_\Omega) + \rho_{b_{exc}}\Theta(q^2 - s_h). \tag{5.21}$$

Note that we made use of the definition for the decay constant given in the last section, such that $\Gamma_{b0}^2 = (2\pi)^4\langle M|\mathcal{J}_\mu|b_0\rangle\langle b_0|\mathcal{J}_\nu|M\rangle$. Moreover, we introduced the threshold parameter s_h separating the ground state contribution from the excited and continuum part.

Coming back to our example of emission and absorption of a $b\overline{u}$ quark pair, we have to deal with the following diagrams in the region $q^2 \gg m_b^2$:
At $q^2 = m_B^2$ the lowest real on-shell hadronic state, the B-meson, starts contributing to the loop (figure (5.3 b))). For increasing momentum transfer, higher resonances are created and get even overlapped with multihadronic states (figures (5.3 c) and d)).

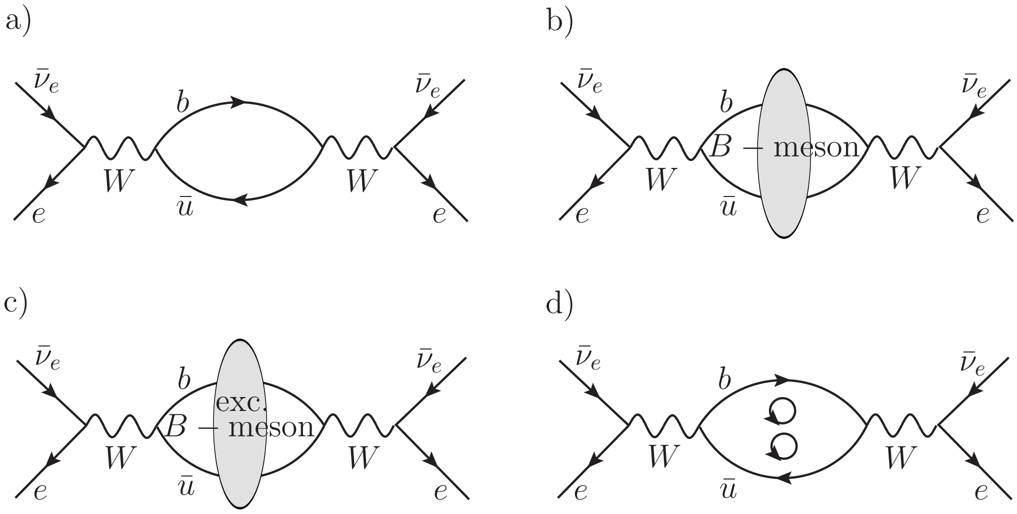

Figure 5.3: Diagrams corresponding to the correlation function $\Pi_{\mu\nu}$ in terms of hadrons. Here, different energies of $\bar{\nu}_e e$ scattering are considered. Figure reproduced from [178].

Given the analyticity of the correlation function (5.16) in the momentum transfer variable q, the OPE result and the sum over hadronic states can be matched via a dispersion relation. In order to get this dispersion relation we will first have a look at the analytic properties of the correlation function (5.16). Writing out the time-orderd product explicitly,

$$\Pi(q^2) = \mathrm{i}\int \mathrm{d}^4x\ \mathrm{e}^{\mathrm{i}qx}\left\{\Theta(x_0)\langle\Omega|\mathcal{J}(x)\mathcal{J}(0)|\Omega\rangle + \Theta(-x_0)\langle\Omega|\mathcal{J}(0)\mathcal{J}(x)|\Omega\rangle\right\}. \tag{5.22}$$

Inserting a complete set of momentum eigenstates $|P\rangle$, exploiting translational invariance and employing the Fourier representation of the Heavidside function,

$$\Theta(x_0) = \frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\mathrm{d}\omega\frac{\mathrm{e}^{\mathrm{i}\omega x_0}}{\omega-\mathrm{i}\varepsilon}, \tag{5.23}$$

we get

$$\begin{aligned}\Pi(q^2) \;=\; \frac{1}{2\pi}\int \mathrm{d}^4x\ \mathrm{e}^{\mathrm{i}qx}\Bigg\{&\int_{-\infty}^{\infty}\mathrm{d}\omega\frac{\mathrm{e}^{\mathrm{i}\omega x_0}}{\omega-\mathrm{i}\varepsilon}\sum_P \mathrm{e}^{\mathrm{i}(p_P-p_\Omega)x}|\langle\Omega|\mathcal{J}(0)|P\rangle|^2 \\ &+\int_{-\infty}^{\infty}\mathrm{d}\omega\frac{\mathrm{e}^{-\mathrm{i}\omega x_0}}{\omega-\mathrm{i}\varepsilon}\sum_P \mathrm{e}^{\mathrm{i}(p_P-p_\Omega)x}|\langle\Omega|\mathcal{J}(0)|P\rangle|^2\Bigg\}.\end{aligned} \tag{5.24}$$

If all integrations are performed we are left with

$$\begin{aligned}\Pi(q^2) \;=\; (2\pi)^3\sum_P\Bigg\{&\delta^{(3)}(\boldsymbol{q}-\boldsymbol{p}_P+\boldsymbol{p}_\Omega)\frac{|\langle\Omega|\mathcal{J}(0)|P\rangle|^2}{(q_0-p_{P0}+p_{\Omega0}-\mathrm{i}\varepsilon)} \\ &+\delta^{(3)}(\boldsymbol{q}+\boldsymbol{p}_P-\boldsymbol{p}_\Omega)\frac{|\langle\Omega|\mathcal{J}(0)|P\rangle|^2}{(q_0+p_{P0}-p_{\Omega0}-\mathrm{i}\varepsilon)}\Bigg\},\end{aligned} \tag{5.25}$$

which reveals the singular nature mentioned above. For a physical process $q_0 \geq 0$ and hence $q_0 + p_{P0} - p_0 \neq 0$ due to kinematical reasons. Therefore, $\Pi(q^2)$ has only simple poles and a branch cut on the positive real axis. By virtue of Cauchy's theorem we can give (5.25) by an integral representation,

$$\Pi(q^2) = \frac{1}{2\pi\mathrm{i}} \oint_C \mathrm{d}s \frac{\Pi(s)}{s-q^2} \tag{5.26}$$

$$= \frac{1}{2\pi\mathrm{i}} \oint_{|s|+R} \mathrm{d}s \frac{\Pi(s)}{s-q^2} + \frac{1}{2\pi\mathrm{i}} \int_0^R \mathrm{d}s \frac{\Pi(s+\mathrm{i}\varepsilon) - \Pi(s-\mathrm{i}\varepsilon)}{s-q^2}, \tag{5.27}$$

where the integration contour is chosen as depicted in figure (5.4).

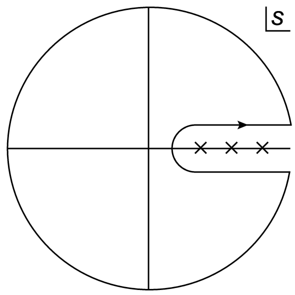

Figure 5.4: Integration contour in the complex s-plane used to derive the dispersion relation connectiong OPE results and hadronic parts. Figure adapted from [177].

Taking the radius R to infinity, the integral over the circle tends to zero provided that the correlation function vanishes sufficiently fast [177]. The second integral of (5.27), on the other hand, can be replaced by an integration over the imaginary part of the correlation function because

$$\Pi(s+\mathrm{i}\varepsilon) - \Pi(s-\mathrm{i}\varepsilon) = 2\mathrm{i}\,\mathrm{Im}\Pi(s) \tag{5.28}$$

for momentum transfers $q^2 \geq \min\{p_{0b} - p_\Omega, s_h\}$[177]. Consequently,

$$\Pi(q^2) = \frac{1}{\pi} \int_{\min\{p_{0b}-p_\Omega, s_h\}}^{\infty} \mathrm{d}s \frac{\mathrm{Im}\Pi(s)}{s-q^2}. \tag{5.29}$$

The gained relation is called sum rule. It allows to constrain, if not to predict, observable characteristics of the hadronic ground state. Vice versa, parameters of QCD such as quark masses and condensate densities can be extracted from sum rules by employing experimentally known hadronic parts.

Problems concerning sum rules

However, there are some caveats that have to be mentioned. Calculating only a first few terms of the OPE, as is done usually, it has to be clear that the accuracy of the sum rule approach is diminished. Put differently, the sum rules heavily rely on an assumption known as quark-hadron duality [182]. Besides, we have ignored the fact that the correlation function (5.16) might be UV divergent, which would imply that the integral in (5.29) diverges. In order to fix this problem subtraction terms would have to be introduced. This would lead to quite complicated formulae. Apart from this, being interested in the features of the lowest hadronic contribution, it is desirable that this very contribution dominates over the contributions coming from excited and continuum states. This is not guaranteed a priori.

In order to overcome these drawbacks, Shifman, Vainshtein, and Zakharov (SVZ) suggested to apply a Borel transformation [165]. This transformation is formally defined by

$$\mathcal{B}_{M^2} f(Q^2) = \lim_{\substack{-q^2, n \to \infty \\ -q^2/n = M^2}} \frac{(-q^2)^{n+1}}{n!} \left(\frac{d}{dq^2} \right)^n f(Q^2), \tag{5.30}$$

with M^2 being the so called Borel parameter. We will not go into details of this transformation here. However, we want to point out that the Borel transformation not only kills the subtraction terms needed for convergence but indeed suppresses excited contributions if an appropriate Borel parameter is chosen [183].

Still, due to the truncation of the OPE and the hadronic sum being approximated with quark-hadron duality, SVZ sum rules cannot provide exact results. Additionally, there might be non-perturbative vacuum fluctuations at large momenta, i.e. direct instantons. Indeed, concerning the calculation of the proton's charge radius or vector meson masses, for example, the sum rules have not provided satisfactory results (see e.g. [184]).

Nevertheless, the sum rule approach represents a powerful complementary method to other techniques developed to get a better understanding of QCD like e.g. numerical lattice simulations.

However, none of these methods is capable of giving an analytic solution to IR QCD, the reason lying in the non-linearity of the strong force and the large coupling constant in this regime. There is no way we know to write down the exact S-matrix of this theory. Sum rules, for instance, presume quark confinement rather than providing a proof. And phenomena like chiral symmetry breaking and the corresponding generation of masses are far from being understood.

To gain more insight into these problems of QCD it was proposed to consider its large-N_c limit [154, 155], N_c denoting the number of color charges.

5.3.3 Large-N_c bound states in QCD

Against intuition, taking the limit of an infinite number of colors N_c, and vanishing coupling constant $g \equiv 4\pi\alpha_s$, while holding the t'Hooft coupling $\lambda = g^2 N_c$ fixed, offers considerable simplifications.

Concerning the theory in two dimensions, the t'Hooft limit even allows for exact solutions as was demonstrated by the computation of the meson spectrum in [185].

Unfortunately, this is not the case in four dimensions. Nevertheless, t'Hooft was able to show that in this limit Feynman graphs can be arranged in a $1/N_c$ expansion accounting for the finiteness of color charges with planar ($g = 0$) graphs dominating. He

also showed that this expansion corresponds to a perturbative expansion on the string theory side with the string coupling $g_s \propto 1/N_c$ [154].
Therefore, in the spirit of gauge/gravity duality, there is hope that QCD might be solved in the large-N_c limit and that the $N_c = 3$ case might be close to this limit. Besides, it was shown that static and dynamical properties of baryons made from heavy quarks can be discussed in the framework of mean field theory when the large-N limit is employed [155].

5.4 Transferring QCD methods to gravity

Contrary to QCD, gravity does neither comprise asymptotic freedom nor confinement. However, the description of black holes in terms of bound states of N gravitons as proposed in [64] is based on large-N physics. A common feature of such large-N systems is their non-perturbative character. The reason is the following: Even if the individual constituent gravitons are interacting weakly, the large number of them will overcompensate this effect. According to the N portrait each graviton will experience a large collective potential rendering the black hole as a bound state. In this respect, the collective effects caused by the large-N nature are in one-to-one correspondence to the confining situation in QCD.
This will entail that the vacuum state should not be identified with the perturbative one but rather with a vacuum supporting the bound state sector of the Hilbert space with proper currents acting on it.
Furthermore, there should be no obstruction preventing us from using condensates just like in sum rules. In our case, these condensates would not mimick confinement effects. Instead, they would parametrize the mean field experienced by probe particles in the background of the relativistic large-N bound state.

5.4.1 ACD for gravitational bound states

In section (5.3.1) we have introduced the auxiliary current description appropriate for QCD bound states. If we want to describe now gravitational bound states, such as black holes, on Minkowski spacetime the above construction has to be extended.

Above all, now not only intrinsic symmetries have to be considered, but also extrinsic identifiers, namely transformations which keep the classical bound state metric invariant. These bound state isometries can be represented via their associated symmetry generator[6] $\mathcal{K}$. An invariance of the bound state $|\mathcal{B}\rangle$ under some spacetime isometry should imply the same invariance for $\mathcal{J}|\Omega\rangle$. Employing Lorentz invariance of the non-perturbative vacuum this condition is equivalent to the statement that the commutator of the current with the generator $\mathcal{K}$ is zero,

$$[\mathcal{J}, \mathcal{K}] = 0. \tag{5.31}$$

This algebraic condition leads to a differential equation restricting the spacetime dependence of the current $\mathcal{J}(x)$ (and the fields it comprises) in a way that is compatible with

[6]Notice that these generators $\mathcal{K}$ can be identified with the Killing vectors of the spacetime considered.

the symmetries of the given spacetime. Let us be more specific and restrict ourselves to Schwarzschild black holes in the following.

According to [64] black holes can be modeled by bound states of N gravitons. Working in a relativistic regime, the number of constituents only represents a mean number that is subject to fluctuations. Taking the idea of [64] seriously, however, the individual couplings are extremely weak. Consequently, the particles can be treated as being almost free and fluctuations around N become negligible. Thinking in terms of a Fock state expansion of the bound state $|\mathcal{B}\rangle$ in a basis of gravitons of different occupation number with respect to the non-perturbative vacuum $|\Omega\rangle$ the overlap between $|\mathcal{B}\rangle$ and an N-graviton state should be large. Fock states containing a number of gravitons strongly deviating from N, on the other hand, should have a negligible overlap with the bound state.

Being dominated by potential energy, Schwarzschild black holes can approximately be described in pure gravity. Therefore, the N gravitons can be represented by massless spin-2 fields $h_{\mu\nu}$[7]. For convenience, however, we will use scalar fields for their description. At the partonic level, where gravitons are non-interacting, we can safely proceed that way due to the degeneracy of the choice of currents. Taking the current to be $\mathcal{J}(x) = (h^{\mu}_{\mu})^N$, for example, and working in de-Donder gauge, $\partial^{\mu} h_{\mu\nu} = 1/2\partial_{\nu} h^{\mu}_{\mu}$, the graviton propagator in momentum space is in principle given by the scalar propagator of massless scalar field,

$$\tilde{\Delta}_{\mu\nu\alpha\beta}(p^2) = \tfrac{1}{2}\tilde{\Delta}^{(0)}(p^2)(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta}). \tag{5.32}$$

The surrounding tensor structure subject to contractions will only give rise to numerical prefactors. As long as such numerical differences are unimportant we will use the scalar representation.

Concerning gauge quantum numbers there are none in our case. For electrically charged black holes, such as the Reissner-Nordström solution, the situation would be different and $U(1)$-fields should be included in the auxiliary current. For a Schwarzschild black hole, however, we can write the current as[8] $\mathcal{J}(x) = \phi^N(x)$.

What is missing is the precise spacetime dependence of the current. Being the unique spherically symmetric solution to Einstein's equations in vacuum [186] the Schwarzschild solution admits an $SO(3)$ group of isometries. The Killing vectors that correspond to this symmetry, i.e. the generators of $SO(3)$, are the three angular momentum operators $J_i = x_j \partial_k \epsilon_{ijk}$, $i, j, k = 1, 2, 3$. Apart from spatial isotropy the Schwarzschild solution is also distinguished by temporal homogeneity. Therefore, using (5.31), the Schwarzschild auxiliary current is given by

$$\mathcal{J}(x) = \phi^N(|\boldsymbol{r}|), \tag{5.33}$$

where $|\boldsymbol{r}|$ stands for the Euclidean distance.

Concerning the inclusion of spacetime isometries in the current we might have created the impression that these isometries are due to geometrical concepts. In our framework, however, they rather have to be interpreted as a consequence of the explicit breaking of certain Lorentz symmetries in the presence of bound states.

[7]Had we coupled electromagnetism to gravity, spin-1 fields would need to be included as well.

[8]Note that we switched the notation of the scalar field to ϕ.

Being interested in the calculation of observables associated with the black hole structure, we have to realize that also these observables $\mathcal{O}$ are subject to the corresponding isometries stored in Q. Using Ward's identity this implies

$$0 = \langle \mathcal{B}|\partial_\mu j^\mu|\mathcal{B}\rangle = \langle \mathcal{B}|\delta\mathcal{O}|\mathcal{B}\rangle = \delta\langle \mathcal{B}|\mathcal{O}|\mathcal{B}\rangle. \tag{5.34}$$

Here, j, not to be confused with the auxiliary current $\mathcal{J}$, denotes a conserved current corresponding to an isometry. In practice, (5.34) implies that observables can be calculated in a fully Lorentz covariant way and it suffices to impose the symmetry constraints in the end.

5.4.2 Multilocal current construction

If we want to describe large macroscopic bound states like black holes within the auxiliary current description just like hadrons in QCD we might wonder whether local currents still represent an appropriate approximation. After all, the current fields may be localized anywhere within the Schwarzschild radius. To answer this question it is important to know what kind of information concerning black holes we are interested in. In particular, this will be their interior structure, of course. Since we plan to gather this information within an S-matrix framework we will treat black holes as asymptotic states. Therefore, multilocal currents will reduce to local ones anyway (see [169]). However, we expect to require a multilocal ACD when studying properties not related to such an asymptotic framework. In the case of black holes this would concern questions about their Schwarzschild radius, for example.

5.4.3 Sum rules with regard to gravitational bound states

Contrary to the auxiliary current description the sum rule technique does not seem appropriate for gravitational bound states. This is due to the fact that the sum-rule method requires a single lowest-lying state which is well separated from the rest of the spectrum. For gravity we simply do not know the spectral density.

Concerning black holes the assumption that these are bound states of $N \gg 1$ gravitons already indicates that the above requirement cannot be fulfilled. Of course the black hole mass might be quantized in steps of M_p or the inverse size of the Schwarzschild radius but the separation of scales concerning the energy of an individual constituent compared to the energy of the black hole bound state itself seems too high for employing a dual description.

However, many techniques used in the sum rule approach, such as the operator product expansion, are universally applicable and might be helpful for the description of gravitational bound states as well.

5.4.4 Large-N physics

Apart from these techniques, the large-N nature of black holes might prove helpful in our further studies. As indicated above in the case of QCD a lifting of the gauge group from $SU(3)$ to $SU(N_c)$ entails many simplifications involving bound states. Indeed, this feature is not tied to QCD itself but rather turns out to be a generic property of large-N bound states. The crucial point here is that large-N physics allows for a systematic expansion of physical quantities in terms of $1/N$. Take, for example, a Bose-Einstein condensate. Here, deviations from the Gross-Pitaevskii equation naturally appear at order $1/N$. Of course, N is not connected to the color charge here, but denotes the number of bosons in the condensate. In the case of black holes N denotes the number of field components. As already mentioned in the previous chapters, the large-N nature of black holes is in so far important as that quantum corrections appearing in a $1/N$ expansion could account for the many mysteries in the (semi)classical treatment. Besides, large-N physics allows for mean-field techniques based on the Hartree ansatz.

We are now prepared to pursue the many questions we have concerning black holes in a constituent framework. In consequence we will now try to gather information of black holes which General Relativity would withhold us by construction.

Chapter 6

Applying the constituent description to Schwarzschild black holes

Borrowing the QCD techniques introduced in the last section we want to put forward the ideas of [64] in a more quantitative framework in the following. The aim is to calculate generic bound state properties beyond kinematical scaling relations.

However, there is still one obstruction we have to overcome. Although the methods introduced, like the mean-field concept, for instance, work well for non-relativistic systems, a generalization to bound states of relativistic constituents is not straight forward. One of the reasons is the absence of particle number conservation in relativistic systems. Nevertheless, viewed as bound states, black holes can be expanded in terms of Fock eigenstates describing multigraviton states. Then we can always project onto a sector of Fock space of a given number N of particles. Provided that the eigenstates carry the same quantum numbers and isometries as the true black hole state there will be a non-zero overlap (see (5.3.1)).

The first steps in order to test our formalism were taken by Stefan Hofmann and Tehseen Rug in [169]. Amongst other things they predicted observables at the partonic level, such as the light cone distribution of bound state constituents, their number density and energy density. After reviewing their results we will present calculations going beyond the partonic level.

6.1 Observables at the partonic level

6.1.1 Black holes' constituent number density

In order to measure the constituent number density at the partonic level, i.e. for non-interacting constituents, we have to evaluate the particle density operator $n(\boldsymbol{k}) = a^\dagger(\boldsymbol{k})a(\boldsymbol{k})$ in the bound state $|\mathcal{B}\rangle$. Inverting the mode expansion for scalar fields representing the constituent gravitons[1] and replacing the creation and annihilation operators

[1]As we have seen in section (5.4) taking scalar instead of spin-2 constituents at the partonic level solely amounts to an irrelevant numerical prefactor.

accordingly the expectation for the free number density can be expressed as

$$\begin{aligned}\langle\mathcal{B}|n(k)|\mathcal{B}\rangle &= \frac{1}{\Gamma_{\mathcal{B}}^2}\int\frac{\mathrm{d}^4P}{(2\pi)^4}|\mathcal{B}(P,Q)|^2\int\mathrm{d}^4u\mathrm{d}^4z\mathrm{d}^4x\mathrm{d}^4y \\ &\quad\times\mathrm{e}^{\mathrm{i}Pu}\mathrm{e}^{-\mathrm{i}Pz}\mathrm{e}^{-\mathrm{i}k(x-y)}\langle\Omega|\mathrm{T}\phi^N(z)\phi(x)\phi(y)\phi^N(u))|\Omega\rangle. \end{aligned} \tag{6.1}$$

Note that we plugged in the explicit form of the current corresponding to a black hole. Constant factors have been absorbed in the definition of the decay constant $\Gamma_{\mathcal{B}}$. The appearance of time ordering is justified by the fact that all operators involved are commuting. We can interpret the operator $n(k)$ as a measuring device of size $|\boldsymbol{x}-\boldsymbol{y}|$. In the next step, we have to evaluate the vacuum expectation value

$$\langle\Omega|\mathrm{T}\phi^N(z)\phi(x)\phi(y)\phi^N(u)|\Omega\rangle. \tag{6.2}$$

As we have learnt in section (5.3.2) such a correlation function involving the non-perturbative vacuum can be calculated by means of the operator product expansion. In the case of free fields a simple example of this expansion is Wick's theorem

$$\begin{aligned}\langle\Omega|\mathrm{T}\phi(x)\phi(y)|\Omega\rangle &= \langle 0|\mathrm{T}\phi(x)\phi(y)|0\rangle + \langle\Omega|:\phi(x)\phi(y):|\Omega\rangle & (6.3)\\ &= \quad \phi(x)\phi(y) \quad + \langle\Omega|:\phi(x)\phi(y):|\Omega\rangle, & (6.4)\end{aligned}$$

where

$$\phi(x)\phi(y) = -\mathrm{i}\triangle(x,y) = \mathrm{i}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\mathrm{e}^{-\mathrm{i}k(x-y)}}{k^2+\mathrm{i}\varepsilon} \tag{6.5}$$

is the Fourier representation of the propagator and :: stands for the normal ordered product[2]. Contrary to normal ordered products evaluated in the perturbative vacuum the operator product considered here does not vanish. A comparison with (5.18) rather shows, that this contribution amounts to the vacuum condensate in which the constituents of the black hole are immersed.

A priori, there will be three possible types of diagrams resulting from Wick's theorem: Purely perturbative ones, i.e. those with all fields contracted, purely non-perturbative ones, i.e. those with no current fields contracted, and mixed diagrams. Calculating these, we will, for convenience, use the double scaling limit of infinitely heavy black holes and infinitely many constituents:

$$M_{\mathcal{B}}\to\infty, \qquad N\to\infty, \qquad N/M_{\mathcal{B}}\ \text{fixed}, \tag{6.6}$$

with the mass of the bound state being large in comparison to the typical energy of the graviton constituents. Note that this corresponds to the limit taken in the quantum-N portrait. Besides, this limit assures that the number density, and any other observable we wish to calculate, neither vanishes nor diverges. Though working with free fields we expect our results to be non-trivial due to the occurrence of non-perturbative effects

[2]Note that we will abbreviate $\langle\Omega|:\phi(x)\phi(y):|\Omega\rangle$ often as $\langle\phi(x)\phi(y)\rangle$ in the following.

associated with the large-N limit taken. Indeed, connectedness requires $N \geq 2$ current fields such that at least one correlation between these fields can be realized. Of course, multiple connections are possible as well.

As demonstrated in [169] for $N = 2$ the only graph that is connected and non-zero in the double scaling limit (6.6) is the purely perturbative one. For any higher number of fields constituting the current the situation is quite different. While purely perturbative and mixed diagrams inevitably involve loop contributions that vanish in the double scaling limit, there exist connected non-perturbative contributions for every $N \geq 3$. In fact, these are the only surviving contributions. Figure (6.1) shows the generic perturbative and mixed diagrams.

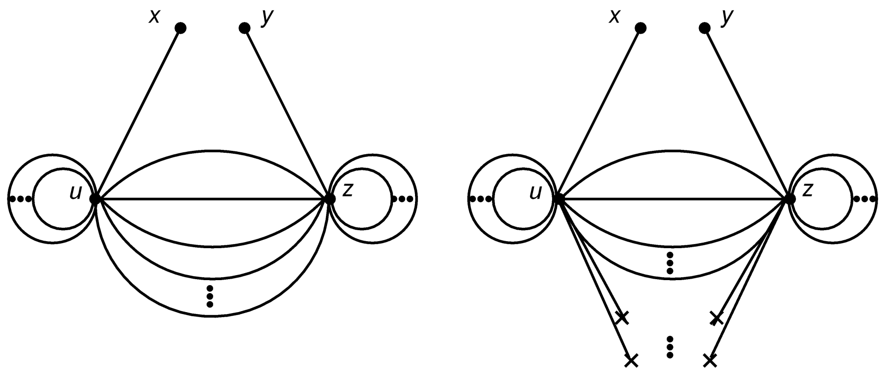

Figure 6.1: Generic diagrams corresponding to the calculation of the constituent number density. L.h.s.: purely perturbative diagram. R.h.s: mixed diagram. Due to composite operator renormalization both contributions vanish in the limit of large black hole masses. Figure adapted from [169].

There are either one-point loops or loops that occur if there are more than two propagators between the spacetime points z and u of the current insertions. The one-point loops corresponding to propagators of the form

$$\underbracket{\phi(z)\phi}(z) = \mathrm{i} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 + \mathrm{i}\varepsilon} \tag{6.7}$$

would lead to divergences if not properly renormalized. At this point composite operator renormalization comes to help dictating that such a propagator cannot occur in the first place because both fields $\phi(z)$ must be subject to normal ordering and therefore condense [169, 187]. The same is true for all other kinds of other loops if the double scaling limit is executed. In this case all loop-internal lines shrink to a point.

Let us for example have a look at the perturbative part in the case $N = 3$:

$$\begin{aligned}
&\int \frac{\mathrm{d}^4 P}{(2\pi)^4} |\mathcal{B}(P)|^2 \int \mathrm{d}^4 u \mathrm{d}^4 z \mathrm{d}^4 x \mathrm{d}^4 y \, \mathrm{e}^{\mathrm{i}Pu} e^{-\mathrm{i}Pz} \mathrm{e}^{-\mathrm{i}k(x-y)} \langle \Omega | \mathrm{T} \phi^3(z) \phi(x) \phi(y) \phi^3(u) | \Omega \rangle_p \\
\propto &\int \frac{\mathrm{d}^4 P}{(2\pi)^4} |\mathcal{B}(P)|^2 \int \mathrm{d}^4 u \mathrm{d}^4 z \mathrm{d}^4 x \mathrm{d}^4 y \int \frac{\mathrm{d}^4 q_{(4)}}{(2\pi)^{16}} \frac{1}{\prod_{j=1}^{4} q_j^2} \\
&\times \mathrm{e}^{-\mathrm{i}z(P+q_1+q_2+q_3)} \mathrm{e}^{\mathrm{i}x(q_3-k)} \mathrm{e}^{-\mathrm{i}y(q_4-k)} \mathrm{e}^{\mathrm{i}u(P+q_1+q_2+q_4)}
\end{aligned} \tag{6.8}$$

where $\mathrm{d}^4q_{(4)} = \mathrm{d}^4q_1...\mathrm{d}^4q_4$ and the momenta q_j correspond to the propagators connecting the different spacetime locations. Note that we have neglected the mass of the graviton constituents. Therefore, the replacements

$$q_3 \to \tilde{q}_3 - P - q_1 - q_2 \quad \text{and} \quad q_4 \to \tilde{q}_4 - P - q_1 - q_2 \tag{6.9}$$

lead to

$$\frac{1}{M_{\mathcal{B}}^4}\int \frac{\mathrm{d}^4P}{(2\pi)^4}|\mathcal{B}(P)|^2 \int \mathrm{d}^4u\mathrm{d}^4z\mathrm{d}^4x\mathrm{d}^4y \int \frac{\mathrm{d}^4q_{(2)}\mathrm{d}^4\tilde{q}_{(2)}}{(2\pi)^{16}}\frac{1}{\prod\limits_{j=1}^{2} q_j^2} \tag{6.10}$$

$$\times \mathrm{e}^{-\mathrm{i}z\tilde{q}_3}\mathrm{e}^{\mathrm{i}x(\tilde{q}_3-P-q_1-q_2-k)}\mathrm{e}^{-\mathrm{i}y(\tilde{q}_4-P-q_1-q_{2}4-k)}\mathrm{e}^{\mathrm{i}u\tilde{q}_4}. \tag{6.11}$$

Here, we made use of the on-shell condition $P^2 = -M_{\mathcal{B}}^2$ of the black hole wave function. This replacement actually has a nice interpretation. In our picture the constituents are associated with the black hole bound state. Consequently, their propagation is subject to in-medium effects. Since the bound state is characterized by its scale $M_{\mathcal{B}}$ the constituents, though being inherently massless, will acquire an effective mass set by this scale due to their propagation in a non-trivial background. Sending $M_{\mathcal{B}} \to \infty$ the propagator shrinks to a point. Eventually, this yields a contact interaction with an effective coupling $1/M_{\mathcal{B}}$ [169].
Performing the integrals over $\tilde{q}_3$ and $\tilde{q}_4$, z and u, q_1 and x we are left with the same integral as in the one-point loop case,

$$\frac{1}{M_{\mathcal{B}}^6}\int \frac{\mathrm{d}^4P}{(2\pi)^4}|\mathcal{B}(P)|^2 \int \mathrm{d}^4y \int \frac{\mathrm{d}^4q_2}{(2\pi)^4}\frac{1}{q_2^2}. \tag{6.12}$$

As we explained before this contribution can consistently be set to zero when properly renormalizing the operator at parton level. It is now easy to transfer the conclusion of vanishing loops if an arbitrary number of auxiliary current fields are involved. Purely non-perturbative diagrams do not contribute either. This is due to the fact that the light cone correlation is effectively disconnected in these cases. Hence, at the parton level only mixed diagrams without loops have to be considered if the bound state constituents are taken to be massless. Then, effectively we are left with a single diagram:

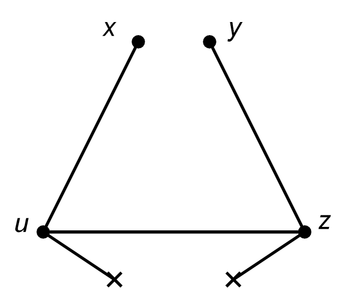

Figure 6.2: The only diagram contributing to the constituent number density for the case $N = 3$.

The corresponding amplitude reads

$$\langle\mathcal{B}|n(k)|\mathcal{B}\rangle = 2\binom{N}{2}^2(\mathrm{i})^3\Gamma^{-2}\mathcal{B}\int\frac{\mathrm{d}^4P}{(2\pi)^4}|\mathcal{B}(P)|^2\int\mathrm{d}^4u\mathrm{d}^4z\mathrm{d}^4x\mathrm{d}^4y\,\mathrm{e}^{\mathrm{i}Pu}e^{-\mathrm{i}Pz}\mathrm{e}^{-\mathrm{i}k(x-y)}$$
$$\times\int\frac{\mathrm{d}^4q_{(3)}}{(2\pi)^{12}}\frac{\mathrm{e}^{-\mathrm{i}q_1(z-u)}\mathrm{e}^{-\mathrm{i}q_2(z-x)}\mathrm{e}^{-\mathrm{i}q_3(y-u)}}{\prod_{j=1}^{3}q_j^2}\langle\phi^{N-2}(z)\phi^{N-2}(u)\rangle. \tag{6.13}$$

The factor 2 accounts for exchanging the spacetime points z and u and the combinatorical factor $\binom{N}{2}^2$ comes from choosing which two of the N fields of the current are chosen for contraction with field operators of our diagnostic device at the respective spacetime points.

Using the same manipulations as before we observe that the expectation value (6.13) for the particle number density inside the black hole scales as $(N/M_{\mathcal{B}})^4$ and therefore remains finite in the double scaling limit. The authors of [169] furthermore point out that $1/N$-corrections to this leading scaling behavior come from the binomial coefficient $\binom{N}{2}^2$ and are as such entirely caused by combinatorics associated with the large-N nature of the considered bound state.

Integrating the number density (6.13) we will gain the number of constituents making up the bound state N_c. Note that N_c is not be confused with the number N counting the field content of the auxiliary current $\mathcal{J}$. In fact, the constituent number N_c includes virtual gravitons of the background as well. It was found in [169] that $N_c \propto N^4\langle\phi^{2(N-2)}\rangle/\Gamma_{\mathcal{B}}^2$, where the momentum integration was cut off at $M_{\mathcal{B}}$ for consistency reasons[3]. Note that the total number of constituents diverges if the semiclassical limit is taken.

6.1.2 Black holes' energy density and mass

Apart from this result, Hofmann and Rug also calculated the expectation value for the energy density inside the black hole,

$$\mathcal{E}(x) = \frac{c}{2}\,(N/M_{\mathcal{B}})^{\,2}\,|\mathcal{B}(x)|^2\;\Gamma_{\mathcal{B}}^{-2}\langle\phi^{2(N-1)}\rangle, \tag{6.14}$$

where c denotes a dimensionless constant of order one. An integration of this quantity over a spatial slice yields the mass of the black hole [169]

$$M_{\mathcal{B}}^2 = \frac{1}{16}\frac{\langle\phi^{2(N-1)}\rangle}{\langle\phi^{2(N-2)}\rangle}\frac{N_{\mathrm{c}}}{N^2} \propto \langle\phi^{2(N-1)}\rangle N^2. \tag{6.15}$$

Here, we made use of the constituent number's scaling with N. While $M_{\mathcal{B}}$ denotes a macroscopic property of the black hole, N represents a microscopic quantity belonging to the auxiliary current. For this reason, formula (6.15) provides a connection between the microscopic and the macroscopic sector of our bound state picture. We also want to mention, that this result bears comparison with the N scaling of baryons composed

[3]Obviously, a single constituent can at most carry a momentum corresponding to the mass of the black hole bound state.

of heavy quark fields [155]. Besides, the ratio of condensates in (6.15) becomes N independent in the double scaling limit. Being of mass dimension two, this factor could be identified with the squared inverse size, r^{-2}, of the considered bound state. Since we are trying to describe black holes, this scale would actually be set by the Schwarzschild radius r_s implying that black holes correspond to maximally packed systems. This again, would be in accordance with [64]. However, as the authors of [169] admit, this identification is pure speculation because the condensates are non-perturbative objects and thus require experimental input.

6.2 Going beyond the partonic level - Scattering processes

In order to gather structural information about the interior of the modeled black hole bound state, we will focus on applications within S-matrix theory in this section. As probing devices for a deep inelastic scattering process we will use virtual gravitons emitted outside the black hole and absorbed by its interior. Here, the horizon of the black hole, or otherwise the surface of an arbitrary bound state, serves as a boundary separating the outer region, that can be treated perturbatively, from the non-perturbative interior. Information about the constituents inside the bound state can be extracted from the scattering angle between the emitter asymptotics.

At this point we want to stress again that within our field theoretical approach the common geometrical concept based on general relativity is by no means fundamental. This is to be contrasted with the semiclassical treatment introduced in chapter 3, where only probe particles are quantized but the geometrical background is left untouched. In that case we still have to deal with the usual horizon problem prohibiting an observer outside the black hole to get any information about the internal structure of the system. Our procedure, instead, will allow for a complementary, non-geometrical, description of the black hole interior based on observables which can be measured by an outside observer. Indeed, spacetime geometry and the associated existence of a horizon in the Schwarzschild case are to be understood as an effective macroscopic phenomenon. On the microscopic level, this perception should break down allowing for the resolution of the bound state.

In fact, quantum field theory distinguishes between two source types, external and internal sources, referring to the absence and presence of sources in the physical Hilbert space, respectively. Clearly, an external source is structureless, while small-scale structure can be assigned to an internal composite source. Viewing black holes as external sources, i.e. not resolved in a physical Hilbert space, small-scale structure of their interior is a void concept. Then, scattering experiments only allow to extract observables localized in the exterior region. In particular, the $1/r$-potential can be recovered for $r > r_s$. Furthermore, a resummation of tree scattering processes sourced by the external black hole gives rise to geodesic motion in the respective Schwarzschild background [164]. If black holes are treated as internal sources, on the other hand, their interior quantum structure can be resolved by employing probes of sufficient virtuality, $-q^2 > r_s^{-2}$. As long as $-q^2 < M_p^2$ a description within in a weakly coupled field theory is possible. We will now consider such a process in more detail.

6.2.1 Probing the black hole structure

Consider an ingoing scalar field $\phi(x_1)$ of (on-shell) momentum k, emitting a graviton $h_{\mu\nu}$ at z_1 of appropriate virtuality $-q^2$, which subsequently gets absorbed at z_2 by another scalar field being part of the black hole. Figure (6.3) illustrates this process which is captured within the linearized Einstein-Hilbert action coupled to the energy-momentum tensor of a massless scalar, $T = \mathrm{d}\phi \otimes \mathrm{d}\phi - (\mathrm{d}\phi, \mathrm{d}\phi)\eta/2$,

$$S = \int \mathrm{d}^4 x \left[\frac{1}{2} h_{\mu\nu} \epsilon^{\mu\nu}_{\alpha\beta} h^{\alpha\beta} + \frac{1}{M_\mathrm{P}} h_{\mu\nu} T^{\mu\nu}\right] . \tag{6.16}$$

Here $\epsilon^{\mu\nu}_{\alpha\beta}$ is the linearized graviton kinetic operator expanded around flat spacetime.

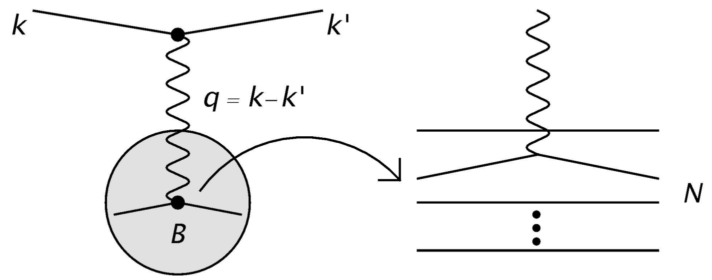

Figure 6.3: Feynman diagram for the scattering of a scalar on a black hole bound state at tree level. The wiggly line corresponds to the exchange of a virtual graviton. On the right hand side, the corresponding absorption is resolved into the microscopic constituents spectating and participating in the scattering process.

The corresponding amplitude reads

$$a^{(2)}(x_1, x_2) = \tfrac{\mathrm{i}^2}{M_\mathrm{P}^4} \int \mathrm{d}^4 z_1 \mathrm{d}^4 z_2 \; \mathcal{P}^{\mu\nu}(z_1, z_2; x_1, x_2) \mathcal{N}_{\mu\nu}(z_2),$$

where $\mathcal{P}$ contains all correlations with respect to the perturbative vacuum state $|0\rangle$, and $\mathcal{N}$ carries local, non-perturbative information about the black hole quantum state $|\mathcal{B}\rangle$. Put differently, $\mathcal{P}$ describes spacetime events that originate outside the bound state, while $\mathcal{N}$ is localized in its interior:

$$\begin{aligned} \mathcal{P}^{\mu\nu} &= \langle 0|\mathrm{T}\phi(x_2) T_{\alpha\beta}(z_1)\phi(x_1)|0\rangle \; \Delta^{\alpha\beta\mu\nu}(z_1, z_2) \; , \\ \mathcal{N}_{\mu\nu} &= \langle \mathcal{B}'| \, {:}T_{\mu\nu}{:} \, (z_2)|\mathcal{B}\rangle \; . \end{aligned} \tag{6.17}$$

Here, $\Delta^{\alpha\beta\mu\nu}$ denotes the free graviton propagator and $|\mathcal{B}'\rangle$ is the black hole quantum state after absorbing the graviton.

Note that this process is reminiscent of the familiar deep inelastic lepton-nucleon scattering in QCD, where $\mathcal{P}^{\mu\nu}$ corresponds to the leptonic tensor and $\mathcal{N}^{\mu\nu}$ to the hadronic tensor incorporating the current correlator and hence non-trivial information about the bound state structure (see e.g. [181]).

Using the auxiliary current description, and provided that the bound state wave function $\mathcal{B}(\mathcal{L})$ has a sufficiently compact support in $\mathcal{L}$–space, the graviton absorption event can be translated to the origin:

$$\mathcal{N}(z_2) \approx \mathrm{e}^{\mathrm{i}(P'-P)z_2} \langle \mathcal{B}' | :T: (0) | \mathcal{B} \rangle \,, \tag{6.18}$$

with P' and P denoting the central momenta of wave-packets corresponding to the black hole quantum states after and before the graviton absorption, respectively.

The evaluation of $\mathcal{P}$ is straightforward. Truncating the ingoing and outgoing emitter legs, the one-graviton exchange amplitude becomes

$$\begin{aligned} \langle \mathcal{B}'\phi' | \mathcal{B}\phi \rangle^{(2)} &= \mathrm{i}^2 \int \mathrm{d}^4x \mathrm{d}^4y \, \mathrm{e}^{-\mathrm{i}(k'-k)x} D(x) D(y) a^{(2)}(z_1, z_2) \\ &= -\mathrm{i}(2\pi)^4 \delta(k' + P' - k - P) \; \alpha_{\mathrm{g}}^{\,2} \\ &\quad \times \langle \mathcal{B}' | :T_{\mu\nu}: (0) | \mathcal{B} \rangle \; \Delta^{\mu\nu\alpha\beta}(k' - k) \; G_{\alpha\beta\rho\sigma} \, \tfrac{k'^{\rho} k^{\sigma}}{k'^{0} k^{0}} \,, \end{aligned} \tag{6.19}$$

where $D = \Box$ is the kinetic operator and the coupling $\alpha_{\mathrm{g}} \equiv 1/(4\pi M_p^{\,2})$ has been introduced. $G_{\alpha\beta\rho\sigma} = \eta_{\beta(\alpha}\eta_{\rho)\sigma} - \eta_{\beta\sigma}\eta_{\alpha\rho}$ is the Wheeler–DeWitt metric.

The total cross section $\sigma(\mathcal{B}'\phi' \leftarrow \mathcal{B}\phi)$ involves the absolute square of this amplitude and an integration over all intermediate bound states in the spectrum of the theory. Therefore, the differential cross section can be written as

$$k'^{0} \frac{\mathrm{d}\sigma}{\mathrm{d}^3 k'} = \frac{2}{\mathcal{F}(\phi)} \; |\alpha_{\mathrm{g}} \Delta(k' - k)|^2 \; \mathcal{E}^{\alpha\beta\mu\nu}(k, k') \mathcal{A}_{\alpha\beta\mu\nu}(\mathcal{B}; k, k'). \tag{6.20}$$

Here, $\mathcal{F}$ denotes the ingoing flux factor and Δ the scalar part of the graviton propagator. The emission tensor $\mathcal{E}$ captures the virtual graviton emission in the exterior of the bound state and the absorption tensor $\mathcal{A}$ its subsequent absorption by a black hole constituent. That is, the emission tensor $\mathcal{E} \equiv Q \otimes Q$ is build from

$$Q^{\mu\nu} = 4\pi^2 \; \Pi^{\mu\nu\alpha\beta}(k' - k) \; G_{\alpha\beta\rho\sigma} \, \tfrac{k'^{\rho} k^{\sigma}}{k'^{0} k^{0}} \,, \tag{6.21}$$

with the graviton polarization tensor $\Pi_{\mu\nu\alpha\beta}(q) \equiv \pi_{\mu(\alpha}\pi_{\beta)\nu} - \pi_{\mu\nu}\pi_{\alpha\beta}$, where $\pi_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$. Conversely, graviton absorption is described as the energy momentum correlation of black hole constituents:

$$\mathcal{A} = \tfrac{1}{2\pi} \int \mathrm{d}^4x \; \mathrm{e}^{-\mathrm{i}(k'-k)x} \langle \mathcal{B} | T(x) \otimes T(0) | \mathcal{B} \rangle. \tag{6.22}$$

Note that the information $\mathcal{A}$ contains about the black hole interior is actually not yet resolved in terms of chronologically ordered subprocesses. For practical calculations, $\mathcal{A}$ will be related to the corresponding time ordered amplitude in the next section.

6.2.2 Chronological ordering

Given that the graviton absorption tensor is not directly subject to time ordering, the question arises whether it can be deconstructed into causal correlations. The method to achieve this is well-known in the context of scattering processes on bound states in QCD and will be adapted to the problem at hand in the following discussion.

As a first step, let us relate $\mathcal{A}$ to a tensor built from $T(x) \wedge T(0)$. Inserting a complete set of physical states $\tilde{\mathcal{B}}$ in between the energy-momentum tensors at x and 0 in (6.22) and exploiting appropriate spacetime translations we arrive at

$$\mathcal{A} = \tfrac{1}{2\pi} \sum\!\!\!\!\!\!\int_{\tilde{\mathcal{B}}} (2\pi)^4 \delta(q + P - P') \langle \mathcal{B}|T(0)|\tilde{\mathcal{B}}\rangle\langle\tilde{\mathcal{B}}|T(0)|\mathcal{B}\rangle, \tag{6.23}$$

with $q \equiv k - k'$. Standard kinematical arguments[4] allow to replace (6.22) with

$$\mathcal{A} = \tfrac{1}{2\pi} \int \mathrm{d}^4x \; \mathrm{e}^{\mathrm{i}qx} \langle \mathcal{B}|[T(x), T(0)]|\mathcal{B}\rangle. \tag{6.24}$$

It can be shown that the absorption tensor (6.24) is related to the absorptive part of the Compton-like amplitude $\mathcal{C}$ for the forward scattering of a virtual graviton off a black hole [181],

$$\mathcal{C} = \mathrm{i} \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x} \langle \mathcal{B}|\mathrm{T}\; T(x) \otimes T(0)|\mathcal{B}\rangle. \tag{6.25}$$

In order to see this, let us make the discontinuity of $\mathcal{C}$ manifest. Expressing the time-ordered product by means of Heaviside functions and repeating the steps that allowed to extract the kinematical support of $\mathcal{A}$, i.e. inserting a complete set of physical states and exploiting translational invariance, we arrive at

$$\mathcal{C} = \sum\!\!\!\!\!\!\int_{\tilde{\mathcal{B}}} \tfrac{(2\pi)^3 \delta(\mathbf{P}' - \mathbf{P} - \mathbf{q})}{P'^0 - P^0 - q^0 - \mathrm{i}\varepsilon} \langle \mathcal{B}|T(0)|\tilde{\mathcal{B}}\rangle\langle\tilde{\mathcal{B}}|T(0)|\mathcal{B}\rangle \tag{6.26}$$

if the Fourier representation of the Heaviside function is employed and all integrations are performed. Using furthermore

$$\mathrm{Abs}\frac{1}{\omega} \equiv \frac{1}{2\mathrm{i}} \left[\frac{1}{\omega - \mathrm{i}\varepsilon} - \frac{1}{\omega + \mathrm{i}\varepsilon}\right] = \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}x_0 \, \mathrm{e}^{-\mathrm{i}\omega x_0}, \tag{6.27}$$

it follows directly that $\mathrm{Abs}(P'^0 - P^0 - q^0 - \mathrm{i}\varepsilon)^{-1} = \pi\delta(P'^0 - P^0 - q^0)$ and therefore

$$\pi\mathcal{A}(\mathcal{B}; q) = \mathrm{Abs}\, \mathcal{C}(\mathcal{B}; q). \tag{6.28}$$

This way the absorption tensor $\mathcal{A}$ can be deconstructed in terms of handier chronologically ordered correlations.

6.2.3 Constituent representation of $\mathcal{A}$

As we have already seen in the illustration of observables at the partonic level in section (6.1), the time-ordered product of energy-momentum tensors in $\mathcal{C}$ gives rise to three contributions: The first corresponds to maximal connectivity between the tensors, resulting in a purely perturbative contribution void of any structural information. The second represents a disconnected contribution. Finally, the third contribution allows

[4] Physical processes require $q_0 > 0$. Since $P_0' > P_0$ in our case, the artificially added part $\int \mathrm{d}^4x \; \mathrm{e}^{\mathrm{i}qx} \langle \mathcal{B}|T(0), T(x)|\mathcal{B}\rangle$ vanishes.

for perturbative correlations between the energy-momentum tensors and, in addition, carries structural information. Dropping the contributions without any information of the interior,

$$\mathrm{T}\, T_{\alpha\beta}(x)T_{\mu\nu}(0) = \tfrac{1}{4}G^{ab}_{\alpha\beta}G^{mn}_{\mu\nu}\; C_{bm}(x)\; \mathcal{O}_{an}(x,0), \tag{6.29}$$

where $C(x) \equiv 4\langle 0|\mathrm{T}\ \mathrm{d}\phi(x)\otimes \mathrm{d}\phi(0)|0\rangle$ denotes the correlation with respect to the perturbative vacuum,

$$C(x) = -\frac{2}{\pi^2}\frac{x^2\eta - 4x\otimes x}{(x^2)^3} \tag{6.30}$$

in free field theory, and $\mathcal{O}(x,0) \equiv :\mathrm{d}\phi(x)\otimes \mathrm{d}\phi(0):$ is the bilocal operator allowing to extract certain structural information when anchored in a quantum bound state.

In order to extract local observables, $\mathcal{O}(x,0)$ has to be expanded in a series of local operators. In principle, this amounts to a Laurent-series expansion of the corresponding Green's function. Let us first focus on its Taylor part:

$$\phi(x) = \exp\left(x\cdot\partial_z\right)\phi(z)|_{z=0}. \tag{6.31}$$

The ordinary partial derivative is appropriate in the free field theory context, otherwise $\mathcal{O}(x,0)$ requires a gauge invariant completion. Then,

$$\mathcal{O}(x,0) = \sum_{j=0}^{\infty}\tfrac{1}{j!}\;\mathcal{O}^{[j]}(0), \tag{6.32}$$

with $\mathcal{O}^{[j]}(0) \equiv\ :(x\partial_z)^j\mathrm{d}\phi(0)\otimes\mathrm{d}\phi(0):$ and $[j]\equiv 4$ denoting the mass dimension of the local operator. Note that we suppressed the spacetime point x appearing in the directional derivative in order to stress the local character of the operator expansion.

The fast track to relate $\mathcal{C}$ to constituent observables is to evaluate $\mathcal{O}(x,0)$ in a black hole quantum state using the auxiliary current description. We find for the local operators

$$\langle\mathcal{B}|\mathcal{O}^{[j]}(0)|\mathcal{B}\rangle = \kappa\,(xP)^j\;\langle\mathcal{B}|\phi(r)\phi(0)|\mathcal{B}\rangle P{\otimes}P, \tag{6.33}$$

where κ denotes a combinatoric factor and a point-split regularization ($r^2\to 0$) was employed. The operator appearing on the right hand side of (6.33) measures the constituent number density - a quantity we already encountered in section (6.1).

As a consequence, the absorptive part of the forward virtual graviton scattering amplitude $\mathcal{C}$ or, equivalently, the graviton absorption tensor $\mathcal{A}$ can be directly interpreted in terms of the black hole constituent distribution.

6.2.4 Analytic properties of $\mathcal{C}$

The Ward-Takahashi identity associated with the underlying gauge symmetry fixes the tensorial structure $\theta_{\alpha\beta\mu\nu}(q,P)\equiv \Pi^{ab}_{\alpha\beta}\Pi^{mn}_{\mu\nu}\eta_{bm}P_aP_n$ of the amplitude $\mathcal{C}(q,P)$ in accordance with source conservation. The Laurent-series expansion of $\mathcal{O}(x,0)$ in local operators gives to leading order in $(qP)/P^2$

$$\mathcal{C}(q,P) \quad = \quad -\tfrac{\mathrm{i}}{2\pi^2}\theta(q,P)\langle\mathcal{B}|\phi(r)\phi(0)|\mathcal{B}\rangle\sum_{j=-\infty}^{\infty}C_j(q)w^j. \tag{6.34}$$

Here, the coefficients C_j are calculable and turn out to be momentum independent, and the expansion parameter u is given by $u \equiv -P^2/q^2 \gg 1$. Note that this parameter is the analogue of the inverse Bjorken scaling variable known from deep inelastic scattering (see e.g. [181]). There is a profound difference between these two parameters, however. While in standard discussions of deep inelastic scattering in the infinite momentum frame asymptotic freedom is exploited, this is not possible in gravity. For the problem at hand, however, there is a natural limit and correspondingly an appropriate expansion parameter: Considering black holes of large mass and momentum transfers smaller than M_p (which is needed in order to trust the perturbative expansion) we are naturally led to the expansion parameter u.
The discontinuity of $\mathcal{C}$ for fixed $q^2 = -Q^2$ turns out to be at

$$u_* = \frac{M_\mathcal{B}}{2(M'_\mathcal{B} - M_\mathcal{B})} \left(1 - \frac{M'^2_\mathcal{B} - M^2_\mathcal{B}}{Q^2}\right) \gg 1. \tag{6.35}$$

That is, $\mathcal{C}$ has an isolated pole at $u_* \gg 1$ and, in particular, no branch cut in leading order, corresponding to the statement that $M'_\mathcal{B}/M_\mathcal{B} - 1 \approx 0$. Of course, the presence of a branch cut beyond leading order poses no obstacle. On the contrary, it has an evident interpretation in terms of intermediate black hole excitations.
In order to project onto the Laurent coefficients, a path enclosing $[-u_*, u_*] \subset \mathbb{R}$ in the complex u plane has to be chosen.

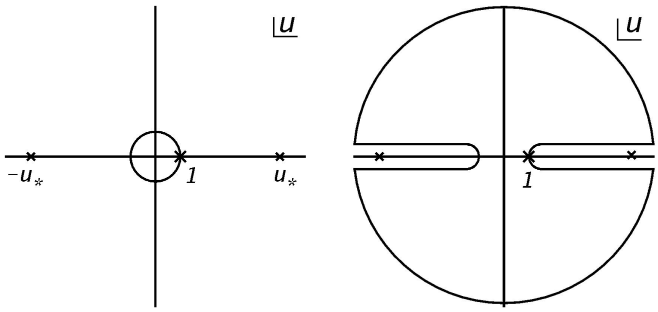

Figure 6.4: Integration contour in the complex u-plane. The left figure displays an integration contour corresponding to the radius of convergence. In order to relate this to the physical u-region ($P^2 > Q^2$) we perform a contour deformation (right figure). The radius of the circle is sent to infinity.

This covers the physical u region, while the radius of convergence of the corresponding Taylor series would only allow for unphysical $u \in [-1, 1]$ (see Figure 6.4). We find

$$\int_0^1 \mathrm{d}\zeta\ \zeta^{k-2} \mathcal{A}(q, P, \zeta) = \frac{C_{k-1}}{4\pi^2} \langle \mathcal{B}|\phi(r)\phi(0)|\mathcal{B}\rangle\ \theta(q, P), \tag{6.36}$$

with $\zeta \equiv 1/u$ denoting the graviton virtuality relative to the black hole target mass. Hence, all moments of the absorption tensor with respect to ζ are directly proportional to the constituent distribution inside the black hole. This implies that

$$\mathrm{d}\sigma/\mathrm{d}^3\mathrm{k}' \propto \langle \mathcal{B}|\phi(\mathrm{r})\phi(0)|\mathcal{B}\rangle. \tag{6.37}$$

Consequently, black hole constituent distributions are observables and, in principle, can be extracted from scattering experiments.
Note that effectively only the moment of the absorption tensor corresponding to $k = 2$ contributes in the above equation. This is due to the fact that we are working in a limit where ${M_{\mathcal{B}}}^2 \gg {M_p}^2 \gg -q^2$. Furthermore, drawing an analogy to deep inelastic scattering in QCD, we can identify the black hole distribution function with a Lorentz-invariant structure function. While the most general tensor decomposition of the hadronic tensor in QCD leads to several structure functions we here have to deal with only one such function. However, if we did not restrict our considerations to the Schwarzschild case but added features like charge to the black hole this would also lead to further structure functions.

Dealing with a non-perturbative object, $\langle\mathcal{B}|\phi(r)\phi(0)|\mathcal{B}\rangle$ cannot be determined from first principles. However, we will adopt a simple toy model for the black hole wave function in the next section and compute $\mathcal{D}(|\mathbf{r}|)$. Assuming that the wave function is localized within the Schwarzschild radius will lead to a qualitative understanding of the distribution of quanta inside $|\mathcal{B}\rangle$.
In order to allow quantitative statements the distribution should be measured at some scale $\Lambda \ll M_p$, where the effective field theory description is valid. Predicting the cross section at a different scale can then be achieved by means of renormalization group techniques. Higher order radiative corrections to the scattering process leading to evolution equations for the distribution function should be considered in the future, too. Notice that this procedure is similar to the DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) evolution [188, 189, 190] of quark and gluon distributions within the framework of perturbative QCD.

6.2.5 Constituent distribution at parton level

In sections (6.2.3) and (6.2.4) we presented a constituent interpretation for virtual graviton absorption by a black hole quantum bound state. Central for this interpretation was the constituent distribution $\mathcal{D}(r) \equiv \langle\mathcal{B}|\phi(r)\phi(0)|\mathcal{B}\rangle$. According to section (5.4), $\mathcal{D}(r)$ can, in the spherically symmetric case of a black hole, only depend on the spatial distance $|\boldsymbol{r}|$, but not on time. The spatial length scale $|\boldsymbol{r}|$ is at the observers disposal. It can be interpreted as the necessarily finite spatial extent of an apparatus that emits a ϕ quantum at one end and subsequently absorbs it at the other end. In between emission and absorption the quantum probes the medium in which the apparatus has been submerged, in our case the black hole interior. At the partonic level, no individual interactions between the probe and black hole constituents take place, therefore the only relevant scale remains $|\boldsymbol{r}|$. Effectively, then, there is only the correlation across the apparatus between emission and absorption events, which scales as $|\boldsymbol{r}|^{-2}$. The calculation of the light-cone distribution of black hole constituents has been outlined in section (6.1.1) calculated in [169] at the parton level,

$$\mathcal{D}(|\mathbf{r}|) = 2\left(\frac{N}{M_{\mathcal{B}}}\right)^4 \Gamma_{\mathcal{B}}^{-2}\langle\phi^{2(N-2)}\rangle \frac{1}{4\pi^2|\boldsymbol{r}|^2}\int \mathrm{d}^3P\cos(\boldsymbol{P}\boldsymbol{r}/2)|\mathcal{B}(\boldsymbol{P},\mathcal{Q})|^2, \tag{6.38}$$

where $\langle\phi^{2(N-2)}\rangle$ is a condensate parametrizing the non-perturbative vacuum structure inside a black hole quantum bound state.

At the diagrammatic level, $\mathcal{D}(|\boldsymbol{r}|)$ can be represented by figure (6.5).

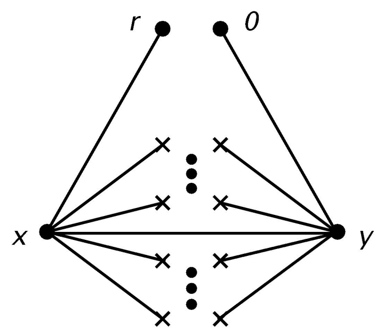

Figure 6.5: The only diagram contributing to $\mathcal{D}(|\boldsymbol{r}|)$. Straight lines represent free propagators, while lines ending in crosses correspond to non-perturbative condensation processes.

As we have learnt before, even in the absence of gauge interactions, N carries non-perturbative information via its dependence on $\Gamma_{\mathcal{B}}$ and, in addition, its dependence on ϕ condensates. The latter dependence deserves elaboration. It can be traced back to the fact that $N \gg 1$ for black holes, implying minimal connectivity between the spacetime events at which the auxiliary currents are operative. At the level of $\mathcal{D}(|\boldsymbol{r}|)$, this can be seen as follows: The constituent distribution is generated by a four-point correlator, where two spacetime points are associated with the read-in events, i.e. the auxiliary current locations, and one point split for localizing an apparatus of finite extent, consisting of an emitter and an absorber. The measurement process requires altogether six ϕ fields at four spacetime locations. The vast majority of fields composing the auxiliary currents has two options. Either they enhance the connectivity between the locations of the currents, or they condense. In the double scaling limit (6.6), $N, M_{\mathcal{B}} \to \infty$ and $N/M_{\mathcal{B}}$ fixed, condensation of ϕ quanta turns out to be the favored option as we have explained in detail before.

Violations of this limit are not exponentially suppressed, but of order $1/N \ll 1$. This indicates the essential quantum character of black holes and shows that black holes are essentially beyond a semiclassical description. Since in our approach the cross section is given in terms of the number density, these corrections are in principle measurable in scattering experiments.

Let us now calculate the constituent distribution $\mathcal{D}(|\mathbf{r}|)$ assuming a Gaussian wave function $\mathcal{B}(\boldsymbol{P}, \mathcal{Q})$ for the black hole peaked around $M_{\mathcal{B}}$ with a standard deviation given by $1/r_s$. This choice reflects those features of the a priori unknown black hole state that are relevant for the qualitative behavior of the constituent distribution. For instance, the non-perturbative ground state has compact support characterized by the Schwarzschild surface itself. In Figure (6.6) we show the constituent distribution in wavelength space, $\widetilde{D}(\lambda) \propto \lambda\, \mathrm{erf}(2r_s/\lambda)$, where erf denotes the error function and $\lambda = 1/|\boldsymbol{k}|$. Here $|\boldsymbol{k}|$ is the absolute value of the constituent three momentum.

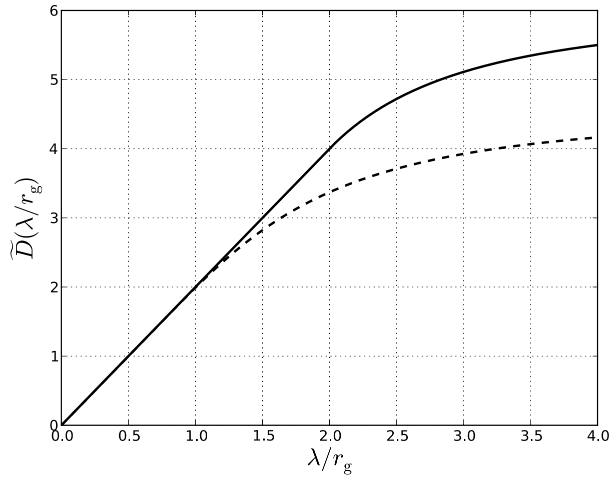

Figure 6.6: Parton distribution as a function of the wavelength for a Gaussian wave function of variance r_s (dashed line) and a Heaviside profile (solid line). Here $\tilde{\mathcal{D}}(\lambda/r_s)$ is normalized to $\frac{r_s}{2}(N/M_{\mathcal{B}})^4\Gamma_{\mathcal{B}}^{-2}\langle\phi^{2(N-2)}\rangle$. The distribution is plotted only for wavelengths $\lambda < 4r_s$ since the condensates are not supported outside the black hole. By construction our analysis is valid only up to $\lambda = 2r_s$. Note that r_s is relabeled by r_g in the figure.

As can be seen, black hole constituents prefer to occupy long wavelength modes under the chosen assumptions. Then black hole interiors are dominated by soft physics which is in accordance with the postulates of the quantum-N portrait [64].

6.2.6 Including corrections

The above results are only gained in the double scaling limit (6.6). In the next step it will therefore be important to include effects of finite masses and constituent numbers. In particular, this will be important to shed light on the purification of Hawking radiation within our framework[5]. At the same time it will be important to capture these effects when considering the explicit emergence of geometry from our framwork.
Before doing so, however, it is of paramount importance to recover the Schwarzschild geometry itself. In fact, current research proves that such an emergence takes place. In [191] the authors show explicitly that a summation over all tree-level contributions describing the interactions between gravitons and the black hole constituent source leads to the Schwarzschild metric. In the double scaling limit these considerations resemble Duff's computation [164] which uses a classical source. Furthermore, a finite bound state mass provides $1/N$ departures from the classical metric. Let us stress again that these quantum corrections are beyond semiclassicality. Concerning the singularity problem these considerations show once more that not only quantum objects are unaffected by spacetime singularities, but that the singularities themselves are evaded due to quantum corrections of the background and hence altered geodesics. As mentioned before, this also underscorses the irrelevance of semiclassical controversies.

[5]A prescription for the derivation of Hawking radiation will be given in the next section.

Besides, we only considered a one-graviton exchange so far. Multiple graviton exchanges should be taken into account as well. These corrections can either be radiative or higher-order in tree level. In particular, the latter corrections are interesting as they are able to provide new geomtric information. Concerning our picture an important application can be found in [191].

Furthermore, we have exclusively worked in a free-field context. Taking gauge interactions into account, we will encounter a more versatile condensate structure. Indeed, the operator $\mathcal{O}(x,0)$ encountered in our calculations then requires a gauge-invariant completion. This can be achieved by attaching a path-ordered exponential of the gauge field, i.e. a Wilson line, to the operator. While in QCD the gauge field is given by the gluon field A_μ, in the context of gravity gauge invariance is replaced by diffeomorphism invariance and we rather have to deal with the Levi-Civita connection Γ. In analogy to the Fock-Schwinger gauge in QCD (see section (5.3.2)) the condition $x^\lambda x^\sigma \Gamma^\mu_{\lambda\sigma} = 0$ can then be employed such that all condensate terms can be expressed via (covariant) curvature condensates (see [169] for details.).

Apart from calculating our previous results more precisely, there is a whole plethora of other interesting black hole aspects that are equally worth investigating. Among these are the formation of black holes and their Hawking radiation (see also chapter (3).

6.3 Prospects for the future

6.3.1 Deriving black hole formation, entropy and Hawking radiation

Within our approach black hole formation and Hawking radiation should be understood in terms of scattering processes as well.
According to Thorne's hoop conjecture the scattering, for example, of two massless scalars at center of mass energy $s \gg M_p^2$ and impact parameter smaller than the Schwarzschild radius associated to this energy, $r_s = 2G_N\sqrt{s}$, should lead to the formation of a black hole. The corresponding S-matrix element schematically corresponds to the process $2 \to N$ where it is important to stress that the outgoing N-particle state should be a non-perturbative bound state. This is to be contrasted to the analogous perturbative processes where the outgoing particles are free asymptotic gravitons [192].

The S-matrix element corresponding to Hawking radiation will be of the form $\langle \mathcal{B}', p_1, p_2, ...|\mathcal{B}\rangle$ with the p's denoting the momenta of the evaporated particles and $\mathcal{B}'$ the remaining black hole state. To leading order, the thermal spectrum found by Hawking should be reproduced. Quantum corrections are then expected to allow for a purification of the radiation.

6.3.2 Demonstrating the emergence of geometry

To confirm our approach we do not only have the possibility to compare our results with semiclassical considerations in the large-N limit. In fact, we should also be able to show explicitly how spacetime geometry itself emerges from graviton condensation.

As we have seen this has already been accomplished for the Schwarzschild case [191]. However, a generalization to other spacetimes is desirable as well. This brings us to our next point, namely to extend our method to other spacetimes for a start.

6.3.3 Applications of our formalism to other spacetimes - A generalization of the ACD

Our view on the emergence of spacetime geometry by graviton condensation is a far-reaching assumption. Though dissolving many puzzles a restriction to black holes to substantiate this idea is not sufficient.
Of course, there are many other spacetimes we might apply our formalism to. The most relevant spacetime solutions to consider, however, are those which open the possibility to bear comparison with preexisting (possibly experimental) results. Concerning the cosmological constant problem (see Introduction) the application of our formalism to de Sitter (dS) spacetime is of particular interest. As far as current observations of the cosmic microwave background are regarded, such as the angular radiation temperature anisotropy power spectrum, a constituent resolution of inflationary spacetimes might be thought of. In order to understand the holographic principle in more detail and especially the AdS/CFT correspondence an auxiliary current description for Anti-de Sitter space will be vital.
Either way, it will be crucial to supplement the previous ACD (5.3.1). The reason behind is the following: So far, we did not consider any perturbations of the considered spacetime. For the scattering of a graviton off a black hole this was not necessary since we could easily restrict the scattering to happen in radial direction in order to preserve the rotational symmetry. However, if we are interested in calculating the cosmological constant within our framework, for example, the expectation value of the energy-momentum tensor of test particles moving in the de Sitter vacuum state must be determined. The motion of these particles will inherently break the present spacetime isometries. Therefore, a split of the current into background-isometries preserving and breaking parts must be made, i.e.

$$J(x) = J_0(x) + \delta J(x) \equiv \sum_{j=0} \delta_{(j)} J(x) \tag{6.39}$$

with $J_0(x) = J_0$ in the special case of dS. Of course, in order not to destabilize the background the fluctuations must not increase alarmingly. Sticking to dS spacetime we must also take into consideration that, like many other spacetimes, this solution of Einstein's equations is not asymptotically flat. Contrary to the ACD presented in section (5.3.1) this implies that P^2 does no longer represent an appropriate state labeling. A generic bound state $|\mathcal{B}\rangle$ is therefore given by

$$\begin{aligned} |\mathcal{B}\rangle &= \sum\!\!\!\!\!\!\int_{\mathcal{L}(\mathcal{B})} \mathcal{B}(\mathcal{L}) \int \mathrm{d}^4x\, F_{\mathcal{L}}(x) J(x) |\Omega\rangle \\ &= \sum_{j=0} \sum\!\!\!\!\!\!\int_{\mathcal{L}_j(\mathcal{B}_j)} \mathcal{B}_j(\mathcal{L}_j) \int \mathrm{d}^4x\, F_{\mathcal{L}_j}(x) \delta_{(j)} J(x) |\Omega\rangle, \end{aligned} \tag{6.40}$$

where instead of a plane wave the function $F_{\mathcal{L}}(x)$ has been introduced. Inserting the above generalized current $F_{\mathcal{L}}(x)$ can be understood as a weight function weighting the different currents $\delta_{(j)}J(x)$ leading to geometric perturbations.
From this point on the calculation of observables of interest is straightforward. In fact, it turns out that only diagonal matrix elements, i.e. with $i = j$, contribute. The reason is that background states with differing fluctuations are represented by different quantum numbers and therefore are orthogonal to each other.
In general, however, there is one remaining caveat to applying the above modified ACD. Dealing with spacetimes that are not asymptotically flat the current can no longer be approximated to be local and therefore S-matrix calculations can no longer be executed. This, unfortunately, diminishes the number of computable observables significantly.
Nevertheless, we believe that our formalism can provide important contributions to the solution of a wide range of problems. As far as the evaluation of the cosmological constant is concerned, for example, we expect a technically natural small energy density. This is due to the fact that the contribution of vacuum bubbles should vanish in the limit $N \to \infty$ guaranteeing a natural suppression for higher order terms. The reason behind is that we regard Minkowski spacetime as fundamental and the fact that vacuum energy on such a background can be renormalized to zero.

6.3.4 Applications to QCD

Last but not least it should be mentioned that our framework can also be tested further on flat spacetime. Though being heavily influenced by QCD methods our formalism is advanced in the sense that it does not rely on sum rules (see section (5.4)). Instead, we have a theory that comprises a wave function for the bound state at hand indicating at which point in space non-trivial vacuum structures are arising. We might therefore have the possibility to predict observables for which QCD methods have failed so far or have been rather imprecise. Among these are vector meson masses, the nucleon charge radii or glueballs (see also section (5.3.2)). Conventional glueball sum rules, for example, seem to require an instanton-improved operator product expansion with instantons providing non-perturbative contributions to the Wilson coefficients [193]. Charge radius considerations, on the other hand, might have failed because of currents assumed to be local. A multilocal construction as introduced in section (5.4.2) might solve this problem.
In this respect our formalism is also capable of complementing QCD lattice simulations.

As can be seen there exists a wide range of possible applications of our method. Having in mind a new perspective on spacetime with geometry emerging from graviton condensation on flat spacetime we constructed our framework especially with regard to black holes. After all, these objects of gravity are surrounded by many unresolved problems we intended to attack. To realize our concept we helped ourselves with auxiliary currents common in QCD.

Having the same line of reasoning in mind, there exists yet another approach which uses coherent states [194]. Though also applicable to black holes, we applied this alternative approach directly to another spacetime, namely Anti-de Sitter spacetime.

As pointed out in chapter (3) this spacetime is of particular interest regarding AdS/CFT duality. Since the main focus of this thesis is placed on black holes to give a guideline for a constituent description of spacetime we will introduce this alternative, so-called corpuscular description, in the appendix (see (B)).

Conclusions and Outlook

Ever since its existence General Relativity has emanated an irrevocable fascination. The possibility of black holes illustrates this point impressively. With regard to this fact we have shown extensively that one is confronted with many inconsistencies when taking the premise that spacetime curvature is of fundamental nature.

From a purely classical point of view the existence of spacetime singularities, as present in black holes, is an intriguing feature. First regarded as an artifact of (idealized) highly symmetric systems, Hawking and Penrose once and for all demonstrated that a singular behavior can occur under very general assumptions. Their seminal theorems prove that the existence of trapped surfaces and the avoidance of closed timelike curves inevitably lead to geodesic incompleteness, which can be used as an indicator of singular behavior. Though unassailable mathematically, we have stressed that the physical relevance of these theorems is limited. In the end this is due to the fact that the probes used are (classical) objects in the test-particle limit. Taking finite masses and spatial extents into account, however, geodesic motion is no longer guaranteed. After all, we demonstrate that a realistic detector will break down far before the Planck distance where GR, from an effective field theory point of view, becomes strongly coupled anyway.

These considerations have prompted us to perform a semiclassical investigation of spacetime singularities. Using quantum test particles and fields for static and dynamical spactimes, respectively, we reviewed quantum theoretical criteria corresponding to geodesic incompleteness. In particular we transferred the preexisting criterion for a unitary time evolution of quantum mechanical test particles to quantum fields in static spacetimes. Having this, we explained that a singularity analysis of black holes with respect to quantum field theory requires the previous formalism to be modified for dynamical spacetimes. We reviewed the further development of our ansatz which has meanwhile shown that the cosmological relevant Friedmann-Robertson-Walker and the Schwarzschild spacetime indeed behave regularly when tested by quantum fields. These results have deprived the singularity theorems their physical foundation once more.

Albeit downsizing the physical impact of the singularity theorems, however, we had to admit that semiclassical considerations concerning black holes give rise to new puzzles. Within this approach neither the origin of their entropy can be explained nor their untypical negative heat capacity. Let alone the famous information paradox which caused an excess of solution proposals. Most of them, operating in semiclassical regimes themselves, however, are ensuing new problems.

Therefore, the insight that spacetime geometry might not be fundamental, but rather a result of bound state formation of fundamental particles on flat spacetime,

offers a totally new approach to these puzzles. Budding ideas in such a direction were first pursued by Dvali and Gomez illustrating the Quantum-N portrait where black holes are viewed as (leaky) bound states of gravitons at the verge of a quantum phase transition. Contrary to semiclassical considerations, where the metric remains classical and the only quantum corrections are of exponential form, this picture discloses $1/N$ corrections due to the microscopic resolution of the background. It is these corrections that tip the scale such that the above mentioned semiclassical mysteries are arising in the first place.
To capture the basic ideas of this proposal we developed a toy model based on Bose-Einstein condensates. Contrary to other toy models in this area we have used bosons that are derivatively coupled. The intention behind was to mimick properties of gravity more precisely. A Bogoliubov analysis indicated that our model indeed stays at the critical point of a phase transition. This initiated a subsequent numerical analysis which, unfortunately however, was not capable to confirm our expectations. After all, representing a non-relativistic theory, we could not hope to model many gravitational effects that are comprised by the N portrait. To mention here, for example, is the process of depletion which is assumed to correspond to Hawking radiation.

We were thus led to establish a description of quantum bound states that is able to substantiate the N-portrait idea in more detail within a quantum field theoretical framework. Reminding ourselves that especially in QCD substantial developments have been made to describe hadronic bound states we aimed at bringing influences from QCD and GR together in order to describe gravitational bound states. As it turned out, many techniques applied in QCD, like the auxiliary current description or the operator product expansion, were also beneficial for our purposes. On the other hand, the absence of confinement and asymptotic freedom in gravity prevented us from taking over sum rules and quark-hadron duality. Our proposal for generating bound states resembling black holes is therefore based on the existence of individually weakly coupled constituents subject to a strong collective potential. This is achieved by assuming the vacuum to support condensation such that the constituents are immersed in a non-trivial medium. Assuming furthermore that black holes are large-N objects a collective effect ensures the binding of the constituents. In this respect, we can consider black holes in a relativistic Hartree-like framework. In particular, we constructed auxiliary currents composed of the fundamental constituent fields in such a way that a black hole quantum state was created that is compatible with the real black hole state as far as isometries and quantum numbers are concerned. This implies that the currents must be invariant under the isometry group corresponding to the Schwarzschild solution[6]. Of course, there exists a plethora of possible states having a non-zero overlap with the true one. Here, the label N becomes an important indicator. However, as long as we cannot resort to experimental input the overlap has to be parametrized.
Before coming to the main result of this thesis we first reviewed how our construction can be used to calculate a general formula of the constituent distribution as well as the number density and energy density of black hole constituents at tree level.
Subsequently, we probed the constituent structure of black hole interiors by means of virtual graviton absorption. Analogous to the scattering off hadrons in QCD the cor-

[6]Given that the black hole is neither charged nor rotating.

responding amplitude and cross section, respectively, separate into perturbative and non-perturbative contributions. While the perturbative part could be calculated in a straight forward manner we had to employ an operator product expansion for the non-perturbative part to extract physical information. Being not able to exploit the QCD feature of asymptotic freedom further calculations were performed under the assumption of large black hole masses and momentum transfers smaller than the Planck scale. This way we were able to show that the cross section for the considered scattering process is proportional to the black hole constituent distribution. Hence, within our framework observables tied to the interior of black holes can be captured by outside observers. Although the constituent distribution cannot be determined from first principles, we showed that black holes are dominated by long-wavelength gravitons if we assume the black hole wave function to be Gaussian and peaked around its mass with a standard deviation set by its extension, i.e. the Schwarzschild radius. In that sense our results are not only intriguing theoretically. They also offer an explicit possibility to check experimentally whether our perceptions on black holes are justified. Unfortunately, at the technical level of today such experiments are not feasible yet.
Besides, our calculations were in accordance with the expectations for such a treatment of black holes as put forward in the N portrait. Here, we have to mention that all considerations were made in the double scaling limit of sending the black hole mass and constituent number to infinity while keeping there ratio fixed. Violations to this limit are indeed given by $1/N$ and are not exponentially suppressed as in a semiclassical treatment.
Taking finite masses and constituent numbers into account explicitly was left for future research. However, we mentioned that recently our colleagues have not only succeeded in reproducing the Schwarzschild geometry from the introduced microscopic description but also in demonstrating $1/N$ corrections to the emergent geometry. Apart from investigating such corrections we pointed out that further considerations beyond tree level will give rise to a richer condensate structure.
Clearly, being constructed in a universal manner our formalism can and should be applied to many other issues concerning black holes. Following this spirit we have illustrated ways how the formation of black holes or their Hawking radiation might be calculated explicitly.
These objectives, however, do not yet exhaust our formalism. Indeed other spacetimes, like de Sitter or Anti-de Sitter might equally be described. In particular, this could shed light on the cosmological constant problem.
Besides, our method might be an important complement to QCD methods, especially with regard to multilocal currents.

Making this journey from the classical theory of General Relativity over semiclassical considerations to a picture where quantum constituents form spacetime in an emergent way we were able to demonstrate how oversimplifications of an underlying theory can lead to seemingly unresolvable paradoxes - black holes representing a prime example.

The idea itself that spacetime geometry is an emergent concept can be realized in various ways. In this thesis, we developed a way using auxiliary currents.
Another possibility, however, is given by coherent states. We refer the interested reader to the appendix for this alternative approach (see appendix (B)). At this point we just

want to mention that an application of this method to Anti-de Sitter spacetime shows how so-called corpuscular corrections arise and that these corrections obey the same scaling as those found in the auxiliary current picture.
In fact, both the auxiliary current method and the coherent state approach might be used interchangeably. After all, they are both based on a Fock state expansion.

Appendix A

Driving a detector towards its limits

Let us consider the motion of a point particle with mass m in the interior of a Schwarzschild black hole towards the singularity. The action for such a particle is given by

$$S = -m \int_\gamma \mathrm{d}s = -m \int \mathrm{d}^4x \int_\gamma \mathrm{d}\tau \delta^4(x - z(\tau)) \sqrt{-g_{\mu\nu}(z) \dot{z}^\mu \dot{z}^\nu} \tag{A.1}$$

where $g_{\mu\nu}$ represents the Schwarzschild metric and $\dot{z}^\mu = \frac{\mathrm{d}z^\mu}{\mathrm{d}\tau}$ is the tangent to the particle's world line $\gamma$with τ being an arbitrary evolution parameter[1]. Variation of S gives

$$\delta S = -\frac{1}{2} m \int \mathrm{d}^4x \int_\gamma \mathrm{d}\tau \delta^4(x - z(\tau)) \frac{\dot{z}^\mu \dot{z}^\nu}{\sqrt{-g_{\alpha\beta}(z) \dot{z}^\alpha \dot{z}^\beta}} \delta g_{\mu\nu} \tag{A.2}$$

such that the energy momentum tensor, defined as $T_{\mu\nu} := -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$, can then be read off to be

$$T_{\mu\nu} = m \int_\gamma \mathrm{d}\tau \delta^2(x - z(\tau)) \frac{\dot{z}_\mu \dot{z}_\nu}{\sqrt{-g_{\alpha\beta}(z) \dot{z}^\alpha \dot{z}^\beta}}. \tag{A.3}$$

Note that we took the particle to fall radially towards the singularity here such that $\sqrt{-g} = 1$ (see (1.1)). In particular, we are interested in the energy density an observer of velocity $\dot{z}^\mu$ measures as the particle approaches the singularity. In order to evaluate $T_{\mu\nu} \dot{z}^\mu \dot{z}^\nu$ we first need to know the particle's geodesic described by the geodesic equation

$$\frac{\mathrm{d}^2 z^i}{\mathrm{d}\lambda^2} + \Gamma^i_{jk} \frac{\mathrm{d}z^j}{\mathrm{d}\lambda} \frac{\mathrm{d}z^k}{\mathrm{d}\lambda} \tag{A.4}$$

where we introduced the affine parameter λ. For the Schwarzschild case it is described by the differential equations

$$\frac{\mathrm{d}^2 t}{\mathrm{d}\lambda^2} + \frac{r_s}{r(r - r_s)} \frac{\mathrm{d}t}{\mathrm{d}\lambda} \frac{\mathrm{d}r}{\mathrm{d}\lambda} = 0 \tag{A.5}$$

[1]Note that we neglect back reaction here.

and

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\lambda^2} + \frac{r_s\,(r-r_s)}{2r^3}\left(\frac{\mathrm{d}t}{\mathrm{d}\lambda}\right)^2 - \frac{r_s}{2r\,(r-r_s)}\left(\frac{\mathrm{d}r}{\mathrm{d}\lambda}\right)^2 \;=\; 0. \tag{A.6}$$

For a particle freely falling in the interior region of the black hole we furthermore have

$$1 \;=\; -\left(1-\frac{r_s}{r}\right)\left(\frac{\mathrm{d}t}{\mathrm{d}\tau}\right)^2 + \frac{1}{\left(1-\frac{r_s}{r}\right)}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2. \tag{A.7}$$

Setting $\lambda = \tau$ equation (A.5) can be solved by $\frac{\mathrm{d}t}{\mathrm{d}\tau} = k\frac{r}{r-r_s}$ with k being a constant of integration. Plugged into (A.7), $\frac{\mathrm{d}r}{\mathrm{d}\tau} = \sqrt{\frac{r-r_s}{r} + k^2}$. The coordinate invariant energy density of the particle is then measured to be

$$T_{\mu\nu}u^\mu u^\nu \;=\; m\int \mathrm{d}\tau\,\delta(t-t(\tau))\left(\frac{\mathrm{d}t}{\mathrm{d}\tau}\right)^4 + m\int \mathrm{d}\tau\,\delta(r-r(\tau))\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^4 \tag{A.8}$$

$$\overset{!}{\leq}\; M_p \tag{A.9}$$

where we made use of (A.7). Demanding that this energy density be smaller than the Planck mass a minimum radius r_{min} can be determined until which the infalling particle can be described safely within the GR framework. Below this boundary, i.e. in a sphere smaller than r_{min}, the detector theory breaks down. As we can infer from (A.8) the energy density is proportional to the mass of the probe and its velocity. The parameters can now be adjusted such that there is a breakdown of the detector theory before reaching the Planck distance. Here, of course, we have to mind that the particle's velocity must stay below the speed of light.

Appendix B

Introducing an alternative: A corpuscular description of Anti-de Sitter spacetime

As already mentioned in the introduction of this thesis there exists an alternative proposal for the constituent description of spacetime. Instead of introducing auxiliary currents that source the generation of an effective spacetime geometry classical solutions can also be viewed as the large N-limit of an underlying coherent state description [194].

Coherent states, or Glauber states, $|\alpha\rangle$ are defined as the eigenstates of the annihilation operator a with eigenvalue $\alpha \in \mathbb{C}$,

$$a|\alpha\rangle \;=\; \alpha|\alpha\rangle. \tag{B.1}$$

In order to study the particle aspect of coherent states we will consider them now in Fock space. Inserting the completeness relation of the occupation number states $|n\rangle$,

$$|a\rangle \;=\; \sum_n |n\rangle\langle n|a\rangle. \tag{B.2}$$

Using $a|n\rangle = \sqrt{n}|n-1\rangle$ and iteration we get

$$|\alpha\rangle \;=\; \langle 0|\alpha\rangle \sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{B.3}$$

i.e. we can express the coherent state in terms of an exact expansion in Fock space. The remaining overlap matrix element $\langle 0|\alpha\rangle$ can be computed by means of normalization and is given by

$$\langle 0|\alpha\rangle \;=\; \exp\left(-\frac{1}{2}|\alpha|^2\right). \tag{B.4}$$

Clearly, the average constituent number in a coherent state is given by

$$N = \langle\alpha|n|\alpha\rangle = \langle\alpha|a^\dagger a|\alpha\rangle = |\alpha|^2 \tag{B.5}$$

and therefore

$$|\alpha\rangle \quad = \quad e^{-\frac{N}{2}}\sum_n \frac{N^{n/2}}{\sqrt{n!}}|n\rangle. \tag{B.6}$$

Generalizing this expression to the QFT case, i.e. taking different occupation numbers for different modes into account, we have

$$\prod_k \otimes\, |\alpha_k\rangle. \tag{B.7}$$

Note that coherent states are said to be the most classical states as they exactly fulfill the minimum uncertainty principle ($\triangle x \triangle p = \hbar/2$). However, we want to point out that this is no longer the case if higher-point correlators are considered. We will investigate the so called corpuscular corrections in such cases in the following.

Having discussed black holes and partly de Sitter spacetime in terms of auxiliary currents we will now discuss Anti-de Sitter spacetime by means of coherent states. In particular, we are interested in the scalar propagator in AdS.

Before moving on, however, we will first present the characteristics of AdS spacetime. Apart from being a maximally symmetric spacetime of constant negative curvature this spacetime prominently occurs in string theory and brane models of cosmology [4, 195]. In recent years, its importance has been further increased through the development of holography and the AdS/CFT correspondence which relates physics of gravitationally interacting particles in the bulk to a conformal field theory (CFT) on the boundary [56, 134, 196].

B.1 AdS - an overview

Anti-de Sitter spacetime is a Lorentzian manifold with constant negative scalar curvature. It is maximally symmetric, i.e. it has the maximum number of possible Killing vectors $n(n+1)/2$, where n stands for the number of dimensions. To get a rough idea about the geometry it is worth mentioning that this spacetime can be understood as the Lorentzian analogue of hyperbolic space, just like Minkowski spacetime can be viewed as the analogue of Euclidean space or de Sitter spacetime as the analogue of elliptical space. Hence, the metric of $n+1$-dimensional AdS spacetime,

$$\mathrm{d}s^2 = -\mathrm{d}{x_0}^2 + \sum_{i=1}^{n-1} \mathrm{d}{x_i}^2 - \mathrm{d}{x_n}^2, \tag{B.8}$$

can be obtained from the embedding of an hyperboloid,

$$-{x_0}^2 + \sum_{i=1}^{n-1} {x_i}^2 - {x_n}^2 = -R^2, \tag{B.9}$$

with curvature radius R into $(n+2)$-dimensional flat spacetime[1]. The constraint (B.9) can be solved if

$$x_0 = R\cosh\rho\cos\tau \qquad x_i = R\sinh\rho\,\Omega_i \qquad x_n = R\cosh\rho\sin\tau \tag{B.10}$$

[1]Notice, that R is connected to the cosmological constant Λ via $\Lambda = -(n-1)(n-2)/(2R^2)$.

where $\rho \in \mathbb{R}^+$, $\tau \in [0, 2\pi]$ and Ω_i with $i = 1, ..., n-2$ are angular coordinates subject to the condition $\sum_i \Omega_i^2 = 1$. These coordinates (ρ, τ, Ω_i) take all points of the hypersurface (B.9) into account exactly once and are therefore called global coordinates. The following figure depicts the case of AdS_2.

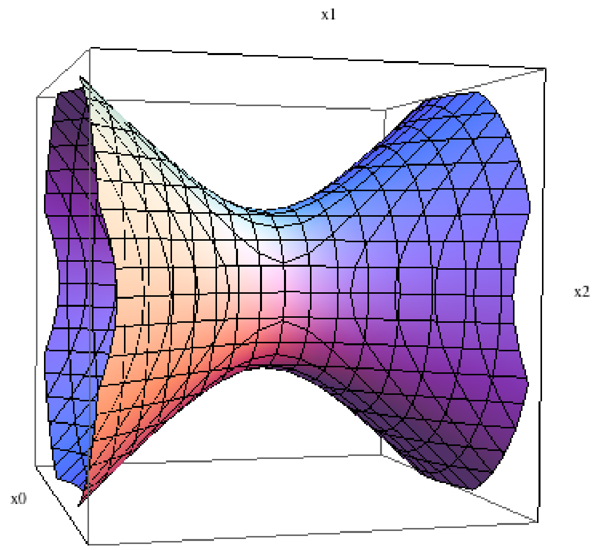

Figure B.1: Anti-de Sitter spacetime in $(1+1)$ dimensions embedded in $(1+2)$ dimensional space.

Here, the circles and hyperbolas are lines of constant ρ and τ, respectively. As τ is periodic in 2π the embedded surface contains closed timelike curves circling the x_1 axis. However, this inconsistency can be avoided by taking the universal cover, i.e. by unwrapping the embedding. If we now introduce a new coordinate θ by $\tan\theta = \sinh\rho$ the metric (B.8) becomes

$$\mathrm{d}s^2 \quad = \quad \frac{R^2}{\cos^2\theta}\left[-\mathrm{d}\tau^2 + \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\Omega_{n-2}^2\right]. \tag{B.11}$$

Here, $0 \leq \theta \leq \pi/2$ and the metric only covers half the spacetime. The hypersurface $\theta = \pi/2$ displays the AdS boundary which corresponds to spatial infinity. Let us now introduce the Poincaré coordinate system by defining the light cone coordinates

$$U \equiv \frac{x_0 - x_{n-1}}{R^2} \qquad V \equiv \frac{x_0 + x_{n-1}}{R^2} \tag{B.12}$$

and redefining $x_i \to \frac{x_i}{RU}$ and $x_0 \to \frac{x_0}{RU}$. Introducing $z \equiv 1/U$ the metric (B.8) becomes

$$\mathrm{d}s^2 = \frac{R^2}{z^2}(\mathrm{d}x^\mu \mathrm{d}x_\mu + \mathrm{d}z^2) \quad \mu, \nu = 0, ..., n-1. \tag{B.13}$$

Note that $z > 0$, i.e. Poincaré coordinates only cover half the spacetime. The region $z = 0$ corresponds to $\theta = \pi/2$ in global coordinates and therefore does not belong to AdS space, but is part of its boundary. For $z \to \infty$ a particle horizon is approached

[197]. (B.13) makes it also evident, that AdS is conformally flat and indeed reproduces flat spacetime if the limit of zero curvature is taken, i.e. for $R \to \infty$.

For a deeper inside into Anti-de Sitter spacetime we refer the interested reader to [198] and [199]. [199] is especially interesting with respect to the AdS/CFT correspondence.

We will use the Poincaré patch for the discussion of the scalar propagator in AdS spacetime in the following.

B.2 Scalar 2-point function in the corpuscular description of AdS

In order to see how corpuscular effects correct observables in Anti de-Sitter spacetime or how they modify it, we will study the 2-point function of a scalar field ϕ of mass[2] m.

Before addressing the full non-linear propagator let us for simplicity first compute the scattering of a scalar on AdS spacetime linearized around a flat background.

B.2.1 Scattering on linearized AdS

In the static patch of AdS linearized around $(d+1)$-dimensional Minkowski spacetime we have

$$h_{00} = -\frac{\Lambda}{6} r^2, \; h_{0i} = 0, \; h_{ij} = -\frac{\Lambda}{6} x_i x_j \tag{B.14}$$

with Λ denoting the cosmological constant and $r^2 = x_i x_i$ [11]. Notice that this approximation is only valid for $r^2 \ll R^2$, R standing for the curvature radius of AdS spacetime. In order to get a corpuscular interpretation of this solution, we expand it in Fourier space,

$$h_{\mu\nu}(z) = \frac{L^d}{M_p^{(d-1)/2}} \int \frac{\mathrm{d}^d k}{\sqrt{(2\pi)^d 2\omega_k L^d}} (e^{\mathrm{i}kx} a_k \epsilon_{\mu\nu}(k) + \text{h.c.}). \tag{B.15}$$

Here M_p denotes the $(d+1)$-dimensional Planck mass, L^d a regulating spatial volume, $\epsilon_{\mu\nu}(k)$ the polarization tensor and ω_k the dispersion relation. At the classical level, a_k is simply a Fourier coefficient and $\epsilon_{\mu\nu}$ the polarization tensor. Quantum mechanically, however, the metric itself, and thus also its expansion coefficients, should become operators. This implies $[a_k, a_q^\dagger] = L^{-d}\delta^{(d)}(k-q)$ for the annihilation and creation operators. Note that these operators are not the creation and annihilation operators belonging to asymptotic states, but rather to the (interacting) corpuscular quanta considered. For a thorough discussion see [200]. Following the last section we have for AdS described by a coherent state of corpuscles

$$|AdS\rangle = \prod_k |N_k\rangle = \prod_k e^{-\frac{N_k}{2}} \sum_{n_k} \frac{N_k^{\frac{n_k}{2}}}{\sqrt{n_k!}} |n_k\rangle, \tag{B.16}$$

[2]A generalization to massless higher bosonic spins is straightforward. Choosing holographic gauge, i.e. $A_z = 0$, it is always possible to write the equation of motion for massless higher spin fields propagating in AdS in the same form as the scalar field equation. Thus our considerations will apply to arbitrary massless bosonic higher spin fields in AdS space-time.

where $|n_k\rangle$ is an eigenstate of the number operator $a_k^\dagger a_k$. Therefore,

$$a_k|AdS\rangle \;=\; \sqrt{N_k}|AdS\rangle. \tag{B.17}$$

This implies that $\sqrt{N_k}$ can be identified with the expectation value $\langle N_k|a_k|N_k\rangle$ which is equivalent to the classical Fourier coefficient. Thus, N_k can be interpreted as the mean number of gravitons in the AdS coherent state with momentum k.
Actually, a coherent state representing a quantum resolution of the above metric can be thought of in two equivalent ways:
One way is in terms of longitudinal, off-shell, massless gravitons of Einstein's theory with a cosmological constant expanded on Minkowski spacetime.
Alternatively, one could deform Einstein gravity by adding a Fierz-Pauli mass term. Then the deformed solution reduces to the metric above in the limit[3] $m^2r^2 \ll 1$ where m is the mass deformation parameter [11, 194]. This theory now propagates five on-shell degrees of freedom. Thus, a second way to think about the coherent state is in terms of massive, on-shell gravitons. Since the metric is static globally, these gravitons should have zero frequency. From the dispersion relation of massive gravitons we can then conclude that the gravitons in the coherent state can be interpreted as on-shell tachyons.

In order to get an intuition how corpuscular corrections arise, let us now consider the scattering of a massive probe scalar on AdS to leading order. This process will serve as a motivation for the analogous full non-linear computation of the scalar propagator in the corpuscular description of AdS which we will discuss next.

The interaction Lagrangian for this process is given by

$$\mathcal{L} = \frac{1}{M_p} h_{\mu\nu} T^{\mu\nu}(\phi), \tag{B.18}$$

where $T_{\mu\nu}(\phi)$ is the standard linearized energy momentum tensor of a massive scalar. To leading order the amplitude takes the form

$$\begin{aligned}\mathcal{A}(q,p) \quad &= \mathrm{i}\langle AdS'| \otimes \langle 0|\mathrm{T} b_q S_{int} b_p^\dagger|0\rangle \otimes |AdS\rangle \\ &= \frac{\mathrm{i}}{M_p}\int \mathrm{d}^{(d+1)}x \langle AdS'|h_{\mu\nu}|AdS\rangle\langle 0|\mathrm{T} b_q T^{\mu\nu}(\phi) b_p^\dagger|0\rangle\end{aligned} \tag{B.19}$$

with b_q and $b_p^\dagger$ denoting the annihilation and creation operators of the scalar field, respectively. In accordance to calculations of the last chapters corrections to the classical result are encoded in the correlator $\langle AdS'|h_{\mu\nu}|AdS\rangle$. In particular, setting $|AdS'\rangle = |AdS\rangle$ the classical result would be recovered by construction. Of course, within a corpuscular treatment this cannot be the case as back reactions due to interactions with corpuscles in the coherent state have to be taken into account. In other words, the scattered AdS state $|AdS'\rangle$, can no longer be identified with the original state $|AdS\rangle$. In general, it is not even clear that the scattered state can still be modeled as a coherent state. Taking the back reaction to be small, however, it seems to be a good approximation to describe $|AdS\rangle$ still as a coherent superposition, but with the occupations in different

[3]Notice that the authors of [11] show that this is the case for dS spacetime. A similar result can be obtained for AdS.

modes changed and disturbed dispersion relations. Under these assumptions the matrix element takes the form

$$\langle AdS'|AdS\rangle = \frac{L^d}{M_p^{(d-1)/2}}\int\frac{\mathrm{d}^d k}{\sqrt{(2\pi)^d 2\omega_k L^d}}\Big(e^{\mathrm{i}kx}\sqrt{N_k}\epsilon_{\mu\nu}(k) + e^{-\mathrm{i}kx}\sqrt{N_k'}\epsilon^*_{\mu\nu}(k)\Big). \tag{B.20}$$

This form makes the origin of the corrections very transparent. We see that the source of the corpuscular corrections is encoded in the difference between N_k' and N_k. Thus, first of all the integral in (B.20) will no longer give the classical field. Secondly, the overlap between $|AdS\rangle$ and $|AdS'\rangle$ is not exactly one. Although we will not compute the explicit form of the correction, let us be a bit more precise. Consider first the deviation of N_k' from N_k. Assuming that the back reaction is small, we can parametrize $N_k' = (N+\delta)_k$ where $|\delta_k| \ll N_k$. Then we can expand $\sqrt{N_k'} \simeq \sqrt{N_k}(1+\delta_k/(2N_k)+...)$. Furthermore, the overlap between the coherent states is given by

$$\langle AdS'|AdS\rangle = \exp\bigg\{-\frac{1}{2}\int \mathrm{d}^d k\Big(N_k + N_k' - 2\sqrt{N_k N_k'}\Big)\bigg\} \simeq 1 - \frac{1}{2}\int \mathrm{d}^d k \frac{\delta_k^2}{4N_k}. \tag{B.21}$$

On the one hand, these expressions are consistent with our earlier remark that to leading order we recover the classical result.
On the other hand, corrections naturally appear as powers of $1/N$. Notice that these effects are entirely of corpuscular nature. In particular, the physics of such corrections can never be uncovered in a semiclassical treatment where instead corrections are expected to be exponentially suppressed.

Being motivated by these results let us now consider the full non-linear propagator on an AdS background.

B.2.2 Scalar propagator in AdS

We are now prepared to discuss the full non-linear propagator of a massive scalar field in AdS spacetime. For the calculations to come we will use the Poincaré patch introduced above. Since the (classical[4]) AdS metric is conformally flat, i.e. $-g^c_{tt} = g^c_{ii} = g^c_{zz}$, we will express all components in terms of g^c_{zz}. In analogy to the linear case the classical AdS solution to Einstein's field equations should be understood as expectation value of a quantum operator acting on $|AdS\rangle$. Following this logic we expand the metric in terms of plane waves,

$$g^c_{zz} = \sqrt{L}\int\frac{\mathrm{d}k_z}{\sqrt{(2\pi)2\omega_{k_z}}}(b_{k_z}e^{ik_z z} + \text{h.c.}), \tag{B.22}$$

with the classical Fourier coefficients

$$b_{k_z} = -\sqrt{\pi}\sqrt{\frac{\omega_{k_z}}{L}}R^2|k_z|. \tag{B.23}$$

Note that due to conformal flatness it suffices to consider the z-component only. Let us furthermore stress, that the dispersion relation ω_k is not that of free particles. As

[4]Denoted by the index "c" in the following.

demonstrated in [201] it is independent of k. Promoting (B.22) to an operator equation and using (B.16) to describe the background geometry a proper classical limit again leads to the interpretation of b_k and $b_k^\dagger$ as annihilation and creation operators of corpuscles in the AdS state with commutation relation similar to the a_k and $a_k^\dagger$ introduced in the last section. In particular we have $b_k^\dagger b_k|n_k\rangle = n_k|n_k\rangle$ and $b_k|N_k\rangle = \sqrt{N_k}|N_k\rangle$. Notice that since $\sqrt{N_k}$ corresponds to the classical Fourier coefficients in (B.22) we see that the occupation of corpuscles becomes large at high momenta.

Let us now consider the classical equation for the propagator G_c of a massive scalar in an AdS background, namely,

$$\left\{\frac{1}{\sqrt{-g}}\partial_A(\sqrt{-g}g^{AB}\partial_B) + m^2\right\}G_c(X,Y) = \frac{1}{\sqrt{-g}}\delta^{(d+1)}(X-Y), \tag{B.24}$$

with g denoting the determinant of the metric, $X = \{x_\mu, z\}$ and A, B are $(d+1)$-dimensional indices. Restricting to the case $d = 3$ (B.24) becomes in Poincaré coordinates

$$\mathcal{O}_c(X)G_c(X,Y) = \delta^{(4)}(X,Y), \tag{B.25}$$

where we defined $\mathcal{O}_c(X) \equiv -g_{zz}^3\Box_M - 2g_{zz}^2(\partial^z g_{zz})\partial^z + g_{zz}^2 m^2$ and $\Box_M = \partial^\mu\partial_\mu + (\partial^z)^2$.

Within the corpuscular theory (B.25) must be replaced by an operator statement evaluated in the state (B.16). In particular, using the expansion (B.22) we see that we need to evaluate the action of powers of the creation and annihilation operators on the coherent state. To see qualitatively how corpuscular corrections arise within this approach, however, let us for simplicity first consider

$$\begin{aligned} g_{zz}|AdS\rangle &= \sum_{\{n_k\}}\sqrt{L}\int\frac{\mathrm{d}k_z}{\sqrt{(2\pi)^2 2\omega_{k_z}}}\left(b_{k_z}e^{\mathrm{i}k_z z} + b_{k_z}^\dagger e^{-\mathrm{i}k_z z}\right)\prod_k e^{-\frac{N_k}{2}}\frac{N_k^{\frac{n_k}{2}}}{\sqrt{n_k!}}|\{n_k\}\rangle \\ &= \sqrt{L}\int\frac{\mathrm{d}k_z}{\sqrt{(2\pi)^2 2\omega_{k_z}}}\sqrt{N_k}e^{\mathrm{i}k_z z}|AdS\rangle \\ &\quad + \sum_{\{n_k\}}\sqrt{L}\int\frac{\mathrm{d}k_z}{\sqrt{(2\pi)^2 2\omega_{k_z}}}e^{-\mathrm{i}k_z z}\prod_k e^{-\frac{N_k}{2}}\frac{N_k^{\frac{n_k}{2}}}{\sqrt{n_k!}}\frac{n_k}{\sqrt{N_k}}|\{n_k\}\rangle. \end{aligned} \tag{B.26}$$

We find that there is no way we can shift the summation index such that an eigenstate equation could be formed for the creation operator. We can therefore infer from (B.26) that the origin of corpuscular corrections can be traced back to the fact that $|AdS\rangle$ is not an eigenstate of $b_k^\dagger$. Besides, using for convenience the saddle point approximation $n_k = N_k - 1/2$ as already used in [202] it becomes clear that in the limit $N_k \to \infty$ the classical result is reproduced, i.e. corpuscular corrections vanish.

Concerning the operator $\mathcal{O}_c$ higher powers of the metric occur which simply implies higher order corrections. In fact, we can rearrange the terms as a normal ordered part plus quantum corrections from commutators. Notice that by construction the normal ordered part simply reduces to its classical value. Quantum corrections, however, give departures from (B.25). These quantum effects can then be absorbed in a redefinition of the propagator. In other words, we now demand that the full propagator G_f containing corpuscular corrections should satisfy the following equation

$$\langle AdS|\mathcal{O}_f(X)G_f(X,Y)|AdS\rangle \stackrel{!}{=} \delta^{(4)}(X-Y), \tag{B.27}$$

where $\mathcal{O}_f = \mathcal{O}_c + \mathcal{O}_q$ and $G_f = G_c + G_q$, and $\mathcal{O}_q$ and G_q denote the quantum parts of the equation of motion operator and Green's function, respectively. As it turns out, (B.27) can be solved by iteration, $G_q = \sum_{j=1}^{\infty} G_{q,j}$. Hence,

$$G_f = G_c - G_c\mathcal{O}_qG_c + G_c\mathcal{O}_qG_c\mathcal{O}_qG_c - ... \tag{B.28}$$

where we used $\mathcal{O}_cG_c = 1$ and it is understood that $\mathcal{O}_q$ is evaluated in the AdS state. Notice that (B.28) corresponds to a Dyson series which can be summed up explicitly. Thus, we find

$$G_f = \frac{G_c}{1+\mathcal{O}_qG_c}. \tag{B.29}$$

Let us stress that this derivation is not restricted to AdS spacetime. Rather it applies to all Green's functions for arbitrary backgrounds which are resolved by means of coherent states. Furthermore, the result is consistent with the semiclassical limit, since in that case $\mathcal{O}_q = 0$.
Finally, we can interpret $(\mathcal{O}_c + \mathcal{O}_q)^{-1}$ as propagator which can be derived from a corpuscular effective action $S = S_{\text{classical}} + S_{\text{corpuscular}}$ with $S_{\text{corpuscular}} \sim \int \mathrm{d}^4X\, \phi\mathcal{O}_q\phi$. This action is of true corpuscular origin. In fact, taking backreaction from quantum loops on a classical AdS into account we would still be blind to $S_{\text{corpuscular}}$. We can only uncover this structure in a corpuscular approach.

An explicit evaluation of $\mathcal{O}_q$ for AdS as performed by [203] uncovers that the quantum effects mix classical and quantum contributions. In particular, an expression containing the classical metric is multiplied by an integral coming from commutator terms. As it turns out this integral diverges. According to [203] this divergence, however, can be interpreted as follows: Since we used the classical N_k to perform our computation, we can think of it as a bare distribution characterizing the geometry. Then, taking commutators is equivalent to considering loop effects. As usual, the infinities we encounter when performing such computations should be reabsorbed in a redefinition of the classical parameters. Thus, what we are uncovering could be interpreted as a corpuscular renormalization of the N_k and subsequently of the metric. In other words, on top of the usual wave function renormalization one encounters in perturbation theory, we uncover a novel, corpuscular source that renormalizes the classical field.

To complete the analysis $\mathcal{O}_q$ would have to be applied to the classical Green's function to find G_f. This computation, however, shall not be part of this thesis.

B.3 Further applications

Having discussed the physics of corpuscular corrections to the propagator in AdS spacetime we can now think of further applications of our technique to uncover corpuscular effects.

One very interesting phenomenon of quantum field theory is the Unruh effect which also occurs in AdS spacetime. Concerning flat spacetime the Unruh effect [204, 205] is the prediction that a uniformly accelerated observer will detect thermal radiation while an inertial observer won't. That is, from the viewpoint of an accelerating observer, the vacuum of the inertial observer will look like a many-particle state in thermal equilibrium, i.e. a so called Kubo-Martin-Schwinger (KMS) state [206, 207, 208]. This effect should also be affected by corpuscular corrections. To analyze these we had a look at the Wightman function for Anti-de Sitter space. Being subject to the so-called KMS condition [206, 207, 208], i.e. a statement about Green's functions in thermal equilibrium, the question was whether this condition was still fulfilled in a corpuscular framework. The analysis was completed by Lukas Gruending and Tehseen Rug. They found that a corpuscular resolution of AdS is incompatible with a perfect thermal spectrum of Unruh radiation.

Concerning the coherent state picture of AdS also the before mentioned AdS/CFT correspondence is worth investigation. See [201] for more details.

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