Modeling the Dynamics of Large Conditional Heteroskedastic Covariance Matrices

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Abstract

Many economic and financial time series exhibit time-varying volatility. GARCH models are tools for forecasting and analyzing the dynamics of this volatility. The co-movements in financial markets and financial assets around the globe have recently become the main area of interest of financial econometricians; hence, multivariate GARCH models have been introduced in order to capture these co-movements. A large variety of multivariate GARCH models exists in the financial world, and each of these models has its advantages and limitations. An important goal in constructing multivariate GARCH models is to make them parsimonious enough without compromising their adequacy in real-world applications. Another aspect is to ensure that the conditional covariance matrix is a positive-definite one.

Motivated by the idea that volatility in financial markets is driven by a few latent variables, a new parametrization in multivariate context is proposed in this thesis. The factors in our proposed model are obtained through a recursive use of the singular value decomposition (SVD). This recursion enables us to sequentially extract the volatility clustering from the data set; accordingly, our model is called Sequential Volatility Extraction (SVX model in short). Logarithmically transformed singular values and the components of their corresponding singular vectors were modeled using the ARMA approach. We can say that in terms of basic idea and modeling approach our model resembles a stochastic volatility model.

Empirical analysis and the comparison with the already existing multivariate GARCH models show that our proposed model is parsimonious because it requires lower number of parameters to estimate when compared to the two alternative models (i.e., DCC and GOGARCH). At the same time, the resulting covariance matrices from our model are positive-(semi)-definite. Hence, we can argue that our model fulfills the basic requirements of a multivariate GARCH model. Based on the findings, it can be concluded that SVX model can be applied to financial data of dimensions ranging from low to high.

Zusammenfassung

Zahlreiche ökonomische und Finanz-Zeitreihen weisen zeitlich variierende Volatilitäten auf. GARCH-Modelle sind Werkzeuge zur Analyse und Prognose solcher Volatilitätsdynamiken. In jüngerer Vergangenheit sind die *gemeinsamen* Bewegungen unterschiedlicher Finanzmärkte und -instrumente in das Zentrum des Interesses finanzökonometrischer Forschung gerückt. Zur Modellierung solcher Phänomene wurden diverse multivariate GARCH-Prozesse entwickelt, welche jeweils mit spezifischen Vor- und Nachteilen einhergehen. All diesen Ansätzen ist das zentrale Ziel gemeinsam, einerseits sparsame Parametrisierungen zu finden, die zugleich hinreichend komplex sind, um reale Renditedynamiken adäquat abzubilden. Zudem sollen solche Modelle stets eine positiv definite bedingte Kovarianzmatrix implizieren.

In der vorliegenden Arbeit wird eine neue multivariate GARCH-Parametrisierung vorgeschlagen, welche durch die Idee motiviert ist, dass die Volatilitätsdynamik der Finanzmärkte durch (wenige) latente Variablen oder *Faktoren* angetrieben wird. Die Faktoren in unserem Modell werden durch rekursive Singulärwertzerlegungen (SVD) generiert. Diese Rekursion ermöglicht es, die vorhandenen Volatilitätscluster *sequentiell* aus den Daten zu extrahieren. Das Modell wird daher als *Sequential Volatility Extraction*, kurz *SVX*, bezeichnet. Logarithmisch transformierte Singulärwerte und die Komponenten ihrer zugehörigen Singulärvektoren werden hierbei mit Hilfe eines ARMA-Ansatzes modelliert. Aufgrund der Kernidee und der Modellierungsstrategie ähnelt der Ansatz in mancher Hinsicht dem stochastischen Volatilitätsmodell.

Empirische Analysen und der Vergleich mit existierenden multivariaten GARCH-Modellen zeigen, dass das vorgeschlagene Modell im Vergleich zu bestehenden Ansätzen wie DCC oder GOGARCH sparsam parametrisiert ist. Zugleich impliziert es positiv (semi-) definite bedingte Kovarianzmatrizen, erfüllt also beide zentralen Erfordernisse multivariater GARCH-Modelle. Das SVX-Modell ist daher eine nützliche Alternative zu bestehenden Ansätzen, sowohl für hoch- als auch für niedrigdimensionale Probleme.

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“ Ashmaan, I wish you a good start in life. ”

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Chapter 1

Introduction

1.1 Basic idea and research questions

Applications of time series modeling have extensively grown in the economic and financial world, where volatility, measured in terms of standard deviation, is of vital importance in such fields as asset allocation, portfolio optimization, risk management, etc. Despite the fact that volatility is not directly observable, it is well known that the volatility of economic and financial data often varies overtime. Therefore, the modeling of the time-varying volatility is essential in the analysis of economic and financial time series. Since the introduction of the Autoregressive Conditional Heteroskedasticity (ARCH) model by Engle (1982) and its generalization (GARCH: Generalized Autoregressive Conditional Heteroskedasticity) by Bollerslev (1986), financial econometricians have become more concerned about the volatility modeling. Consequently, the term 'volatility' has gained more attention in the financial world than ever before. Extending the basic idea presented in these models, a variety of GARCH models has been proposed in the literature. Bollerslev et al. (1992), Teräsvirta (2006) and Rossi (2010) have presented brief surveys about univariate GARCH models.

One of the stylized facts of financial time series is that volatility changes over time and these changes tend to appear ín clusters, implying state-dependent time-varying movements. As noticed by Mandelbrot (1963), "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes." Such a phenomenon is called "volatility clustering". While returns themselves are uncorrelated, absolute returns $\mid r_t \mid$ and squared returns (r_t^2) display positive, significant and slow decaying autocorrelations pattern, which is quantitative manifestation of the fact. $Corr\left(r_t^2, r_{t-1}^2\right) > 0$ for t ranging from a few seconds to a several weeks or months. With the primary purpose of modeling the phenomenon of volatility clustering, numerous stochastic models have been developed in finance such as GARCH models, stochastic volatility models, and realized volatility models.

During the last two decades or so, the focus in volatility modeling has partly shifted from univariate to multivariate modeling due to the co-movements in the financial returns and financial markets. Hence, multivariate volatility or covariance matrices have been taken into consideration in order to capture the dynamics of these co-movements. Among other tasks, a financial econometrician has to predict the dependence in the co-movements among the asset returns and also among the financial markets. Recognizing this feature through a multivariate model leads to a more relevant empirical model than working with separate univariate models. In financial applications, the extension from univariate to multivariate models gives the opportunity to make better decision in various areas such as asset pricing, portfolio optimization, risk management, etc. Models describing the dynamics of co-movements in the volatility are categorized in three mutually exclusive classes: 1) Multivariate GARCH (MGARCH) models, 2) Multivariate Stochastic Volatility (MSV) models, and 3) Multivariate Realized Volatility (MRV) models. A brief summary about two of these classes (MGARCH and MSV) is given in the following paragraphs. There is no further information about MRV models in this thesis as these are not relevant in the presented research; however, a brief description of these models can be found in Andersen and Bollerslev (1998), and, Chiriac and Voev (2011) (and the references therein).

Lately, in the last few decades, numerous variants and extensions of univariate GARCH to multivariate GARCH models have been proposed. Among others, Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2008) have presented brief surveys about multivariate GARCH models. According to the survey of Bauwens et al. (2006), multivariate GARCH models are constructed by using three non mutually exclusive approaches: (i) a direct generalization of univariate GARCH model of Bollerslev (1986) such as vectorized (VEC) model proposed by Bollerslev et al. (1988), Baba Engle Kraft and Kroner (BEKK) model of Engle & Kroner (1995), and Factor-GARCH model of Engle et al. (1990) [1]; (ii) a linear combination of univariate GARCH models which includes Orthogonal GARCH (OGARCH) model which was initially proposed by Kariya (1988) and popularized by Alexander and Chibumba (1997), Generalized Orthogonal GARCH (GO-GARCH) model which was introduced by van der Weide. R. (2002) and latent factor model of Diebold and Nerlove (1989); (iii) a nonlinear combination of univariate GARCH models such as Constant Conditional Correlation (CCC) model of Bollerslev (1990), Dynamic Conditional Correlation (DCC) model of Christodoulakis ans Satchell (2002), Engle (2002), Tse and Tsui (2002), and Copula-GARCH models of Patton (2000) and Jondeau & Rockinger (2001). All of these multivariate models impose different restrictions to capture the dynamics of variances, covariances, and correlations. Along with several

[1]It should be noted that the Factor-GARCH of Engle assumes that factors are observable.

good properties, these models also possess the following two major drawbacks: curse of dimensionality and positive (semi) definiteness.

1. Curse of dimensionality

Among others, Ding and Engle (1996) reported that the first difficulty that is ever encountered while modeling the dynamics of large conditional covariance matrix is that the model usually involves a large number of parameters. Engle and Sheppard (2002) reported that the number of estimated parameters increases quadratically $O\left(K^4\right)$ as the dimensions of the data set increases (here K refers to the dimensions of the data set). The (quasi) maximum likelihood method is mostly used to obtain the estimates of these parameters. The interaction among these parameters causes the convergence problem (i.e., optimization algorithm does not converge properly), due to which the model becomes difficult to estimate and is not useful in terms of prediction. To overcome this problem to some extent, the diagonal and the scalar versions for a few of the the previously mentioned models are proposed and are frequently mentioned in the literature as well. However, still the application of diagonal and scalar versions of these models in high dimensional date set leaves us with a large number of parameters. Besides this, these solutions make the model too restrictive in the sense that no interaction is allowed between variances and covariances (the terms evolve independently). The following table gives the idea the number of parameters increases as the dimensionality of the data set increases if any of the previously mentioned MGARCH models is used with the exception of Integrated GARCH (IGARCH) and EWMA (*exponentially weighted moving average*) models because these two models assume that all the variables in the data set follow the same dynamics.

Table 1.1: Summary of MGARCH(1,1) models ($S_{t-1} = \varepsilon_{t-1}\varepsilon_{t-1}^{'}$)

Model	*Functional Form*	*# Parameters*	$K=5$	$K=20$	$K=29$
$VECH-GARCH$	$vech(H_t) = vech(C) + Avech(S_{t-1}) + Bvech(H_{t-1})$	$K(K+1)(K(K+1)+1)/2$	465	88410	378885
$DVECH-GARCH$	$vech(H_t) = vech(C) + Avech(S_{t-1}) + Bvech(H_{t-1})$	$K(K+5)/2$	25	250	493
$BEKK$	$H_t = CC' + AS_{t-1}A' + BH_{t-1}B'$	$(5K^2+K)/2$	65	1010	2117
$Diag\ BEKK$	$H_t = CC' + AS_{t-1}A' + BH_{t-1}B'$	$(K^2+5K)/2$	25	250	493
$Scalar\ BEKK$	$H_t = CC' + \alpha^2 S_{t-1} + \beta^2 H_{t-1}$	$K(K+1)/2+2$	17	212	437
$EWMA$	$H_t = (1-\alpha)/(1-\alpha^{t-1})\,\Sigma_{i=1}^{t-1}\alpha^{i-1}S_{t-i}$	1	1	1	1
$Integrated\ MGARCH$	$H_t = \alpha S_{t-1} + (1-\alpha)H_{t-1}$	1	1	1	1
CCC	$H_t = D_t R D_t,\ D_t = diag\left(h_{1t}^{1/2}, ..., h_{Kt}^{1/2}\right)$	$K(K+5)/2$	25	250	493
DCC	$H_t = D_t R_t D_t,\ D_t = diag\left(h_{1t}^{1/2}, ..., h_{Kt}^{1/2}\right)$	$(K+1)(K+4)/2$	27	252	495
$O-GARCH$	$H_t = P\Lambda_t P',\ \Lambda_t = diag(h_{1t}, ..., h_{Kt})$	$K(K+5)/2^*$	25	250	493
$GO-GARCH$	$H_t = P\Lambda_t P',\ \Lambda_t = diag(h_{1t}, ..., h_{Kt})$	$K(K+5)/2^*$	25	250	493

Source. Ding ans Engle (2001)

* Depending on number of factors to be used (here $m = K$). In all models $\mu_t = 0$.

2. Positive (semi) definiteness

Besides the curse of dimensionality, there is one more complexity: every model must ensure that the estimated conditional covariance matrix is positive definite (PD) or positive semi-definite (PSD). This condition is needed because, by definition, the covariance matrix possesses the positive (non-negative) diagonal elements which are called variances. This could be achieved by deriving the conditions under which the model itself produces positive (semi) definite covariance matrices. Two examples of such models are the VEC model of Bollerslev, Engle and Wooldridge (1988) and the BEKK model (a restricted version of VEC model) of Engle and Kroner (1995). These models are formulated in such a way that positive definiteness is implied by the model structure by adding some simple constraints. Due to these constraints, the model may fail to capture some important features in the data set and becomes inadequate in terms of its applicability to real-world data.

In general, it can be concluded that very few models described in the literature are capable of handling the above mentioned problems appropriately such as Stochastic Volatility[2] (SV hereinafter) and Factor-GARCH models. In GARCH models, the conditional variance of returns is assumed to be a function of past returns, whereas in univariate stochastic volatility model of Taylor (1986) volatility does not depend of past observations but on some unobserved stochastic process. The introduction of the additional error term makes the SV model more adequate in real world applications than the GARCH type models because these models overcome the difficulties encountered with GARCH models. SV models allow the log of volatility to evolve, and it is ensured that the variance of the process always remains positive without a further need for constraints. The logarithm of this unobserved variance is modeled through a linear stochastic process such as an autoregression. The multivariate version of this model was proposed by Harvey et al. (1994). A substantial review regarding specification, estimation, and evaluation of multivariate stochastic volatility models was presented by Asai et al. (2006).

In this thesis, we propose a new parametrization to capture the dynamics of large conditional heteroskedastic covariance matrices using the dimension reduction technique. The applications of dimension reduction technique in finance has been set off from Principal Component Analysis (PCA) [3] which is based on the eigenvalue decomposition of the unconditional correlation matrix of the original data or the unconditional covariance matrix of the standardized data.[4] The aim of PCA is to reduce

[2]Although it is not easy to obtain the exact likelihood function for such stochastic volatility model.

[3]PCA was introduced by Karl Pearson in 1901, while the first Factor-GARCH model was proposed in 1990 by Robert Engle.

[4]Correlation matrix of the original data is equal to the covariance matrix of the standardized data.

the number of variables in a data set and still accounts for as much of the total variation in the data set as possible. Since the number of principal components (also known as factors) are intended to be much smaller than the number of variables in the original data set, it makes the model more feasible in terms of applications even in high dimensional data sets. In general, all factor models proposed in finance are representative of this technique. All of these models assume that the co-movement in volatility is a function of some underlying common variables, called factors, which are unconditionally uncorrelated but conditionally they are correlated and heteroskedastic, possessing GARCH-type structure (Silvennoinen and Teräsvirta, 2006). Some factor models assume that the factors are observable; the first model of this type was introduced by Engle et al. (1990) and was called Factor-GARCH model. However, in other models the factors are assumed to be latent and to follow ARCH processes such as the latent factor ARCH model of Diebold and Nerlove (1989). Since the model of Diebold and Nerlove assumes that factors are latent, their model is rather a multivariate stochastic volatility (MSV) model than a MGRACH one. The factor-GARCH model has a parsimonious[5] parametrization and positive definiteness of conditional covariance matrices are easily guaranteed.

The presence of correlated factors in Factor-GARCH models is undesirable property in the sense that many factors can capture very similar characteristics of the data set. If the factors are uncorrelated, they would definitely present different common components driving the volatility. With this motivation, several factor models have been proposed in the literature, in which the observed data are assumed to be linked to unobserved and conditionally uncorrelated factors through a linear, invertible transformation such as MTV (multivariate time series) model of Kariya (1988), OG-ARCH model of Alexander & Chibumba (1997), and GO-GARCH model of van der Weide (2002). All of these models work through a decomposition of the unconditional covariance matrix of asset returns. In the literature, different decompositions have been described as means of extracting the factors and achieving a more parsimonious model for practical applications. These includes Eigenvalue decomposition (Alexander & Chibumba, 1997), Cholesky decomposition (Tsay, 2002; Kawakatsu, 2003; and Mittnik et al., 2013) and Polar decomposition (Lanne and Saikkonen, 2005), etc. The most important decomposition among them is the Eigenvalue decomposition which is sometimes also known as spectral decomposition.[6]

Our proposed model is based on the idea that volatility in financial markets is driven by a few latent variables called factors, which is basically the same idea

[5]A parsimonious model accomplishes a desired level of explanation or prediction with as few predictor variables as possible.

[6]If one chooses the eigenvectors of $\mathbf{A}$ as an orthonormal basis, the matrix representation of $\mathbf{A}$ in this basis is diagonal. Equivalently, $\mathbf{A}$ can be written as a linear combination of pairwise orthogonal projections. This combination is called its spectral decomposition (Halmos, 1963).

as the one behind the latent factor model of Diebold and Nerlove (1989). Based on this fact, we can say that our methodology is related to SV models. However, the way of obtaining factors is quite different; the factors are extracted by a recursive use of the Singular Value Decomposition (SVD). A detailed explanation of SVD is presented in the next chapter. A fact about SVD worth mentioning is that it leaves us with three matrices, one of which is diagonal having singular values on its main diagonal, while the others are orthogonal and contain corresponding singular vectors in their columns. In each recursion of SVD, we extracted the volatility clustering from the data set. When all volatility clustering was extracted, we modeled the dynamics of the co-volatility matrix. For this purpose, we needed to model the dynamics of singular values and the corresponding singular vectors as well. The dynamics of singular values (which play the role of variances) are modeled using the logarithmic transformation as in stochastic volatility models. Depending on whether the singular vectors show some temporal dependence, we needed to model their dynamics, which could be done by using any of the time series models such as ARIMA, ARFIMA, etc. Singular value decomposition allows the sequential extraction of these volatility factors by preserving the orthogonality and the orthonormality in the factor loadings (called singular vectors). Because of this sequential extraction of the factors, we called the proposed model Sequential Volatility Extraction, in short SVX, model.

In general, several important questions were addressed while developing this new model. In particular, we have to answer the following questions:

- What is the difference between singular value decomposition and eigenvalue decomposition?
- Can we adequately extract the volatility clustering from the data set using our proposed model?
- If so, how to forecast the covariance matrices using our proposed methodology?
- To what extent does our model overcome the two major drawbacks presented earlier regarding the existing MGARCH model?
- What are the similarities and differences between our proposed model and the already existing MGARCH models in finance especially DCC and GO-GARCH models?
- What are the limitations of the proposed model?

1.2 Structure of the thesis

This thesis is structured as follows:

Chapter 2

In this chapter, we introduce a new parametrization approach to model the dynamics of conditional heteroskedastic covariance matrices when dealing with high dimensional data set. This chapter is divided into four sections. Section 2.1 is devoted for the detailed description of the newly proposed methodology. To facilitate the reader, we first list all the steps used in the methodology and then describe them in detail. The arguments presented are supported with definitions and theorems from linear algebra. This section is also accompanied by a brief description of different tests used during the formulation of the methodology such as Ljung-Box, McLeod-Li, and Engle ARCH test. A small-scale simulation study which helps to explain the methodology is also described in this section. In the second section of this chapter, we present a forecasting scheme which is based on the nascent modeling technique outlined in the first section. Despite of the fact that forecasting is not a part of the methodology and usually is treated as a separate topic, we placed it in the methodology chapter because our model deals with a sequential extraction of volatility clustering; for this reason, the resulting matrices are somehow difficult to tackle and ultimately make the forecasting a bit tricky in comparison to the already existing ones. Since we used the block matrices to forecast the covariance matrices, the manipulation of these type of matrices is not as easy task especially when dealing with high dimensional data set. Indeed, a special care is needed to perform the basic mathematical operations, which are used to construct these forecasted covariance matrices such as addition, multiplication, and finding the inverse, etc. In the third section of this chapter, we present a conclusion based on the arguments presented in the previous two sections of this chapter. Different proofs of theorems and some supporting graphics related to the earlier sections are included in this chapter.

Chapter 3

This chapter is devoted for the empirical applications of our proposed methodology. Like Chapter 2, this chapter is also divided into four sections. The first section describes the data set used in the present study and highlights some of the stylized facts of the financial time series such as volatility clustering, lepto-kurtic behavior of return's distribution, etc. The second section deals with the applications of proposed methodology using the data set described in the first section of this chapter. The detection and modeling of the factors driving the volatility in the data set are the main concerns of this section. The results of these two sections are supported by tables and graphs. The major findings of the previous two sections are summarized in the third section. We provide the estimates of such models as ARMA and ARFIMA, which are used to capture the dynamics of the resulting singular values and also the corresponding singular vectors in the last section of this chapter.

Chapter 4

Forecasting of covariance matrices and their comparison with the already existing MGARCH models are the main subjects of this chapter. The first section of this chapter is devoted to the introduction of some ideas regarding the point and interval forecasts and how to calculate these. The second section explains the forecasting scheme in detail and describes how the interval forecasts are constructed using our proposed methodology. The third section explains in detail different tests used in the forecast evaluation. In the fourth section of this chapter, we provide a detailed description of the two most extensively used MGARCH models in finance, i.e., DCC-GARCH and GO-GARCH, especially focusing on the estimation and forecasting when using these models. In the fifth section, we present the forecasting results and evaluate our newly proposed model with the already existing ones such as DCC-GARCH and GO-GARCH by comparing their out-of-sample forecasting performances. Conclusions are presented in the last section.

Chapter 5

This chapter is divided into two sections. The first section summarizes our main results presented in Chapter 3 and 4 with comprehensive conclusions. This section also contains a summary about the contribution of the present thesis to the already existing literature related to the financial world. In the second section, we discuss the possible directions for future research.

Softwares

For computational purposes, two programming languages, MATLAB (MathWorks, 2014b) and R (R Development Core Team, 2014), were used. The estimation and forecasting of ARMA model with Gaussian assumption was done using Kevin Sheppard's MFE Toolbox[7] and Alexandros Gabrielsen's ARMAX-GARCH-K. We used Toolbox[8], which facilitates the estimation and forecasting of ARMA models by using Student's t and its skewed version. The estimation and forecasting of DCC-MGARCH and GO-GARCH models were done in R using 'rmgarch' [9] and 'gogarch' [10] packages written by Alexious Ghalanos and Bernhard Pfaff, respectively.

[7]https://www.kevinsheppard.com/MFE_Toolbox
[8]https://de.mathworks.com/matlabcentral/fileexchange/32882-armax-garch-k-toolbox
[9]https://cran.r-project.org/web/packages/rmgarch/index.html
[10]https://cran.r-project.org/web/packages/gogarch/index.html

Chapter 2

Methodology

In this chapter, we describe a new approach to model the dynamics of large conditional heteroskedastic covariance matrices. Our intention is to extract and model the volatility clustering from the data set while keeping the model adequate for real-world applications. Similarities and differences between eigenvalue and singular value decomposition are also among the main concerns of this chapter since our proposed model is based on the latter. Thereafter, we present a forecasting scheme, which shows how to forecast covariance matrices using our proposed methodology. In this chapter, we try to resolve the first three research questions presented earlier.

2.1 Modeling strategy

We have divided the formulation of our proposed methodology in several steps as we think it is better to first list all these steps before describing them in detail. This helps the reader to get the general idea of the methodology. The steps of the methodology are as follows:

1. Decompose the outer product of the innovations using Singular Value Decomposition (SVD) to obtain factor(s) $\lambda_{i,t}$ and the loadings u_t;

2. Check whether loadings $u_1, u_2, ..., u_T$ are still clustered;

3. If clustering is still present, repeat extraction until there is no clustering in loadings;

4. Model the dynamics of λ_s & u_t;

5. Forecast the covariance matrices.

A detailed description follows below:

Due to the fact that in practice, we do not observe demeaned vectors[1], so we denote $\widetilde{Y}_t = (\widetilde{y}_{1t}, \ldots, \widetilde{y}_{Kt})$ as a vector of observed data set in time period t. Let $Y_t = (y_{1t}, y_{2t}, ..., y_{Kt})^{'}$ refer to the K-dimensional centered random vector of observations in time period t implied by:

$$\widetilde{Y}_t = \mu_t + \varepsilon_t, \quad t = 1, 2, ..., T \tag{2.1.1}$$

$$\varepsilon_t = \widetilde{Y}_t - \mu_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim D\,(0,\, H_t)$$

and

$$H_t = E\left(\varepsilon_t \varepsilon_t^{'} \mid \Omega_{t-1}\right) = U_t \Lambda_t U_t^{'} \tag{2.1.2}$$

where $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Kt})^{'}$ is the $K \times 1$ vector of innovations, Ω_{t-1} refers to the information set up to and including time $t-1$. Furthermore, since we are not using the conventional notations, so in our case innovations could follow any of the multivariate distribution D, where D ={Gaussian, Student's t, and Skewed t distribution} with mean 0, i.e., $E\,(\varepsilon_t \mid \Omega_{t-1}) = 0$ and H_t is a $K \times K$ conditional covariance matrix. Also, let h_{ijt} denote the ij^{th} element in H_t, and y_{it} and ε_{it} are the i^{th} element in y_t and ε_t, respectively. U_t and Λ_t are obtained via singular value decomposition, which is described in detail during the step of our proposed methodology. Autoregressive Moving-Average (ARMA(p,q)) model is used to capture the dynamics of the conditional mean equation μ_t. Thus, the following conditional-mean equation is considered:

$$y_{k,t} = \alpha_{0,k} + \sum_{i=1}^{p} \alpha_{i,k} y_{t-i,k} + \sum_{j=0}^{q} \beta_{j,k} \varepsilon_{t-j,k} + \varepsilon_{k,t}, \quad \varepsilon_{k,t} \sim D\left(0, \sigma^2_{\varepsilon_k}\right) \tag{2.1.3}$$

Several likelihood functions are commonly used to estimate ARMA models depending on the distributional assumptions of innovations such as maximum likelihood, quasi-maximum likelihood, composite maximum likelihood, etc. Most extensively used ones are the Maximum Likelihood Estimation (MLE) or the Quasi-Maximum Likelihood Estimation (QMLE). The problem of maximizing the multivariate likelihood function for higher dimensions most often is convergence. As an alternative, different non-and semi-parametric approaches have been suggested by different researchers (see Linton 2008 and the references therein). These approaches have the advantage of not imposing a particular restrictions on the data set. In this thesis, we limit our consideration to the parametric estimation.

[1]Vector resulted from the subtraction of unconditional mean of the data set from all data points

To account for the linear dependence in time series, we use ARMA model, whereas for nonlinear dependence the (G)ARCH model is required. Therefore, it is necessary to first check which kind of dependence is present in the data set and which model is suitable to account for that. To accomplish this, the following three tests are broadly used in time series analysis.

Ljung-Box test

This test is used to check if the residuals from the estimated ARMA(p,q) model behave like a white noise (Enders, 2010). Recall that for a white noise process the sample autocorrelations are approximately independently and normally distributed with 0 means and variances $1/T$. Hence, a good check of the correlation structure of the innovations is to plot the sample autocorrelations $\hat{\rho}_{\varepsilon}$ versus m (where m shows particular lag) along with the error bounds of $\pm 2/\sqrt{T}$. Instead of testing independence at each distinct lag, Ljung-Box test helps us to check the white noise property collectively using a number of lags and is therefore known as a portmanteau test.[2] According to the literature, this type of test was first proposed by Box and Pierce (1970) to test whether the autocorrelations in a given time series are significantly different from zero or not. However, due to the low power of this test[3], a modified version was introduced by Ljung and Box (1978). Both univariate and multivariate versions of Ljung-Box tests have been described in the literature. In this study, we applied this test in a univariate fashion, which helps us to find out separately which series of residuals exhibits white noise property. The test statistic is formulated as follows::

$$Q_{M,k} = \frac{T(T+2)\sum_{m=1}^{M} \hat{\rho}_{\varepsilon_k}(m)}{T-m},$$

where $(\hat{\rho}_{\varepsilon_k}(m))$ is the estimated sample autocorrelations at lag m and T is the sample size. We reject the joint null hypothesis of no significant autocorrelations $H_0 : \hat{\rho}_{\varepsilon}(1) = \hat{\rho}_{\varepsilon}(2) = ... = \hat{\rho}_{\varepsilon}(m) = 0$ against the alternative that at least one of $\hat{\rho}_{\varepsilon}(1), ...\hat{\rho}_{\varepsilon}(m) \neq 0$, is nonzero at a conventional level of significance ($\alpha =$ 0.05, & 0.01). The test statistic follows Chi-square distribution with $m-p-q$ degrees of freedom, i.e., $Q_m \sim \chi^2_{(m-p-q)}$(where p and q refer to the autoregressive and the moving average lags of the ARMA(p,q) process respectively.[4] Selecting an appropriate lag length is crucial when applying this test as the number of selected lags (m)

[2]Portmanteau tests are residual-based tests that do not have a specific alternative hypothesis

[3]The power of a test is the probability that the test correctly rejects the null hypothesis (H_0) when the alternative hypothesis (H_A) is true. The test statistics performs poorly even in moderately large samples as probability of correctly accepting the alternate hypothesis is low.

[4]Nevertheless, a common practice is to simply report results using m degrees of freedom.

affects the power of the test. If m is too small, the test will not detect high-order autocorrelations. If it is too large, the test will lose power when significant correlation at one lag is washed out by insignificant correlations at other lags. The default value of $m = 20$ has been suggested by Box, Jenkins, and Reinsel (1994), while Tsay (1994) showed simulation evidence that a value approximating $ln(T)$ provides better power performance.[5]

McLeod-Li test[6]

As discussed earlier, while estimating an ARMA model, the autocorrelations function can help us to select the proper values of p and q, and the ACF of the residuals is an important diagnostic tool. Since autocorrelation coefficient measures the degree of linear association and does not account for any non-linearity in the process, ACF my fail to detect important non-linear relationships present in the data set. However, the autocorrelation function (ACF) of the squared process is found to be a useful tool to account for the non linearity in the data set. Based on this idea, McLeod and Li (1983) derived a test to detect the significant autocorrelations in the squared residuals from a linear equation. They demonstrated that for a univariate time series, their approach yields a portmanteau test statistic which is asymptotically distributed as χ^2 with m degrees of freedom if the errors are independent. Their proposed test statistic is as follows:

$$Q_{M,k} = \frac{T(T+2)\sum_{m=1}^{M} \hat{\rho}_{\varepsilon_k^2}(m)}{T-m},$$

Unlike Ljung-Box test, this test is based on the autocorrelations of the squared residuals $\hat{\rho}_{\varepsilon^2}$. It should be noted that the actual form of the non-linearity is not specified by the test, i.e., rejecting the null hypothesis of linearity does not tell you the nature of the non-linearity present in the data set (Enders, 2010).

Engle ARCH test

Lagrange multiplier (LM) tests usually have a higher power than portmanteau tests when the alternative is correct (although they can be asymptotically equivalent in certain cases), but they may have low power against the other alternatives. Engle (1982) proposed a test for conditional heteroskedasticity for univariate GARCH models, while Bollerslev et al. (1988) and Engle and Kroner (1995) developed LM tests for multivariate GARCH models. Among these tests, the one proposed by Engle (1982) is most widely used in financial world and is known as Lagrange Multiplier

[5] In this study, following Box, Jenkins, and Reinsel (1994) we chose $m = 20$. If we use $ln\,(T)$as suggested by Tsay (1994) it could be very small number even for large sample size for example, $ln\,(3000) = 8$, and so the test will fail to detect high-order autocorrelations. For a daily data set, $m = 20$ is treated as sufficient to provide the effects of today's volatility on the volatility after one month.

[6] It is also the exact Lagrange multiplier (LM) test for ARCH errors; however, the test has substantial power to detect various forms of non linearity as compared to Engle ARCH test.

Engle ARCH test (Engle's LM ARCH test). This test assesses the null hypothesis ($H_0 = \alpha_1 = ... = \alpha_m = 0$) that a series of residuals z_t^2 exhibits no conditional heteroskedasticity (ARCH effects). The alternate hypothesis ($H_a = \alpha_i \neq 0,\ i = 1, ..., m$) states that series follows an ARCH(M) model. The test is equivalent to the usual F statistic for testing $\alpha_i = 0\ (i = 1, ..., m)$ in the linear regression

$$z_{k,t}^2 = \alpha_{0,k} + \alpha_{1,k} z_{t-1,k}^2 + \ \ldots \alpha_{m,k} z_{t-m,k}^2 + e_{t,k}, \quad k = 1, 2, ..., K, \quad t = m + 1, ..., T$$

where e_t denotes the error term, m is the prespecified positive integer, and T is the sample size. Engle ARCH test estimates the ARCH(M) model for a specified number of lags m and then computes the Lagrange Multiplier(LM) test statistic ($T \times R^2$), where T is the sample size and R^2 is the coefficient of determination of the regression. Under the null hypothesis, the test statistic is asymptotically χ^2 distributed with m degrees of freedom.

From now on our proposed methodology is divided in several steps. Wherever necessary, the steps are elaborated with illustrative examples. The data used for the illustrative purposes were generated using a simulation study described further.

A three-dimensional small scale ($K = 3$) simulation study was conducted to generate a data set with volatility clustering. The chosen length of simulated data was 3294, which is approximately equivalent to 13 years of daily data. The parameters used to simulate the data were estimated from BEKK-MGARCH model using normality assumption $H_t \sim N(0, I_k)$. For estimation purpose, three components (Johnson and Johnson (JNJ), Pfizer (PFE) and Merck (MRK)) of the Dow 30 were used. The BEKK-MGARCH model has the following form.

$$H_t = CC' + A^{'} \varepsilon_{t-1} \varepsilon_{t-1}^{'} A + B^{'} H_{t-1} B,$$

where A, B, C are $K \times K$ parameter matrices (where K shows the dimensions of the data set), and C is lower triangular. The estimated $\hat{A}$, $\hat{B}$, and $\hat{C}$ are as follows:

$$C = \begin{bmatrix} 0.0007 & 0 & 0 \\ 0.0035 & -0.0000 & 0 \\ 0.0077 & -0.0000 & 0.0000 \end{bmatrix} \qquad A = \begin{bmatrix} 0.3400 & 0.1881 & 0.6015 \\ -0.0415 & -0.1082 & -0.0576 \\ 0.0245 & 0.1713 & 0.5036 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.6800 & 0.3803 & -0.1912 \\ -0.3316 & 0.8331 & -0.0175 \\ 0.0997 & -0.0802 & 0.9035 \end{bmatrix}$$

Figure 2.1 shows the simulated data from BEKK-GARCH (1,1) model. Three simulated series clearly show the volatility clustering (periods of high and low volatility). The summary statistics along with the tests of normality and ARCH effects (described earlier) are reported in Table 2.1. The sample skewness, taken together with the sample kurtosis, indicates a substantial violation of normality (the p-values of the Jarque Bera test statistics also favor this). The Ljung-Box test statistics show that there is no linear dependence in the simulated data. However, McLead-Li and Engle ARCH tests indicate non-linearity in the data set, hence suggesting the use of GARCH model to account for this non-linearity. The autocorrelation functions (ACFs) of the simulated series also show no serial correlation as the model used to simulate data assumes that series has zero autocorrelation, while the squares of the simulated series show high autocorrelation (see Appendix). This is in line with the findings presented in the financial literature (returns are uncorrelated but squared returns are not).

Figure 2.1.1: Combined and individual plots of simulated data

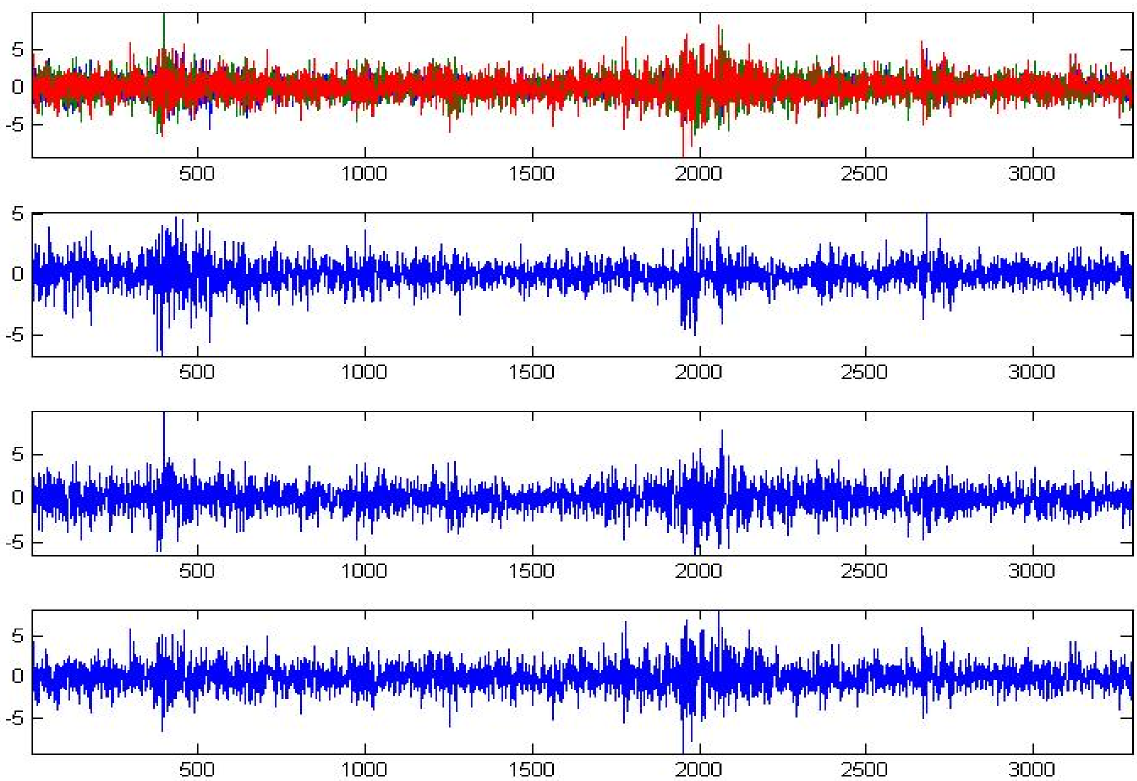


Table 2.1: Summary statistics and testing of $ARCH$ effects in simulated data

	Mean	Variance	Skewness	Kurtosis	Jarque-Bera Test	Ljung-Box (Q_{20})	McLeod-Li(Q_{20})	Engle ARCH (Q_{20})
y_{1t}	0.011	1.158	-0.253	5.70	1,035.90 (0.000)	17.85 (0.597)	1,677.48 (0.000)	519.26 (0.000)
y_{2t}	-0.007	2.219	0.009	4.45	288.30 (0.000)	29.84 (0.073)	1,014.01 (0.000)	362.5 (0.000)
y_{3t}	0.001	2.615	0.143	4.67	393.75 (0.000)	28.17 (0.106)	1,124.54 (0.000)	403.1 (0.000)

Using $\alpha = 0.05$, critical values of Jarque-Bera test is 5.98 while for other three tests this value is 31.41.

2.1.1 Step 1: decompose outer product of innovations

After the white noise property of the innovations is checked and the results are in favor of the null hypothesis, the next step is to decompose the outer product of the innovations. The outer product of the innovations $\varepsilon_t \varepsilon_t^{'}$, where the superscript $'$ shows the transposition of a vector or matrix, is given as follows:

$$S_t^{(1)} = \varepsilon_t \varepsilon_t^{'} \quad t = 1, ..., T.$$

.Since the outer product of two vectors always leaves a rank deficient matrix, S_t suffers from rank deficiency and has at most rank 1. It can be translated as only the first column of S_t is independent while the others are not. To decompose this outer product, we used a Singular Value Decomposition (SVD). It is of worth importance to briefly explain the SVD and highlight some of its best features in order to justify its use in our proposed methodology. In the following, we use some definitions and theorems from linear algebra to support our arguments.

Singular Value Decomposition (Golub and Van Loan, 1989)

Theorem 1. If **A** is a real **m-by-n** matrix then there exist orthogonal matrices $U = [u_1, ..., u_m] \in \mathbb{R}^{\mathbf{m \times m}}$ and $V = [v_1, ..., v_n] \in \mathbb{R}^{\mathbf{n \times n}}$

such that

$U^{'}AV = diag(\lambda_1, ..., \lambda_p) \in \mathbb{R}^{m \times n} \quad p = min\{m, n\}$

where $\lambda_1 \geq \lambda_2 \geq .... \geq \lambda_p \geq 0$. The λ_i are the singular values of **A** and the vectors u_i & v_i are the *ith* left singular vector and the *ith* right singular vector respectively. It is easy to verify by comparing columns in the equations $AV = \Lambda U$ and $A^{'}U = \Lambda^{'} V$ that

$$\left.\begin{array}{l} Av_i = \lambda_i u_i \\ A^{'}u_i = \lambda_i v_i \end{array}\right\} \quad i = 1 : \min\{m, n\}$$

Proof. See Appendix A.2.4.1.

Example. $A = \begin{bmatrix} 0.96 & 1.72 \\ 2.28 & 0.96 \end{bmatrix} = U\Lambda V^{'} = \begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{bmatrix}^{'}$

The SVD reveals a great deal about the structure of a matrix. If the SVD of A is given by Theorem 1, and we define r (where r is the rank of a matrix) by

$$\lambda_1 \geq ..... \geq \lambda_r > \lambda_{r+1} = ... = \lambda_p = 0,$$

then

$$\begin{aligned} rank\,(A) &= r \\ null\,(A) &= span\,\{v_{r+1},\,\ldots,\,v_n\} \\ range\,(A) &= span\,\{u_1,\,\ldots,\,u_r\} \end{aligned}$$

Moreover, if $U_r = U\,(:,\,1:r)$, $\Lambda_r = \Lambda\,(1:r\,,\,1:r)$, and $V_r = V\,(:,\,1:r)$, then we have the SVD *expansion*

$$A = U_r^{'}\Lambda_r V_r^{'} = \sum_{i=1}^{r} \lambda_i u_i v_i^{'}$$

Finally, both the 2-norm and the Frobenius norm are neatly characterized in terms of the SVD.

$$\begin{aligned} \|A\|_F^2 &= \lambda_1^2 + ... + \lambda_p^2 \qquad p = min\{m, n\} \\ \|A\|_2 &= \lambda_1. \end{aligned}$$

As stated earlier, the resultant matrix from outer product is rank deficient so it is of core importance to describe how SVD deals with rank-deficient matrices.

Definition. Any matrix $A \in \mathbb{R}^{m\times n}$ is *rank deficient* if rank(A) $\leq$ min$\{m, n\}$. The rank of a matrix is the maximum number of columns (or rows) which are linearly independent.

SVD: A remedy to rank deficiency and the least squares (LS) problem

One of the most valuable aspects of SVD is that it enables us to handle the rank deficient matrices efficiently. Numerous theorems in linear algebra have the form of "if such-and-such a matrix has a full rank, then such-and-such a property holds". The results of this statement do not help us to address the numerical difficulties frequently encountered in situations where rank deficiency prevails. If matrix S_t is rank deficient, then there are an infinite number of solutions (also known as the minimizers) to the least squares problem. Among these minimizers, we have to choose a particular solution, and this choice is entirely based on the following rule.

*"**The optimal solution is the one that has minimum length**."*

To put it simply, the optimal solution is the one which provides the minimum norm solution. With the help of the following theorem, Golub and Van Loan (1989, p.241) reported that among the different minimizers, this optimal solution can only be obtained with Singular Value Decomposition (SVD).

Theorem 2. Let the SVD of $A \in \mathbb{R}^{m \times n}$ be given by Theorem 1. If $k < r = rank\,(A)$ (where k refers to the rank of a matrix which is rank deficient) and

$$A_k = \sum_{i=1}^{k} \lambda_i u_i v_i^{'}$$

then

$$\overset{min}{rank\,(B)} = k \;\; \|A - B\|_2 = \|A - A_k\|_2 = \lambda_{k+1}.$$

Proof. See Appendix 2.4.2.

Theorem 2 says that the smallest singular value of A is the 2-norm distance of A to the set of all rank deficient matrices. It also follows from theorem that the set of full rank matrices in $\mathbb{R}^{m \times n}$ is both open and dense.

Orthogonality and SVD

The matrix 2-norm and the Frobenius norm are invariant with respect to the Orthogonal transformation (SVD uses Orthogonal transformation). Since SVD leaves us with three matrices (as defined earlier), two of them are Orthogonal matrices: they never change the length of a vector. Such a statement cannot be made for the other decompositions such as *eigenvalue decompositio*n: This means that in eigenvalue decomposition the resultant matrices may not necessarily be orthogonal. Orthogonal transformation is defined as the transformation in which the linear mapping matrix is orthogonal. The *orthogonal* matrix is defined as follows:

Definition. Any matrix $Q \in \mathbb{R}^{m \times n}$ is orthogonal if and only if it is singular, i.e., having determinant $\mid Q \mid \neq 0$ and for which $Q^{'}Q = I$, where I is the identity matrix. To put it simply, an orthogonal matrix is a square matrix whose columns and rows are orthogonal unit vectors, i.e., orthonormal vectors.

$$Q^{'}Q = Q\,Q^{'} = I$$

This leads to an equivalent characterization: a matrix is orthogonal if its transpose is equal to its inverse i.e., $Q^{'} = Q^{-1}$.

Orthonormality and SVD

Besides orthogonality, SVD also preserves ***orthonormality***: this means that all the singular vectors are mutually orthogonal and all of them are of unit length. Due to this property we do not need to normalize the matrix, whereas in the case of *eigenvalue decomposition*, the eigenvectors are not orthonormal and usually need to be normalized (Sheldon, 1997). This property is one of the main advantages of SVD and places this decomposition above the eigenvalue decomposition.

Keeping in view these features of SVD, we decompose the outer product (S_t) using Singular Value Decomposition (SVD). Since S_t is a real, symmetric ($k \times k$) matrix, SVD of S_t yields:

$$S^{(1)}_{t_{(K\times K)}} = U^{(1)}_{t_{(K\times K)}} \Lambda^{(1)}_{t_{(K\times K)}} U^{(1)'}_{t_{(K\times K)}}, \tag{2.1.4}$$

where $\Lambda_t^{(1)} = diag\left(\lambda_{1t}^{(1)}, \lambda_{2t}^{(1)}, ..., \lambda_{Kt}^{(1)}\right)$ having elements such that $\lambda_{1t}^{(1)} \geq \lambda_{2t}^{(1)} = ... = \lambda_{Kt}^{(1)} = 0$, meaning that only the first λ has non-zero values; otherwise, all other singular values are zeros. This indicates that one can extract only one λ whatever the dimensions of innovation vector ε_t. Furthermore, because U_t and U_t' are orthogonal matrices (unitary matrices with orthonormal basis), their multiplication will yield identity matrix i.e.,

$$U_t^{(1)} U_t^{(1)'} = I_K$$

Due to the this restriction. we have in columns of $U_t^{(1)}$

$$u_{1,t}^{(1)2} + u_{2,t}^{(1)2} + ... + u_{K,t}^{(1)2} = 1$$

which implies

$$u_{1,t}^{(1)} = \sqrt{1 - \left(u_{1,t}^{(1)2} + u_{2,t}^{(1)2} + ... + u_{K,t}^{(1)2}\right)} = \sqrt{1 - \sum_{k=2}^{K} u_{k,t}^{(1)2}} \tag{2.1.5}$$

Whatever dimensions the data set has, one can easily extract the one whole component in each iteration of SVD.

2.1.2 Step 2: check u_{kt} for remaining clustering

Since matrix $U^{(1)}$ is unitary, all of its vectors $u_{kt}^{(1)}$ are bounded [−1, 1], making it hard to detect volatility clustering in $u^{(1)}s$ as shown in the following figure. Based on the results from simulated data, each panel of Figure 2.1.2 shows the time series for $u_{1t}^{(1)}$, $u_{2t}^{(1)}$ and $u_{3,t}^{(1)}$, respectively. Looking at these plots, one cannot find whether the presented time series exhibit volatility clustering or not.

Therefore, we apply component-wise Fisher transformation on $U_t^{(1)}$, which brings these components in the bounds of $[-\infty, \infty]$.

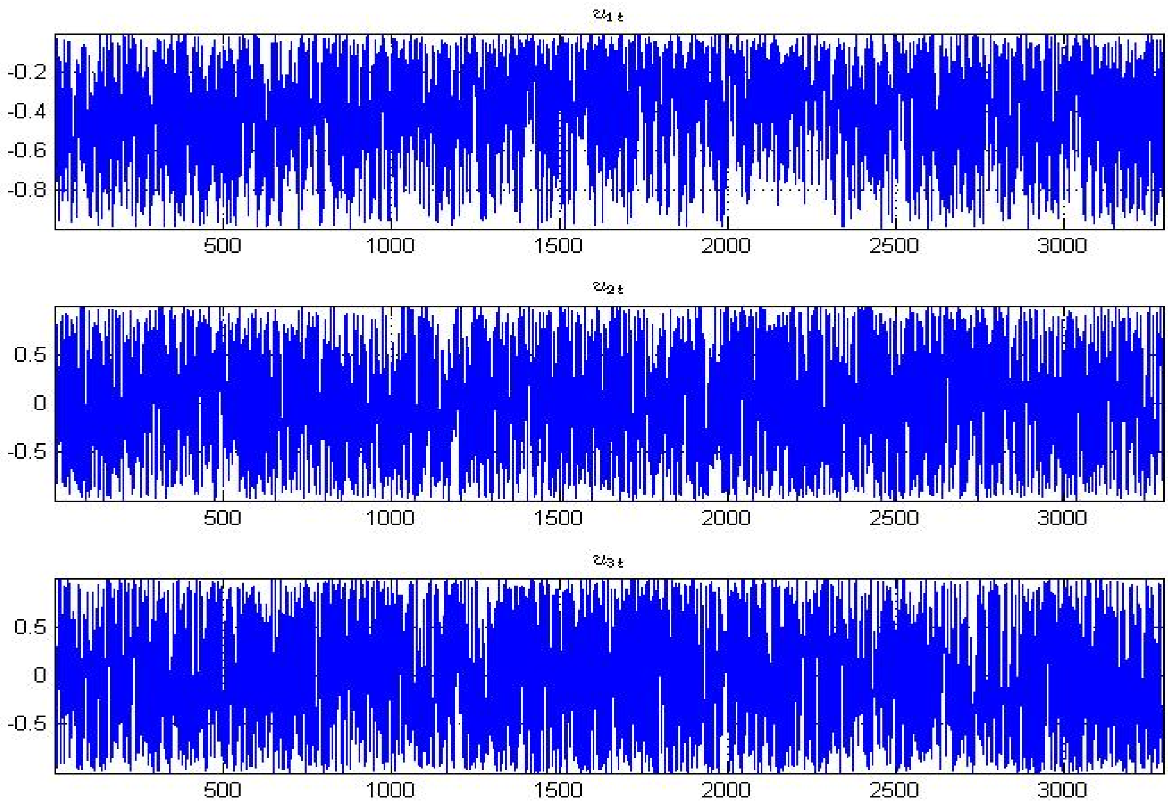
Fisher transformation

Fisher (1915) proposed this transformation to stabilize the variance of correlation coefficients. His transformation converts Pearson's correlation coefficient into the normally distributed variable Z. The transformation is defined as follows:

$$z_{k,t} = \frac{1}{2} ln \left(\frac{1 + U_{K,t}^{(1)}}{1 - U_{K,t}^{(1)}} \right), \qquad \forall -1 < U_{K,t}^{(1)} < 1$$

where ln is the natural logarithmic function and $z_{k,t} = (z_{1,t}, ..., z_{K,t})'$ is $K \times 1$ vector of Fisher-transformed $U_{K,t}^{(1)}$, and t is the time index. It is not important to understand how Fisher came up with this formula. What is important are two attributes of the distribution of the z statistic: (1) It is normal, and (2) It has a known standard error of $1/\sqrt{T-3}$.

Figure 2.1.2: Time plot of singular vectors u_{1t}, u_{2t} and $u_{3,t}$



When this transformation is applied to the preceding time series of $u's$, it results in the time series presented in Figure 2.1.3 with their histograms in Figure 2.1.4. It can be easily concluded from the histograms that the Fisher-transformed residuals are substantially normally distributed.

2.1.3 Step 3: repetition of Steps 1 and 2

The need for this step solely depends on the results of the Step 2 in the sense that if Engle ARCH and McLeod-Li tests show that volatility clustering is still present in the data set, then it is necessary to repeat Steps 1 and 2 in order to extract the remaining volatility clustering. This repetition indicates that volatility in the data set is driven by more than one factor. It must be noted that in this second round we compute the outer product for only those Fisher transformed singular vector components which exhibit volatility clustering.

Let $z_{k,t}^{(1)} = (z_{1,t}^{(1)}, z_{2,t}^{(1)}, \ldots.., z_{k_1,t}^{(1)})'$ be the vector of Fisher-transformed singular vectors with volatility clustering effects from the first round, then we repeat Step 1, i.e.,

$$S_t^{(2)} = z_{k,t}^{(1)} z_{k,t}^{(1)'}, \quad t = 1, \ldots., T$$

and then decompose this outer product again using SVD in order to extract $\lambda_{1,t}^{(2)}$. Repeat Step 3 until the $z_{k,t}^{(i)}$ have no volatility clustering. Because of repetition, this step is called "Repetition of Steps 1 and 2". The number inside the parenthesis indicates how many times we need to repeat Step 3 in order to extract all volatility clustering from the data set. Suppose that after $i-th$ iteration none of the Fisher-transformed singular vector components exhibit volatility clustering, meaning that there will be no more $\lambda_{1,t}^{(i)}$ to extract. In terms of financial modeling, it shows that $\lambda_{1,t}^{(i)}$ $(i = 1, 2, \ldots., K)$ has captured all volatility clustering from the data set. In fact, this step is the most important step of our methodology in the sense that it helps us to identify how many factors are driving the volatility in the data set.

If the Fisher-transformed singular vectors exhibit no volatility clustering after the first round of SVD, it means that $\lambda_{1,t}^{(i)}$ has captured all volatility clustering from the data set (as in the case of simulation study). If this happens, we do not need Step 3 and directly proceed to the next step.

Figure 2.1.5 shows the results of SVD after the first round for the simulated data. The first panel shows the time plot of simulated data, the second shows the time series of singular value $\lambda_{1,t}^{(i)}$. The clustered high and low spikes in $\lambda_{1,t}^{(i)}$ exactly correspond to the regimes of low and high volatility in the data (Y_t), which, in turn, show the capability of singular values to capture the volatility of the data set. The third panel shows the log-transformed singular values $ln\,(\lambda_{1,t})$. The reason for using Log transformation is explained in Step 4. The fourth and fifth panels show the time plots of singular vector components U_K and their Fisher-transformed series Z_k, respectively.

Figure 2.1.3: Plot of Fisher-transformed singular vectors $Z_{K,t}$

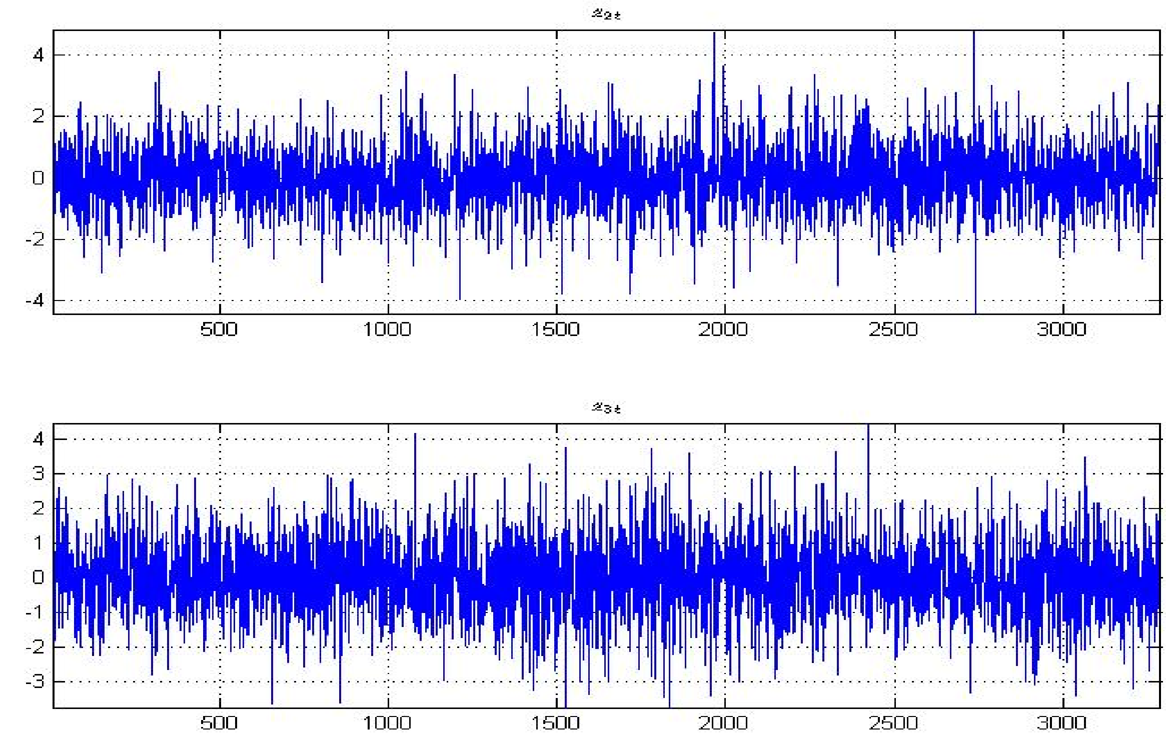


Figure 2.1.4: Histogram of the Fisher-transformed singular vectors $Z_{K,t}$

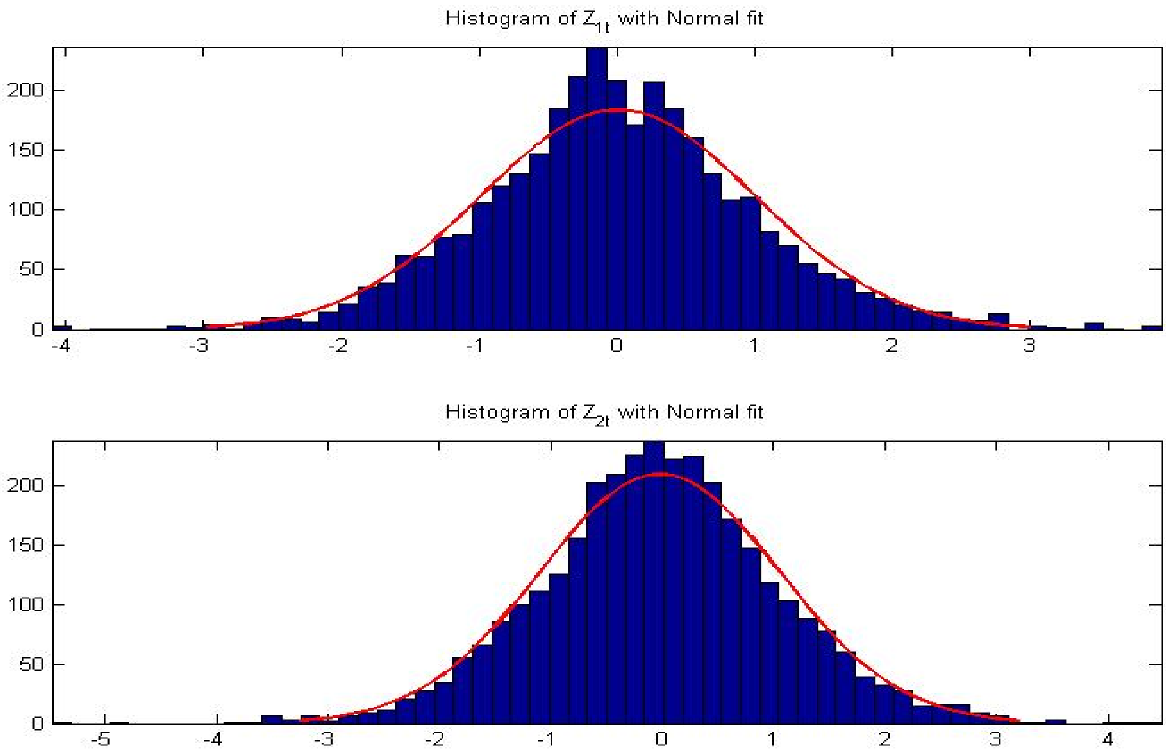


Figure 2.1.5: Results of SVD after the first round for simulated data

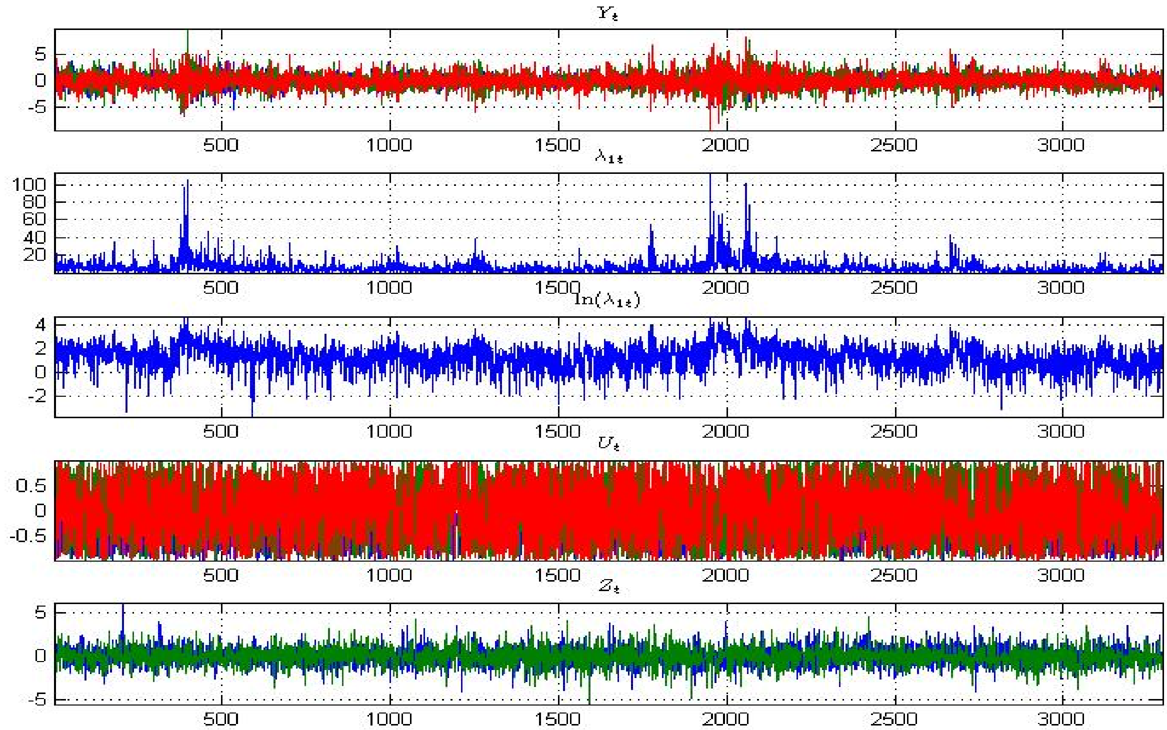


Table 2.2 shows the summary statistics and the results of all tests described to detect the presence of volatility clustering in Fisher-transformed singular vectors resulting from the first round of SVD for the simulated data. The means and variances of $z_{2,t}$, and $z_{3,t}$ are nearly 0 and 1, respectively. The skewness, and kurtosis of these series are pretty close to those of normal distribution. The results from McLeod-Li, and Engle ARCH also show no evidence of volatility clustering, which suggests that these series behave like white noise processes.

Table 2.2: Summary statistics and testing of ARCH effects in $z_{k,t}$

	Mean	**Variance**	**Skewness**	**Kurtosis**	**McLeod-Li**(Q_{20})	**Engle ARCH**
z_{2t}	0.007	0.996	0.107	3.25	28.07 (0.108)	27.55 (0.121)
z_{3t}	-0.021	1.144	0.062	3.48	26.59 (0.147)	24.49 (0.222)

At 5% significance level, the critical value of χ^2_{20} for McLeod Li and Engle ARCH is 31.41

2.1.4 Step 4: modeling the dynamics

After following Steps 1 to 3, we are left with the vector of singular values $\lambda^{(i)}_{1,t} = \left(\lambda^{(1)}_{1,t}, \lambda^{(2)}_{1,t}, \ldots, \lambda^{(K)}_{1,t}\right)$ and the matrix containing singular vectors $U^{(i)}_{i,t} = \left(u^{(1)}_{1,t}, \ldots, u^{(k)}_{K,t}\right)$. This step of our proposed methodology is subdivided into two steps, which consist of modeling the dynamics of the resulting singular values and the corresponding singular vectors as described below:

4a. Singular values $\left(\lambda_{1,t}^{(i)}\right)$

Each repetition leaves us with one sequence of singular values $\left\{\lambda_{1,t}^{(i)}\right\}$, where the subscript i denotes the number of iterations (repetition of Steps 1and 2) and t is the time index. This process is called 'Sequential Volatility Extraction' and, similarly, the resulting singular vectors are called 'Sequentially Derived Singular Vectors'. Depending on these two, we named our new proposed model "**Sequential Volatility Extraction (SVX)**" model (SVX, hereinafter).

After obtaining white noise Fisher-transformed singular vectors, the next step is to model the dynamics of vector of the corresponding singular values $\left\{\lambda_{1,t}^{(i)},\ i = 1, 2, ..., K\right\}$. Any of the time series models could be thought of as a plausible candidate for this purpose. This model selection depends on the pattern of temporal dependence in the singular values, e.g., if the autocorrelation function (ACF) of singular values indicates long memory phenomena, then autoregressive fractionally integrated moving-average ARFIMA(p,d,q) model could perform reasonably well. Otherwise, one could use any of the ARMA(p,q) models. A detailed description of these models with their estimation procedures is given below.

2.1.4.1 Autoregressive Fractionally Integrated Moving Average (ARFIMA) model

Before discussing the ARFIMA model, which has been proposed to capture the long memory behavior of the time series, it is important to describe briefly some preliminary concepts regarding long memory.

Definition of long memory

Long memory is the property of many economic and financial time series. As we know, for a stationary time series the ACF decays exponentially to zero with the increase in lag length. There are some time series whose ACF decay hyperbolically (at a much slower rate) rather than exponentially. Such time series are referred to as long memory time series in the economic and financial literature. Several possible definitions were found in the literature regarding the long memory phenomenon. Among these, the most widely used is the one provided by McLeod and Hipel (1978). According to them, given a discrete time series y_t with autocorrelations function ρ_j at lag j, the process under consideration possesses long memory if the quantity

$$\lim_{n\to\infty} \sum_{j=-n}^{n} \mid \rho_j \mid = \infty \tag{2.1.6}$$

is infinite. Thus the autocorrelations are non-summable and decay to zero quite slowly. Generally speaking, long memory implies that the process contains a long-range time dependence. A stationary process with slowly decaying correlations is therefore called a stationary process with long memory or long-range dependence or strong dependence (in contrast to processes with summable correlations, which are also known as processes with short memory or short-range dependence or weak dependence).

Brockwell and Davis (1995) expended the definition of long memory as

$$\rho_j \sim C \mid j \mid^{2d-1}, \quad j \to \infty \quad C \neq 0 \quad 0 < d < \frac{1}{2}$$

where $\sim$ means that the ratio of the right hand side and the left hand side converges to unity as $i \to \infty$. Hence, we get that

$$\underset{n\to\infty}{lim} \sum_{j=-n}^{n} \rho_j = \pm\infty, \tag{2.1.7}$$

The definition of long memory provided by Brockwell and Davis is suitable when considering the idea of long memory in the frequency domain. By restricting attention to real-valued processes and assuming that $C > 0$, the condition reduces to

$$\underset{n\to\infty}{lim} \sum_{j=1}^{n} \rho_j = \infty,$$

because $\rho_j = -\rho_j$. The spectral density of y_t (where y_t is a discrete time process) is given by the following definition:

$$f_y(\omega) = \frac{1}{2\pi} + \frac{1}{\pi} \sum_{j=1}^{\infty} \rho_j cos(j\omega),$$

If y_t contains long memory, it is clear that the spectral density of y_t is finite at $\omega = 0$. This observation provides another characteristic of long memory processes: The spectral density has a pole at $\omega = 0$.

Since the definition of long memory in frequency domain involves spectral density, we must first define the spectral density. Priestly (1981) defines spectral density f as follows:

$$f(\omega) = \frac{\sigma^2}{2\pi} \sum_{k=-\infty}^{\infty} \rho(k)\, e^{jk\omega},$$

which implies that

$$f(\omega) \approx c_f \mid k \mid^{\alpha-1} \tag{2.1.8}$$

as $\omega \to 0$, where c_f is positive constant which shows the number of Fourier frequencies.

Thus, for $\alpha < 1$, $f \to \infty$ at the origin.

Let y_t be a stationary process for which the following equality holds. There exists a real number $\beta \in (0, 1)$ and a constant $c_f > 0$ such that

$$\lim_{\omega \to 0} f(\lambda) / \left[c_f |\omega|^{-\beta} \right] = 1,$$

Then y_t is called a stationary process with long memory or long range dependence, or strong dependence (Beran, 1994), where the spectral density $f(\omega)$ will be unbounded at lower frequencies.

It should be noted that in the following we use continuous time processes, even though the time series under consideration followed discrete time processes. The reason is that a proper description of the long memory includes the concept of Brownian motion, which is defined in a continuous time frame.

Introduction to ARFIMA model

The ARFIMA model was proposed by Granger and Joyeux (1980) and by Hosking (1981). It was introduced as the generalization of the well-known ARIMA(p,d,q) model of Box and Jenkins (1976). To be more precise, an ARIMA model is an ARFIMA model where the innovations are fractionally differentiated white noise (fractional white noise may contain long memory). Simply put, the difference between ARIMA and ARFIMA model is that the differencing parameter d in the previous model can take only integer values while in the latter model d can take any fractional value, and the property of long memory lies in the region $0 \leq d \leq \frac{1}{2}$. The ARFIMA model has attractive features of modeling long term persistence and, capturing short- and long-term correlation structures of time series.

An autoregressive fractionally-integrated moving average time series model may be written in lag operator notation as follows:

$$\phi(L)(1-L)^d y_t = \theta(L)\varepsilon_t, \quad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right), \tag{2.1.9}$$

where L is the backward-shift operator, $\phi(L) = 1 + \phi_1 L + \phi_2 L^2 + ... + \phi_p L^p$, $\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + ... + \theta_q L^q$, and $(1-L)^d$ is the fractionally differenced operator, in which d stands for fractionally differenced parameter. Hosking (1981) defined this fractional differencing operator as an infinite binomial series expansion in powers of the backward-shift operator.

$$(1-L)^d = \sum_{k=0}^{\infty} \left(\begin{array}{c} d \\ k \end{array} \right) (-L)^k = 1 - dL - \frac{1}{2}d(1-d)L^2 - \frac{1}{6}d(1-d)(2-d)L^3 - ...., \tag{2.1.10}$$

where the sequence of coefficients

$$\begin{pmatrix} d \\ k \end{pmatrix} (-1)^j = \frac{\Gamma(d+1)(-1)^j}{\Gamma(d-j+1)\Gamma(j+1)} = \frac{\Gamma(-d+j)}{\Gamma(-d)\Gamma(j+1)}$$

is square summable. $\Gamma(.)$ denotes the gamma (generalized factorial) function. For values $d > \frac{1}{2}$, $(1-L)^d$ can be defined by combining integer differencing and (2.1.10).

The lag operator is not a variable in a strict mathematical sense, but the mathematical theorem from Lindstrom (1995) yields the convergence of the expansion.

Some of the properties of ARFIMA(p,d,q) are summarized as follows:

1. When $d < \frac{1}{2}$, y_t is a stationary process and has the infinite moving average $MA(\infty)$ representation.

2. When $d > -\frac{1}{2}$, y_t is invertible and has the infinite autoregressive $AR(\infty)$ representation.

3. The spectral density of $\{y_t\}$ is

$$f(\omega) = \left(2sin\tfrac{1}{2}\omega\right)^{-2d} \text{ for } 0 < \omega \leq \pi$$

and

$$f(\omega) \sim \omega^{-2d} \text{ as } \omega \to 0.$$

4. The process is both stationary and invertible if all roots of $\phi(L)$ and $\theta(L)$ lie outside the unit circle and $-0.5 < d < 0.5 =| d |< 0.5$.

5. The covariance function of $\{y_t\}$ is

$$\gamma_j = E(y_t y_{t-j}) = \frac{(-1)^j (-2d)!}{(j-d)!(-j-d)!}$$

and the correlation function of $\{y_t\}$ is

$$\rho_j = \frac{\gamma_j}{\gamma_0} = \frac{(-d)!(j+d-1)!}{(d-1)!(j-d)!} \quad (j = 0, \pm 1, ...),$$

$$\rho_j = \frac{d(1+d)...(j-1+d)}{(1-d)(2-d)...(j-d)} \quad (j = 1, 2, ...). \tag{2.1.11}$$

In particular, $\gamma_0 = (-2d)!/\{(-d)!\}^2$ and $\rho_1 = {}^{d}/_{(1-d)}$. As $j \to \infty$,

$$\rho_j \approx \frac{(-d)!}{(d-1)!} j^{2d-1}.$$

6. The inverse autocorrelations of $\{y_t\}$ are

$$\rho_{inv,j} = \frac{d!(j-d-1)!}{(-d-1)!(j+d)!} \sim \frac{d!}{(-d-1)!} j^{-1-2d} \tag{2.1.12}$$

as $j \to \infty$.

7. The partial correlations of $\{y_t\}$ are

$$\phi_{jj} = {}^{d}\!/\!_{(j-d)} \quad (j=1,2,...).$$

8. When $d = 0$, the process under consideration is a white noise process with zero correlations and constant spectral density.

9. When $-0.5 < d < 0$, the process has a short memory, or long-range negative dependence. It has been referred to as "*anti-persistent*" by Mandelbrot (1977). The autocorrelations and partial autocorrelations are all *negative*, except for $\rho_0 = 1$, and decay hyperbolically to zero as number of lags increases.

10. When $0 < d < 0.5$, the process is a long-memory stationary process. All correlations and partial correlations are *positive* and decay *hyperbolically* to zero as lag order increases.

11. For $d \geq 0.5$, the process is non-stationary and possesses infinite variance as reported by Granger and Joyeux (1980).

Estimation of ARFIMA(p,d,q) model

Several methods for the estimation of ARFIMA models have been proposed in the literature. Detailed surveys regarding these procedure have been provided by Baillie (1996), Ooms & Doornik (1999) and Beran (1994). These estimation procedures are divided into two types: one-step estimation and two-step estimation.

1. One-step estimation procedure (parametric estimation)

In one-step estimation procedure, the differencing parameter d and all ARMA(p,q) parameters are estimated simultaneously using one of the proposed Maximum Likelihood methods such as Exact Maximum Likelihood (EML) by Sowell (1992), Modified Profile Likelihood (MPL) by Barndorff-Nielson (1983), Conditional Sum of Squares (CSS) by Beran (1995), and many others as reported by Baillie (1996).

1.a Exact Maximum Likelihood Estimation (Start from Here)

Sowell (1992) derived the exact MLE of a stationary univariate ARFIMA process with unconditional normally distributed disturbances ε_t. The autocovariance function of a stationary ARMA process with mean μ,

$$\gamma_i = E\left[(y_t - \mu)(y_{t-i} - \mu)\right],$$

defines the variance matrix of the joint distribution of $y_t = (y_1, ..., y_T)'$.

$$V\left[y_t\right] = \begin{bmatrix} \gamma_0 & \gamma_1 & \cdot & \cdot & \cdot & \gamma_{T-1} \\ \gamma_1 & \gamma_0 & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot & \cdot \\ \cdot & & & \cdot & & \gamma_1 \\ \gamma_{T-1} & \cdot & \cdot & \cdot & \gamma_1 & \gamma_0 \end{bmatrix} = \Sigma \tag{2.1.13}$$

which is a symmetric Toeplitz matrix, denoted by $\tau = [\gamma_0, ..., \gamma_{T-1}]$. Under normality

$$y_t \sim N_T\left(\mu, \Sigma\right)$$

Let Z_T be a sample of T observations such that $Z_T = (z_1, ..., z_T)'$ and $Z_T \sim N_T\left(0, \Sigma\right)$, with probability density function

$$f\left(Z_T, \Sigma\right) = (2\pi)^{-T/2} \mid \Sigma \mid^{-1/2} exp\left\{-\frac{1}{2} Z_T' \Sigma^{-1} Z_T\right\},$$

To estimate the parameters of the model by maximum likelihood, the evaluation of the likelihood function for a given set of parameter values is required. The log-likelihood function is given by

$$ln\left(\mathcal{L}\left(Z_T, \Sigma\right)\right) = -\frac{T}{2} log\left(2\pi\right) - \frac{1}{2} log \mid \Sigma \mid -\frac{1}{2}\left[Z_T' \Sigma^{-1} Z\right], \tag{2.1.14}$$

Since MLE procedure is computationally demanding because it requires the inversion of the covariance matrix at each iteration during the maximization of the likelihood function, the procedure was found inadequate in terms of its real-world applicability.

1.b Whittle's Approximate Maximum Likelihood Estimation

Another widely used alternative approximate MLE for the ARFIMA(p,d,q) model under normality is Whittle's approximate MLE. Whittle (1953) noted that the covariance matrix can be diagonalized by transforming the vector y into the frequency domain and that it can approximate the log-likelihood as follows:

$$ln\left(\mathcal{L}\left(z, \Sigma\right)\right) = \sum_{j=1}^{T-1} log\left[(2\pi) f\left(\omega_j\right)\right] + \sum_{j=1}^{T-1} \left[\frac{I_T\left(\omega_j\right)}{f\left(\omega_j\right)}\right], \tag{2.1.15}$$

The above approximation has been used by different researchers. Tschering (1992) studied the small sample properties of this estimator using simulation and showed that this estimator is strongly consistent, asymptotically normally distributed, and efficient. An alternative frequency-domain approximate MLE, which numerically minimizes the following quantity has been suggested by Fox and Taqqu (1986):

$$\sum \{I(\omega_j)\} / f(\omega_j; \xi),$$

where $I(\omega_j)$ is the spectral density function evaluated at frequency ω_j, ξ denotes the vector of unknown parameters, and the summation is over m frequencies.

Limitation of one-step estimation procedures

All of the above mentioned estimation procedures require specification of p and q values, and estimation of the full ARFIMA model conditioned by those choices. These methods involve the challenge of choosing an appropriate ARMA specification. Since we do not know the true specification of the model, the ML estimates tends to be inconsistent, whereas the semi-parametric procedure presented in the next paragraph is consistent even though the short-run dynamics are not specified.

2. Two-step estimation method (semi-parametric estimation)

Why should we ever used semi-parametric procedure to estimate the ARFIMA models ? If we have a full parametric specification of the model, the semi-parametric procedure must be inefficient due to the loss of information. However, if we do not know the true specification of the model, the earlier described parametric procedures tend to be inconsistent (see Robinson, 1995), whereas the semi-parametric procedures presented below are consistent even though the short-run dynamics are not specified.

A major part of these procedures can be motivated by estimating d from the behavior of the spectrum at low frequencies. As mentioned earlier, for the ARFIMA processes d is identified by the asymptotic behavior of the spectrum near zero frequency.

The earliest methods used to estimate the parameters of ARFIMA(p,d,q) model are based on a semi-parametric approach (which is also known as the two-step estimation procedure). In these methods, the differencing parameter d is estimated in the first step, whereas all other parameters of the model are estimated in the second step. The differencing parameter d can be estimated both in time and frequency domain. In time domain, sample autocorrelations at higher lags are used (see Robinson, 1990), but the disadvantage of this procedure is that it requires all autocovariances γ_k to be positive.

These two-step procedures differ only in their first step. In the second step, the estimated differencing parameter $\hat{d}$ is used to transform the observed series y_t into a series $u_t = (1-L)^d y_t$, which is assumed to follow ARMA(p,q) process $\phi(L) u_t = \theta(L) \varepsilon_t$. The remaining parameters of the model are estimated by standard time series procedures applied to this transformed series. There are the four ways to estimate the fractional differencing parameter in the first step:

1. The first is the estimated resealed range exponent, H [see McLeod and Hipel (1978) for a survey of this statistic]. The differencing parameter can be identified as $\hat{d} = \hat{H} - \frac{1}{2}$. A generalization, which is designed to be robust to 'short-term dependence', is given by Lo (1989).

2. Janacek (1982) obtained the differencing parameter estimate by numerical integration of the log periodogram.

3. Geweke and Porter-Hudak (1983) exploit the behavior of the spectral density around zero. Using frequencies near zero, a univariate regression of the log periodogram on the log of the frequencies is performed. The slope estimate is an estimate of the differencing parameter.

4. The fourth procedure presented by Shea (1990) is the maximum likelihood estimation of the regression model used by Geweke and Porter-Hudak (1983), termed as "GPH-Method".

In this study, we only consider one parametric estimation via Whittle's Approximate Maximum Likelihood Method (WMLE).

2.1.4.2 Autoregressive Moving Average (ARMA) model

Box and Jenkins (1971) introduced Autoregressive Moving Average ARMA(p,q) models. These models are extensively used in time series analysis to describe the stationary time series processes. These models are formed as a resultant of the combination of the p Autoregressive AR(p) processes with the q Moving Average MA(q) processes, hence termed as ARMA(p,q) models. If the time series process is denoted by y_t, the ARMA(p,q) model is described mathematically as follows;

$$y_t = c + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t, \tag{2.1.16}$$

where ε_t are white noise error terms ($\varepsilon_t \sim WN\left(0, \sigma^2\right)$). Using lag operator (back-shift operator), the above model can be written as follows

$$\phi\left(L\right) y_t = \theta\left(L\right) \varepsilon_t,$$

where $\phi\left(L\right) = 1 - \phi_1 L - .... - \phi_p L^p = 1 - \sum_{i=1}^{p} \phi_i L^i$ is the autoregressive and $\theta\left(L\right) = 1 + \theta_1 L + ... + \theta_q L^q = 1 + \sum_{j=1}^{q} \theta_j L^j$ is the moving average polynomials respectively. ARMA model is appropriate when y_t is a linear function of its own past values (lags of AR polynomial) and past values of shocks (lags of MA polynomial). If the dependence of y_t is nonlinear, the process is called nonlinear autoregressive moving average (NARMA) model. The ARMA model is said to be stationary if all the characteristics roots of the autoregressive polynomial lie outside the unit circle.

Estimation of ARMA(p,q) model

After choosing the correct order $(p, q)$of the model, the estimates of ϕ_i , θ_j, and $\sigma^2$are obtained by using one of the following methods:
1. Ordinary Least Square (OLS)
2. Maximum Likelihood Estimation (MLE) and
3. Yule-Walker equations

We will not go into the details of all above listed methods, but will describe here only the maximum likelihood method, which is most widely used. A Gaussian ARMA(p,q) model takes the following form:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + ... + \theta_q \varepsilon_{t-q},$$

The goal is to estimate the vector of population parameters
$\psi = (c, \phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q, \sigma^2)'$. Under the normality assumption, the conditional log-likelihood function for the ARMA(p,q) process is given by

$$\mathcal{L}\left(\phi_i, \theta_j, \sigma^2\right) = -\frac{T}{2} log\left(2\pi\right) - \frac{T}{2} log\left(\sigma_y^2\right) - \sum_{t=1}^{T} \frac{\varepsilon_t^2}{2\sigma_y^2}, \tag{2.1.17}$$

where $\mathcal{L}$ stands for log-likelihood function, σ_y^2 shows the variance of y_t and $\varepsilon_t = (y_t - \mu)$.
After selecting the appropriate model (ARMA(p,q) or ARFIMA(p,d,q)), the values of p and q are selected using Bayesian Information Criterion (BIC). This selection leaves us with AR(p) , MA(q) or their combinations of ARMA(p,q) models.

Following the results of simulation study $log\left(\lambda_{1,t}^{(i)}\right)$ performs reasonably well. Hence, our resulting model takes the following mathematical form:

$$log\left(\lambda_{1,t}^{(i)}\right) = \alpha_{0,i} + \sum_{p=1}^{p} \alpha_{p,k} log\left(\lambda_{1,t-p}^{(i)}\right) + \sum_{q=0}^{q} \beta_{q,k} \eta_{k,t-j} + \eta_{k,t}, \quad \eta_{k,t} \sim NID\left(0, \sigma_{\eta_{k,t}}^2\right), \tag{2.1.18}$$

where η_t is generated independently of ε_t for all t. Following the dynamics of the above equation, one can think that our proposed model bears a certain resemblance to EGARCH model of Nelson (1991), in which the dynamics of conditional variance are modeled using $log\left(h_t\right)$and its lags. In fact, our proposed model differs from EGARCH model in the sense that it does not depend on the past observations of the observed variable $y_{i,t}$ but on some unobserved latent variable instead. However, based on this fact, we can say that our model resembles SV models, where the volatility in the data set is assumed to be a latent variable. We know from the standard theory that all the moments of $log\,(\lambda_t)$exist if and only if

$$\sum_{i=1}^{p} \mid \alpha_i \mid < 1,$$

The following table shows the summary statistics of singular values along with different transformations such as Log and Square-root and the results of the Ljung Box test. The skewness and kurtosis of $\lambda_{1,t}$ shows departure from normality; for this reason, we apply some transformation in order to achieve normality.

Table 2.3: Summary statistics of $\lambda_{1,t}$ along with different transformations and test for serial correlations

	Mean	Variance	Skewness	Kurtosis	Ljung-Box (Q_{20})
λ_{1t}	5.968	59.899	4.883	44.588	4,657.80 (0.000)
$log(\lambda_{1t})$	1.239	1.214	-0.389	3.53	2,358.70 (0.000)
$\sqrt{\lambda_{1t}}$	2.143	1.377	1.576	7.743	4,539.60 (0.000)

As we know by construction, all the values of $\lambda_{1,t}$ are positive, and logarithmic and square root transformations are commonly used to normalize the positively skewed data. Due to this, we also use these two transformations. Perhaps it would also be better if one uses the Box-Cox transformation. A brief summary about these transformations is given below:

Logarithmic transformation(s)

Logarithmic transformations are actually a class of transformations rather than a single transformation. Log-normal variables (i.e, normally distributed after log transformation) are relatively common in many fields of science. In brief, a logarithm is the power (exponent) a base number must be raised to in order to get the original number. Any given number can be expressed as y^x in an infinite number of ways depending on the base of the log used. For example, the base could be 1, 2, 10, and so on. Another common option is the Natural Logarithm, where the constant e (2.7182818. . .) is taken as the base. Thus, $Log_{10}\,(100) = 2$ and $Log_e\,(100) = 4.605$. As this example illustrates, a base in a logarithm can be almost any number, which present an infinite number of options for transformation.

It should be noted that whenever logarithm transformation is used in this study, it means *natural logarithm*.

Square root transformation

This transformation is a member of a class of transformations called '*Power Transformations.* Power transformations are simply transformations that raise numbers to an exponent (power). For example, a square root transformation can be characterized as $y^{1/2}$. Most readers will be familiar with this procedure– when one applies a square root transformation, the square root of every value is taken (technically, a special case of a power transformation where all values are raised to the one-half power). Square root transformations are traditionally thought of as good for normalizing Poisson distributions and equalizing variance of data. However, as one cannot take the square root of a negative number, a constant must be added to move the minimum value of the distribution above 0, preferably to 1.00. In this study, we use this transformation for values which are positive by construction hence, there is no not need to add any constants.

Box-Cox transformation

Box-Cox transformation is also a member of the power transformations family. Statisticians George Box and David Cox developed a procedure to identify an appropriate exponent (Lambda = l) to transform data into a "normal shape." The Lambda value indicates the power to which all data should be raised. In order to do this, the Box-Cox power transformation searches from Lambda = -5 to Lambda = +5 until the best value is found. It should be noted that for Lambda = 0, the transformation is NOT y^0 (because this would be 1 for every value) but the logarithm of Y instead. This transformation takes the following form[7].

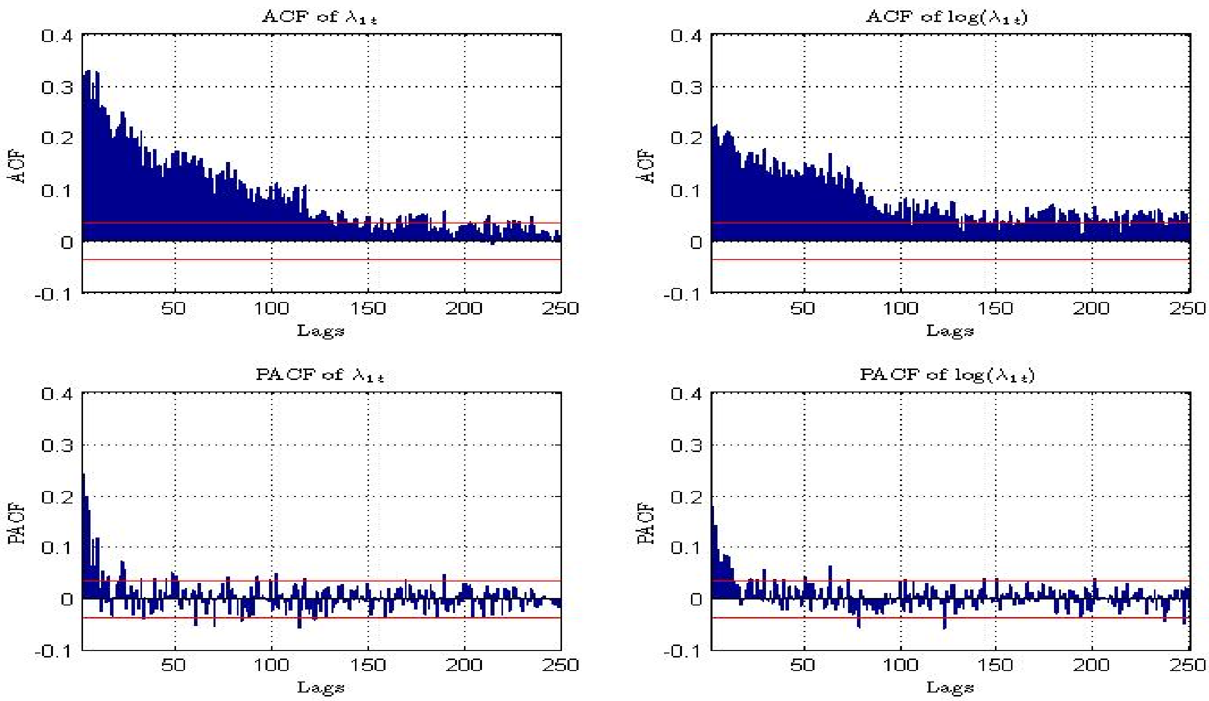
$$y^{(\lambda)} = \begin{cases} \frac{y^{\lambda}-1}{\lambda} & (\lambda \neq 0) \\ log_e\left(y\right) & (\lambda = 0) \end{cases},$$

Does Box-Cox always work?

The Box-Cox power transformation is not a guarantee for normality. This is because it actually does not real check for normality; the method checks for the smallest standard deviation. The assumption is that among all transformations with Lambda values between -5 and +5, the transformed data has the highest likelihood – but not a guarantee – to be normally distributed when standard deviation is the smallest.

[7] Since Box and Cox (1964) other authors have introduced modifications of this transformations for special applications and circumstances (e.g., John & Draper, 1980), but for most researchers, the original Box-Cox transformation suffices and is preferable due to its computational simplicity.

Figure 2.1.6: Autocorrelations and partial autocorrelations functions of $\lambda_{1,t}$ and $log\left(\lambda_{1,t}\right)$



Therefore, it is absolutely necessary to always check the transformed data for normality using a probability plot. Additionally, the Box-Cox Power transformation only works if all the data is positive and greater than 0.

Depending on the skewness and kurtosis of the Log-transformed singular values resulted from simulation study (which are pretty close to a normal distribution), we select this transformation to model the dynamics of singular values $log\left(\lambda_{i,t}\right)$.

The following figure shows the autocorrelations and partial autocorrelations functions (ACFs and PACFs) of singular values along with logarithmic transformation for the simulated data. The ACFs are indicating long memory phenomena and one can think of ARFIMA model as a good choice in order to model the dynamics of the presented time series. On the basis of this argument, different ARFIMA(p,d,q) models were estimated. For all these models, the results were rejecting the null of long memory phenomena as the estimates for d are insignificant (see Appendix 2.4.4). The estimation of ARFIMA model is further discussed in the next chapter. However, the tails-off behavior in ACFs and PACFs indicates that ARMA(p,q) model would be a good choice to capture the dynamics of singular values. Based on BIC, we select ARMA(1,1)model to capture the dynamics of $log\left(\lambda_{1,t}\right)$. The selected model also possesses the desirable property that the residuals from selected model behave like a white noise process as reported by the Ljung-Box test statistic (see Appendix 2.4.5).

Table 2.4: Summary statistics of $u_{i,t}$ and testing of serial dependence

	Mean	Variance	Skewness	Kurtosis	Ljung-Box (Q_{20})
$u_{1,t}$	-0.397	0.075	-0.422	2.07	91.70 (0.000)
$u_{2,t}$	-0.002	0.367	0.023	1.74	16.68 (0.674)
$u_{3,t}$	-0.014	0.400	0.033	1.66	12.36 (0.903)

4b. Singular vectors ($u_{i,t}$)

Modeling of the singular vectors depends on whether they show any temporal dependence or not. If yes, we model the dynamics of these singular vectors also using ARMA(p,q) approach. For example, in the case of simulated study, the Ljung-Box in Table 2.4 shows that there is a temporal dependence in the first component of U_t. Hence, require ARMA model to account for this temporal dependence, while the second and third components behave like a white noise and therefore do not need any modeling. Like the modeling of a singular vector through ARMA model, the leg length selection is based on BIC, and ARMA(1,1) model was chosen to model the dynamics of singular vectors (see Appendices 2.4.5). The dynamics of u_{it} is modeled using the following ARMA(p,q) structure:

$$u_{k,t}^{(i)} = \alpha_{0,k} + \sum_{i=1}^{p} \alpha_{i,k} u_{t-i,k}^{(i)} + \sum_{j=0}^{q} \beta_{j,k} \nu_{t-j,k} + \nu_{k,t}, \quad \nu_{k,t} \sim NID\left(0, \sigma^2_{\nu_{k,t}}\right), \qquad (2.1.19)$$

2.2 Forecasting

After estimation the important task of any time series model is to forecast the future values. This section deals with issues on forecasting a conditional covariance matrices based on the methodology presented earlier. Like DCC-MGARCH model, using our proposed model, the h-step-ahead forecasts of covariance matrices are obtained by combining the forecasts of the singular values $\hat{\Lambda}_{t+h|t}$ and the singular vectors $\hat{U}_{t+h|t}$ respectively.[8]

$$H_{t+h|t} = S_{t+h|t} = U_{t+h|t} \Lambda_{t+h|t} U'_{t+h|t} \qquad (2.2.1)$$

However, unlike the forecasts from DCC-MGARCH model, the forecasts of any of these two quantities do not depend on each other (for forecasting from DCC-MGARCH model see Chapter 4).

[8]Where $U'_{t+h|t}$ refers to the transposition of $U_{t+h|t}$.

As stated in the previous section, we get the white noise Fisher-transformed singular vector components after i iterations, so the dimensions of the two matrices, $\Lambda_{t+h|t}$ and $U_{t+h|t}$, depend on how many times we need to repeat Step 3 of SVX model in order to extract the factors driving volatility in the data set. The number of columns shows the total number of iterations (repetition of Step 3), and the individual row refers to a particular stock. All entries in the first column of this matrix always contain non-zero elements, while the subsequent columns possess the first $m - K_i$ rows with 0 entries, which shows that the corresponding components of the data vector have no volatility clustering and remaining non-zero elements represent the components with volatility clustering. Simply put, the number of rows that have zeros as an element are equal to the elements in the vector $z_K^{(i)}$ for each iteration i, where $i = 2, 3, ..., K$. On this basis of the methodology described in earlier section it is easy to understand that the number of columns varies with iterations, whereas the number of rows, remains same (which is equal to the dimensions of the data set).

$$
\begin{bmatrix}
u_{1,1,t} & 0_{1,2,t} & 0_{1,3,t} & . & . & 0_{1,i,t} \\
. & . & . & . & . & . \\
. & . & . & & & . \\
. & . & . & & & . \\
. & 0_{m-K_1,2,t} & . & & & . \\
. & u_{1,2,t} & 0_{m-K_2,2,t} & & & . \\
. & . & u_{1,3,t} & . & & . \\
. & . & & & . & . \\
. & . & & & & . \\
 & & & & & \\
u_{K,1,t} & u_{K_1,2,t} & u_{K_2,3,t} & . & . & u_{K_i,i,t}
\end{bmatrix}_{m \times n}
\times
\begin{bmatrix}
\lambda_{1,t} & 0 & . & . & . & . & . & . & 0 \\
0 & \lambda_{2,t} & . & . & . & . & . & . & . \\
. & . & . & & & & & & . \\
. & . & & . & & & & & . \\
. & . & & & . & & & & . \\
. & . & & & & . & & & . \\
. & . & & & & & . & & . \\
. & . & & & & & & . & 0 \\
0 & . & . & . & . & . & . & . & \lambda_{K,t}
\end{bmatrix}_{m \times m}
$$

$$
\times
\begin{bmatrix}
u_{1,1,t} & . & . & . & . & . & . & . & . & u_{K,1,t} \\
0_{1,2,t} & . & . & . & 0_{m-K_1,2,t} & u_{1,2,t} & . & . & . & u_{K_1,2,t} \\
0_{1,3,t} & . & . & . & . & 0_{m-K_2,2,t} & u_{1,3,t} & & & u_{K_2,3,t} \\
. & . & & & & & . & & & . \\
. & . & & & & & & . & & . \\
0_{1,i,t} & . & . & . & . & . & . & . & . & u_{K_i,i,t}
\end{bmatrix}_{n \times m}
$$

The above matrix multiplication can be expressed in the form of the following relationship:

$$S^{(i)}_{t+h|t} = \sum_{i=1}^{K} \lambda_{i,t+h|t} U_{i,t+h|t} U^{'}_{i,t+h|t},$$

Expanding the above relation for $i = 1, 2, ...K$, we get

$$S^{(1)}_{t+h|t} = \lambda_{1,t+h|t} U_{1,t+h|t} U^{'}_{1,t+h|t},$$

$$\begin{aligned}
S^{(2)}_{t+h|t} &= \lambda_{1,t+h|t} U_{1,t+h|t} U^{'}_{1,t+h|t} + \lambda_{2,t+h|t} U_{2,t+h|t} U^{'}_{2,t+h|t}, \\
&= \ldots, \\
&= \ldots, \\
&= \ldots, \\
S^{(K)}_{t+h|t} &= \lambda_{1,t+h|t} U_{1,t+h|t} U^{'}_{1,t+h|t} + \ldots + \lambda_{K,t+h|t} U_{K,t+h|t} U^{'}_{K,t+h|t},
\end{aligned}$$

2.3 Conclusion

In this chapter, we proposed a new multivariate modeling strategy to model the dynamics of conditional covariance matrices when dealing with high dimensional data sets. An attempt was made to overcome the drawbacks possessed by the existing multivariate GARCH models in finance in a way that the proposed model remains adequate for real-world applications. Based on the following two considerations: (1) the basic idea of our model that volatility in financial markets is a function of some latent variables (called factors), and (2) our approach of modeling the dynamics of these factors, we can conclude that our proposed model resembles Stochastic Volatility (SV) models. However, the way of obtaining factors is quite different. We used a singular value decomposition recursively, which makes our approach innovative. The properties described in the first section regarding SVD, such as orthogonality and orthonormality, provide us with a solid foundation to distinguish the proposed model from the already existing ones, especially considering the O-GARCH and the GO-GARCH models which are based on eigenvalue decomposition.

On one hand, the sequential extraction of volatility clustering making our modeling approach very intuitive in the sense that all the factor models presented in the financial literature are suggestive of more than one factor to account for volatility clustering in the high dimensional data set, especially when the data vector consists of more than five dimensions $(K > 5)$, e.g., Harvey et al. (1994), Alexander and Chibumba (1997), and Jingjing (2011). On the other hand, it helps us to find out for which variables in the data set the volatility is driven by only one factor and for which by two factors, and so on. Besides this, using our approach one might think that in each iteration the variables that show no volatility clustering might belong to the same sector such as finance, information technology, pharmaceutical, and so on. For example, if we consider the case of simulation study, all three stocks (Merck, Pfizer, and Johnson & Johnson) that are used to simulate the data set belong to the pharmaceutical group, and we can conclude that there is only one common factor driving the volatility of these stocks.

Our presented results from small scale simulation study ($K = 3$), which was carried out to elaborate the different steps of our methodology, encouraged us to apply and evaluate the proposed methodology using a high dimensional real-world data set. The remaining part of this thesis is mainly concerned with the application of our proposed methodology in high dimensional real-world data set.

2.4 Appendix

2.4.1 Proof of Theorem 1

Let $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$ be unit 2-norm vectors that satisfy $Ax = \sigma y$ with $\sigma = \|A\|_2$. Because any orthonormal set can be extended to form an orthonormal basis for the whole space, it is possible to find $V_1 \in \mathbb{R}^{n(n-1)}$ and $U_1 \in \mathbb{R}^{m(m-1)}$so $V = [x\, V_1] \in \mathbb{R}^{n\times n}$ and $U = [y\, U_1] \in \mathbb{R}^{m\times m}$ are orthogonal. It is not hard to show that $U^T AV$ has the following structure.

$$U^T AV = \begin{bmatrix} \sigma & \omega^T \\ 0 & B \end{bmatrix} \equiv A_1.$$

Since

$$\|A_1 \left(\begin{bmatrix} \sigma \\ \omega \end{bmatrix} \right) \|_2^2 \; \geq \left(\sigma^2 \dotplus \omega^T \omega \right)^2$$

we have $\|A_1\|_2^2 \geq \left(\sigma^2 \dotplus \omega^T\omega\right)$. But $\sigma^2 = \|A\|_2^2 = \|A_1\|_2^2$, and so we must have $\omega = 0$. An obvious induction argument completes the proof of the theorem. □

2.4.2 Proof of Theorem 2

Since $U^{'}\Lambda V = diag(\lambda_1, ..., \lambda_k, 0, ..., 0)$ it follows that rank$(A) = k$ and that $U^{'}\,(A - A_k)\,V = diag\,(0, ..., 0, \lambda_{k+1}, ..., \lambda_p)$ and so $\|A - A_k\|_2 = \lambda_{k+1}$. Now suppose rank$(B) = k$ for some $B \in \mathbb{R}^{m\times n}$. It follows that we can find the orthonormal vectors $x_1, ..., x_k$ so null(B) =span$\{x_1, ..., x_{n-k}\}$. A dimension argument shows that

$$span\,\{x_1, ..., x_{n-k}\} \,\cap\, span\,\{v_1, ..., v_{k+1}\} \,\neq \{0\}\,.$$

Let z be a unit 2-norm vector in this intersection. Since $Bz = 0$ and

$$Az = \sum_{i=1}^{k+1} \lambda_i \left(v_i^{'} z\right) u_i$$

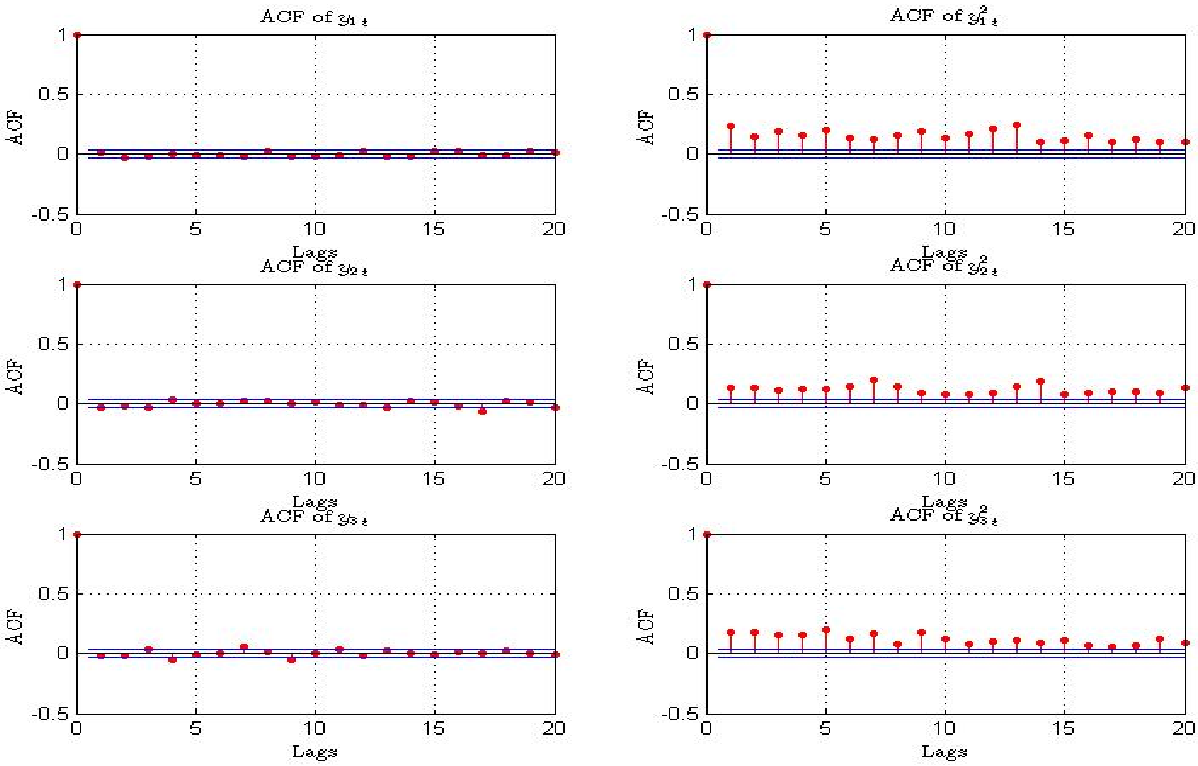
we have

$$\|A - B\|_2^2 \,\geq \|A - Bz\|_2^2 \,= \|Az\|_2^2 \,= \sum_{i=1}^{k+1} \lambda_i^2 \left(v_i^{'} z\right)^2 \,\geq \lambda_{k+1}^2$$

completing the proof of the theorem. □

2.4.3 ACFs of simulated data and their squares

Figure 2.4.1: Autocorrelation functions of y_t and y_t^2



2.4.4 Whittle's approximate estimates of ARFIMA (p,d,q) model for $log\left(\lambda_{1,t}\right)$

Table 2.5: Estimates of the $ARFIMA(p,d,q)$ model using Whittle's approximate maximum likelihood method (WMLE)

$ARFIMA(p,d,q)$	$(0,d,0)$	$(1,d,0)$	$(2,d,0)$	$(3,d,0)$	$(0,d,1)$	$(0,d,2)$	$(0,d,3)$	$(1,d,1)$	$(1,d,2)$	$(2,d,1)$	$(2,d,2)$
d	**0.002** (0.007)	**0.003** (0.010)	**0.003** (0.213)	**0.004** (0.015)	**0.003** (0.015)	**0.005** (0.431)	**0.005** (0.037)	**0.003** (0.017)	**0.004** (0.039)	**0.004** (0.039)	**0.004** (0.041)
ϕ_0		-0.782 (1.053)	-1.094 (0.970)	-0.956 (0.923)	-0.520 (0.744)	-0.526 (1.909)		-0.548 (0.973)	-0.690 (0.543)	-0.407 (2.096)	-0.367 (19.859)
ϕ_1		-0.154 (0.014)	-0.208 (0.016)	-0.257 (0.018)				0.987 (0.003)	0.987 (0.003)	0.976 (0.031)	1.014 (0.020)
ϕ_2			-0.085 (0.013)	-0.127 (0.015)						0.011 (0.030)	-0.027 (0.020)
ϕ_3				-0.074 (0.012)							
θ_1					-0.236 (0.022)	-0.976 (0.043)	-0.395 (0.040)	-0.929 (0.010)	-0.944 (0.041)	-0.933 (0.013)	-0.973 (0.028)
θ_2						0.025 (0.032)	-0.093 (0.013)		0.011 (0.028)		0.037 (0.017)
θ_3							-0.056 (0.011)				
σ_u^2	1.310	1.292	1.281	1.280	1.287	1.293	1.276	1.275	1.275	1.275	1.275
LL	-1,766.437	-1,744.218	-1,736.122	-1,729.502	-1,737.412	-1,796.101	-1,724.607	-1,722.872	-1,722.745	-1,722.764	-1,722.745
AIC	8.9688	8.7649	8.9526	8.9501	8.6831	8.9243	8.8803	8.6831	8.6833	8.6788	8.9161
BIC	8.9707	8.7686	8.9563	8.8801	8.6868	8.9280	8.9512	8.6887	8.6888	8.6843	8.9217
GoF	0.296 (0.998)	0.286 (1.000)	0.282 (1.000)	0.2804 (1.000)	0.283 (1.000)	0.267 (0.000)	0.279 (1.000)	0.280 (1.000)	0.280 (1.000)	0.280 (1.000)	0.280 (1.000)

Note: Standard errors are given in parenthesis.

2.4.5 ARMA estimates for log-transformed singular values

Table 2.6: Estimates of ARMA(p,q) model for $log\left(\lambda_{1,t}\right)$

$Parameter$	$AR(1)$	$AR(2)$	$AR(3)$	$MA(1)$	$MA(2)$	$MA(3)$	$ARMA(0,0)$	$ARMA(1,1)$	$ARMA(1,2)$	$ARMA(1,3)$	$ARMA(2,1)$	$ARMA(2,2)$	$ARMA(2,3)$	$ARMA(3,1)$	$ARMA(3,2)$	$ARMA(3,3)$
α_0	0.958	0.787	0.686	1.241	1.241	1.241	1.241	0.021	0.020	0.020	0.017	0.031	0.027	0.017	0.026	0.033
$s.e$	0.028	0.032	0.035	0.022	0.024	0.026	0.019	0.006	0.006	0.007	0.005	0.011	0.011	0.006	0.009	0.012
α_1	0.228	0.188	0.165					0.983	0.983	0.984	0.996	0.221	0.227	0.999	0.285	0.052
$s.e$	0.018	0.018	0.018					0.005	0.005	0.006	0.014	0.082	0.092	0.024	0.094	0.018
α_2		0.178	0.153								-0.010	0.754	0.751	-0.002	0.723	0.763
$s.e$		0.017	0.017								0.015	0.080	0.088	0.004	0.099	0.067
α_3			0.128											-0.012	-0.030	0.158
$s.e$			0.018											0.020	0.019	0.063
β_1				0.173	0.172	0.159		-0.904	-0.895	-0.895	-0.913	-0.152	-0.140	-0.915	-0.198	0.035
$s.e$				0.013	0.019	0.019		0.012	0.018	0.012	0.011	0.079	0.093	0.016	0.096	0.013
β_2					0.141	0.139			-0.012	-0.005		-0.695	-0.690		-0.667	-0.690
$s.e$					0.014	0.015			0.017	0.010		0.069	0.081		0.092	0.068
β_3						0.101				-0.009			-0.028			-0.173
$s.e$						0.016				0.019			0.019			0.059
LL	-4906.04	-4852.63	-4824.55	-4930.74	-4890.01	-4871.32	-4995.47	-4714.88	-4714.65	-4714.53	-4712.59	-4706.68	-4705.49	-4709.97	-4705.44	-4705.37
AIC	0.1421	0.1103	0.0938	0.1571	0.1330	0.1222	0.1958	0.0267	0.0271	0.0277	0.0259	0.0229	0.0228	0.0249	0.0227	0.0233
BIC	0.1458	0.1158	0.1013	0.1608	0.1385	0.1297	0.1977	0.0322	0.0345	0.0369	0.0333	0.0321	0.0339	0.0341	0.0339	0.0363
Q_{20}	872.20	403.96	240.62	1284.08	816.89	634.20	2358.75	22.49	21.62	21.22	21.48	23.39	20.34	21.25	20.40	20.36
$p-val$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.36	0.38	0.37	0.27	0.44	0.38	0.43	0.44

2.4.6 Estimates of ARMA(p,q) Model for singular vector

Table 2.7: Estimates of $ARMA(p,q)$ models for $u_{1,t}$

$Parameter$	$AR(1)$	$AR(2)$	$AR(3)$	$MA(1)$	$MA(2)$	$MA(3)$	$ARMA(0,0)$	$ARMA(1,1)$	$ARMA(1,2)$	$ARMA(1,3)$	$ARMA(2,1)$	$ARMA(2,2)$	$ARMA(2,3)$	$ARMA(3,1)$	$ARMA(3,2)$	$ARMA(3,3)$
α_0	-0.390	-0.377	-0.355	-0.397	-0.397	-0.397	-0.397	-0.013	-0.023	-0.038	-0.001	-0.001	-0.031	-0.032	-0.033	0.000
$s.e$	0.008	0.011	0.012	0.005	0.005	0.005	0.005	0.013	0.019	0.028	0.001	0.001	0.038	0.032	0.026	0.000
α_1	0.017	0.016	0.014					0.968	0.942	0.905	0.990	0.860	0.922	0.866	0.934	2.151
$s.e$	0.018	0.018	0.018					0.032	0.048	0.071	0.021	0.065	0.093	0.108	0.796	0.023
α_2		0.036	0.035								0.009	0.139	-0.001	0.021	-0.047	-1.444
$s.e$		0.018	0.018								0.021	0.065	0.003	0.025	0.709	0.036
α_3			0.056											0.032	0.031	0.293
$s.e$			0.017											0.031	0.040	0.014
β_1				0.016	0.013	0.008		-0.937	-0.941	-0.904	-0.992	-0.861	-0.921	-0.865	-0.933	-2.165
$s.e$				0.017	0.018	0.025		0.045	0.049	0.072	0.004	0.068	0.095	0.107	0.796	0.026
β_2					0.030	0.028			0.041	0.022		-0.130	0.023		0.067	1.508
$s.e$					0.016	0.016			0.025	0.022		0.067	0.024		0.706	0.041
β_3						0.050				0.032			0.027			-0.342
$s.e$						0.017				0.026			0.034			0.015
LL	-4852.63	-411.64	-406.40	-414.29	-412.58	-408.23	-414.71	-391.76	-389.82	-388.71	-386.15	-385.79	-387.51	-387.66	-387.64	-376.33
AIC	0.1103	-2.586	-2.589	-2.585	-2.586	-2.588	-2.586	-2.598	-2.599	-2.599	-2.601	-2.601	-2.599	-2.600	-2.599	-2.605
BIC	0.1158	-2.581	-2.581	-2.581	-2.580	-2.580	-2.584	-2.593	-2.591	-2.590	-2.594	-2.591	-2.588	-2.590	-2.588	-2.592
Q_{20}	403.96	73.52	56.86	86.71	76.77	62.96	91.70	24.14	18.54	14.90	33.45	33.40	15.82	16.16	15.61	14.45
$p-val$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.55	0.78	0.03	0.03	0.73	0.71	0.74	0.81

Chapter 3

Empirical applications

Some remarks about this chapter

Throughout this chapter, we use a significance level of 5% i.e., $\alpha = 0.05$, to evaluate our null hypothesis. Only p-values of the test statistics are presented to make a decision regarding the null hypothesis. With the exception of the first table, all stocks used in the analysis are represented with their symbols (which is known as *Ticker Symbol*[1] in the financial world). The numerical quantities are mostly rounded to three decimal places at most for precision purpose.

3.1 Data description

The Dow 30 (high yielding 30 components of Dow Jones) was used in this study for empirical purposes. Dow Jones updates its components such as the Dow 5, the Dow10, and the Dow 30 on the basis of performance,. These components belong to 10 different sectors, which are defined by the proprietary classification systems of Dow Jones[2]. These sectors include Finance, Industry, Health Care, Technology, etc. It is worth mentioning that components are added and deleted on an as-needed basis; hence, do not remain the same and their positions are also changing in time. Component selection is not governed by quantitative rules; a particular component is typically added if the company has an excellent reputation, demonstrates sustained growth, and is of interest to a large number of investors. When one component is replaced, all of them are reviewed.

The data set, which consists of Total Return Index (TRI) , was initially down-

[1]An arrangement of characters (usually letters) representing a particular security listed on an exchange or otherwise traded publicly. When a company issues securities to the public marketplace, it selects an available ticker symbol for its securities which investors use to place trade orders. Every listed security has a unique ticker symbol, facilitating the vast array of trade orders that flow through the financial markets every day. http.//www.investopedia.com/terms/t/tickersymbol.asp

[2]https.//www.djindexes.com

loaded in March, 2014 from an official website of Thomson Reuters [3] and provided to me by one of my Ph.D fellows Mr. Zurab Kotchlamazashvili. At that time, the Dow 30 was comprised of the stocks used in this study. The stocks cover almost all the sectors with the exception of Utilities. A brief introduction regarding these stocks is provided in Appendix 3.4.1.

The sample period spans from January 02, 2001, to December 31, 2013, which amounts to 3387 observations. The data for the stock "Visa (A)" were not available until March 18, 2008, hence we discarded this stock from 30 components and considered the remaining 29 for empirical purposes. Removal of financial holidays[4]and computation of the continuously compounded percentage log returns[5] using the formula displayed below yielded a total of $T = 3268$ observations. We chose this sample period because it covers the three time periods of high volatility, i.e., 9/11/2001, the market crash of 2002, and the financial crisis of 2008-09. Figure 3.1 shows the plots of the Total Return Index (TRI) and the percentage log returns for all of these stocks. Effects of these events on the volatility behavior are shown in Figure 3.1 .

$$r_{k,t} = 100 \times ln\left(\frac{TRI_{k,t}}{TRI_{k,t-1}}\right) = 100\left(ln\left(TRI_{k,t}\right) - ln\left(TRI_{k,t-1}\right)\right),\ k = 1, 2, ..., 29.$$

Besides these three highly volatile regimes, the graph also consists of a few negative large spikes which do not fall in these time periods. Out of 29 stocks, 23 have high depreciation (maximum negative returns) during the above-mentioned high volatile periods, while for the remaining ones this tumbling is caused by "Bad News" which is called as "Bad News Impact" in the financial world.

Table 3.1 and Figure 3.1.2 highlight some facts regarding the causes for the tumbling of stock returns during the sampling period. Table 3.1 reports the dates on which the returns from these stocks were at their minimal level. Figure 3.1.2 shows the distribution of these causes in terms of percentages. It can be inferred from the table and the figure that the majority (approximately 40%) of these stocks reached their minimum level during the financial crisis of 2008-09, followed by the Stock Market Crash of 2002, during which 30% of the stock returns fell down. Interestingly, the impact of bad news takes the third position as the cause for stock returns sinking as described further. The final position in this regard is taken by the 9/11 terrorist attack. A brief summary about this impact of bad news is described in the following

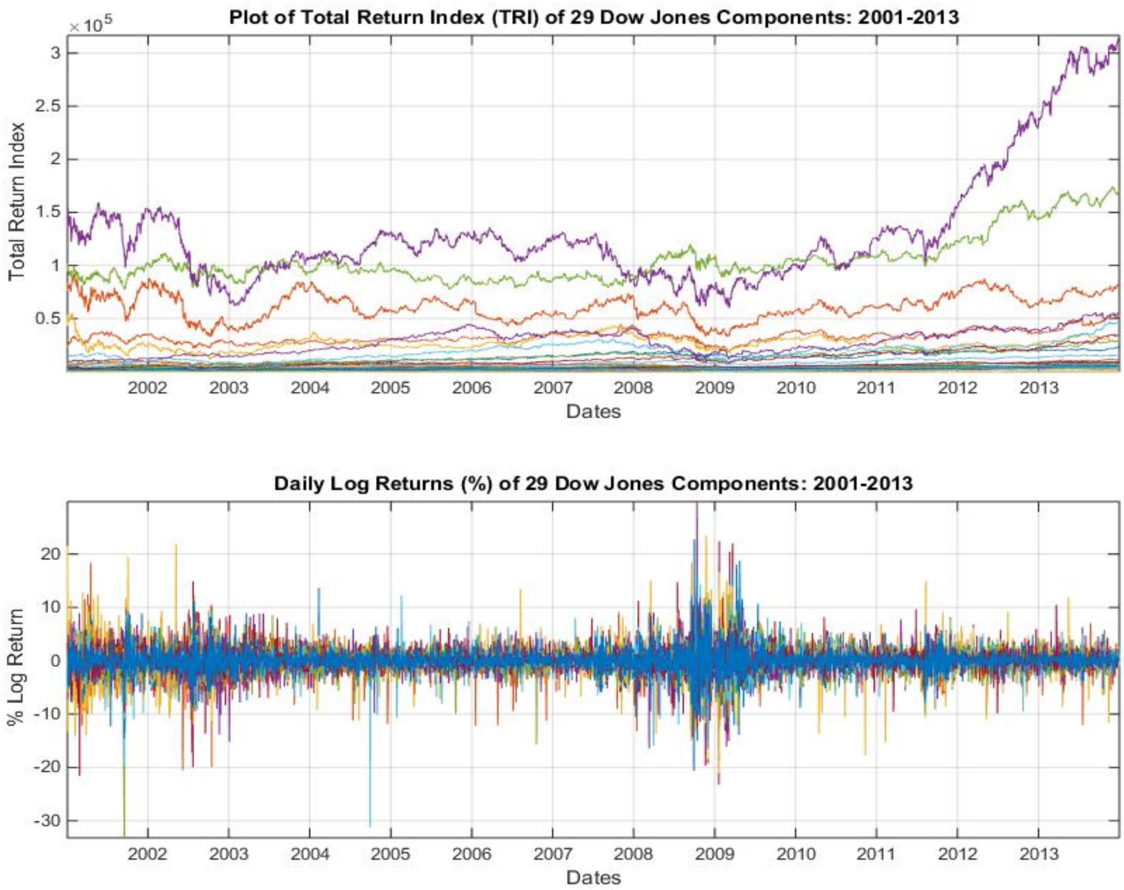
[3]http.//thomsonreuters.com

[4]Financial holidays were removed with the help of MATLAB's command "holidays" which indicates New York Stock Exchange holidays and non-trading days since 1985 to date (based on a modern five-day working week).

[5]The natural logarithm of the simple gross returns of an asset is called the continuously compounded returns or *log returns* (Tsay, 2010, p. 5).

paragraphs.

Figure 3.1.1: Daily total return index and percent log returns of 29 components of Dow Jones: 2001-2013



For example, shares in Nike (NKE), a manufacturer of athletic shoes and clothing, fell down on Tuesday, 27 February, 2001, after the company announced it expected the slowdown in the United States economy to slice at least 24% from its third-quarter profits[6].

Similarly, on September 30, 2004, Merck and company announced that it was immediately pulling its arthritis and acute pain medication Vioxx[7] from the market worldwide due to the findings that it increases heart attack and stroke risk in patients. This bad news plunged the stock.

On December 17, 2004, CNN (Cable News Network) reported that Pfizer

[6]http://news.bbc.co.uk/2/hi/business/1191894.stm

[7]A company spokesman said that there are two million current users of Vioxx worldwide and that 84 million prescriptions had been written for the drug since 1999.

has no plans to pull the popular painkiller Celebrex[8] off the market despite the data showing that patients using the drug in a long-term cancer study had more than double the risk of a heart attack. This bad news sent stocks tumbling.

Caterpillar (CAT) reported disappointing news regarding company's quarterly earnings and cut its 2006 and 2007 forecast on slowing demand for its earth-moving equipments and rising raw material prices that are eroding margins [9].

Table 3.1: Stocks with their minimum of percentage log returns: 2001-2013

S.No.	Component's Name	Ticker	Minimum	Date
1	Exxon Mobil Corporation	XOM	-15.03	October 15, 2008
2	Microsoft Corporation	MSFT	-12.46	January 22, 2009
3	General Electric Company	GE	-13.68	April 11, 2008
4	Johnson & Johnson	JNJ	-17.25	July 19, 2002
5	Wal-Mart Stores Incorporated	WMT	-8.41	October 15, 2008
6	Chevron Corporation	CVX	-13.34	October 15, 2008
7	JPMorgan Chase & Company.	JPM	-23.23	January 20, 2009
8	Procter & Gamble Company	PG	-8.32	September 21, 2001
9	Pfizer Inc.	PFE	-11.82	December 17, 2004
10	International Business Machines Corporation	IBM	-10.67	April 8, 2002
11	AT&T Incorporated	T	-10.75	July 22, 2002
12	Coca-Cola Company	KO	-10.60	October 16, 2002
13	Merck & Company Incorporated	MRK	-31.17	September 30, 2004
14	Verizon Communications Incorporated	VZ	-12.61	July 22, 2002
15	Walt Disney Company	DIS	-20.29	September 17, 2001
16	Intel Corporation	INTC	-20.48	June 7, 2002
17	Cisco Systems Incorporated	CSCO	-17.69	November 11, 2010
18	Home Depot Incorporated	HD	-15.16	January 3, 2003
19	United Technologies Corporation	UTX	-33.20	September 17, 2001
20	Boeing Company	BA	-19.39	September 17, 2001
21	McDonald's Corporation	MCD	-13.72	September 17, 2002
22	American Express Company	AXP	-19.35	September 29, 2008
23	3M Company	MMM	-9.38	July 7, 2006
24	Goldman Sachs Group Incorporated	GS	-21.03	January 20, 2009
25	UnitedHealth Group Incorporated	UNH	-20.62	September 29, 2008
26	Caterpillar Inc.	CAT	-15.69	October 20, 2006
27	E. I. du Pont de Nemours and Company	DD	-12.03	December 1, 2008
28	NIKE Incorporation	NKE	-21.65	February 27, 2001
29	Travelers Companies Incorporated	TRV	-20.07	September 29, 2008

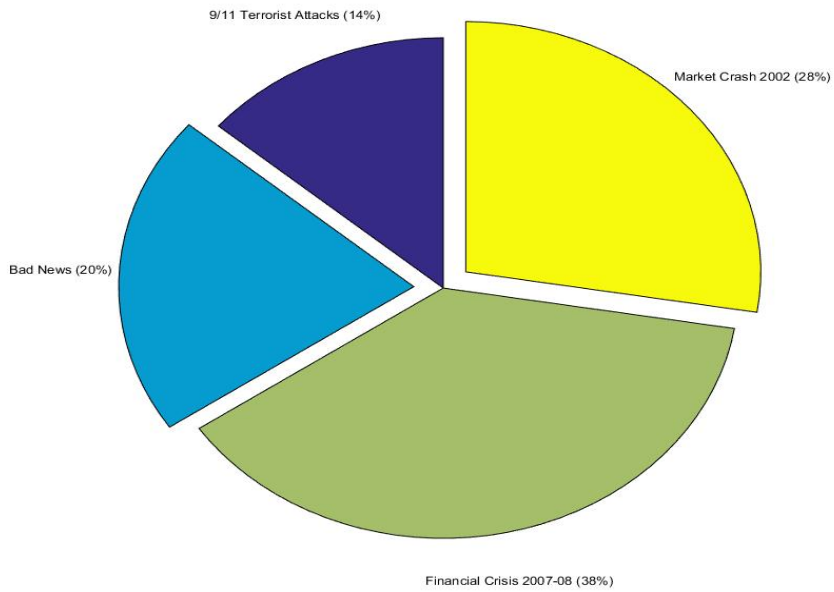
9/11/2001 | Stock Market Crash (2002)

Financial Crisis (2008-09) | Bad News Impact

[8]According to Pfizer, it is "the world's most-prescribed arthritis drug" and has been prescribed to 27 million Americans.

[9]http.//articles.latimes.com/2006/oct/21/business/fi-earns21.2

Figure 3.1.2: Pie-chart showing the distribution of causes of depreciation in stock returns



Similarly, in a conference with financial analysts, Cisco CEO said that Cisco expects that its revenue will grow only by 3-5 % in the current quarter of 2002 compared to the last year. This news caused Cisco shares to sink by about 18% [10].

Shares in 3M Co. sank on July 7, 2006, after the company warned that its second-quarter profit will miss earlier guidance on weaker-than-expected demand for optical films used in flat-screen televisions and computer monitors. [11]

3.2 Application of proposed methodology

After describing the data set under consideration in detail, we now proceed to the empirical application of our proposed methodology. Table 3.2 presents some relevant summary statistics of the percentage log returns, a test of normality (Jarque-Bera), and a test of linear dependence (Ljung-Box). It can be seen that almost all stocks have positive returns over the whole sample period with the exception of General Electric (GE), Merck (MRK), and Cisco (CSCO). The sample skewness $\hat{\mu}_3/\hat{\mu}_2^{3/2}$ indicates considerable asymmetry which, taken together with the sample kurtosis $\hat{\mu}_4/\hat{\mu}_2^2$, indicates a substantial violation of normality. The Jarque-Bera test shows the same results regarding normality assumption of the returns since the test statistic is based on these two moments of the distribution.

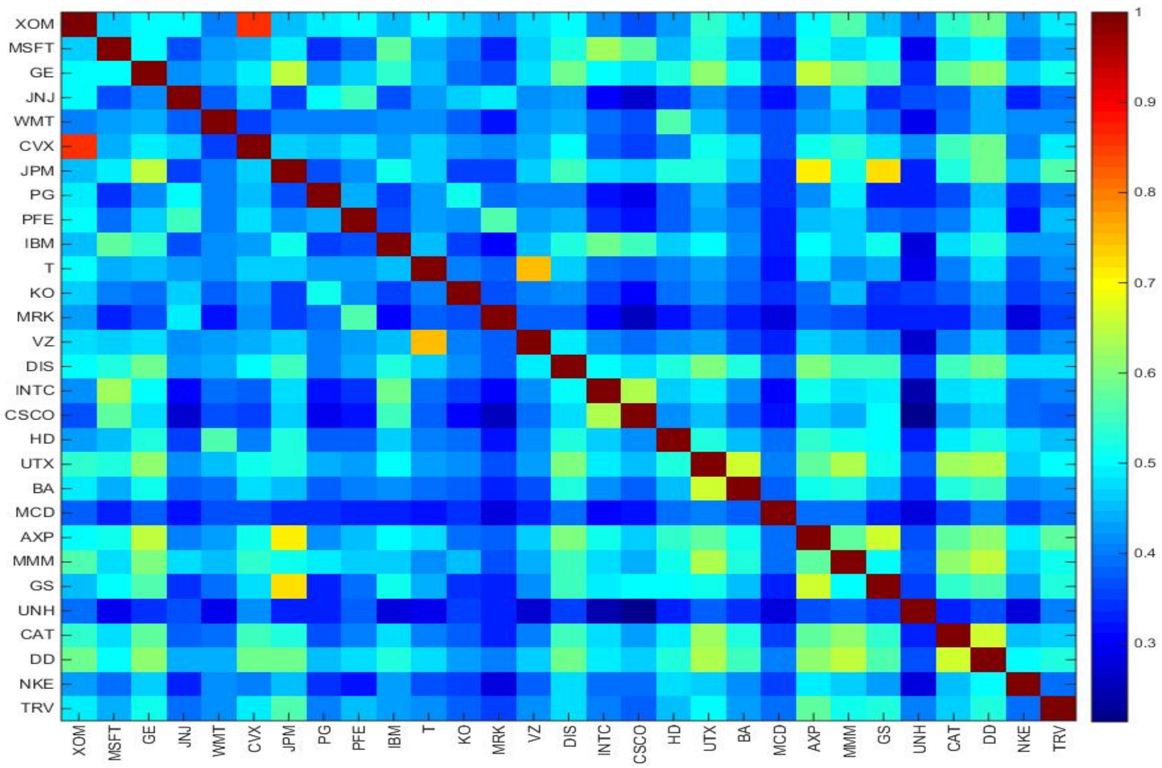
[10] http.//www.sify.com/finance/cisco-shares-plummet-sink-broader-market-news-technology-kllvumgdhfh.html?tag=Electric%20Equipment%20%20%20Switchgear,Circuit

[11] http.//www.wsj.com/articles/SB115227770129300606

Table 3.2: Summary statistics of percent log returns of Dow Jones 29 components

Ticker	Min.	Max.	Mean	Variance	Skewness	Kurtosis	Jarque Bera	Ljung-Box Q_{20}
XOM	-15.03	15.86	0.034	2.540	-0.02	14.8	0.000	0.000
MSFT	-12.46	17.06	0.026	3.605	0.15	10.0	0.000	0.000
GE	-13.68	17.98	-0.001	4.148	0.05	11.6	0.000	0.000
JNJ	-17.25	11.54	0.028	1.400	-0.58	23.3	0.000	0.000
WMT	-8.41	10.50	0.018	1.918	0.20	7.8	0.000	0.000
CVX	-13.34	18.94	0.046	2.725	0.02	16.0	0.000	0.000
JPM	-23.23	22.39	0.021	7.533	0.27	15.6	0.000	0.000
PG	-8.32	9.73	0.032	1.428	-0.22	9.5	0.000	0.000
PFE	-11.82	9.69	0.001	2.609	-0.34	8.8	0.000	0.000
IBM	-10.67	11.35	0.030	2.564	0.21	9.6	0.000	0.000
T	-10.75	15.08	0.009	2.787	0.18	9.4	0.000	0.013
KO	-10.60	13.00	0.019	1.613	0.03	12.7	0.000	0.013
MRK	-31.17	12.25	-0.002	3.331	-1.93	36.1	0.000	0.024
VZ	-12.61	13.66	0.021	2.540	0.11	9.1	0.000	0.000
DIS	-20.29	14.82	0.036	3.976	-0.02	11.2	0.000	0.003
INTC	-20.48	18.33	0.003	5.690	-0.23	10.0	0.000	0.000
CSCO	-17.69	21.82	-0.010	6.677	0.14	11.8	0.000	0.000
HD	-15.16	13.16	0.026	3.955	0.10	8.6	0.000	0.011
UTX	-33.20	12.79	0.042	3.150	-1.97	44.7	0.000	0.000
BA	-19.39	14.38	0.032	3.832	-0.33	9.2	0.000	0.004
MCD	-13.72	8.97	0.042	2.194	-0.20	9.4	0.000	0.083
AXP	-19.35	18.77	0.026	6.057	0.00	12.5	0.000	0.000
MMM	-9.38	9.42	0.036	2.195	-0.16	8.0	0.000	0.001
GS	-21.03	23.48	0.021	6.086	0.34	16.0	0.000	0.000
UNH	-20.62	29.83	0.052	4.495	0.29	24.6	0.000	0.000
CAT	-15.69	13.74	0.051	4.436	-0.11	7.6	0.000	0.000
DD	-12.03	10.86	0.024	3.243	-0.23	8.5	0.000	0.000
NKE	-21.65	11.88	0.059	3.401	-0.36	14.0	0.000	0.001
TRV	-20.07	22.76	0.028	3.859	0.27	19.5	0.000	0.000

Figure 3.2.1: Sample correlation matrix of percentage log returns



Only p-values of the test statistics are reported here, which are all highly significant at the conventional level of significance, i.e., $\alpha = 0.05$. The presence of serial correlation in returns was also checked with a portmanteau test using first 20 lags. The results of the Ljung-Box test show that returns also possess high significant autocorrelations, which is an indication to filter these returns through $ARMA(p,q)$ model, with the exception of McDonald's (MCD), for which the p-value is greater than the selected $\alpha-$level. However, in this study apply this filtration process to all the stocks under consideration.

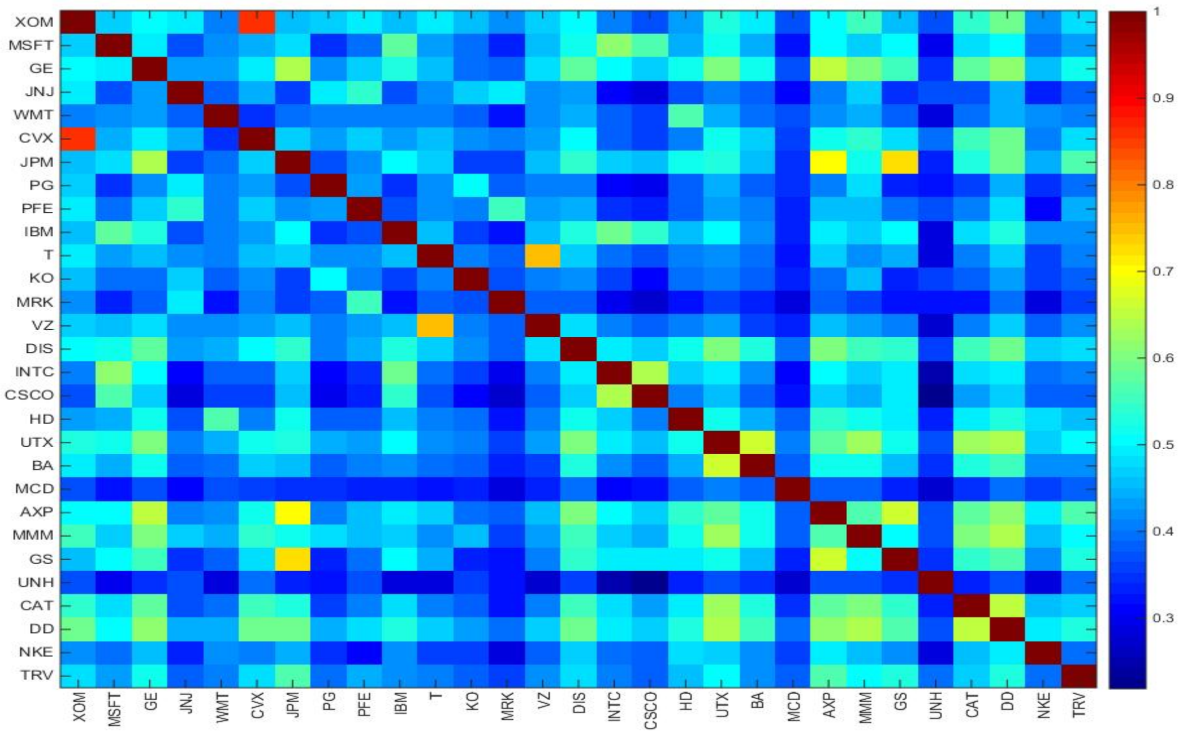
Figure 3.1.3 shows the sample correlation matrix of the returns. It can be seen that all stocks are positively correlated, i.e., the individual stock returns tend to move in the same direction with each other. It can also be observed that stocks belonging to the same sector possess strong correlation with each other, e.g., XOM and CVX, which belong to oil and gas sector, $\hat{\rho} = 0.86$, T and VZ, which belong to the telecommunications sector, are highly correlated with $\hat{\rho} = 0.75$, and for JPM and GS, which belong to financial sector, $\hat{\rho} = 0.73$, and so on. The converse of the previous statement regarding correlation is true if the stocks belong to different sectors, e.g., for MRK and CSCO $\hat{\rho} = 0.26$; similarly, for UNH & INTC $\hat{\rho} = 0.25$, etc.

Following the results presented in Table 3.2, different $ARMA(p,q)$ specifications were used to account for the dependence structure in mean returns. The lag length selection is purely based on information criteria such as AIC, and BIC. Since the main purpose of the present study is to forecast the future, $AR(2)$ model was selected on the basis of BIC. The ARMA (p,q) estimates for all stock returns are given in Appendix 3.4.2. It can be seen that BIC selects the model whose residuals possess the white noise property. Table 3.3 presents the summary statistics of $AR(2)-$filtered returns along with Ljung-Box, Engle ARCH, and McLeod-Li tests used to detect the presence of volatility clustering in the filtered returns and their squares respectively. In contrast to the previous table, now all stocks have exactly zero mean returns without any negative or positive values, which is in line with the findings reported in the financial literature that stocks have zero mean returns. Like the previous table, sample skewness and kurtosis still suggest the violation of normality, also proved by Jarque-Bera normality test. From the p-values of Ljung-Box test statistics it can be seen that there are no autocorrelations left in the filtered returns. However, Engle ARCH and McLeod-Li tests show the presence of highly significant autocorrelations in the squared filtered returns causes volatility clustering. All p-values are highly significant, and as we are dealing with multivariate data set, multivariate GARCH($MGARCH$) model is required in order to account for this dependence in squared returns.

Table 3.3: Summary statistics of $AR(2)-$filtered returns and testing for ARCH effects

Ticker	Min.	Max.	Mean	Variance	Skewness	Kurtosis	Jarque Bera	Ljung-Box Q_{20}	McLeod-Li Q_{20}	Engle ARCH Q_{20}
XOM	-13.25	13.07	0.000	2.438	-0.32	11.4	0.000	0.475	0.000	0.000
MSFT	-12.61	16.02	0.000	3.480	-0.02	9.4	0.000	0.972	0.000	0.000
GE	-13.52	17.72	0.000	4.062	-0.02	11.4	0.000	0.147	0.000	0.000
JNJ	-17.41	10.54	0.000	1.366	-0.75	23.7	0.000	0.941	0.000	0.000
WMT	-8.15	9.35	0.000	1.848	0.07	7.2	0.000	0.790	0.000	0.000
CVX	-12.15	16.43	0.000	2.649	-0.23	12.8	0.000	0.553	0.000	0.000
JPM	-24.08	20.70	0.000	7.337	0.01	14.9	0.000	0.917	0.000	0.000
PG	-8.38	9.12	0.000	1.382	-0.29	9.1	0.000	0.970	0.000	0.000
PFE	-11.35	9.47	0.000	2.551	-0.39	8.5	0.000	0.931	0.000	0.000
IBM	-10.63	10.41	0.000	2.458	0.01	8.4	0.000	0.985	0.000	0.000
T	-11.21	14.57	0.000	2.753	0.13	9.1	0.000	0.999	0.000	0.000
KO	-10.51	12.44	0.000	1.589	-0.07	12.1	0.000	0.978	0.000	0.000
MRK	-31.14	11.90	0.000	3.294	-2.02	36.6	0.024	0.997	0.006	0.011
VZ	-12.53	13.45	0.000	2.488	0.09	9.0	0.000	0.985	0.000	0.000
DIS	-20.53	14.26	0.000	3.881	-0.13	11.3	0.000	0.609	0.000	0.000
INTC	-20.21	18.71	0.000	5.553	-0.22	9.6	0.000	0.879	0.000	0.000
CSCO	-17.60	21.39	0.000	6.377	0.00	10.4	0.000	0.995	0.000	0.000
HD	-15.28	12.92	0.000	3.831	-0.01	8.4	0.000	0.985	0.000	0.000
UTX	-33.03	12.43	0.000	3.079	-2.03	45.1	0.000	0.902	0.000	0.000
BA	-19.26	13.35	0.000	3.794	-0.34	8.9	0.000	0.808	0.000	0.000
MCD	-13.80	8.88	0.000	2.164	-0.23	9.3	0.000	0.989	0.000	0.000
AXP	-19.43	19.00	0.000	5.927	-0.08	12.4	0.000	0.100	0.000	0.000
MMM	-9.28	8.94	0.000	2.158	-0.21	7.8	0.000	0.956	0.000	0.000
GS	-21.48	22.37	0.000	5.925	0.08	14.4	0.000	0.639	0.000	0.000
UNH	-20.74	27.61	0.000	4.402	0.09	21.7	0.000	0.975	0.000	0.000
CAT	-15.72	13.78	0.000	4.386	-0.10	7.5	0.000	0.238	0.000	0.000
DD	-11.66	10.60	0.000	3.195	-0.25	8.3	0.000	0.374	0.000	0.000
NKE	-21.78	10.55	0.000	3.330	-0.45	14.1	0.000	0.994	0.000	0.000
TRV	-20.51	18.53	0.000	3.702	0.04	16.2	0.000	0.652	0.000	0.000

Figure 3.2.2: Sample correlation matrix of mean-filtered returns



The sample correlation matrix of the mean-filtered returns is also shown after Table 3.3. A quick look at the correlation matrix presented in Figure 3.2.2 allows to draw a general conclusion that mean filtering process preserves the same correlation structure between the stocks as shown in the earlier correlation plot of the original percent log returns (Figure 3.2.1). Even the estimates for the correlations are exactly the same as those presented earlier.

3.2.1 Results of SVX after the first round

The following figure shows the results of our proposed methodology described in the previous chapter. The first panel of the Figure 3.2.3 shows the plot of $AR\,(2)$ −filtered returns ε_t, which is very similar to the plot of original log returns as the mean of the returns (which is equal to 0) was subtracted from each value of the vector of returns. The second panel shows the time plot of singular values λ_{1t} resulted from the first round of singular value decomposition (SVD). It can be clearly seen that these singular values, which play the role of returns variance, are good representatives of the volatility in the data set. We can also notice how well these singular values capture the volatility clustering from the data set, i.e., the clusters of spikes in λ_{1t} correspond exactly to the clusters in the $AR\,(2)$ −filtered returns.

Following the results of the simulation study from the methodology chapter, the third panel of the figure shows the log transformed singular values $log\,(\lambda_{1t})$, which are used later in this chapter when modeling the dynamics of singular values. Since the large spikes in λ_{1t} tell us about the positively skewed distribution of a singular values, log-transformation helps us to make it less skewed. The last two panels show the time plot of singular vector components $U_{i,t}$ and the corresponding Fisher-transformed singular vector components $Z_{i,t}$, respectively. The values of $U_{i,t}$ lie between -1 and $+1$, making it hard to check whether the $u's$ are clustered or not. We use Fisher-transformation, which bounded these singular vector components $+\infty, -\infty$, making it easier to test for the presence of volatility clustering. Form the last panel, one can see that Fisher-transformed singular vector components are now bounded between -3 and +4.

Since the question regarding the presence of volatility clustering in the data set is addressed through checking the presence of this phenomenon in the resulting singular vector $U_{1,t}$, thus following the description of our proposed methodology from the first section of the previous chapter, we use Fisher transformation to stabilize the variance in the components of the singular vector $U_{1,i,t}$. The summary statistics and the results of the test of the ARCH effects presence in singular vector components along with the Fisher transformation are presented in Tables 3.4 and 3.5, respectively.

Figure 3.2.3: Results of SVX model after the first round of SVD

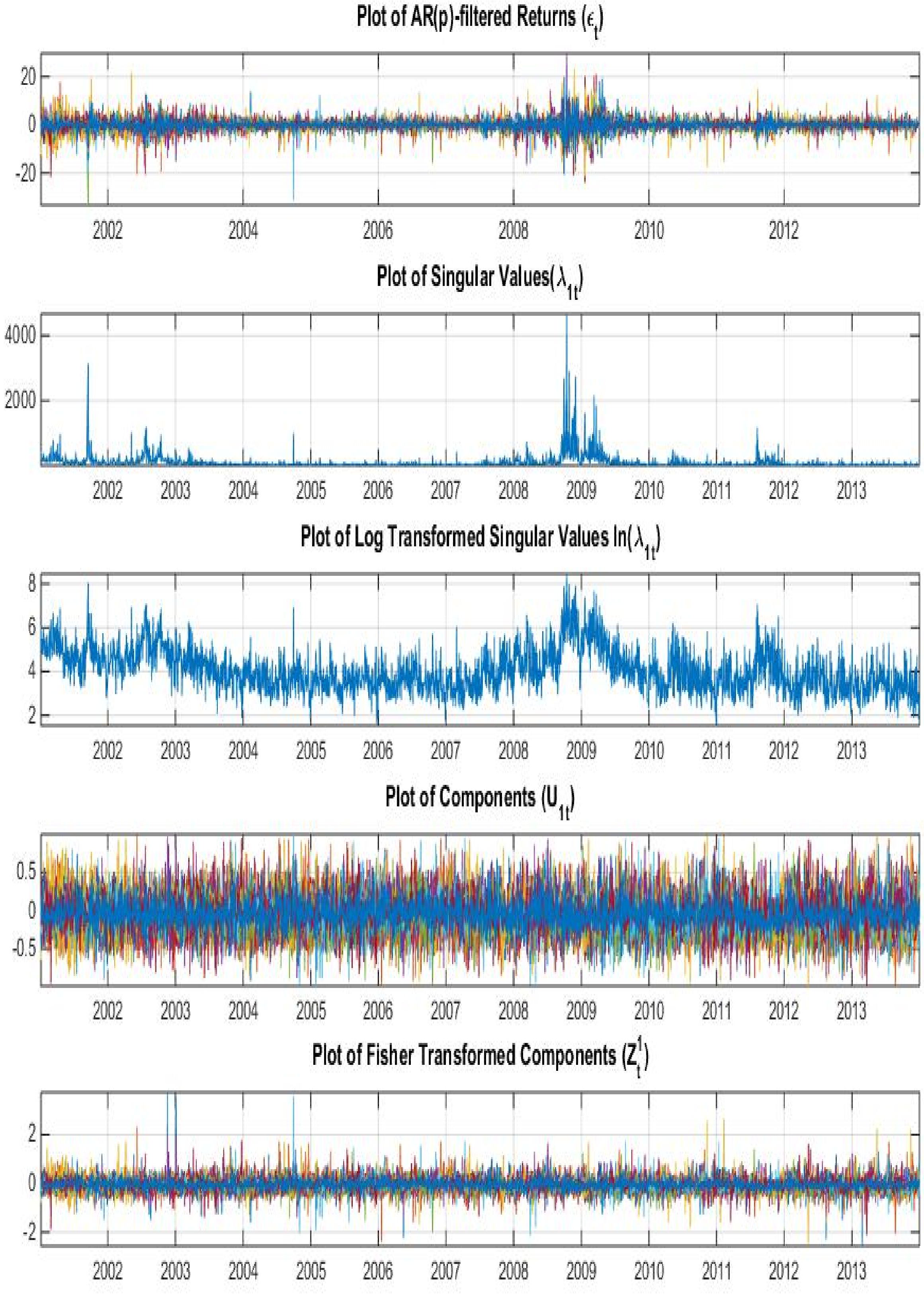


Table 3.4: Summary statistics and testing of ARCH effects in components of first singular vector $U_{1,t}$

Ticker	Mean	Variance	Skewness	Kurtosis	Ljung-Box Q_{20}	McLeod Li Q_{20}	Engle ARCH Q_{20}
XOM	-0.129	0.010	-1.359	5.80	0.00	0.00	0.00
MSFT	-0.045	0.033	0.385	5.34	0.00	0.00	0.00
GE	-0.061	0.029	0.295	5.08	0.00	0.00	0.00
JNJ	-0.031	0.014	0.238	5.78	0.00	0.00	0.00
WMT	-0.029	0.023	0.345	6.05	0.00	0.00	0.00
CVX	-0.107	0.018	0.004	4.71	0.00	0.00	0.00
JPM	-0.070	0.046	0.268	3.79	0.00	0.00	0.00
PG	-0.033	0.016	0.121	6.35	00.17	0.17	0.04
PFE	-0.045	0.027	0.350	5.25	0.12	0.12	0.00
IBM	-0.042	0.023	0.548	5.78	0.00	0.00	0.04
T	-0.046	0.026	0.223	4.42	0.33	0.33	0.00
KO	-0.036	0.017	0.105	6.44	0.00	0.00	0.00
MRK	-0.038	0.033	0.302	6.64	0.01	0.01	0.00
VZ	-0.042	0.025	0.201	4.28	0.04	0.04	0.00
DIS	-0.051	0.035	0.304	4.17	0.00	0.00	0.41
INTC	-0.055	0.054	0.431	3.90	0.02	0.02	0.00
CSCO	-0.055	0.057	0.374	4.58	0.27	0.27	0.00
HD	-0.049	0.038	0.317	4.49	0.01	0.01	0.00
UTX	-0.055	0.024	0.354	4.09	0.02	0.02	0.00
BA	-0.056	0.040	0.463	4.50	0.13	0.13	0.00
MCD	-0.032	0.028	0.269	6.19	0.15	0.15	0.00
AXP	-0.063	0.040	0.313	3.98	0.00	0.00	0.00
MMM	-0.052	0.020	0.326	7.71	0.00	0.00	0.02
GS	-0.067	0.046	0.114	3.41	0.06	0.06	0.00
UNH	-0.040	0.048	0.296	4.52	0.11	0.11	0.00
CAT	-0.077	0.041	0.283	4.07	0.10	0.10	0.00
DD	-0.064	0.028	0.123	4.04	0.00	0.00	0.01
NKE	-0.041	0.035	0.101	5.63	0.31	0.31	0.27
TRV	-0.046	0.031	0.217	4.54	0.00	0.00	0.00

Table 3.5: Summary statistics & testing of ARCH effects in Fisher transformed components of first singular vector $Z_{K_1,t}$

Ticker	Mean	Variance	Skewness	Kurtosis	Ljung-Box Q_{20}	McLeod Li Q_{20}	Engle ARCH Q_{20}
XOM	-0.048	0.059	1.435	46.60	0.15	1.00	1.00
MSFT	-0.045	0.041	0.521	13.59	0.00	0.35	0.39
GE	-0.062	0.034	0.175	10.56	0.00	0.00	0.00
JNJ	-0.032	0.014	0.217	7.13	0.00	0.00	0.00
WMT	-0.029	0.026	0.435	10.14	0.00	0.00	0.00
CVX	-0.110	0.020	-0.120	5.51	0.00	0.00	0.00
JPM	-0.073	0.054	0.361	5.88	0.00	0.00	0.00
PG	-0.033	0.018	0.153	10.22	0.26	0.07	0.13
PFE	-0.046	0.031	0.660	12.06	0.07	0.00	0.00
IBM	-0.042	0.026	1.007	11.48	0.02	0.68	0.71
T	-0.047	0.029	0.159	5.73	0.32	0.00	0.00
KO	-0.036	0.019	-0.277	11.44	0.00	0.01	0.01
MRK	-0.038	0.048	1.126	40.21	0.01	0.92	0.94
VZ	-0.043	0.028	0.095	5.40	0.04	0.00	0.00
DIS	-0.053	0.040	0.138	7.01	0.00	0.88	0.86
INTC	-0.056	0.067	0.549	9.92	0.06	0.21	0.32
CSCO	-0.056	0.080	0.905	15.48	0.40	0.36	0.65
HD	-0.050	0.045	0.543	10.55	0.03	0.98	0.98
UTX	-0.056	0.026	0.290	5.18	0.04	0.00	0.00
BA	-0.057	0.048	0.678	8.20	0.21	0.06	0.07
MCD	-0.033	0.034	0.447	13.17	0.10	0.00	0.00
AXP	-0.065	0.046	0.227	5.42	0.00	0.00	0.00
MMM	-0.053	0.024	0.230	18.59	0.00	0.79	0.83
GS	-0.071	0.053	-0.102	4.92	0.07	0.00	0.00
UNH	-0.041	0.059	0.367	7.05	0.14	0.00	0.01
CAT	-0.080	0.049	0.025	7.44	0.23	0.02	0.06
DD	-0.066	0.031	-0.302	7.23	0.00	0.65	0.66
NKE	-0.043	0.044	-0.497	14.12	0.71	0.15	0.16
TRV	-0.048	0.035	0.120	6.25	0.01	0.00	0.01

The values of skewness and kurtosis are still away from symmetry and indicate that any asymmetric distribution would be an ideal candidate to account for this asymmetry in a singular vector (as shown in the next chapter). Their histograms with normal and Student's t-density (location-scale) fit are presented in Appendices 3.4.3 and 3.4.4, respectively. As can be seen from these histograms, those with Student's t-density function provide a better fit when compared to those with normal ones. So far as the results of McLeod-Li and Engle ARCH tests are concerned, it can be seen from Table 3.4 that in case of the former test, for 10 components out of 29 we do not reject the null of serial autocorrelations (as shown by the green boxes), whereas in case of the latter test, this number reduces to only 2 (as shown by the cyan-colored boxes).

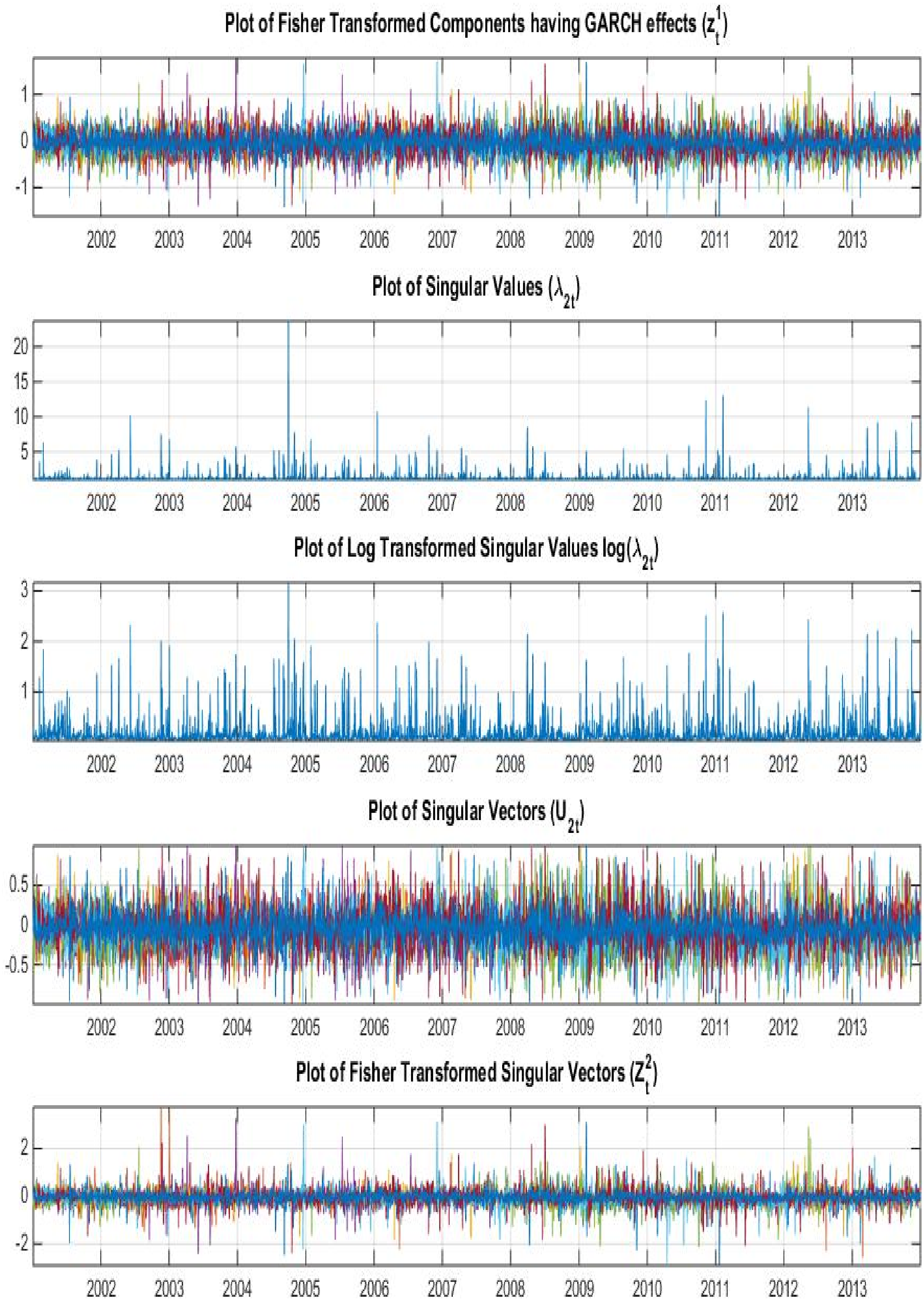
However, when Fisher transformation was applied to all these components, the results for these two tests are changed to a great extent, i.e., at 5% $\alpha-$level nearly half of the total stocks (14 out of 29) exhibits no volatility clustering as represented by the green-and cyan-colored boxes in the last two columns of Table 3.4. Since for these stocks the p-values are greater than the significance level, accepting the null hypothesis of no significant autocorrelations and no conditional heteroskedasticity in those Fisher-transformed singular vector components. It is also interesting to note that the stocks exhibiting no volatility clustering belong to seven different sectors such as Technology, Consumer Goods and Services, Industry, etc., which means that even though they have different sectoral classification, there is one common factor responsible for driving the volatility in these stocks.

So far as the results of Ljung-Box test regarding singular vector components are concerned which are reported in for Table 3.4 10 out of 29 components are not showing high significant autocorrelations. The Ljung-Box test results for the Fisher-transformed singular vector components which are reported in Table 3.5 this number increases to 12. Furthermore, these components are not exactly the same for which their Fisher-transformed singular vector components have passed through the other two tests which are used to detect the volatility clustering, i.e, McLeod-Li and Engle ARCH test.

3.2.2 Results of SVX after the second round

For the remaining fifteen stocks presented in Table 3.5, the p-values of the tests statistics show that these stocks still exhibit volatility clustering and hence require another round of the third step of our proposed methodology (which consists of a repetition of Steps 1 and 2). Following the methodology, only those fifteen components which exhibit volatility clustering from the last round were taken into the second round

Figure 3.2.4: Results of SVX model after the second round of SVD



of SVD; the results from this round are presented in the following Figure3.2.4 and Table 3.6.

Unlike the previous figure, the first panel shows the time series of only those Fisher transformed components which exhibit volatility clustering from the first round of SVD. The second and the third panel show the time plots of the second sequence of the resulting singular values λ_{2t} and the log-transformed values $log\left(\lambda_{2t}\right)$, respectively.

Table 3.6: Summary statistics and testing of ARCH effects in components of second singular vector $U_{2,t}$

Ticker	Mean	Variance	Skewness	Kurtosis	Ljung-Box Q_{20}	McLeod Li Q_{20}	Engle ARCH Q_{20}
GE	-0.058	0.028	0.190	7.39	0.00	0.00	0.00
JNJ	-0.029	0.013	0.176	8.61	0.00	0.00	0.00
WMT	-0.027	0.022	0.367	9.51	0.00	0.00	0.00
CVX	-0.100	0.018	-0.330	6.08	0.00	0.00	0.00
JPM	-0.070	0.047	0.169	5.02	0.00	0.00	0.00
PFE	-0.043	0.026	0.263	7.57	0.08	0.00	0.00
T	-0.044	0.025	0.102	6.28	0.12	0.00	0.00
KO	-0.033	0.016	-0.042	10.20	0.00	0.04	0.05
VZ	-0.040	0.024	0.031	5.93	0.03	0.00	0.00
UTX	-0.052	0.023	0.270	5.63	0.01	0.00	0.00
MCD	-0.031	0.028	0.195	8.95	0.10	0.00	0.00
AXP	-0.062	0.041	0.229	5.48	0.00	0.00	0.00
GS	-0.067	0.047	-0.062	4.60	0.03	0.00	0.00
UNH	-0.038	0.052	0.257	6.14	0.07	0.00	0.00
TRV	-0.045	0.031	0.088	6.67	0.01	0.00	0.01

Table 3.7: Summary statistics & testing of ARCH effects in Fisher transformed components of second singular vector $Z_{K_2,t}$

Ticker	Mean	Variance	Skewness	Kurtosis	Ljung-Box Q_{20}	McLeod Li Q_{20}	Engle ARCH Q_{20}
GE	-0.060	0.042	-0.334	45.18	0.00	0.56	0.59
JNJ	-0.029	0.014	0.097	13.71	0.00	0.18	0.31
WMT	-0.027	0.030	0.785	32.71	0.00	0.00	0.00
CVX	-0.103	0.021	-0.482	8.79	0.00	0.00	0.00
JPM	-0.073	0.065	0.811	17.60	0.00	0.00	0.00
PFE	-0.043	0.037	2.086	57.00	0.15	0.00	0.00
T	-0.045	0.030	-0.120	12.73	0.08	0.00	0.00
KO	-0.035	0.021	-2.182	46.91	0.00	0.68	0.67
VZ	-0.048	0.059	1.435	46.60	0.00	0.06	0.14
UTX	-0.053	0.026	-0.054	12.08	0.05	0.07	0.06
MCD	-0.031	0.045	1.165	50.70	0.09	0.07	0.15
AXP	-0.065	0.053	-0.215	12.79	0.00	0.00	0.00
GS	-0.073	0.063	-0.987	14.12	0.05	0.00	0.00
UNH	-0.039	0.080	0.620	18.54	0.12	0.12	0.13
TRV	-0.047	0.039	-0.323	14.79	0.12	0.46	0.32

As it can be clearly seen from the plot of singular values λ_{2t} and its log-transformed counterpart these two sequences are now lower in magnitude and also less clustered as compared to the time plot of $\lambda_{1,t}$, presented in Figure 3.2.3. This is because of the lower magnitude of the components presented in the first panel of the figure since the values are now bounded between -2 to +2. However, the spikes in singular values become more visible when log-transformation is used, and it looks that more asymmetry is present in the singular values. The fourth and the fifth panel show the plot of singular vector components $U_{2,i,t}$ and the corresponding Fisher-transformed singular vector components $Z_t^{(2)}$, respectively, which allows to describe them in the same fashion as we described for the first round of the SVD. The only difference is that the time plot of these singular vector components is now much clearer than the time plot of singular vector components from the first round. This is because in the second round of SVD we have a smaller number of singular vector components, i.e, only 15.

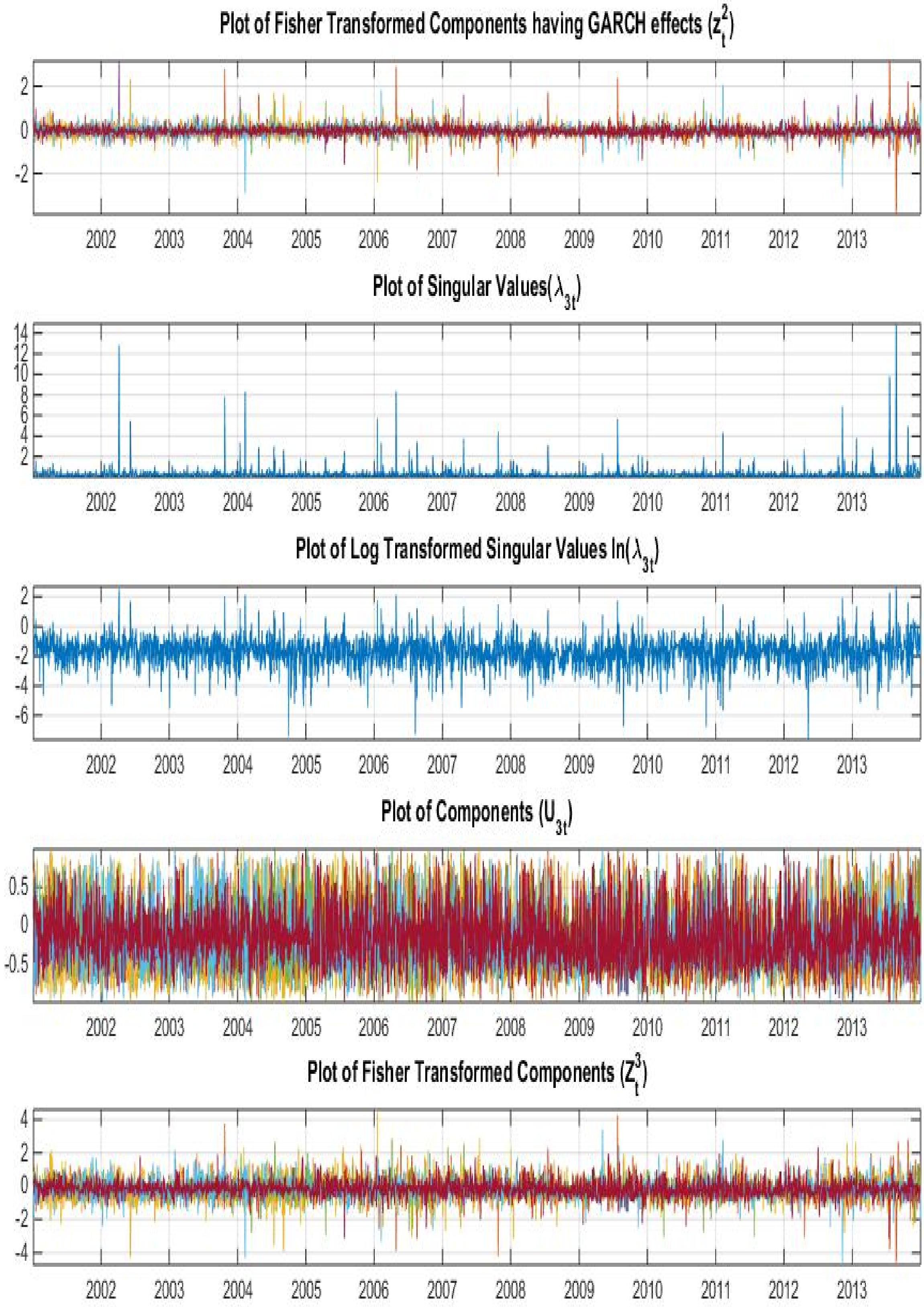
Table 3.6 and 3.7 represent summary statistics and results of the tests used to detect significant autocorrelations and volatility clustering in the components of the second singular vector and the Fisher-transformed ones. All of these components have a negative mean and they are away from symmetry as shown by the values of skewness and kurtosis. Out of 15, the null of no significant autocorrelations is accepted only for 4 components. It can be seen from the last two columns of Table 3.6 that none of the components pass the tests used to detect volatility clustering. However, after applying Fisher transformation, it can be observed that eight components exhibit no volatility clustering (as shown with green-and cyan-colored boxes), while this is not true for the remaining 7 stocks which still possess volatility clustering. Every two of these stocks belong to one particular sector, e.g., GE and UTX belong to the Industrial sector, JNJ and UNH belong to Health Care, KO and MCD belong to Consumer Services, and only VZ and TRV belong to two different sectors, i.e., Telecommunications and Finance, respectively.

The PFE and T after Fisher transformation show no significant autocorrelations (shown by magenta-colored boxes) but autocorrelations in their squares are still very significant as shown by the corresponding p-values of McLeod-Li and Engle ARCH test statistics.

3.2.3 Results of SVX after the third round

The results shown in Table 3.7 are suggesting to use the third step of our proposed methodology one more time in order to account for volatility clustering from the remaining stocks. For this reason, we applied SVD one more time by considering only those Fisher-transformed singular vector components which exhibit volatility clustering from the second round of SVD. The results obtained are presented in the Figure3.2.5 and the Table3.8. The first panel of the Figure 3.2.5 shows the time plot of those components which exhibit volatility clustering after the previous round. The

Figure 3.2.5: Results of SVX model after third round of SVD



second panel of the figure under discussion shows the time plot of the singular values $\lambda_{3,t}$. It can be easily seen from the figure that the values are less clustered as compared to the previous two rounds of the SVD and only a few large spikes are observed. These singular values are even smaller in magnitude than those presented in Figure 3.2.4. This fact might lead us to conclude that 1) $\lambda_{3,t}$ has captured all volatility clustering from the remaining stocks, and 2) the series exhibit no serial correlation as the large spikes are not clustered. However, we verify this statement by using the tests devised to check the presence of volatility clustering phenomenon. The plot of $U_{3,t}$ is more dense as compared to the previous one and the corresponding values of $Z_{3,t}$ are bounded between -5 to +5 which are bit bigger than those of previous one.

Table 3.8 shows the summary statistics and the results regarding the presence of volatility clustering in the resulting singular vector components. The kurtosis of singular vector components are very close to the normal distribution, however, the skewness is away from normality. It can be seen that all p-values of the Ljung-Box test statistics are below the significance level, hence rejecting the null of no significant autocorrelations. Similarly, the null of no significant autocorrelations in squared singular vector components is rejected in most of the cases as p-values of the McLeod-Li test statistic are reported in Table 3.8. Similar conclusion can be drawn for the ARCH type of heteroskedasticity results of which are presented in the last column of the Table 3.8.

Table 3.8: Summary statistics and testing of ARCH effects in components of third singular vector$U_{3,t}$

Ticker	Mean	Variance	Skewness	Kurtosis	Ljung-Box Q_{20}	McLeod-Li Q_{20}	Engle ARCH Q_{20}
WMT	-0.271	0.030	-0.507	2.63	0.03	0.22	0.31
CVX	-0.134	0.120	0.553	3.64	0.00	0.00	0.00
JPM	-0.187	0.208	0.595	2.65	0.00	0.00	0.00
PFE	-0.242	0.026	-0.674	3.05	0.00	0.05	0.15
T	-0.104	0.103	0.405	3.35	0.02	0.00	0.00
AXP	-0.123	0.144	0.440	3.08	0.00	0.16	0.17
GS	-0.134	0.142	0.467	2.92	0.00	0.00	0.00

Table 3.9: Summary statistics & testing of ARCH effects in Fisher transformed components of third singular vector $Z_{K_3,t}$

Ticker	Mean	Variance	Skewness	Kurtosis	Ljung-Box Q_{20}	McLeod-Li Q_{20}	Engle ARCH Q_{20}
WMT	-0.290	0.041	-0.949	4.27	0.09	0.54	0.53
CVX	-0.148	0.231	0.011	16.05	0.38	0.18	0.25
JPM	-0.224	0.410	0.353	6.49	0.00	0.02	0.06
PFE	-0.256	0.035	-1.111	4.76	0.01	0.15	0.28
T	-0.114	0.165	-0.082	11.04	0.06	0.22	0.28
AXP	-0.140	0.260	-0.087	9.96	0.00	0.95	0.94
GS	-0.152	0.227	0.235	5.98	0.07	0.29	0.38

However, the results shown in Table 3.9 are quite different when Fisher-transformation is used for the singular vector components. More than half of the components are now behaving like a white noise as shown by the p-values of the

Ljung-Box test statistics. As far as the autocorrelations in squared Fisher-transformed components are concerned, there is one exception for which the null of no autocorrelations can not be accepted, and this is true for the stock JPM since the p-value of McLeod Li test statistic is less than the chosen significance level (as indicated in magenta-colored box). However, the results of Engle ARCH test regarding this and all other stocks also favor the null hypothesis of no conditional heteroskedasticity (as can be seen in the adjacent cyan-colored boxes). The results in the table suggest of no more further repetitions of Step 3 of our proposed methodology. Hence, no further extraction of λ is needed.

Once again, the results of Ljung-Box test indicate the presence of serial correlations in $U_{3,t}$ and in a few components of $Z_{3,t}$, and ultimately suggesting us to use ARMA(p,q) model in order to account for this serial dependence, which is considered in the next section.

3.2.4 Modeling the dynamics

After extracting all volatility clustering from the data set, the next step of our methodology consists of modeling the dynamics of resulting singular values and also their corresponding singular vectors. Following the division of this step into two sub-steps, we first describe the modeling of singular value dynamics and then singular vector dynamics.

3.2.4.1 Singular values

Since three rounds of singular value decomposition were required in order to capture all volatility clustering from the data set, we are left with three sequences of singular values $\lambda_{1,t}$, $\lambda_{2,t}$, and $\lambda_{3,t}$. As described in the methodology section, before modeling we first look for the properties of resulting singular values. The following table gives a summary of the singular value statistics along with different nonlinear transformations[12] such as log, square root, and Box-Cox. Their respective histograms are displayed in Figure 3.2.6. As can be seen from Table 3.10, for all singular values $(\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t})$ the skewness and kurtosis indicate a departure from normality. The Jarque-Bera test also rejects this null of normality assumption. Therefore, the above-mentioned nonlinear transformations were used in order to achieve the normality. In terms of the three transformations, it can be seen that the logarithmic and Box-Cox transformations help to make the data less skewed as compared to the square root transformation.

[12] A nonlinear transformation changes (increases or decreases) linear relationships between variables and, thus, changes the correlation between variables. Examples of a nonlinear transformation of variable x would be taking the square root of x or the reciprocal of x.

It can be seen that histograms of singular values with the log and Box-Cox transformation are pretty close to that of normal distribution with the exception of the sequence of second singular value. Since Ljung-Box test assumes normality of the data set and could not account for nonlinear type of dependence, since the null of no significant autocorrelations is accepted in the second sequence of singular values. However, when using the square root and Box-Cox transformations, the test shows a presence of highly significant autocorrelations. Besides the argument presented in the methodology when describing Box-Cox transformation, we consider only logarithmic transformation due to the simplicity in handling with the logarithmic function.

Table 3.10: Summary statistics and the test of serial dependence in singular values along with different transformations

	Mean	Variance	Skewness	Kurtosis	Jarque-Bera	Ljung-Box $Q_{20}(r_k)$
$\lambda_{1,t}$	103.94	41,827.65	9.16	136.30	0.000	0.000
$\lambda_{2,t}$	1.280	0.699	11.53	212.63	0.000	0.912
$\lambda_{3,t}$	0.297	0.346	13.23	246.18	0.000	0.888
$ln(\lambda_{1,t})$	4.043	0.978	0.660	3.49	0.000	0.000
$ln(\lambda_{2,t})$	0.184	0.277	4.426	27.44	0.000	0.837
$ln(\lambda_{3,t})$	-1.707	0.953	0.491	5.81	0.000	0.000
$\sqrt{\lambda_{1,t}}$	8.615	29.730	3.053	19.05	0.001	0.000
$\sqrt{\lambda_{2,t}}$	1.110	0.049	6.559	64.00	0.001	0.037
$\sqrt{\lambda_{3,t}}$	0.478	0.068	3.825	33.89	0.001	0.000
$BoxCox(\lambda_{1,t})$	2.573	0.139	0.023	2.76	0.012	0.000
$BoxCox(\lambda_{1,t})$	0.085	0.002	0.866	2.69	0.000	0.000
$BoxCox(\lambda_{1,t})$	-1.553	0.696	0.066	5.48	0.000	0.000

In Figure 3.2.6, each column shows the histograms of the original and the log-transformed singular values. It can be seen from the first column that the resulting singular values are positively skewed. In the histograms with square root transformation, the same positive skewness can also be clearly observed. However, the histograms with the logarithmic and Box-Cox transformation look similar to the normal ones with the exception of second singular values in both cases.

The autocorrelation function is the most widely-used device to detect any kind of linear dependence in the data set. The ACFs of singular values along with different transformations are presented in Figure 3.2.7. The first panel of the figure shows the ACFs of $\lambda_{1,t}$, $log(\lambda_{1t})$, $\sqrt{\lambda_{1t}}$, and Box-Cox (λ_{1t}). Due to the very slow decaying pattern in the ACFs shown in the first panel, one might think that these slowly decaying autocorrelations are indicating the presence of long memory in the process; hence, ARFIMA (p,d,q) might be plausible in order to capture the dynamics of the first sequence of singular values.

Figure 3.2.6: Histograms of singular values$\lambda_{i,t}$ with different transformations

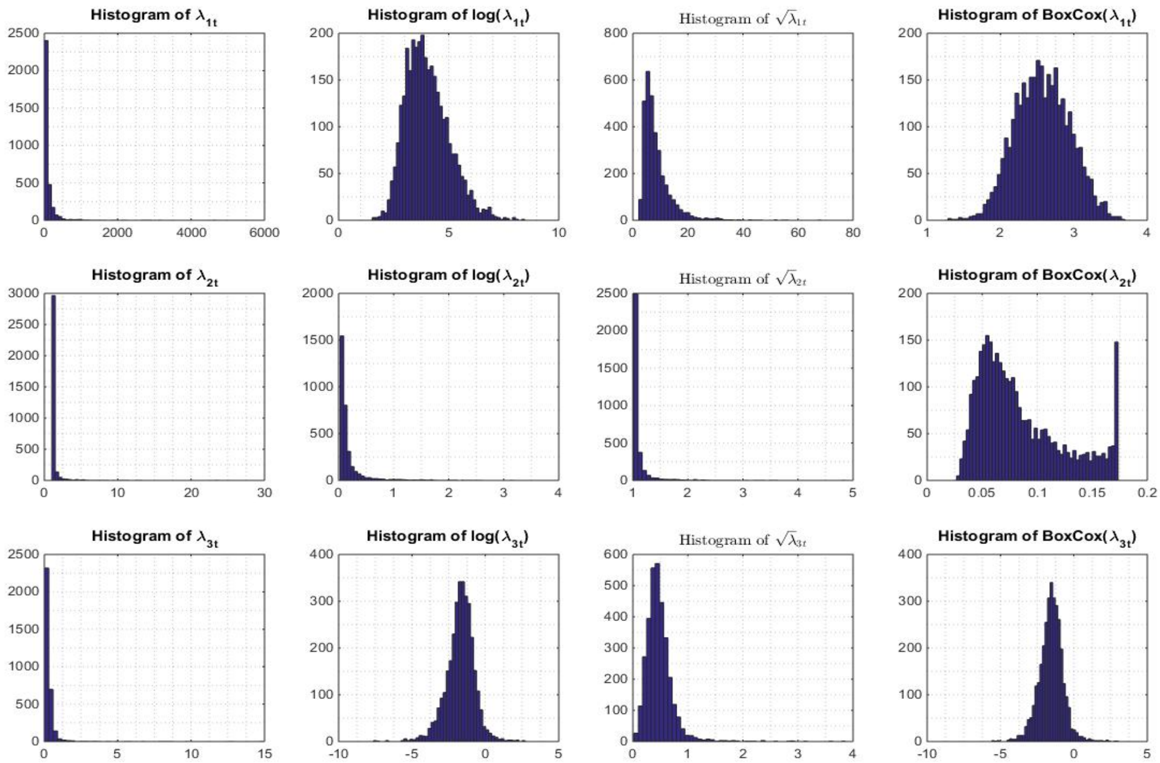
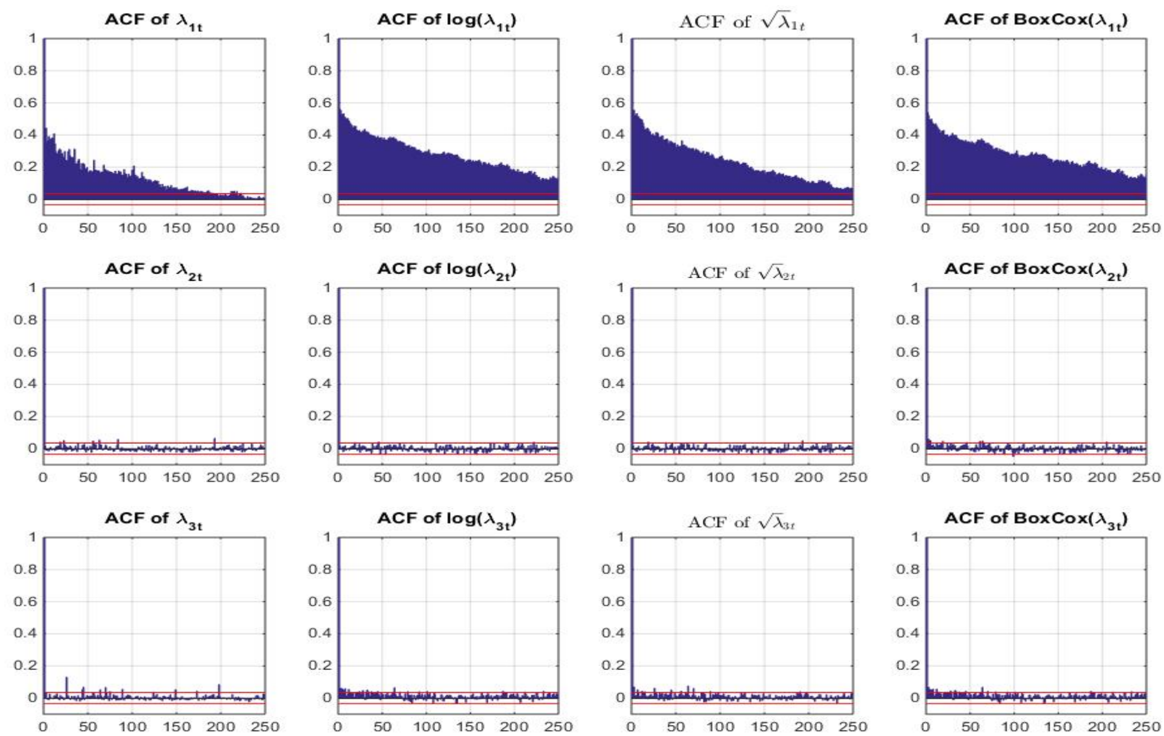


Figure 3.2.7: ACFs of singular values $\lambda_{i,t}$ with different transformations



Based on this argument, we also tested the presence of long memory phenomena and applied ARFIMA models with different specifications. We used Whittle's approximate MLE method to estimate the differencing parameter $\hat{d}$. The estimates were found to be very close to zero, i.e., they are rejecting the presence of long memory in the sequence as shown in Appendix 3.4.12. This rejection leads us to use ARMA(p,q) model in order to account for the dependence structure in the resulting λs.

However, the ACFs of the sequence of second singular values are within the bounds of $\pm\,^{2}/\sqrt{T}$, i.e., they show no significant autocorrelations; for this reason, there is no need to model its dynamics. More than 5% of the ACFs of third singular values are lying outside the bounds of two standard errors; therefore, it can be concluded that there are significant serial correlations in this sequence, which need to be corrected by any ARMA model.

The following three consecutive tables give the estimates of different ARMA specifications in order to select the lag length p and q for the log-transformed singular values. Mostly parameters are significant at conventional level (α= 0.05) as their t-statistics are greater than 1.96 (as shown in parenthesis). As usual, log-likelihood always selects the model with a large number of parameters, i.e., ARMA(3,3), and so does the AIC. However, keeping in view the main objective of the present research which is the forecasting of the future returns, our decision on lag-length selection solely depends on BIC. In all three cases, BIC selects ARMA(1,1) model. It can also be seen from tables 3.8-3.10 that the residuals from the selected model behave like a white noise as shown by the Ljung-Box test statistics and their corresponding p-values.

3.2.4.2 Singular vectors

In this sub-section, we address the modeling of the dynamics of singular vector components, which result from each round of singular value decomposition (SVD). application. Since three rounds of SVD were required to capture all the volatility clustering from the data set under consideration and, hence we are left with three singular vectors of different dimensions, i.e., the first singular vector consists of 29 components, while for the second and the third vectors, this number reduces to 15 and 7, respectively. The summary statistics of these singular vector components is already presented in Tables 3.4, 3.6 and 3.8, while for Fisher-transformed singular vector components these values were presented adjacent to these three tables.

The histograms of these components with normal and Student's t-density fit are presented in the Appendices 3.4.3, 3.4.6, and 3.4.9. As it can be seen from these figures that data do not fit normal distribution well as that there are still a number of

Table 3.11: Estimates of ARMA(p,q) model for $log\,(\lambda_{1,t})$

Parameters	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	1.761	1.153	0.901	4.034	4.034	4.034	4.034	0.048	0.044	0.039	0.040	0.043	0.049	0.034	0.041	0.058
	(29.322)	(17.772)	(13.428)	(189.340)	(164.014)	(148.370)	(233.106)	(3.439)	(3.355)	(3.075)	(2.939)	(3.363)	(3.054)	(2.694)	(2.872)	(2.246)
α_1	0.563	0.369	0.294					0.988	0.989	0.990	1.023	0.966	0.716	1.039	0.860	0.793
	(37.077)	(21.953)	(17.198)					(286.469)	(304.053)	(310.946)	(42.107)	(53.089)	(4.126)	(34.941)	(10.441)	(4.221)
α_2		0.345	0.265								-0.033	0.023	0.272	-0.014	0.171	-0.084
		(20.035	(14.433)								(-1.437)	(1.171)	(1.574)	(-0.401)	(2.175)	(-0.941)
α_3			0.217											-0.034	-0.041	0.277
			(12.461)											(-1.431)	(-2.061)	(1.141)
β_1				0.371	0.373	0.372		-0.840	-0.821	-0.821	-0.855	-0.799	-0.548	-0.872	-0.692	-0.625
				(33.371)	(18.576)	(18.879)		(-55.762)	(-44.232)	(-53.526)	(-47.500)	(-26.811)	(-3.127)	(-44.898)	(-8.389)	(-3.342)
β_2					0.302	0.321			-0.027	-0.005		-0.046	-0.230		-0.154	0.114
					(20.448)	(20.725)			(-1.375)	(-0.960)		(-1.409)	(-1.581)		(-2.366)	(1.000)
β_3						0.209				-0.032			-0.039			-0.274
						(13.403)				(-1.866)			(-1.836)			(-1.296)
LL	-3972.44	-3765.55	-3686.78	-4245.19	-4055.59	-3966.42	-4597.79	-3518.86	-3517.71	-3516.29	-3515.82	-3515.79	-3513.85	-3514.00	-3513.94	-3513.47
AIC	-0.4041	-0.5301	-0.5778	-0.2370	-0.3525	-0.4065	-0.0217	-0.6812	-0.6813	-0.6815	-0.6824	-0.6818	-0.6824	-0.6829	-0.6824	-0.6820
BIC	-0.4003	-0.5245	-0.5703	-0.2333	-0.3469	-0.3991	-0.0198	-0.6756	-0.6738	-0.6722	-0.6750	-0.6725	-0.6712	-0.6736	-0.6712	-0.6690
Ljung-Box Q_{20}	1296.51	369.95	223.67	6734.47	3691.26	2423.21	15870.53	20.85	19.36	17.71	19.48	19.66	17.54	18.21	17.81	16.03
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.50	0.61	0.49	0.48	0.62	0.57	0.60	0.72

Maximum likelihood parameter estimates (t-statistics are given in parenthesis)

Table 3.12: Estimates of ARMA(p,q) model for $log\,(\lambda_{2,t})$

Parameters	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	0.181	0.181	0.182	0.184	0.184	0.184	0.184	0.177	0.317	0.300	0.005	0.533	0.518	0.285	0.012	0.012
	(31.358)	(28.460)	(26.080)	(37.439)	(37.350)	(37.580)	(38.072)	(9.363)	(7.936)	(8.084)	(1.526)	(3.727)	(3.797)	(7.792)	(1.063)	(2.178)
α_1	0.017	0.017	0.017					0.041	-0.721	-0.629	0.982	-1.183	-1.154	-0.546	0.223	-0.394
	(0.871)	(0.869)	(0.868)					(0.411)	(-3.439)	(-3.171)	(35.906)	(-4.264)	(-4.982)	(-2.748)	(1.359)	(-2.395)
α_2		0.002	0.002								-0.010	-0.706	-0.657	0.012	0.725	0.361
		(0.122)	(0.131)								(-0.562)	(-1.250)	(-1.055)	(0.632)	(4.201)	(1.181)
α_3			-0.006											-0.012	-0.014	0.966
			(-0.434)											(-0.819)	(-0.756)	(8.872)
β_1				0.017	0.017	0.017		-0.024	0.739	0.647	-0.965	1.202	1.172	0.563	-0.206	0.415
				(0.885)	(0.844)	(0.846)		(-0.282)	(3.484)	(3.195)	(-54.110)	(4.527)	(5.199)	(2.840)	(-1.248)	(6.769)
β_2					0.002	0.002			0.021	0.013		0.727	0.674		-0.717	-0.343
					(0.500)	(0.166)			(1.171)	(0.469)		(1.343)	(1.109)		(-4.219)	(-2.835)
β_3						-0.006				-0.012			-0.004			-0.985
						(-0.346)				(-0.573)			(-0.354)			(-6.093)
LL	-438.07	-438.04	-437.91	-438.10	-438.09	-438.03	-438.57	-438.07	-437.81	-437.62	-437.02	-437.04	-437.02	-437.68	-437.01	-399.43
AIC	-2.5684	-2.5678	-2.5673	-2.5684	-2.5678	-2.5672	-2.5687	-2.5678	-2.5673	-2.5668	-2.5678	-2.5672	-2.5666	-2.5668	-2.5666	-2.5690
BIC	-2.5547	-2.5522	-2.5598	-2.5546	-2.5522	-2.5597	-2.5608	-2.5622	-2.5599	-2.5575	-2.5603	-2.5579	-2.5554	-2.5575	-2.5554	-2.5601
Ljung-Box Q_{20}	12.70	12.67	12.58	12.69	12.65	12.59	13.86	12.69	12.19	11.89	11.54	10.73	10.66	11.99	11.45	11.63
p-value	0.89	0.89	0.89	0.89	0.89	0.89	0.84	0.89	0.91	0.92	0.93	0.95	0.95	0.92	0.93	0.93

Maximum likelihood parameter estimates (t-statistics are given in parenthesis)

Table 3.13: Estimates of ARMA(p,q) model for $log\left(\lambda_{3,t}\right)$

Parameters	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-1.666	-1.562	-1.554	-1.707	-1.707	-1.707	-1.707	-0.039	-0.042	-0.037	-0.028	-0.055	-0.044	-0.031	-0.038	-0.041
	(-45.239)	(-33.264)	(-28.415)	(-97.862)	(-92.820)	(-92.518)	(-99.977)	(-2.132)	(-1.603)	(-2.003)	(-1.985)	(-1.946)	(-1.471)	(-2.502)	(-2.657)	(-2.043)
α_1	0.024	0.023	0.023					0.977	0.975	0.978	0.970	0.276	0.397	0.967	0.377	0.265
	(1.274)	(1.196)	(1.179)					(91.773)	(63.201)	(89.709)	(58.931)	(2.364)	(1.925)	(44.315)	(3.586)	(2.438)
α_2		0.062	0.062								0.013	0.692	0.577	0.040	0.628	0.637
		(3.540)	(3.542)								(0.956)	(5.959)	(2.939)	(1.510)	(6.106)	(6.415)
α_3			0.005											-0.025	-0.027	0.074
			(0.300)											(-1.243)	(-1.547)	(1.757)
β_1				0.022	0.023	0.022		-0.949	-0.962	-0.964	-0.960	-0.278	-0.384	-0.960	-0.368	-0.254
				(1.401)	(1.297)	(1.368)		(-66.603)	(-14.411)	(-44.667)	(-77.127)	(-2.373)	(-1.811)	(-93.236)	(-3.492)	(-2.339)
β_2					0.057	0.057			0.016	0.037		-0.646	-0.525		-0.578	-0.586
					(3.598)	(3.654)			(0.274)	(1.604)		(-5.701)	(-2.782)		(-5.611)	(-5.907)
β_3						0.004				-0.024			-0.027			-0.100
						(1.358)				(-1.108)			(-0.989)			(-2.149)
LL	-4553.81	-4547.49	-4547.00	-4554.05	-4548.28	-4548.26	-4554.91	-4534.71	-4534.27	-4533.36	-4529.86	-4529.56	-4528.57	-4525.76	-4523.23	-4523.19
AIC	-0.0480	-0.0513	-0.0510	-0.0479	-0.0508	-0.0502	-0.0480	-0.0591	-0.0588	-0.0587	-0.0615	-0.0611	-0.0610	-0.0634	-0.0643	-0.0637
BIC	-0.0443	-0.0457	-0.0435	-0.0442	-0.0452	-0.0428	-0.0461	-0.0535	-0.0513	-0.0494	-0.0540	-0.0517	-0.0499	-0.0541	-0.0531	-0.0507
Ljung-Box Q_{20}	72.99	51.61	50.39	73.38	53.98	53.48	79.03	23.97	23.00	20.99	23.31	19.11	17.53	20.63	17.26	16.96
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.29	0.40	0.27	0.51	0.62	0.42	0.64	0.66

Maximum likelihood parameter estimates (t-statistics are given in parenthesis)

bins in the middle and also on both sides, which are left uncovered with a normal fit. However, the Student's t-distribution provides better a fit by covering these bins in quite a good way as compared to the previous one. On the basis of these histograms, we can argue that Student's t-distribution is a better representative of the empirical distribution of singular vector components.

The ACFs of the resulting singular vector components are consecutively shown in the following three figures. A careful insight allows us to conclude that for most components these autocorrelations are positive and lie outside the bounds of the two standard error. In spite of being small in magnitudes, i.e.,< 0.2, these are significantly different from zero as shown by the results of Ljung-Box test for $U's$ in Tables 3.4, 3.6, and 3.8. Hence, it can be easily concluded from these tables and graphs that these components do not behave like a white noise and exhibit serial dependence, which needs to be accounted for with the help of ARMA(p,q) models.

Keeping in view the above mentioned arguments, ARMA models with different specifications of p and q were estimated in order to select a parsimonious model. These models along with their estimated parameters, log likelihoods (LL), different information criterion and the residual's properties are presented in various tables in the Appendices 3.4.5, 3.4.8, and 3.4.11 depending on the rounds of SVD. Model selection was purely based on significant parameters, BIC, and the white noise properties of the residuals.

It can be seen from these tables that in most of the cases BIC selects the ARMA(1,1) model (as shown by cyan-colored boxes) and the residuals from this model behave like a white noise. Although the AR and MA coefficients in ARMA(1,1) model for almost all components of $\mathbf{U's}$ cancel out each other. This cancellation reduces the ARMA(1,1) model to only ARMA(0,0) indicating that a model with the only constant term would be good choice. However, Ljung-Box test results show that the residuals from ARMA(0,0) model do not behave like a white noise (as shown by pink-colored boxes). Similarly, the estimates for the components of the first, the second, and the third Fisher-transformed singular vector of the ARMA(p,q) model are presented in the Appendices 3.4.5, 3.4.8, and 3.4.11, respectively. Overall, it can be easily seen that even for Fisher-transformed singular vector components, ARMA(1,1) model was selected based on BIC. We did not completely rely on the results of Ljung-Box test and used more advanced tests which are used to compare the out-of-sample forecasting accuracy of two models (i.e., ARMA(1,1) and ARMA(0,0)).

One of the most-widely used test among them is the Diebold-Mariano (DM) test. As shown by Clark and McCracken (2001), the Diebold-Mariano test statistic follows a t-distribution only when the underlying forecasting models are not nested. For example, the DM test might not work when comparing forecasts from an ARMA(0,0)

Figure 3.2.8: Autocorrelation functions of singular vector components $U_{1,t}$ from first round of SVX model

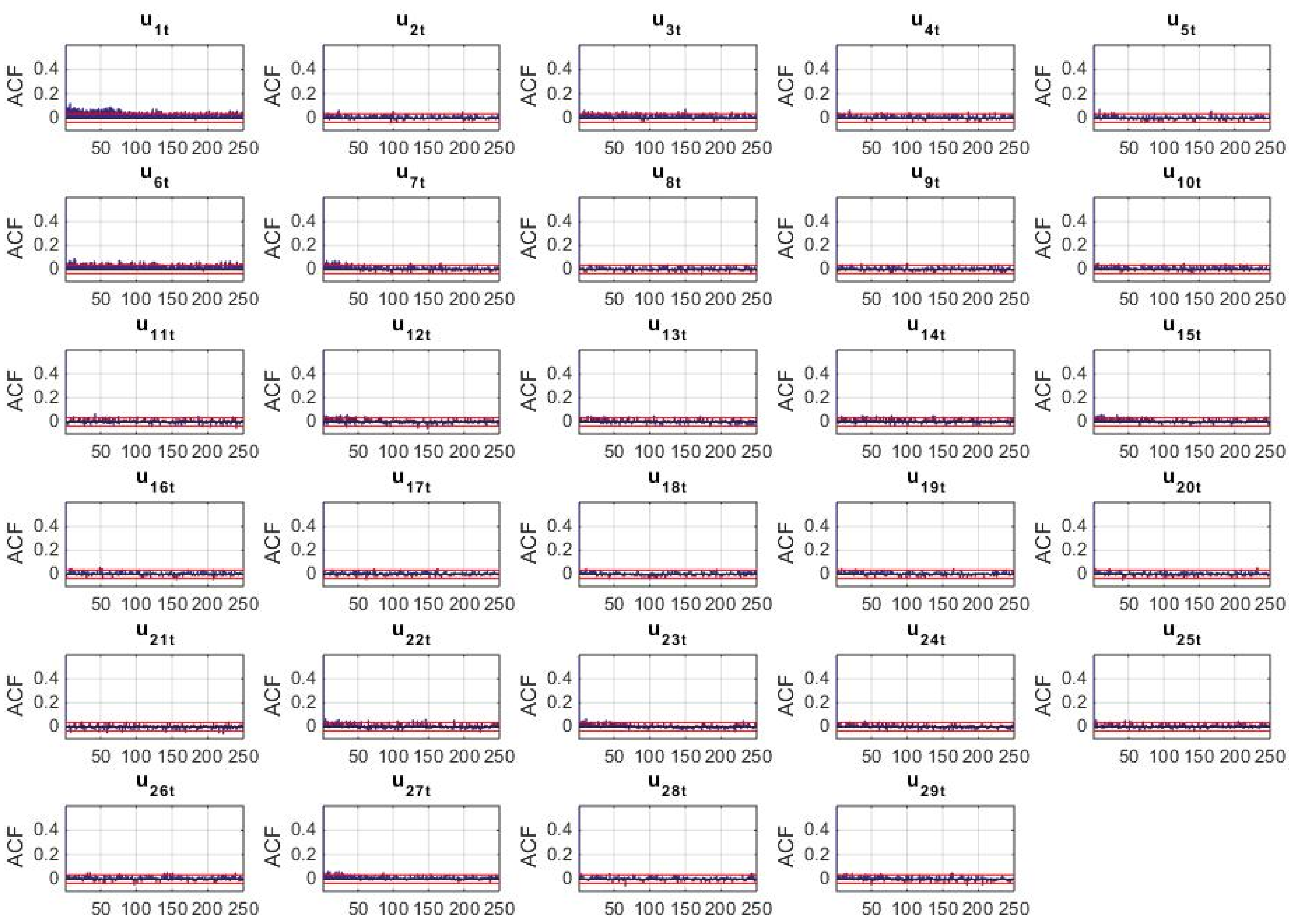


Figure 3.2.9: Autocorrelation functions for singular vector components $U_{2,t}$ from second round of SVX model

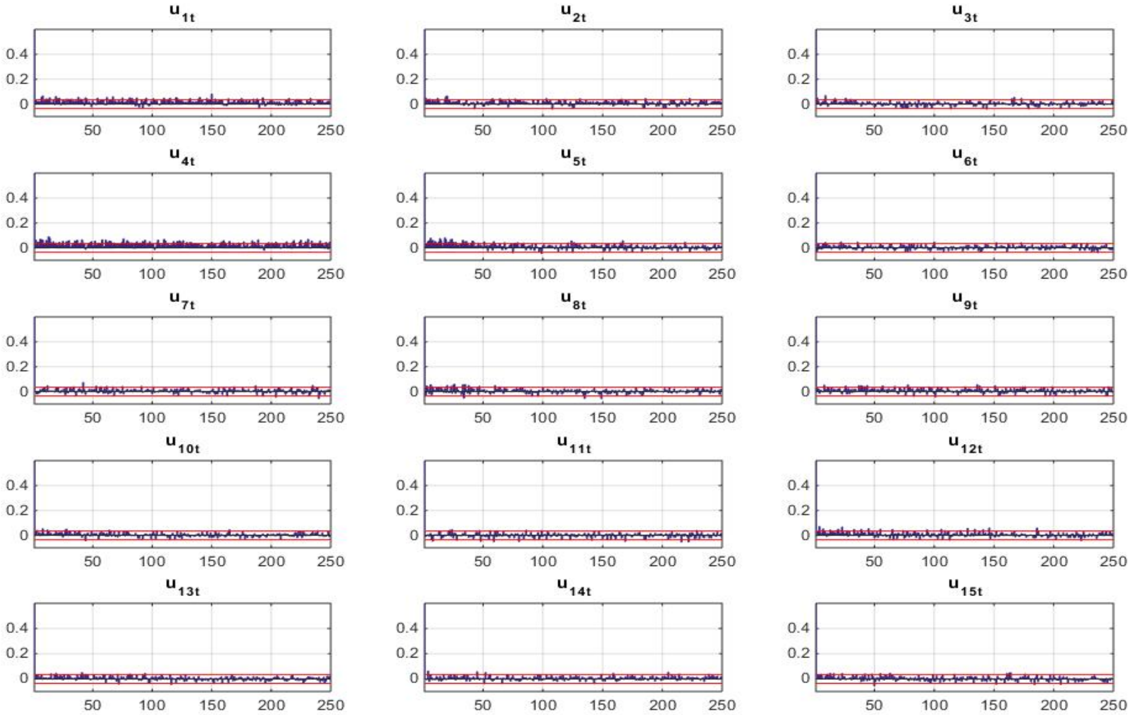
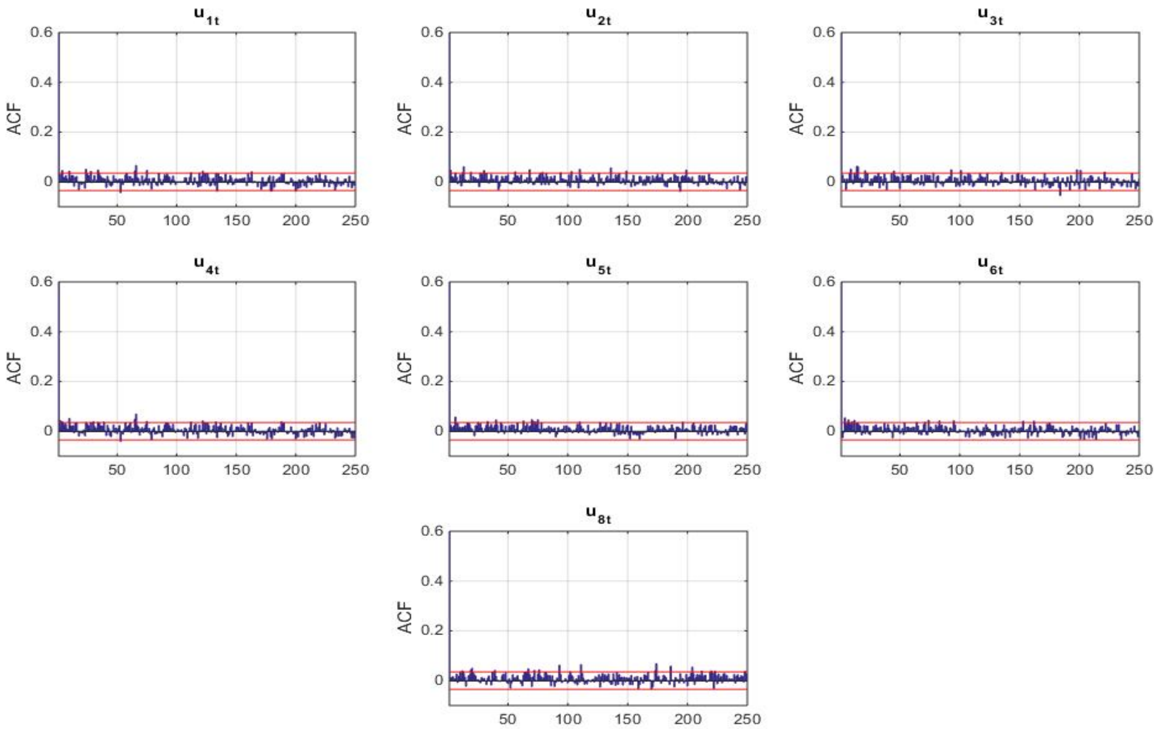


Figure 3.2.10: Autocorrelation functions for singular vector components $U_{3,t}$ from third round of SVX model



model to those obtained from an ARMA(1,1) model. Clearly, ARMA(0,0) model can be obtained from the ARMA(1,1) model specification by setting $\alpha_1 = \beta_1 = 0$. The problem with nested model is that under the null hypothesis of equal forecasting performance (so that the data are generated by the smaller model and the larger) the two models should predict equally well. However, the larger model will always contain some extra error as it contains unnecessary parameters. Hence, in order to test whether the data are actually generated from the different models, one needs to control the parameter uncertainty. Clark and West (2007) developed a simple procedure to adjust the forecast errors from the large model so that a simple variant of DM statistic can be used with nested models. This variant is called the Modified Diebold Mariano (MDM) test, which is described below.

Modified Diebold-Mariano (MDM) test

Diebold and Mariano (1995) introduced a new parametric test, which by virtue of its robustness to all the error properties, has been shown to be superior to its predecessors in a simulation. The test takes a very general specification, performing a test on the sample mean of a loss differential function, which can be arbitrarily defined, e.g., as the difference between two mean square forecast errors. The Diebold-Mariano test is straightforward to implement and has very attractive robustness properties compared to its rivals (Harvey *et al.*, 1997).

The Diebold-Mariano test statistic divides the loss differential sample mean by an estimate of its standard error found to be biased in finite samples. The Modified Diebold-Mariano (MDM) test embodies two amendments. First, a finite sample correction to the variance estimator in the test statistic to obtain unbiased estimate of the standard error, and second, the use of critical t-distribution values rather than a standard normal distribution. A comprehensive simulation study showed that the Modified Diebold-Mariano (MDM) test exhibits significant and substantial improvements to the original test (Harvey *et al.*, 1997).

Suppose two competing forecasts are made from the estimated model 1 and model 2 with errors e_{1t} and e_{2t}, respectively, for $t = 1, 2, ..., n$. Deriving the loss function from these forecast errors in period t by $g(e_{1t})$ and $g(e_{2t})$, respectively, the corresponding differential loss function can be constructed as $d_t = g(e_{1t}) - g(e_{2t})$. Our aim now is to test the null hypothesis of equal forecast accuracy, $H_0 : E(d_t) = \bar{d} = 0$. The Diebold-Mariano test statistic is then defined as:

$$H_1 = \frac{\bar{d}}{\sqrt{\hat{V}(\bar{d})}}, \tag{3.2.1}$$

The test statistic H_1 follows the standard normal distribution

$\bar{d} = \frac{1}{n}\sum_{t=1}^{n} d_t$, $\hat{V}(\bar{d}) = \frac{1}{n}\left[\hat{\gamma}_0 + 2\sum_{i=1}^{n-1}\hat{\gamma}_i\right]$ and $\hat{\gamma}_i = \frac{1}{n}\sum_{i=1}^{n-1}(d_t - \bar{d})(d_{t-i} - \bar{d})$,

where $\hat{\gamma}_i$ denote ith autocovariance of the d_t series.

A general optimal h-steps-ahead forecast error is now a function of future white noise terms forming an $MA(h-1)$ type process, and this renders zero autocovariance for all lags greater than $h-1$. Harvey (1997) argued that practically this result may not hold but would be expected for reasonably well-conceived forecasts. By using this result to the estimate the variance the loss differential sample mean, the following estimator for an h-steps-ahead forecast is obtained:

$$\hat{V}(\bar{d}) = \frac{1}{n}\left[\hat{\gamma}_0 + 2\sum_{i=1}^{h-1}\hat{\gamma}_i\right],$$

The problem with Diebold-Mariano test statistic is that its empirical size significantly exceeds its nominal size in cases of moderate and small sample sizes. As a result, one should correct the statistic of the Modified Diebold-Mariano (MDM) test for small sample bias in original variance estimate, which is given as:

$$MDM = H_1\sqrt{\frac{(n+1-2h+(\frac{h}{n})(h-1))}{n}}, \tag{3.2.2}$$

Compare the sample value of MDM test statistic to a tabulated t- value with $n-1$ degrees of freedom.

Table 3.14 shows the results of the MDM test, which is used to compare the out-of-sample forecasting accuracy of ARMA(0,0) vs ARMA(1,1). The entries presented in the table are the p-values of the MDM test for forecasting horizons 1 through 5. The table shows that in most cases forecasts provided by ARMA(0,0) model are superior to those provided by ARMA(1,1) model with only a very few exceptions such as 3^{rd}, 8^{th}, and 27^{th} components of $\mathbf{U_1}$ and the first component of $\mathbf{U_2}$ where the null hypothesis is rejected. To sum up, ARMA(0,0) model mostly outperforms ARMA(1,1) model for almost all forecasting horizons. Based on these results, ARMA(0,0) model was selected in order to account for the dependence structure in the singular vector components resulted from all rounds of SVD.

Table 3.14: Point forecast evaluation of ARMA(1,1) and ARMA(0,0) for singular vector components after three rounds of SVD: P-values of the MDM test

Components	After 1st Round of SVD					After 2nd Round of SVD					After 3rd Round of SVD				
	U_1					U_2					U_3				
	h=1	h=2	h=3	h=4	h=5	h=1	h=2	h=3	h=4	h=5	h=1	h=2	h=3	h=4	h=5
u_1	0.933	0.616	0.285	0.116	0.045	0.001	0.000	0.000	0.000	0.000	0.150	0.957	0.109	0.934	0.102
u_2	0.182	0.155	0.135	0.126	0.121	0.118	0.104	0.088	0.074	0.068	0.450	0.417	0.402	0.394	0.391
u_3	0.001	0.000	0.000	0.000	0.000	0.766	0.650	0.599	0.577	0.570	0.328	0.317	0.312	0.690	0.699
u_4	0.088	0.078	0.066	0.055	0.050	0.000	1.000	0.000	1.000	0.000	0.060	0.931	0.070	0.925	0.079
u_5	0.679	0.572	0.515	0.486	0.470	0.999	0.001	0.999	0.003	0.997	0.036	0.970	0.968	0.965	0.960
u_6	0.000	1.000	0.000	1.000	0.000	0.977	0.044	0.973	0.048	0.972	0.920	0.918	0.915	0.894	0.883
u_7	0.999	0.001	0.999	0.003	0.997	0.815	0.368	0.744	0.318	0.714	0.006	0.995	0.006	0.994	0.006
u_8	0.006	0.008	0.010	0.010	0.011	0.829	0.799	0.755	0.713	0.674	—	—	—	—	—
u_9	0.981	0.030	0.978	0.032	0.979	0.521	0.568	0.455	0.551	0.452	—	—	—	—	—
u_{10}	0.282	0.979	0.086	0.945	0.082	0.006	0.008	0.010	0.989	0.988	—	—	—	—	—
u_{11}	0.752	0.438	0.665	0.388	0.638	0.529	0.458	0.421	0.401	0.388	—	—	—	—	—
u_{12}	0.931	0.916	0.882	0.848	0.815	0.995	0.025	0.993	0.037	0.986	—	—	—	—	—
u_{13}	0.202	0.896	0.153	0.867	0.159	0.145	0.854	0.845	0.838	0.840	—	—	—	—	—
u_{14}	0.520	0.547	0.470	0.534	0.468	0.987	0.988	0.988	0.984	0.985	—	—	—	—	—
u_{15}	0.931	0.911	0.877	0.844	0.816	0.304	0.292	0.287	0.280	0.274	—	—	—	—	—
u_{16}	0.986	0.256	0.955	0.179	0.916	—	—	—	—	—	—	—	—	—	—
u_{17}	1.000	0.213	0.974	0.083	0.938	—	—	—	—	—	—	—	—	—	—
u_{18}	0.997	0.010	0.994	0.013	0.990	—	—	—	—	—	—	—	—	—	—
u_{19}	0.004	0.005	0.993	0.992	0.991	—	—	—	—	—	—	—	—	—	—
u_{20}	0.926	0.898	0.863	0.833	0.805	—	—	—	—	—	—	—	—	—	—
u_{21}	0.531	0.451	0.407	0.381	0.364	—	—	—	—	—	—	—	—	—	—
u_{22}	0.997	0.027	0.995	0.039	0.990	—	—	—	—	—	—	—	—	—	—
u_{23}	0.542	0.521	0.501	0.484	0.469	—	—	—	—	—	—	—	—	—	—
u_{24}	0.186	0.161	0.151	0.148	0.138	—	—	—	—	—	—	—	—	—	—
u_{25}	0.992	0.993	0.993	0.990	0.991	—	—	—	—	—	—	—	—	—	—
u_{26}	0.182	0.086	0.048	0.032	0.023	—	—	—	—	—	—	—	—	—	—
u_{27}	0.020	0.020	0.021	0.023	0.023	—	—	—	—	—	—	—	—	—	—
u_{28}	0.283	0.115	0.064	0.054	0.046	—	—	—	—	—	—	—	—	—	—
u_{29}	0.242	0.234	0.235	0.232	0.229	—	—	—	—	—	—	—	—	—	—

Note. Under the null hypothesis in pairwise MDM test, we assume that ARMA(0,0) model has significantly smaller MSFE than the corresponding MSFE of the ARMA(1,1) model.

3.3 Conclusion

This chapter deals with the empirical analysis of our newly proposed methodology described in the previous chapter. The first section of this chapter briefly explained the important features of the data set by indicating the causes of the tumbling of returns throughout the sample period. The financial-crisis of 2008-09 and the stock market crash of 2002 were found as the two major causes for the sinking of returns to their lowest, followed by the Bad News impact and the terrorist attack of 9/11/2001.

Due to the presence of highly significant autocorrelations in the returns, different ARMA(p,q) models were estimated to account for this linear dependence. Based on Bayesian Information Criterion (BIC), ARMA(2,0) was selected as the parsimonious model, i.e, a model with lower number of parameters, and the residuals from the selected model also exhibit white noise property. For further analysis, these AR(2)filtered returns were used.

Since our proposed methodology consists of the extraction of volatility clustering from the data set in a sequential way, after three rounds of SVD we successfully extracted all volatility clustering from the data set in hand. In the second and third round of SVD, only those Fisher-transformed singular vector components which exhibit volatility clustering from the previous round were used. These three rounds lead us to conclude that there are three latent factors responsible for driving the volatility in the Dow 30 components. To model the dynamics of resultant factors, we needed to model the dynamics of singular values and the corresponding singular vectors as well. Following the Box-Jenkins (1976) modeling approach, the ACFs of the resulting sequences of singular values were used in order to identify and estimate time series models. Depending on the hyperbolically decaying pattern in ACFs of the first sequence of singular values, ARFIMA(p,d,q) models were estimated using Whittle's Approximate MLE method. On the basis of estimates, the null of long memory was rejected and ARMA(p,q) models then estimated as an alternative to capture the dynamics of the three sequences of resulting singular values. Once again, BIC was used to select the appropriate lag lengths p and q. We finally ended up with ARMA(1,1) model, which exhibits white noise property of the residuals.

The answer to the question whether the resulting singular vector components need modeling or not depends on the presence of significant autocorrelations in them. Based on this, in this study the Ljung-Box test reported the presence of autocorrelations significantly different from zero and, hence require time series models to account for this dependence structure. Since their ACFs do not show hyperbolic decay (showing no long memory pattern), no fractionally integrated ARMA model was used, but simple ARMA model was considered for modeling purposes. Based on the out-of-sample forecasting performance of ARMA(0,0) versus ARMA(1,1), the results of MDM test suggested that ARMA(0,0) model outperforms ARMA(1,1) model; for this reason, it was selected to model the dynamics of the resulting singular vector components.

To sum up, we can argue that using our proposed methodology, one can extract and model the volatility clustering in a high dimensional data set. We can also conclude that the problem of rapidly increasing number of parameters when modeling the dynamics of large data set can be overcome using our proposed model.

The question regarding the goodness-of-fit of the proposed model is answered in the next chapter.

3.4 Appendix

3.4.1 A brief description of Dow Jones 29 stocks

1. Exxon Mobil Corporation (XOM) ***Sector: Oil & Gas***

Exxon Mobil Corporation was founded in 1870 and its headquarter is located in Irving, Texas. It produces crude oil and natural gas and also manufactures and markets commodity petrochemicals, including olefin, aromatics, polyethylene and polypropylene plastics, and specialty products; and transports and sells crude oil, natural gas, and petroleum products.

2. Microsoft Corporation (MSFT) ***Sector: Technology***

It develops, licenses, and supports software products, services, and devices worldwide. The company's Devices and Consumer (D&C) Licensing segment licenses Windows operating system and related software; Microsoft Office for consumers; and Windows Phone operating system. Microsoft Corporation was founded in 1975 and is based in Redmond, Washington.

3. General Electric Company (GE) ***Sector: Industrial***

GE was founded in 1892 and is headquartered in Fairfield, Connecticut. It provides infrastructure and financial services worldwide through its different segments. Power and Water segment offers gas, steam and aeroderivative turbines, nuclear reactors, generators, combined cycle systems, controls, and related services; Oil and Gas segment provides surface and sub-sea drilling and production systems; Energy Management segments offers plant automation hardware, software, and embedded computing systems and so on.

4. Johnson & Johnson (JNJ) ***Sector: Health Care***

Founded in 1885 and based in New Brunswick, New Jersey, JNJ operates in three segments. Consumer segment offers baby care products and oral care products, Pharmaceutical segment provides various products in the areas of immunology, infectious diseases, neuroscience, oncology, and cardiovascular and metabolic diseases and The Medical segment offers orthopedic and trauma and neurological products, products to treat cardiovascular disease, diagnostics products; blood glucose monitoring and insulin delivery products; and disposable contact lenses.

5. Wal-Mart Stores Inc. (WMT) ***Sector: Consumer Services***

Wal-Mart Stores, Inc. operates retail stores in various formats worldwide. It operates discount stores, supermarkets, super centers, hypermarkets, warehouse clubs, cash and carry stores, home improvement stores, specialty electronics stores, restaurants, apparel stores, drug stores, and convenience stores. The company was founded in 1945 and is headquartered in Bentonville, Arkansas.

6. Chevron Corporation (CVX) ***Sector: Oil & Gas***

Established in 1879 and is headquartered in San Ramon, California. The company through its subsidiaries, engages in the petroleum, chemicals, and power and energy operations worldwide. The company operates in two segments, Upstream and Downstream. The Upstream segment is involved in the exploration, development, and production of crude oil and natural gas; processing, liquefaction, transportation, and regasification associated with liquefied natural gas; transportation of crude oil through pipelines; and transportation, storage, and marketing of natural gas, as well as holds interest in a gas-to-liquids plant. The Downstream segment engages in refining crude oil into petroleum products; marketing crude oil and refined products; transporting crude oil and refined products through pipeline, marine vessel, motor equipment, and rail car; and manufacturing and marketing commodity petrochemicals and fuel and lubricant additives, as well as plastics for industrial uses.

7. JPMorgan Chase & Co. (JPM) ***Sector: Financial***

Founded in 1799 and based in New York, JPM provides various financial services worldwide. The company operates through four segments. Consumer & Community Banking, Corporate & Investment Bank, Commercial Banking, and Asset Management. The Consumer & Community Banking segment offers deposit and investment products and services to consumers; lending, deposit, and cash management and payment solutions to small businesses; and residential mortgages and home equity loans, as well as provides credit cards, payment services, payment processing services, and auto and student loans. The Corporate & Investment Bank segment provides investment banking, market-making, prime brokerage, and treasury and securities products and services to corporations, investors, financial institutions, and government and municipal entities. The Commercial Banking segment offers financial solutions, including lending, treasury, investment banking, and asset management to corporations, municipalities, financial institutions, and nonprofit entities, as well as finances real estate investors and owners. The Asset Management segment provides investment and wealth management services across various asset classes, such as equities, fixed income, alternatives, and money market funds; multi-asset investment management services; retirement products and services; and brokerage and banking services comprising trusts and estates, loans, mortgages, and deposits.

8. Procter & Gamble Company (PG) ***Sector: Consumer Goods***

The Procter & Gamble Company was founded in 1837 and is headquartered in Cincinnati, Ohio.The Procter & Gamble Company, together with its subsidiaries, manufactures and sells branded consumer packaged products worldwide. It operates through five segments. The Beauty, Hair and Personal Care segment offers antiperspirants and deodorants, personal cleansing, cosmetics, skin care, hair care and color and salon professional products. The Grooming segment provides blades and razors, pre and post-shave products, and other shave care products, as well as electronic hair removal devices. The Health Care segment offers gastrointestinal, rapid diagnostics, respiratory, vitamins/minerals/supplements, and other personal health care products. The Fabric Care and Home Care segment provides fabric care products, such as laundry additives, fabric enhancers, and laundry detergents; and home care products comprising air care, dish care, P&G professional, and surface care products. The Baby, Feminine and Family Care segment offers adult incontinence and feminine care products; baby wipes, diapers, and pants; and family care products, such as paper towels, tissues, and toilet papers. The company was founded in 1837 and is headquartered in Cincinnati, Ohio.

9. Pfizer Inc. (PFE) *Sector: Health Care*

A bio-pharmaceutical company, founded in 1849 with its headquarter located in New York, discovers, develops, manufactures, and sells healthcare products worldwide. The company operates through Global Innovative Pharmaceutical (GIP); Global Vaccines, Oncology and Consumer Healthcare (VOC); and Global Established Pharmaceutical (GEP) segments. The GIP segment develops, registers, and commercializes medicines for various therapeutic areas, including inflammation, cardiovascular/ metabolic, neuroscience and pain, rare diseases, and women/ men health. The VOC segment develops and commercializes vaccines, as well as products for oncology. The GEP segment offers patent-protected products that have lost marketing exclusivity in various markets; and generic pharmaceuticals, and sterile injectable and biosimilar development products. Pfizer Inc. was founded in 1849 and is headquartered in New York, New York.

10. International Business Machines Corporation (IBM) *Sector: Technology*

International Business Machines Corporation provides information technology (IT) products and services worldwide. The company's Global Technology Services segment provides IT infrastructure and business process services, such as outsourcing, processing, integrated technology, cloud, and technology support. Its Global Business Services segment offers consulting and systems integration services for strategy and transformation, application innovation services, enterprise applications, and smarter analytics; and application management, maintenance, and support services. Its Software segment provides middle-ware and operating systems software, including WebSphere software to integrate and manage business processes; and information management software that enables clients to integrate, manage, and analyze data from various sources.

11. AT&T, Inc. (T) ***Sector: Telecommunications***

AT&T Inc. provides telecommunications services in the United States and internationally. The company operates through two segments, Wireless and Wireline. The Wireless segment offers data and voice services, including local, long-distance, and network access services, as well as roaming services to youth, family, professionals, small businesses, government, and business customers. The Wireline segment provides switched and dedicated transport, DSL Internet access, network integration, managed Web-hosting, packet, and enterprise networking services, as well as intrastate, interstate, and international wholesale networking capacity to other service providers. The company was founded in 1910 and is headquartered in Armonk, New York.

12. Coca-Cola Company (KO) ***Sector: Consumer Goods***

With its headquarter located in Atlanta, Georgia ,the Coca-Cola founded in 1886, manufactures and distributes various nonalcoholic beverages worldwide. The company primarily offers sparkling beverages and still beverages. Its sparkling beverages include nonalcoholic ready-to-drink beverages with carbonation, such as carbonated energy drinks, and carbonated waters and flavored waters. The company's still beverages comprise nonalcoholic beverages without carbonation, including non-carbonated waters, flavored and enhanced waters, non-carbonated energy drinks, juices and juice drinks, ready-to-drink teas and coffees, and sports drinks.

13. Merck & Co. Inc. (MRK) ***Sector: Health Care***

Merck & Co., Inc. provides health care solutions worldwide. The company offers therapeutic and preventive agents to treat cardiovascular, type 2 diabetes, asthma, nasal allergy symptoms, allergic rhinitis, chronic hepatitis C virus, HIV-1 infection, fungal infections, intra-abdominal infections, hypertension, arthritis and pain, inflammatory, osteoporosis, male pattern hair loss, and fertility diseases. Further, it offers animal health products, for pneumonia in cattle, horses, and swine; vaccines for poultry. Additionally, the company provides companion animal products, such as diabetes mellitus treatment drugs and vaccines for dogs and cats; ointments for acute and chronic otitis; anthelmintic products; chewable tablets to kill fleas and ticks in dogs; and products for protection against bites from fleas, ticks, mosquitoes, and sandflies. rinarians, distributors, and animal producers, as well as managed health care providers. The company was founded in 1891 and is headquartered in Kenilworth, New Jersey.

14. Verizon Communications Inc. (VZ) ***Sector: Telecommunication***

Verizon Communications Inc., founded in 1983 and is based in New York, through its subsidiaries, provides communications, information, and entertainment products and services to consumers, businesses, and governmental agencies worldwide. The company's Wireless segment offers wireless voice and data services; messaging services; service that enables its customers to access the Internet on smart phones, basic phones, notebook computers, and tablets. It also offers voice and data services, such as conferencing and contact center solutions, and private line and data access networks, as well as customer premise equipment, installation, maintenance, and site services; and data, voice, local dial tone, and broadband services primarily to local, long distance, and other carriers.

15. Walt Disney Company (DIS) ***Sector: Consumer Services***

The Walt Disney Company, together with its subsidiaries, operates as an entertainment company worldwide. The company operates in five segments. Media Networks, Parks and Resorts, Studio Entertainment, Consumer Products, and Interactive. The Media Networks segment operates broadcast and cable television networks, domestic television stations, and radio networks and stations; and is involved in the television production and television distribution operations. The Parks and Resorts segment owns and operates the Walt Disney World Resort in Florida that includes theme parks; hotels; vacation club properties. The Studio Entertainment segment produces and acquires live-action and animated motion pictures, direct-to-video content, musical recordings, and live stage plays. The Consumer Products segment licenses trade names, characters, and visual and literary properties to retailers, show promoters, and publishers. The Interactive segment creates and delivers entertainment and lifestyle content across interactive media platforms. The company was founded in 1923 and is based in Burbank, California.

16. Intel Corporation (INTC) ***Sector: Technology***

Intel Corporation designs, manufactures, and sells integrated digital technology platforms worldwide, founded in 1968 and is based in Santa Clara, California. It operates through PC Client Group, Data Center Group, Internet of Things Group, Mobile and Communications Group, Software and Services. The company's platforms are used in various computing applications comprising notebooks, desktops, servers, tablets, smart phones, wireless and wired connectivity products, transportation systems, and retail devices. It offers microprocessors that processes system data and controls other devices in the system; chipsets, which send data between the microprocessor and input, display, and storage devices, such as keyboard, mouse, monitor, hard drive or solid-state drive, and optical disc drives; system-on-chip products that integrate its central processing units with other system components onto a single chip; and wired network connectivity products. The company also provides mobile communications components, such as base band processors, radio frequency transceivers, Wi-Fi products, Blue tooth technology, global navigation satellite systems, and power management chips, as well as home gateway and set-top box components. In addition, it offers software products for endpoint security, network and content security, risk and compliance, and consumer and mobile security; NAND flash memory products, which are used in solid-state drives; and custom foundry services, including custom silicon, packaging, and manufacturing test services.

17. Cisco Systems, Inc. (CSCO) ***Sector: Technology***

Cisco Systems, Inc. designs, manufactures, and sells Internet Protocol (IP) based networking products and services related to the communications and information technology industry worldwide. It provides switching products, including fixed-configuration and modular switches, and storage products that provide connectivity to end users, workstations, IP phones, wireless access points, and servers; and NGN routing products that interconnect public and private wireline and mobile networks for mobile, data, voice, and video applications. The company was founded in 1984 and is headquartered in San Jose, California.

18. Home Depot, Inc. (HD) ***Sector: Consumer Services***

The Home Depot, Inc. was founded in 1978 and is based in Atlanta, Georgia. It operates as a home improvement retailer. It operates "The Home Depot" stores that sell various building materials, home improvement products, and lawn and garden products, as well as provide installation, home maintenance, and professional service programs to do-it-yourself, do-it-for-me, and professional customers. The company offers installation programs that include flooring, cabinets, countertops, water heaters, and sheds; and professional installation in various categories sold through its in-home sales programs, such as roofing, siding, windows, kitchen and bath refacing, furnaces, and central air systems, as well as act as a contractor to provide installation services to its do-it-for-me customers through third-party installers.

19. United Technologies Corporation (UTX) ***Sector: Industrial***

United Technologies Corporation provides technology products and services to build systems and aerospace industries worldwide. Its Otis segment designs, manufactures, sells, and installs passenger and freight elevators, escalators, and moving walkways; modernization products to upgrade elevators and escalators; and maintenance and repair services. The company's UTC Climate, Controls & Security segment provides heating, ventilating, air conditioning, and refrigeration solutions, such as controls for residential, commercial, industrial, and transportation applications. Its Pratt & Whitney segment supplies aircraft engines for commercial, military, business jet, and general aviation markets; and provides aftermarket maintenance, repair, and overhaul, as well as fleet management services.The company offers its services through manufacturers representatives, distributors, wholesalers, dealers, retail outlets, and sales representatives, as well as directly to customers. It was founded in 1934 and is headquartered in Hartford, Connecticut.

20. Boeing Company (BA) ***Sector: Industrial***

Founded in 1916 ans based in Chicago, the Boeing Company, together with its subsidiaries, designs, develops, manufactures, sells, services, and supports commercial jetliners, military aircraft, satellites, missile defense, human space flight, and launch systems and services worldwide. The company operates in five segments. Commercial Airplanes, Boeing Military Aircraft, Network & Space Systems, Global Services & Support, and Boeing Capital. The Commercial Airplanes segment develops, produces, and markets commercial jet aircraft for various passenger and cargo requirements, as well as provides related support services to the commercial airline industry. The Boeing Military Aircraft segment is involved in the research, development, production, and modification of manned and unmanned military aircraft and weapons systems. The Network & Space Systems segment engages in the research, development, production, and modification of electronics and information solutions; strategic missile and defense systems; space and intelligence systems; and space exploration products. The Global Services and Support segment offers integrated logistics, while the Boeing Capital segment facilitates, arranges, structures, and provides financing solutions, such as equipment under operating leases, finance leases, notes and other receivables, assets held for sale or re-lease, and investments for its commercial airplanes customers.

21. McDonald's Corp. (MCD) ***Sector: Consumer Services***

McDonald's Corporation operates and franchises McDonald's restaurants in almost all over the world. McDonald's is the world's largest chain of hamburger fast food restaurants, serving around 68 million customers daily in 119 countries across 35,000 outlets. The company's restaurants offer various food products, soft drinks, coffee, and other beverages. The company was founded in 1940 and is based in Oak Brook, Illinois.

22. American Express Company (AXP) ***Sector: Financial***

The company was founded in 1850 and is headquartered in New York, together with its subsidiaries, it provides charge and credit payment card products and travel-related services to consumers and businesses worldwide. The company operates through four segments. U.S. Card Services, International Card Services, Global Commercial Services, and Global Network & Merchant Services. Its products and services include charge and credit card products; payments and expense management products and services; consumer and business travel services; stored value products, such as travelers cheque and other prepaid products; and network services.

23. 3M Company (MMM) ***Sector: Industrial***

3M Company operates as a diversified technology company worldwide. Its Industrial segment offers tapes; coated, non-woven, and bonded abrasives; adhesives; ceramics; sealants; specialty materials; filtration products; closure systems for personal hygiene products; acoustic systems products; automotive components; abrasion-resistant films; structural adhesives; and paint finishing and detailing products. The company's Safety and Graphics segment provides personal protection products; traffic safety and security products; commercial graphics systems; commercial cleaning and protection products; floor matting; and roofing granules. Its Electronics and Energy segment offers optical films; packaging and interconnection devices; insulating and splicing solutions for the electronics, telecommunications, and electrical industries; touch screens and monitors; renewable energy component solutions; and infrastructure protection products. The Health Care segment provides medical and surgical supplies; skin health and infection prevention products; drug delivery and health information systems; and dental, orthodontic, and food safety products. Its Consumer segment offers sponges, scouring pads, cloths, consumer and office tapes, re-positionable notes, indexing systems, construction and home improvement products, home care products, protective materials, and consumer and office tapes and adhesives. 3M Company was founded in 1902 and is headquartered in St. Paul, Minnesota.

24. Goldman Sachs Group Incorporated (GS) ***Sector: Financial***

Founded in 1869 with its headquarter located in New York, it operates as an investment banking, securities, and investment management company worldwide. The company operates through four segments. Investment Banking, Institutional Client Services, Investing & Lending, and Investment Management. The first segment provides financial advisory services, such as strategic advisory assignments related to mergers and acquisitions, divestitures, corporate defense activities, restructurings, spin-offs, and risk management; and underwriting services, including public offerings and private placements of a range of securities and other financial instruments, as well as derivative transactions entered into with public and private sector clients. The second segment is involved in client execution activities related to making markets in interest rate products, credit products, mortgages, currencies, commodities, and equities; and provides securities services, such as financing, securities lending, and other prime brokerage services, as well as markets in and clears client transactions on primary stock, options, and futures exchanges. The third segment invests in and originates longer-term loans to provide financing to clients; and makes investments in debt securities and loans, public and private equity securities, and real estate entities. The fourth segment offers investment management products and services; and wealth advisory services, including portfolio management and financial counseling, and brokerage and other transaction services.

25. UnitedHealth Group Incorporated (UNH) ***Sector: Health Care***

UnitedHealth Group Incorporated operates as a diversified health and well-being company in the United States. The company's United Healthcare segment offers consumer-oriented health benefit plans and services for national employers, public sector employers, mid-sized employers, small businesses, and individuals; and health care coverage, and health and well-being services to individuals aged 50 and older addressing their needs for preventive and acute health care services. It also provides services dealing with chronic disease and other specialized issues for older individuals; Medicaid plans, Children Health Insurance Program, and health care programs; and health services, including commercial health and dental benefits. The company was founded in 1974 and is based in Minnetonka, Minnesota.

26. Caterpillar Inc. (CAT) ***Sector: Industrial***

Caterpillar Inc. manufactures and sells construction and mining equipment, diesel and natural gas engines, industrial gas turbines, and diesel-electric locomotives worldwide. The company's Construction Industries segment offers backhoe, small wheel, skid steer, multi-terrain, compact track, medium and compact wheel, and track-type loaders; mini, wheel, and track excavators; track-type tractors; and select work tools, motor graders, telehandlers, soil compactors, and pipelayers. Its Energy & Transportation segment offers reciprocating engines; reciprocating engine powered generator sets; integrated systems and solutions for the electric power generation industry; and turbines and its related services, centrifugal gas compressors, diesel-electric locomotives and components, and rail-related products and services. Its Financial Products segment provides retail and wholesale financing primarily for Caterpillar equipment, machinery, and engines; offers property, casualty, life, accident, and health insurance; insurance brokerage services; and purchases short-term trade receivables. The company was founded in 1925 and is headquartered in Peoria, Illinois.

27. E. I. du Pont de Nemours and Company (DD) ***Sector: Basic Materials***

The company was founded in 1802 and is headquartered in Wilmington, Delaware. It operates as a science and technology based company worldwide. The company's Agriculture segment offers corn hybrid, soybean, canola, sunflower, sorghum, inoculants, seed products, wheat, rice, herbicides, fungicides, and insecticides. Its Electronics & Communications segment provides various materials and systems, including photo-polymers and electronic materials for photovoltaic products, consumer electronics, displays, and advanced printing. The company's Industrial Bio-sciences segment develops and manufactures a portfolio of enzymes and bio-based materials. Its Nutrition & Health segment offers cultures, probiotics, texturants, emulsifiers, natural sweeteners, and soy-based food ingredients.

28. NIKE, Inc. (NKE) ***Sector: Consumer Goods***

NIKE, Inc., together with its subsidiaries, designs, develops, markets, and sells athletic footwear, apparel, equipment, and accessories for men, women, and kids worldwide. The company offers products in eight categories, including running, basketball, football, men's training, women's training, sportswear, action sports, and golf under the NIKE and Jordan brand names. It also markets products designed for kids, as well as for other athletic and recreational uses, such as cricket, lacrosse, tennis, volleyball, wrestling, walking, and outdoor activities. In addition, the company sells sports apparel and accessories; and markets apparel with licensed college and professional team and league logos. Further, it sells a line of performance equipment, including bags, socks, sport balls, eye-wear, timepieces, digital devices, bats, gloves, protective equipment, golf clubs. The company was founded in 1964 and is headquartered in Beaverton, Oregon.

29. Travelers Companies, Inc. (TRV) ***Sector: Financial***

With its foundation in 1853 and based in New York, TRV through its subsidiaries, provides a range of commercial and personal property, and casualty insurance products and services to businesses, government units, associations, and individuals in the Unites states and internationally. It operates in three segments. Business and International Insurance segment offers property and casualty products, including commercial multi-peril, commercial property, general liability, commercial automobile, and worker's compensation; and personal property, employer's liability, public and product liability, professional indemnity, commercial property, surety, marine, aviation, personal accident, and kidnap and ransom insurance. The Bond & Specialty Insurance segment provides fidelity and surety, general liability, and others, such as property, worker's compensation, commercial automobile, and commercial multi-peril insurance products. The Personal Insurance segment offers property and casualty insurance covering personal risks, primarily automobile and homeowners insurance to individuals.

3.4.2 Estimation Results for ARMA(p,q) Model for Dow Jones 29 Components

Table 3.15: Exxon Mobile (XOM): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box Q_{20}**
AR(1)	0.0398 (0.0281)	-0.1231 (0.0383)						-6130.8	0.915	0.919	0.125
AR(2)	0.0451 (0.0280)	-0.1382 (0.0384)	-0.1075 (0.0452)					-6109.4	0.903	0.909	0.475
AR(3)	0.0440 (0.0281)	-0.1353 (0.0376)	-0.1037 (0.0438)	0.0292 (0.0374)				-6108.0	0.903	0.910	0.562
MA(1)	0.0344 (0.0289)				-0.1494 (0.0463)			-6129.2	0.914	0.918	0.001
MA(2)	0.0346 (0.0197)				-0.1301 (0.0382)	-0.0831 (0.0403)		-6118.7	0.909	0.914	0.082
MA(3)	0.0345 (0.0212)				-0.1324 (0.0368)	-0.0837 (0.0401)	0.0437 (0.0373)	-6115.6	0.907	0.915	0.002
ARMA(0,0)	0.0342 (0.0008)							-6159.8	0.933	0.934	0.081
ARMA(1,1)	0.0232 (0.0136)	0.3640 (0.1561)			-0.5067 (0.1408)			-6116.4	0.907	0.913	0.000
ARMA(1,2)	0.0424 (0.0195)	-0.1957 (0.1355)			0.0631 (0.1365)	-0.1142 (0.0480)		-6113.1	0.906	0.913	0.000
ARMA(1,3)	0.0322 (0.0254)	0.1028 (0.1808)			-0.2364 (0.1864)	-0.0707 (0.0413)	0.0480 (0.0469)	-6110.6	0.905	0.914	0.000
ARMA(2,1)	0.0527 (0.0309)	-0.3309 (0.2271)	-0.1324 (0.0545)		0.1953 (0.2258)			-6108.4	0.903	0.910	0.000
ARMA(2,2)	0.0543 (0.0354)	-0.3111 (0.4923)	-0.2005 (1.0636)		0.1760 (0.5118)	0.0736 (1.0448)		-6108.2	0.903	0.913	0.000
ARMA(2,3)	0.0535 (0.0549)	-0.2941 (0.1910)	-0.1960 (0.3407)		0.1591 (0.1861)	0.0717 (0.3352)	0.0029 (0.0065)	-6108.2	0.904	0.915	0.000
ARMA(3,1)	0.0452 (0.2266)	-0.1630 (0.5466)			-0.1076 (0.0520)	0.0263 (0.1554)	0.0278 (0.5989)	-6108.0	0.903	0.913	0.000
ARMA(3,2)	0.0963 (0.0925)	-1.2633 (0.1609)	-0.4668 (0.2137)		-0.1443 (0.0534)	1.1321 (0.1620)	0.2096 (0.1754)	-6103.8	0.901	0.913	0.000
ARMA(3,3)	0.0067 (0.0058)	0.5780 (0.1896)	-0.0045 (0.0052)	0.2491 (0.0987)	-0.7176 (0.1773)	-0.0048 (0.0046)	-0.1588 (0.1075)	-6104.6	0.902	0.916	0.000

Note. Standard Errors are given in parenthesis.

Table 3.16: Microsoft (MSFT): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box Q_{20}**
AR(1)	0.0248 (0.0330)	-0.0716 (0.0270)						-6709.6	1.2696	1.2733	0.032
AR(2)	0.0256 (0.0331)	-0.0765 (0.0270)	-0.0482 (0.0255)					-6705.4	1.2676	1.2732	0.972
AR(3)	0.0245 (0.0331)	-0.0759 (0.0270)	-0.0485 (0.0255)	0.0182 (0.0246)				-6704.3	1.2676	1.2750	0.900
MA(1)	0.0258 (0.1348)				-0.0783 (0.0273)			-6722.8	1.2776	1.2814	0.042
MA(2)	0.0256 (0.0316)				-0.0736 (0.0265)	-0.0441 (0.0275)		-6719.9	1.2765	1.2821	0.150
MA(3)	0.0257 (0.0228)				-0.0729 (0.0272)	-0.0450 (0.0292)	0.0172 (0.0487)	-6719.4	1.2768	1.2843	0.183
ARMA(0,0)	0.0260 (0.0011)							-6.7320	1.2827	1.2845	0.000
ARMA(1,1)	0.0124 (0.6566)	0.4150 (2.0946)			-0.4971 (2.0628)			-6705.7	1.2678	1.2734	0.172
ARMA(1,2)	0.0208 (0.0140)	0.0810 (0.4292)			-0.1568 (0.4268)	-0.0389 (0.0420)		-6705.3	1.2682	1.2756	0.169
ARMA(1,3)	0.0227 (0.0942)	0.0069 (0.0402)			-0.0813 (0.0513)	-0.0467 (0.0295)	0.0170 (0.0447)	-6704.9	1.2685	1.2779	0.215
ARMA(2,1)	0.0351 (0.0284)	-0.4374 (0.2253)	-0.0768 (0.0315)		0.3628 (0.2255)			-6704.7	1.2678	1.2753	0.310
ARMA(2,2)	0.0127 (0.0590)	-0.1373 (0.5488)	0.5016 (0.4454)		0.0765 (0.5008)	-0.5668 (0.3952)		-6701.3	1.2664	1.2757	0.242
ARMA(2,3)	0.0035 (0.0062)	0.3300 (0.3013)	0.4492 (0.2229)		-0.4083 (0.3011)	-0.4806 (0.2378)	0.0633 (0.0238)	-6696.6	1.2641	1.2753	0.432
ARMA(3,1)	0.0406 (0.0822)	-0.7336 (0.2191)	-0.1004 (0.0294)	-0.0144 (0.0168)	0.6584 (0.2171)			-6702.5	1.2671	1.2764	0.277
ARMA(3,2)	0.0024 (0.0041)	0.3483 ((0.2734)	0.4304 (0.2258)	0.0730 (0.0261)	-0.4285 (0.2706)	-0.4558 (0.2353)		-6697.3	1.2645	1.2757	0.436
ARMA(3,3)	0.0038 (0.0094)	0.2220 (0.3003)	0.2815 (0.3866)	0.2593 (0.2499)	-0.3032 (0.3040)	-0.3115 (0.4103)	-0.2010 (0.2686)	-6696.5	1.2646	1.2777	0.578

Note. Standard Errors are given in parenthesis.

Table 3.17: General Electric (GE): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	-0.0039 (0.0355)	-0.0103 (0.0335)						-6951.5	1.4176	1.4238	0.098
AR(2)	-0.0040 (0.0355)	-0.0107 (0.0337)	0.0074 (0.0341)					-6951.4	1.4182	1.4214	0.147
AR(3)	-0.0036 (0.0355)	-0.0104 (0.0337)	0.0083 (0.0344)	-0.0187 (0.0285)				-6950.5	1.4182	1.4257	0.142
MA(1)	-0.0012 (0.0274)				-0.0102 (0.7570)			-6961.1	1.4235	1.4272	0.131
MA(2)	-0.0011 (0.0031)				-0.0100 (0.0280)	0.0070 (0.0136)		-6961.0	1.4240	1.4296	0.115
MA(3)	-0.0012 (0.0255)				-0.0098 (0.1466)	0.0053 (0.0128)	-0.0169 (0.1911)	-6960.5	1.4244	1.4318	0.115
ARMA(0,0)	-0.0011 (0.0013)							-6961.2	1.4230	1.4248	0.000
ARMA(1,1)	-0.0038 (0.0016)	0.0278 (0.0239)			-0.0380 (0.0063)			-6951.5	1.4182	1.4238	0.000
ARMA(1,2)	-0.0038 (0.0047)	0.0269 (0.0705)			-0.0373 (0.0676)	0.0080 (0.0133)		-6951.4	1.4188	1.4262	0.000
ARMA(1,3)	-0.0038 (0.2482)	0.0266 (2.4106)			-0.0368 (2.1177)	0.0070 (0.5102)	-0.0144 (0.2325)	-6951.0	1.4192	1.4285	0.000
ARMA(2,1)	-0.0035 (0.0082)	-0.9080 (0.0510)	0.0247 (0.0245)		0.9064 (0.0467)			-6944.2	1.4144	1.4219	0.000
ARMA(2,2)	-0.0062 (0.0301)	-0.7898 (0.2517)	0.1428 (0.2275)		0.7818 (0.2689)	-0.1240 (0.2408)		-6943.3	1.4145	1.4238	0.000
ARMA(2,3)	-0.0061 (0.0064)	-0.7682 (0.1524)	0.1515 (0.1321)		0.7640 (0.1510)	-0.1506 (0.1415)	-0.0263 (0.0421)	-6942.3	1.4145	1.4256	0.000
ARMA(3,1)	-0.0129 (0.1395)	-0.8747 (0.0647)	-0.0082 (0.1483)	-0.0357 (0.1363)	0.8682 (0.0664)			-6945.4	1.4157	1.4250	0.000
ARMA(3,2)	-0.0011 (0.0008)	-0.0933 (0.0876)	0.7264 (0.1206)	-0.0363 (0.0319)	0.0897 (0.0852)	-0.7139 (0.1172)		-6944.1	1.4156	1.4268	0.000
ARMA(3,3)	-0.0029 (0.0025)	0.0208 (0.0201)	0.3224 (0.1360)	-0.5361 (0.0988)	-0.0322 (0.0189)	-0.3014 (0.1321)	0.5074 (0.0967)	-6939.6	1.4134	1.4264	0.000

Note. Standard Errors are given in parenthesis.

Table 3.18: Johnson and Johnson(JNJ): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	0.0301 (0.0208)	-0.0402 (0.0310)						-5179.8	0.3334	0.3371	0.844
AR(2)	0.0334 (0.0208)	-0.0452 (0.0309)	-0.0845 (0.0334)					-5166.1	0.3256	0.3312	0.941
AR(3)	0.0336 (0.0210)	-0.0463 (0.0303)	-0.0846 (0.0335)	-0.0174 (0.0402)				-5165.3	0.3257	0.3332	0.592
MA(1)	0.0281 (0.0246)				-0.0482 (0.0409)			-5183.1	0.3354	0.3391	0.000
MA(2)	0.0282 (0.0178)				-0.0467 (0.0307)	-0.0847 (0.0342)		-5171.4	0.3289	0.3345	0.068
MA(3)	0.0282 (0.0271)				-0.0454 (0.0320)	-0.0848 (0.0330)	-0.0141 (0.0415)	-5171.1	0.3293	0.3367	0.087
ARMA(0,0)	0.0280 (0.0004)							-5186.3	0.3367	0.3386	0.000
ARMA(1,1)	0.0120 (0.0068)	0.6033 (0.1767)			-0.6719 (0.1717)			-5170.1	0.3280	0.3336	0.015
ARMA(1,2)	0.0216 (0.0135)	0.2669 (0.2765)			-0.3136 (0.2763)	-0.0692 (0.0390)		-5166.3	0.3263	0.3338	0.078
ARMA(1,3)	0.0173 (0.1027)	0.4177 (3.0351)			-0.4649 (3.0369)	-0.0641 (0.1109)	0.0198 (0.3476)	-5166.0	0.3268	0.3361	0.082
ARMA(2,1)	0.0280 (0.0210)	0.1306 (0.3124)	-0.0782 (0.0334)		-0.1773 (0.3110)			-5165.8	0.3260	0.3335	0.065
ARMA(2,2)	0.0271 (229.90)	0.0906 (4480.28)	-0.0000 (0.0217)		-0.0001 (4.4551)	-0.0001 (0.3086)		-5165.7	0.3266	0.3359	0.071
ARMA(2,3)	0.0386 (0.0449)	-0.6219 (0.5045)	0.3029 (0.7693)		0.5797 (0.5073)	-0.4063 (0.6991)	-0.0799 (0.0803)	-5163.0	0.3256	0.3367	0.258
ARMA(3,1)	0.0227 (0.0175)	0.2848 (0.3137)	-0.0699 (0.0421)	0.0073 (0.0291)	-0.3317 (0.3131)			-5165.2	0.3263	0.3356	0.067
ARMA(3,2)	0.0253 (0.0165)	0.0827 (0.1369)	0.0605 (0.0815)	-0.0040 (0.0035)	-0.1296 (0.1451)	-0.1399 (0.0911)		-5164.9	0.3267	0.3379	0.068
ARMA(3,3)	0.0375 (0.0215)	-0.6474 (0.1105)	-0.0971 (0.0393)	0.4896 (0.0873)	0.6015 (0.1113)	-0.0083 (0.0058)	-0.5599 (0.0787)	-5158.4	0.3233	0.3364	0.137

Note. Standard Errors are given in parenthesis.

Table 3.19: Wal-Mart Stores (WMT): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box** Q_{20}
AR(1)	0.0168 (0.0241)	-0.0709 (0.0255)						-5675.5	0.6367	0.6404	0.410
AR(2)	0.0188 (0.0241)	-0.0704 (0.0253)	-0.0542 (0.0255)					-5667.6	0.6325	0.6381	0.790
AR(3)	0.0201 (0.0240)	-0.0730 (0.0252)	-0.0497 (0.0255)	-0.0075 (0.0227)				-5663.3	0.6305	0.6391	0.802
MA(1)	0.0179 (0.0296)				-0.0792 (0.0280)			-5691.8	0.6467	0.6504	0.781
MA(2)	0.0178 (0.0905)				-0.0760 (0.0276)	-0.0533 (0.0308)		-5687.5	0.6447	0.6503	0.821
MA(3)	0.0178 (0.0463)				-0.0760 (0.0249)	-0.0531 (0.0299)	-0.0071 (0.0334)	-5687.4	0.6452	0.6527	0.791
ARMA(0,0)	0.0280 (0.0004)							-5186.3	0.3367	0.3386	0.000
ARMA(1,1)	0.0172 (0.0203)	-0.0945 (0.0477)			0.0255 (0.0420)			-5675.4	0.6373	0.6429	0.000
ARMA(1,2)	0.0195 (0.5941)	-0.2185 (1.2468)			0.1462 (1.2710)	-0.0605 (0.0855)		-5670.3	0.6347	0.6422	0.002
ARMA(1,3)	0.0199 (0.0315)	-0.2373 (0.1626)			0.1651 (0.1621)	-0.0648 (0.0282)	-0.0135 (0.0120)	-5670.0	0.6352	0.6445	0.002
ARMA(2,1)	0.0074 (0.0082)	0.6020 (0.1088)	0.0025 (0.0063)		-0.6756 (0.1025)			-5662.2	0.6298	0.6393	0.005
ARMA(2,2)	0.0080 (0.0039)	0.6621 (0.1012)	-0.0872 (0.0862)		-0.7381 (0.1058)	0.0940 0.0831)		-5661.7	0.6301	0.6394	0.005
ARMA(2,3)	0.0078 (0.0140)	0.6618 (0.1560)	-0.0734 (0.1171)		-0.7372 (0.1544)	0.0765 (0.1193)	0.0057 (0.0422)	-5661.7	0.6307	0.6419	0.005
ARMA(3,1)	0.0097 (0.0132)	0.4929 (0.1613)	-0.0120 (0.0370)	0.0089 (0.0190)	-0.5687 (0.1618)			-5661.5	0.6300	0.6393	0.005
ARMA(3,2)	0.0128 (0.0597)	0.0669 (0.5225)	0.2470 (0.2079)	0.0128 (0.0769)	-0.1429 (0.5323)	-0.2921 (0.2325)		-5660.7	0.6301	0.6413	0.005
ARMA(3,3)	0.0110 (0.0099)	0.0838 (0.1010)	0.1644 (0.1441)	0.1738 (0.1461)	-0.1588 (0.0987)	-0.2073 (0.1433)	-0.1622 (0.1323)	-5659.9	0.6303	0.6433	0.006

Note. Standard Errors are given in parenthesis.

Table 3.20: Chevron (CVX): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box** Q_{20}
AR(1)	0.0512 (0.0294)	-0.0909 (0.0389)						-6258.3	0.9934	0.9971	0.258
AR(2)	0.0558 (0.0296)	-0.0989 (0.0396)	-0.0751 (0.0472)					-6247.8	0.9876	0.9932	0.553
AR(3)	0.0536 (0.0299)	-0.0958 (0.0396)	-0.0710 (0.0464)	0.0406 (0.0347)				-6245.1	0.9866	0.9940	0.579
MA(1)	0.0460 (0.0350)				-0.1039 (0.0468)			-6259.2	0.9939	0.9977	0.442
MA(2)	0.0461 (0.0257)				-0.0921 (0.0386)	-0.0589 (0.0442)		-6253.8	0.9913	0.9968	0.456
MA(3)	0.0460 (0.0295)				-0.0943 (0.0392)	-0.0598 (0.0445)	0.0463 (0.0346)	-6250.1	0.9896	0.9971	0.456
ARMA(0,0)	0.0459 (0.0008)							-6274.8	1.0029	1.0047	0.018
ARMA(1,1)	0.0337 (0.0159)	0.2916 (0.1543)			-0.3940 (0.1592)			-6252.6	0.9905	0.9961	0.108
ARMA(1,2)	0.0581 (0.0380)	-0.2435 (0.2055)			0.1500 (0.2051)	-0.0885 (0.0521)		-6249.6	0.9893	0.9968	0.098
ARMA(1,3)	0.0438 (0.0225)	0.0726 (0.1310)			-0.1676 (0.1452)	-0.0532 (0.0451)	0.0490 (0.0286)	-6246.8	0.9882	0.9975	0.098
ARMA(2,1)	0.0712 (0.0336)	-0.3963 (0.1420)	-0.1057 (0.0492)		0.2994 (0.1444)			-6246.0	0.9871	0.9945	0.098
ARMA(2,2)	0.0806 (0.0484)	-0.4383 (0.2294)	-0.2735 (0.2191)		0.3428 (0.2401)	0.1699 (0.2159)		-6244.9	0.9870	0.9964	0.087
ARMA(2,3)	0.0799 (0.0867)	-0.4247 (0.2847)	-0.2706 (0.3430)		0.3292 (0.2875)	0.1686 (0.3726)	0.0015 (0.0104)	-6244.9	0.9876	0.9988	0.090
ARMA(3,1)	0.0552 (0.0443)	-0.1253 (0.3077)	-0.0739 (0.0395)	0.0384 (0.0838)	0.0296 (0.2975)			-6245.1	0.9872	0.9965	0.090
ARMA(3,2)	0.1499 (0.1317)	-1.3670 (0.2008)	-0.8039 (0.2317)	-0.1122 (0.0685)	1.2755 (0.2002)	0.6109 (0.2632)		-6240.9	0.9852	0.9964	0.090
ARMA(3,3)	0.1323 (0.0673)	-1.1945 (0.2536)	-0.6592 (0.3254)	0.0539 (0.1321)	1.1027 (0.2547)	0.4808 (0.3328)	-0.1486 (0.1311)	-6240.2	0.9854	0.9984	0.090

Note. Standard Errors are given in parenthesis.

Table 3.21: JPMorgan (JPM): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	0.0181 (0.0478)	-0.0876 (0.0415)						-7909.1	2.0037	2.0074	0.902
AR(2)	0.0176 (0.0478)	-0.0924 (0.0424)	-0.0274 (0.0379)					-7906.8	2.0029	2.0042	0.917
AR(3)	0.0200 (0.0479)	-0.0925 (0.0425)	-0.0273 (0.0385)	-0.0358 (0.0360)				-7902.7	2.0010	2.0084	0.908
MA(1)	0.0204 (0.2668)				-0.0928 (0.0476)			-7922.8	2.0121	2.0158	0.903
MA(2)	0.0203 (0.0239)				-0.0926 (0.0430)	-0.0280 (0.0328)		-7921.6	2.0119	2.0175	0.906
MA(3)	0.0202 (0.5817)				-0.0939 (0.0443)	-0.0274 (0.1027)	-0.0404 (0.1359)	-7919.0	2.0110	2.0184	0.903
ARMA(0,0)	0.0208 (0.0023)							-7936.1	2.0196	2.0214	0.508
ARMA(1,1)	0.0120 (0.1137)	0.2158 (0.2737)			-0.3127 (0.2610)			-7906.6	2.0028	2.0084	0.603
ARMA(1,2)	0.0129 (0.0251)	0.1695 (0.4478)			-0.2635 (0.4609)	-0.0168 (0.0169)		-7906.3	2.0032	2.0106	0.604
ARMA(1,3)	0.0127 (0.0093)	0.1840 (0.1982)			-0.2799 (0.2027)	-0.0066 (0.0067)	-0.0412 (0.0390)	-7903.4	2.0021	2.0114	0.602
ARMA(2,1)	0.0048 (0.0027)	0.6596 (0.1616)	0.0213 (0.0364)		-0.7527 (0.1522)			-7903.5	2.0015	2.0085	0.605
ARMA(2,2)	0.0034 (0.0030)	0.9166 (0.1676)	-0.1281 (0.1077)		-1.0083 (0.1684)	0.1700 (0.0992)		-7902.6	2.0015	2.0109	0.044
ARMA(2,3)	0.0077 (0.0076)	0.8334 (0.1551)	-0.2707 (0.2837)		-0.9306 (0.1577)	0.3353 (0.3139)	-0.0569 (0.0556)	-7900.4	2.0008	2.0120	0.107
ARMA(3,1)	0.0115 (0.0134)	0.3798 (0.2277)	0.0139 (0.0180)	-0.0343 (0.0413)	-0.4748 (0.2264)			-7901.2	2.0007	2.0047	0.048
ARMA(3,2)	0.0107 (0.0195)	0.0429 (0.0449)	0.3504 (0.2285)	-0.0076 (0.0104)	-0.1386 (0.0624)	-0.3751 (0.2345)		-7899.9	2.0005	2.0045	0.064
ARMA(3,3)	0.0101 (0.0398)	-0.0159 (0.0134)	0.2719 (0.2277)	0.1329 (0.1368)	-0.0795 (0.0490)	-0.2993 (0.2282)	-0.1488 (0.1447)	-7899.2	2.0007	2.0137	0.072

Note. Standard Errors are given in parenthesis.

Table 3.22: Procter and Gamble (PG): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	0.0368 (0.0209)	-0.0876 (0.0266)						-5197.0	0.3439	0.3476	0.902
AR(2)	0.0404 (0.0210)	-0.0975 (0.0266)	-0.0671 (0.0285)					-5184.8	0.3370	0.3426	0.970
AR(3)	0.0387 (0.0211)	-0.0940 (0.0264)	-0.0621 (0.0285)	0.0170 (0.0250)				-5181.4	0.3355	0.3430	0.913
MA(1)	0.0325 (0.0220)				-0.0993 (0.0309)			-5204.4	0.3484	0.3522	0.905
MA(2)	0.0327 (0.0202)				-0.0912 (0.0271)	-0.0572 (0.0313)		-5199.3	0.3459	0.3515	0.900
MA(3)	0.0326 (0.0178)				-0.0916 (0.0263)	-0.0592 (0.0282)	0.0224 (0.0225)	-5198.5	0.3460	0.3535	0.913
ARMA(0,0)	0.0324 (0.0004)							-5218.7	0.3566	0.3584	0.001
ARMA(1,1)	0.0251 (0.0134)	0.2742 (0.1311)			-0.3766 (0.1298)			-5190.2	0.3404	0.3459	0.036
ARMA(1,2)	0.0295 (0.0162)	0.1406 (0.1834)			-0.2359 (0.1830)	-0.0353 (0.0304)		-5189.3	0.3404	0.3479	0.061
ARMA(1,3)	0.0286 (0.0201)	0.1664 (0.2017)			-0.2609 (0.2005)	-0.0394 (0.0371)	0.0279 (0.0289)	-5188.0	0.3402	0.3496	0.110
ARMA(2,1)	0.0508 (0.0258)	-0.3769 (0.1634)	-0.0920 (0.0303)		0.2814 (0.1615)			-5183.0	0.3365	0.3440	0.147
ARMA(2,2)	0.0576 (0.0344)	-0.3621 (0.2031)	-0.3076 (0.1565)		0.2711 (0.2035)	0.2325 (0.1747)		-5181.4	0.3361	0.3455	0.099
ARMA(2,3)	0.0606 (0.0313)	-0.4011 (0.2722)	-0.3612 (0.1901)		0.3064 (0.2698)	0.2812 (0.2021)	-0.0172 (0.0402)	-5181.0	0.3366	0.3477	0.109
ARMA(3,1)	0.0302 (0.0276)	0.1201 (0.4392)	-0.0415 (0.0837)	0.0261 (0.0711)	-0.2148 (0.4405)			-5181.0	0.3359	0.3452	0.114
ARMA(3,2)	0.0357 (0.0510)	0.0876 (0.4580)	-0.1593 (0.1031)	0.0122 (0.0380)	-0.1821 (0.4698)	0.1201 (0.0936)		-5180.7	0.3364	0.3475	0.082
ARMA(3,3)	0.0255 (0.0230)	0.2312 (0.6638)	-0.1943 (0.3092)	0.2140 (0.2785)	-0.3290 (0.6671)	0.1715 (0.3604)	-0.2027 (0.2550)	-5179.7	0.3364	0.3494	0.161

Note. Standard Errors are given in parenthesis.

Table 3.23: Pfizer (PFE): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	0.0030 (0.0282)	-0.0307 (0.0263)						-6195.6	0.9550	0.9588	0.851
AR(2)	0.0044 (0.0281)	-0.0362 (0.0260)	-0.0833 (0.0273)					-6180.7	0.9465	0.9521	0.931
AR(3)	0.0041 (0.0281)	-0.0347 (0.0262)	-0.0823 (0.0274)	0.0131 (0.0243)				-6180.3	0.9469	0.9543	0.907
MA(1)	0.0012 (0.0048)				-0.0366 (0.0354)			-6201.6	0.9587	0.9624	0.900
MA(2)	0.0014 (0.0021)				-0.0310 (0.0233)	-0.0826 (0.0279)		-6190.7	0.9526	0.9582	0.904
MA(3)	0.0014 (0.0019)				-0.0321 (0.0396)	-0.0823 (0.0276)	0.0147 (0.0369)	-6190.3	0.9530	0.9605	0.904
ARMA(0,0)	0.0012 (0.0008)							-6203.5	0.9592	0.9611	0.001
ARMA(1,1)	0.0023 (0.0005)	0.5160 (0.1410)			-0.5742 (0.1434)			-6188.2	0.9511	0.9567	0.005
ARMA(1,2)	0.0029 (0.0222)	0.1574 (1.4582)			-0.1910 (1.4692)	-0.0726 (0.1598)		-6183.8	0.9490	0.9565	0.003
ARMA(1,3)	0.0026 (0.0048)	0.3777 (0.1897)			-0.4123 (0.1911)	-0.0690 (0.0312)	0.0390 (0.0168)	-6182.1	0.9486	0.9579	0.005
ARMA(2,1)	0.0050 (0.0061)	-0.2520 (0.1857)	-0.0890 (0.0266)		0.2179 (0.1852)			-6180.1	0.9468	0.9542	0.007
ARMA(2,2)	0.0050 (0.0341)	-0.2400 (1.0416)	-0.0347 (0.0521)		0.2052 (1.0410)	-0.0548 (0.0679)		-6180.0	0.9473	0.9566	0.006
ARMA(2,3)	0.0058 (0.0374)	-0.6786 (0.1451)	-0.0813 (0.0362)		0.6462 (0.1445)	-0.0265 (0.0156)	-0.0554 (0.0214)	-6179.2	0.9474	0.9586	0.007
ARMA(3,1)	0.0057 (0.0143)	-0.5122 (0.1847)	-0.0986 (0.0311)	-0.0299 (0.0240)	0.4779 (0.1863)			-6180.0	0.9473	0.9566	0.006
ARMA(3,2)	0.0003 (0.0008)	0.6907 (0.3964)	0.1481 (0.3166)	0.0721 (0.0362)	-0.7258 (0.4032)	-0.2075 (0.3211)		-6178.4	0.9470	0.9582	0.016
ARMA(3,3)	0.0101 (0.0201)	-1.3740 (0.0977)	-0.2988 (0.0750)	0.1199 (0.0848)	1.3447 (0.0920)	0.1713 (0.0721)	-0.2279 (0.0769)	-6170.7	0.9428	0.9559	0.443

Note. Standard Errors are given in parenthesis.

Table 3.24: International Business Machines (IBM): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	0.0264 (0.0279)	-0.0061 (0.0269)						-6151.8	0.9282	0.9341	0.780
AR(2)	0.0266 (0.0279)	-0.0041 (0.0272)	0.0088 (0.0274)					-6151.3	0.9285	0.9320	0.985
AR(3)	0.0263 (0.0280)	-0.0040 (0.0272)	0.0078 (0.0278)	0.0028 (0.0253)				-6151.1	0.9290	0.9365	0.986
MA(1)	0.0296 (0.0210)				-0.0060 (0.0093)			-6175.3	0.9426	0.9463	0.820
MA(2)	0.0296 (0.0901)				-0.0061 (0.1913)	0.0085 (0.0238)		-6175.1	0.9431	0.9487	0.831
MA(3)	0.0296 (0.0974)				-0.0062 (0.0615)	0.0088 (0.0095)	0.0039 (0.0444)	-6175.1	0.9437	0.9512	0.885
ARMA(0,0)	0.0296 0.0008							-6175.3	0.9420	0.9439	0.005
ARMA(1,1)	0.0311 (0.0401)	-0.1629 (0.0717)			0.1579 (0.0611)			-6151.3	0.9285	0.9341	0.019
ARMA(1,2)	0.0311 (0.0286)	-0.1630 (0.0319)			0.1591 (0.0354)	0.1591 (0.0354)		-6151.2	0.9291	0.9365	0.098
ARMA(1,3)	0.0310 (0.0643)	-0.1611 (0.0391)			0.1571 (0.0462)	0.0068 (0.0090)	0.0003 (0.0005)	-6151.2	0.9297	0.9390	0.098
ARMA(2,1)	0.0470 (0.1618)	-0.9064 (0.0505)	0.0144 (0.0332)		0.8988 (0.0360)			-6147.0	0.9265	0.9340	0.048
ARMA(2,2)	0.0577 (1.7737)	-1.0436 (0.5660)	-0.1075 (0.5623)		1.0465 (0.6571)	0.1303 (0.6551)		-6145.2	0.9260	0.9354	0.051
ARMA(2,3)	0.0019 (0.0019)	0.6481 (0.1365)	0.2639 (0.1143)		-0.6536 (0.1425)	-0.2552 (0.1170)	-0.0117 (0.0166)	-6148.6	0.9287	0.9399	0.000
ARMA(3,1)	0.0012 (0.0014)	0.9458 (0.0437)	0.0071 (0.0112)	-0.0088 (0.0132)	-0.9503 (0.0450)			-6148.8	0.9282	0.9375	0.000
ARMA(3,2)	0.0598 (0.0598)	-0.3556 (0.0554)	-0.9354 (0.0421)	0.0113 (0.0317)	0.3522 (0.0504)	0.9404 (0.0418)		-6143.5	0.9256	0.9368	0.002
ARMA(3,3)	0.0530 (0.0359)	0.0269 (0.0268)	-0.7896 (0.1997)	-0.3207 (0.1039)	-0.0365 (0.0361)	0.7985 (0.1796)	0.3277 (0.1076)	-6142.4	0.9255	0.9386	0.002

Note. Standard Errors are given in parenthesis.

Table 3.25: AT&T (T): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box Q_{20}**
AR(1)	0.0084 (0.0292)	-0.0315 (0.0261)						-6309.4	1.0247	1.0284	0.934
AR(2)	0.0084 (0.0292)	-0.0334 (0.0261)	-0.0543 (0.0306)					-6304.2	1.0221	1.0277	0.999
AR(3)	0.0096 (0.0292)	-0.0336 (0.0262)	-0.0542 (0.0307)	-0.0129 (0.0269)				-6301.9	1.0213	1.0288	0.940
MA(1)	0.0085 (0.0140)				-0.0353 (0.0268)			-6309.5	1.0247	1.0285	0.936
MA(2)	0.0085 (0.0054)				-0.0346 (0.0278)	-0.0550 (0.0335)		-6304.6	1.0224	1.0280	0.928
MA(3)	0.0085 (0.0215)				-0.0342 (0.0373)	-0.0547 (0.0315)	-0.0103 (0.0338)	-6304.5	1.0229	1.0303	0.929
ARMA(0,0)	0.0085 0.0009							-6311.3	1.0252	1.0278	0.189
ARMA(1,1)	0.0022 (0.0012)	0.7079 (0.1515)			-0.7530 (0.1468)			-6304.7	1.0224	1.0280	0.283
ARMA(1,2)	0.0052 (0.0038)	0.3424 (0.7070)			-0.3766 (0.7057)	-0.0411 (0.0510)		-6304.1	1.0226	1.0301	0.312
ARMA(1,3)	0.0022 (0.0197)	0.7040 (0.2938)			-0.7383 (0.3210)	-0.0302 (0.0604)	0.0214 (0.0409)	-6303.6	1.0230	1.0323	0.391
ARMA(2,1)	0.0078 (0.0107)	0.0470 (0.3135)	-0.0524 (0.0334)		-0.0807 (0.3137)			-6304.2	1.0227	1.0302	0.287
ARMA(2,2)	0.0049 (0.0054)	0.0837 (0.0628)	0.2684 (0.3439)		-0.1180 (0.0617)	-0.3193 (0.3362)		-6303.7	1.0230	1.0323	0.369
ARMA(2,3)	0.0124 (0.0146)	-0.5971 (0.3531)	-0.0093 (0.0220)		0.5631 (0.3611)	-0.0658 (0.0318)	-0.0390 (0.0252)	-6303.4	1.0234	1.0346	0.298
ARMA(3,1)	0.0098 (0.0074)	-0.0666 (0.0654)	-0.0553 (0.0304)	-0.0143 (0.0134)	0.0331 (0.0647)			-6301.9	1.0219	1.0312	0.292
ARMA(3,2)	0.0131 (0.0223)	0.1351 (0.1302)	-0.5782 (0.2382)	-0.0257 (0.0188)	-0.1689 (0.1269)	0.5421 (0.2518)		-6300.7	1.0218	1.0330	0.290
ARMA(3,3)	0.0128 (0.0258)	0.1521 (0.2989)	-0.5915 (0.2501)	-0.0016 (0.0031)	-0.1866 (0.2840)	0.5560 (0.2583)	-0.0254 (0.0518)	-6300.7	1.0224	1.0355	0.299

Note. Standard Errors are given in parenthesis.

Table 3.26: Coca-Cola (KO): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box Q_{20}**
AR(1)	0.0211 (0.0223)	-0.0407 (0.0331)						-5412.5	0.4757	0.4800	0.906
AR(2)	0.0223 (0.0224)	-0.0428 (0.0335)	-0.0152 (0.0342)					-5409.2	0.4744	0.4795	0.978
AR(3)	0.0225 (0.0226)	-0.0429 (0.0337)	-0.0155 (0.0347)	-0.0072 (0.0319)				-5409.2	0.4750	0.4824	0.973
MA(1)	0.0195 (0.0185)				-0.0419 (0.0354)			-5414.5	0.4770	0.4807	0.956
MA(2)	0.0195 (0.0391)				-0.0416 (0.0326)	-0.0147 (0.0276)		-5414.2	0.4774	0.4830	0.969
MA(3)	0.0195 (0.0132)				-0.0418 (0.0362)	-0.0143 (0.0221)	-0.0075 (0.0265)	-5414.1	0.4780	0.4854	0.748
ARMA(0,0)	0.0195 (0.0005)							-5417.3	0.4781	0.4800	0.005
ARMA(1,1)	0.0111 (0.0128)	0.4723 (0.4730)			-0.5138 (0.4727)			-5410.3	0.4751	0.4806	0.084
ARMA(1,2)	0.0107 (0.0112)	0.4956 (0.4163)			-0.5386 (0.4218)	0.0040 (0.0060)		-5410.3	0.4757	0.4831	0.085
ARMA(1,3)	0.0107 (0.0394)	0.4927 (0.7172)			-0.5360 (0.7041)	0.0079 (0.0386)	-0.0070 (0.0553)	-5410.2	0.4762	0.4855	0.089
ARMA(2,1)	0.0218 (0.3279)	-0.0169 (0.3965)	-0.0144 (0.0631)		-0.0260 (0.5679)			-5409.2	0.4750	0.4825	0.070
ARMA(2,2)	0.0241 (0.0321)	0.6512 (0.0647)	-0.8591 (0.0596)		-0.6479 (0.0544)	0.8703 (0.0679)		-5404.4	0.4726	0.4820	0.015
ARMA(2,3)	0.0237 (0.0252)	0.6819 (0.0590)	-0.8630 (0.0528)		-0.7291 (0.0683)	0.9110 (0.0688)	-0.0593 (0.0338)	-5399.1	0.4700	0.4812	0.193
ARMA(3,1)	0.0023 (0.0053)	0.8639 (0.0852)	0.0232 (0.0433)	0.0023 (0.0053)	-0.9083 (0.0967)			-5407.9	0.4748	0.4841	0.123
ARMA(3,2)	0.0258 (0.0369)	0.6127 (0.0627)	-0.8266 (0.0726)	-0.0541 (0.0346)	-0.6565 (0.0537)	0.8726 (0.0803)		-5399.4	0.4702	0.4814	0.166
ARMA(3,3)	0.0159 (0.0286)	1.0329 (0.5984)	-1.1212 (0.4271)	0.3220 (0.5440)	-1.0799 (0.5926)	1.1695 (0.4220)	-0.3839 (0.5451)	-5397.9	0.4699	0.4829	0.246

Note. Standard Errors are given in parenthesis.

Table 3.27: Merck (MRK): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	-0.0009 (0.0319)	-0.0020 (0.0257)						-6600.0	1.2025	1.2062	0.902
AR(2)	0.0005 (0.0319)	-0.0039 (0.0256)	-0.0258 (0.0272)					-6595.5	1.2003	1.2054	0.997
AR(3)	0.0012 (0.0319)	-0.0048 (0.0257)	-0.0266 (0.0272)	-0.0005 (0.0252)				-6594.8	1.2005	1.2080	0.904
MA(1)	-0.0022 (0.0013)				-0.0021 (0.0018)			-6602.7	1.2041	1.2079	0.902
MA(2)	-0.0021 (0.0011)				-0.0021 (0.0024)	-0.0261 (0.0179)		-6601.6	1.2041	1.2097	0.940
MA(3)	-0.0021 (0.0011)				-0.0020 (0.0002)	-0.0261 (0.0255)	-0.0008 (0.0001)	-6601.6	1.2047	1.2122	0.804
ARMA(0,0)	-0.0022 (0.0010)							-6602.7	1.2035	1.2059	0.800
ARMA(1,1)	0.0007 (0.0001)	0.6796 (0.1057)			-0.6959 (0.1112)			-6596.1	1.2007	1.2063	0.840
ARMA(1,2)	0.0006 (0.0011)	0.6528 (0.1765)			-0.6576 (0.1913)	-0.0173 (0.0385)		-6595.6	1.2011	1.2085	0.847
ARMA(1,3)	0.0006 (0.0003)	0.6653 (0.1074)			-0.6701 (0.1124)	-0.6701 (0.1124)	0.0106 (0.0171)	-6595.4	1.2016	1.2109	0.849
ARMA(2,1)	0.0008 (0.0006)	0.3404 (0.2400)	-0.0232 (0.0288)		-0.3454 (0.2455)			-6594.9	1.2006	1.2073	0.848
ARMA(2,2)	0.0010 (0.0002)	0.2434 (0.3905)	0.0773 (0.0295)		-0.2480 (0.3939)	-0.1026 (0.0444)		-6594.9	1.2012	1.2105	0.851
ARMA(2,3)	0.0009 (0.0004)	0.3165 (0.3457)	0.0294 (0.0202)		-0.3213 (0.3539)	-0.0551 (0.0352)	0.0049 (0.0020)	-6594.8	1.2018	1.2130	0.850
ARMA(3,1)	0.0012 (0.0022)	-0.0246 (0.0370)	-0.0267 (0.0430)	-0.0009 (0.0020)	0.0198 (0.0328)			-6594.8	1.2012	1.2105	0.847
ARMA(3,2)	0.0012 (0.0005)	-0.0518 (0.0426)	-0.0004 (0.0002)	-0.0017 (0.0010)	0.0470 (0.0491)	-0.0266 (0.0198)		-6594.8	1.2018	1.2129	0.847
ARMA(3,3)	0.0027 (0.0349)	-0.9962 (0.3236)	0.0326 (0.3382)	0.4907 (0.3756)	0.9913 (0.3143)	-0.0681 (0.3311)	-0.5157 (0.3672)	-6592.6	1.2011	1.2141	0.806

Note. Standard Errors are given in parenthesis.

Table 3.28: Verizon Communications (VZ): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	0.0203 (0.0278)	-0.0427 (0.0315)						-6150.6	0.9275	0.9312	0.780
AR(2)	0.0207 (0.0277)	-0.0473 (0.0313)	-0.0587 (0.0298)					-6143.0	0.9235	0.9290	0.985
AR(3)	0.0212 (0.0278	-0.0488 (0.0317)	-0.0599 (0.0300)	-0.0231 (0.0275)				-6142.1	0.9235	0.9310	0.807
MA(1)	0.0210 (0.0136)				-0.0484 (0.0332)			-6156.3	0.9310	0.9347	0.000
MA(2)	0.0209 (0.0213)				-0.0479 (0.0311)	-0.0591 (0.0308)		-6150.6	0.9281	0.9337	0.005
MA(3)	0.0208 (0.0933)				-0.0469 (0.0352)	-0.0587 (0.0373)	-0.0185 (0.0198)	-6150.0	0.9284	0.9358	0.008
ARMA(0,0)	0.0211 (0.0008)							-6159.7	0.9324	0.9343	0.000
ARMA(1,1)	0.0082 (0.0203)	0.5351 (0.1085)			-0.5989 (0.1065			-6143.3	0.9236	0.9292	0.005
ARMA(1,2)	0.0110 (0.0142)	0.3965 (0.1764)			-0.4458 (0.1751)	-0.0374 (0.0285)		-6141.9	0.9234	0.9308	0.008
ARMA(1,3)	0.0103 (0.0274)	0.4311 (0.2874)			-0.4805 (0.2978)	-0.0375 (0.0466)	0.0066 (0.0106)	-6141.8	0.9240	0.9333	0.008
ARMA(2,1)	0.0121 (0.0722)	0.3762 (0.6563)	-0.0401 (0.0755)		-0.4256 (0.6653)			-6141.9	0.9234	0.9308	0.008
ARMA(2,2)	0.0126 (0.0118)	0.2755 (0.3025)	0.0277 (0.0319)		-0.3250 (0.3000)	-0.0736 (0.0541)		-6141.8	0.9239	0.9333	0.007
ARMA(2,3)	0.0216 (0.2062)	-0.5673 (0.1337)	0.3939 (0.1341)		0.5236 (0.1408)	-0.4721 (0.1207)	-0.0554 (0.0389)	-6135.4	0.9206	0.9318	0.209
ARMA(3,1)	0.0080 (0.0310)	0.5803 (0.6234)	-0.0305 (0.0440)	0.0171 (0.0543)	-0.6298 (0.6214)			-6141.8	0.9239	0.9332	0.008
ARMA(3,2)	0.0108 (0.0368)	0.3383 (0.8640)	0.0671 (0.1612)	0.0064 (0.0265)	-0.3877 (0.8577)	-0.1089 (0.1008)		-6141.7	0.9245	0.9357	0.007
ARMA(3,3)	0.0406 (0.1091)	-1.2407 (0.0853)	-0.1524 (0.0455)	0.1221 (0.0687)	1.1954 (0.0754)	0.0305 (0.0177)	-0.2133 (0.0713)	-6133.1	0.9198	0.9329	0.488

Note. Standard Errors are given in parenthesis.

Table 3.29: Walt Disney (DIS): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	0.0337 (0.0349)	-0.0301 (0.0276)						-6876.5	1.3717	1.3755	0.389
AR(2)	0.0346 (0.0349)	-0.0342 (0.0277)	-0.0497 (0.0272)					-6871.2	1.3691	1.3747	0.609
AR(3)	0.0343 (0.0350)	-0.0333 (0.0278)	-0.0486 (0.0274)	0.0144 (0.0282)				-6870.8	1.3695	1.3769	0.523
MA(1)	0.0357 (0.0286)				-0.0334 (0.0294)			-6890.5	1.3803	1.3840	0.006
MA(2)	0.0355 (0.0247)				-0.0304 (0.0291)	-0.0487 (0.0277)		-6886.7	1.3786	1.3842	0.042
MA(3)	0.0356 (0.0323)				-0.0308 (0.0278)	-0.0479 (0.0274)	0.0131 (0.0490)	-6886.4	1.3790	1.3865	0.043
ARMA(0,0)	0.0358 (0.0012)							-6892.1	1.3807	1.3825	0.000
ARMA(1,1)	0.0245 (0.0318)	0.2264 (0.0943)			-0.2693 (0.1114)			-6874.5	1.3711	1.3767	0.006
ARMA(1,2)	0.0290 (0.0180)	0.0997 (0.1143			-0.1323 (0.1191)	-0.0424 (0.0289)		-6872.3	1.3704	1.3778	0.020
ARMA(1,3)	0.0277 (0.0485)	0.1342 (0.0738)			-0.1669 (0.0769)	-0.0419 (0.0380)	0.0209 (0.0270)	-6871.6	1.3706	1.3799	0.024
ARMA(2,1)	0.0398 (0.0471	-0.1858 (0.2221)	-0.0552 (0.0273)		0.1522 (0.2261)			-6871.0	1.3696	1.3770	0.028
ARMA(2,2)	0.0431 (0.0541)	-0.0848 (0.1116)	-0.2447 (0.1535)		0.0485 (0.1137)	0.1977 (0.1644)		-6870.6	1.3699	1.3793	0.030
ARMA(2,3)	0.0394 (0.5606)	0.0202 (0.1256)	-0.2361 (0.4838)		-0.0532 (0.2340)	0.1952 (0.5065)	0.0178 (0.1321)	-6870.1	1.3703	1.3815	0.030
ARMA(3,1)	0.0288 (0.1373)	0.1326 (3.2254)	-0.0424 (0.2180)	0.0200 (0.1631)	-0.1659 (3.2232)			-6870.8	1.3701	1.3794	0.028
ARMA(3,2)	0.0844 (0.0612)	-0.7608 (0.0497)	-0.9569 (0.0353)	-0.0324 (0.0311)	0.7285 (0.0384)	0.9161 (0.0369)		-6862.4	1.3656	1.3768	0.022
ARMA(3,3)	0.0886 (0.0598)	-0.7983 (0.1203)	-0.9752 (0.0576)	-0.0681 (0.1066)	0.7683 (0.1226)	0.9354 (0.0605)	0.0374 (0.1071)	-6862.4	1.3661	1.3792	0.022

Note. Standard Errors are given in parenthesis.

Table 3.30: Intel Corporation (INTC): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	-0.0002 (0.0416)	-0.0436 (0.0271)						-7466.1	1.7326	1.7366	0.590
AR(2)	0.0002 (0.0416)	-0.0452 (0.0273)	-0.0459 (0.0263)					-7462.6	1.7310	1.7363	0.879
AR(3)	0.0018 (0.0415)	-0.0462 (0.0273)	-0.0441 (0.0264)	-0.0176 (0.0265)				-7460.0	1.7300	1.7375	0.7021
MA(1)	0.0026 (0.0013)				-0.0480 (0.0290)			-7474.2	1.7375	1.7412	0.098
MA(2)	0.0024 (0.0023)				-0.0476 (0.0266)	-0.0476 (0.0243)		-7470.6	1.7360	1.7415	0.051
MA(3)	0.0024 (0.0019)				-0.0474 (0.0268)	-0.0477 (0.0340)	-0.0177 (0.0350)	-7470.1	1.7362	1.7437	0.029
ARMA(0,0)	0.0027 (0.0017)							-7477.6	1.7390	1.7409	0.000
ARMA(1,1)	-0.0004 (0.0002)	0.1187 (0.6882)			-0.1710 (0.7360)			-7465.7	1.7329	1.7385	0.001
ARMA(1,2)	-0.0002 (0.0001)	0.0007 (0.0005)			-0.0476 (0.0268)	-0.0448 (0.0285)		-7462.8	1.7318	1.7392	0.009
ARMA(1,3)	-0.0001 (0.0005)	-0.0459 (0.0355)			-0.0010 (0.0170)	-0.0473 (0.0765)	-0.0188 (0.1541)	-7462.2	1.7320	1.7413	0.008
ARMA(2,1)	0.0006 (0.0008)	0.5763 (0.1496)	-0.0176 (0.0096)		-0.6223 (0.1414)			-7460.5	1.7304	1.7378	0.001
ARMA(2,2)	0.0008 (0.0011)	0.7306 (0.1658)	-0.1516 (0.0889)		-0.7774 (0.1714)	0.1419 (0.0806)		-7460.0	1.7307	1.7400	0.001
ARMA(2,3)	0.0009 (0.0016)	0.6878 (0.1875)	-0.1919 (0.2213)		-0.7354 (0.1888)	0.1854 (0.2031)	-0.0109 (0.0202)	-7459.9	1.7312	1.7424	0.001
ARMA(3,1)	0.0016 (0.0016)	0.2855 (0.1692)	-0.0298 (0.0236)	-0.0099 (0.0195)	-0.3327 (0.1604)			-7459.6	1.7304	1.7397	0.001
ARMA(3,2)	0.0016 (0.0011)	0.0564 (0.0694)	0.1836 (0.2447)	-0.0091 (0.0120)	-0.1037 (0.0734)	-0.2263 (0.2393)		-7459.3	1.7308	1.7420	0.001
ARMA(3,3)	0.0012 0.0005)	0.1888 (0.1897)	0.0417 (0.0668)	0.1137 (0.0900)	-0.2371 (0.1926)	-0.0787 (0.0811)	-0.1191 (0.0790)	-7459.1	1.7313	1.7444	0.001

Note. Standard Errors are given in parenthesis.

Table 3.31: Cisco (CSCO): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	-0.0171 (0.0447)	-0.0272 (0.0242)						-7702.7	1.8774	1.8811	0.995
AR(2)	-0.0184 (0.0446)	-0.0306 (0.0247)	-0.0516 (0.0278)					-7698.0	1.8751	1.8807	0.995
AR(3)	-0.0145 (0.0444	-0.0300 (0.0246)	-0.0395 (0.0255)	-0.0089 (0.0260)				-7686.0	1.8684	1.8758	0.931
MA(1)	-0.0105 (0.0073)				-0.0303 (0.0280)			-7737.7	1.8988	1.9025	0.900
MA(2)	-0.0108 (0.0201)				-0.0297 (0.0199)	-0.0496 (0.0294)		-7733.6	1.8969	1.9024	0.900
MA(3)	-0.0108 (0.0104)				-0.0289 (0.0220)	-0.0501 (0.0256)	-0.0092 (0.0102)	-7733.4	1.8974	1.9048	0.911
ARMA(0,0)	-0.0103 (0.0020)							-7739.1	1.8990	1.9009	0.000
ARMA(1,1)	-0.0164 (0.0231)	0.0452 (0.0273)			-0.0780 (0.0290)			-7702.3	1.8777	1.8833	0.001
ARMA(1,2)	-0.0166 (0.0231)	0.0204 (0.0257)			-0.0510 (0.0510)	-0.0373 (0.0282)		-7700.0	1.8769	1.8844	0.003
ARMA(1,3)	-0.0166 (0.0817)	0.0131 (0.0169)			-0.0434 (0.0204)	-0.0376 (0.0221)	-0.0075 (0.0107)	-7699.9	1.8775	1.8868	0.003
ARMA(2,1)	-0.0212 (0.0095)	-0.1625 (0.1039)	-0.0527 (0.0278)		0.1307 (0.0977)			-7697.8	1.8756	1.8830	0.002
ARMA(2,2)	-0.0143 (0.0303)	-0.0215 (0.0855)	-0.0392 (0.0428)		-0.0083 (0.0211)	-0.0085 (0.0524)		-7686.0	1.8690	1.8783	0.003
ARMA(2,3)	-0.0159 (0.0356)	-0.0572 (0.0295)	-0.0954 (0.1290)		-0.0100 (0.0151)	0.0274 (0.0279)	0.0550 (0.1271)	-7686.0	1.8696	1.8808	0.003
ARMA(3,1)	-0.0143 (0.0303)	-0.0215 (0.0855)	-0.0392 (0.0428)	-0.0083 (0.0211)	-0.0085 (0.0524)			-7686.0	1.8690	1.8783	0.003
ARMA(3,2)	-0.0159 (0.0356)	-0.0572 (0.0295)	-0.0954 (0.1290)	-0.0100 (0.0151)	0.0274 (0.0279)	0.0550 (0.1271)		-7686.0	1.8696	1.8808	0.003
ARMA(3,3)	-0.0113 (0.0277)	0.0745 (0.1030)	-0.0670 (0.1689)	0.1683 (0.2485)	-0.1005 (0.0904)	0.0247 (0.1618)	-0.1732 (0.2451)	-7685.3	1.8698	1.8829	0.005

Note. Standard Errors are given in parenthesis.

Table 3.32: Home Depot (HD): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	0.0222 (0.0346)	0.0004 (0.0256)						-6864.6	1.3645	1.3682	0.633
AR(2)	0.0229 (0.0347)	-0.0000 (0.0258)	-0.0315 (0.0266)					-6862.9	1.3640	1.3642	0.985
AR(3)	0.0237 (0.0347)	0.0003 (0.0259)	-0.0282 (0.0267)	0.0063 (0.0258)				-6861.3	1.3637	1.3711	0.942
MA(1)	0.0259 (0.0922)				0.0004 (0.0013)			-6883.2	1.3759	1.3796	0.911
MA(2)	0.0258 (0.0127)				0.0008 (0.0006)	-0.0348 (0.0289)		-6881.4	1.3754	1.3810	0.940
MA(3)	0.0258 (0.0229)				0.0011 (0.0039)	-0.0346 (0.0485)	0.0054 (0.0339)	-6881.4	1.3760	1.3834	0.940
ARMA(0,0)	0.0259 (0.0012)							-6883.2	1.3752	1.3771	0.000
ARMA(1,1)	0.0217 (0.0093)	0.0211 (0.0283)			-0.0218 (0.0084)			-6864.6	1.3651	1.3706	0.033
ARMA(1,2)	0.0222 (0.0484)	0.0023 (0.0039)			-0.0018 (0.0007)	-0.0310 (0.0397)		-6863.2	1.3648	1.3723	0.090
ARMA(1,3)	0.0223 (0.0469)	-0.0025 (0.0034)			0.0036 (0.0029	-0.0306 (0.0321)	0.0095 (0.0229)	-6863.0	1.3653	1.3746	0.093
ARMA(2,1)	0.0369 (0.0394)	-0.6562 (0.2376)	-0.0316 (0.0278)		0.6564 (0.2383)			-6861.6	1.3638	1.3713	0.138
ARMA(2,2)	0.0179 (0.0128)	1.0049 (0.0854)	-0.7017 (0.0930)		-0.9975 (0.0839)	0.6916 (0.0971)		-6852.7	1.3590	1.3683	0.060
ARMA(2,3)	0.0178 (0.0188)	1.0079 (0.0839)	-0.7014 (0.0939)		-1.0142 (0.0911)	0.7109 (0.1117)	-0.0190 (0.0459)	-6852.1	1.3593	1.3704	0.072
ARMA(3,1)	0.0080 (0.0092)	0.7144 (0.3473)	-0.0281 (0.0265)	0.0057 (0.0084)	-0.7166 (0.3508)			-6858.7	1.3627	1.3720	0.114
ARMA(3,2)	0.0068 (0.0134)	0.2536 (0.3262)	0.4758 (0.2948)	0.0133 (0.0320)	-0.2536 (0.3265)	-0.5159 (0.2953)		-6857.9	1.3628	1.3740	0.218
ARMA(3,3)	0.0107 (0.0128)	1.5509 (0.0754)	-1.3008 (0.0916)	0.2566 (0.0634)	-1.5664 (0.0821)	1.3210 (0.1014)	-0.2721 (0.0704)	-6837.9	1.3512	1.3696	0.080

Note. Standard Errors are given in parenthesis.

Table 3.33: United Technologies (UTX): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box** Q_{20}
AR(1)	0.0449 (0.0310)	-0.0793 (0.0230)						-6500.9	1.1419	1.1456	0.489
AR(2)	0.0474 (0.0313)	-0.0825 (0.0226)	-0.0392 (0.0312)					-6497.7	1.1405	1.1461	0.777
AR(3)	0.0465 (0.0317)	-0.0815 (0.0228)	-0.0357 (0.0311)	0.0437 (0.0461)				-6492.5	1.1379	1.1454	0.902
MA(1)	0.0416 (0.0346)				-0.0844 (0.0235)			-6500.2	1.1415	1.1462	0.890
MA(2)	0.0416 (0.0380)				-0.0799 (0.0245)	-0.0259 (0.0240)		-6499.2	1.1414	1.1470	0.888
MA(3)	0.0415 (0.0434)				-0.0802 (0.0224)	-0.0233 (0.0338)	0.0466 (0.0517)	-6495.7	1.1399	1.1474	0.808
ARMA(0,0)	0.0416 (0.0010)							-6511.3	1.1476	1.1495	0.480
ARMA(1,1)	0.0354 (0.0382)	0.1529 (0.2084)			-0.2361 (0.2059)			-6499.7	1.1417	1.1473	0.589
ARMA(1,2)	0.0591 (0.0508)	-0.4187 (0.2178)			0.3391 (0.2207)	-0.0674 (0.0320)		-6497.9	1.1413	1.1487	0.708
ARMA(1,3)	0.0441 (0.0430)	-0.0578 (0.1924)			-0.0229 (0.1847)	-0.0279 (0.0352)	0.0458 (0.0673)	-6495.7	1.1405	1.1498	0.608
ARMA(2,1)	0.0570 (0.0621)	-0.2989 (0.3108)	-0.0625 (0.0530)		0.2165 (0.3118)			-6497.3	1.1409	1.1483	0.666
ARMA(2,2)	0.0563 (0.0375)	0.2995 (0.3168)	-0.7045 (0.0771)		-0.3842 (0.2962)	0.7340 (0.1154)		-6487.7	1.1356	1.1469	0.649
ARMA(2,3)	0.0572 (0.0305)	0.2634 (0.2656)	-0.6889 (0.0915)		-0.3449 (0.2540)	0.7100 (0.1164)	0.0084 (0.0226)	-6487.6	1.1362	1.1474	0.656
ARMA(3,1)	0.0477 (0.0288)	-0.1061 (0.0885)	-0.0378 (0.0311)	0.0425 (0.0359)	0.0246 (0.0876)			-6492.5	1.1385	1.1479	0.789
ARMA(3,2)	0.0573 (0.0340)	0.3519 (0.0885)	-0.7388 (0.0679)	-0.0039 (0.0017)	-0.4331 (0.0769)	0.7786 (0.0701)		-6485.3	1.1348	1.1459	0.732
ARMA(3,3)	0.0200 (0.0282)	1.0437 (0.2664)	-1.0408 0.1407)	0.5400 (0.2243)	-1.1385 (0.2649)	1.1353 (0.1425)	-0.5942 (0.2320)	-6480.8	1.1326	1.1457	0.736

Note. Standard Errors are given in parenthesis.

Table 3.34: Boeing Company (BA): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box** Q_{20}
AR(1)	0.0321 (0.0344)	0.0015 (0.0279)						-6831.4	1.3441	1.3479	0.804
AR(2)	0.0325 (0.0346)	0.0017 (0.0279)	0.0106 (0.0278)					-6830.4	1.3441	1.3454	0.808
AR(3)	0.0323 (0.0347)	0.0004 (0.0276)	0.0108 (0.0276)	0.0417 (0.0284)				-6825.6	1.3418	1.3493	0.806
MA(1)	0.0324 (0.1271				0.0015 (0.0556)			-6831.5	1.3442	1.3479	0.804
MA(2)	0.0324 (0.0649)				0.0006 (0.0017)	0.0113 (0.0369)		-6831.3	1.3447	1.3503	0.804
MA(3)	0.0323 (0.1263)				0.0040 (0.0313)	0.0147 (0.0461)	0.0428 (0.0318)	-6828.3	1.3435	1.3509	0.808
ARMA(0,0)	0.0324 (0.0012)							-6831.5	1.3436	1.3497	0.458
ARMA(1,1)	0.0444 (0.1453)	-0.3815 (2.1931			0.3853 (2.2188)			-6831.3	1.3447	1.3503	0.404
ARMA(1,2)	0.0249 (0.0343)	0.2276 (0.7464)			-0.2270 (0.7563)	0.0187 (0.0374)		-6831.0	1.3451	1.3526	0.489
ARMA(1,3)	0.0400 (0.0574)	-0.2520 (0.1236)			0.2548 (0.1168)	0.0161 (0.0217)	0.0490 (0.0250)	-6827.5	1.3436	1.3529	0.450
ARMA(2,1)	0.0187 (0.0148)	0.4332 (0.1474)	0.0147 (0.0133)		-0.4320 (0.1541)			-6829.3	1.3440	1.3515	0.339
ARMA(2,2)	0.0213 (0.0296)	1.2627 (0.0429)	-0.9171 (0.0453)		-1.2434 (0.0479)	0.9139 (0.0407)		-6820.2	1.3391	1.3485	0.327
ARMA(2,3)	0.0453 (0.0429)	0.4277 (0.0389)	-0.8913 (0.0591)		-0.4273 (0.0316)	0.9150 (0.0528)	0.0111 (0.0071)	-6822.7	1.3413	1.3525	0.360
ARMA(3,1)	0.0409 (0.1283)	-0.2686 (0.4304)	0.0112 (0.0271)	0.0482 (0.0332)	0.2700 (0.4206)			-6824.8	1.3419	1.3513	0.240
ARMA(3,2)	0.0463 (0.0503)	-0.1275 (0.2229)	-0.3140 (0.1958)	0.0507 (0.0273)	0.1285 (0.2242)	0.3331 (0.1938)		-6824.0	1.3420	1.3532	0.239
ARMA(3,3)	0.0571 (0.6757)	-0.2106 (0.3295)	-0.3271 (0.4443)	-0.1949 (0.1272)	0.2138 (0.3305)	0.3473 (0.4304)	0.2490 (0.1217)	-6823.3	1.3422	1.3553	0.255

Note. Standard Errors are given in parenthesis.

Table 3.35: McDonald's (MCD): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	0.0431 (0.0260)	-0.0262 (0.0282)						-5919.2	0.7859	0.7896	0.923
AR(2)	0.0458 (0.0260)	-0.0273 (0.0279)	-0.0477 (0.0248)					-5914.5	0.7836	0.7892	0.989
AR(3)	0.0466 (0.0261)	-0.0282 (0.0278)	-0.0481 (0.0248)	-0.0155 (0.0244)				-5914.1	0.7840	0.7914	0.957
MA(1)	0.0421 (0.0261)				-0.0291 (0.0485)			-5919.2	0.7859	0.7896	0.712
MA(2)	0.0422 (0.0242)				-0.0289 (0.0328)	-0.0485 (0.0237)		-5915.4	0.7842	0.7897	0.758
MA(3)	0.0422 (0.0341)				-0.0285 (0.0292)	-0.0479 (0.0250)	-0.0123 (0.0275)	-5915.1	0.7846	0.7921	0.758
ARMA(0,0)	0.0421 (0.0007)							-5920.4	0.7860	0.7879	0.000
ARMA(1,1)	0.0183 (0.0160)	0.5656 (0.2246)			-0.6060 (0.2147)			-5916.8	0.7850	0.7906	0.400
ARMA(1,2)	0.0341 (0.0365)	0.1905 (0.3853)			-0.2188 (0.3778)	-0.0432 (0.0305)		-5915.2	0.7846	0.7921	0.626
ARMA(1,3)	0.0640 (0.0438)	-0.5279 (1.3535)			0.4994 (1.3587)	-0.0635 (0.0476)	-0.0327 (0.0567)	-5915.0	0.7852	0.7945	0.597
ARMA(2,1)	0.0335 (0.0160)	0.2573 (0.5906)	-0.0410 (0.0470)		-0.2856 (0.5925)			-5914.1	0.7840	0.7914	0.583
ARMA(2,2)	0.0334 (0.0169)	0.2470 (0.1774)	-0.0268 (0.0202)		-0.2753 (0.1713)	-0.0145 (0.0047)		-5914.1	0.7846	0.7939	0.583
ARMA(2,3)	0.0374 (0.0203)	0.1417 (0.0945)	-0.0183 (0.0434)		-0.1700 (0.0955)	-0.0254 (0.0451)	-0.0067 (0.0088)	-5914.1	0.7852	0.7964	0.581
ARMA(3,1)	0.0345 (0.0402)	0.2354 (0.3596)	-0.0409 (0.0318)	-0.0041 (0.0130)	-0.2637 (0.3563)			-5914.0	0.7845	0.7939	0.584
ARMA(3,2)	0.0119 (0.0246)	1.0803 (0.7117)	-0.3860 (0.3503)	0.0251 (0.0239)	-1.1091 (0.7188)	0.3696 (0.3890)		-5913.8	0.7850	0.7962	0.627
ARMA(3,3)	0.0279 (0.0245)	0.6278 (0.4055)	-0.0063 (0.0215)	-0.2772 (0.2132)	-0.6566 (0.4034)	-0.0243 (0.0252)	0.2933 (0.2115)	-5913.3	0.7853	0.7984	0.722

Note. Standard Errors are given in parenthesis.

Table 3.36: American Express (AXP): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	0.0262 (0.0429)	-0.0756 (0.0325)						-7565.8	1.7936	1.7973	0.091
AR(2)	0.0280 (0.0429)	-0.0774 (0.0328)	-0.0376 (0.0339)					-7562.9	1.7924	1.7980	0.100
AR(3)	0.0298 (0.0430)	-0.0785 (0.0328)	-0.0365 (0.0342)	-0.0094 (0.0336)				-7560.7	1.7917	1.7991	0.100
MA(1)	0.0263 (0.0675)				-0.0812 (0.0384)			-7569.6	1.7959	1.7996	0.087
MA(2)	0.0262 (0.1736)				-0.0789 (0.0330)	-0.0315 (0.0325)		-7568.0	1.7955	1.8011	0.087
MA(3)	0.0262 (0.0197)				-0.0786 (0.0333)	-0.0314 (0.0247)	-0.0036 (0.0192)	-7567.9	1.7961	1.8036	0.088
ARMA(0,0)	0.0265 (0.0019)							-7579.7	1.8015	1.8033	0.000
ARMA(1,1)	0.0224 (0.0370)	0.0741 (0.1815)			-0.1548 (0.1787)			-7565.3	1.7939	1.7995	0.000
ARMA(1,2)	0.0285 (0.2174)	-0.1623 (0.5995)			0.0840 (0.5858)	-0.0408 (0.0402)		-7563.6	1.7934	1.8009	0.000
ARMA(1,3)	0.0317 (0.0816)	-0.2823 (0.3042)			0.2052 (0.3119)	-0.0518 (0.0282)	-0.0172 (0.0416)	-7563.3	1.7939	1.8032	0.000
ARMA(2,1)	0.0073 (0.0073)	0.7084 (0.2372)	0.0387 (0.0617)		-0.7845 (0.2446)			-7560.4	1.7915	1.7990	0.000
ARMA(2,2)	0.0108 (0.0073)	0.9873 (0.1234)	-0.3604 (0.1984)		-1.0639 (0.1094)	0.4037 (0.1949)		-7556.8	1.7899	1.7992	0.000
ARMA(2,3)	0.0116 (0.0380)	1.0554 (0.0982)	-0.4564 (0.1370)		-1.1374 (0.0988)	0.5212 (0.1590)	-0.0213 (0.0367)	-7556.4	1.7903	1.8015	0.000
ARMA(3,1)	0.0111 (0.0158)	0.5910 (0.1576)	0.0164 (0.0337)	0.0187 (0.0403)	-0.6714 (0.1680)			-7557.6	1.7904	1.7997	0.000
ARMA(3,2)	0.0113 (0.0762)	1.3246 (0.1291)	-0.6875 (0.1361)	-0.0504 (0.0338)	-1.4058 (0.1231)	0.7853 (0.1426)		-7555.7	1.7898	1.8010	0.000
ARMA(3,3)	0.0195 (0.0173)	0.2552 (0.3114)	0.3950 (0.1971)	-0.3253 (0.2650)	-0.3338 (0.2901)	-0.3976 (0.2234)	0.3470 (0.2495)	-7556.5	1.7909	1.8040	0.000

Note. Standard Errors are given in parenthesis.

Table 3.37: 3M Company (MMM): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box Q_{20}**
AR(1)	0.0382 (0.0260)	-0.0632 (0.0236)						-5914.2	0.7828	0.7865	0.920
AR(2)	0.0395 (0.0261)	-0.0657 (0.0237)	-0.0423 (0.0273)					-5911.1	0.7815	0.7861	0.956
AR(3)	0.0408 (0.0261)	-0.0653 (0.0237)	-0.0430 (0.0275)	-0.0021 (0.0255)				-5907.7	0.7801	0.7875	0.911
MA(1)	0.0356 (0.0183)				-0.0687 (0.0256)			-5913.9	0.7826	0.7864	0.942
MA(2)	0.0356 (0.0182				-0.0661 (0.0235)	-0.0386 (0.0257)		-5911.5	0.7818	0.7874	0.916
MA(3)	0.0356 (0.0399)				-0.0661 (0.0268)	-0.0386 (0.0282)	-0.0010 (0.0026)	-5911.5	0.7824	0.7899	0.916
ARMA(0,0)	0.0356 (0.0007)							-5921.0	0.7864	0.7882	0.380
ARMA(1,1)	0.0175 (0.0102)	0.5196 (0.2514)			-0.5846 (0.2385)			-5910.9	0.7814	0.7870	0.538
ARMA(1,2)	0.0396 (0.0236)	-0.1019 (0.0614)			0.0358 (0.0447)	-0.0454 (0.0312)		-5911.1	0.7822	0.7896	0.587
ARMA(1,3)	0.0640 (0.0472)	-0.7762 (0.0960)			0.7127 (0.0973)	-0.0901 (0.0413)	-0.0492 (0.0359)	-5909.2	0.7816	0.7909	0.545
ARMA(2,1)	0.0449 (0.0275)	-0.2119 (0.0947)	-0.0502 (0.0244)		0.1466 (0.0941)			-5911.0	0.7820	0.7895	0.644
ARMA(2,2)	0.0322 (0.0311)	-0.2312 (0.3379)	0.3369 (0.5361)		0.1636 (0.3345)	-0.3819 (0.5158)		-5910.3	0.7823	0.7916	0.627
ARMA(2,3)	0.1166 (0.0577)	-1.7315 (0.0353)	-0.9405 (0.0394)		1.6678 (0.0448)	0.8154 (0.0648)	-0.0775 (0.0264)	-5904.7	0.7795	0.7906	0.613
ARMA(3,1)	0.0516 (0.0979)	-0.3378 (2.1308)	-0.0611 (0.1337)	-0.0171 (0.1312)	0.2729 (2.1365)			-5907.6	0.7806	0.7899	0.519
ARMA(3,2)	0.0178 (0.0181)	0.2648 (0.2632)	0.2420 (0.2594)	0.0230 (0.0217)	-0.3312 (0.2604)	-0.2625 (0.2395)		-5906.7	0.7807	0.7919	0.526
ARMA(3,3)	0.0210 (0.0124)	0.1619 (0.1994)	0.1126 (0.1206)	0.1679 (0.1685)	-0.2262 (0.2038)	-0.1443 (0.1122)	-0.1500 (0.1427)	-5906.5	0.7812	0.7942	0.508

Note. Standard Errors are given in parenthesis.

Table 3.38: Goldman Sachs (GS): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box Q_{20}**
AR(1)	0.0180 (0.0430)	-0.0481 (0.0373)						-7567.1	1.7944	1.7998	0.418
AR(2)	0.0192 (0.0431)	-0.0484 (0.0378)	-0.0326 (0.0364)					-7565.1	1.7938	1.7994	0.639
AR(3)	0.0207 (0.0433)	-0.0491 (0.0379)	-0.0302 (0.0370)	-0.0094 (0.0304)				-7563.6	1.7934	1.8009	0.798
MA(1)	0.0211 (0.0499)				-0.0514 (0.0535)			-7583.4	1.8044	1.8081	0.059
MA(2)	0.0210 (0.0430)				-0.0506 (0.0361)	-0.0345 (0.0331)		-7581.7	1.8039	1.8095	0.067
MA(3)	0.0210 (0.0553)				-0.0514 (0.0393)	-0.0346 (0.0356)	-0.0139 (0.0219)	-7581.4	1.8043	1.8118	0.080
ARMA(0,0)	0.0213 (0.0019)							-7587.5	1.8062	1.8081	0.000
ARMA(1,1)	0.0176 (0.0063)	-0.0308 (0.0527)			-0.0182 (0.0211)			-7567.1	1.7950	1.8006	0.000
ARMA(1,2)	0.0192 (0.3655)	-0.1043 (0.4647)			0.0551 (0.4761)	-0.0351 (0.1569)		-7565.4	1.7946	1.8020	0.000
ARMA(1,3)	0.0192 (0.0573)	-0.1005 (0.0596)			0.0507 (0.0322)	-0.0363 (0.0410)	-0.0118 (0.0171)	-7565.2	1.7951	1.8044	0.000
ARMA(2,1)	0.0066 (0.0120)	0.6663 (0.1174)	-0.0041 (0.0102)		-0.7166 (0.1155)			-7561.4	1.7921	1.7996	0.000
ARMA(2,2)	0.0067 (0.0032)	0.7139 (0.0964)	-0.0502 (0.0321)		-0.7645 (0.1004)	0.0482 (0.0460)		-7561.3	1.7927	1.8020	0.000
ARMA(2,3)	0.0080 (0.0069)	0.7198 (0.0997)	-0.1175 (0.2103)		-0.7721 (0.0991)	0.1286 (0.2211)	-0.0203 (0.0222)	-7560.9	1.7930	1.8042	0.000
ARMA(3,1)	0.0094 (0.1133)	0.5497 (0.1828)	-0.0038 (0.0395)	-0.0126 (0.0350)	-0.6011 (0.1915)			-7561.4	1.7927	1.8020	0.000
ARMA(3,2)	0.0127 (0.2188)	-0.1295 (0.7488)	0.4846 (0.2528)	0.0096 (0.1992)	0.0784 (0.8269)	-0.5377 (0.2166)		-7560.4	1.7927	1.8039	0.000
ARMA(3,3)	0.0123 (0.0079)	-0.1478 (0.0579(	0.4755 (0.1146)	0.0510 (0.0234)	0.0962 (0.0638)	-0.5288 (0.1256)	-0.0418 (0.0368)	-7560.3	1.7933	1.8063	0.000

Note. Standard Errors are given in parenthesis.

Table 3.39: UnitedHealth (UNH): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box Q_{20}**
AR(1)	0.0538 (0.0376)	-0.0215 (0.0368)						-7090.5	1.5027	1.5064	0.325
AR(2)	0.0570 (0.0379)	-0.0234 (0.0368)	-0.0164 (0.0330)					-7084.0	1.4993	1.5049	0.975
AR(3)	0.0544 (0.0385)	-0.0221 (0.0369)	-0.0152 (0.0331)	0.0422 (0.0321)				-7080.9	1.4981	1.5055	0.950
MA(1)	0.0518 (0.0381)				-0.0222 (0.0293)			-7091.5	1.5033	1.5070	0.112
MA(2)	0.0518 (0.0432)				-0.0206 (0.0277)	-0.0143 (0.0299)		-7091.2	1.5037	1.5093	0.319
MA(3)	0.0517 (0.0451)				-0.0208 (0.0279)	-0.0119 (0.0303)	0.0440 (0.0322)	-7088.2	1.5025	1.5100	0.326
ARMA(0,0)	0.0518 (0.0014)							-7092.3	1.5032	1.5050	0.042
ARMA(1,1)	0.0213 (0.0145)	0.6114 (0.3498)			-0.6273 (0.3579)			-7088.6	1.5022	1.5078	0.547
ARMA(1,2)	0.0212 (0.0190)	0.6131 (0.2454)			-0.6369 (0.2685)	0.0131 (0.0511)		-7088.4	1.5026	1.5101	0.554
ARMA(1,3)	0.0423 (0.0411)	0.2071 (0.4166)			-0.2285 (0.4175)	-0.0071 (0.0189)	0.0412 (0.0442)	-7086.6	1.5022	1.5115	0.508
ARMA(2,1)	0.0737 (0.0617)	-0.3279 (0.1034)	-0.0322 (0.0455)		0.3058 (0.1040)			-7083.2	1.4994	1.5069	0.622
ARMA(2,2)	0.0880 (0.6678)	-0.4320 (12.8786)	-0.2123 (11.3548)		0.4060 (12.8110)	0.1746 (10.8886)		-7082.8	1.4998	1.5091	0.622
ARMA(2,3)	0.0661 (0.0415)	0.3137 (0.1855)	-0.5730 (0.5154)		-0.3359 (0.1949)	0.5826 (0.5360)	0.0449 (0.0388)	-7079.8	1.4986	1.5098	0.621
ARMA(3,1)	0.0560 (0.0564)	-0.0505 (0.1170)	-0.0159 (0.0187)	0.0417 (0.0301)	0.0284 (0.1097)			-7080.9	1.4987	1.5080	0.560
ARMA(3,2)	0.0593 (0.0751)	0.0005 (0.0015)	-0.1334 (0.3779)	0.0432 (0.0301)	-0.0228 (0.0428)	0.1197 (0.3861)		-7080.7	1.4991	1.5103	0.721
ARMA(3,3)	0.0990 (0.0727)	-0.1741 (0.2039)	-0.3620 (0.3684)	-0.3564 (0.2927)	0.1497 (0.1879)	0.3569 (0.3619)	0.4116 (0.2978)	-7079.1	1.4988	1.5118	0.671

Note. Standard Errors are given in parenthesis.

Table 3.40: Caterpillar (CAT): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box Q_{20}**
AR(1)	0.0513 (0.0369)	0.0016 (0.0231)						-7070.7	1.4906	1.4943	0.118
AR(2)	0.0510 (0.0370)	0.0016 (0.0231)	-0.0218 (0.0288)					-7067.6	1.4893	1.4918	0.238
AR(3)	0.0505 (0.0371)	0.0044 (0.0229)	-0.0217 (0.0286)	0.0432 (0.0249)				-7060.5	1.4855	1.4930	0.010
MA(1)	0.0515 (0.0325)				0.0017 (0.0006)			-7070.7	1.4906	1.4943	0.000
MA(2)	0.0514 (0.0340)				0.0036 (0.0081)	-0.0221 (0.0308)		-7069.9	1.4907	1.4963	0.001
MA(3)	0.0514 (0.0867)				0.0026 (0.0505)	-0.0197 (0.0979)	0.0427 (0.0555)	-7067.0	1.4895	1.4970	0.011
ARMA(0,0)	0.0515 (0.0014)							-7070.7	1.4900	1.4949	0.000
ARMA(1,1)	0.0341 (0.4395)	0.3324 (5.4033)			-0.3334 (5.3704)			-7070.6	1.4912	1.4968	0.000
ARMA(1,2)	0.0751 (0.0533)	-0.4349 (0.2305)			0.4381 (0.2281)	-0.0287 (0.0235)		-7069.0	1.4908	1.4982	0.003
ARMA(1,3)	0.0505 (0.0898)	0.0160 (0.0260)			-0.0134 (0.0535)	-0.0196 (0.0571)	0.0429 (0.0280)	-7067.0	1.4901	1.4995	0.011
ARMA(2,1)	0.0769 (0.0632)	-0.5065 (0.1172)	-0.0297 (0.0256)		0.5099 (0.1121)			-7064.2	1.4878	1.4953	0.003
ARMA(2,2)	0.1293 (0.1654)	-1.0210 (0.4445)	-0.5251 (0.3346)		1.0287 (0.4488)	0.5064 (0.3578)		-7061.7	1.4869	1.4963	0.003
ARMA(2,3)	0.0918 (0.2862)	-0.3321 (0.2658)	-0.5064 (0.3887)		0.3354 (0.2759)	0.4836 (0.3919)	0.0358 (0.0328)	-7062.1	1.4878	1.4990	0.031
ARMA(3,1)	0.0500 (0.0340)	0.0129 (0.0255)	-0.0217 (0.0153)	0.0434 (0.0241)	-0.0085 (0.0227)			-7060.5	1.4861	1.4955	0.010
ARMA(3,2)	0.0637 (0.0501)	-0.0558 (0.1464)	-0.2242 (0.1551)	0.0454 (0.0207)	0.0599 (0.1479)	0.2038 (0.1587)		-7060.2	1.4866	1.4978	0.019
ARMA(3,3)	0.1237 (0.0836)	-0.3582 (0.1859)	-0.6455 (0.1377)	-0.3821 (0.1668)	0.3592 (0.1807)	0.6259 (0.1300)	0.4195 (0.1545)	-7058.8	1.4863	1.4994	0.034

Note. Standard Errors are given in parenthesis.

Table 3.41: E. I. du Pont de Nemours (DD): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box Q_{20}**
AR(1)	0.0247 (0.0316)	-0.0453 (0.0279)						-6555.8	1.1755	1.1792	0.000
AR(2)	0.0244 (0.0316)	-0.0463 (0.0279)	-0.0226 (0.0282)					-6553.9	1.1749	1.1788	0.374
AR(3)	0.0247 (0.0318)	-0.0451 (0.0279)	-0.0215 (0.0284)	0.0228 (0.0273)				-6552.0	1.1744	1.1818	0.010
MA(1)	0.0236 (0.0149)				-0.0472 (0.0292)			-6555.7	1.1754	1.1791	0.001
MA(2)	0.0236 (0.0263)				-0.0455 (0.0305)	-0.0186 (0.0220)		-6555.1	1.1757	1.1813	0.001
MA(3)	0.0236 (0.0350)				-0.0456 (0.0301)	-0.0181 (0.0247)	0.0225 (0.0340)	-6554.3	1.1758	1.1832	0.001
ARMA(0,0)	0.0236 (0.0010)							-6559.2	1.1769	1.1805	0.000
ARMA(1,1)	0.0186 (0.0980)	0.2080 (2.0551)			-0.2552 (2.0335)			-6555.4	1.1759	1.1814	0.001
ARMA(1,2)	0.0330 (0.4224)	-0.3822 (0.2301)			0.3367 (0.2190)	-0.0393 (0.0496)		-6554.7	1.1760	1.1835	0.001
ARMA(1,3)	0.0259 (0.0316)	-0.0956 (0.3070)			0.0499 (0.3100)	-0.0224 (0.0203)	0.0210 (0.0292)	-6554.3	1.1764	1.1857	0.001
ARMA(2,1)	0.0373 (0.0383)	-0.5950 (0.3131)	-0.0442 (0.0303)		0.5498 (0.3174)			-6552.3	1.1746	1.1820	0.001
ARMA(2,2)	0.0414 (0.5196)	-0.6661 (0.3373)	-0.1494 (0.8010)		0.6221 (0.3464)	0.1023 (0.7752)		-6552.2	1.1751	1.1844	0.001
ARMA(2,3)	0.0807 (0.0436)	-1.4954 (0.1158)	-0.7297 (0.1483)		1.4546 (0.1174)	0.6354 (0.1657)	-0.0568 (0.0294)	-6546.8	1.1724	1.1836	0.003
ARMA(3,1)	0.0381 (0.0348)	-0.5671 (0.2490)	-0.0463 (0.0290)	-0.0031 (0.0099)	0.5224 (0.2506)			-6551.8	1.1748	1.1841	0.001
ARMA(3,2)	0.0037 (0.0025)	0.5196 (0.4317)	0.3091 (0.4201)	0.0374 (0.0304)	-0.5655 (0.4333)	-0.3094 (0.4343)		-6550.0	1.1744	1.1856	0.001
ARMA(3,3)	0.0770 (0.2150)	-0.5108 (0.2011)	-0.6635 (0.0594)	-0.7658 (0.1966)	0.4738 (0.1867)	0.6646 (0.0584)	0.7719 (0.1805)	-6542.1	1.1701	1.1832	0.012

Note. Standard Errors are given in parenthesis.

Table 3.42: Nike (NKE): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	**LL**	**AIC**	**BIC**	**Ljung-Box Q_{20}**
AR(1)	0.0596 (0.0324	-0.0218 (0.0249)						-6634.2	1.2234	1.2272	0.871
AR(2)	0.0610 (0.0325)	-0.0233 (0.0250)	-0.0349 (0.0266)					-6631.2	1.2222	1.2251	0.994
AR(3)	0.0658 (0.0325)	-0.0244 (0.0251)	-0.0349 (0.0266)	-0.0588 (0.0238)				-6622.8	1.2177	1.2278	0.997
MA(1)	0.0593 (0.0385)				0.0593 (0.0385)			-6635.7	1.2243	1.2281	0.005
MA(2)	0.0592 (0.0294)				-0.0276 (0.0312)	-0.0398 (0.0286)		-6633.2	1.2235	1.2291	0.002
MA(3)	0.0591 (0.0316)				-0.0284 (0.0249)	-0.0440 (0.0295)	-0.0608 (0.0234)	-6627.0	1.2203	1.2277	0.075
ARMA(0,0)	0.0593 (0.0010)							-6636.5	1.2242	1.2261	0.000
ARMA(1,1)	0.0177 (0.0088)	0.6876 (0.0799)			-0.7405 (0.0806)			-6627.2	1.2197	1.2253	0.036
ARMA(1,2)	0.0219 (0.0104)	0.6163 (0.1103)			-0.6460 (0.1144)	-0.0396 (0.0296)		-6624.9	1.2190	1.2264	0.077
ARMA(1,3)	0.0310 (0.0178)	0.4597 (0.2542)			-0.4884 (0.2575)	-0.0267 (0.0299)	-0.0435 (0.0265)	-6622.7	1.2182	1.2276	0.173
ARMA(2,1)	0.0218 (0.0105)	0.6602 (0.1063)	-0.0412 (0.0266)		-0.6874 (0.1052)			-6625.0	1.2190	1.2265	0.080
ARMA(2,2)	0.0124 (0.0111)	1.4071 (0.3621)	-0.6213 (0.2365)		-1.4382 (0.3812)	0.6187 (0.2800)		-6622.5	1.2181	1.2275	0.178
ARMA(2,3)	0.0490 (0.0289)	-0.3250 (0.1596)	0.4661 (0.1513)		0.2979 (0.1579)	-0.5227 (0.1471)	-0.0663 (0.0246)	-6620.4	1.2175	1.2287	0.395
ARMA(3,1)	0.0795 (0.1105)	-0.2515 (1.1913)	-0.0398 (0.0325)	-0.0604 (0.0298)	0.2287 (1.2087)			-6622.6	1.2182	1.2275	0.020
ARMA(3,2)	0.0413 (0.0211)	-0.0109 (0.0452)	0.3634 (0.2292)	-0.0595 (0.0257)	-0.0157 (0.0748)	-0.4040 (0.2224)		-6619.5	1.2169	1.2281	0.257
ARMA(3,3)	0.0435 (0.0222)	-0.2403 (0.1277)	-0.1201 (0.0805)	0.6088 (0.0838)	0.2129 (0.1244)	0.0605 (0.0761)	-0.6708 (0.0788)	-6612.0	1.2129	1.2260	0.172

Note. Standard Errors are given in parenthesis.

Table 3.43: Travelers (TRV): Estimation results for ARMA(p,q) models

Model	α_0	α_1	α_2	α_3	β_1	β_2	β_3	LL	AIC	BIC	Ljung-Box Q_{20}
AR(1)	0.0317 (0.0344)	-0.1240 (0.0450)						-6817.8	1.3358	1.3395	0.662
AR(2)	0.0351 (0.0343)	-0.1319 (0.0456)	-0.0606 (0.0347)					-6808.6	1.3308	1.3364	0.652
AR(3)	0.0351 (0.0345)	-0.1315 (0.0457)	-0.0578 (0.0349)	0.0239 (0.0312)				-6806.8	1.3303	1.3377	0.652
MA(1)	0.0280 (0.0207)				-0.1367 (0.0486)			-6815.2	1.3342	1.3380	0.487
MA(2)	0.0280 (0.0247)				-0.1295 (0.0456)	-0.0400 (0.0399)		-6812.8	1.3334	1.3390	0.412
MA(3)	0.0280 (0.0278)				-0.1288 (0.0448)	-0.0412 (0.0370)	0.0258 (0.0282)	-6811.7	1.3333	1.3408	0.400
ARMA(0,0)	0.0279 (0.0012)							-6843.3	1.3508	1.3526	0.058
ARMA(1,1)	0.0217 (0.0209)	0.2481 (0.2677)			-0.3797 (0.2528)			-6812.8	1.3333	1.3389	0.005
ARMA(1,2)	0.0393 (0.0339)	-0.4038 (0.3210)	0.2744 (0.3146)		-0.0976 (0.0554)			-6811.7	1.3333	1.3407	0.010
ARMA(1,3)	0.0337 (0.0275)	-0.1968 (0.1507)			0.0671 (0.1393)	-0.0667 (0.0465)	0.0225 (0.0474)	-6811.2	1.3336	1.3429	0.025
ARMA(2,1)	0.0374 (0.0379)	-0.2061 (0.1238)	-0.0712 (0.0444)		0.0745 (0.1117)			-6808.5	1.3314	1.3388	0.031
ARMA(2,2)	0.0420 (0.0225)	-0.1187 (0.0452)	-0.3250 (0.2176)		-0.0105 (0.0097)	0.2788 (0.2338)		-6807.0	1.3311	1.3404	0.039
ARMA(2,3)	0.0431 (0.0264)	-0.1305 (0.0430)	-0.3538 (0.2638)		-0.0000 (0.0000)	0.3061 (0.2798)	-0.0080 (0.0178)	-6807.0	1.3316	1.3428	0.039
ARMA(3,1)	0.0468 (0.0199)	-0.4559 (0.2579)	-0.1009 (0.0443)	0.0027 (0.0024)	0.3245 (0.2560)			-6806.3	1.3306	1.3399	0.057
ARMA(3,2)	0.0710 (0.1029)	-0.8455 (0.3055)	-0.5161 (0.3556)	-0.0605 (0.0632)	0.7157 (0.3134)	0.3680 (0.3419)		-6804.7	1.3303	1.3414	0.061
ARMA(3,3)	0.0237 (0.0652)	0.1345 (1.1878)	-0.2228 (0.6204)	0.3124 (0.3002)	-0.2649 (1.1844)	0.1951 (0.4749)	-0.3035 (0.2626)	-6803.9	1.3304	1.3434	0.069

Note. Standard Errors are given in parenthesis.

3.4.3 Histograms of 29 components of $U_{1,t}$ with Different Distributions Fit

Figure 3.4.1: Histograms of 29 singular vector components with Gaussian fit from first round of SVX model

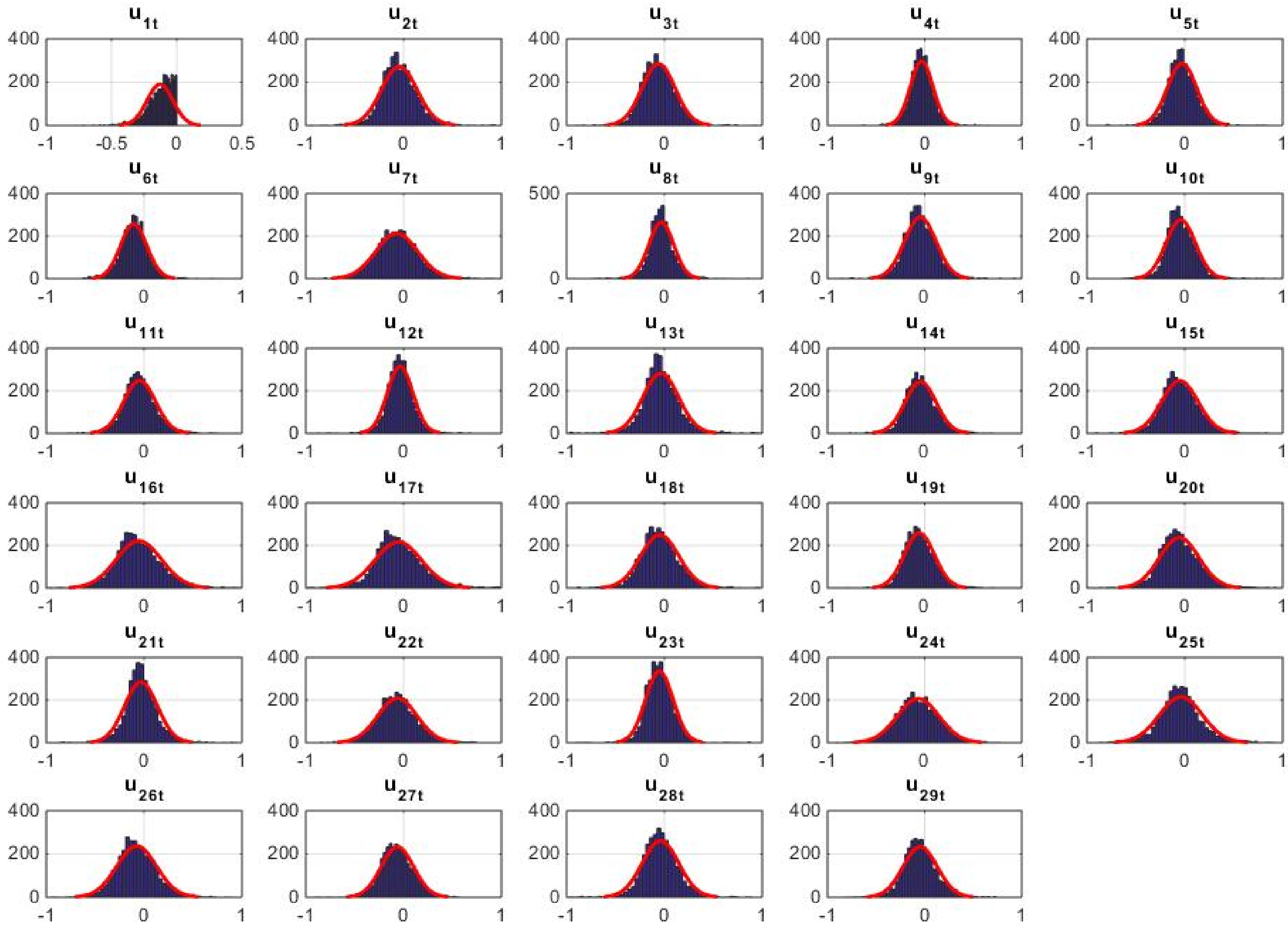
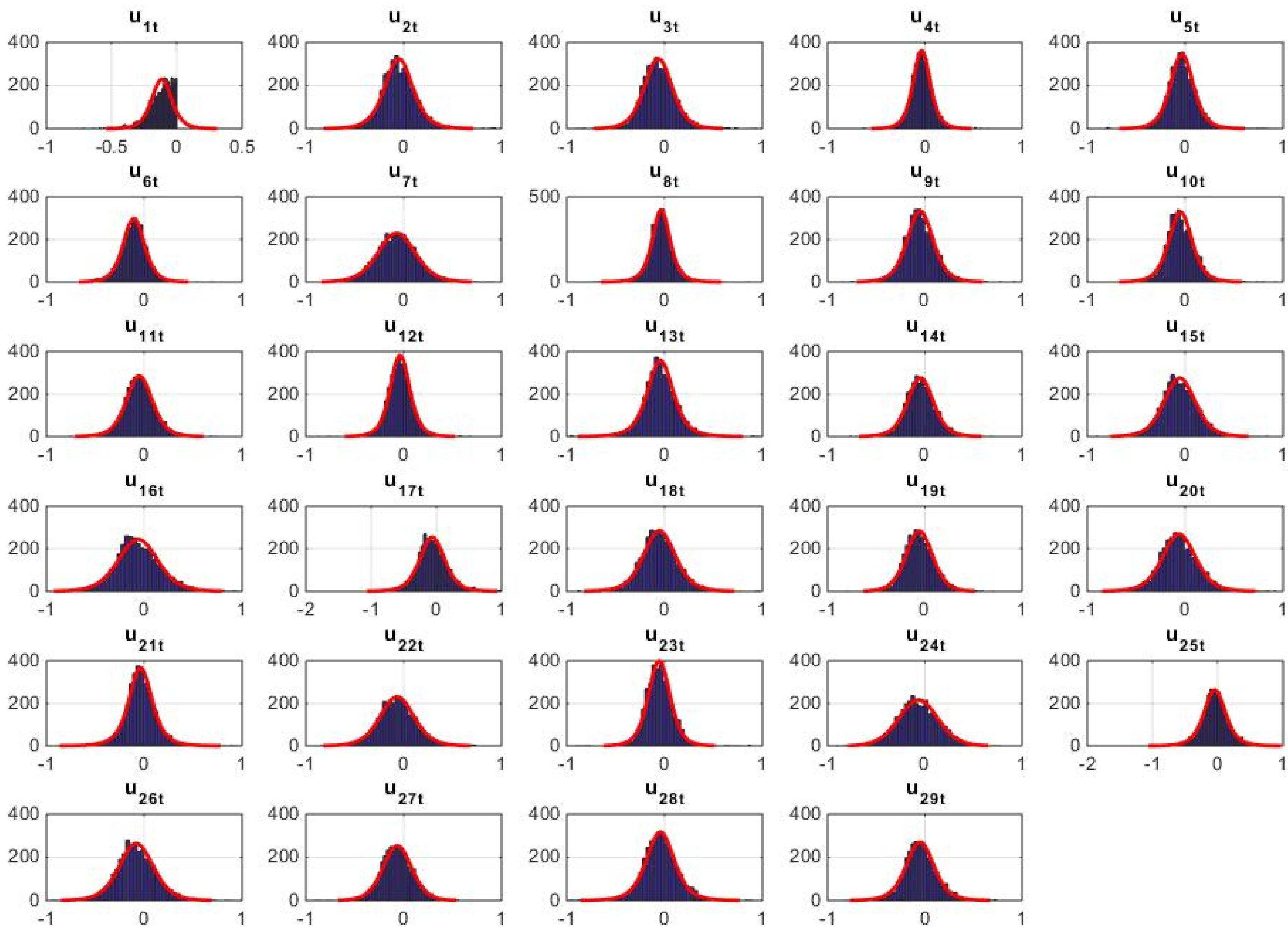


Figure 3.4.2: Histograms of 29 singular vector components with Student's t fit from first round of SVX model



3.4.4 Estimates of ARMA(p,q) models for 29 components of $\mathbf{U_{1,t}}$ from First Round

Table 3.44: Estimates of ARMA(p,q) model for first component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.116	-0.107	-0.100	-0.129	-0.129	-0.129	-0.129	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
	(0.003)	(0.004)	(0.004)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
α_1	0.101	0.094	0.089					0.992	0.992	0.993	1.020	1.237	1.242	1.020	0.731	0.134
	(0.020)	(0.020)	(0.020)					(0.004)	(0.004)	(0.004)	(0.022)	(0.198)	(0.155)	(0.021)	(0.251)	(0.000)
α_2		0.072	0.066								-0.025	-0.241	-0.247	-0.021	0.270	0.993
		(0.018)	(0.018)								(0.021)	(0.197)	(0.156)	(0.025)	(0.242)	(0.000)
α_3			0.068											-0.004	-0.009	-0.137
			(0.018)											(0.008)	(0.012)	(0.000)
β_1				0.089	0.087	0.088		-0.963	-0.942	-0.942	-0.970	-1.188	-1.193	-0.971	-0.682	-0.079
				(0.018)	(0.021)	(0.021)		(0.010)	(0.021)	(0.019)	(0.008)	(0.200)	(0.157)	(0.008)	(0.243)	(0.000)
β_2					0.065	0.064			-0.023	-0.020		0.210	0.215		-0.280	-0.980
					(0.017)	(0.018)			(0.020)	(0.021)		(0.194)	(0.155)		(0.237)	(0.000)
β_3						0.055				-0.004			0.001			0.112
						(0.015)				(0.005)			(0.001)			(0.000)
LL	2918.2	2926.8	2934.3	2915.5	2922.8	2928.6	2900.8	2992.9	2993.8	2993.8	2995.8	2996.0	2996.0	2995.9	2995.9	3012.2
AIC	-4.624	-4.628	-4.632	-4.622	-4.626	-4.629	-4.614	-4.669	-4.669	-4.668	-4.670	-4.670	-4.669	-4.669	-4.669	-4.678
BIC	-4.620	-4.623	-4.625	-4.618	-4.620	-4.621	-4.612	-4.663	-4.661	-4.659	-4.663	-4.660	-4.658	-4.660	-4.658	-4.665
Ljung-Box Q_{20}	256.18	184.88	138.56	280.85	218.67	178.30	413.83	14.98	14.00	14.12	14.27	14.45	14.42	14.33	14.32	14.80
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78	0.83	0.82	0.82	0.81	0.81	0.81	0.81	0.79

Note. Standard Errors are given in parenthesis.

Table 3.45: Estimates of ARMA(p,q) model for second component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.042	-0.042	-0.042	-0.045	-0.045	-0.045	-0.045	-0.001	-0.001	-0.001	0.000	-0.001	-0.001	-0.004	-0.034	-0.003
	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.077)	(0.001)
α_1	0.053	0.052	0.052					0.987	0.988	0.988	1.025	0.483	0.314	0.932	0.382	0.122
	(0.019)	(0.019)	(0.019)					(0.006)	(0.005)	(0.005)	(0.020)	(0.367)	(0.186)	(0.081)	(0.421)	(0.084)
α_2		0.014	0.014								-0.031	0.505	0.673	-0.032	-0.141	0.026
		(0.018)	(0.018)								(0.019)	(0.361)	(0.183)	(0.027)	(1.408)	(0.016)
α_3			0.004											0.016	0.011	0.793
			(0.019)											(0.019)	(0.066)	(0.075)
β_1				0.051	0.052	0.052		-0.975	-0.947	-0.947	-0.984	-0.450	-0.272	-0.889	-0.331	-0.084
				(0.018)	(0.019)	(0.019)		(0.008)	(0.020)	(0.020)	(0.007)	(0.349)	(0.188)	(0.084)	(0.429)	(0.080)
β_2					0.016	0.016			-0.030	-0.035		-0.522	-0.675		0.134	-0.007
					(0.017)	(0.016)			(0.019)	(0.026)		(0.338)	(0.175)		(1.379)	(0.007)
β_3						0.002				0.005			-0.023			-0.810
						(0.011)				(0.016)			(0.020)			(0.070)
LL	923.5	923.8	927.6	923.1	923.5	923.5	918.7	939.4	940.9	940.9	937.4	939.3	939.9	930.3	928.3	946.7
AIC	-3.402	-3.402	-3.404	-3.402	-3.402	-3.401	-3.400	-3.411	-3.412	-3.411	-3.410	-3.410	-3.410	-3.405	-3.403	-3.413
BIC	-3.398	-3.396	-3.396	-3.398	-3.396	-3.394	-3.398	-3.406	-3.404	-3.402	-3.402	-3.401	-3.399	-3.395	-3.392	-3.400
Ljung-Box Q_{20}	30.72	29.36	28.28	31.44	29.80	29.61	43.17	18.28	15.59	15.48	15.37	15.99	15.48	15.76	28.01	9.89
p-value	0.06	0.08	0.10	0.05	0.07	0.08	0.00	0.57	0.74	0.75	0.75	0.72	0.75	0.73	0.11	0.97

Note. Standard Errors are given in parenthesis.

Table 3.46: Estimates of ARMA(p,q) model for third component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.059	-0.057	-0.056	-0.061	-0.061	-0.061	-0.061	0.000	0.000	0.000	-0.003	-0.002	-0.002	-0.001	0.000	-0.001
	(0.003)	(0.003)	(0.003)	(0.003)	(.003)	(0.003)	(0.002)	(0.000)	(0.000)	(0.000)	(0.002)	(0.001)	(0.001)	(0.001)	(0.000)	(0.001)
α_1	0.027	0.026	0.025					0.997	0.997	0.997	0.933	0.112	0.105	0.988	1.411	0.635
	(0.020)	(0.020)	(0.020)					(0.003)	(0.003)	(0.003)	(0.034)	(0.113)	(0.098)	(0.023)	(0.079)	(0.043)
α_2		0.025	0.025								0.021	0.857	0.863	-0.007	-0.415	-0.552
		(0.019)	(0.019)								(0.018)	(0.116)	(0.101)	(0.014)	(0.080)	(0.191)
α_3			0.029											0.009	0.003	0.898
			(0.017)											(0.026)	(0.005)	(0.217)
β_1				0.026	0.025	0.025		-0.987	-0.988	-0.988	-0.918	-0.092	-0.091	-0.972	-1.402	-0.631
				(0.018)	(0.020)	(0.020)		(0.007)	(0.011)	(0.010)	(0.031)	(0.116)	(0.105)	(0.017)	(0.082)	(0.055)
β_2					0.024	0.024			0.001	-0.001		-0.833	-0.839		0.411	0.568
					(0.017)	0.018)			(0.008)	(0.006)		(0.118)	0.102)		(0.084)	(0.181)
β_3						0.026				0.002			0.007			-0.878
						(0.016)				(0.010)			(0.013)			(0.221)
LL	1158.0	1160.2	1161.7	1157.9	1158.9	1160.1	1156.8	1186.7	1186.7	1186.7	1169.8	1177.1	1177.2	1182.0	1188.6	1184.2
AIC	-3.546	-3.547	-3.547	-3.546	-3.546	-3.546	-3.546	-3.563	-3.562	-3.562	-3.552	-3.556	-3.555	-3.559	-3.562	-3.559
BIC	-3.542	-3.541	-3.539	-3.542	-3.540	-3.538	-3.544	-3.557	-3.555	-3.552	-3.544	-3.546	-3.544	-3.549	-3.551	-3.546
Ljung-Box Q_{20}	81.06	74.37	66.98	81.60	75.65	75.65	90.01	24.60	24.56	24.45	25.92	23.55	23.39	22.99	23.73	21.15
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.22	0.22	0.17	0.26	0.27	0.29	0.25	0.39

Note. Standard Errors are given in parenthesis.

Table 3.47: Estimates of ARMA(p,q) model for fourth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.030	-0.029	-0.028	-0.031	-0.031	-0.031	-0.031	-0.001	0.000	0.000	0.000	0.000	0.000	-0.001	-0.001	-0.001
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.000)	(0.001)
α_1	0.049	0.048	0.047					0.984	0.988	0.987	1.021	0.063	0.050	0.998	0.071	-0.978
	(0.020)	(0.020)	(0.020)					(0.021)	(0.012)	(0.017)	(0.022)	(0.051)	(0.039)	(0.040)	(0.048)	(0.035)
α_2		0.018	0.016								-0.026	0.922	0.937	-0.028	0.931	0.972
		(0.019	(0.019)								(0.020)	(0.048)	(0.054)	(0.028)	(0.035)	(0.021)
α_3			0.039											0.010	-0.018	0.973
			(0.019)											(0.035)	(0.022)	(0.039)
β_1				0.047	0.047	0.045		-0.965	-0.949	-0.948	-0.984	-0.040	-0.011	-0.960	-0.032	0.996
				(0.019)	(0.019)	(0.021)		(0.033)	(0.021)	(0.027)	(0.010)	(0.048)	(0.046)	(0.033)	(0.035)	(0.035)
β_2					0.016	0.016			-0.023	-0.029		-0.920	-0.933		-0.927	-0.945
					(0.015)	(0.022)			(0.019)	(0.023)		(0.042)	(0.053)		(0.032)	(0.037)
β_3						0.036				0.007			-0.020			-0.963
						(0.019)				(0.011)			(0.023)			(0.042)
LL	2390.2	2390.7	2393.5	2389.9	2390.3	2392.4	2386.1	2404.8	2405.6	2405.7	2407.1	2409.1	2409.6	2404.6	2409.8	2413.1
AIC	-4.300	-4.300	-4.301	-4.300	-4.300	-4.300	-4.298	-4.309	-4.309	-4.308	-4.309	-4.310	-4.310	-4.307	-4.310	-4.311
BIC	-4.297	-4.294	-4.294	-4.296	-4.294	-4.293	-4.297	-4.303	-4.301	-4.299	-4.302	-4.301	-4.299	-4.298	-4.299	-4.298
Ljung-Box Q_{20}	58.05	54.72	46.72	59.17	56.34	49.55	75.97	23.31	21.57	21.31	22.49	22.00	21.07	21.66	20.87	23.99
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.36	0.38	0.31	0.34	0.39	0.36	0.40	0.24

Note. Standard Errors are given in parenthesis.

Table 3.48: Estimates of ARMA(p,q) model for fifth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.028	-0.027	-0.027	-0.029	-0.029	-0.029	-0.029	-0.001	-0.001	-0.001	-0.023	-0.001	-0.001	-0.001	-0.002	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.001)	(0.001)	(0.001)	(0.029)	(0.001)	(0.001)	(0.001)	(0.001)	(0.011)
α_1	0.045	0.044	0.043					0.954	0.956	0.956	0.188	0.777	0.667	0.970	0.474	0.667
	(0.022)	(0.022)	(0.022)					(0.018)	(0.017)	(0.017)	(1.069)	(0.404)	(0.359)	(0.024)	(0.150)	(28.451)
α_2		0.027	0.027								0.021	0.187	0.295	-0.013	0.486	-0.571
		(0.018)	(0.018)								(0.054)	(0.390)	(0.347)	(0.018)	(0.160)	(25.944)
α_3			0.005											-0.003	-0.013	0.816
			(0.017)											(0.007)	(0.023)	(2.124)
β_1				0.042	0.043	0.044		-0.930	-0.917	-0.917	-0.145	-0.741	-0.628	-0.932	-0.435	-0.628
				(0.020)	(0.023)	(0.026)		(0.021)	(0.035)	(0.032)	(1.068)	(0.392)	(0.355)	(0.021)	(0.161)	(28.156)
β_2					0.029	0.029			-0.017	-0.013		-0.200	-0.295		-0.480	0.573
					(0.017)	(0.018)			(0.028)	(0.021)		(0.369)	(0.344)		(0.155)	(23.654)
β_3						0.007				-0.004			-0.013			-0.809
						(0.041)				(0.008)			(0.017)			(3.627)
LL	1557.2	1558.5	1559.0	1556.5	1557.9	1558.0	1553.4	1564.0	1564.5	1564.5	1558.5	1566.2	1566.5	1564.2	1565.5	1566.9
AIC	-3.790	-3.790	-3.790	-3.790	-3.790	-3.789	-3.789	-3.794	-3.793	-3.793	-3.790	-3.794	-3.793	-3.793	-3.793	-3.793
BIC	-3.787	-3.785	-3.783	-3.786	-3.784	-3.782	-3.787	-3.788	-3.786	-3.784	-3.782	-3.785	-3.782	-3.783	-3.782	-3.780
Ljung-Box Q_{20}	42.69	39.45	38.71	43.38	39.63	39.18	54.11	29.08	27.14	27.12	39.27	27.18	26.78	27.58	26.97	22.67
p-value	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.09	0.13	0.13	0.01	0.13	0.14	0.12	0.14	0.31

Note. Standard Errors are given in parenthesis.

Table 3.49: Estimates of ARMA(p,q) model for sixth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.101	-0.099	-0.094	-0.107	-0.107	-0.107	-0.107	0.000	0.000	0.000	0.000	-0.001	-0.001	0.000	0.000	-0.001
	(0.003)	(0.003)	(0.004)	(0.002)	(0.003)	(0.003)	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
α_1	0.058	0.056	0.055					0.996	0.996	0.996	1.011	0.264	0.334	1.009	0.270	-0.638
	(0.019)	(0.019)	(0.019)					(0.002)	(0.002)	(0.002)	(0.015)	(0.098)	(0.119)	(0.020)	(0.076)	(0.130)
α_2		0.023	0.021								-0.014	0.730	0.660	-0.035	0.713	0.844
		(0.019)	(0.019)								(0.015)	(0.098)	(0.119)	(0.028)	(0.083)	(0.055)
α_3			0.044											0.023	0.013	0.785
			(0.020)											(0.018)	(0.024)	(0.096)
β_1				0.055	0.055	0.056		-0.983	-0.969	-0.969	-0.986	-0.229	-0.308	-0.985	-0.244	0.663
				(0.019)	(0.020)	(0.019)		(0.004)	(0.017)	(0.020)	(0.003)	(0.094)	(0.119)	(0.005)	(0.074)	(0.137)
β_2					0.021	0.019			-0.014	-0.035		-0.745	-0.683		-0.732	-0.838
					(0.018)	(0.017)			(0.017)	(0.028)		(0.093)	(0.112)		(0.073)	(0.049)
β_3						0.040				0.021			0.015			-0.778
						(0.018)				(0.020)			(0.018)			(0.102)
LL	1914.8	1915.8	1919.1	1914.2	1915.0	1917.9	1909.0	1964.2	1964.5	1965.2	1966.3	1967.4	1967.7	1963.5	1967.4	1970.0
AIC	-4.009	-4.009	-4.011	-4.009	-4.009	-4.010	-4.006	-4.039	-4.038	-4.038	-4.040	-4.040	-4.039	-4.037	-4.039	-4.040
BIC	-4.006	-4.004	-4.003	-4.005	-4.003	-4.002	-4.004	-4.033	-4.031	-4.029	-4.032	-4.030	-4.028	-4.028	-4.028	-4.027
Ljung-Box Q_{20}	128.58	117.92	99.84	131.76	122.58	106.57	167.73	31.17	30.49	29.04	30.52	27.11	26.79	29.51	27.04	23.50
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.06	0.09	0.06	0.13	0.14	0.08	0.13	0.26

Note. Standard Errors are given in parenthesis.

Table 3.50: Estimates of ARMA(p,q) model for seventh component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.070	-0.068	-0.064	-0.070	-0.070	-0.070	-0.070	-0.001	-0.002	-0.002	-0.001	-0.001	-0.002	-0.001	-0.002	-0.002
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.002)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
α_1	0.004	0.004	0.002					0.980	0.978	0.976	0.928	1.013	0.150	0.923	0.087	0.348
	(0.021)	(0.021)	(0.021)					(0.007)	(0.008)	(0.009)	(0.023)	(1.102)	(0.078)	(0.024)	(0.079)	(0.189)
α_2		0.032	0.032								0.053	-0.031	0.817	0.029	0.821	0.814
		(0.020)	(0.020)								(0.021)	(1.098)	(0.076)	(0.029)	(0.080)	(0.080)
α_3			0.052											0.027	0.057	-0.189
			(0.020)											(0.020)	(0.020)	(0.156)
β_1				0.004	0.001	-0.003		-0.950	-0.998	-0.995	-0.950	-1.037	-0.172	-0.946	-0.107	-0.370
				(0.010)	(0.007)	(0.074)		(0.009)	(0.023)	(0.022)	(0.012)	(1.114)	(0.080)	(0.013)	(0.078)	(0.185)
β_2					0.028	0.023			0.054	0.029		0.085	-0.797		-0.800	-0.798
					(0.018)	(0.020)			(0.021)	(0.029)		(1.089)	(0.072)		(0.076)	(0.074)
β_3						0.046				0.025			0.058			0.241
						(0.022)				(0.019)			(0.020)			(0.148)
LL	405.6	407.3	411.8	405.2	406.7	410.4	405.2	438.8	443.3	444.4	448.3	448.4	450.5	449.3	450.2	450.9
AIC	-3.085	-3.085	-3.088	-3.085	-3.085	-3.087	-3.085	-3.105	-3.107	-3.107	-3.110	-3.109	-3.110	-3.110	-3.110	-3.110
BIC	-3.081	-3.080	-3.080	-3.081	-3.079	-3.079	-3.084	-3.099	-3.099	-3.098	-3.102	-3.100	-3.099	-3.101	-3.099	-3.097
Ljung-Box Q_{20}	151.00	133.95	108.42	150.60	136.92	118.03	152.53	30.80	24.10	21.70	24.60	23.96	21.70	21.61	22.19	21.08
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.24	0.36	0.22	0.24	0.36	0.36	0.33	0.39

Note. Standard Errors are given in parenthesis.

Table 3.51: Estimates of ARMA(p,q) model for eight component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.031	-0.031	-0.031	-0.033	-0.033	-0.033	-0.033	-0.020	-0.008	-0.008	-0.043	-0.055	-0.055	-0.028	-0.052	-0.039
	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.011)	(0.008)	(0.007)	(0.008)	(0.005)	(0.004)	(0.005)	(0.005)	(0.259)
α_1	0.041	0.040	0.040					0.377	0.753	0.758	-0.330	0.198	0.201	0.151	0.253	0.489
	(0.020)	(0.020)	(0.020)					(0.321)	(0.230)	(0.206)	(0.240)	(0.036)	(0.037)	(0.147)	(0.051)	(4.676)
α_2		0.009	0.009								0.026	-0.891	-0.897	0.005	-0.898	-0.948
		(0.020)	(0.020)								(0.026)	(0.065)	(0.041)	(0.026)	(0.049)	(0.977)
α_3			-0.004											-0.004	0.039	0.253
			(0.018)											(0.020)	(0.019)	(4.242)
β_1				0.040	0.041	0.041		-0.339	-0.714	-0.718	0.370	-0.192	-0.162	-0.110	-0.213	-0.449
				(0.020)	(0.020)	(0.023)		(0.329)	(0.229)	(0.210)	(0.240)	(0.037)	(0.042)	(0.148)	(0.047)	(4.720)
β_2					0.010	0.010			-0.023	-0.020		0.892	0.891		0.892	0.941
					(0.016)	(0.025)			(0.025)	(0.019)		(0.064)	(0.038)		(0.046)	(0.959)
β_3						-0.004				-0.006			0.038			-0.215
						(0.048)				(0.019)			(0.020)			(4.303)
LL	2112.0	2113.6	2114.2	2111.1	2111.3	2111.3	2108.5	2112.6	2112.9	2113.0	2114.0	2117.4	2119.8	2114.3	2119.6	2119.7
AIC	-4.130	-4.130	-4.130	-4.130	-4.129	-4.128	-4.128	-4.130	-4.129	-4.129	-4.130	-4.132	-4.132	-4.130	-4.132	-4.132
BIC	-4.126	-4.125	-4.123	-4.126	-4.123	-4.121	-4.127	-4.124	-4.122	-4.119	-4.123	-4.122	-4.121	-4.120	-4.121	-4.119
Ljung-Box Q_{20}	21.38	21.10	21.03	21.32	20.84	20.92	25.89	20.93	21.15	21.01	21.02	24.72	21.70	20.99	21.52	20.94
p-value	0.37	0.39	0.40	0.38	0.41	0.40	0.17	0.40	0.39	0.40	0.40	0.21	0.36	0.40	0.37	0.40

Note. Standard Errors are given in parenthesis.

Table 3.52: Estimates of ARMA(p,q) model for ninth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.046	-0.045	-0.045	-0.045	-0.045	-0.045	-0.045	-0.083	-0.002	-0.084	-0.082	-0.001	-0.001	-0.078	-0.005	-0.005
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.007)	(0.001)	(0.029)	(0.006)	(0.001)	(0.001)	(0.007)	(0.003)	(0.003)
α_1	-0.023	-0.023	-0.022					-0.829	0.961	-0.865	-0.820	0.188	0.133	-0.736	0.057	-0.327
	(0.021)	(0.021)	(0.021)					(0.083)	(0.024)	(0.612)	(0.086)	(0.114)	(0.092)	(0.162)	(0.052)	(0.188)
α_2		0.025	0.024								0.000	0.788	0.838	0.008	0.801	0.833
		(0.022)	(0.022)								(0.002)	(0.120)	(0.088)	(0.030)	(0.063)	(0.050)
α_3			-0.007											0.003	0.026	0.394
			(0.020)											(0.046)	(0.022)	(0.199)
β_1				-0.022	-0.022	-0.022		0.808	-0.987	0.844	0.799	-0.204	-0.159	0.716	-0.081	0.311
				(0.019)	(0.020)	(0.025)		(0.094)	(0.031)	(0.829)	(0.096)	(0.126)	(0.097)	(0.161)	(0.051)	(0.202)
β_2					0.024	0.024			0.042	0.005		-0.755	-0.810		-0.768	-0.801
					(0.022)	(0.026)			(0.021)	(0.568)		(0.133)	(0.104)		(0.071)	(0.053)
β_3						-0.007				0.011			0.018			-0.369
						(0.151)				(0.444)			(0.024)			(0.212)
LL	1288.3	1289.4	1291.1	1288.1	1289.1	1289.2	1287.3	1291.0	1293.7	1291.1	1291.2	1297.5	1297.8	1292.9	1296.0	1296.8
AIC	-3.626	-3.626	-3.626	-3.625	-3.625	-3.625	-3.626	-3.627	-3.628	-3.625	-3.626	-3.629	-3.629	-3.627	-3.628	-3.628
BIC	-3.622	-3.620	-3.619	-3.622	-3.620	-3.617	-3.624	-3.621	-3.620	-3.616	-3.619	-3.620	-3.618	-3.617	-3.617	-3.615
Ljung-Box Q_{20}	26.57	23.08	23.32	26.69	23.29	23.39	27.38	22.51	15.04	22.12	22.47	12.26	12.13	22.22	13.39	12.95
p-value	0.15	0.28	0.27	0.14	0.27	0.27	0.12	0.31	0.77	0.33	0.32	0.91	0.91	0.33	0.86	0.88

Note. Standard Errors are given in parenthesis.

Table 3.53: Estimates of ARMA(p,q) model for tenth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.041	-0.041	-0.040	-0.042	-0.042	-0.042	-0.042	-0.001	-0.049	-0.001	-0.051	0.000	0.000	0.000	0.000	-0.001
	(0.003	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.000)	(0.006)	(0.000)	(0.005)	(0.000)	(0.001)	(0.000)	(0.000)	(0.001)
α_1	0.030	0.030	0.030					0.986	-0.161	0.985	-0.204	0.562	0.730	1.006	0.611	-0.434
	(0.019)	(0.019)	(0.019)					(0.008)	(0.131)	(0.009)	(0.082)	(0.258)	(0.887)	(0.027)	(1.382)	(0.249)
α_2		-0.004	-0.005								0.000	0.429	0.261	-0.036	0.371	0.495
		(0.018)	(0.017)								(0.000)	(0.250)	(0.869)	(0.027)	(1.500)	(0.044)
α_3			0.029											0.025	0.012	0.921
			(0.018)											(0.020)	(0.124)	(0.203)
β_1				0.030	0.030	0.028		-0.972	0.192	-0.964	0.234	-0.537	-0.709	-0.983	-0.589	0.468
				(0.018)	(0.019)	(0.019)		(0.011)	(0.130)	(0.020)	(0.080)	(0.249)	(0.897)	(0.017)	(1.397)	(0.220)
β_2					-0.004	-0.003			-0.001	-0.032		-0.439	-0.285		-0.392	-0.488
					(0.040)	(0.048)			(0.002)	(0.024)		(0.235)	(0.798)		(1.384)	(0.028)
β_3						0.023				0.026			0.016			-0.931
						(0.031)				(0.016)			(0.069)			(0.185)
LL	1551.6	1551.6	1553.0	1551.5	1551.5	1552.5	1550.0	1566.0	1551.7	1567.2	1551.7	1565.7	1566.0	1562.1	1563.0	1573.1
AIC	-3.787	-3.786	-3.786	-3.787	-3.786	-3.786	-3.786	-3.795	-3.786	-3.795	-3.786	-3.794	-3.793	-3.791	-3.791	-3.797
BIC	-3.783	-3.781	-3.779	-3.783	-3.781	-3.779	-3.785	-3.789	-3.778	-3.785	-3.778	-3.784	-3.782	-3.782	-3.780	-3.784
Ljung-Box Q_{20}	35.90	36.10	32.00	36.04	36.25	32.90	40.24	19.39	36.15	17.55	36.09	18.77	17.89	18.27	18.66	8.54
p-value	0.02	0.02	0.04	0.02	0.01	0.03	0.00	0.50	0.01	0.62	0.02	0.54	0.59	0.57	0.54	0.99

Note. Standard Errors are given in parenthesis.

Table 3.54: Estimates of ARMA(p,q) model for eleventh component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.046	-0.045	-0.045	-0.045	-0.045	-0.045	-0.045	-0.083	-0.002	-0.084	-0.082	-0.001	-0.001	-0.078	-0.005	-0.005
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.007)	(0.001)	(0.029)	(0.006)	(0.001)	(0.001)	(0.007)	(0.003)	(0.003)
α_1	-0.023	-0.023	-0.022					-0.829	0.961	-0.865	-0.820	0.188	0.133	-0.736	0.057	-0.327
	(0.021)	(0.021)	(0.021)					(0.083)	(0.024)	(0.612))	(0.086)	(0.114)	(0.092)	(0.162)	(0.052)	(0.188)
α_2		0.025	0.024								0.000	0.788	0.838	0.008	0.801	0.833
		(0.022)	(0.022)								(0.002)	(0.120)	(0.088)	(0.030)	(0.063)	(0.050)
α_3			-0.007											0.003	0.026	0.394
			(0.020)											(0.046)	(0.022)	(0.199)
β_1				-0.022	-0.022	-0.022		0.808	-0.987	0.844	0.799	-0.204	-0.159	0.716	-0.081	0.311
				(0.019)	(0.020)	(0.025)		(0.094)	(0.031)	(0.829)	(0.096)	(0.126)	(0.097)	(0.161)	(0.051)	(0.202)
β_2					0.024	0.024			0.042	0.005		-0.755	-0.810		-0.768	-0.801
					(0.022)	(0.026)			(0.021)	(0.568)		(0.133)	(0.104)		(0.071)	(0.053)
β_3						-0.007				0.011			0.018			-0.369
						(0.151)				(0.444)			(0.024)			(0.212)
LL	1288.3	1289.4	1291.1	1288.1	1289.1	1289.2	1287.3	1291.0	1293.7	1291.1	1291.2	1297.5	1297.8	1292.9	1296.0	1296.8
AIC	-3.626	-3.626	-3.626	-3.625	-3.625	-3.625	-3.626	-3.627	-3.628	-3.625	-3.626	-3.629	-3.629	-3.627	-3.628	-3.628
BIC	-3.622	-3.620	-3.619	-3.622	-3.620	-3.617	-3.624	-3.621	-3.620	-3.616	-3.619	-3.620	-3.618	-3.617	-3.617	-3.615
Ljung-Box Q_{20}	26.57	23.08	23.32	26.69	23.29	23.39	27.38	22.51	15.04	22.12	22.47	12.26	12.13	22.22	13.39	12.95
p-value	0.15	0.28	0.27	0.14	0.27	0.27	0.12	0.31	0.77	0.33	0.32	0.91	0.91	0.33	0.86	0.88

Note. Standard Errors are given in parenthesis.

Table 3.55: Estimates of ARMA(p,q) model for twelfth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.037	-0.035	-0.034	-0.036	-0.036	-0.036	-0.036	-0.048	-0.001	-0.062	-0.003	-0.003	-0.003	-0.058	-0.006	-0.006
	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.005)	(0.000)	(0.005)	(0.002)	(0.002)	(0.002)	(0.005)	(0.004)	(0.004)
α_1	-0.031	-0.029	-0.031					-0.352	0.976	-0.754	0.846	0.650	0.649	-0.738	-0.014	-0.655
	(0.020)	(0.020)	(0.020)					(0.098)	(0.013)	(0.078)	(0.057)	(0.100)	(0.095)	(0.083)	(0.038)	(0.036)
α_2		0.047	0.048								0.064	0.268	0.269	0.028	0.773	0.600
		(0.018)	(0.018)								(0.019)	(0.119)	(0.106)	(0.023)	(0.079)	(0.053)
α_3			0.032											0.073	0.071	0.888
			(0.018)											(0.019)	(0.021)	(0.063)
β_1				-0.028	-0.032	-0.030		0.315	-1.011	0.727	-0.882	-0.685	-0.684	0.709	-0.021	0.642
				(0.019)	(0.019)	(0.020)		(0.097)	(0.023)	(0.078)	(0.054)	(0.104)	(0.099)	(0.082)	(0.027)	(0.041)
β_2					0.051	0.048			0.053	0.024		-0.202	-0.203		-0.755	-0.562
					(0.019)	(0.019)			(0.019)	(0.025)		(0.121)	(0.108)		(0.078)	(0.060)
β_3						0.025				0.068			0.001			-0.837
						(0.020)				(0.019)			(0.000)			(0.082)
LL	2020.5	2025.0	2027.2	2020.3	2024.5	2025.5	2018.9	2021.4	2033.5	2028.5	2031.1	2031.7	2031.7	2030.7	2032.8	2041.6
AIC	-4.074	-4.076	-4.077	-4.074	-4.076	-4.076	-4.074	-4.074	-4.081	-4.077	-4.079	-4.079	-4.078	-4.078	-4.079	-4.084
BIC	-4.070	-4.070	-4.069	-4.070	-4.070	-4.068	-4.072	-4.068	-4.073	-4.068	-4.072	-4.070	-4.067	-4.069	-4.068	-4.071
Ljung-Box Q_{20}	55.72	44.26	39.03	56.06	42.73	39.69	58.60	52.98	30.17	32.16	30.32	29.04	29.09	31.01	25.84	21.60
p-value	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.07	0.04	0.06	0.09	0.09	0.06	0.17	0.36

Note. Standard Errors are given in parenthesis.

Table 3.56: Estimates of ARMA(p,q) model for thirteenth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.036	-0.036	-0.035	-0.038	-0.038	-0.038	-0.038	-0.057	-0.058	-0.027	-0.058	-0.001	-0.001	-0.001	-0.001	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.011)	(0.009)	(0.028)	(0.010)	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)
α_1	0.038	0.038	0.039					-0.517	-0.549	0.287	-0.543	0.395	0.606	0.998	0.428	-0.118
	(0.022)	(0.022)	(0.022)					(0.263)	(0.208)	(0.751)	(0.234)	(0.195)	(0.328)	(0.026)	(0.566)	(0.020)
α_2		-0.007	-0.008								0.003	0.585	0.375	-0.045	0.528	0.129
		(0.020)	(0.020)								(0.008)	(0.192)	(0.324)	(0.031))	(0.599)	(0.054)
α_3			0.040											0.028	0.018	0.946
			(0.019)											(0.020)	(0.040)	(0.038)
β_1				0.039	0.039	0.038		0.555	0.588	-0.249	0.582	-0.355	-0.573	-0.965	-0.395	0.143
				(0.023)	(0.022)	(0.028)		(0.255)	(0.204)	(0.752)	(0.227)	(0.193)	(0.327)	(0.014)	(0.568)	(0.036)
β_2					-0.008	-0.007			0.003	-0.018		-0.605	-0.411		-0.556	-0.134
					(0.025)	(0.145)			(0.010)	(0.032)		(0.188)	(0.312)		(0.562)	(0.057)
β_3						0.038				0.040			0.022			-0.924
						(0.031)				(0.018)			(0.020)			(0.027)
LL	934.5	934.6	937.3	934.4	934.5	937.0	932.0	935.2	935.2	937.2	935.2	945.0	945.5	944.5	945.4	950.5
AIC	-3.409	-3.408	-3.409	-3.409	-3.408	-3.409	-3.408	-3.409	-3.408	-3.409	-3.408	-3.414	-3.413	-3.413	-3.413	-3.416
BIC	-3.405	-3.403	-3.402	-3.405	-3.403	-3.402	-3.406	-3.403	-3.401	-3.399	-3.401	-3.404	-3.402	-3.404	-3.402	-3.403
Ljung-Box Q_{20}	33.02	33.05	25.96	32.90	32.93	26.48	39.93	31.75	31.58	25.65	31.57	16.67	16.06	17.65	16.14	15.02
p-value	0.03	0.03	0.17	0.03	0.03	0.15	0.01	0.05	0.05	0.18	0.05	0.67	0.71	0.61	0.71	0.78

Note. Standard Errors are given in parenthesis.

Table 3.57: Estimates of ARMA(p,q) model for fourteenth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.041	-0.040	-0.041	-0.042	-0.042	-0.042	-0.042	0.000	-0.062	-0.058	-0.058	-0.001	-0.001	-0.023	-0.002	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.000)	(0.009)	(0.009)	(0.011)	(0.000)	(0.000)	(0.053)	(0.001)	(0.001)
α_1	0.023	0.023	0.023					0.992	-0.473	-0.370	-0.412	0.289	0.410	0.451	0.298	-0.105
	(0.020)	(0.020)	(0.020)					(0.004)	(0.177)	(0.180)	(0.237)	(0.100)	(0.136)	(1.251)	(0.246)	(0.197)
α_2		0.024	0.024								0.037	0.696	0.577	0.014	0.670	0.267
		(0.019)	(0.019)								(0.019)	(0.099)	(0.135)	(0.026)	(0.244)	(0.102)
α_3			-0.016											-0.016	-0.014	0.792
			(0.019)											(0.034)	(0.024)	(0.128)
β_1				0.022	0.023	0.024		-0.982	0.496	0.394	0.435	-0.285	-0.393	-0.429	-0.281	0.128
				(0.017)	(0.021)	(0.021)		(0.006)	(0.178)	(0.179)	(0.237)	(0.104)	(0.136)	(1.260)	(0.243)	(0.174)
β_2					0.024	0.024			0.038	0.033		-0.683	-0.556		-0.648	-0.233
					(0.021)	(0.018)			(0.019)	(0.020)		(0.101)	(0.139)		(0.256)	(0.095)
β_3						-0.016				-0.011			-0.021			-0.812
						(0.018)				(0.023)			(0.024)			(0.113)
LL	1365.9	1366.9	1368.1	1365.8	1366.8	1367.2	1364.9	1377.3	1367.4	1367.5	1367.2	1377.2	1377.9	1368.3	1374.2	1379.8
AIC	-3.673	-3.673	-3.673	-3.673	-3.673	-3.673	-3.673	-3.679	-3.673	-3.672	-3.673	-3.678	-3.678	-3.673	-3.676	-3.679
BIC	-3.669	-3.667	-3.666	-3.669	-3.667	-3.665	-3.671	-3.674	-3.665	-3.663	-3.665	-3.669	-3.667	-3.663	-3.665	-3.665
Ljung-Box Q_{20}	29.28	26.51	26.22	29.47	26.69	26.84	31.87	16.39	26.01	26.22	25.92	16.18	15.23	26.23	16.27	8.46
p-value	0.08	0.15	0.16	0.08	0.14	0.14	0.04	0.69	0.17	0.16	0.17	0.71	0.76	0.16	0.70	0.99

Note. Standard Errors are given in parenthesis.

Table 3.58: Estimates of ARMA(p,q) model for fifteenth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.050	-0.048	-0.046	-0.051	-0.051	-0.051	-0.051	-0.001	-0.001	-0.001	-0.008	-0.004	-0.005	-0.001	-0.001	-0.001
	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.004)	(0.002)	(0.000)	(0.000)	(0.000)	(0.005)	(0.003)	(0.006)	(0.000)	(0.000)	(0.001)
α_1	0.019	0.019	0.017					0.986	0.985	0.986	0.799	0.068	0.057	0.973	1.155	1.166
	(0.019)	(0.019)	(0.019)					(0.006)	(0.007)	(0.007)	(0.109)	(0.064)	(0.094)	(0.021)	(0.141)	(0.347)
α_2		0.040	0.039								0.042	0.861	0.846	0.018	-0.161	-0.419
		(0.018)	(0.018)								(0.018)	(0.080)	(0.102)	(0.034)	(0.145)	(1.379)
α_3			0.044											-0.005	-0.008	0.237
			(0.017)											(0.033)	(0.020)	(1.699)
β_1				0.018	0.016	0.016		-0.966	-0.981	-0.981	-0.788	-0.037	-0.045	-0.965	-1.152	-1.161
				(0.017)	(0.020)	(0.020)		(0.010)	(0.019)	(0.022)	(0.110)	(0.067)	(0.096)	(0.011)	(0.142)	(0.370)
β_2					0.038	0.035			0.016	0.022		-0.830	-0.814		0.183	0.442
					(0.017)	(0.018)			(0.018)	(0.024)		(0.087)	(0.109)		(0.143)	(1.363)
β_3						0.039				-0.006			0.026			-0.242
						(0.016)				(0.025)			(0.022)			(1.673)
LL	856.5	860.6	863.8	856.4	858.8	861.4	855.8	880.1	880.5	880.6	866.3	872.9	873.9	883.0	884.2	884.3
AIC	-3.361	-3.363	-3.364	-3.361	-3.362	-3.363	-3.361	-3.375	-3.375	-3.374	-3.366	-3.369	-3.369	-3.376	-3.376	-3.375
BIC	-3.357	-3.358	-3.357	-3.357	-3.356	-3.356	-3.359	-3.369	-3.367	-3.365	-3.359	-3.360	-3.358	-3.366	-3.365	-3.362
Ljung-Box Q_{20}	75.28	65.81	51.89	75.75	66.13	54.91	80.34	22.38	21.07	21.16	29.12	23.07	20.63	21.35	20.87	20.67
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.39	0.39	0.09	0.29	0.42	0.38	0.40	0.42

Note. Standard Errors are given in parenthesis.

Table 3.59: Estimates of ARMA(p,q) model for sixteenth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.057	-0.057	-0.056	-0.055	-0.055	-0.055	-0.055	-0.064	-0.005	-0.006	-0.001	0.000	0.000	-0.001	-0.001	-0.001
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.002)	(0.020)	(0.003)	(0.003)	(0.001)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)
α_1	-0.039	-0.038	-0.039					-0.167	0.900	0.889	0.937	1.355	1.389	0.933	1.084	0.326
	(0.019)	(0.019)	(0.019)					(0.363)	(0.049)	(0.049)	(0.029)	(0.124)	(0.127)	(0.021)	(3.563)	(0.197)
α_2		0.008	0.008								0.052	-0.363	-0.396	0.046	-0.099	0.893
		(0.019)	(0.019)								(0.020)	(0.124)	(0.128)	(0.026)	(3.327)	(0.112)
α_3			0.005											0.009	0.003	-0.236
			(0.018)											(0.019)	(0.227)	(0.175)
β_1				-0.038	-0.039	-0.039		0.128	-0.942	-0.931	-0.977	-1.395	-1.435	-0.978	-1.130	-0.369
				(0.019)	(0.019)	(0.019)		(0.364)	(0.053)	(0.052)	(0.020)	(0.119)	(0.129)	(0.010)	(3.571)	(0.193)
β_2					0.009	0.009			0.064	0.041		0.409	0.462		0.150	-0.886
					(0.018)	(0.019)			(0.019)	(0.025)		(0.118)	(0.140)		(3.535)	(0.119)
β_3						0.005				0.026			-0.014			0.286
						(0.020)				(0.019)			(0.020)			(0.170)
LL	149.3	149.4	149.6	149.0	149.2	149.2	146.6	149.4	152.6	153.6	157.8	161.1	161.4	161.4	161.5	161.4
AIC	-2.928	-2.928	-2.927	-2.928	-2.927	-2.927	-2.927	-2.928	-2.929	-2.929	-2.932	-2.934	-2.933	-2.934	-2.933	-2.932
BIC	-2.924	-2.922	-2.920	-2.924	-2.922	-2.919	-2.925	-2.922	-2.921	-2.920	-2.925	-2.924	-2.922	-2.924	-2.922	-2.919
Ljung-Box Q_{20}	33.63	32.99	32.41	33.56	32.75	32.41	35.75	33.22	19.12	17.51	18.31	18.11	17.66	17.87	17.66	17.48
p-value	0.03	0.03	0.04	0.03	0.04	0.04	0.02	0.03	0.51	0.62	0.57	0.58	0.61	0.60	0.61	0.62

Note. Standard Errors are given in parenthesis.

Table 3.60: Estimates of ARMA(p,q) model for seventeenth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.057	-0.055	-0.055	-0.055	-0.055	-0.055	-0.055	-0.071	-0.058	-0.057	-0.055	-0.118	-0.115	-0.001	-0.135	-0.001
	(0.004)	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)	(0.002)	(0.014)	(0.007)	(0.004)	(0.005)	(0.019)	(0.016)	(0.001)	(0.020)	(0.001)
α_1	-0.025	-0.024	-0.024					-0.273	-0.044	-0.026	-0.030	-0.565	-0.463	0.951	-0.563	0.711
	(0.020)	(0.020)	(0.020)					(0.233)	(0.098)	(0.022)	(0.023)	(0.151)	(0.299)	(0.022)	(0.153)	(0.351)
α_2		0.034	0.035								0.034	-0.568	-0.617	0.056	-0.854	0.245
		(0.020)	(0.020)								(0.020)	(0.212)	(0.180)	(0.028)	(0.193)	(0.258)
α_3			0.000											-0.022	-0.026	0.032
			(0.020)											(0.021)	(0.026)	(0.245)
β_1				-0.023	-0.024	-0.024		0.243	0.021	0.002	0.006	0.548	0.440	-0.979	0.539	-0.738
				(0.019	(0.019	(0.020		(0.221	(0.096	(0.011	(0.007	(0.147	(0.297	(0.010	(0.152	(0.350
β_2					0.034	0.035			0.034	0.035		0.592	0.635		0.864	-0.193
					(0.020)	(0.020)			(0.020)	(0.020)		(0.218)	(0.176)		(0.188)	(0.266)
β_3						-0.002				-0.002			-0.020			-0.049
						(0.013)				(0.011)			(0.038)			(0.241)
LL	59.9	61.8	62.0	56.8	58.7	58.7	55.8	60.3	61.9	61.9	61.8	63.0	63.4	67.7	64.1	68.0
AIC	-2.873	-2.874	-2.873	-2.871	-2.872	-2.871	-2.871	-2.873	-2.873	-2.873	-2.873	-2.873	-2.873	-2.876	-2.874	-2.875
BIC	-2.870	-2.868	-2.866	-2.868	-2.866	-2.864	-2.870	-2.867	-2.866	-2.863	-2.866	-2.864	-2.862	-2.867	-2.862	-2.862
Ljung-Box Q_{20}	20.87	16.81	16.93	21.27	17.20	17.24	23.29	19.33	16.84	16.87	16.81	17.05	16.60	15.63	14.14	15.29
p-value	0.40	0.67	0.66	0.38	0.64	0.64	0.27	0.50	0.66	0.66	0.67	0.65	0.68	0.74	0.82	0.76

Note. Standard Errors are given in parenthesis.

Table 3.61: Estimates of ARMA(p,q) model for eighteenth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.048	-0.047	-0.046	-0.049	-0.049	-0.049	-0.049	-0.002	-0.002	-0.002	-0.003	-0.004	-0.004	-0.073	-0.001	-0.008
	(0.003)	(0.004)	(0.004)	(0.003)	(0.004)	(0.004)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.009)	(0.000)	(0.003)
α_1	0.002	0.002	0.000					0.968	0.965	0.966	0.904	0.559	0.458	-0.572	1.690	-0.616
	(0.019)	(0.019)	(0.019)					(0.013)	(0.015)	(0.014)	(0.032)	(0.130)	(0.167)	(0.149)	(0.132)	(0.029)
α_2		0.031	0.031								0.028	0.366	0.463	0.032	-0.693	0.533
		(0.018)	(0.018)								(0.019)	(0.130)	(0.168)	(0.021)	(0.129)	(0.034)
α_3			0.022											0.036	-0.008	0.904
			(0.018)											(0.018)	(0.015)	(0.030)
β_1				0.002	0.001	0.001		-0.951	-0.969	-0.971	-0.909	-0.561	-0.462	0.573	-1.695	0.633
				(0.003)	(0.001)	(0.002)		(0.016)	(0.023)	(0.023)	(0.027)	(0.139)	(0.169)	(0.148)	(0.136)	(0.032)
β_2					0.031	0.031			0.023	0.030		-0.336	-0.439		0.713	-0.504
					(0.018)	(0.018)			(0.018)	(0.029)		(0.137)	(0.184)		(0.129)	(0.037)
β_3						0.022				-0.008			0.011			-0.879
						(0.018)				(0.023)			(0.021)			(0.034)
LL	712.8	715.8	717.5	712.7	714.3	715.0	712.7	723.2	724.0	724.1	722.0	723.0	723.1	718.6	723.9	726.9
AIC	-3.273	-3.274	-3.275	-3.273	-3.273	-3.273	-3.274	-3.279	-3.279	-3.278	-3.278	-3.278	-3.277	-3.275	-3.278	-3.279
BIC	-3.269	-3.269	-3.267	-3.269	-3.268	-3.266	-3.272	-3.273	-3.271	-3.269	-3.270	-3.268	-3.266	-3.266	-3.266	-3.266
Ljung-Box Q_{20}	36.79	30.73	27.69	37.06	31.77	29.15	37.27	16.06	14.96	14.73	17.07	16.23	16.19	28.26	14.78	15.31
p-value	0.01	0.06	0.12	0.01	0.05	0.08	0.01	0.71	0.78	0.79	0.65	0.70	0.70	0.10	0.79	0.76

Note. Standard Errors are given in parenthesis.

Table 3.62: Estimates of ARMA(p,q) model for nineteenth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.054	-0.052	-0.051	-0.055	-0.055	-0.055	-0.055	-0.002	-0.057	-0.103	-0.084	-0.002	-0.002	-0.093	-0.002	-0.003
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.001)	(0.018)	(0.006)	(0.025)	(0.001)	(0.001)	(0.008)	(0.001)	(0.002)
α_1	0.017	0.017	0.017					0.966	-0.035	-0.880	-0.577	0.149	0.172	-0.785	0.173	-0.829
	(0.020)	(0.020)	(0.020)					(0.020)	(0.329)	(0.055)	(0.430)	(0.087)	(0.153)	(0.107)	(0.130)	(0.012)
α_2		0.042	0.041								0.040	0.822	0.801	0.055	0.806	0.801
		(0.018)	(0.018)								(0.029)	(0.084)	(0.151)	(0.024)	(0.099)	(0.019)
α_3			0.004											0.032	-0.007	0.970
			(0.018)											(0.019)	(0.040)	(0.013)
β_1				0.016	0.016	0.016		-0.945	0.051	0.898	0.596	-0.143	-0.161	0.802	-0.162	0.841
				(0.016)	(0.020)	(0.019)		(0.022)	(0.339)	(0.059)	(0.429)	(0.095)	(0.154)	(0.105)	(0.118)	(0.017)
β_2					0.040	0.040			0.040	0.054		-0.801	-0.778		-0.784	-0.773
					(0.017)	(0.019)			(0.021)	(0.024)		(0.090)	(0.165)		(0.108)	(0.027)
β_3						0.005				0.034			-0.008			-0.955
						(0.020)				(0.017)			(0.025)			(0.018)
LL	1451.4	1454.8	1455.2	1451.1	1453.8	1453.9	1450.7	1456.8	1454.1	1456.0	1455.4	1466.2	1466.3	1456.7	1465.9	1470.4
AIC	-3.725	-3.727	-3.727	-3.725	-3.726	-3.726	-3.726	-3.728	-3.726	-3.726	-3.727	-3.733	-3.732	-3.727	-3.732	-3.734
BIC	-3.722	-3.721	-3.719	-3.722	-3.721	-3.718	-3.724	-3.723	-3.718	-3.717	-3.719	-3.723	-3.721	-3.718	-3.721	-3.721
Ljung-Box Q_{20}	32.74	25.34	24.97	32.92	25.94	25.66	34.89	14.49	26.08	25.95	27.51	13.19	12.91	25.37	12.93	11.44
p-value	0.04	0.19	0.20	0.03	0.17	0.18	0.02	0.80	0.16	0.17	0.12	0.87	0.88	0.19	0.88	0.93

Note. Standard Errors are given in parenthesis.

Table 3.63: Estimates of ARMA(p,q) model for twentieth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.055	-0.053	-0.052	-0.056	-0.056	-0.056	-0.056	-0.002	-0.051	-0.082	-0.043	-0.004	-0.003	-0.002	-0.075	-0.003
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.002)	(0.001)	(0.018)	(0.011)	(0.038)	(0.003)	(0.002)	(0.002)	(0.129)	(0.003)
α_1	0.012	0.012	0.011					0.956	0.084	-0.467	0.193	0.351	0.651	0.954	-0.690	0.612
	(0.020)	(0.020)	(0.020)					(0.020)	(0.305)	(0.166)	(0.696)	(0.161)	(0.265)	(0.040)	(1.155)	(0.412)
α_2		0.050	0.050								0.047	0.580	0.297	0.038	0.320	-0.167
		(0.019)	(0.019)								(0.027)	(0.165)	(0.270)	(0.026)	(1.182)	(0.394)
α_3			0.008											-0.031	0.039	0.492
			(0.018)											(0.020)	(0.040)	(0.221)
β_1				0.011	0.011	0.011		-0.938	-0.073	0.478	-0.182	-0.349	-0.643	-0.944	0.701	-0.604
				(0.017)	(0.022)	(0.024)		(0.022)	(0.302)	(0.166)	(0.696)	(0.161)	(0.262)	(0.034)	(1.156)	(0.399)
β_2					0.051	0.051			0.050	0.056		-0.552	-0.257		-0.263	0.214
					(0.019)	(0.019)			(0.020)	(0.022)		(0.165)	(0.285)		(1.216)	(0.380)
β_3						0.007				0.033			-0.027			-0.519
						(0.020)				(0.019)			(0.027)			(0.212)
LL	603.8	608.0	608.1	603.6	607.7	607.8	603.4	608.7	608.0	608.2	608.1	610.5	611.1	610.9	608.8	612.6
AIC	-3.206	-3.208	-3.208	-3.206	-3.208	-3.208	-3.207	-3.209	-3.208	-3.207	-3.208	-3.209	-3.208	-3.209	-3.207	-3.209
BIC	-3.203	-3.203	-3.200	-3.203	-3.203	-3.200	-3.205	-3.203	-3.200	-3.198	-3.200	-3.199	-3.197	-3.200	-3.196	-3.196
Ljung-Box Q_{20}	26.29	16.90	16.39	26.46	16.91	16.51	27.12	14.71	16.51	16.05	16.38	13.10	11.27	10.87	17.18	7.61
p-value	0.16	0.66	0.69	0.15	0.66	0.68	0.13	0.79	0.68	0.71	0.69	0.87	0.94	0.95	0.64	0.99

Note. Standard Errors are given in parenthesis.

Table 3.64: Estimates of ARMA(p,q) model for twenty first component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.033	-0.033	-0.033	-0.032	-0.032	-0.032	-0.032	0.000	-0.061	0.000	-0.053	-0.036	-0.041	0.000	-0.044	-0.019
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.000)	(0.007)	(0.000)	(0.016)	(0.003)	(0.004)	(0.000)	(0.004)	(0.000)
α_1	-0.011	-0.011	-0.011					0.998	-0.873	0.998	-0.614	0.893	0.706	0.984	0.651	1.122
	(0.022)	(0.022)	(0.022)					(0.000)	(0.165)	(0.000)	(0.472)	(0.008)	(0.049)	(0.000)	(0.051)	(0.000)
α_2		-0.005	-0.005								-0.015	-0.987	-0.974	0.006	-0.980	-1.309
		(0.022)	(0.022)								(0.022)	(0.011)	(0.075)	(0.000)	(0.084)	(0.000)
α_3			0.003											0.003	-0.008	0.601
			(0.021)											(0.000)	(0.034)	(0.000)
β_1				-0.011	-0.011	-0.011		-1.005	0.862	-1.013	0.603	-0.901	-0.718	-1.005	-0.663	-1.137
				(0.032)	(0.025)	(0.029)		(0.000)	(0.164)	(0.000)	(0.473)	(0.007)	(0.053)	(0.000)	(0.046)	(0.000)
β_2					-0.005	-0.005			-0.016	0.003		0.990	0.981		0.987	1.332
					(0.044)	(0.040)			(0.019)	(0.000)		(0.010)	(0.074)			0.000)
β_3						0.003				0.005			-0.004			-0.618
						(0.008)				(0.000)			(0.017)			(0.000)
LL	1206.2	1206.3	1206.3	1206.2	1206.2	1206.3	1206.0	1219.1	1206.5	1220.3	1206.3	1210.3	1208.0	1225.9	1209.0	1236.3
AIC	-3.575	-3.575	-3.574	-3.575	-3.575	-3.574	-3.576	-3.583	-3.574	-3.582	-3.574	-3.576	-3.574	-3.586	-3.575	-3.591
BIC	-3.572	-3.569	-3.567	-3.572	-3.569	-3.567	-3.574	-3.577	-3.567	-3.573	-3.567	-3.567	-3.563	-3.576	-3.563	-3.578
Ljung-Box Q_{20}	26.34	26.10	26.02	26.35	26.13	26.11	26.48	27.02	25.89	26.56	25.77	26.94	24.83	26.20	24.96	24.78
p-value	0.15	0.16	0.17	0.15	0.16	0.16	0.15	0.13	0.17	0.15	0.17	0.14	0.21	0.16	0.20	0.21

Note. Standard Errors are given in parenthesis.

Table 3.65: Estimates of ARMA(p,q) model for twenty second component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.064	-0.061	-0.057	-0.063	-0.063	-0.063	-0.063	-0.002	-0.003	-0.003	-0.017	-0.019	-0.025	-0.032	-0.031	-0.010
	(0.004)	(0.004)	(0.004)	(0.003)	(0.004)	(0.004)	(0.002)	(0.001)	(0.002)	(0.002)	(0.006)	(0.006)	(0.010)	(0.042)	(0.020)	(0.014)
α_1	-0.003	-0.003	-0.007					0.965	0.955	0.954	0.669	0.868	0.377	0.408	0.283	0.806
	(0.020)	(0.020)	(0.020)					(0.019)	(0.033)	(0.036)	(0.097)	(0.162)	(0.143)	(0.701)	(0.303)	(0.192)
α_2		0.034	0.033								0.060	-0.163	0.230	0.035	0.168	-0.698
		(0.020)	(0.020)								(0.019)	(0.149)	(0.164)	(0.022)	(0.162)	(0.154)
α_3			0.072											0.055	0.062	0.735
			(0.019)											(0.045)	(0.024)	(0.177)
β_1				-0.002	-0.007	-0.006		-0.939	-0.966	-0.965	-0.677	-0.881	-0.386	-0.417	-0.292	-0.811
				(0.003)	(0.026)	(0.055)		(0.021)	(0.038)	(0.041)	(0.096)	(0.165)	(0.143)	(0.703)	(0.303)	(0.199)
β_2					0.034	0.033			0.042	0.040		0.223	-0.197		-0.135	0.741
					(0.021)	(0.020)			(0.020)	(0.023)		(0.145)	(0.165)		(0.161)	(0.151)
β_3						0.066				0.002			0.053			-0.703
						(0.018)				(0.004)			(0.019)			(0.188)
LL	620.3	624.8	634.5	619.9	621.8	629.5	619.9	633.3	636.0	636.0	632.5	633.2	635.3	635.3	635.5	639.1
AIC	-3.216	-3.219	-3.224	-3.216	-3.217	-3.221	-3.217	-3.224	-3.225	-3.224	-3.223	-3.223	-3.223	-3.224	-3.223	-3.225
BIC	-3.213	-3.213	-3.217	-3.213	-3.211	-3.213	-3.215	-3.218	-3.217	-3.215	-3.215	-3.213	-3.212	-3.215	-3.212	-3.212
Ljung-Box Q_{20}	68.43	62.07	39.48	68.02	61.50	40.60	67.64	29.00	24.59	24.57	35.71	37.28	31.09	34.01	32.18	16.32
p-value	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.09	0.22	0.22	0.02	0.01	0.05	0.03	0.04	0.70

Note. Standard Errors are given in parenthesis.

Table 3.66: Estimates of ARMA(p,q) model for twenty third component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.051	-0.050	-0.048	-0.052	-0.052	-0.052	-0.052	-0.002	-0.002	-0.002	-0.001	-0.001	-0.001	-0.002	-0.002	-0.004
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
α_1	0.019	0.019	0.017					0.967	0.962	0.954	0.956	0.665	0.579	0.945	0.336	-0.839
	(0.020)	(0.020)	(0.020)					(0.018)	(0.025)	(0.040)	(0.026)	(0.292)	(0.375)	(0.036)	(0.138)	(0.036)
α_2		0.027	0.026								0.021	0.311	0.393	0.012	0.607	0.832
		(0.019)	(0.019)								(0.022)	(0.291)	(0.366)	(0.030)	(0.130)	(0.019)
α_3			0.046											0.013	0.023	0.939
			(0.017)											(0.024)	(0.023)	(0.045)
β_1				0.018	0.016	0.013		-0.938	-0.954	-0.945	-0.951	-0.655	-0.573	-0.940	-0.330	0.859
				(0.016)	(0.018)	(0.015)		(0.023)	(0.031)	(0.054)	(0.018)	(0.304)	(0.373)	(0.033)	(0.133)	(0.038)
β_2					0.023	0.021			0.024	0.009		-0.291	-0.384		-0.596	-0.794
					(0.017)	(0.016)			(0.021)	(0.032)		(0.298)	(0.367)		(0.134)	(0.028)
β_3						0.044				0.017			0.018			-0.913
						(0.018)				(0.021)			(0.024)			(0.047)
LL	1775.1	1776.5	1780.0	1774.7	1775.7	1778.8	1774.2	1791.0	1791.8	1792.3	1796.9	1797.5	1797.9	1795.0	1797.2	1801.5
AIC	-3.924	-3.924	-3.925	-3.923	-3.923	-3.925	-3.924	-3.933	-3.933	-3.932	-3.936	-3.936	-3.935	-3.934	-3.935	-3.937
BIC	-3.920	-3.918	-3.918	-3.920	-3.918	-3.917	-3.922	-3.927	-3.925	-3.923	-3.928	-3.926	-3.924	-3.925	-3.924	-3.924
Ljung-Box Q_{20}	75.41	68.08	54.41	75.85	70.12	57.81	80.72	19.50	17.46	16.49	16.94	17.79	16.31	16.02	16.12	16.05
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.49	0.62	0.69	0.66	0.60	0.70	0.72	0.71	0.71

Note. Standard Errors are given in parenthesis.

Table 3.67: Estimates of ARMA(p,q) model for twenty fourth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.067	-0.066	-0.064	-0.067	-0.067	-0.067	-0.067	-0.002	-0.002	-0.002	-0.001	0.000	0.000	-0.075	0.000	-0.005
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)	(0.000)	(0.032)	(0.000)	(0.003)
α_1	0.001	0.001	0.000					0.971	0.969	0.969	0.959	1.761	1.757	-0.178	1.916	-0.262
	(0.018)	(0.018)	(0.018)					(0.019)	(0.022)	(0.021)	(0.024)	(0.062)	(0.070)	(0.481)	(0.042)	(0.025)
α_2		0.024	0.024								0.019	-0.764	-0.760	0.024	-0.905	0.249
		(0.018)	(0.018)								(0.018)	(0.062)	(0.069)	(0.018)	(0.052)	(0.070)
α_3			0.031											0.035	-0.012	0.942
			(0.018)											(0.018)	(0.016)	(0.073)
β_1				0.001	0.000	0.000		-0.953	-0.970	-0.971	-0.962	-1.758	-1.762	0.178	-1.922	0.272
				(0.001)	(0.000)	(0.000)		(0.020)	(0.029)	(0.028)	(0.014)	(0.065)	(0.072)	(0.481)	(0.035)	(0.028)
β_2					0.024	0.023			0.020	0.024		0.764	0.780		0.924	-0.234
					(0.018)	(0.018)			(0.018)	(0.022)		(0.064)	(0.071)		(0.035)	(0.081)
β_3						0.029				-0.004			-0.012			-0.914
						(0.017)				(0.014)			(0.021)			(0.081)
LL	396.6	397.6	399.3	396.5	397.4	398.9	396.5	400.7	401.4	401.4	403.5	407.8	408.0	399.3	407.2	405.0
AIC	-3.080	-3.080	-3.080	-3.079	-3.079	-3.080	-3.080	-3.081	-3.081	-3.081	-3.083	-3.085	-3.084	-3.079	-3.084	-3.082
BIC	-3.076	-3.074	-3.072	-3.076	-3.074	-3.072	-3.078	-3.076	-3.074	-3.071	-3.075	-3.075	-3.073	-3.070	-3.072	-3.069
Ljung-Box Q_{20}	31.14	28.36	23.63	30.82	28.05	23.85	30.89	16.55	15.76	15.68	15.52	15.85	15.44	23.94	14.72	12.34
p-value	0.05	0.10	0.26	0.06	0.11	0.25	0.06	0.68	0.73	0.74	0.75	0.73	0.75	0.25	0.79	0.90

Note. Standard Errors are given in parenthesis.

Table 3.68: Estimates of ARMA(p,q) model for twenty fifth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.040	-0.040	-0.038	-0.040	-0.040	-0.040	-0.040	0.000	-0.070	-0.057	-0.073	-0.120	-0.023	-0.054	-0.022	0.000
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.000)	(0.008)	(0.011)	(0.008)	(0.015)	(0.010)	(0.012)	(0.010)	(0.000)
α_1	-0.001	-0.001	-0.001					0.994	-0.732	-0.415	-0.791	-1.310	0.007	-0.410	-0.011	-0.292
	(0.019)	(0.019)	(0.019)					(0.004)	(0.100)	(0.222)	(0.101)	(0.145)	(0.041)	(0.267)	(0.090)	(0.198)
α_2		-0.002	-0.002								-0.029	-0.677	0.412	-0.002	0.405	0.621
		(0.019)	(0.019)								(0.020)	(0.138)	(0.232)	(0.007)	(0.212)	(0.097)
α_3			0.061											0.060	0.063	0.660
			(0.019)											(0.018)	(0.020)	(0.168)
β_1				-0.001	-0.001	0.001		-0.988	0.734	0.416	0.791	1.303	-0.006	0.410	0.010	0.285
				(0.030)	(0.002)	(0.003)		(0.007)	(0.099)	(0.221)	(0.099)	(0.151)	(0.016)	(0.267)	(0.081)	(0.206)
β_2					-0.002	-0.006			-0.029	-0.004		0.644	-0.419		-0.413	-0.639
					(0.004)	(0.017)			(0.017)	(0.020		(0.140)	(0.233)		(0.215)	(0.097)
β_3						0.060				0.057			0.060			-0.619
						(0.019)				(0.019)			(0.023)			(0.177)
LL	312.2	312.2	318.3	312.1	312.1	318.0	312.1	317.8	315.2	319.1	315.6	318.0	319.7	319.0	320.0	325.3
AIC	-3.028	-3.027	-3.030	-3.028	-3.027	-3.030	-3.028	-3.031	-3.028	-3.030	-3.029	-3.030	-3.030	-3.030	-3.030	-3.033
BIC	-3.024	-3.022	-3.023	-3.024	-3.022	-3.023	-3.026	-3.025	-3.021	-3.021	-3.021	-3.020	-3.019	-3.021	-3.019	-3.020
Ljung-Box Q_{20}	27.97	27.99	15.00	28.04	28.07	15.03	28.08	24.32	21.06	12.98	20.75	16.38	11.59	12.77	11.17	11.15
p-value	0.11	0.11	0.78	0.11	0.11	0.77	0.11	0.23	0.39	0.88	0.41	0.69	0.93	0.89	0.94	0.94

Note. Standard Errors are given in parenthesis.

Table 3.69: Estimates of ARMA(p,q) model for twenty sixth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.075	-0.074	-0.071	-0.077	-0.077	-0.077	-0.077	-0.007	-0.008	-0.013	0.000	0.000	0.000	0.000	0.000	0.000
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.002)	(0.006)	(0.010)	(0.032)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)
α_1	0.020	0.020	0.019					0.914	0.897	0.836	1.001	0.932	0.934	1.004	0.310	0.294
	(0.018)	(0.018)	(0.018)					(0.082)	(0.131)	(0.414)	(0.014)	(0.113)	(0.120)	(0.010)	(2.864)	(0.636)
α_2		0.018	0.018								-0.005	0.065	0.063	-0.003	0.686	0.615
		(0.018)	(0.018)								(0.014)	(0.114)	(0.121)	(0.010)	(2.857)	(0.787)
α_3			0.033											-0.004	-0.001	0.086
			((0.018)											(0.008)	(0.003)	(0.182)
β_1				0.019	0.019	0.018		-0.886	-0.880	-0.819	-0.990	-0.920	-0.922	-0.991	-0.298	-0.282
				(0.019)	(0.018)	(0.019)		(0.087)	(0.131)	(0.420)	(0.008)	(0.113)	(0.122)	(0.007)	(2.846)	(0.642)
β_2					0.017	0.016			0.014	0.001		-0.071	-0.065		-0.687	-0.616
					(0.01)7	(0.019)			(0.021)	(0.003)		(0.115)	(0.132)		(2.818)	(0.783)
β_3						0.031				0.020			-0.004			-0.086
						(0.018)				(0.032)			(0.019)			(0.172)
LL	567.9	568.5	570.3	567.4	567.8	569.4	566.7	571.6	571.9	572.4	580.2	580.4	580.5	580.4	580.4	580.5
AIC	-3.184	-3.184	-3.185	-3.184	-3.184	-3.184	-3.184	-3.186	-3.186	-3.185	-3.191	-3.190	-3.190	-3.190	-3.190	-3.189
BIC	-3.181	-3.179	-3.177	-3.180	-3.178	-3.177	-3.182	-3.181	-3.178	-3.176	-3.183	-3.181	-3.179	-3.181	-3.178	-3.176
Ljung-Box Q_{20}	25.68	23.48	18.65	25.63	23.79	19.52	28.45	7.51	7.58	8.26	8.43	8.68	8.67	8.53	8.45	8.50
p-value	0.18	0.27	0.54	0.18	0.25	0.49	0.10	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99

Note. Standard Errors are given in parenthesis.

Table 3.70: Estimates of ARMA(p,q) model for twenty seventh component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.063	-0.060	-0.058	-0.064	-0.064	-0.064	-0.064	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003
	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)	(0.002)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
α_1	0.016	0.015	0.014					0.986	0.985	0.985	0.952	0.412	0.317	0.925	0.162	-0.411
	(0.019)	(0.019)	(0.019)					(0.006)	(0.007)	(0.007)	(0.022)	(0.288)	(0.684)	(0.025)	(0.208)	(0.266)
α_2		0.049	0.048								0.031	0.568	0.661	0.039	0.790	0.638
		(0.018)	(0.018)								(0.019)	(0.287)	(0.678)	(0.043)	(0.210)	(0.157)
α_3			0.024											0.002	0.018	0.729
			(0.018)											(0.052)	(0.020)	(0.161)
β_1				0.014	0.013	0.013		-0.962	-0.987	-0.987	-0.957	-0.413	-0.321	-0.929	-0.166	0.412
				(0.013)	(0.018)	(0.018)		(0.009)	(0.022)	(0.019)	(0.013)	(0.297)	(0.678)	(0.017)	(0.209)	(0.273)
β_2					0.046	0.045			0.028	0.033		-0.532	-0.629		-0.755	-0.599
					(0.017)	(0.017)			(0.020)	(0.025)		(0.291)	(0.691)		(0.209)	(0.166)
β_3						0.019				-0.006			0.010			-0.695
						(0.016)				(0.016)			(0.045)			(0.153)
LL	1232.2	1236.4	1238.4	1232.1	1235.7	1236.4	1231.8	1261.5	1262.8	1262.8	1264.8	1266.3	1266.4	1249.9	1261.6	1265.3
AIC	-3.591	-3.593	-3.594	-3.591	-3.593	-3.593	-3.592	-3.609	-3.609	-3.608	-3.610	-3.610	-3.610	-3.600	-3.607	-3.608
BIC	-3.587	-3.588	-3.586	-3.587	-3.587	-3.585	-3.590	-3.603	-3.601	-3.599	-3.603	-3.601	-3.599	-3.591	-3.596	-3.595
Ljung-Box Q_{20}	95.12	75.42	69.27	95.57	77.92	73.48	100.54	22.27	20.26	20.18	20.22	19.66	19.56	20.99	19.47	18.61
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.44	0.45	0.44	0.48	0.49	0.40	0.49	0.55

Note. Standard Errors are given in parenthesis.

Table 3.71: Estimates of ARMA(p,q) model for twenty eight component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.040	-0.039	-0.038	-0.041	-0.041	-0.041	-0.041	-0.062	-0.063	-0.048	-0.018	-0.017	-0.025	-0.042	-0.060	-0.032
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.002)	(0.012)	(0.011)	(0.012)	(0.006)	(0.006)	(0.016)	(0.007)	(0.046)	(0.011)
α_1	0.023	0.023	0.022					-0.523	-0.549	-0.180	0.554	0.538	0.342	-0.081	-0.292	-0.084
	(0.019)	(0.019)	(0.019)					(0.257)	(0.242)	(0.277)	(0.152)	(0.185)	(0.296)	(0.146)	(0.585)	(0.156)
α_2		0.012	0.011								0.016	0.040	0.047	0.014	-0.239	-0.126
		(0.019)	(0.019)								(0.017)	(0.128)	(0.115)	(0.021)	(0.553)	(0.179)
α_3			0.046											0.047	0.050	0.416
			(0.017)											(0.018)	(0.017)	(0.122)
β_1				0.022	0.022	0.022		0.544	0.573	0.203	-0.533	-0.516	-0.320	0.103	0.315	0.106
				(0.019)	(0.018)	(0.019)		(0.256)	(0.243)	(0.278)	(0.151)	(0.183)	(0.297)	(0.147)	(0.589)	(0.160)
β_2					0.010	0.011			0.005	0.015		-0.025	-0.044		0.256	0.145
					(0.016)	(0.018)			(0.009)	(0.021)		(0.118)	(0.121)		(0.563)	(0.186)
β_3						0.044				0.044			0.037			-0.367
						(0.016)				(0.017)			(0.020)			(0.124)
LL	852.3	855.7	860.1	851.9	852.1	855.4	851.1	853.0	853.0	856.0	857.6	857.6	859.5	860.2	860.3	861.9
AIC	-3.359	-3.360	-3.362	-3.358	-3.358	-3.359	-3.358	-3.358	-3.358	-3.359	-3.361	-3.360	-3.361	-3.362	-3.361	-3.361
BIC	-3.355	-3.354	-3.355	-3.355	-3.352	-3.352	-3.357	-3.353	-3.350	-3.350	-3.353	-3.351	-3.349	-3.352	-3.350	-3.348
Ljung-Box Q_{20}	19.87	18.93	11.30	19.99	19.44	12.03	22.64	19.94	19.70	12.43	14.67	14.61	11.44	11.44	12.26	9.52
p-value	0.47	0.53	0.94	0.46	0.49	0.91	0.31	0.46	0.48	0.90	0.80	0.80	0.93	0.93	0.91	0.98

Note. Standard Errors are given in parenthesis.

Table 3.72: Estimates of ARMA(p,q) model for twenty ninth component of $\mathbf{U_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.046	-0.044	-0.043	-0.046	-0.046	-0.046	-0.046	-0.001	-0.001	-0.001	-0.005	-0.004	-0.004	-0.002	-0.002	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.001)	(0.001)	(0.001)	(0.003)	(0.002)	(0.003)	(0.001)	(0.001)	(0.001)
α_1	0.006	0.006	0.005					0.972	0.968	0.971	0.853	0.234	0.166	0.940	0.625	0.607
	(0.020)	(0.020)	(0.020)					(0.013)	(0.018)	(0.015)	(0.060)	(0.229)	(0.196)	(0.029)	(0.243)	(0.260)
α_2		0.040	0.040								0.038	0.684	0.743	0.037	0.341	0.300
		(0.018)	(0.018)								(0.020)	(0.253)	(0.210)	(0.025)	(0.214)	(0.445)
α_3			0.029											-0.011	-0.004	0.054
			(0.018)											(0.019)	(0.054)	(0.398)
β_1				0.005	0.004	0.002		-0.952	-0.969	-0.972	-0.853	-0.226	-0.166	-0.943	-0.627	-0.608
				(0.010)	(0.149)	(0.003)		(0.016)	(0.026)	(0.024)	(0.058)	(0.240)	(0.197)	(0.022)	(0.239)	(0.262)
β_2					0.037	0.036			0.023	0.035		-0.648	-0.710		-0.309	-0.267
					(0.025)	(0.017)			(0.020)	(0.025)		(0.264)	(0.215)		(0.219)	(0.442)
β_3						0.028				-0.012			0.016			-0.056
						(0.019)				(0.017)			(0.020)			0.387)
LL	1038.6	1042.2	1043.8	1038.5	1041.0	1042.3	1038.5	1049.9	1050.8	1051.0	1047.9	1050.6	1050.9	1051.7	1052.9	1052.9
AIC	-3.473	-3.474	-3.475	-3.473	-3.473	-3.474	-3.473	-3.479	-3.479	-3.478	-3.477	-3.478	-3.478	-3.479	-3.479	-3.478
BIC	-3.469	-3.469	-3.467	-3.469	-3.468	-3.466	-3.471	-3.473	-3.471	-3.469	-3.470	-3.469	-3.467	-3.470	-3.468	-3.465
Ljung-Box Q_{20}	46.02	38.00	32.84	46.14	38.89	34.29	46.87	17.86	16.19	15.95	17.66	16.52	15.89	15.99	16.00	16.01
p-value	0.00	0.01	0.04	0.00	0.01	0.02	0.00	0.60	0.70	0.72	0.61	0.68	0.72	0.72	0.72	0.72

Note. Standard Errors are given in parenthesis.

3.4.5 Estimates of ARMA(p,q) model for all Components of $\mathbf{Z_{1,t}}$ from First Round of SVD

Table 3.73: Estimates of ARMA(p,q) model for first component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.048	-0.047	-0.046	-0.048	-0.048	-0.048	-0.048	-0.062	-0.002	-0.087	-0.003	-0.003	-0.004	-0.075	-0.115	-0.008
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.031)	(0.001)	(0.012)	(0.001)	(0.001)	(0.002)	(0.010)	(0.026)	(0.003)
α_1	-0.009	-0.009	-0.010					-0.291	0.963	-0.830	0.907	0.622	0.068	-0.633	-0.982	-0.583
	(0.017)	(0.017)	(0.017)					(0.646)	(0.013)	(0.178)	(0.029)	(0.141)	(0.131)	(0.153)	(0.235)	(0.151)
α_2		0.020	0.020								0.030	0.310	0.846	0.015	-0.467	0.542
		(0.018)	(0.018)								(0.017)	(0.137)	(0.131)	(0.023)	(0.259)	(0.048)
α_3			0.014											0.032	0.021	0.872
			(0.019(											(0.019(	(0.019(	(0.121)
β_1				-0.009	-0.010	-0.009		0.279	-0.976	0.821	-0.920	-0.632	-0.081	0.624	0.974	0.586
				(0.015)	(0.017)	(0.016)		(0.643)	(0.022)	(0.179)	(0.024)	(0.150)	(0.132)	(0.152)	(0.236)	(0.169)
β_2					0.021	0.021			0.027	0.013		-0.280	-0.839		0.480	-0.522
					(0.018)	(0.018)			(0.017)	(0.020)		(0.145)	(0.133)		(0.258)	(0.050)
β_3						0.013				0.032			0.034			-0.846
						(0.018)				(0.016))			(0.017)			(0.135)
LL	-18.1	-16.5	-15.6	-18.2	-17.5	-17.2	-18.3	-18.1	-11.3	-16.3	-12.2	-11.6	-11.1	-14.3	-13.5	-8.4
AIC	-2.826	-2.826	-2.826	-2.826	-2.825	-2.825	-2.826	-2.825	-2.829	-2.825	-2.828	-2.828	-2.827	-2.826	-2.826	-2.828
BIC	-2.822	-2.820	-2.818	-2.822	-2.820	-2.817	-2.824	-2.819	-2.821	-2.816	-2.820	-2.818	-2.816	-2.817	-2.815	-2.815
Ljung-Box Q_{20}	26.69	23.98	22.64	26.81	24.36	23.35	26.56	26.47	15.25	21.73	16.41	16.02	14.64	21.55	22.37	14.42
p-value	0.14	0.24	0.31	0.14	0.23	0.27	0.15	0.15	0.76	0.36	0.69	0.72	0.80	0.37	0.32	0.81

Note. Standard Errors are given in parenthesis.

Table 3.74: Estimates of ARMA(p,q) model for second component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.042	-0.042	-0.042	-0.045	-0.045	-0.045	-0.045	-0.001	-0.001	-0.001	0.000	0.000	-0.001	-0.030	-0.039	-0.003
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.014)	(0.023)	(0.001)
α_1	0.059	0.059	0.059					0.987	0.988	0.988	1.036	0.663	0.304	0.348	0.294	0.155
	(0.020)	(0.020)	(0.020)					(0.006)	(0.005)	(0.005)	(0.021)	(0.360)	(0.168)	(0.303)	(0.973)	(0.072)
α_2		0.015	0.015								0.020	0.327	0.683	-0.003	-0.165	0.007
		(0.018)	(0.018)								(0.003)	(0.356)	(0.166)	(0.004)	(1.315)	(0.007)
α_3			-0.002											0.001	0.006	0.774
			(0.019)											(0.003)	(0.119)	(0.066)
β_1				0.058	0.059	0.059		-0.976	-0.938	-0.938	-0.986	-0.620	-0.253	-0.290	-0.236	-0.114
				(0.019)	(0.019)	(0.020)		(0.008)	(0.021)	(0.021)	(0.006)	(0.340)	(0.170)	(0.310)	(0.963)	(0.066)
β_2					0.018	0.018			-0.040	-0.039		-0.359	-0.685		0.163	0.009
					(0.017)	(0.019)			(0.020)	(0.020)		(0.332)	(0.157)		(1.223)	(0.011)
β_3						-0.002				-0.001			-0.034			-0.794
						(0.008)				(0.001)			(0.022)			(0.058)
LL	595.2	595.6	599.1	594.8	595.3	595.3	589.2	606.6	609.2	609.2	606.1	607.2	608.5	599.5	599.6	614.4
AIC	-3.201	-3.201	-3.202	-3.201	-3.201	-3.200	-3.198	-3.208	-3.209	-3.208	-3.207	-3.207	-3.207	-3.202	-3.201	-3.210
BIC	-3.197	-3.195	-3.195	-3.197	-3.195	-3.193	-3.196	-3.202	-3.201	-3.199	-3.199	-3.197	-3.196	-3.193	-3.190	-3.197
Ljung-Box Q_{20}	26.16	24.84	24.35	26.86	25.07	25.19	40.06	19.76	15.22	15.22	15.12	15.97	15.14	23.47	24.77	10.08
p-value	0.16	0.21	0.23	0.14	0.20	0.19	0.00	0.47	0.76	0.76	0.77	0.72	0.77	0.27	0.21	0.97

Note. Standard Errors are given in parenthesis.

Table 3.75: Estimates of ARMA(p,q) model for third component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.061	-0.059	-0.058	-0.062	-0.062	-0.062	-0.062	0.000	0.000	0.000	-0.003	-0.002	-0.002	0.000	0.000	0.000
	(0.003)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.000)	(0.000)	(0.000)	(0.002)	(0.001)	(0.002)	(0.000)	(0.000)	(0.000)
α_1	0.021	0.020	0.020					0.998	0.998	0.998	0.926	0.108	0.102	0.994	1.381	1.386
	(0.020)	(0.020)	(0.020)					(0.002)	(0.002)	(0.001)	(0.042)	(0.083)	(0.085)	(0.016)	(0.070)	(0.075)
α_2		0.023	0.023								0.021	0.864	0.868	-0.004	-0.382	-0.449
		(0.020)	(0.020)								(0.019)	(0.087)	(0.088)	(0.009)	(0.070)	(0.087)
α_3			0.025											0.005	0.000	0.062
			(0.017)											(0.012)	(0.000)	(0.055)
β_1				0.020	0.019	0.019		-0.990	-0.993	-0.993	-0.914	-0.091	-0.091	-0.980	-1.377	-1.382
				(0.020)	(0.019)	(0.020)		(0.004)	(0.051)	(0.008)	(0.041)	(0.086)	(0.088)	(0.014)	(0.073)	(0.078)
β_2					0.022	0.023			0.003	0.003		-0.842	-0.846		0.383	0.451
					(0.019)	(0.018)			(0.055)	(0.008)		(0.091)	(0.092)		(0.074)	(0.089)
β_3						0.024				0.000			0.008			-0.062
						(0.017)				(0.000)			(0.033)			(0.058)
LL	906.2	908.2	909.4	906.1	907.0	907.9	905.5	932.1	932.1	932.1	914.4	921.4	921.4	926.7	933.7	933.7
AIC	-3.392	-3.392	-3.392	-3.392	-3.391	-3.391	-3.392	-3.407	-3.406	-3.406	-3.395	-3.399	-3.398	-3.402	-3.406	-3.405
BIC	-3.388	-3.387	-3.385	-3.388	-3.386	-3.384	-3.390	-3.401	-3.399	-3.396	-3.388	-3.390	-3.387	-3.393	-3.395	-3.392
Ljung-Box Q_{20}	66.50	61.91	56.63	66.76	62.63	57.58	71.46	24.69	24.60	24.59	26.48	24.19	23.99	23.99	24.25	24.29
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.22	0.22	0.15	0.23	0.24	0.24	0.23	0.23

Note. Standard Errors are given in parenthesis.

Table 3.76: Estimates of ARMA(p,q) model for fourth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.030	-0.029	-0.028	-0.032	-0.032	-0.032	-0.032	-0.001	0.000	0.000	0.000	0.000	0.000	-0.001	-0.001	-0.001
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)
α_1	0.049	0.048	0.047					0.984	0.988	0.987	1.022	0.064	0.050	1.000	0.071	-0.980
	(0.02)0	(0.020)	(0.020)					(0.021)	(0.013)	(0.019)	(0.021)	(0.043)	(0.032)	(0.034)	(0.032)	(0.025)
α_2		0.018	0.016								-0.027	0.921	0.937	-0.028	0.932	0.973
		(0.019)	(0.019)								(0.019)	(0.045)	(0.033)	(0.025)	(0.026)	(0.018)
α_3			0.039											0.010	-0.018	0.975
			(0.019)											(0.016)	(0.020)	(0.029)
β_1				0.047	0.047	0.045		-0.966	-0.949	-0.948	-0.985	-0.041	-0.011	-0.961	-0.031	0.997
				(0.019)	(0.020)	(0.021)		(0.034)	(0.027)	(0.029)	(0.009)	(0.039)	(0.028)	(0.033)	(0.025)	(0.028)
β_2					0.015	0.016			-0.023	-0.030		-0.920	-0.934		-0.929	-0.947
					(0.013)	(0.019)			(0.022)	(0.034)		(0.041)	(0.031)		(0.024)	(0.032)
β_3						0.036				0.007			-0.020			-0.966
						(0.019)				(0.047)			(0.009)			(0.033)
LL	2290.1	2290.7	2293.3	2289.8	2290.2	2292.3	2286.0	2304.5	2305.3	2305.4	2306.8	2308.7	2309.3	2304.3	2309.4	2312.7
AIC	-4.239	-4.239	-4.240	-4.239	-4.239	-4.239	-4.237	-4.247	-4.247	-4.247	-4.248	-4.249	-4.248	-4.246	-4.248	-4.250
BIC	-4.235	-4.233	-4.232	-4.235	-4.233	-4.232	-4.235	-4.242	-4.240	-4.237	-4.241	-4.239	-4.237	-4.237	-4.237	-4.237
Ljung-Box Q_{20}	57.28	54.09	46.40	58.36	55.65	49.15	75.07	23.74	21.89	21.64	22.83	22.27	21.26	21.95	21.06	24.55
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.35	0.36	0.30	0.33	0.38	0.34	0.39	0.22

Note. Standard Errors are given in parenthesis.

Table 3.77: Estimates of ARMA(p,q) model for fifth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.028	-0.027	-0.027	-0.029	-0.029	-0.029	-0.029	-0.001	-0.001	-0.039	-0.022	-0.001	-0.001	-0.001	-0.002	-0.018
	(0.003)	(0.003)	(0.00)3	(0.003)	(0.003)	(0.003)	(0.003)	(0.001)	(0.001)	(0.008)	(0.009)	(0.003)	(0.000)	(0.001)	(0.001)	(0.006)
α_1	0.051	0.050	0.050					0.951	0.954	-0.350	0.213	0.826	0.675	0.977	0.482	1.236
	(0.025)	(0.025)	(0.025)					(0.023)	(0.019)	(0.223)	(0.315)	(2.467)	(0.276)	(0.033)	(0.123)	(0.286)
α_2		0.023	0.022								0.014	0.139	0.286	-0.024	0.476	-0.978
		(0.018)	(0.018)								(0.015)	(2.373)	(0.268)	(0.025)	(0.108)	(0.327)
α_3			0.005											-0.001	-0.015	0.120
			(0.017)											(0.000)	(0.023)	(0.249)
β_1				0.049	0.050	0.050		-0.929	-0.908	0.400	-0.163	-0.782	-0.629	-0.931	-0.436	-1.187
				(0.024)	(0.026)	(0.025)		(0.025)	(0.034)	(0.225)	(0.318)	(2.442)	(0.274)	(0.023)	(0.117)	(0.281)
β_2					0.025	0.025			-0.026	0.043		-0.163	-0.296		-0.478	0.937
					(0.020)	(0.018)			(0.026)	(0.024)		(2.297)	(0.263)		(0.111)	(0.323)
β_3						0.008				0.020			-0.013			-0.102
						(0.017)				(0.018)			(0.015)			(0.244)
LL	1350.0	1350.8	1351.3	1349.3	1350.3	1350.4	1345.2	1354.4	1355.5	1351.0	1350.8	1357.0	1357.2	1355.2	1356.4	1355.3
AIC	-3.663	-3.663	-3.663	-3.663	-3.663	-3.662	-3.661	-3.665	-3.666	-3.662	-3.663	-3.666	-3.665	-3.665	-3.665	-3.664
BIC	-3.660	-3.658	-3.655	-3.659	-3.657	-3.655	-3.659	-3.660	-3.658	-3.653	-3.655	-3.656	-3.654	-3.655	-3.654	-3.650
Ljung-Box Q_{20}	35.49	33.48	32.82	36.12	33.63	33.19	48.28	28.03	24.53	32.92	33.35	24.57	24.30	24.92	24.52	29.10
p-value	0.02	0.03	0.04	0.01	0.03	0.03	0.00	0.11	0.22	0.03	0.03	0.22	0.23	0.20	0.22	0.09

Note. Standard Errors are given in parenthesis.

Table 3.78: Estimates of ARMA(p,q) model for sixth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.103	-0.101	-0.097	-0.110	-0.110	-0.110	-0.110	0.000	0.000	0.000	0.000	-0.001	-0.001	0.000	0.000	-0.001
	(0.003)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
α_1	0.059	0.058	0.057					0.996	0.996	0.996	1.013	0.274	0.332	1.011	0.274	-0.640
	(0.020)	(0.020)	(0.020)					(0.002)	(0.002)	(0.002)	(0.024)	(0.101)	(0.180)	(0.020)	(0.089)	(0.129)
α_2		0.022	0.019								-0.016	0.720	0.662	-0.037	0.709	0.847
		(0.020)	(0.020)								(0.023)	(0.100)	(0.179)	(0.029)	(0.095)	(0.059)
α_3			0.045											0.023	0.012	0.784
			(0.020)											(0.020)	(0.018)	(0.091)
β_1				0.057	0.056	0.057		-0.983	-0.968	-0.968	-0.986	-0.238	-0.305	-0.985	-0.248	0.666
				(0.019)	(0.020)	(0.020)		(0.004)	(0.018)	(0.019)	(0.003)	(0.097)	(0.187)	(0.005)	(0.086)	(0.136)
β_2					0.020	0.018			-0.015	-0.037		-0.737	-0.685		-0.729	-0.841
					(0.021)	(0.021)			(0.018)	(0.030)		(0.095)	(0.163)		(0.085)	(0.053)
β_3						0.040				0.022			0.014			-0.779
						(0.019)				(0.022)			(0.030)			(0.097)
LL	1765.5	1766.4	1769.6	1764.9	1765.5	1768.4	1759.4	1814.5	1814.8	1815.6	1816.5	1817.6	1817.9	1814.0	1817.7	1820.2
AIC	-3.918	-3.918	-3.919	-3.917	-3.917	-3.918	-3.915	-3.947	-3.947	-3.947	-3.948	-3.948	-3.947	-3.946	-3.947	-3.948
BIC	-3.914	-3.912	-3.912	-3.914	-3.912	-3.911	-3.913	-3.942	-3.939	-3.937	-3.940	-3.939	-3.936	-3.936	-3.936	-3.935
Ljung-Box Q_{20}	126.49	116.61	98.55	129.62	121.15	105.15	166.23	31.05	30.22	28.67	30.26	26.71	26.39	29.15	26.65	23.25
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.07	0.09	0.07	0.14	0.15	0.08	0.15	0.28

Note. Standard Errors are given in parenthesis.

Table 3.79: Estimates of ARMA(p,q) model for seventh component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.072	-0.070	-0.067	-0.073	-0.073	-0.073	-0.073	-0.001	-0.002	-0.002	-0.001	-0.001	-0.003	-0.002	-0.003	-0.002
	(0.004)	(0.004)	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
α_1	0.006	0.006	0.004					0.980	0.977	0.975	0.930	1.039	0.151	0.924	0.083	0.373
	(0.022)	(0.022)	(0.022)					(0.007)	(0.009)	(0.010)	(0.025)	(0.156)	(0.067)	(0.026)	(0.067)	(0.196)
α_2		0.030	0.030								0.050	-0.058	0.814	0.025	0.824	0.809
		(0.020)	(0.020)								(0.022)	(0.156)	(0.066)	(0.030)	(0.067)	(0.073)
α_3			0.051											0.029	0.057	-0.211
			(0.020)											(0.021)	(0.021)	(0.163)
β_1				0.006	0.003	0.000		-0.950	-0.996	-0.992	-0.949	-1.061	-0.170	-0.944	-0.100	-0.392
				(0.019)	(0.012)	(0.000)		(0.010)	(0.024)	(0.024)	(0.013)	(0.164)	(0.071)	(0.015)	(0.065)	(0.191)
β_2					0.027	0.022			0.052	0.025		0.109	-0.796		-0.804	-0.796
					(0.019)	(0.018)			(0.023)	(0.030)		(0.163)	(0.063)		(0.064)	(0.067)
β_3						0.045				0.028			0.058			0.262
						(0.019)				(0.020)			(0.021)			(0.154)
LL	148.1	149.6	154.0	147.8	149.1	152.6	147.7	181.3	185.4	186.6	189.7	189.9	192.1	190.9	191.8	192.6
AIC	-2.927	-2.928	-2.930	-2.927	-2.927	-2.929	-2.928	-2.947	-2.949	-2.949	-2.952	-2.951	-2.952	-2.952	-2.952	-2.952
BIC	-2.924	-2.922	-2.922	-2.923	-2.922	-2.921	-2.926	-2.941	-2.941	-2.940	-2.944	-2.942	-2.941	-2.942	-2.940	-2.938
Ljung-Box Q_{20}	149.16	133.64	109.20	148.90	136.47	118.37	151.94	32.06	25.78	22.78	26.23	25.32	22.79	22.79	23.30	21.84
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.17	0.30	0.16	0.19	0.30	0.30	0.27	0.35

Note. Standard Errors are given in parenthesis.

Table 3.80: Estimates of ARMA(p,q) model for eight component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.032	-0.032	-0.032	-0.033	-0.033	-0.033	-0.033	-0.023	-0.008	-0.008	-0.043	-0.056	-0.056	-0.028	-0.054	-0.045
	(0.002)	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.019)	(0.007)	(0.009)	(0.010)	(0.005)	(0.005)	(0.009)	(0.005)	(0.047)
α_1	0.037	0.036	0.037					0.313	0.752	0.750	-0.309	0.196	0.199	0.144	0.246	0.405
	(0.020)	(0.020)	(0.020)					(0.572)	(0.201)	(0.258)	(0.318)	(0.034)	(0.037)	(0.270)	(0.050)	(0.843)
α_2		0.004	0.005								0.020	-0.898	-0.901	0.001	-0.903	-0.936
		(0.021)	(0.021)								(0.035)	(0.056)	(0.039)	(0.001)	(0.047)	(0.175)
α_3			-0.005											-0.004	0.035	0.180
			(0.018)											(0.271)	(0.021)	(0.761)
β_1				0.037	0.037	0.037		-0.278	-0.716	-0.713	0.346	-0.190	-0.163	-0.107	-0.209	-0.369
				(0.020)	(0.020)	(0.021)		(0.585)	(0.201)	(0.235)	(0.321)	(0.035)	(0.042)	(0.011)	(0.047)	(0.849)
β_2					0.006	0.005			-0.025	-0.022		0.898	0.895		0.896	0.928
					(0.011)	(0.007)			(0.024)	(0.028)		(0.055)	(0.038)		(0.044)	(0.171)
β_3						-0.005				-0.005			0.034			-0.145
						(0.012)				(0.123)			(0.021)			(0.767)
LL	1960.6	1962.0	1962.6	1959.8	1959.9	1959.9	1957.6	1961.0	1961.3	1961.4	1962.3	1966.3	1968.2	1962.6	1968.0	1968.0
AIC	-4.037	-4.037	-4.037	-4.037	-4.036	-4.036	-4.036	-4.037	-4.036	-4.036	-4.037	-4.039	-4.039	-4.037	-4.039	-4.039
BIC	-4.034	-4.032	-4.030	-4.033	-4.031	-4.028	-4.034	-4.031	-4.029	-4.027	-4.030	-4.030	-4.028	-4.027	-4.028	-4.026
Ljung-Box Q_{20}	20.07	20.09	20.00	19.98	19.83	19.91	23.66	19.99	20.18	20.08	20.00	22.63	20.29	19.95	20.17	19.94
p-value	0.45	0.45	0.46	0.46	0.47	0.46	0.26	0.46	0.45	0.45	0.46	0.31	0.44	0.46	0.45	0.46

Note. Standard Errors are given in parenthesis.

Table 3.81: Estimates of ARMA(p,q) model for ninth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.047	-0.046	-0.047	-0.046	-0.046	-0.046	-0.046	-0.082	-0.082	-0.083	-0.082	-0.171	-0.001	-0.079	-0.005	-0.005
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.008)	(0.007)	(0.881)	(0.022)	(0.012)	(0.001)	(0.007)	(0.003)	(0.003)
α_1	-0.038	-0.037	-0.036					-0.785	-0.801	-0.823	-0.800	-1.882	0.159	-0.742	0.068	-0.308
	(0.028)	(0.028)	(0.028)					(0.123)	(0.097)	(19.336)	(0.393)	(0.080)	(0.088)	(0.108)	(0.078)	(0.405)
α_2		0.027	0.026								-0.005	-0.893	0.813	0.000	0.790	0.836
		(0.028)	(0.028)								(0.079)	(0.074)	(0.089)	(0.000)	(0.069)	(0.058)
α_3			-0.009											0.001	0.031	0.374
			(0.025)											(0.001)	(0.033)	(0.393)
β_1				-0.036	-0.036	-0.036		0.753	0.765	0.788	0.764	1.855	-0.198	0.707	-0.106	0.278
				(0.026)	(0.029)	(0.029)		(0.139)	(0.113)	(16.311)	(0.358)	(0.094)	(0.093)	(0.118)	(0.071)	(0.432)
β_2					0.027	0.027			-0.005	-0.003		0.866	-0.778		-0.753	-0.802
					(0.028)	(0.029)			(0.014)	(4.249)		(0.088)	(0.109)		(0.082)	(0.061)
β_3						-0.008				0.007			0.023			-0.339
						(0.019)				(2.022)			(0.034)			(0.415)
LL	1048.9	1050.1	1051.7	1048.6	1049.8	1050.0	1046.4	1051.9	1051.9	1052.0	1052.1	1054.6	1058.4	1053.6	1056.5	1057.1
AIC	-3.479	-3.479	-3.479	-3.479	-3.479	-3.478	-3.478	-3.480	-3.480	-3.479	-3.480	-3.481	-3.482	-3.480	-3.481	-3.481
BIC	-3.475	-3.474	-3.472	-3.475	-3.473	-3.471	-3.476	-3.475	-3.472	-3.470	-3.472	-3.471	-3.471	-3.471	-3.470	-3.468
Ljung-Box Q_{20}	26.52	22.53	22.68	26.84	22.85	22.87	30.23	20.79	21.03	20.80	21.00	20.03	11.22	20.99	12.39	12.51
p-value	0.15	0.31	0.30	0.14	0.30	0.30	0.07	0.41	0.40	0.41	0.40	0.46	0.94	0.40	0.90	0.90

Note. Standard Errors are given in parenthesis.

Table 3.82: Estimates of ARMA(p,q) model for tenth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.041	-0.041	-0.040	-0.042	-0.042	-0.042	-0.042	-0.001	-0.048	-0.001	-0.050	-0.001	0.000	0.000	0.000	-0.001
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.000)	(0.009)	(0.000)	(0.006)	(0.001)	(0.001)	(0.000)	(0.000)	(0.001)
α_1	0.027	0.027	0.027					0.986	-0.148	0.985	-0.177	0.157	0.735	1.006	0.640	-0.438
	(0.020)	(0.020)	(0.020)					(0.010)	(0.210)	(0.010)	(0.127)	(0.507)	(0.437)	(0.023)	(0.437)	(0.196)
α_2		-0.005	-0.006								-0.002	0.828	0.257	-0.035	0.342	0.497
		(0.017)	(0.017)								(0.004)	(0.498)	(0.428)	(0.026)	(0.470)	(0.019)
α_3			0.025											0.023	0.012	0.924
			(0.019)											(0.018)	(0.039)	(0.178)
β_1				0.027	0.027	0.025		-0.973	0.176	-0.965	0.205	-0.147	-0.715	-0.985	-0.619	0.471
				(0.020)	(0.020)	(0.020)		(0.012)	(0.211)	(0.022)	(0.129)	(0.500)	(0.441)	(0.012)	(0.436)	(0.167)
β_2					-0.005	-0.004			-0.003	-0.031		-0.820	-0.279		-0.363	-0.491
					(0.006)	(0.012)			(0.007)	(0.026)		(0.485)	(0.396)		(0.428)	(0.015)
β_3						0.020				0.024			0.014			-0.934
						(0.018)				(0.016)			(0.033)			(0.158)
LL	1350.2	1350.2	1351.3	1350.1	1350.1	1350.9	1348.9	1362.3	1350.3	1363.4	1350.3	1361.7	1362.2	1358.5	1359.4	1369.5
AIC	-3.663	-3.663	-3.663	-3.663	-3.663	-3.663	-3.663	-3.670	-3.662	-3.670	-3.662	-3.669	-3.668	-3.667	-3.667	-3.672
BIC	-3.660	-3.657	-3.655	-3.660	-3.657	-3.655	-3.661	-3.665	-3.655	-3.660	-3.655	-3.659	-3.657	-3.657	-3.655	-3.659
Ljung-Box Q_{20}	32.54	32.73	29.84	32.63	32.81	30.41	35.59	20.64	32.70	19.09	32.65	20.46	19.47	19.88	20.08	10.10
p-value	0.04	0.04	0.07	0.04	0.04	0.06	0.02	0.42	0.04	0.52	0.04	0.43	0.49	0.47	0.45	0.97

Note. Standard Errors are given in parenthesis.

Table 3.83: Estimates of ARMA(p,q) model for eleventh component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.047	-0.048	-0.049	-0.047	-0.047	-0.047	-0.047	-0.056	-0.035	-0.045	-0.021	-0.017	-0.016	-0.032	-0.026	-0.029
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.009)	(0.016)	(0.005)	(0.010)	(0.004)	(0.008)	(0.018)	(0.030)	(0.013)
α_1	-0.001	-0.001	-0.001					-0.181	0.257	0.044	0.583	1.316	1.329	0.360	0.707	0.595
	(0.021)	(0.021)	(0.021)					(0.183)	(0.333)	(0.097)	(0.185)	(0.190)	(0.166)	(0.337)	(1.218)	(0.548)
α_2		-0.017	-0.016								-0.019	-0.669	-0.661	-0.016	-0.256	0.019
		(0.020)	(0.020)								(0.024)	(0.169)	(0.176)	(0.025)	(0.670)	(0.063)
α_3			-0.024											-0.016	-0.009	-0.230
			(0.019)											(0.030)	(0.052)	(0.397)
β_1				-0.001	-0.001	-0.002		0.182	-0.259	-0.046	-0.584	-1.323	-1.331	-0.362	-0.709	-0.599
				(0.001)	(0.004)	(0.004)		(0.200)	(0.334)	(0.102)	(0.183)	(0.194)	(0.164)	(0.340)	(1.220)	(0.554)
β_2					-0.017	-0.016			-0.020	-0.016		0.658	0.640		0.239	-0.037
					(0.022)	(0.023)			(0.021)	(0.023)		(0.170)	(0.195)		(0.654)	(0.073)
β_3						-0.023				-0.023			0.008			0.220
						(0.020)				(0.019)			(0.042)			(0.413)
LL	1152.5	1153.1	1155.7	1152.3	1152.8	1153.6	1152.3	1152.5	1153.1	1153.8	1154.5	1156.7	1156.8	1156.2	1156.2	1156.4
AIC	-3.542	-3.542	-3.543	-3.542	-3.542	-3.542	-3.543	-3.542	-3.542	-3.541	-3.542	-3.543	-3.543	-3.543	-3.542	-3.542
BIC	-3.539	-3.537	-3.536	-3.539	-3.536	-3.534	-3.541	-3.536	-3.534	-3.532	-3.535	-3.534	-3.531	-3.534	-3.531	-3.529
Ljung-Box Q_{20}	22.48	22.00	20.34	22.40	22.01	20.80	22.38	22.39	21.75	20.92	21.53	18.44	18.05	20.27	19.84	19.26
p-value	0.32	0.34	0.44	0.32	0.34	0.41	0.32	0.32	0.35	0.40	0.37	0.56	0.58	0.44	0.47	0.50

Note. Standard Errors are given in parenthesis.

Table 3.84: Estimates of ARMA(p,q) model for twelfth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.037	-0.036	-0.035	-0.036	-0.036	-0.036	-0.036	-0.048	-0.001	-0.064	-0.003	-0.003	-0.003	-0.060	-0.006	-0.006
	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)	(0.003)	(0.002)	(0.005)	(0.000)	(0.005)	(0.002)	(0.002)	(0.002)	(0.005)	(0.026)	(0.003)
α_1	-0.032	-0.030	-0.032					-0.332	0.976	-0.751	0.850	0.664	0.677	-0.747	-0.010	-0.657
	(0.020)	(0.020)	(0.020)					(0.098)	(0.012)	(0.072)	(0.058)	(0.086)	(0.128)	(0.078)	(1.018)	(0.033)
α_2		0.047	0.048								0.063	0.257	0.244	0.027	0.773	0.602
		(0.018)	(0.018)								(0.020)	(0.098)	(0.140)	(0.020)	(0.325)	(0.050)
α_3			0.034											0.077	0.073	0.895
			(0.018)											(0.018)	(0.020)	(0.058)
β_1				-0.029	-0.033	-0.031		0.295	-1.011	0.723	-0.887	-0.699	-0.713	0.717	-0.027	0.643
				(0.018)	(0.02)1	(0.020)		(0.097)	(0.023)	(0.073)	(0.056)	(0.089)	(0.131)	(0.076)	(1.065)	(0.038)
β_2					0.051	0.048			0.053	0.023		-0.192	-0.176		-0.758	-0.564
					(0.019)	(0.019)			(0.018)	(0.024)		(0.099)	(0.143)		(0.294)	(0.058)
β_3						0.027				0.071			-0.003			-0.844
						(0.020)				(0.018)			(0.003)			(0.076)
LL	1846.0	1850.3	1852.7	1845.8	1850.0	1851.1	1844.3	1846.8	1858.3	1854.9	1856.0	1856.6	1856.6	1857.0	1858.1	1867.9
AIC	-3.967	-3.969	-3.970	-3.967	-3.969	-3.969	-3.967	-3.967	-3.973	-3.971	-3.972	-3.972	-3.971	-3.972	-3.972	-3.977
BIC	-3.963	-3.964	-3.963	-3.963	-3.963	-3.962	-3.965	-3.961	-3.966	-3.961	-3.965	-3.962	-3.960	-3.963	-3.961	-3.964
Ljung-Box Q_{20}	57.28	45.86	40.38	57.67	44.17	41.13	60.79	54.51	33.06	31.89	33.13	31.79	31.66	30.47	27.62	23.38
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.04	0.03	0.05	0.05	0.06	0.12	0.27

Note. Standard Errors are given in parenthesis.

Table 3.85: Estimates of ARMA(p,q) model for thirteenth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.036	-0.037	-0.036	-0.038	-0.038	-0.038	-0.038	-0.056	-0.053	-0.001	-0.053	-0.001	-0.001	-0.001	-0.001	-0.002
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.010)	(0.011)	(0.001)	(0.011)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
α_1	0.041	0.042	0.043					-0.464	-0.390	0.975	-0.389	0.472	0.728	0.996	0.668	-0.124
	(0.025)	(0.025)	(0.025)					(0.218)	(0.257)	(0.015)	(0.242)	(0.190)	(0.249)	(0.031)	(0.271)	(0.016)
α_2		-0.018	-0.019								-0.008	0.502	0.247	-0.059	0.270	0.114
		(0.021)	(0.021)								(0.020)	(0.187)	(0.245)	(0.034)	(0.277)	(0.029)
α_3			0.042											0.037	0.031	0.948
			((0.021)											(0.021)	(0.024)	(0.032)
β_1				0.043	0.044	0.041		0.508	0.433	-0.934	0.432	-0.425	-0.689	-0.958	-0.629	0.154
				(0.027)	(0.025)	(0.025)		(0.208)	(0.256)	(0.032)	(0.238)	(0.186)	(0.246)	(0.020)	(0.268)	(0.021)
β_2					-0.018	-0.016			-0.007	-0.060		-0.530	-0.298		-0.319	-0.122
					(0.019)	(0.019)			(0.025)	(0.033)		(0.180)	(0.232)		(0.259)	0.037)
β_3						0.038				0.036			0.030			-0.926
						(0.021)				(0.020)			(0.021)			(0.029)
LL	331.4	331.9	335.0	331.5	332.1	334.5	328.6	332.7	332.7	338.3	332.7	339.2	340.0	339.4	340.0	351.9
AIC	-3.040	-3.039	-3.041	-3.040	-3.039	-3.040	-3.038	-3.040	-3.039	-3.042	-3.039	-3.043	-3.042	-3.043	-3.042	-3.049
BIC	-3.036	-3.034	-3.033	-3.036	-3.034	-3.033	-3.037	-3.034	-3.032	-3.033	-3.032	-3.033	-3.031	-3.033	-3.031	-3.036
Ljung-Box Q_{20}	32.921	31.939	23.580	32.639	31.429	24.620	39.412	29.715	29.726	18.315	29.745	19.240	17.176	18.589	16.976	13.169
p-value	0.034	0.044	0.261	0.037	0.050	0.216	0.006	0.075	0.074	0.567	0.074	0.506	0.642	0.549	0.655	0.870

Note. Standard Errors are given in parenthesis.

Table 3.86: Estimates of ARMA(p,q) model for fourteenth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.042	-0.041	-0.042	-0.043	-0.043	-0.043	-0.043	0.000	-0.064	-0.058	-0.060	-0.127	-0.001	-0.030	-0.002	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.000)	(0.009)	(0.008)	(0.009)	(0.011)	(0.000)	(0.023)	(0.001)	(0.001)
α_1	0.025	0.024	0.025					0.992	-0.490	-0.360	-0.441	-1.088	0.420	0.303	0.304	-0.100
	(0.020)	(0.020)	(0.020)					(0.005)	(0.171)	(0.163)	(0.186)	(0.081)	(0.133)	(0.519)	(0.196)	(0.223)
α_2		0.022	0.022								0.037	-0.875	0.567	0.015	0.668	0.258
		(0.019)	(0.019)								(0.019)	(0.067)	(0.132)	(0.021)	(0.190)	(0.101)
α_3			-0.018											-0.019	-0.016	0.798
			(0.019)											(0.020)	(0.024)	(0.142)
β_1				0.024	0.025	0.025		-0.982	0.515	0.385	0.466	1.105	-0.401	-0.279	-0.284	0.125
				(0.019)	(0.022)	(0.020)		(0.006)	(0.173)	(0.163)	(0.187)	(0.073)	(0.134)	(0.525)	(0.192)	(0.197)
β_2					0.022	0.022			0.039	0.031		0.907	-0.548		-0.647	-0.225
					(0.019)	(0.018)			(0.020)	(0.021)		(0.057)	(0.133)		(0.199)	(0.091)
β_3						-0.018				-0.014			-0.023			-0.820
						(0.018)				(0.017)			(0.021)			(0.124)
LL	1222.2	1223.0	1224.4	1222.1	1222.9	1223.4	1221.1	1232.9	1223.6	1223.8	1223.4	1227.7	1233.5	1224.4	1229.8	1235.8
AIC	-3.585	-3.585	-3.585	-3.585	-3.585	-3.585	-3.585	-3.591	-3.585	-3.584	-3.585	-3.587	-3.590	-3.585	-3.587	-3.590
BIC	-3.581	-3.579	-3.578	-3.581	-3.579	-3.577	-3.583	-3.585	-3.577	-3.575	-3.577	-3.577	-3.578	-3.575	-3.576	-3.577
Ljung-Box Q_{20}	29.52	27.22	26.72	29.70	27.33	27.28	32.30	18.09	26.48	26.62	26.45	21.33	16.85	27.02	17.84	9.40
p-value	0.08	0.13	0.14	0.07	0.13	0.13	0.04	0.58	0.15	0.15	0.15	0.38	0.66	0.13	0.60	0.98

Note. Standard Errors are given in parenthesis.

Table 3.87: Estimates of ARMA(p,q) model for fifteenth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.052	-0.050	-0.048	-0.053	-0.053	-0.053	-0.053	-0.001	-0.001	-0.001	-0.009	-0.004	-0.005	-0.001	-0.001	-0.002
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)	(0.000)	(0.000)	(0.000)	(0.007)	(0.003)	(0.005)	(0.000)	(0.000)	(0.001)
α_1	0.021	0.020	0.019					0.986	0.986	0.986	0.795	0.065	0.058	0.976	1.149	0.683
	(0.019)	(0.019)	(0.019)					(0.006)	(0.007)	(0.007)	(0.129)	(0.074)	(0.076)	(0.023)	(0.139)	(0.029)
α_2		0.035	0.034								0.037	0.862	0.851	0.012	-0.159	-0.629
		(0.019	(0.019)								(0.018)	(0.091)	(0.100)	(0.021)	(0.145)	(0.024)
α_3			0.040											-0.002	-0.004	0.913
			(0.017)											(0.004)	(0.036)	(0.021)
β_1				0.020	0.018	0.018		-0.967	-0.978	-0.978	-0.781	-0.036	-0.044	-0.966	-1.143	-0.663
				(0.017	(0.018)	(0.020)		(0.010)	(0.022)	(0.021)	(0.130)	(0.077)	(0.075)	(0.011)	(0.140)	(0.032)
β_2					0.033	0.031			0.012	0.015		-0.833	-0.821		0.174	0.634
					(0.018)	(0.019)			(0.019)	(0.021)		(0.096)	(0.107)		(0.140)	(0.028)
β_3						0.036				-0.003			0.021			-0.891
						(0.016)				(0.011)			(0.021)			(0.022)
LL	638.1	641.6	644.3	638.0	639.8	642.0	637.4	659.7	660.0	660.0	646.0	652.9	653.4	662.3	663.4	666.9
AIC	-3.227	-3.229	-3.230	-3.227	-3.228	-3.229	-3.228	-3.240	-3.240	-3.239	-3.231	-3.235	-3.234	-3.240	-3.240	-3.242
BIC	-3.224	-3.223	-3.222	-3.224	-3.222	-3.221	-3.226	-3.234	-3.232	-3.230	-3.224	-3.225	-3.223	-3.231	-3.229	-3.229
Ljung-Box Q_{20}	66.89	59.75	48.23	67.31	59.88	50.47	71.85	20.77	19.93	19.98	28.78	21.48	19.86	20.12	19.77	19.16
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.46	0.46	0.09	0.37	0.47	0.45	0.47	0.51

Note. Standard Errors are given in parenthesis.

Table 3.88: Estimates of ARMA(p,q) model for sixteenth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.058	-0.058	-0.058	-0.056	-0.056	-0.056	-0.056	-0.065	-0.006	-0.060	-0.001	0.000	-0.146	-0.001	-0.133	-0.001
	(0.005)	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)	(0.005)	(0.075)	(0.003)	(0.005)	(0.001)	(0.000)	(0.026)	(0.000)	(0.011)	(0.000)
α_1	-0.046	-0.046	-0.046					-0.162	0.885	-0.068	0.936	1.346	-0.669	0.929	-0.348	0.881
	(0.019)	(0.019)	(0.019)					(1.334)	(0.057)	(0.032)	(0.026)	(0.128)	(0.688)	(0.021)	(0.044)	(0.469)
α_2		0.006	0.006								0.055	-0.353	-0.917	0.052	-0.980	0.196
		(0.019)	(0.019)								(0.020)	(0.128)	(0.124)	(0.027)	(0.022)	(0.489)
α_3			-0.002											0.008	-0.053	-0.090
			(0.019)											(0.020)	(0.019)	(0.089)
β_1				-0.046	-0.046	-0.046		0.116	-0.934	0.022	-0.982	-1.393	0.625	-0.981	0.302	-0.934
				(0.019)	(0.019)	(0.019)		(1.342)	(0.060)	(0.021)	(0.015)	(0.123)	(0.679)	(0.008)	(0.040)	(0.470)
β_2					0.007	0.008			0.065	0.005		0.405	0.897		0.968	-0.147
					(0.017)	(0.015)			(0.020)	(0.013)		(0.122)	(0.043)		(0.019)	(0.513)
β_3						-0.002				-0.002			-0.060			0.101
						(0.006)				(0.010)			(0.111)			(0.099)
LL	-215.4	-215.4	-215.3	-215.7	-215.6	-215.6	-219.2	-215.4	-213.7	-215.4	-209.3	-206.3	-209.2	-206.0	-212.2	-205.8
AIC	-2.705	-2.704	-2.704	-2.705	-2.704	-2.703	-2.703	-2.704	-2.705	-2.703	-2.707	-2.708	-2.706	-2.709	-2.704	-2.708
BIC	-2.701	-2.699	-2.696	-2.701	-2.698	-2.696	-2.701	-2.699	-2.697	-2.694	-2.700	-2.699	-2.695	-2.699	-2.693	-2.694
Ljung-Box Q_{20}	26.243	25.922	25.894	26.257	25.791	25.859	31.009	25.939	17.485	25.985	17.048	17.021	23.701	16.694	26.374	16.298
p-value	0.158	0.168	0.169	0.158	0.173	0.171	0.055	0.168	0.621	0.166	0.650	0.652	0.256	0.673	0.154	0.698

Note. Standard Errors are given in parenthesis.

Table 3.89: Estimates of ARMA(p,q) model for seventeenth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.057	-0.055	-0.056	-0.056	-0.056	-0.056	-0.056	-0.073	-0.060	-0.057	-0.060	-0.116	-0.114	-0.062	-0.143	-0.022
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.014)	(0.018)	(0.005)	(0.033)	(0.024)	(0.016)	(0.016)	(0.013)	(0.022)
α_1	-0.027	-0.026	-0.026					-0.306	-0.065	-0.024	-0.100	-0.555	-0.449	-0.140	-0.609	0.227
	(0.019)	(0.019)	(0.019)					(0.224)	(0.307)	(0.022)	(0.575)	(0.153)	(0.180)	(0.271)	(0.044)	(0.341)
α_2		0.034	0.035								0.032	-0.508	-0.588	0.032	-0.929	0.030
		(0.020)	(0.020)								(0.023)	(0.284)	(0.196)	(0.024)	(0.088)	(0.123)
α_3			-0.003											0.001	-0.027	0.350
			(0.020)											(0.007)	(0.022)	(0.247)
β_1				-0.025	-0.026	-0.026		0.273	0.039	-0.002	0.074	0.535	0.424	0.115	0.583	-0.247
				(0.017)	(0.019)	(0.019)		(0.215)	(0.308)	(0.011)	(0.577)	(0.154)	(0.179)	(0.270)	(0.041)	(0.340)
β_2					0.035	0.035			0.034	0.035		0.533	0.606		0.932	0.014
					(0.020)	(0.020)			(0.022)	(0.021)		(0.291)	(0.191)		(0.079)	(0.113)
β_3						-0.005				-0.005			-0.022			-0.361
						(0.018)				(0.004)			(0.029)			(0.248)
LL	-498.8	-496.9	-496.7	-502.1	-500.1	-500.1	-503.2	-498.4	-496.8	-496.8	-496.9	-496.0	-495.6	-496.7	-494.6	-496.0
AIC	-2.531	-2.532	-2.531	-2.529	-2.530	-2.529	-2.529	-2.531	-2.531	-2.531	-2.531	-2.531	-2.531	-2.531	-2.531	-2.530
BIC	-2.527	-2.526	-2.524	-2.525	-2.524	-2.522	-2.527	-2.525	-2.524	-2.521	-2.524	-2.522	-2.520	-2.521	-2.520	-2.517
Ljung-Box Q_{20}	17.96	13.88	14.05	18.29	14.18	14.15	20.99	16.01	13.89	13.92	13.88	14.10	13.60	14.05	11.64	13.40
p-value	0.59	0.84	0.83	0.57	0.82	0.82	0.40	0.72	0.84	0.83	0.84	0.83	0.85	0.83	0.93	0.86

Note. Standard Errors are given in parenthesis.

Table 3.90: Estimates of ARMA(p,q) model for eighteenth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.050	-0.048	-0.047	-0.050	-0.050	-0.050	-0.050	-0.002	-0.002	-0.002	-0.003	-0.004	-0.004	-0.075	-0.001	-0.009
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.009)	(0.000)	(0.003)
α_1	-0.002	-0.002	-0.004					0.967	0.964	0.965	0.903	0.580	0.423	-0.590	1.670	-0.609
	(0.018)	(0.018)	(0.018)					(0.012)	(0.014)	(0.014)	(0.031)	(0.129)	(0.369)	(0.147)	(0.132)	(0.048)
α_2		0.027	0.027								0.029	0.345	0.496	0.025	-0.675	0.533
		(0.018)	(0.018)								(0.018)	(0.128)	(0.361)	(0.022)	(0.129)	(0.035)
α_3			0.020											0.035	-0.008	0.896
			(0.019)											(0.018)	(0.017)	(0.045)
β_1				-0.002	-0.003	-0.003		-0.952	-0.972	-0.972	-0.911	-0.585	-0.431	0.587	-1.679	0.622
				(0.02)8	(0.011)	(0.026)		(0.015)	(0.023)	(0.023)	(0.026)	(0.138)	(0.367)	(0.146)	(0.136)	(0.054)
β_2					0.027	0.027			0.025	0.030		-0.315	-0.475		0.697	-0.508
					(0.018)	(0.018)			()0.018)	(0.024)		(0.135)	(0.386)		(0.130)	(0.037)
β_3						0.020				-0.005			0.015			-0.871
						(0.019)				(0.016)			(0.035)			(0.052)
LL	419.2	421.7	423.1	419.1	420.3	420.9	419.1	428.0	429.0	429.0	427.2	428.0	428.2	424.3	429.0	431.6
AIC	-3.093	-3.094	-3.095	-3.093	-3.093	-3.093	-3.094	-3.098	-3.098	-3.098	-3.097	-3.097	-3.096	-3.095	-3.097	-3.098
BIC	-3.090	-3.089	-3.087	-3.090	-3.088	-3.086	-3.092	-3.093	-3.091	-3.088	-3.090	-3.088	-3.085	-3.085	-3.086	-3.085
Ljung-Box Q_{20}	33.053	28.377	25.860	33.275	29.197	27.055	33.104	16.523	15.197	15.068	17.022	16.374	16.242	25.794	14.799	15.030
p-value	0.033	0.101	0.170	0.031	0.084	0.134	0.033	0.684	0.765	0.773	0.652	0.693	0.702	0.173	0.788	0.775

Note. Standard Errors are given in parenthesis.

Table 3.91: Estimates of ARMA(p,q) model for nineteenth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.055	-0.053	-0.053	-0.056	-0.056	-0.056	-0.056	-0.002	-0.089	-0.105	-0.086	-0.002	-0.002	-0.095	-0.002	-0.003
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.001)	(0.022)	(0.006)	(0.019)	(0.001)	(0.001)	(0.008)	(0.001)	(0.002)
α_1	0.015	0.015	0.015					0.966	-0.598	-0.883	-0.584	0.152	0.182	-0.785	0.172	-0.830
	(0.021)	(0.021)	(0.021)					(0.020)	(0.386)	(0.056)	(0.315)	(0.100)	(0.154)	(0.117)	(0.096)	(0.011)
α_2		0.040	0.040								0.038	0.818	0.790	0.052	0.806	0.800
		(0.018)	(0.018)								(0.023)	(0.099)	(0.152)	(0.029)	(0.094)	(0.018)
α_3			0.001											0.030	-0.007	0.970
			(0.018)											(0.023)	(0.012)	(0.012)
β_1				0.014	0.014	0.014		-0.945	0.613	0.899	0.601	-0.147	-0.172	0.801	-0.162	0.842
				(0.022)	(0.018)	(0.022)		(0.022)	(0.386)	(0.060)	(0.317)	(0.109)	(0.166)	(0.116)	(0.103)	(0.016)
β_2					0.039	0.039			0.038	0.052		-0.797	-0.766		-0.783	-0.772
					(0.017)	(0.017)			(0.025)	(0.026)		(0.106)	(0.164)		(0.102)	(0.026)
β_3						0.002				0.033			-0.009			-0.956
						(0.002)				(0.017)			(0.015)			(0.016)
LL	1323.6	1326.8	1327.1	1323.3	1325.9	1325.9	1323.0	1328.3	1326.6	1328.0	1327.5	1337.3	1337.3	1328.6	1337.0	1341.7
AIC	-3.647	-3.649	-3.648	-3.647	-3.648	-3.647	-3.647	-3.649	-3.648	-3.648	-3.648	-3.654	-3.653	-3.648	-3.653	-3.655
BIC	-3.643	-3.643	-3.641	-3.643	-3.642	-3.640	-3.646	-3.644	-3.640	-3.639	-3.641	-3.644	-3.642	-3.639	-3.642	-3.642
Ljung-Box Q_{20}	30.56	23.81	23.62	30.70	24.29	24.20	32.26	14.66	25.60	24.27	25.46	13.24	12.89	23.82	12.99	11.61
p-value	0.06	0.25	0.26	0.06	0.23	0.23	0.04	0.80	0.18	0.23	0.18	0.87	0.88	0.25	0.88	0.93

Note. Standard Errors are given in parenthesis.

Table 3.92: Estimates of ARMA(p,q) model for twentieth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.056	-0.053	-0.053	-0.057	-0.057	-0.057	-0.057	-0.003	-0.055	-0.083	-0.050	-0.064	-0.124	-0.002	-0.088	-0.004
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.004)	(0.011)	(0.010)	(0.019)	(0.069)	(0.002)	(0.054)	(0.003)
α_1	0.020	0.019	0.019					0.950	0.024	-0.468	0.074	0.002	-0.706	0.961	-0.800	0.682
	(0.021)	(0.021)	(0.021)					(0.026)	(0.024)	(0.165)	(0.173)	(0.003)	(0.564)	(0.042)	(0.470)	(0.419)
α_2		0.050	0.050								0.049	-0.124	-0.469	0.031	0.211	-0.199
		(0.020)	(0.020)								(0.021)	(0.321)	(0.645)	(0.028)	(0.485)	(0.334)
α_3			0.003											-0.034	0.040	0.451
			(0.019)											(0.020)	(0.025)	(0.250)
β_1				0.018	0.019	0.019		-0.932	-0.005	0.488	-0.055	0.017	0.725	-0.943	0.820	-0.665
				(0.019)	(0.022)	(0.021)		(0.029)	(0.003)	(0.165)	(0.173)	(0.023)	(0.564)	(0.034)	(0.469)	(0.407)
β_2					0.051	0.051			0.051	0.061		0.175	0.531		-0.145	0.239
					(0.020)	(0.020)			(0.020)	(0.023)		(0.323)	(0.633)		(0.498)	(0.318)
β_3						0.003				0.031			0.042			-0.482
						(0.014)				(0.020)			(0.027)			(0.237)
LL	322.0	326.2	326.3	321.8	326.0	326.1	321.2	325.8	326.2	326.4	326.2	326.3	326.9	328.0	327.0	329.5
AIC	-3.034	-3.036	-3.035	-3.034	-3.036	-3.035	-3.034	-3.036	-3.035	-3.035	-3.035	-3.035	-3.034	-3.036	-3.034	-3.035
BIC	-3.030	-3.030	-3.028	-3.030	-3.030	-3.028	-3.032	-3.030	-3.028	-3.025	-3.028	-3.025	-3.023	-3.026	-3.023	-3.022
Ljung-Box Q_{20}	23.174	14.129	14.027	23.405	14.114	13.996	24.883	14.367	13.960	13.659	14.023	13.847	12.211	10.291	14.942	7.576
p-value	0.280	0.824	0.829	0.269	0.825	0.831	0.206	0.811	0.833	0.847	0.829	0.838	0.909	0.963	0.780	0.994

Note. Standard Errors are given in parenthesis.

Table 3.93: Estimates of ARMA(p,q) model for twenty first component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.033	-0.033	-0.033	-0.033	-0.033	-0.033	-0.033	-0.035	0.000	-0.054	0.000	0.000	-0.041	-0.055	-0.110	-0.001
	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.000)	(0.013)	(0.000)	(0.000)	(0.006)	(0.020)	(0.011)	(0.000)
α_1	-0.012	-0.012	-0.012					-0.071	0.998	-0.649	0.984	0.871	0.713	-0.681	-1.470	-0.516
	(0.023)	(0.023)	(0.023)					(0.067)	(0.000)	(0.361)	(0.000)	(0.000)	(0.016)	(0.518)	(0.070)	(0.000)
α_2		0.002	0.002								0.011	0.124	-0.980	-0.006	-0.918	0.533
		(0.02)5	(0.025)								(0.000)	(0.000)	(0.135)	(0.051)	(0.071)	(0.000)
α_3			0.010											0.015	0.007	0.968
			(0.023)											(0.034)	(0.032)	(0.000)
β_1				-0.012	-0.012	-0.012		0.058	-1.013	0.637	-1.005	-0.889	-0.726	0.669	1.460	0.495
				(0.019)	(0.052)	(0.020)		(0.064)	(0.000)	(0.364)	(0.000)	(0.000)	(0.044)	(0.503)	(0.063)	(0.000)
β_2					0.003	0.002			0.008	-0.006		-0.116	0.987		0.909	-0.549
					(0.071)	(0.012)			(0.000)	(0.015)		(0.000)	(0.138)		(0.055)	(0.000)
β_3						0.010				0.015			-0.004			-0.964
						(0.022)				(0.020)			(0.062)			(0.000)
LL	905.5	905.6	905.7	905.5	905.5	905.7	905.3	905.5	919.6	906.0	923.1	921.9	908.0	906.2	909.7	928.4
AIC	-3.391	-3.391	-3.390	-3.391	-3.391	-3.390	-3.392	-3.391	-3.399	-3.390	-3.401	-3.399	-3.390	-3.390	-3.391	-3.402
BIC	-3.387	-3.385	-3.383	-3.387	-3.385	-3.383	-3.390	-3.385	-3.391	-3.380	-3.393	-3.390	-3.379	-3.380	-3.380	-3.389
Ljung-Box Q_{20}	28.06	28.08	27.75	28.07	28.12	27.82	28.24	28.07	28.58	27.03	28.27	28.24	26.74	26.95	24.82	24.95
p-value	0.11	0.11	0.12	0.11	0.11	0.11	0.10	0.11	0.10	0.13	0.10	0.10	0.14	0.14	0.21	0.20

Note. Standard Errors are given in parenthesis.

Table 3.94: Estimates of ARMA(p,q) model for twenty second component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.066	-0.063	-0.059	-0.065	-0.066	-0.066	-0.065	-0.002	-0.003	-0.003	-0.020	-0.021	-0.028	-0.041	-0.036	-0.011
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.002)	(0.002)	(0.006)	(0.007)	(0.011)	(0.040)	(0.026)	(0.024)
α_1	-0.003	-0.003	-0.007					0.967	0.959	0.958	0.639	0.833	0.383	0.280	0.263	0.790
	(0.021)	(0.021)	(0.021)					(0.018)	(0.030)	(0.031)	(0.096)	(0.163)	(0.157)	(0.644)	(0.317)	(0.295)
α_2		0.031	0.031								0.058	-0.158	0.185	0.032	0.122	-0.687
		(0.022)	(0.022)								(0.021)	(0.136)	(0.151)	(0.023)	(0.155)	(0.176)
α_3			0.071											0.062	0.062	0.721
			(0.020)											(0.037)	(0.024)	(0.238)
β_1				-0.003	-0.007	-0.006		-0.942	-0.970	-0.969	-0.647	-0.846	-0.392	-0.289	-0.272	-0.798
				(0.015)	(0.028)	(0.041)		(0.020)	(0.035)	(0.036)	(0.095)	(0.166)	(0.157)	(0.647)	(0.318)	(0.305)
β_2					0.031	0.032			0.039	0.038		0.216	-0.154		-0.091	0.730
					(0.023)	(0.022)			(0.021)	(0.021)		(0.132)	(0.153)		(0.155)	(0.178)
β_3						0.065				0.002			0.052			-0.689
						(0.018)				(0.003)			(0.020)			(0.262)
LL	397.3	401.8	411.2	397.0	398.6	406.1	397.0	409.5	411.9	411.9	408.6	409.4	411.5	411.7	411.9	415.9
AIC	-3.080	-3.082	-3.087	-3.080	-3.080	-3.084	-3.080	-3.087	-3.088	-3.087	-3.086	-3.085	-3.086	-3.087	-3.086	-3.088
BIC	-3.076	-3.076	-3.080	-3.076	-3.075	-3.077	-3.078	-3.081	-3.080	-3.078	-3.078	-3.076	-3.075	-3.078	-3.075	-3.075
Ljung-Box Q_{20}	66.92	61.98	40.47	66.54	61.37	41.33	66.16	32.03	28.07	28.04	39.75	40.64	34.51	37.21	35.56	18.27
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.11	0.11	0.01	0.00	0.02	0.01	0.02	0.57

Note. Standard Errors are given in parenthesis.

Table 3.95: Estimates of ARMA(p,q) model for twenty third component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.052	-0.051	-0.049	-0.053	-0.053	-0.053	-0.053	-0.002	-0.002	-0.002	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
α_1	0.016	0.016	0.015					0.968	0.964	0.959	0.959	0.666	0.588	0.950	0.333	0.218
	(0.021)	(0.021)	(0.021)					(0.016)	(0.020)	(0.032)	(0.025)	(0.306)	(0.526)	(0.031)	(0.133)	(0.136)
α_2		0.019	0.018								0.019	0.310	0.384	0.006	0.611	0.635
		(0.019)	(0.019)								(0.021)	(0.304)	(0.516)	(0.050)	(0.130)	(0.109)
α_3			0.038											0.016	0.023	0.110
			(0.016)											(0.027)	(0.019)	(0.097)
β_1				0.016	0.015	0.012		-0.942	-0.957	-0.951	-0.955	-0.655	-0.583	-0.945	-0.327	-0.213
				(0.026)	(0.018)	(0.029)		(0.020)	(0.029)	(0.040)	(0.015)	(0.321)	(0.529)	(0.026)	(0.133)	(0.133)
β_2					0.017	0.015			0.020	0.004		-0.294	-0.379		-0.603	-0.624
					(0.021)	(0.024)			(0.020)	(0.008)		(0.313)	(0.509)		(0.130)	(0.113)
β_3						0.037				0.018			0.020			-0.086
						(0.017)				(0.015)			(0.020)			(0.093)
LL	1481.8	1482.5	1485.0	1481.4	1481.9	1484.1	1481.0	1494.1	1494.8	1495.3	1499.4	1499.9	1500.5	1497.9	1499.9	1499.9
AIC	-3.744	-3.744	-3.745	-3.744	-3.744	-3.744	-3.744	-3.751	-3.751	-3.750	-3.754	-3.753	-3.753	-3.752	-3.753	-3.752
BIC	-3.740	-3.738	-3.737	-3.740	-3.738	-3.737	-3.742	-3.745	-3.743	-3.741	-3.746	-3.744	-3.742	-3.743	-3.742	-3.739
Ljung-Box Q_{20}	62.178	58.480	48.546	62.490	59.643	50.455	65.751	20.158	18.767	17.409	18.120	18.835	16.998	16.962	16.903	17.076
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.448	0.537	0.626	0.579	0.533	0.653	0.655	0.659	0.648

Note. Standard Errors are given in parenthesis.

Table 3.96: Estimates of ARMA(p,q) model for twenty fourth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.070	-0.069	-0.067	-0.071	-0.071	-0.071	-0.071	-0.002	-0.002	-0.080	-0.001	0.000	0.000	-0.082	-0.058	-0.004
	(0.004)	(0.004)	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.001)	(0.295)	(0.001)	(0.000)	(0.000)	(0.025)	(0.040)	(0.003)
α_1	0.006	0.005	0.005					0.975	0.974	-0.131	0.967	1.759	1.757	-0.224	-0.023	-0.255
	(0.019)	(0.019)	(0.018)					(0.016)	(0.017)	(4.154)	(0.022)	(0.065)	(0.069)	(0.357)	(0.279)	(0.031)
α_2		0.022	0.022								0.014	-0.762	-0.760	0.023	0.178	0.253
		(0.019)	(0.019)								(0.019)	(0.064)	(0.069)	(0.019)	(0.405)	(0.049)
α_3			0.031											0.036	0.032	0.939
			(0.018)											(0.018)	(0.020)	(0.057)
β_1				0.005	0.004	0.005		-0.958	-0.971	0.136	-0.965	-1.756	-1.758	0.229	0.028	0.264
				(0.016)	(0.088)	(0.026)		(0.016)	(0.023)	(4.150)	(0.012)	(0.069)	(0.073)	(0.357)	(0.283)	(0.035)
β_2					0.022	0.021			0.015	0.022		0.762	0.770		-0.156	-0.240
					(0.020)	(0.019)			(0.016)	(0.029)		(0.068)	(0.072)		(0.407)	(0.055)
β_3						0.029				0.032			-0.006			-0.912
						(0.017)				(0.071)			(0.015)			(0.063)
LL	159.6	160.5	162.1	159.5	160.3	161.8	159.4	164.2	164.6	162.0	166.8	170.9	171.0	162.3	162.2	168.7
AIC	-2.934	-2.934	-2.935	-2.934	-2.934	-2.934	-2.935	-2.937	-2.936	-2.934	-2.938	-2.939	-2.939	-2.934	-2.934	-2.937
BIC	-2.931	-2.929	-2.927	-2.931	-2.929	-2.927	-2.933	-2.931	-2.929	-2.925	-2.930	-2.930	-2.928	-2.925	-2.922	-2.924
Ljung-Box Q_{20}	29.90	27.51	22.89	29.64	27.29	23.09	30.08	16.21	15.87	23.35	15.74	15.81	15.66	23.12	22.30	12.38
p-value	0.07	0.12	0.29	0.08	0.13	0.28	0.07	0.70	0.72	0.27	0.73	0.73	0.74	0.28	0.32	0.90

Note. Standard Errors are given in parenthesis.

Table 3.97: Estimates of ARMA(p,q) model for twenty fifth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.041	-0.041	-0.039	-0.041	-0.041	-0.041	-0.041	0.000	-0.071	-0.057	-0.075	-0.124	-0.035	-0.055	-0.030	-0.001
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.000)	(0.008)	(0.011)	(0.009)	(0.016)	(0.032)	(0.012)	(0.025)	(0.000)
α_1	-0.001	-0.001	-0.001					0.995	-0.724	-0.392	-0.779	-1.297	-0.118	-0.388	-0.095	-0.282
	(0.019)	(0.019)	(0.019)					(0.004)	(0.102)	(0.217)	(0.103)	(0.125)	(0.403)	(0.243)	(0.336)	(0.169)
α_2		-0.004	-0.004								-0.030	-0.706	0.274	-0.004	0.301	0.587
		(0.019)	(0.019)								(0.019)	(0.148)	(0.396)	(0.016)	(0.314)	(0.106)
α_3			0.061											0.059	0.062	0.684
			(0.019)											(0.019)	(0.019)	(0.144)
β_1				-0.001	-0.001	0.001		-0.989	0.726	0.393	0.779	1.289	0.118	0.389	0.095	0.276
				(0.007)	(0.006)	(0.050)		(0.005)	(0.102)	(0.218)	(0.101)	(0.130)	(0.402)	(0.244)	(0.336)	(0.177)
β_2					-0.004	-0.007			-0.029	-0.006		0.673	-0.281		-0.309	-0.607
					(0.022)	(0.037)			(0.017)	(0.018)		(0.150)	(0.401)		(0.321)	(0.107)
β_3						0.060				0.057			0.060			-0.644
						(0.019)				(0.018)			(0.019)			(0.151)
LL	-19.6	-19.5	-13.4	-19.6	-19.6	-13.7	-19.6	-14.9	-16.5	-12.6	-16.0	-13.4	-12.5	-12.6	-12.2	-7.4
AIC	-2.825	-2.824	-2.827	-2.825	-2.824	-2.827	-2.825	-2.827	-2.825	-2.827	-2.826	-2.827	-2.827	-2.827	-2.827	-2.829
BIC	-2.821	-2.818	-2.820	-2.821	-2.818	-2.820	-2.823	-2.821	-2.818	-2.818	-2.818	-2.817	-2.815	-2.818	-2.816	-2.816
Ljung-Box Q_{20}	26.56	26.55	13.91	26.63	26.64	13.94	26.68	25.16	19.41	11.79	19.06	14.12	11.42	11.56	11.08	11.53
p-value	0.15	0.15	0.84	0.15	0.15	0.83	0.14	0.20	0.50	0.92	0.52	0.82	0.93	0.93	0.94	0.93

Note. Standard Errors are given in parenthesis.

Table 3.98: Estimates of ARMA(p,q) model for twenty sixth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.078	-0.077	-0.074	-0.080	-0.080	-0.080	-0.080	-0.008	-0.011	-0.019	0.000	0.000	0.000	0.000	0.000	0.000
	(0.004)	(0.004)	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.008)	(0.015)	(0.060)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
α_1	0.020	0.020	0.019					0.896	0.866	0.756	1.005	0.931	0.934	1.007	0.017	0.374
	(0.018)	(0.018)	(0.018)					(0.107)	(0.193)	(0.752)	(0.020)	(0.101)	(0.100)	(0.013)	(0.026)	(1.079)
α_2		0.019	0.019								-0.007	0.067	0.064	-0.002	0.988	0.541
		(0.018)	(0.018)								(0.020)	(0.101)	(0.101)	(0.010)	(0.014)	(1.073)
α_3			0.031											-0.007	-0.010	0.082
			(0.018)											(0.012)	(0.020)	(0.140)
β_1				0.020	0.019	0.018		-0.867	-0.848	-0.738	-0.993	-0.918	-0.920	-0.993	-0.004	-0.361
				(0.018)	(0.019)	(0.018)		(0.111)	(0.194)	(0.747)	(0.005)	(0.101)	(0.102)	(0.004)	(0.014)	(1.082)
β_2					0.018	0.017			0.014	0.004		-0.075	-0.065		-0.982	-0.543
					(0.018)	(0.017)			(0.019)	(0.039)		(0.101)	(0.106)		(0.016)	(1.080)
β_3						0.029				0.020			-0.008			-0.085
						(0.017)				(0.037)			(0.016)			(0.131)
LL	295.2	295.9	297.5	294.8	295.3	296.7	294.1	298.1	298.4	298.8	305.7	306.0	306.1	306.0	306.1	305.9
AIC	-3.017	-3.017	-3.018	-3.017	-3.017	-3.017	-3.017	-3.019	-3.018	-3.018	-3.023	-3.022	-3.022	-3.022	-3.022	-3.021
BIC	-3.014	-3.012	-3.010	-3.013	-3.011	-3.010	-3.016	-3.013	-3.011	-3.008	-3.015	-3.013	-3.010	-3.013	-3.010	-3.008
Ljung-Box Q_{20}	21.914	19.844	15.663	21.900	20.138	16.346	24.401	7.703	8.055	9.290	8.934	9.133	9.014	8.839	9.135	9.120
p-value	0.345	0.468	0.737	0.346	0.449	0.695	0.225	0.994	0.991	0.979	0.984	0.981	0.983	0.985	0.981	0.981

Note. Standard Errors are given in parenthesis.

Table 3.99: Estimates of ARMA(p,q) model for twenty seventh component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.065	-0.062	-0.061	-0.066	-0.066	-0.066	-0.066	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003
	(0.003)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
α_1	0.009	0.009	0.008					0.987	0.985	0.985	0.950	0.445	0.315	0.923	0.152	-0.409
	(0.019)	(0.019)	(0.019)					(0.006)	(0.007)	(0.011)	(0.021)	(0.289)	(0.461)	(0.023)	(0.227)	(0.236)
α_2		0.044	0.044								0.034	0.536	0.664	0.039	0.797	0.638
		(0.018)	(0.018)								(0.018)	(0.287)	(0.455)	(0.036)	(0.223)	(0.151)
α_3			0.020											0.004	0.021	0.729
			(0.018)											(0.034)	(0.019)	(0.146)
β_1				0.008	0.007	0.007		-0.964	-0.992	-0.992	-0.960	-0.450	-0.324	-0.931	-0.160	0.407
				(0.017)	(0.046)	(0.034)		(0.009)	(0.020)	(0.028)	(0.013)	(0.299)	(0.460)	(0.017)	(0.226)	(0.243)
β_2					0.042	0.041			0.031	0.034		-0.499	-0.633		-0.764	-0.602
					(0.017)	(0.018)			(0.019)	(0.038)		(0.292)	(0.463)		(0.223)	(0.159)
β_3						0.015				-0.004			0.013			-0.694
						(0.018)				(0.067)			(0.026)			(0.140)
LL	1046.7	1050.1	1051.8	1046.6	1049.6	1050.0	1046.5	1073.1	1074.7	1074.7	1076.5	1077.8	1078.0	1061.9	1073.2	1076.9
AIC	-3.478	-3.479	-3.480	-3.478	-3.479	-3.478	-3.478	-3.493	-3.494	-3.493	-3.495	-3.495	-3.494	-3.485	-3.491	-3.493
BIC	-3.474	-3.474	-3.472	-3.474	-3.473	-3.471	-3.476	-3.488	-3.486	-3.484	-3.487	-3.486	-3.483	-3.476	-3.480	-3.480
Ljung-Box Q_{20}	84.34	68.90	64.43	84.49	70.50	67.49	86.90	23.53	21.17	21.15	21.12	20.73	20.57	21.81	20.56	19.44
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.39	0.39	0.39	0.41	0.42	0.35	0.42	0.49

Note. Standard Errors are given in parenthesis.

Table 3.100: Estimates of ARMA(p,q) model for twenty eight component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.042	-0.042	-0.040	-0.043	-0.043	-0.043	-0.043	-0.067	-0.066	-0.050	-0.021	-0.020	-0.047	-0.045	-0.071	-0.038
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.012)	(0.012)	(0.018)	(0.009)	(0.013)	(0.119)	(0.020)	(0.026)	(0.012)
α_1	0.020	0.021	0.020					-0.569	-0.563	-0.186	0.505	0.469	0.113	-0.086	-0.362	-0.097
	(0.019)	(0.019)	(0.019)					(0.228)	(0.232)	(0.404)	(0.196)	(0.202)	(1.514)	(0.470)	(0.326)	(0.207)
α_2		0.002	0.002								0.007	0.064	-0.221	0.004	-0.350	-0.172
		(0.021)	(0.021)								(0.017)	(0.400)	(1.323)	(0.019)	(0.294)	(0.139)
α_3			0.039											0.039	0.043	0.386
			(0.018)											(0.018)	(0.019)	(0.123)
β_1				0.020	0.020	0.020		0.589	0.585	0.206	-0.486	-0.448	-0.093	0.105	0.383	0.116
				(0.020)	(0.024)	(0.019)		(0.228)	(0.233)	(0.405)	(0.195)	(0.226)	(1.524)	(0.470)	(0.326)	(0.210)
β_2					0.001	0.002			0.000	0.007		-0.057	0.223		0.359	0.179
					(0.044)	(0.007)			(0.000)	(0.029)		(0.424)	(1.363)		(0.297)	(0.144)
β_3						0.038				0.036			0.039			-0.343
						(0.017)				(0.019)			(0.045)			(0.124)
LL	466.4	469.2	472.5	466.0	466.1	468.5	465.4	467.2	467.2	468.9	470.2	470.2	472.0	472.6	473.0	474.1
AIC	-3.122	-3.123	-3.125	-3.122	-3.121	-3.122	-3.122	-3.122	-3.122	-3.122	-3.123	-3.123	-3.123	-3.124	-3.124	-3.124
BIC	-3.119	-3.118	-3.117	-3.118	-3.116	-3.115	-3.120	-3.117	-3.114	-3.113	-3.116	-3.113	-3.112	-3.115	-3.113	-3.111
Ljung-Box Q_{20}	14.26	13.93	8.58	14.33	14.32	9.12	16.12	13.57	13.56	9.35	12.23	12.09	9.27	8.63	8.58	7.17
p-value	0.82	0.83	0.99	0.81	0.81	0.98	0.71	0.85	0.85	0.98	0.91	0.91	0.98	0.99	0.99	1.00

Note. Standard Errors are given in parenthesis.

Table 3.101: Estimates of ARMA(p,q) model for twenty ninth component of $\mathbf{Z_{1,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.048	-0.046	-0.045	-0.048	-0.048	-0.048	-0.048	-0.001	-0.001	-0.001	-0.006	-0.004	-0.004	-0.002	-0.002	-0.002
	(0.003)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.001)	(0.001)	(0.001)	(0.003)	(0.003)	(0.003)	(0.001)	(0.001)	(0.001)
α_1	0.003	0.003	0.003					0.973	0.969	0.971	0.850	0.210	0.150	0.941	0.616	0.607
	(0.020)	(0.020)	(0.020)					(0.013)	(0.018)	(0.015)	(0.065)	(0.202)	(0.205)	(0.028)	(0.236)	(0.242)
α_2		0.035	0.035								0.035	0.708	0.759	0.034	0.351	0.339
		(0.018)	(0.018)								(0.020)	(0.224)	(0.226)	(0.029)	(0.234)	(0.248)
α_3			0.022											-0.009	-0.004	0.017
			(0.020)											(0.021)	(0.010)	(0.018)
β_1				0.003	0.002	0.001		-0.955	-0.971	-0.973	-0.852	-0.203	-0.152	-0.945	-0.618	-0.609
				(0.023)	(0.033)	(0.033)		(0.016)	(0.025)	(0.025)	(0.064)	(0.211)	(0.204)	(0.021)	(0.240)	(0.245)
β_2					0.032	0.032			0.022	0.031		-0.675	-0.729		-0.321	-0.309
					(0.017)	(0.018)			(0.019)	(0.025)		(0.234)	(0.231)		(0.232)	(0.247)
β_3						0.022				-0.010			0.014			-0.020
						(0.019)				(0.014)			(0.024)			(0.031)
LL	840.6	843.4	844.4	840.5	842.4	843.1	840.5	849.5	850.2	850.4	847.5	850.1	850.3	851.0	852.1	852.1
AIC	-3.351	-3.353	-3.353	-3.351	-3.352	-3.352	-3.352	-3.356	-3.356	-3.356	-3.354	-3.355	-3.355	-3.356	-3.356	-3.355
BIC	-3.348	-3.347	-3.345	-3.348	-3.346	-3.344	-3.350	-3.351	-3.349	-3.346	-3.347	-3.346	-3.344	-3.347	-3.345	-3.342
Ljung-Box Q_{20}	39.19	33.52	30.39	39.26	34.00	31.26	39.56	17.84	16.39	16.23	18.29	16.69	16.21	16.31	16.22	16.23
p-value	0.01	0.03	0.06	0.01	0.03	0.05	0.01	0.60	0.69	0.70	0.57	0.67	0.70	0.70	0.70	0.70

Note. Standard Errors are given in parenthesis.

3.4.6 Histograms of all components of $U_{2,t}$ with Different Distributions Fit

Figure 3.4.3: Histograms of 15 components of singular vector with Normal fit after second round of SVD

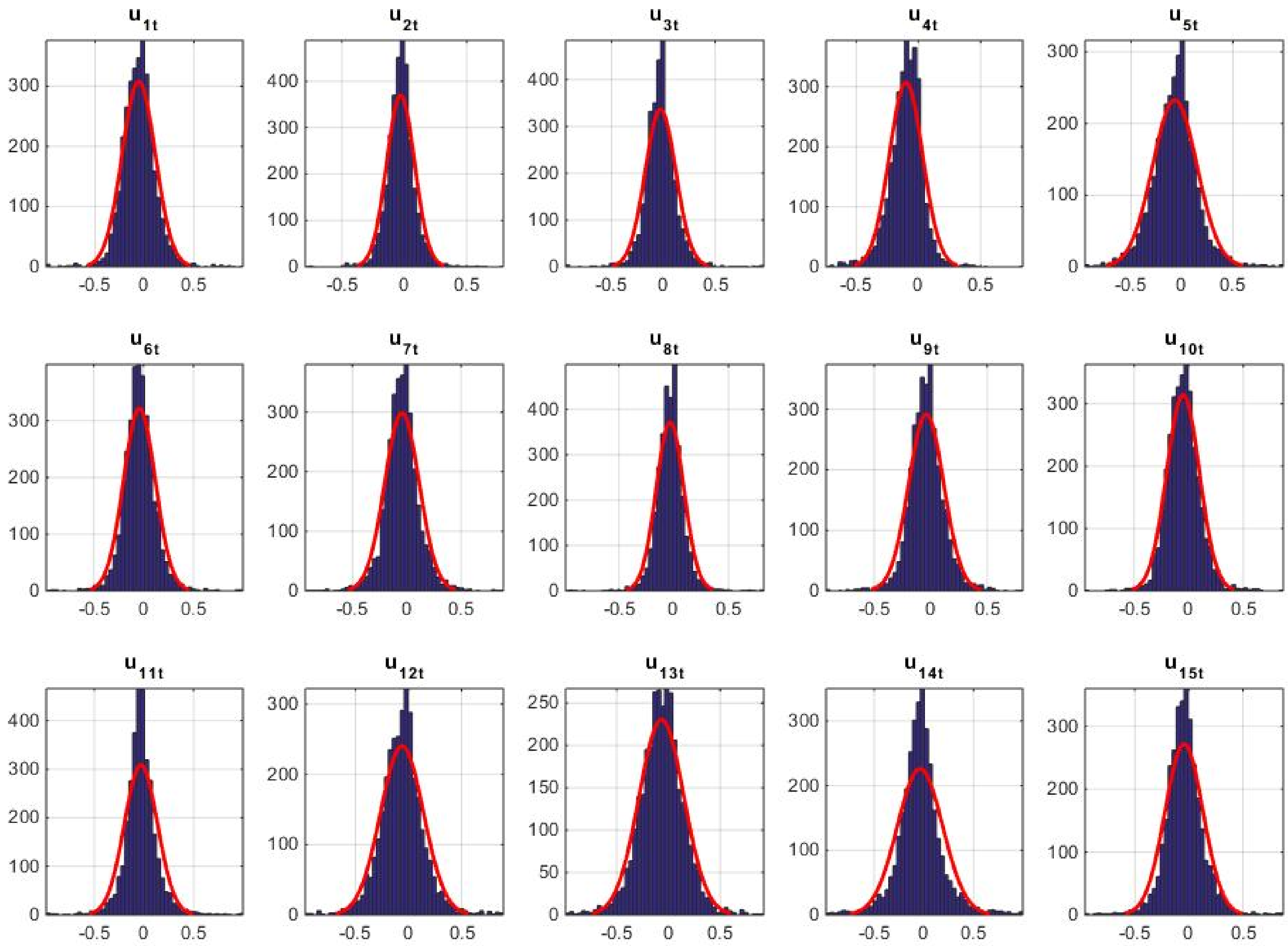
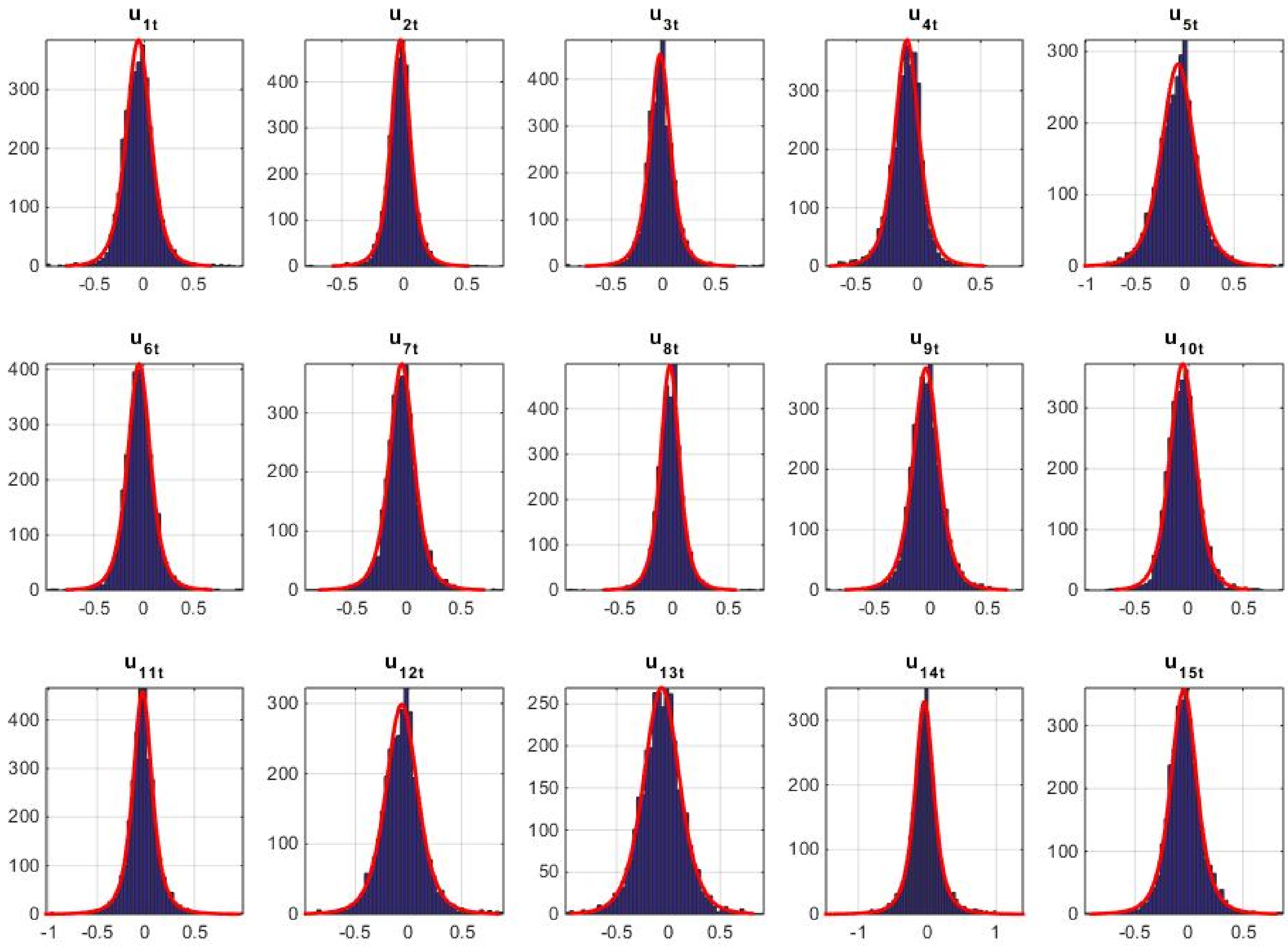


Figure 3.4.4: Histograms of 15 components of singular vector with Student's t fit after second round of SVD



3.4.7 Estimates of ARMA(p,q) model for all Components of $\mathbf{U_{2,t}}$ from Second Round

Table 3.102: Estimates of ARMA(p,q) model for first component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.056	-0.055	-0.054	-0.058	-0.058	-0.058	-0.058	0.000	0.000	0.000	-0.002	-0.001	-0.002	0.000	0.000	-0.001
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.000)	(0.000)	(0.001)
α_1	0.029	0.029	0.028					0.998	0.998	0.998	0.944	0.115	0.114	0.995	1.397	0.423
	(0.020)	(0.020)	(0.020)					(0.002)	(0.002)	(0.002)	(0.032)	(0.089)	(0.104)	(0.031)	(0.077)	(0.179)
α_2		0.019	0.018								0.015	0.859	0.859	-0.016	-0.409	-0.260
		(0.020)	(0.020)								(0.023)	(0.091)	(0.102)	(0.033)	(0.086)	(0.201)
α_3			0.021											0.013	0.010	0.816
			(0.017)											(0.021)	(0.020)	(0.114)
β_1				0.028	0.028	0.028		-0.988	-0.986	-0.986	-0.926	-0.095	-0.097	-0.976	-1.386	-0.411
				(0.020)	(0.021)	(0.020)		(0.005)	(0.049)	(0.030)	(0.026)	(0.092)	(0.111)	(0.016)	(0.078)	(0.194)
β_2					0.018	0.018			-0.003	-0.010		-0.839	-0.840		0.394	0.259
					(0.018)	(0.023)			(0.052)	(0.098)		(0.094)	(0.103)		(0.080)	(0.207)
β_3						0.019				0.007			0.004			-0.785
						(0.016)				(0.124)			(0.009)			(0.115)
LL	1182.2	1183.9	1184.8	1182.2	1182.7	1183.4	1180.9	1209.7	1209.7	1209.8	1192.8	1200.4	1200.4	1205.1	1211.3	1206.6
AIC	-3.561	-3.561	-3.561	-3.561	-3.560	-3.560	-3.560	-3.577	-3.576	-3.576	-3.566	-3.570	-3.569	-3.573	-3.576	-3.572
BIC	-3.557	-3.555	-3.553	-3.557	-3.555	-3.553	-3.559	-3.571	-3.569	-3.566	-3.558	-3.561	-3.558	-3.563	-3.565	-3.559
Ljung-Box Q_{20}	76.88	72.56	67.99	77.34	73.37	69.13	86.21	26.82	26.83	26.45	28.18	25.82	25.80	25.12	25.65	23.29
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.14	0.15	0.11	0.17	0.17	0.20	0.18	0.27

Note. Standard Errors are given in parenthesis.

Table 3.103: Estimates of ARMA(p,q) model for second component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.028	-0.027	-0.026	-0.029	-0.029	-0.029	-0.029	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	0.000	-0.001
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)
α_1	0.048	0.048	0.047					0.985	0.989	0.988	1.023	0.052	0.044	0.995	0.064	-0.968
	(0.020)	(0.020)	(0.020)					(0.024)	(0.012)	(0.024)	(0.022)	(0.054)	(0.017)	(0.049)	(0.032)	(0.031)
α_2		0.016	0.014								-0.028	0.933	0.944	-0.030	0.939	0.973
		(0.019)	(0.019)								(0.021)	(0.048)	(0.022)	(0.035)	(0.028)	(0.019)
α_3			0.035											0.012	-0.019	0.964
			(0.019)											(0.034)	(0.026)	(0.032)
β_1				0.047	0.047	0.045		-0.967	-0.950	-0.949	-0.986	-0.029	-0.004	-0.957	-0.024	0.986
				(0.019)	(0.019)	(0.020)		(0.038)	(0.022)	(0.037)	(0.010)	(0.053)	(0.003)	(0.046)	(0.024)	(0.031)
β_2					0.014	0.014			-0.025	-0.031		-0.932	-0.942		-0.937	-0.948
					(0.020)	(0.017)			(0.017)	(0.046)		(0.042)	(0.023)		(0.027)	(0.033)
β_3						0.032				0.007			-0.020			-0.956
						(0.019)				(0.046)			(0.016)			(0.032)
LL	2521.5	2521.9	2524.1	2521.2	2521.5	2523.2	2517.5	2534.8	2535.7	2535.8	2536.9	2539.0	2539.6	2534.5	2539.8	2542.5
AIC	-4.381	-4.380	-4.381	-4.381	-4.380	-4.381	-4.379	-4.388	-4.388	-4.388	-4.389	-4.390	-4.389	-4.387	-4.389	-4.391
BIC	-4.377	-4.375	-4.374	-4.377	-4.375	-4.373	-4.377	-4.383	-4.381	-4.378	-4.382	-4.380	-4.378	-4.378	-4.378	-4.377
Ljung-Box Q_{20}	56.11	53.45	47.09	57.09	54.76	49.49	73.30	24.90	22.96	22.67	23.96	23.41	22.18	23.02	21.90	24.75
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.29	0.31	0.24	0.27	0.33	0.29	0.35	0.21

Note. Standard Errors are given in parenthesis.

Table 3.104: Estimates of ARMA(p,q) model for third component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.026	-0.025	-0.025	-0.027	-0.027	-0.027	-0.027	-0.001	-0.001	-0.037	-0.021	-0.001	-0.001	-0.001	-0.002	-0.017
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.001)	(0.001)	(0.006)	(0.029)	(0.000)	(0.001)	(0.001)	(0.001)	(0.004)
α_1	0.051	0.050	0.049					0.956	0.958	-0.365	0.217	0.816	0.645	0.976	0.472	1.269
	(0.025)	(0.025)	(0.025)					(0.019)	(0.017)	(0.173)	(1.117)	(0.371)	(0.765)	(0.040)	(0.136)	(0.253)
α_2		0.021	0.021								0.013	0.150	0.315	-0.025	0.483	-0.930
		(0.018)	(0.018)								(0.059)	(0.359)	(0.733)	(0.028)	(0.159)	(0.142)
α_3			0.006											0.000	-0.013	0.051
			(0.017)											(0.000)	(0.037)	(0.051)
β_1				0.049	0.049	0.050		-0.934	-0.912	0.415	-0.167	-0.772	-0.599	-0.930	-0.427	-1.220
				(0.024)	(0.025)	(0.025)		(0.022)	(0.028)	(0.175)	(1.121)	(0.360)	(0.764)	(0.027)	(0.154)	(0.255)
β_2					0.024	0.024			-0.026	0.042		-0.173	-0.325		-0.486	0.877
					(0.019)	(0.018)			(0.022)	(0.022)		(0.341)	(0.712)		(0.148)	(0.109)
β_3						0.009				0.021			-0.012			-0.027
						(0.016)				(0.018)			(0.016)			(0.050)
LL	1604.4	1605.2	1605.7	1603.9	1604.8	1605.0	1599.9	1609.1	1610.2	1605.5	1605.2	1611.2	1611.4	1609.3	1610.5	1610.1
AIC	-3.819	-3.819	-3.819	-3.819	-3.819	-3.818	-3.817	-3.821	-3.821	-3.818	-3.818	-3.821	-3.821	-3.820	-3.820	-3.820
BIC	-3.815	-3.813	-3.811	-3.815	-3.813	-3.811	-3.815	-3.816	-3.814	-3.809	-3.811	-3.812	-3.810	-3.811	-3.809	-3.806
Ljung-Box Q_{20}	36.13	34.27	33.55	36.69	34.39	33.87	48.80	29.08	25.48	33.56	34.14	25.50	25.29	25.95	25.53	30.20
p-value	0.01	0.02	0.03	0.01	0.02	0.03	0.00	0.09	0.18	0.03	0.03	0.18	0.19	0.17	0.18	0.07

Note. Standard Errors are given in parenthesis.

Table 3.105: Estimates of ARMA(p,q) model for fourth component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.094	-0.092	-0.088	-0.100	-0.100	-0.100	-0.100	0.000	0.000	0.000	0.000	-0.001	-0.001	0.000	0.000	-0.001
	(0.003)	(0.003)	(0.004)	(0.002)	(0.003)	(0.003)	(0.002)	(0.000)	(0.000	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
α_1	0.062	0.061	0.060					0.996	0.996	0.996	1.017	0.307	0.364	1.015	0.300	-0.665
	(0.020)	(0.020)	(0.020)					(0.002)	(0.002)	(0.002)	(0.019)	(0.110)	(0.135)	(0.020)	(0.096)	(0.133)
α_2		0.020	0.018								-0.020	0.687	0.630	-0.041	0.687	0.863
		(0.020)	(0.020)								(0.019)	(0.110)	(0.135)	(0.032)	(0.114)	(0.055)
α_3			0.045											0.023	0.009	0.792
			(0.021)											(0.023)	(0.033)	(0.090)
β_1				0.060	0.059	0.060		-0.983	-0.963	-0.963	-0.986	-0.269	-0.332	-0.984	-0.268	0.696
				(0.019)	(0.020)	(0.020)		(0.004)	(0.020)	(0.020)	(0.004)	(0.106)	(0.138)	(0.005)	(0.100)	(0.137)
β_2					0.019	0.017			-0.020	-0.041		-0.706	-0.655		-0.708	-0.854
					(0.024)	(0.029)			(0.020)	(0.031)		(0.104)	(0.124)		(0.098)	(0.049)
β_3						0.040				0.022			0.011			-0.794
						(0.019)				(0.022)			(0.025)			(0.096)
LL	1930.9	1931.7	1935.0	1930.4	1930.9	1933.9	1924.3	1976.8	1977.5	1978.2	1978.8	1979.9	1980.1	1976.1	1979.5	1982.1
AIC	-4.019	-4.019	-4.020	-4.019	-4.018	-4.020	-4.016	-4.047	-4.046	-4.046	-4.047	-4.047	-4.047	-4.045	-4.046	-4.047
BIC	-4.015	-4.013	-4.013	-4.015	-4.013	-4.012	-4.014	-4.041	-4.039	-4.037	-4.040	-4.038	-4.036	-4.036	-4.035	-4.034
Ljung-Box Q_{20}	120.36	111.76	94.51	123.43	116.05	100.70	161.06	32.88	31.42	29.96	31.51	28.25	28.13	30.41	28.41	25.06
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.05	0.07	0.05	0.10	0.11	0.06	0.10	0.20

Note. Standard Errors are given in parenthesis.

Table 3.106: Estimates of ARMA(p,q) model for fifth component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.069	-0.067	-0.063	-0.070	-0.070	-0.070	-0.070	-0.001	-0.001	-0.002	-0.001	-0.001	-0.002	-0.001	-0.002	-0.002
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.000)	(0.001	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)
α_1	0.013	0.013	0.011					0.982	0.980	0.978	0.934	1.079	0.152	0.929	0.097	0.419
	(0.02)2	(0.022)	(0.022)					(0.007)	(0.008)	(0.009)	(0.025)	(0.174)	(0.077)	(0.026)	(0.084)	(0.195)
α_2		0.031	0.031								0.047	-0.096	0.814	0.020	0.814	0.802
		(0.020)	(0.020)								(0.022)	(0.174)	(0.076)	(0.030)	(0.083)	(0.079)
α_3			0.052											0.031	0.054	-0.247
			(0.020)											(0.021)	(0.021)	(0.164)
β_1				0.012	0.010	0.007		-0.951	-0.994	-0.991	-0.948	-1.097	-0.167	-0.944	-0.110	-0.434
				(0.023)	(0.021)	(0.027)		(0.010)	(0.024)	(0.023)	(0.012)	(0.183)	(0.081)	(0.014)	(0.083)	(0.191)
β_2					0.028	0.022			0.048	0.020		0.144	-0.796		-0.795	-0.791
					(0.019)	(0.019)			(0.023)	(0.029)		(0.180)	(0.071)		(0.079)	(0.073)
β_3						0.044				0.029			0.055			0.295
						(0.019)				(0.020)			(0.021)			(0.156)
LL	345.2	346.9	351.2	345.0	346.3	349.7	344.7	383.2	386.7	388.0	389.9	390.1	392.4	391.2	392.1	393.0
AIC	-3.048	-3.048	-3.051	-3.048	-3.048	-3.050	-3.048	-3.071	-3.072	-3.072	-3.074	-3.074	-3.075	-3.074	-3.074	-3.074
BIC	-3.044	-3.043	-3.043	-3.044	-3.043	-3.042	-3.046	-3.065	-3.065	-3.063	-3.067	-3.064	-3.063	-3.065	-3.063	-3.061
Ljung-Box Q_{20}	162.30	144.71	118.59	162.43	148.48	129.59	169.83	32.16	27.15	23.73	27.65	26.44	24.17	23.79	24.65	22.83
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.13	0.25	0.12	0.15	0.23	0.25	0.22	0.30

Note. Standard Errors are given in parenthesis.

Table 3.107: Estimates of ARMA(p,q) model for sixth component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.044	-0.042	-0.043	-0.043	-0.043	-0.043	-0.043	-0.077	-0.077	-0.078	-0.076	-0.001	-0.002	-0.073	-0.006	-0.005
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.007)	(0.007)	(0.008)	(0.007)	(0.001)	(0.002)	(0.008)	(0.004)	(0.003)
α_1	-0.027	-0.026	-0.026					-0.800	-0.798	-0.836	-0.784	0.213	0.162	-0.723	0.061	-0.330
	(0.022)	(0.022)	(0.022)					(0.096)	(0.095)	(0.135)	(0.102)	(0.137)	(0.100)	(0.142)	(0.141)	(0.187)
α_2		0.029	0.028								0.003	0.757	0.802	0.010	0.783	0.820
		(0.024)	(0.024)								(0.003)	(0.137)	(0.093)	(0.031)	(0.091)	(0.055)
α_3			-0.009											0.004	0.026	0.392
			(0.022)											(0.012)	(0.032)	(0.204)
β_1				-0.025	-0.026	-0.026		0.774	0.773	0.811	0.759	-0.233	-0.190	0.699	-0.088	0.310
				(0.021)	(0.019)	(0.022)		(0.107)	(0.107)	(0.135)	(0.114)	(0.151)	(0.103)	(0.129)	(0.133)	(0.200)
β_2					0.028	0.028			0.002	0.007		-0.717	-0.768		-0.747	-0.786
					(0.022)	(0.024)			(0.006)	(0.018)		(0.151)	(0.109)		(0.107)	(0.059)
β_3						-0.009				0.011			0.017			-0.364
						(0.023)				(0.029)			(0.027)			(0.215)
LL	1352.4	1353.8	1355.5	1352.2	1353.6	1353.7	1351.1	1355.3	1355.3	1355.4	1355.5	1361.4	1361.7	1357.2	1360.2	1361.0
AIC	-3.665	-3.665	-3.666	-3.665	-3.665	-3.664	-3.665	-3.666	-3.665	-3.665	-3.665	-3.668	-3.668	-3.666	-3.667	-3.667
BIC	-3.661	-3.659	-3.658	-3.661	-3.659	-3.657	-3.663	-3.660	-3.658	-3.656	-3.658	-3.659	-3.657	-3.657	-3.656	-3.654
Ljung-Box Q_{20}	28.13	23.67	23.85	28.31	23.93	23.97	29.61	23.11	22.98	22.51	22.91	13.29	13.20	22.62	14.31	13.86
p-value	0.11	0.26	0.25	0.10	0.25	0.24	0.08	0.28	0.29	0.31	0.29	0.86	0.87	0.31	0.81	0.84

Note. Standard Errors are given in parenthesis.

Table 3.108: Estimates of ARMA(p,q) model for seventh component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.044	-0.045	-0.046	-0.044	-0.044	-0.044	-0.044	-0.055	-0.033	-0.041	-0.022	-0.015	-0.012	-0.028	-0.020	-0.030
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.074)	(0.012)	(0.073)	(0.008)	(0.003)	(0.010)	(0.006)	(0.045)	(0.009)
α_1	-0.001	-0.001	-0.002					-0.243	0.243	0.061	0.525	1.383	1.409	0.386	0.955	0.336
	(0.022)	(0.022)	(0.022)					(1.683)	(0.266)	(1.647)	(0.160)	(0.161)	(0.134)	(0.139)	(2.495)	(0.225)
α_2		-0.021	-0.021								-0.021	-0.724	-0.690	-0.020	-0.414	0.588
		(0.021)	(0.021)								(0.023)	(0.148)	(0.226)	(0.022)	(1.554)	(0.096)
α_3			-0.018											-0.009	0.002	-0.609
			(0.020)											(0.015)	(0.066)	(0.174)
β_1				-0.001	-0.002	-0.002		0.246	-0.244	-0.063	-0.526	-1.394	-1.413	-0.388	-0.958	-0.338
				(0.000)	(0.002)	(0.002)		(1.723)	(0.265)	(1.633)	(0.156)	(0.166)	(0.138)	(0.135)	(2.501)	(0.231)
β_2					-0.022	-0.021			-0.024	-0.021		0.716	0.663		0.392	-0.620
					(0.020)	(0.021)			(0.021)	(0.065)		(0.146)	(0.245)		(1.542)	(0.095)
β_3						-0.017				-0.016			0.019			0.609
						(0.021)				(0.021)			(0.048)			(0.174)
LL	1371.6	1372.5	1374.7	1371.4	1372.2	1372.7	1371.4	1371.6	1372.4	1372.8	1373.5	1376.1	1376.4	1375.2	1375.4	1377.5
AIC	-3.677	-3.676	-3.677	-3.676	-3.676	-3.676	-3.677	-3.676	-3.676	-3.675	-3.677	-3.678	-3.677	-3.677	-3.676	-3.677
BIC	-3.673	-3.671	-3.670	-3.673	-3.671	-3.669	-3.675	-3.670	-3.668	-3.666	-3.669	-3.668	-3.666	-3.668	-3.665	-3.664
Ljung-Box Q_{20}	27.56	26.87	26.01	27.47	26.89	26.45	27.43	27.36	26.81	26.57	26.75	22.90	21.55	25.89	24.66	22.44
p-value	0.12	0.14	0.17	0.12	0.14	0.15	0.12	0.13	0.14	0.15	0.14	0.29	0.37	0.17	0.21	0.32

Note. Standard Errors are given in parenthesis.

Table 3.109: Estimates of ARMA(p,q) model for eight component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.034	-0.033	-0.032	-0.033	-0.033	-0.033	-0.033	-0.045	-0.001	-0.059	-0.003	-0.003	-0.003	-0.055	-0.005	-0.005
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.004)	(0.000)	(0.005)	(0.002)	(0.002)	(0.002)	(0.005)	(0.003)	(0.003)
α_1	-0.030	-0.029	-0.030					-0.343	0.977	-0.765	0.856	0.656	0.678	-0.754	-0.006	-0.656
	(0.020)	(0.020)	(0.020)					(0.097)	(0.012)	(0.073)	(0.056)	(0.107)	(0.091)	(0.076)	(0.004)	(0.030)
α_2		0.045	0.045								0.060	0.269	0.247	0.025	0.778	0.597
		(0.019)	(0.019)								(0.020)	(0.121)	(0.108)	(0.028)	(0.091)	(0.044)
α_3			0.028											0.072	0.068	0.904
			(0.018)											(0.022)	(0.018)	(0.043)
β_1				-0.027	-0.031	-0.029		0.308	-1.011	0.738	-0.891	-0.689	-0.711	0.726	-0.028	0.644
				(0.017)	(0.020)	(0.021)		(0.096)	(0.023)	(0.073)	(0.055)	(0.110)	(0.094)	(0.074)	(0.022)	(0.032)
β_2					0.048	0.046			0.050	0.022		-0.207	-0.183		-0.763	-0.560
					(0.020)	(0.021)			(0.019)	(0.022)		(0.122)	(0.107)		(0.090)	(0.050)
β_3						0.021				0.066			-0.002			-0.858
						(0.021)				(0.017)			(0.003)			(0.057)
LL	2111.1	2115.1	2116.9	2110.9	2114.7	2115.3	2109.6	2111.9	2122.6	2118.8	2120.3	2121.0	2121.0	2120.8	2122.1	2132.4
AIC	-4.129	-4.131	-4.132	-4.129	-4.131	-4.131	-4.129	-4.129	-4.135	-4.132	-4.134	-4.134	-4.133	-4.134	-4.134	-4.139
BIC	-4.126	-4.126	-4.124	-4.126	-4.125	-4.123	-4.127	-4.124	-4.128	-4.123	-4.126	-4.124	-4.122	-4.124	-4.123	-4.126
Ljung-Box Q_{20}	53.05	43.00	39.04	53.41	41.57	39.56	56.38	50.49	31.34	30.88	31.49	30.22	30.12	29.63	26.70	23.61
p-value	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.05	0.06	0.05	0.07	0.07	0.08	0.14	0.26

Note. Standard Errors are given in parenthesis.

Table 3.110: Estimates of ARMA(p,q) model for ninth component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.039	-0.038	-0.039	-0.040	-0.040	-0.040	-0.040	0.000	-0.060	-0.054	-0.056	-0.001	-0.001	-0.026	-0.002	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.000)	(0.009)	(0.010)	(0.010)	(0.000)	(0.000)	(0.022)	(0.001)	(0.001)
α_1	0.026	0.026	0.026					0.992	-0.487	-0.339	-0.425	0.014	0.425	0.360	0.191	-0.136
	(0.021)	(0.021)	(0.021)					(0.004)	(0.181)	(0.238)	(0.214)	(0.007)	(0.138)	(0.555)	(0.467)	(0.357)
α_2		0.019	0.020								0.035	0.970	0.563	0.011	0.775	0.272
		(0.019)	(0.019)								(0.019)	(0.010)	(0.138)	(0.033)	(0.447	(0.132)
α_3			-0.017											-0.017	-0.010	0.818
			(0.019)											(0.023)	(0.024)	(0.228)
β_1				0.025	0.026	0.026		-0.983	0.514	0.365	0.451	0.004	-0.404	-0.335	-0.170	0.158
				(0.020)	(0.021)	(0.022)		(0.006)	(0.183)	(0.238)	(0.215)	(0.004)	(0.133)	(0.555)	(0.468)	(0.312)
β_2					0.020	0.020			0.037	0.029		-0.968	-0.547		-0.760	-0.238
					(0.021)	(0.020)			(0.021)	(0.024)		(0.009)	(0.143)		(0.457)	(0.119)
β_3						-0.017				-0.013			-0.021			-0.838
						(0.019)				(0.025)			(0.027)			(0.189)
LL	1442.2	1442.9	1444.1	1442.1	1442.8	1443.2	1441.0	1452.6	1443.3	1443.5	1443.2	1455.0	1453.0	1444.2	1449.3	1455.5
AIC	-3.720	-3.720	-3.720	-3.720	-3.720	-3.719	-3.720	-3.726	-3.719	-3.719	-3.719	-3.726	-3.724	-3.719	-3.722	-3.725
BIC	-3.716	-3.714	-3.712	-3.716	-3.714	-3.712	-3.718	-3.720	-3.712	-3.709	-3.712	-3.716	-3.713	-3.710	-3.711	-3.712
Ljung-Box Q_{20}	30.34	28.48	28.10	30.50	28.58	28.66	33.26	19.18	28.01	28.19	27.95	19.21	18.36	28.29	19.58	10.64
p-value	0.06	0.10	0.11	0.06	0.10	0.09	0.03	0.51	0.11	0.10	0.11	0.51	0.56	0.10	0.48	0.95

Note. Standard Errors are given in parenthesis.

Table 3.111: Estimates of ARMA(p,q) model for tenth component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.051	-0.049	-0.049	-0.052	-0.052	-0.052	-0.052	-0.002	-0.082	-0.098	-0.081	-0.002	-0.002	-0.088	-0.002	-0.003
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.001)	(0.018)	(0.006)	(0.018)	(0.001)	(0.003)	(0.007)	(0.001)	(0.002)
α_1	0.020	0.020	0.020	-0.052				0.971	-0.581	-0.898	-0.595	0.129	0.165	-0.787	0.176	-0.830
	(0.021)	(0.021)	(0.021)	(0.003)				(0.014)	(0.346)	(0.070)	(0.316)	(0.090)	(2.301)	(0.117)	(0.146)	(0.012)
α_2		0.041	0.041								0.042	0.838	0.804	0.057	0.804	0.796
		(0.018)	(0.018)								(0.025)	(0.087)	(2.249)	(0.028)	(0.123)	(0.020)
α_3			0.000											0.030	-0.011	0.968
			(0.019)											(0.024)	(0.031)	(0.013)
β_1				0.019	0.019	0.019		-0.950	0.600	0.919	0.617	-0.122	-0.151	0.809	-0.161	0.843
				(0.021	(0.022)	(0.021)		(0.017)	(0.346)	(0.073)	(0.316)	(0.098)	(2.194)	(0.116)	(0.139)	(0.017)
β_2					0.040	0.040			0.043	0.057		-0.816	-0.779		-0.780	-0.766
					(0.018)	(0.018)			(0.022)	(0.028)		(0.092)	(2.388)		(0.133)	(0.028)
β_3						0.001				0.033			-0.011			-0.952
						(0.002)				(0.018)			(0.302)			(0.018)
LL	1547.6	1550.8	1551.2	1547.4	1550.1	1550.1	1546.8	1554.4	1550.6	1551.9	1551.6	1561.7	1561.9	1552.8	1561.6	1567.2
AIC	-3.784	-3.786	-3.785	-3.784	-3.785	-3.785	-3.784	-3.788	-3.785	-3.785	-3.786	-3.791	-3.791	-3.786	-3.790	-3.793
BIC	-3.781	-3.780	-3.778	-3.780	-3.780	-3.777	-3.783	-3.782	-3.778	-3.776	-3.778	-3.782	-3.779	-3.776	-3.779	-3.780
Ljung-Box Q_{20}	33.96	26.88	26.74	34.21	27.43	27.36	36.77	15.91	28.47	27.36	28.48	14.81	14.28	26.88	14.24	12.96
p-value	0.03	0.14	0.14	0.02	0.12	0.13	0.01	0.72	0.10	0.13	0.10	0.79	0.82	0.14	0.82	0.88

Note. Standard Errors are given in parenthesis.

Table 3.112: Estimates of ARMA(p,q) model for eleventh component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.031	-0.031	-0.031	-0.031	-0.031	-0.031	-0.031	-0.027	0.000	-0.057	0.000	-0.095	0.000	-0.052	-0.102	-0.017
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.018)	(0.000)	(0.009)	(0.000)	(0.009)	(0.000)	(0.028)	(0.010)	(0.000)
α_1	-0.010	-0.010	-0.010					0.120	0.998	-0.869	0.987	-1.127	0.089	-0.699	-1.450	1.314
	(0.024)	(0.024)	(0.024)					(0.581)	(0.000)	(0.249)	(0.000)	(0.013)	(0.000)	(0.765)	(0.099)	(0.000)
α_2		-0.002	-0.002								0.008	-0.977	0.905	-0.008	-0.900	-1.411
		(0.023)	(0.023)								(0.000)	(0.030)	(0.000)	(0.083)	(0.065)	(0.000)
α_3			0.007											0.009	0.008	0.548
			(0.023)											(0.070)	(0.019)	(0.000)
β_1				-0.010	-0.010	-0.009		-0.130	-1.012	0.859	-1.005	1.119	-0.105	0.689	1.443	-1.328
				(0.049)	(0.021)	(0.024)		(0.583)	(0.000)	(0.250)	(0.000)	(0.014)	(0.000)	(0.744)	(0.100)	(0.000)
β_2					-0.001	-0.002			0.007	-0.010		0.971	-0.917		0.895	1.441
					(0.003)	(0.003)			(0.000)	(0.015)		(0.040)	(0.000)		(0.066)	(0.000)
β_3						0.007				0.005			0.013			-0.568
						(0.023)				(0.010)			(0.000)			(0.000)
LL	1205.2	1205.2	1205.3	1205.1	1205.1	1205.2	1205.0	1205.2	1219.4	1205.5	1222.7	1208.6	1220.8	1205.6	1208.3	1239.7
AIC	-3.575	-3.574	-3.574	-3.575	-3.574	-3.573	-3.575	-3.574	-3.582	-3.573	-3.584	-3.575	-3.582	-3.573	-3.574	-3.593
BIC	-3.571	-3.568	-3.566	-3.571	-3.568	-3.566	-3.573	-3.568	-3.575	-3.564	-3.577	-3.566	-3.571	-3.564	-3.563	-3.580
Ljung-Box Q_{20}	28.55	28.49	28.32	28.58	28.53	28.39	28.57	28.52	29.22	28.14	28.92	28.74	28.60	27.81	26.51	29.09
p-value	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.08	0.11	0.09	0.09	0.10	0.11	0.15	0.09

Note. Standard Errors are given in parenthesis.

Table 3.113: Estimates of ARMA(p,q) model for twelfth component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.062	-0.060	-0.056	-0.062	-0.062	-0.062	-0.062	-0.002	-0.002	-0.002	-0.018	-0.020	-0.026	-0.033	-0.031	-0.011
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.001)	(0.001)	(0.006)	(0.006)	(0.009)	(0.037)	(0.020)	(0.027)
α_1	-0.002	-0.003	-0.006					0.972	0.966	0.966	0.643	0.843	0.394	0.379	0.298	0.764
	(0.021)	(0.021)	(0.021)					(0.015)	(0.022)	(0.023)	(0.090)	(0.167)	(0.160)	(0.641)	(0.297)	(0.359)
α_2		0.030	0.030								0.058	-0.169	0.186	0.031	0.130	-0.642
		(0.022)	(0.022)								(0.021)	(0.143)	(0.161)	(0.023)	(0.187)	(0.249)
α_3			0.072											0.059	0.063	0.696
			(0.020)											(0.038)	(0.024)	(0.325)
β_1				-0.002	-0.007	-0.006		-0.949	-0.978	-0.977	-0.651	-0.858	-0.403	-0.388	-0.307	-0.771
				(0.034)	(0.022)	(0.019)		(0.017)	(0.029)	(0.031)	(0.089)	(0.171)	(0.160)	(0.643)	(0.297)	(0.369)
β_2					0.030	0.030			0.038	0.036		0.228	-0.156		-0.100	0.683
					(0.022)	(0.021)			(0.021)	(0.023)		(0.139)	(0.162)		(0.186)	(0.250)
β_3						0.065				0.002			0.054			-0.661
						(0.018)				(0.017)			(0.020)			(0.353)
LL	583.8	588.5	598.2	583.6	585.1	592.7	583.6	597.5	599.8	599.8	595.6	596.4	598.9	599.0	599.2	602.5
AIC	-3.194	-3.196	-3.202	-3.194	-3.194	-3.198	-3.195	-3.202	-3.203	-3.202	-3.200	-3.200	-3.201	-3.202	-3.201	-3.203
BIC	-3.190	-3.191	-3.194	-3.190	-3.189	-3.191	-3.193	-3.196	-3.195	-3.193	-3.193	-3.191	-3.190	-3.192	-3.190	-3.189
Ljung-Box Q_{20}	65.44	60.54	38.58	65.14	59.94	39.60	64.81	29.49	25.56	25.53	37.18	38.26	31.41	33.49	32.14	16.96
p-value	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.08	0.18	0.18	0.01	0.01	0.05	0.03	0.04	0.66

Note. Standard Errors are given in parenthesis.

Table 3.114: Estimates of ARMA(p,q) model for thirteenth component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.067	-0.065	-0.063	-0.067	-0.067	-0.067	-0.067	-0.001	-0.001	-0.061	-0.001	0.000	-0.002	-0.077	-0.001	-0.004
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.001)	(0.035)	(0.001)	(0.000)	(0.002)	(0.023)	(0.001)	(0.002)
α_1	0.007	0.007	0.006					0.980	0.979	0.094	0.969	1.812	-0.007	-0.221	1.283	-0.249
	(0.019)	(0.019)	(0.019)					(0.012)	(0.013)	(0.510)	(0.022)	(0.071)	(0.015)	(0.344)	(0.511)	(0.039)
α_2		0.032	0.032								0.014	-0.814	0.970	0.034	-0.275	0.252
		(0.019)	(0.019)								(0.018)	(0.071)	(0.014)	(0.019)	(0.507)	(0.041)
α_3			0.031											0.038	-0.018	0.941
			(0.018)											(0.019)	(0.018)	(0.049)
β_1				0.006	0.005	0.006		-0.964	-0.976	-0.088	-0.967	-1.805	0.010	0.227	-1.281	0.258
				(0.029)	(0.022)	(0.017)		(0.013)	(0.023)	(0.509)	(0.012)	(0.077)	(0.026)	(0.345)	(0.510)	(0.045)
β_2					0.032	0.031			0.014	0.031		0.810	-0.952		0.302	-0.236
					(0.019)	(0.019)			(0.019)	(0.019)		(0.075)	(0.015)		(0.494)	(0.046)
β_3						0.029				0.026			0.011			-0.914
						(0.017)				(0.024)			(0.017)			(0.055)
LL	360.1	361.9	363.5	360.0	361.7	363.2	359.9	367.2	367.5	363.3	368.9	372.1	368.9	363.6	369.4	370.6
AIC	-3.057	-3.058	-3.058	-3.057	-3.058	-3.058	-3.058	-3.061	-3.060	-3.057	-3.061	-3.063	-3.060	-3.057	-3.060	-3.061
BIC	-3.053	-3.052	-3.051	-3.053	-3.052	-3.050	-3.056	-3.055	-3.053	-3.048	-3.054	-3.053	-3.049	-3.048	-3.049	-3.047
Ljung-Box Q_{20}	32.43	27.78	23.18	32.25	27.74	23.58	32.87	16.40	16.08	23.65	16.00	16.17	15.60	23.54	15.20	13.51
p-value	0.04	0.11	0.28	0.04	0.12	0.26	0.03	0.69	0.71	0.26	0.72	0.71	0.74	0.26	0.76	0.85

Note. Standard Errors are given in parenthesis.

Table 3.115: Estimates of ARMA(p,q) model for fourteenth component of $\mathbf{U_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.039	-0.039	-0.037	-0.038	-0.038	-0.038	-0.038	-0.036	-0.066	-0.054	-0.069	-0.116	-0.027	-0.053	-0.025	0.000
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.007)	(0.009)	(0.008)	(0.015)	(0.037)	(0.010)	(0.012)	(0.000)
α_1	-0.004	-0.004	-0.004					0.072	-0.708	-0.416	-0.755	-1.317	-0.039	-0.433	-0.047	-0.305
	(0.019)	(0.019)	(0.019)					(0.038)	(0.090)	(0.178)	(0.091)	(0.134)	(0.578)	(0.192)	(0.147)	(0.179)
α_2		-0.007	-0.007								-0.037	-0.697	0.335	-0.009	0.342	0.609
		(0.019	(0.019)								(0.020)	(0.121)	(0.391)	(0.019)	(0.193)	(0.084)
α_3			0.063											0.060	0.066	0.684
			(0.019)											(0.019)	(0.019)	(0.147)
β_1				-0.004	-0.003	-0.001		-0.077	0.708	0.414	0.753	1.309	0.037	0.431	0.044	0.298
				(0.107)	(0.010)	(0.006)		(0.043)	(0.090)	(0.178)	(0.089)	(0.140)	(0.577)	(0.192)	(0.146)	(0.187)
β_2					-0.007	-0.012			-0.035	-0.010		0.661	-0.348		-0.354	-0.632
					(0.019	(0.019)			(0.018)	(0.022)		(0.126)	(0.400)		(0.197)	(0.084)
β_3						0.062				0.057			0.063			-0.641
						(0.019)				(0.019)			(0.019)			(0.157)
LL	195.8	195.9	202.4	195.8	195.8	202.3	195.7	195.8	199.7	203.6	200.1	203.4	203.8	203.7	204.2	209.5
AIC	-2.957	-2.956	-2.959	-2.957	-2.956	-2.959	-2.957	-2.956	-2.958	-2.960	-2.958	-2.959	-2.959	-2.960	-2.959	-2.962
BIC	-2.953	-2.950	-2.952	-2.953	-2.950	-2.952	-2.955	-2.950	-2.950	-2.950	-2.951	-2.950	-2.948	-2.950	-2.948	-2.949
Ljung-Box Q_{20}	29.74	29.66	16.09	29.80	29.77	16.02	30.02	29.73	21.07	13.16	20.59	14.40	12.63	12.73	12.07	11.35
p-value	0.07	0.08	0.71	0.07	0.07	0.72	0.07	0.07	0.39	0.87	0.42	0.81	0.89	0.89	0.91	0.94

Note. Standard Errors are given in parenthesis.

Table 3.116: Estimates of ARMA(p,q) model for fifteenth component of $\mathbf{U_{2,t}}$

Model	**AR(1)**	**AR(2)**	**AR(3)**	**MA(1)**	**MA(2)**	**MA(3)**	**ARMA(0,0)**	**ARMA(1,1)**	**ARMA(1,2)**	**ARMA(1,3)**	**ARMA(2,1)**	**ARMA(2,2)**	**ARMA(2,3)**	**ARMA(3,1)**	**ARMA(3,2)**	**ARMA(3,3)**
α_0	-0.045	-0.044	-0.043	-0.045	-0.045	-0.045	-0.045	-0.089	-0.001	-0.001	-0.005	-0.004	-0.004	-0.002	-0.002	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.006)	(0.001)	(0.001)	(0.002)	(0.002)	(0.003)	(0.001)	(0.001)	(0.001)
α_1	-0.001	-0.001	-0.001					-0.991	0.967	0.969	0.856	0.238	0.132	0.933	0.599	0.610
	(0.020)	(0.020)	(0.020)					(0.005)	(0.020)	(0.018)	(0.054)	(0.257)	(0.315)	(0.031)	(0.282)	(0.261)
α_2		0.031	0.031								0.039	0.678	0.770	0.033	0.356	0.358
		(0.018	(0.018)								(0.019)	(0.272)	(0.322)	(0.037)	(0.297	(0.248)
α_3			0.023											-0.004	0.003	-0.009
			(0.019											(0.036)	(0.024)	0.010
β_1				-0.001	-0.002	-0.003		0.989	-0.974	-0.976	-0.862	-0.232	-0.138	-0.941	-0.606	-0.616
				(0.001	(0.011	(0.014		(0.005)	(0.028)	(0.026)	(0.050)	(0.270)	(0.312)	(0.024)	(0.293)	0.262
β_2					0.029	0.028			0.027	0.032		-0.643	-0.740		-0.326	-0.328
					(0.016)	(0.017)			(0.020)	(0.022)		(0.284)	(0.326)		(0.286)	(0.250)
β_3						0.022				-0.005			0.022			0.012
						(0.019)				(0.008)			(0.025)			(0.014)
LL	1041.4	1043.7	1044.7	1041.3	1042.8	1043.6	1041.3	1043.3	1051.6	1051.7	1048.9	1050.8	1051.4	1052.0	1053.0	1053.0
AIC	-3.474	-3.475	-3.475	-3.474	-3.475	-3.474	-3.475	-3.475	-3.479	-3.479	-3.478	-3.478	-3.478	-3.479	-3.479	-3.478
BIC	-3.471	-3.470	-3.468	-3.471	-3.469	-3.467	-3.473	-3.469	-3.472	-3.469	-3.470	-3.469	-3.467	-3.470	-3.468	-3.465
Ljung-Box Q_{20}	39.88	34.85	31.44	39.89	35.23	32.28	39.81	40.13	15.63	15.63	17.20	16.71	15.60	15.64	15.63	15.62
p-value	0.01	0.02	0.05	0.01	0.02	0.04	0.01	0.00	0.74	0.74	0.64	0.67	0.74	0.74	0.74	0.74

Note. Standard Errors are given in parenthesis.

3.4.8 Estimates of ARMA(p,q) model for $\mathbf{Z_{2,t}}$ from Second Round of SVD

Table 3.117: Estimates of ARMA(p,q) model for first component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.059	-0.058	-0.058	-0.060	-0.060	-0.060	-0.060	0.000	0.000	0.000	-0.004	-0.001	-0.001	0.000	0.000	0.000
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.000)	(0.000)	(0.000)	(0.007)	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)
α_1	0.012	0.012	0.012					0.998	0.998	0.998	0.915	0.106	0.099	0.998	1.362	1.364
	(0.020)	(0.020)	(0.020)					(0.001)	(0.001)	(0.001)	(0.116)	(0.063)	(0.123)	(0.017)	(0.082)	(0.075)
α_2		0.015	0.015								0.017	0.880	0.887	-0.005	-0.366	-0.414
		(0.020)	(0.020)								(0.019)	(0.066)	(0.120)	(0.023)	(0.079)	(0.096)
α_3			0.014											0.005	0.003	0.050
			(0.015)											(0.031)	(0.078)	(0.107)
β_1				0.012	0.012	0.011		-0.992	-0.998	-0.998	-0.907	-0.096	-0.092	-0.989	-1.362	-1.363
				(0.021)	(0.019)	(0.016)		(0.002)	(0.018)	(0.071)	(0.123)	(0.067)	(0.068)	(0.004)	(0.106)	(0.080)
β_2					0.015	0.015			0.006	0.003		-0.866	-0.873		0.367	0.416
					(0.021)	(0.018)			(0.018)	(0.057)		(0.071)	(0.130)		(0.105)	(0.089)
β_3						0.013				0.003			0.005			-0.047
						(0.014)				(0.015)			(0.094)			(0.106)
LL	555.9	557.1	557.5	555.9	556.3	556.6	555.7	575.5	575.6	575.6	558.3	564.9	564.9	570.4	576.5	576.5
AIC	-3.177	-3.177	-3.177	-3.177	-3.177	-3.176	-3.178	-3.188	-3.188	-3.187	-3.177	-3.181	-3.180	-3.184	-3.187	-3.187
BIC	-3.173	-3.172	-3.169	-3.173	-3.171	-3.169	-3.176	-3.183	-3.180	-3.178	-3.170	-3.171	-3.169	-3.175	-3.176	-3.174
Ljung-Box Q_{20}	43.24	41.99	40.54	43.28	42.10	40.70	44.48	25.70	25.50	25.39	27.60	25.57	25.46	25.47	25.21	25.25
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.18	0.19	0.12	0.18	0.18	0.18	0.19	0.19

Note. Standard Errors are given in parenthesis.

Table 3.118: Estimates of ARMA(p,q) model for second component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.028	-0.028	-0.027	-0.029	-0.029	-0.029	-0.029	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	0.000	-0.001
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)
α_1	0.048	0.047	0.047					0.985	0.990	0.989	1.024	0.052	0.042	0.997	0.064	-0.972
	(0.020)	(0.020)	(0.020)					(0.036)	(0.010)	(0.016)	(0.021)	(0.034)	(0.016)	(0.050)	(0.023)	(0.034)
α_2		0.015	0.013								-0.028	0.935	0.947	-0.030	0.940	0.974
		(0.019)	(0.019)								(0.019)	(0.031)	(0.020)	(0.032)	(0.022)	(0.017)
α_3			0.034											0.012	-0.019	0.969
			(0.019)											(0.030)	(0.017)	(0.034)
β_1				0.047	0.047	0.045		-0.969	-0.951	-0.950	-0.987	-0.029	-0.002	-0.958	-0.024	0.988
				(0.019	(0.020	(0.020)		(0.057)	(0.021)	(0.030)	(0.008)	(0.033)	(0.001)	(0.051)	(0.019)	(0.034)
β_2					0.013	0.013			-0.026	-0.031		-0.935	-0.946		-0.939	-0.951
					(0.023)	(0.029)			(0.020)	(0.022)		(0.028)	(0.020)		(0.022)	(0.029)
β_3						0.031				0.006			-0.020			-0.961
						(0.019)				(0.009)			(0.015)			(0.034)
LL	2360.5	2360.8	2363.0	2360.2	2360.5	2362.1	2356.6	2373.2	2374.2	2374.3	2375.5	2377.6	2378.2	2373.1	2378.3	2380.7
AIC	-4.282	-4.282	-4.282	-4.282	-4.282	-4.282	-4.280	-4.289	-4.289	-4.289	-4.290	-4.291	-4.291	-4.288	-4.291	-4.291
BIC	-4.278	-4.276	-4.275	-4.278	-4.276	-4.274	-4.278	-4.284	-4.282	-4.279	-4.283	-4.281	-4.279	-4.279	-4.279	-4.278
Ljung-Box Q_{20}	54.23	51.82	45.91	55.13	53.01	48.16	70.75	25.24	23.22	22.96	24.25	23.54	22.19	23.16	21.95	25.27
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.28	0.29	0.23	0.26	0.33	0.28	0.34	0.19

Note. Standard Errors are given in parenthesis.

Table 3.119: Estimates of ARMA(p,q) model for third component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.025	-0.025	-0.025	-0.027	-0.027	-0.027	-0.027	-0.021	-0.001	-0.037	-0.001	-0.001	-0.001	-0.012	-0.016	-0.017
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.010)	(0.001)	(0.006)	(0.000)	(0.001)	(0.001)	(0.187)	(0.002)	(0.002)
α_1	0.068	0.068	0.067					0.241	0.953	-0.354	1.023	0.864	0.551	0.603	1.307	1.272
	(0.036)	(0.036)	(0.036)					(0.336)	(0.024)	(0.162)	(0.042)	(0.778)	(0.849)	(7.309)	(0.099)	(0.079)
α_2		0.010	0.009								-0.052	0.101	0.401	-0.027	-0.952	-0.904
		(0.017)	(0.017)								(0.037)	(0.737)	(0.813)	(0.468)	(0.094)	(0.068)
α_3			0.009											0.000	0.045	0.009
			(0.016)											(0.004)	(0.034)	(0.009)
β_1				0.067	0.067	0.068		-0.174	-0.887	0.422	-0.958	-0.799	-0.486	-0.535	-1.241	-1.206
				(0.036)	(0.037)	(0.037)		(0.356)	(0.045)	(0.167)	(0.017)	(0.764)	(0.853)	(7.316)	(0.094)	(0.090)
β_2					0.013	0.014			-0.049	0.038		-0.151	-0.428		0.871	0.825
					(0.020)	(0.025)			(0.036)	(0.026)		(0.708)	(0.772)		(0.075)	(0.081)
β_3						0.012				0.020			-0.018			0.035
						(0.022)				(0.015)			(0.036)			(0.041)
LL	1102.3	1102.4	1102.9	1101.8	1102.1	1102.4	1094.4	1102.5	1104.9	1102.8	1105.4	1105.6	1105.7	1103.1	1106.6	1106.6
AIC	-3.512	-3.511	-3.511	-3.511	-3.511	-3.510	-3.507	-3.511	-3.512	-3.510	-3.512	-3.512	-3.511	-3.510	-3.512	-3.511
BIC	-3.508	-3.506	-3.503	-3.508	-3.505	-3.503	-3.506	-3.506	-3.505	-3.501	-3.505	-3.503	-3.500	-3.501	-3.501	-3.498
Ljung-Box Q_{20}	23.39	23.15	22.57	23.74	23.22	22.70	41.01	23.07	19.85	22.36	19.93	19.92	19.81	22.64	18.68	18.84
p-value	0.27	0.28	0.31	0.25	0.28	0.30	0.00	0.29	0.47	0.32	0.46	0.46	0.47	0.31	0.54	0.53

Note. Standard Errors are given in parenthesis.

Table 3.120: Estimates of ARMA(p,q) model for fourth component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.097	-0.095	-0.091	-0.103	-0.103	-0.103	-0.103	0.000	0.000	0.000	0.000	-0.001	-0.001	0.000	0.000	-0.001
	(0.003)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
α_1	0.061	0.060	0.059					0.996	0.996	0.996	1.018	0.317	0.377	1.017	0.312	-0.670
	(0.020)	(0.021)	(0.021)					(0.002)	(0.002)	(0.002)	(0.021)	(0.120)	(0.141)	(0.020)	(0.193)	(0.172)
α_2		0.017	0.014								-0.021	0.678	0.618	-0.045	0.674	0.870
		(0.023)	(0.023)								(0.021)	(0.120)	(0.140)	(0.033)	(0.229)	(0.067)
α_3			0.045											0.025	0.010	0.791
			(0.022)											(0.024)	(0.042)	(0.114)
β_1				0.060	0.059	0.060		-0.984	-0.963	-0.963	-0.987	-0.277	-0.344	-0.985	-0.279	0.704
				(0.020)	(0.021)	(0.021)		(0.004)	(0.023)	(0.020)	(0.003)	(0.115)	(0.137)	(0.005)	(0.203)	(0.176)
β_2					0.015	0.013			-0.022	-0.045		-0.699	-0.646		-0.698	-0.861
					(0.021)	(0.032)			(0.022)	(0.033)		(0.113)	(0.131)		(0.202)	(0.058)
β_3						0.041				0.024			0.013			-0.796
						(0.019)				(0.024)			(0.017)			(0.121)
LL	1720.5	1721.0	1724.4	1720.0	1720.3	1723.3	1714.0	1764.4	1765.1	1766.0	1766.3	1767.7	1767.9	1764.1	1767.4	1769.9
AIC	-3.890	-3.890	-3.891	-3.890	-3.890	-3.891	-3.887	-3.916	-3.916	-3.916	-3.917	-3.917	-3.917	-3.915	-3.916	-3.917
BIC	-3.886	-3.884	-3.884	-3.886	-3.884	-3.883	-3.885	-3.911	-3.909	-3.907	-3.910	-3.908	-3.906	-3.906	-3.905	-3.904
Ljung-Box Q_{20}	112.47	106.02	89.08	115.07	109.71	94.63	150.46	32.18	30.59	28.87	30.68	27.03	26.85	29.36	27.11	23.76
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.06	0.09	0.06	0.13	0.14	0.08	0.13	0.25

Note. Standard Errors are given in parenthesis.

Table 3.121: Estimates of ARMA(p,q) model for fifth component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.071	-0.070	-0.066	-0.073	-0.073	-0.073	-0.073	-0.001	-0.002	-0.002	-0.002	-0.002	-0.003	-0.002	-0.002	-0.002
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
α_1	0.016	0.016	0.015					0.980	0.977	0.974	0.939	0.095	0.140	0.931	1.049	0.471
	(0.026)	(0.026)	(0.026)					(0.008)	(0.010)	(0.012)	(0.029)	(0.030)	(0.047)	(0.031)	(0.393)	(0.202)
α_2		0.025	0.024								0.039	0.872	0.820	0.010	-0.103	0.792
		(0.020	(0.020								(0.026)	(0.029)	(0.047)	(0.034)	(0.383)	(0.060)
α_3			0.047											0.034	0.031	-0.293
			(0.021											(0.022)	(0.022)	(0.171)
β_1				0.015	0.014	0.011		-0.949	-0.985	-0.981	-0.946	-0.059	-0.147	-0.939	-1.058	-0.478
				(0.025)	(0.027)	(0.023)		(0.012)	(0.028)	(0.029)	(0.015)	(0.030)	(0.054)	(0.019)	(0.394)	(0.196)
β_2					0.022	0.018			0.041	0.009		-0.854	-0.808		0.113	-0.790
					(0.019)	(0.019)			(0.027)	(0.037)		(0.029)	(0.044)		(0.379)	(0.053)
β_3						0.040				0.034			0.054			0.340
						(0.019)				(0.023)			(0.024)			(0.165)
LL	-172.3	-171.3	-167.7	-172.5	-171.7	-168.8	-172.9	-139.0	-136.6	-134.8	-134.2	-135.2	-130.9	-132.6	-132.5	-130.3
AIC	-2.731	-2.731	-2.733	-2.731	-2.731	-2.732	-2.731	-2.751	-2.752	-2.752	-2.753	-2.752	-2.754	-2.754	-2.753	-2.754
BIC	-2.727	-2.726	-2.725	-2.727	-2.725	-2.725	-2.730	-2.745	-2.744	-2.743	-2.746	-2.743	-2.743	-2.744	-2.742	-2.741
Ljung-Box Q_{20}	147.19	135.69	115.41	147.40	138.20	122.99	155.15	40.22	35.82	31.38	36.16	38.85	30.95	31.68	31.25	28.83
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.01	0.01	0.06	0.05	0.05	0.09

Note. Standard Errors are given in parenthesis.

Table 3.122: Estimates of ARMA(p,q) model for sixth component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.046	-0.044	-0.045	-0.043	-0.043	-0.043	-0.043	-0.072	-0.074	-0.076	-0.075	-0.103	-0.001	-0.075	-0.005	-0.005
	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.016)	(0.010)	(0.029)	(0.014)	(0.728)	(0.001)	(0.021)	(0.003)	(0.003)
α_1	-0.073	-0.070	-0.070					-0.673	-0.739	-0.766	-0.749	-1.137	0.216	-0.737	0.075	-0.207
	(0.056)	(0.056)	(0.057)					(0.325)	(0.180)	(0.715)	(0.242)	(9.929)	(0.130)	(0.266)	(0.144)	(0.592)
α_2		0.038	0.037								-0.015	-0.288	0.750	-0.011	0.762	0.820
		(0.046)	(0.044)								(0.070)	(7.385)	(0.127)	(0.144)	(0.104)	(0.084)
α_3			-0.012											0.004	0.047	0.283
			(0.037)											(0.064)	(0.066)	(0.548)
β_1				-0.068	-0.070	-0.070		0.608	0.671	0.697	0.680	1.069	-0.289	0.669	-0.146	0.141
				(0.051)	(0.057)	(0.058)		(0.370)	(0.200)	(0.871)	(0.244)	(10.017)	(0.152)	(0.217)	(0.126)	(0.636)
β_2					0.039	0.040			-0.015	-0.012		0.250	-0.693		-0.711	-0.780
					(0.042)	(0.043)			(0.067)	(0.587)		(6.936)	(0.169)		(0.139)	(0.092)
β_3						-0.013				0.009			0.033			-0.225
						(0.036)				(0.606)			(0.062)			(0.577)
LL	741.3	743.7	745.0	740.6	743.2	743.5	732.4	745.2	745.3	745.4	745.5	745.6	751.1	746.7	749.5	749.8
AIC	-3.291	-3.291	-3.292	-3.290	-3.291	-3.291	-3.286	-3.292	-3.292	-3.291	-3.292	-3.291	-3.294	-3.292	-3.293	-3.293
BIC	-3.287	-3.286	-3.284	-3.286	-3.286	-3.283	-3.284	-3.287	-3.284	-3.282	-3.284	-3.282	-3.283	-3.283	-3.282	-3.280
Ljung-Box Q_{20}	28.89	22.15	22.02	30.43	22.97	22.62	46.97	19.82	20.01	19.68	19.97	19.92	11.20	19.87	12.20	12.58
p-value	0.09	0.33	0.34	0.06	0.29	0.31	0.00	0.47	0.46	0.48	0.46	0.46	0.94	0.47	0.91	0.89

Note. Standard Errors are given in parenthesis.

Table 3.123: Estimates of ARMA(p,q) model for seventh component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.045	-0.047	-0.048	-0.045	-0.045	-0.045	-0.045	-0.025	-0.034	-0.041	-0.025	-0.014	-0.001	-0.028	-0.019	-0.029
	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.011)	(0.010)	(0.012)	(0.008)	(0.004)	(0.001)	(0.006)	(0.016)	(0.008)
α_1	-0.004	-0.005	-0.005					0.454	0.242	0.084	0.467	1.434	1.215	0.403	0.959	0.382
	(0.024)	(0.024)	(0.024)					(0.231)	(0.217)	(0.257)	(0.161)	(0.118)	(0.336)	(0.132)	(0.859)	(0.251)
α_2		-0.031	-0.031								-0.028	-0.751	-0.240	-0.029	-0.387	0.605
		(0.022)	(0.022)								(0.024)	(0.138)	(0.360)	(0.023)	(0.583)	(0.089)
α_3			-0.016											-0.002	0.009	-0.621
			(0.020)											(0.006)	(0.036)	(0.203)
β_1				-0.005	-0.006	-0.006		-0.471	-0.247	-0.089	-0.472	-1.450	-1.226	-0.409	-0.966	-0.391
				(0.018)	(0.015)	(0.019)		(0.247)	(0.215)	(0.254)	(0.158)	(0.123)	(0.333)	(0.129)	(0.861)	(0.257)
β_2					-0.032	-0.031			-0.032	-0.030		0.746	0.210		0.361	-0.639
					(0.022)	(0.023)			(0.022)	(0.023)		(0.129)	(0.347)		(0.585)	(0.087)
β_3						-0.014				-0.012			0.049			0.624
						(0.021)				(0.014)			(0.022)			(0.203)
LL	1094.7	1096.4	1098.3	1094.6	1096.2	1096.5	1094.6	1094.9	1096.4	1096.6	1097.3	1099.9	1104.4	1098.8	1099.0	1100.7
AIC	-3.507	-3.507	-3.508	-3.507	-3.507	-3.507	-3.508	-3.507	-3.507	-3.506	-3.507	-3.508	-3.510	-3.508	-3.507	-3.508
BIC	-3.503	-3.502	-3.501	-3.503	-3.502	-3.499	-3.506	-3.501	-3.499	-3.497	-3.500	-3.499	-3.499	-3.498	-3.496	-3.495
Ljung-Box Q_{20}	29.52	27.55	26.94	29.48	27.54	27.27	29.33	29.38	27.37	27.33	27.29	22.40	18.61	26.66	25.47	22.72
p-value	0.08	0.12	0.14	0.08	0.12	0.13	0.08	0.08	0.13	0.13	0.13	0.32	0.55	0.15	0.18	0.30

Note. Standard Errors are given in parenthesis.

Table 3.124: Estimates of ARMA(p,q) model for eight component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.036	-0.034	-0.033	-0.035	-0.035	-0.035	-0.035	-0.045	-0.001	-0.061	-0.003	-0.002	-0.090	-0.058	-0.083	-0.005
	(0.003	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.004)	(0.000)	(0.005)	(0.002)	(0.002)	(0.020)	(0.005)	(0.015)	(0.003)
α_1	-0.030	-0.029	-0.030					-0.318	0.977	-0.763	0.866	0.662	-1.167	-0.776	-1.112	-0.657
	(0.019)	(0.019)	(0.019)					(0.090)	(0.011)	(0.061)	(0.060)	(0.095)	(0.301)	(0.067)	(0.197)	(0.030)
α_2		0.043	0.043								0.056	0.269	-0.461	0.023	-0.357	0.601
		(0.018)	(0.018)								(0.019)	(0.103)	(0.266)	(0.025)	(0.193)	(0.043)
α_3			0.029											0.077	0.067	0.910
			(0.018)											(0.020)	(0.019)	(0.046)
β_1				-0.027	-0.031	-0.029		0.283	-1.010	0.738	-0.899	-0.695	1.141	0.749	1.088	0.641
				(0.017)	(0.019)	(0.019)		(0.088)	(0.022)	(0.062)	(0.058)	(0.097)	(0.300)	(0.063)	(0.197)	(0.031)
β_2					0.047	0.044			0.047	0.020		-0.211	0.476		0.377	-0.568
					(0.019)	(0.019)			(0.018)	(0.029)		(0.104)	(0.258)		(0.190)	(0.050)
β_3						0.022				0.070			0.061			-0.865
						(0.017)				(0.020)			(0.018)			(0.061)
LL	1677.2	1680.7	1682.5	1677.0	1680.5	1681.2	1675.7	1677.9	1686.7	1686.4	1684.5	1685.2	1686.7	1688.4	1689.7	1699.4
AIC	-3.864	-3.865	-3.866	-3.864	-3.865	-3.865	-3.863	-3.864	-3.868	-3.868	-3.867	-3.867	-3.867	-3.869	-3.869	-3.874
BIC	-3.860	-3.860	-3.858	-3.860	-3.860	-3.857	-3.862	-3.858	-3.861	-3.858	-3.860	-3.857	-3.856	-3.859	-3.858	-3.861
Ljung-Box Q_{20}	54.22	45.05	41.24	54.63	43.57	41.87	58.53	51.80	37.05	29.47	36.99	35.67	24.28	27.53	23.54	26.19
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	0.01	0.02	0.23	0.12	0.26	0.16

Note. Standard Errors are given in parenthesis.

Table 3.125: Estimates of ARMA(p,q) model for ninth component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.048	-0.047	-0.046	-0.048	-0.048	-0.048	-0.048	-0.062	-0.002	-0.087	-0.003	-0.003	-0.004	-0.075	-0.115	-0.008
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.031)	(0.001)	(0.012)	(0.001)	(0.001)	(0.002)	(0.010)	(0.026)	(0.003)
α_1	-0.009	-0.009	-0.010					-0.291	0.963	-0.830	0.907	0.622	0.068	-0.633	-0.982	-0.583
	(0.017)	(0.017)	(0.017)					(0.646)	(0.013)	(0.178)	(0.029)	(0.141)	(0.131)	(0.153)	(0.235)	(0.151)
α_2		0.020	0.020								0.030	0.310	0.846	0.015	-0.467	0.542
		(0.018)	(0.018)								(0.017)	(0.137)	(0.131)	(0.023)	(0.259)	(0.048)
α_3			0.014											0.032	0.021	0.872
			(0.019)											(0.019)	(0.019)	(0.121)
β_1				-0.009	-0.010	-0.009		0.279	-0.976	0.821	-0.920	-0.632	-0.081	0.624	0.974	0.586
				(0.015)	(0.017)	(0.016)		(0.643)	(0.022)	(0.179)	(0.024)	(0.150)	(0.132)	(0.152)	(0.236)	(0.169)
β_2					0.021	0.021			0.027	0.013		-0.280	-0.839		0.480	-0.522
					(0.018)	(0.018)			(0.017)	(0.020)		(0.145)	(0.133)		(0.258)	(0.050)
β_3						0.013				0.032			0.034			-0.846
						(0.018)				(0.016)			(0.017)			(0.135)
LL	-18.1	-16.5	-15.6	-18.2	-17.5	-17.2	-18.3	-18.1	-11.3	-16.3	-12.2	-11.6	-11.1	-14.3	-13.5	-8.4
AIC	-2.826	-2.826	-2.826	-2.826	-2.825	-2.825	-2.826	-2.825	-2.829	-2.825	-2.828	-2.828	-2.827	-2.826	-2.826	-2.828
BIC	-2.822	-2.820	-2.818	-2.822	-2.820	-2.817	-2.824	-2.819	-2.821	-2.816	-2.820	-2.818	-2.816	-2.817	-2.815	-2.815
Ljung-Box Q_{20}	26.69	23.98	22.64	26.81	24.36	23.35	26.56	26.47	15.25	21.73	16.41	16.02	14.64	21.55	22.37	14.42
p-value	0.14	0.24	0.31	0.14	0.23	0.27	0.15	0.15	0.76	0.36	0.69	0.72	0.80	0.37	0.32	0.81

Note. Standard Errors are given in parenthesis.

Table 3.126: Estimates of ARMA(p,q) model for tenth component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.052	-0.050	-0.051	-0.053	-0.053	-0.053	-0.053	-0.002	-0.085	-0.101	-0.083	-0.002	-0.002	-0.090	-0.002	-0.004
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.001)	(0.015)	(0.007)	(0.015)	(0.001)	(0.001)	(0.010)	(0.001)	(0.002)
α_1	0.015	0.015	0.015					0.969	-0.599	-0.899	-0.603	0.137	0.193	-0.777	0.180	-0.833
	(0.026)	(0.026)	(0.026)					(0.014)	(0.264)	(0.081)	(0.256)	(0.111)	(0.380)	(0.186)	(0.156)	(0.012)
α_2		0.038	0.038								0.039	0.825	0.772	0.050	0.794	0.792
		(0.018)	(0.018)								(0.026)	(0.109)	(0.376)	(0.030)	(0.135)	(0.020)
α_3			-0.005											0.025	-0.010	0.967
			(0.019)											(0.029)	(0.030)	(0.012)
β_1				0.014	0.015	0.015		-0.951	0.614	0.915	0.620	-0.132	-0.182	0.793	-0.169	0.846
				(0.036)	(0.023)	(0.038)		(0.016)	(0.263)	(0.086)	(0.255)	(0.122)	(0.368)	(0.185)	(0.151)	(0.017)
β_2					0.037	0.037			0.040	0.051		-0.803	-0.745		-0.769	-0.763
					(0.018)	(0.017)			(0.023)	(0.031)		(0.118)	(0.403)		(0.147)	(0.028)
β_3						-0.004				0.030			-0.012			-0.953
						(0.040)				(0.019)			(0.053)			(0.017)
LL	1331.8	1334.5	1334.9	1331.6	1333.9	1333.9	1331.2	1337.0	1334.7	1335.5	1335.6	1343.7	1343.8	1336.3	1343.5	1349.3
AIC	-3.652	-3.653	-3.653	-3.652	-3.653	-3.652	-3.652	-3.655	-3.653	-3.653	-3.653	-3.658	-3.657	-3.653	-3.657	-3.660
BIC	-3.648	-3.648	-3.645	-3.648	-3.647	-3.645	-3.651	-3.649	-3.645	-3.643	-3.646	-3.648	-3.646	-3.644	-3.646	-3.647
Ljung-Box Q_{20}	29.71	23.87	23.85	29.86	24.21	24.30	31.36	15.68	24.50	24.09	24.56	14.42	13.76	23.75	13.87	12.89
p-value	0.07	0.25	0.25	0.07	0.23	0.23	0.05	0.74	0.22	0.24	0.22	0.81	0.84	0.25	0.84	0.88

Note. Standard Errors are given in parenthesis.

Table 3.127: Estimates of ARMA(p,q) model for eleventh component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.031	-0.031	-0.030	-0.031	-0.031	-0.031	-0.031	-0.039	-0.021	-0.049	-0.020	-0.016	-0.078	-0.048	-0.104	-0.004
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.008)	(0.008)	(0.012)	(0.007)	(0.003)	(0.035)	(0.012)	(0.012)	(0.006)
α_1	-0.013	-0.012	-0.013					-0.262	0.337	-0.590	0.321	1.377	-1.003	-0.591	-1.495	2.109
	(0.025)	(0.025)	(0.025)					(0.247)	(0.225)	(0.300)	(0.202)	(0.080)	(0.713)	(0.284)	(0.035)	(0.465)
α_2		0.019	0.019								0.026	-0.888	-0.509	0.012	-0.946	-1.854
		(0.032)	(0.032)								(0.035)	(0.136)	(0.381)	(0.042)	(0.047)	(0.772)
α_3			0.022											0.035	0.010	0.622
			(0.026)											(0.038)	(0.024)	(0.491)
β_1				-0.012	-0.013	-0.012		0.247	-0.350	0.578	-0.334	-1.375	0.991	0.578	1.487	-2.123
				(0.022	(0.022)	(0.028)		(0.245)	(0.222)	(0.298)	(0.199)	(0.082)	(0.712)	(0.282)	(0.027)	(0.452)
β_2					0.020	0.020			0.028	0.012		0.906	0.515		0.942	1.897
					(0.033)	(0.032)			(0.036)	(0.043)		(0.130)	(0.351)		(0.033)	(0.742)
β_3						0.022				0.034			0.027			-0.657
						(0.027)				(0.036)			(0.040)			(0.471)
LL	441.9	442.5	443.3	441.9	442.5	443.3	441.6	442.0	442.9	443.9	442.8	447.2	444.3	444.0	448.8	449.1
AIC	-3.107	-3.107	-3.107	-3.107	-3.107	-3.107	-3.108	-3.107	-3.107	-3.107	-3.107	-3.109	-3.106	-3.107	-3.109	-3.109
BIC	-3.104	-3.101	-3.099	-3.104	-3.101	-3.099	-3.106	-3.101	-3.099	-3.097	-3.099	-3.099	-3.095	-3.097	-3.098	-3.096
Ljung-Box Q_{20}	31.29	30.19	28.49	31.31	30.12	28.48	31.49	31.15	29.64	26.93	29.75	18.50	25.47	26.75	25.14	15.77
p-value	0.05	0.07	0.10	0.05	0.07	0.10	0.05	0.05	0.08	0.14	0.07	0.55	0.18	0.14	0.20	0.73

Note. Standard Errors are given in parenthesis.

Table 3.128: Estimates of ARMA(p,q) model for twelfth component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.065	-0.063	-0.059	-0.065	-0.065	-0.065	-0.065	-0.002	-0.002	-0.002	-0.023	-0.025	-0.032	-0.047	-0.088	-0.021
	(0.004)	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.001)	(0.001)	(0.006)	(0.008)	(0.014)	(0.023)	(0.021)	(0.038)
α_1	0.002	0.001	-0.001					0.974	0.971	0.970	0.600	0.787	0.378	0.186	-0.025	0.651
	(0.02)6	(0.026)	(0.026)					(0.014)	(0.018)	(0.019)	(0.095)	(0.161)	(0.179)	(0.363)	(0.129)	(0.419)
α_2		0.020	0.019								0.044	-0.169	0.128	0.019	-0.410	-0.624
		(0.027)	(0.027)								(0.025)	(0.122)	(0.157)	(0.027)	(0.269)	(0.202)
α_3			0.066											0.063	0.072	0.643
			(0.020)											(0.024)	(0.024)	(0.342)
β_1				0.002	-0.001	-0.001		-0.954	-0.977	-0.975	-0.602	-0.796	-0.380	-0.188	0.025	-0.659
				(0.008)	(0.012)	(0.006)		(0.016)	(0.029)	(0.030)	(0.094)	(0.168)	(0.177)	(0.364)	(0.120)	(0.429)
β_2					0.019	0.022			0.028	0.020		0.216	-0.109		0.428	0.660
					(0.023)	(0.027)			(0.026)	(0.045)		(0.119)	(0.158)		(0.266)	(0.212)
β_3						0.061				0.007			0.050			-0.599
						(0.018)				(0.029)			(0.020)			(0.369)
LL	154.4	158.3	166.6	154.2	154.9	161.6	154.2	165.5	166.7	166.8	162.9	163.7	166.2	166.9	167.9	171.7
AIC	-2.931	-2.933	-2.937	-2.931	-2.931	-2.934	-2.932	-2.937	-2.938	-2.937	-2.935	-2.935	-2.936	-2.937	-2.937	-2.939
BIC	-2.927	-2.927	-2.930	-2.927	-2.925	-2.927	-2.930	-2.932	-2.930	-2.928	-2.928	-2.926	-2.925	-2.928	-2.926	-2.926
Ljung-Box Q_{20}	63.21	61.82	43.28	62.97	61.33	44.07	63.14	38.81	36.61	36.22	48.13	48.07	40.77	41.55	38.97	26.26
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.16

Note. Standard Errors are given in parenthesis.

Table 3.129: Estimates of ARMA(p,q) model for thirteenth component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.072	-0.070	-0.068	-0.073	-0.073	-0.073	-0.073	-0.001	-0.001	-0.094	-0.001	-0.002	-0.144	-0.091	-0.165	-0.003
	(0.004)	(0.005)	(0.005)	(0.004)	(0.005)	(0.005)	(0.004)	(0.001)	(0.001)	(0.023)	(0.001)	(0.001)	(0.064)	(0.018)	(0.095)	(0.002)
α_1	0.015	0.015	0.014					0.984	0.984	-0.284	0.984	-0.005	-0.555	-0.309	-0.773	-0.230
	(0.019)	(0.019)	(0.019)					(0.009)	(0.009)	(0.302)	(0.028)	(0.011)	(0.488)	(0.238)	(0.808)	(0.050)
α_2		0.024	0.023								0.003	0.975	-0.418	0.028	-0.537	0.247
		(0.021)	(0.021)								(0.026)	(0.011)	(0.416)	(0.022)	(0.504)	(0.062)
α_3			0.029											0.037	0.044	0.937
			(0.017)											(0.018)	(0.023)	(0.044)
β_1				0.015	0.014	0.015		-0.969	-0.973	0.299	-0.973	0.016	0.570	0.323	0.788	0.234
				(0.018)	(0.020)	(0.020)		(0.010)	(0.020)	(0.303)	(0.009)	(0.012)	(0.491)	(0.239)	(0.812)	(0.056)
β_2					0.024	0.023			0.003	0.028		-0.958	0.449		0.566	-0.233
					(0.022)	(0.022)			(0.018)	(0.023)		(0.013)	(0.415)		(0.492)	(0.069)
β_3						0.028				0.035			0.042			-0.911
						(0.016)				(0.017)			(0.020)			(0.048)
LL	-109.7	-108.7	-107.3	-109.7	-108.8	-107.5	-110.1	-103.0	-103.0	-107.3	-101.8	-101.3	-106.9	-107.0	-106.2	-99.6
AIC	-2.769	-2.769	-2.770	-2.769	-2.769	-2.770	-2.770	-2.773	-2.772	-2.769	-2.773	-2.773	-2.769	-2.769	-2.769	-2.773
BIC	-2.766	-2.764	-2.762	-2.766	-2.764	-2.762	-2.768	-2.767	-2.765	-2.760	-2.766	-2.763	-2.758	-2.760	-2.758	-2.760
Ljung-Box Q_{20}	27.37	24.75	21.08	27.30	24.76	21.25	28.70	16.13	16.14	21.32	16.08	15.40065662	21.2562162	21.11	21.07	14.27
p-value	0.13	0.21	0.39	0.13	0.21	0.38	0.09	0.71	0.71	0.38	0.71	0.753041408	0.382192232	0.39	0.39	0.82

Note. Standard Errors are given in parenthesis.

Table 3.130: Estimates of ARMA(p,q) model for fourteenth component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.039	-0.040	-0.037	-0.039	-0.039	-0.039	-0.039	0.000	-0.067	-0.054	-0.070	-0.123	-0.121	-0.052	-0.069	-0.030
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.000)	(0.009)	(0.010)	(0.009)	(0.018)	(0.021)	(0.011)	(0.056)	(0.019)
α_1	-0.002	-0.002	-0.001					0.995	-0.705	-0.373	-0.746	-1.352	-1.309	-0.382	-0.608	-0.530
	(0.019)	(0.019)	(0.019)					(0.003)	(0.107)	(0.188)	(0.109)	(0.129)	(0.205)	(0.190)	(0.780)	(0.244)
α_2		-0.010	-0.009								-0.033	-0.797	-0.772	-0.010	-0.221	0.217
		(0.017)	(0.017)								(0.020)	(0.137)	(0.175)	(0.015)	(0.615)	(0.226)
α_3			0.058											0.056	0.050	0.544
			(0.018)											(0.018)	(0.031)	(0.240)
β_1				-0.002	-0.001	0.002		-0.992	0.706	0.373	0.746	1.347	1.310	0.382	0.608	0.534
				(0.015)	(0.006)	(0.004)		(0.004)	(0.106)	(0.188)	(0.107)	(0.135)	(0.209)	(0.191)	(0.782)	(0.253)
β_2					-0.010	-0.012			-0.031	-0.010		0.767	0.753		0.212	-0.226
					(0.02)1	(0.018)			(0.017)	(0.018)		(0.147)	(0.183)		(0.613)	(0.231)
β_3						0.057				0.053			0.011			-0.500
						(0.018)				(0.018)			(0.018)			(0.256)
LL	-517.7	-517.6	-512.0	-517.7	-517.6	-512.2	-517.7	-514.5	-514.3	-511.0	-514.0	-510.2	-510.0	-510.9	-510.8	-509.5
AIC	-2.520	-2.519	-2.522	-2.520	-2.519	-2.522	-2.520	-2.521	-2.520	-2.522	-2.521	-2.522	-2.522	-2.522	-2.521	-2.522
BIC	-2.516	-2.514	-2.514	-2.516	-2.513	-2.514	-2.518	-2.515	-2.513	-2.513	-2.513	-2.513	-2.511	-2.513	-2.510	-2.509
Ljung-Box Q_{20}	27.55	27.29	15.96	27.59	27.39	16.02	27.71	29.03	19.71	13.39	19.30	11.87	11.92	12.98	12.64	10.83
p-value	0.12	0.13	0.72	0.12	0.12	0.72	0.12	0.09	0.48	0.86	0.50	0.92	0.92	0.88	0.89	0.95

Note. Standard Errors are given in parenthesis.

Table 3.131: Estimates of ARMA(p,q) model for fifteenth component of $\mathbf{Z_{2,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.047	-0.046	-0.046	-0.047	-0.047	-0.047	-0.047	-0.087	-0.001	-0.001	-0.006	-0.004	-0.005	-0.002	-0.002	-0.001
	(0.004)	(0.004)	(0.004)	(0.003)	(0.004)	(0.004)	(0.003)	(0.010)	(0.001)	(0.001)	(0.004)	(0.003)	(0.004)	(0.001)	(0.001)	(0.075)
α_1	-0.004	-0.004	-0.004					-0.866	0.969	0.970	0.843	0.160	0.111	0.938	0.561	0.672
	(0.020)	(0.020)	(0.020)					(0.174)	(0.018)	(0.017)	(0.075)	(0.189)	(0.184)	(0.029)	(0.357)	(19.538)
α_2		0.021	0.021								0.029	0.746	0.787	0.026	0.400	0.584
		(0.018)	(0.018)								(0.020)	(0.212)	(0.192)	(0.019)	(0.349)	(26.894)
α_3			0.004											-0.002	-0.001	-0.286
			(0.026)											(0.006)	(0.001)	(44.830)
β_1				-0.004	-0.004	-0.005		0.857	-0.976	-0.977	-0.849	-0.159	-0.118	-0.947	-0.569	-0.681
				(0.074)	(0.012)	(0.019)		(0.183)	(0.025)	(0.025)	(0.073)	(0.196)	(0.183)	(0.021)	(0.358)	(19.521)
β_2					0.020	0.020			0.021	0.024		-0.717	-0.761		-0.374	-0.558
					(0.017)	(0.017)			(0.019)	(0.064)		(0.219)	(0.198)		(0.347)	(27.154)
β_3						0.003				-0.003			0.013			0.283
						(0.016)				(0.067)			(0.025)			(44.282)
LL	651.0	652.3	652.5	650.9	651.6	651.6	650.9	651.8	656.2	656.2	654.0	655.9	656.2	656.4	657.4	657.5
AIC	-3.235	-3.235	-3.235	-3.235	-3.235	-3.234	-3.236	-3.235	-3.237	-3.237	-3.236	-3.236	-3.236	-3.237	-3.237	-3.236
BIC	-3.232	-3.230	-3.228	-3.232	-3.229	-3.227	-3.234	-3.230	-3.230	-3.227	-3.228	-3.227	-3.225	-3.227	-3.226	-3.223
Ljung-Box Q_{20}	27.63	25.68	25.42	27.62	25.70	25.55	27.50	26.79	16.10	16.09	18.54	16.39	16.03	16.21	15.96	15.54
p-value	0.12	0.18	0.19	0.12	0.18	0.18	0.12	0.14	0.71	0.71	0.55	0.69	0.71	0.70	0.72	0.74

Note. Standard Errors are given in parenthesis.

3.4.9 Histograms of all components of $U_{3,t}$ with Different Distributions Fit

Figure 3.4.5: Histograms of 7 components of singular vector with Normal fit after third round of SVX model

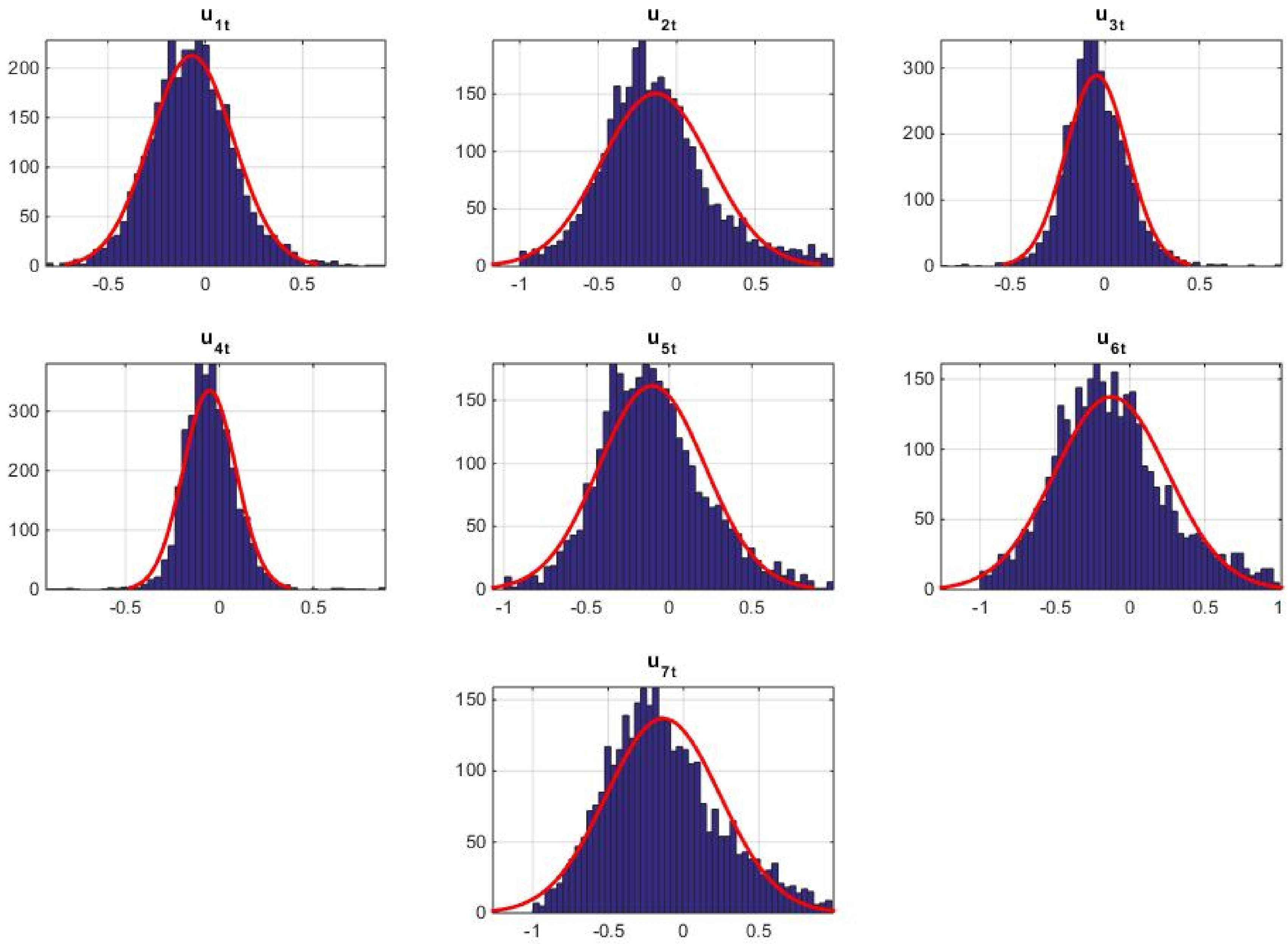
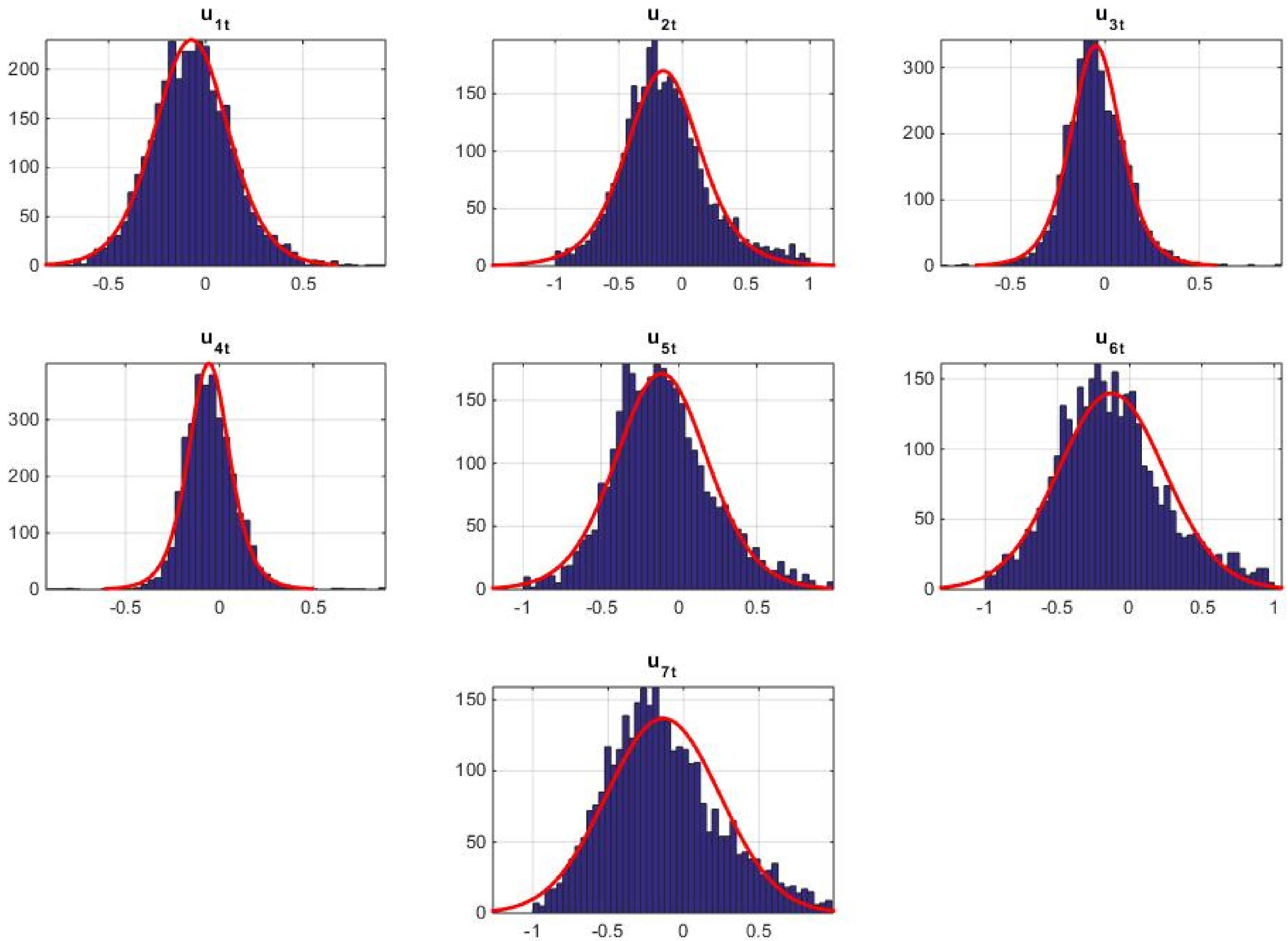


Figure 3.4.6: Histograms of 7 components of singular vector with Student's t fit from third round of SVX model



3.4.10 Estimates of ARMA(p,q) model for $\mathbf{U_{3,t}}$ from Third Round

Table 3.132: Estimates of ARMA(p,q) model for first component of $\mathbf{U_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.263	-0.262	-0.250	-0.271	-0.271	-0.271	-0.271	-0.032	-0.032	-0.041	-0.035	-0.062	-0.056	-0.047	-0.059	-0.094
	(0.006)	(0.007)	(0.009)	(0.003)	(0.003)	(0.003)	(0.003)	(0.014)	(0.015)	(0.022)	(0.017)	(0.029)	(0.032)	(0.031)	(0.036)	(0.035)
α_1	0.030	0.029	0.029					0.884	0.882	0.847	0.866	-0.061	0.461	0.812	0.443	-0.170
	(0.017)	(0.017)	(0.017)					(0.052)	(0.055)	(0.082)	(0.062)	(0.067)	(0.507)	(0.124)	(0.441)	(0.053)
α_2		0.004	0.002								0.004	0.831	0.331	-0.020	0.306	-0.025
		(0.018)	(0.018)								(0.007)	(0.064)	(0.449)	(0.021)	(0.378)	(0.048)
α_3			0.046											0.034	0.035	0.849
			(0.018)											(0.019)	(0.019)	(0.052)
β_1				0.030	0.030	0.028		-0.856	-0.857	-0.820	-0.840	0.096	-0.435	-0.786	-0.416	0.199
				(0.017)	(0.017)	(0.018)		(0.054)	(0.059)	(0.085)	(0.064)	(0.067)	(0.508)	(0.123)	(0.441)	(0.058)
β_2					0.002	0.002			0.002	-0.023		-0.813	-0.343		-0.317	0.042
					(0.002)	(0.003)			(0.005)	(0.026)		(0.063)	(0.429)		(0.362)	(0.051)
β_3						0.043				0.032			0.033			-0.809
						(0.017)				(0.020)			(0.019)			(0.057)
LL	1115.7	1116.2	1120.0	1115.2	1115.2	1118.4	1113.7	1121.1	1121.1	1122.6	1121.0	1122.1	1122.9	1122.6	1122.9	1126.3
AIC	-3.520	-3.520	-3.521	-3.520	-3.519	-3.520	-3.519	-3.523	-3.522	-3.522	-3.522	-3.522	-3.522	-3.522	-3.522	-3.523
BIC	-3.516	-3.514	-3.514	-3.516	-3.513	-3.513	-3.517	-3.517	-3.514	-3.513	-3.514	-3.513	-3.511	-3.513	-3.511	-3.510
Ljung-Box Q_{20}	28.55	28.21	19.81	28.90	28.83	20.90	32.88	19.05	19.04	16.28	19.06	18.32	15.23	16.15	15.18	11.08
p-value	0.10	0.10	0.47	0.09	0.09	0.40	0.03	0.52	0.52	0.70	0.52	0.57	0.76	0.71	0.77	0.94

Note. Standard Errors are given in parenthesis.

Table 3.133: Estimates of ARMA(p,q) model for second component of $\mathbf{U_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.127	-0.125	-0.123	-0.134	-0.134	-0.134	-0.134	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.151	-0.291	-0.074
	(0.007)	(0.007)	(0.007)	(0.006)	(0.006)	(0.007)	(0.006)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.022)	(0.109)	(0.015)
α_1	0.047	0.046	0.045					0.992	0.993	0.994	1.022	0.643	0.443	-0.174	-0.694	-0.285
	(0.019)	(0.019)	(0.019)					(0.007)	(0.006)	(0.006)	(0.020)	(0.403)	(2.184)	(0.170)	(0.511)	(0.069)
α_2		0.020	0.019								-0.029	0.349	0.549	0.029	-0.542	0.028
		(0.019)	(0.019)								(0.020)	(0.402)	(2.173)	(0.023)	(0.344)	(0.010)
α_3			0.017											0.019	0.053	0.703
			(0.018)											(0.020)	(0.039)	(0.055)
β_1				0.045	0.045	0.045		-0.981	-0.954	-0.954	-0.983	-0.609	-0.404	0.220	0.742	0.326
				(0.018)	(0.019)	(0.019)		(0.011)	(0.019)	(0.021)	(0.005)	(0.392)	(2.189)	(0.170)	(0.516)	(0.077)
β_2					0.020	0.020			-0.029	-0.024		-0.371	-0.557		0.598	0.003
					(0.020)	(0.020)			(0.018)	(0.024)		(0.388)	(2.126)		(0.354)	(0.005)
β_3						0.016				-0.005			-0.017			-0.663
						(0.022)				(0.021)			(0.039)			(0.050)
LL	-1163.4	-1162.7	-1158.4	-1163.6	-1162.9	-1162.5	-1167.0	-1154.9	-1153.5	-1153.4	-1154.5	-1153.8	-1153.4	-1157.9	-1156.0	-1152.0
AIC	-2.124	-2.124	-2.126	-2.124	-2.124	-2.124	-2.123	-2.129	-2.129	-2.129	-2.128	-2.128	-2.128	-2.126	-2.126	-2.128
BIC	-2.120	-2.118	-2.119	-2.120	-2.118	-2.116	-2.121	-2.123	-2.122	-2.119	-2.121	-2.119	-2.117	-2.116	-2.115	-2.115
Ljung-Box Q_{20}	34.31	32.53	31.20	34.75	32.95	31.57	43.36	24.47	22.08	22.09	22.00	22.36	22.06	31.26	29.85	23.04
p-value	0.02	0.04	0.05	0.02	0.03	0.05	0.00	0.22	0.34	0.34	0.34	0.32	0.34	0.05	0.07	0.29

Note. Standard Errors are given in parenthesis.

Table 3.134: Estimates of ARMA(p,q) model for third component of $\mathbf{U_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.182	-0.182	-0.180	-0.187	-0.187	-0.187	-0.187	-0.172	-0.170	-0.281	-0.331	-0.001	-0.001	-0.298	-0.005	-0.247
	(0.009)	(0.009)	(0.010)	(0.008)	(0.008)	(0.008)	(0.008)	(0.011)	(0.009)	(0.033)	(0.042)	(0.001)	(0.001)	(0.028)	(0.004)	(0.026)
α_1	0.027	0.027	0.027					0.079	0.091	-0.503	-0.778	1.082	1.092	-0.634	0.208	0.424
	(0.018)	(0.018)	(0.018)					(0.045)	(0.026)	(0.165)	(0.385)	(0.092)	(0.109)	(0.138)	(0.074)	(0.158)
α_2		0.000	-0.001								0.007	-0.089	-0.099	0.017	0.757	-0.109
		(0.018)	(0.018)								(0.169)	(0.092)	(0.109)	(0.021)	(0.075)	(0.176)
α_3			0.011											0.024	0.009	-0.637
			(0.018)											(0.017)	(0.013)	(0.140)
β_1				0.027	0.027	0.028		-0.051	-0.064	0.532	0.807	-1.062	-1.070	0.663	-0.183	-0.384
				(0.018)	(0.008)	(0.019)		(0.040	(0.034)	(0.168)	(0.491)	(0.096)	(0.110)	(0.138)	(0.070)	(0.166)
β_2					0.000	0.000			-0.002	0.014		0.078	0.068		-0.768	0.087
					(0.001)	(0.000)			(0.001)	(0.028)		(0.095)	(0.112)		(0.071)	(0.184)
β_3						0.013				0.025			0.019			0.641
						(0.023)				(0.024)			(0.019)			(0.146)
LL	-2063.9	-2063.9	-2063.5	-2065.8	-2065.8	-2065.5	-2067.0	-2063.9	-2063.9	-2062.8	-2062.9	-2054.9	-2054.3	-2062.1	-2055.8	-2057.9
AIC	-1.573	-1.572	-1.572	-1.572	-1.571	-1.571	-1.571	-1.572	-1.572	-1.572	-1.572	-1.576	-1.576	-1.572	-1.575	-1.573
BIC	-1.569	-1.567	-1.564	-1.568	-1.565	-1.563	-1.570	-1.567	-1.564	-1.562	-1.565	-1.567	-1.565	-1.563	-1.564	-1.560
Ljung-Box Q_{20}	54.26	54.27	53.55	54.72	54.70	53.94	59.38	54.19	54.28	50.60	53.11	41.60	40.81	50.43	39.38	42.56
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00

Note. Standard Errors are given in parenthesis.

Table 3.135: Estimates of ARMA(p,q) model for fourth component of $\mathbf{U_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.231	-0.229	-0.220	-0.242	-0.242	-0.242	-0.242	-0.026	-0.024	-0.031	-0.025	-0.040	-0.035	-0.029	-0.031	-0.072
	(0.005)	(0.007)	(0.007)	(0.003)	(0.003)	(0.003)	(0.003)	(0.016)	(0.015)	(0.021)	(0.018)	(0.036)	(0.031)	(0.026)	(0.019)	(0.068)
α_1	0.044	0.043	0.043					0.895	0.900	0.873	0.903	0.251	0.776	0.879	0.799	-0.158
	(0.018)	(0.018)	(0.018)					(0.064)	(0.062)	(0.085)	(0.079)	(0.684)	(0.867)	(0.123)	(0.194)	(0.091)
α_2		0.010	0.008								-0.006	0.583	0.081	-0.028	0.044	-0.022
		(0.018)	(0.018)								(0.012)	(0.582)	(0.779)	(0.027)	(0.226)	(0.109)
α_3			0.039											0.028	0.027	0.882
			(0.018)											(0.021)	(0.020)	(0.088)
β_1				0.043	0.043	0.042		-0.861	-0.860	-0.833	-0.863	-0.206	-0.736	-0.840	-0.759	0.193
				(0.018)	(0.018)	(0.018)		(0.070)	(0.071)	(0.088)	(0.078)	(0.674)	(0.867)	(0.123)	(0.194)	(0.099)
β_2					0.008	0.009			-0.007	-0.028		-0.575	-0.105		-0.069	0.048
					(0.024)	(0.019)			(0.020)	(0.025)		(0.547)	(0.743)		(0.219)	(0.123)
β_3						0.036				0.028			0.027			-0.840
						(0.017)				(0.018)			(0.020)			(0.098)
LL	1309.5	1310.1	1312.8	1308.9	1309.1	1311.2	1305.8	1315.9	1316.0	1317.2	1315.8	1316.4	1317.0	1317.1	1317.1	1320.8
AIC	-3.639	-3.638	-3.639	-3.638	-3.638	-3.638	-3.637	-3.642	-3.641	-3.641	-3.641	-3.641	-3.641	-3.641	-3.641	-3.642
BIC	-3.635	-3.633	-3.632	-3.634	-3.632	-3.631	-3.635	-3.636	-3.634	-3.632	-3.634	-3.632	-3.630	-3.632	-3.630	-3.629
Ljung-Box Q_{20}	32.08	31.24	24.87	32.57	32.04	26.29	41.90	18.47	18.21	16.16	18.24	17.04	16.08	16.16	16.07	12.39
p-value	0.04	0.05	0.21	0.04	0.04	0.16	0.00	0.56	0.57	0.71	0.57	0.65	0.71	0.71	0.71	0.90

Note. Standard Errors are given in parenthesis.

Table 3.136: Estimates of ARMA(p,q) model for fifth component of $\mathbf{U_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.101	-0.100	-0.099	-0.104	-0.104	-0.104	-0.104	-0.007	-0.008	-0.008	-0.001	-0.001	-0.001	-0.001	-0.001	-0.003
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.004)	(0.004)	(0.004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
α_1	0.025	0.024	0.024					0.930	0.927	0.923	0.996	1.055	1.056	0.985	0.143	-0.017
	(0.018)	(0.018)	(0.018)					(0.037)	(0.038)	(0.040)	(0.012)	(0.114)	(0.280)	(0.021)	(0.055)	(0.218)
α_2		0.015	0.015								-0.005	-0.062	-0.063	-0.014	0.835	0.172
		(0.018)	(0.018)								(0.011)	(0.109)	(0.272)	(0.026)	(0.061)	(0.111)
α_3			0.010											0.016	0.010	0.815
			(0.017)											(0.019)	(0.023)	(0.128)
β_1				0.024	0.024	0.024		-0.906	-0.908	-0.905	-0.981	-1.041	-1.042	-0.970	-0.131	0.031
				(0.017)	(0.018)	(0.019)		(0.039)	(0.045)	(0.042)	(0.009)	(0.118)	(0.280)	(0.013)	(0.058)	(0.217)
β_2					0.015	0.014			0.006	-0.008		0.057	0.054		-0.838	-0.151
					(0.017)	(0.018)			(0.022)	(0.012)		(0.109)	(0.235)		(0.062)	(0.110)
β_3						0.009				0.016			0.005			-0.818
						(0.015)				(0.015)			(0.045)			(0.120)
LL	-920.7	-920.1	-919.9	-922.7	-922.3	-922.2	-923.7	-913.0	-913.0	-912.6	-904.4	-904.4	-904.3	-910.8	-905.4	-902.5
AIC	-2.273	-2.273	-2.272	-2.272	-2.271	-2.271	-2.272	-2.277	-2.276	-2.276	-2.282	-2.281	-2.280	-2.277	-2.280	-2.281
BIC	-2.269	-2.267	-2.265	-2.268	-2.266	-2.263	-2.270	-2.271	-2.269	-2.267	-2.274	-2.272	-2.269	-2.268	-2.269	-2.268
Ljung-Box Q_{20}	30.03	27.96	27.24	30.60	28.71	28.10	34.47	12.14	12.20	11.79	10.94	10.95	10.89	11.34	9.49	9.31
p-value	0.07	0.11	0.13	0.06	0.09	0.11	0.02	0.91	0.91	0.92	0.95	0.95	0.95	0.94	0.98	0.98

Note. Standard Errors are given in parenthesis.

Table 3.137: Estimates of ARMA(p,q) model for sixth component of $\mathbf{U_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.127	-0.127	-0.120	-0.123	-0.123	-0.123	-0.123	-0.134	-0.236	-0.011	-0.220	-0.204	-0.020	-0.010	-0.010	-0.010
	(0.007)	(0.007)	(0.008)	(0.006)	(0.006)	(0.007)	(0.007)	(0.013)	(0.014)	(0.005)	(0.022)	(0.048)	(0.009)	(0.005)	(0.005)	(0.005)
α_1	-0.027	-0.026	-0.026					-0.090	-0.916	0.914	-0.746	-0.118	0.097	0.847	0.873	0.879
	(0.018)	(0.018)	(0.019)					(0.086)	(0.064)	(0.039)	(0.150)	(0.352)	(0.096)	(0.053)	(0.061)	(0.057)
α_2		-0.006	-0.005								-0.035	-0.537	0.743	0.018	-0.006	0.015
		(0.018)	(0.018)								(0.018)	(0.157)	(0.098)	(0.024)	(0.004)	(0.016)
α_3			0.054											0.053	0.052	0.023
			(0.017)											(0.021)	(0.023)	(0.034)
β_1				-0.027	-0.026	-0.030		0.064	0.892	-0.948	0.722	0.080	-0.132	-0.881	-0.907	-0.913
				(0.019)	(0.018)	(0.019)		(0.092)	(0.066)	(0.043)	(0.149)	(0.355)	(0.098)	(0.051)	(0.066)	(0.056)
β_2					-0.002	-0.006			-0.036	0.020		0.523	-0.728		0.025	0.003
					(0.004)	(0.058)			(0.018)	(0.025)		(0.164)	(0.097)		(0.030)	(0.001)
β_3						0.052				0.049			0.079			0.028
						(0.017)				(0.020)			(0.020)			(0.036)
LL	-1465.0	-1462.5	-1457.7	-1465.1	-1465.1	-1460.4	-1466.3	-1465.0	-1461.6	-1450.0	-1461.0	-1460.1	-1449.2	-1446.1	-1446.1	-1446.1
AIC	-1.940	-1.940	-1.943	-1.939	-1.939	-1.941	-1.939	-1.939	-1.940	-1.947	-1.941	-1.941	-1.947	-1.949	-1.949	-1.948
BIC	-1.936	-1.935	-1.935	-1.936	-1.933	-1.934	-1.937	-1.933	-1.933	-1.938	-1.933	-1.931	-1.936	-1.940	-1.937	-1.935
Ljung-Box Q_{20}	54.46	55.52	40.47	54.79	55.06	41.47	54.17	54.30	52.50	16.08	54.28	50.76	17.69	16.02	16.07	16.01
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.71	0.00	0.00	0.61	0.72	0.71	0.72

Note. Standard Errors are given in parenthesis.

Table 3.138: Estimates of ARMA(p,q) model for seventh component of $\mathbf{U_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.133	-0.132	-0.129	-0.134	-0.134	-0.134	-0.134	-0.007	-0.007	-0.007	-0.001	0.000	0.000	-0.118	-0.227	-0.001
	(0.007)	(0.007)	(0.008)	(0.007)	(0.007)	(0.007)	(0.007)	(0.004)	(0.004)	(0.020)	(0.001)	(0.000)	(0.000)	(0.014)	(0.018)	(0.001)
α_1	0.014	0.013	0.013					0.948	0.947	0.946	0.992	1.443	1.443	0.102	0.210	0.528
	(0.018)	(0.018)	(0.018)					(0.026)	(0.026)	(0.144)	(0.007)	(0.067)	(0.070)	(0.085)	(0.052)	(0.130)
α_2		0.009	0.009								-0.001	-0.445	-0.444	0.008	-0.916	-0.408
		(0.018	(0.018)								(0.001)	(0.067)	(0.069)	(0.018)	(0.057)	(0.044)
α_3			0.016											0.016	0.019	0.876
			(0.018)											(0.018)	(0.023)	(0.132)
β_1				0.014	0.013	0.013		-0.931	-0.938	-0.937	-0.983	-1.439	-1.437	-0.088	-0.197	-0.523
				(0.028)	(0.023)	(0.016)		(0.026)	(0.029)	(0.721)	(0.011)	(0.069)	(0.072)	(0.084)	(0.050)	(0.126)
β_2					0.009	0.009			0.008	-0.004		0.442	0.438		0.918	0.424
					(0.018)	(0.024)			(0.017)	(3.609)		(0.068)	(0.072)		(0.058)	(0.041)
β_3						0.016				0.013			0.002			-0.887
						(0.014)				(2.835)			(0.010)			(0.138)
LL	-1441.7	-1441.3	-1440.4	-1442.6	-1442.4	-1442.0	-1442.9	-1435.6	-1435.4	-1435.2	-1431.9	-1428.3	-1428.3	-1440.4	-1438.5	-1426.0
AIC	-1.954	-1.953	-1.953	-1.953	-1.953	-1.952	-1.954	-1.957	-1.956	-1.956	-1.959	-1.960	-1.960	-1.953	-1.953	-1.960
BIC	-1.950	-1.948	-1.946	-1.950	-1.947	-1.945	-1.952	-1.951	-1.949	-1.947	-1.951	-1.951	-1.948	-1.943	-1.942	-1.947
Ljung-Box Q_{20}	38.43	37.44	36.07	38.57	37.75	36.14	40.52	25.49	25.74	25.55	22.86	23.27	23.18	35.99	37.61	20.83
p-value	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.18	0.17	0.18	0.30	0.28	0.28	0.02	0.01	0.41

Note. Standard Errors are given in parenthesis.

3.4.11 Estimates of ARMA(p,q) model for $\mathbf{Z_{3,t}}$ from Third Round of SVD

Table 3.139: Estimates of ARMA(p,q) model for first component of $\mathbf{Z_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.282	-0.282	-0.269	-0.290	-0.290	-0.290	-0.290	-0.035	-0.035	-0.047	-0.038	-0.069	-0.066	-0.055	-0.069	-0.101
	(0.006)	(0.008)	(0.009)	(0.004)	(0.004)	(0.004)	(0.004)	(0.016)	(0.017)	(0.026)	(0.027)	(0.031)	(0.035)	(0.037)	(0.040)	(0.045)
α_1	0.028	0.027	0.027					0.881	0.878	0.837	0.868	-0.058	0.357	0.796	0.339	-0.171
	(0.017)	(0.017)	(0.017)					(0.056)	(0.059)	(0.090)	(0.199)	(0.072)	(0.564)	(0.139)	(0.489)	(0.060)
α_2		-0.002	-0.003								0.002	0.822	0.417	-0.024	0.384	-0.026
		(0.018)	(0.018)								(0.118)	(0.076)	(0.520)	(0.023)	(0.457)	(0.059)
α_3			0.048											0.038	0.038	0.847
			(0.018)											(0.020)	(0.022)	(0.064)
β_1				0.028	0.028	0.027		-0.855	-0.854	-0.812	-0.844	0.094	-0.332	-0.772	-0.314	0.197
				(0.018)	(0.021)	(0.017)		(0.057)	(0.060)	(0.092)	(0.111)	(0.069)	(0.564)	(0.138)	(0.490)	(0.063)
β_2					-0.003	-0.003			0.002	-0.027		-0.808	-0.430		-0.397	0.041
					(0.089)	(0.005)			(0.007)	(0.023)		(0.073)	(0.496)		(0.437)	(0.062)
β_3						0.046				0.036			0.036			-0.809
						(0.018)				(0.019)			(0.023)			(0.069)
LL	592.8	593.3	597.3	592.4	592.4	595.9	591.2	597.3	597.4	599.2	597.2	598.8	599.8	599.2	599.8	602.2
AIC	-3.1997	-3.1993	-3.2012	-3.1994	-3.1988	-3.2004	-3.1993	-3.2018	-3.2012	-3.2018	-3.2012	-3.2015	-3.2015	-3.2018	-3.2015	-3.2024
BIC	-3.1960	-3.1937	-3.1938	-3.1957	-3.1932	-3.1929	-3.1974	-3.1962	-3.1938	-3.1925	-3.1937	-3.1922	-3.1903	-3.1924	-3.1903	-3.1893
Ljung-Box Q_{20}	25.39	25.31	16.47	25.64	25.70	17.25	28.77	17.45	17.44	14.14	17.42	16.02	12.47	14.03	12.45	10.29
p-value	0.19	0.19	0.69	0.18	0.18	0.64	0.09	0.62	0.62	0.82	0.63	0.72	0.90	0.83	0.90	0.96

Note. Standard Errors are given in parenthesis.

Table 3.140: Estimates of ARMA(p,q) model for second component of $\mathbf{Z_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.144	-0.143	-0.143	-0.148	-0.148	-0.148	-0.148	-0.107	-0.001	-0.001	-0.001	-0.174	-0.064	-0.175	-0.340	-0.168
	(0.009)	(0.010)	(0.010)	(0.009)	(0.009)	(0.009)	(0.008)	(0.476)	(0.001)	(0.001)	(0.001)	(0.010)	(0.004)	(0.044)	(0.138)	(0.106)
α_1	0.028	0.028	0.028					0.275	0.993	0.993	1.010	0.808	1.555	-0.199	-0.757	-0.451
	(0.021)	(0.021)	(0.021)					(3.228)	(0.007)	(0.005)	(0.020)	(0.011)	(0.013)	(0.290)	(0.532)	(0.220)
α_2		0.007	0.007								-0.018	-0.983	-0.988	0.013	-0.577	-0.188
		(0.021)	(0.021)								(0.019)	(0.036)	(0.010)	(0.023)	(0.436)	(0.266)
α_3			0.005											0.008	0.038	0.503
			(0.020)											(0.033)	(0.037)	(0.240)
β_1				0.028	0.028	0.028		-0.247	-0.968	-0.968	-0.985	-0.799	-1.529	0.227	0.787	0.476
				(0.021)	(0.021)	(0.022)		(3.247)	(0.021)	(0.019)	(0.006)	(0.016)	(0.020)	(0.285)	(0.535)	(0.221)
β_2					0.008	0.008			-0.019	-0.020		0.976	0.955		0.618	0.211
					(0.023)	(0.186)			(0.020)	(0.020)		(0.040)	(0.025)		(0.442)	(0.278)
β_3						0.005				0.001			0.017			-0.473
						(0.023)				(0.001)			(0.015)			(0.246)
LL	-2237.7	-2237.6	-2233.7	-2237.8	-2237.7	-2237.6	-2239.0	-2237.7	-2233.3	-2233.2	-2234.2	-2235.1	-2232.8	-2233.3	-2231.6	-2230.0
AIC	-1.4663	-1.4658	-1.4676	-1.4663	-1.4658	-1.4652	-1.4661	-1.4658	-1.4679	-1.4672	-1.4672	-1.4661	-1.4669	-1.4672	-1.4676	-1.4680
BIC	-1.4626	-1.4602	-1.4601	-1.4626	-1.4602	-1.4577	-1.4643	-1.4602	-1.4604	-1.4579	-1.4598	-1.4568	-1.4557	-1.4579	-1.4564	-1.4550
Ljung-Box Q_{20}	18.74	18.70	18.64	18.79	18.76	18.59	21.30	18.62	17.02	16.99	17.04	19.18	19.81	18.46	16.77	17.01
p-value	0.54	0.54	0.55	0.54	0.54	0.55	0.38	0.55	0.65	0.65	0.65	0.51	0.47	0.56	0.67	0.65

Note. Standard Errors are given in parenthesis.

Table 3.141: Estimates of ARMA(p,q) model for third component of $\mathbf{Z_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.219	-0.222	-0.218	-0.224	-0.224	-0.224	-0.224	-0.357	-0.352	-0.339	-0.391	-0.298	-0.365	-0.357	-0.646	-0.244
	(0.012)	(0.012)	(0.013)	(0.011)	(0.011)	(0.012)	(0.011)	(0.036)	(0.033)	(0.042)	(0.029)	(0.103)	(0.037)	(0.036)	(0.124)	(0.028)
α_1	0.023	0.024	0.024					-0.595	-0.572	-0.515	-0.741	-0.500	-0.633	-0.616	-1.219	0.662
	(0.018)	(0.018)	(0.018)					(0.143)	(0.129)	(0.172)	(0.104)	(0.300)	(0.149)	(0.141)	(0.373)	(0.188)
α_2		-0.012	-0.012								-0.004	0.172	0.004	0.002	-0.704	-0.132
		(0.018)	(0.018)								(0.004)	(0.172)	(0.008)	(0.012)	(0.181)	(0.246)
α_3			0.015											0.020	0.041	-0.625
			(0.020)											(0.015)	(0.021)	(0.173)
β_1				0.024	0.024	0.025		0.626	0.597	0.541	0.765	0.527	0.658	0.640	1.245	-0.638
				(0.018)	(0.017)	(0.016)		(0.148)	(0.133)	(0.170)	(0.100)	(0.293)	(0.148)	(0.140)	(0.378)	(0.187)
β_2					-0.013	-0.013			-0.011	0.000		-0.181	-0.001		0.735	0.083
					(0.029)	(0.022)			(0.012)	(0.000)		(0.168)	(0.002)		(0.182)	(0.251)
β_3						0.016				0.021			0.018			0.651
						(0.028)				(0.015)			(0.012)			(0.173)
LL	-3177.0	-3176.8	-3176.3	-3178.5	-3178.2	-3177.8	-3179.4	-3176.2	-3176.0	-3175.3	-3175.2	-3175.1	-3174.8	-3174.6	-3172.2	-3167.8
AIC	-0.8911	-0.8907	-0.8903	-0.8902	-0.8898	-0.8894	-0.8903	-0.8911	-0.8906	-0.8904	-0.8910	-0.8905	-0.8900	-0.8908	-0.8916	-0.8937
BIC	-0.8874	-0.8851	-0.8829	-0.8865	-0.8842	-0.8820	-0.8884	-0.8855	-0.8831	-0.8810	-0.8836	-0.8812	-0.8789	-0.8815	-0.8804	-0.8807
Ljung-Box Q_{20}	52.20	52.41	51.46	52.54	52.76	51.78	55.75	49.14	49.63	47.88	49.21	49.00	47.87	47.67	43.26	39.61
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01

Note. Standard Errors are given in parenthesis.

Table 3.142: Estimates of ARMA(p,q) model for fourth component of $\mathbf{Z_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.245	-0.244	-0.235	-0.256	-0.256	-0.256	-0.256	-0.027	-0.025	-0.033	-0.026	-0.040	-0.036	-0.031	-0.033	-0.074
	(0.006)	(0.007)	(0.008)	(0.003)	(0.003)	(0.004)	(0.003)	(0.021)	(0.018)	(0.026)	(0.020)	(0.030)	(0.031)	(0.035)	(0.037)	(0.059)
α_1	0.045	0.045	0.045					0.895	0.903	0.873	0.910	0.301	0.792	0.884	0.806	-0.157
	(0.019)	(0.019)	(0.019)					(0.080)	(0.072)	(0.100)	(0.084)	(0.241)	(0.170)	(0.154)	(0.250)	(0.072)
α_2		0.005	0.003								-0.010	0.542	0.067	-0.034	0.038	-0.023
		(0.018)	(0.018)								(0.019)	(0.215)	(0.102)	(0.025)	(0.136)	(0.085)
α_3			0.037											0.030	0.029	0.891
			(0.019)											(0.021)	(0.020)	(0.076)
β_1				0.045	0.045	0.043		-0.862	-0.862	-0.832	-0.869	-0.253	-0.751	-0.842	-0.764	0.190
				(0.018)	(0.019)	(0.018)		(0.087)	(0.076)	(0.103)	(0.085)	(0.237	(0.172)	(0.155)	(0.253)	(0.078)
β_2					0.004	0.004			-0.011	-0.034		-0.540	-0.098		-0.069	0.049
					(0.009)	(0.008)			(0.018)	(0.024)		(0.204)	((0.098)		(0.132)	(0.094)
β_3						0.033				0.030			0.030			-0.851
						(0.018)				(0.019)			(0.019)			(0.085)
LL	839.3	839.7	842.2	838.8	838.8	840.7	835.5	844.5	844.6	846.0	844.4	845.1	845.9	846.0	846.0	849.3
AIC	-3.3506	-3.3502	-3.3512	-3.3503	-3.3497	-3.3503	-3.3489	-3.3532	-3.3527	-3.3529	-3.3525	-3.3524	-3.3522	-3.3529	-3.3523	-3.3537
BIC	-3.3469	-3.3447	-3.3437	-3.3466	-3.3441	-3.3428	-3.3470	-3.3476	-3.3452	-3.3436	-3.3451	-3.3430	-3.3410	-3.3435	-3.3411	-3.3406
Ljung-Box Q_{20}	27.65	27.30	21.62	27.97	27.81	22.73	37.35	16.07	15.50	13.16	15.59	14.07	13.11	13.14	13.05	11.08
p-value	0.12	0.13	0.36	0.11	0.11	0.30	0.01	0.71	0.75	0.87	0.74	0.83	0.87	0.87	0.88	0.94

Note. Standard Errors are given in parenthesis.

Table 3.143: Estimates of ARMA(p,q) model for fifth component of $\mathbf{Z_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.112	-0.111	-0.111	-0.114	-0.114	-0.114	-0.114	-0.008	-0.008	-0.009	-0.001	-0.001	-0.001	-0.001	-0.001	-0.003
	(0.007)	(0.008)	(0.008)	(0.007)	(0.007)	(0.007)	(0.007)	(0.005)	(0.005)	(0.005)	(0.001)	(0.013)	(0.001)	(0.001)	(0.003)	(0.002)
α_1	0.023	0.022	0.022					0.931	0.931	0.925	1.001	1.049	1.048	0.990	0.174	0.010
	(0.018)	(0.018)	(0.018)					(0.045)	(0.044)	(0.047)	(0.029)	(3.945)	(0.393)	(0.021)	(0.130)	(0.013)
α_2		0.006	0.006								-0.007	-0.055	-0.054	-0.021	0.811	0.184
		(0.019)	(0.019)								(0.029)	(3.839)	(0.382)	(0.023)	(0.252)	(0.090)
α_3			0.005											0.020	0.007	0.780
			(0.016)											(0.017)	(0.142)	(0.092)
β_1				0.023	0.023	0.023		-0.911	-0.912	-0.907	-0.986	-1.035	-1.034	-0.975	-0.162	0.005
				(0.021)	(0.035)	(0.021)		(0.046)	(0.045)	(0.054)	(0.009)	(4.012)	(0.394)	(0.013)	(0.233)	(0.008)
β_2					0.007	0.006			0.001	-0.015		0.047	0.037		-0.818	-0.165
					(0.056)	(0.013)			(0.001)	(0.023)		(3.824)	(0.373)		(0.215)	(0.087)
β_3						0.003				0.019			0.010			-0.789
						(0.011)				(0.017)			(0.015)			(0.085)
LL	-1690.0	-1689.9	-1689.8	-1691.6	-1691.6	-1691.6	-1692.5	-1685.0	-1685.0	-1684.5	-1677.1	-1677.0	-1676.9	-1681.9	-1677.8	-1675.4
AIC	-1.8017	-1.8012	-1.8006	-1.8007	-1.8002	-1.7996	-1.8008	-1.8042	-1.8036	-1.8033	-1.8084	-1.8079	-1.8073	-1.8049	-1.8068	-1.8076
BIC	-1.7980	-1.7956	-1.7932	-1.7970	-1.7946	-1.7921	-1.7990	-1.7986	-1.7961	-1.7940	-1.8010	-1.7985	-1.7962	-1.7956	-1.7956	-1.7946
Ljung-Box Q_{20}	27.59	27.10	26.94	28.02	27.54	27.39	30.71	16.78	16.79	16.04	15.68	15.70	15.38	15.79	14.25	13.49
p-value	0.12	0.13	0.14	0.11	0.12	0.12	0.06	0.67	0.67	0.71	0.74	0.74	0.75	0.73	0.82	0.86

Note. Standard Errors are given in parenthesis.

Table 3.144: Estimates of ARMA(p,q) model for sixth component of $\mathbf{Z_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.146	-0.148	-0.141	-0.140	-0.140	-0.140	-0.140	-0.140	-0.277	-0.017	-0.181	-0.222	-0.089	-0.016	-0.016	-0.014
	(0.009)	(0.010)	(0.010)	(0.009)	(0.008)	(0.009)	(0.009)	(0.008)	(0.017)	(0.010)	(0.031)	(0.109)	(0.451)	(0.011)	(0.009)	(0.010)
α_1	-0.040	-0.041	-0.040					0.006	-0.974	0.879	-0.260	-0.140	0.207	0.814	0.844	0.826
	(0.019)	(0.019)	(0.019)					(0.002)	(0.015)	(0.070)	(0.187)	(0.682)	(0.685)	(0.079)	(0.084)	(0.227)
α_2		-0.019	-0.017								-0.035	-0.444	0.154	0.016	-0.011	-0.130
		(0.018)	(0.018)								(0.025)	(0.194)	(2.555)	(0.030)	(0.011)	(0.212)
α_3			0.047											0.056	0.054	0.201
			(0.016)											(0.023)	(0.023)	(0.381)
β_1				-0.042	-0.040	-0.043		-0.047	0.934	-0.922	0.220	0.090	-0.251	-0.858	-0.889	-0.871
				(0.020)	(0.020)	(0.019)		(0.018)	(0.024)	(0.072)	(0.188)	(0.692)	(0.701)	(0.080)	(0.088)	(0.230)
β_2					-0.013	-0.015			-0.046	0.021		0.418	-0.157		0.029	0.150
					(0.021	(0.016)			(0.019)	(0.027)		(0.192)	(2.454)		(0.039)	(0.215)
β_3						0.047				0.050			0.062			-0.148
						(0.015)				(0.020)			(0.164)			(0.377)
LL	-2430.7	-2423.0	-2419.5	-2430.7	-2430.4	-2426.6	-2433.5	-2430.7	-2427.4	-2422.5	-2422.7	-2420.7	-2418.8	-2413.5	-2413.5	-2413.3
AIC	-1.3482	-1.3522	-1.3538	-1.3482	-1.3477	-1.3495	-1.3471	-1.3476	-1.3489	-1.3513	-1.3519	-1.3525	-1.3530	-1.3568	-1.3562	-1.3557
BIC	-1.3444	-1.3466	-1.3463	-1.3444	-1.3421	-1.3420	-1.3452	-1.3420	-1.3415	-1.3420	-1.3444	-1.3431	-1.3418	-1.3475	-1.3451	-1.3427
Ljung-Box Q_{20}	37.27	37.36	27.40	37.37	37.45	27.62	41.56	37.19	35.24	15.70	36.24	30.68	23.66	14.99	15.09	14.69
p-value	0.01	0.01	0.12	0.01	0.01	0.12	0.00	0.01	0.02	0.74	0.01	0.06	0.26	0.78	0.77	0.79

Note. Standard Errors are given in parenthesis.

Table 3.145: Estimates of ARMA(p,q) model for seventh component of $\mathbf{Z_{3,t}}$

Model	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	ARMA(0,0)	ARMA(1,1)	ARMA(1,2)	ARMA(1,3)	ARMA(2,1)	ARMA(2,2)	ARMA(2,3)	ARMA(3,1)	ARMA(3,2)	ARMA(3,3)
α_0	-0.150	-0.149	-0.147	-0.152	-0.152	-0.152	-0.152	-0.009	-0.009	-0.009	-0.001	0.000	-0.342	-0.143	-0.367	-0.001
	(0.009	(0.009)	(0.010)	(0.008)	(0.008)	(0.009)	(0.008)	(0.006)	(0.006)	(0.006)	(0.001)	(0.000)	(0.040)	(0.019)	(0.023)	(0.000)
α_1	0.011	0.011	0.011					0.939	0.939	0.938	0.997	1.445	-0.441	0.039	-0.505	0.512
	(0.020)	(0.020)	(0.020)					(0.038)	(0.036)	(0.038)	(0.005)	(0.011)	(0.125)	(0.034)	(0.030)	(0.079)
α_2		0.007	0.007								-0.003	-0.446	-0.817	0.007	-0.919	-0.412
		(0.018)	(0.018)								(0.003)	(0.011)	(0.130)	(0.061)	(0.035)	(0.050)
α_3			0.012											0.012	0.014	0.896
			(0.018)											0.061	0.016	0.101
β_1				0.011	0.011	0.011		-0.926	-0.930	-0.929	-0.988	-1.442	0.452	-0.028	0.516	-0.510
				(0.014)	(0.032)	(0.019)		(0.037)	(0.033)	(0.043)	(0.005)	(0.012)	(0.125)	(0.034)	(0.023	(0.078)
β_2					0.007	0.007			0.004	-0.004		0.444	0.836		0.939	0.426
					(0.017)	(0.012)			(0.008)	(0.007)		(0.012)	(0.135)		(0.035)	(0.047)
β_3						0.011				0.008			0.013			-0.907
						(0.021)				(0.012)			(0.023)			(0.106)
LL	-2213.1	-2212.9	-2212.3	-2213.8	-2213.7	-2213.5	-2214.0	-2209.9	-2209.8	-2209.7	-2206.0	-2202.3	-2211.3	-2212.3	-2209.8	-2200.2
AIC	-1.4814	-1.4809	-1.4807	-1.4810	-1.4804	-1.4800	-1.4815	-1.4828	-1.4822	-1.4817	-1.4846	-1.4862	-1.4801	-1.4801	-1.4810	-1.4863
BIC	-1.4777	-1.4754	-1.4732	-1.4773	-1.4749	-1.4725	-1.4796	-1.4772	-1.4747	-1.4723	-1.4771	-1.4769	-1.4689	-1.4707	-1.4698	-1.4732
Ljung-Box Q_{20}	28.73	28.22	27.70	28.68	28.23	27.49	29.82	23.40	23.53	23.59	20.96	21.03	26.45	27.71	25.04	18.62
p-value	0.09	0.10	0.12	0.09	0.10	0.12	0.07	0.27	0.26	0.26	0.40	0.40	0.15	0.12	0.20	0.55

Note. Standard Errors are given in parenthesis.

3.4.12 ARFIMA Models Estimates for $log\left(\lambda_{1,t}\right)$

Table 3.146: Estimates of the $ARFIMA(p,d,q)$ model for first sequence of log transformed singular values using Whittle's approximate MLE

Model	ARFIMA(0,d,0)	ARFIMA(1,d,0)	ARFIMA(2,d,0)	ARFIMA(0,d,1)	ARFIMA(0,d,2)	ARFIMA(1,d,1)	ARFIMA(1,d,2)	ARFIMA(2,d,1)	ARFIMA(2,d,2)
d	**0.031** (0.009)	**0.045** (0.0265)	**0.050** (0.0112)	**0.040** (0.013)	**0.045** (0.017)	**0.066** (0.030)	**0.050** (0.078)	**0.069** (0.074)	**0.061** (0.021)
ϕ_0	0.044 (4.698)	1.281 (1.104)	2.294 (2.161)	5.167 (2.504)	4.748 (1.960)	5.674 (4.323)	6.620 (5.209)	0.047 (0.460)	11.036 (8.129)
ϕ_1		0.346 (0.035)	0.330 (0.019)			0.986 (0.003)	0.985 (0.005)	0.919 (0.047)	0.368 (0.013)
ϕ_2			0.056 (0.015)					0.065 (0.046)	0.610 (0.012)
θ_1				-0.209 (0.020)	-0.264 (0.022)	-0.877 (0.020)	-1.010 (0.081)	-0.917 (0.031)	-0.299 (0.016)
θ_2					-0.086 (0.018)		0.079 (0.049)		-0.534 (0.015)
σ_u^2	0.686	0.663	0.6603	0.6680	0.6645	0.6594	0.6593	0.659	0.659
LL	-22,795.20	-21,987.04	-21,826.98	-21,692.30	-21,668.76	-21,683.13	-21,583.25	-21,434.88	-21,535.01
AIC	11.232	10.716	10.625	10.551	10.538	10.546	10.484	10.393	10.484
BIC	11.184	10.689	10.593	10.508	10.496	10.419	10.446	10.455	10.505
GoF	0.329 (0.093)	0.299 (0.992)	0.301 (0.987)	0.303 (0.971)	0.300 (0.988)	0.306 (0.943)	0.304 (0.969)	0.304 (0.965)	0.3051 (0.954)

Note. Standard Errors are given in parenthesis.

Chapter 4

Forecasting

4.1 Introduction

It is generally impossible to specify a forecast evaluation criterion that is universally accepted (see, e.g., Diebold et al. 1998). This problem becomes more acute when dealing with nonlinear volatility forecasting. In spite of this, in financial econometrics, two popular methods are used for the evaluation of a model performance: in-sample evaluation and out-of-sample evaluation. The method of in-sample evaluation is used to evaluate how well a model fits the data set. It is done by using a maximized value of the log-likelihood (LL) function. One can argue that models with large number of parameters are more likely to have high LL values. Hence, AIC and BIC were introduced to provide a robust in-sample evaluation. Based on the estimation results presented in the previous chapter, we used BIC to select the best model among all the plausible models when modeling the dynamics of singular values and the corresponding singular vectors. This in-sample evaluation can give us information only about the goodness-of-fit of the model, while it cannot tell how well the model performs in terms of forecasting. In line with the methods found in the financial literature, to evaluate the out-of-sample performance of volatility model, we used some standard statistics such as MSE (Mean Square Error), RMSE (Root Mean Square Error) and MAE (Mean Absolute Error), etc. However, a major drawback of MSE is that it relies on the fourth moment (squares of the squares) of the realized returns. We can see that in the following formula:

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(\hat{\sigma}_t - \sigma_t\right)^2,$$

where σ_t is the realized covariance and $\hat{\sigma}_t$ is the forecast of the covariance of the returns. Both of these measures are considered as the second moment of returns. If the distribution of returns has a fat-tailed behavior, the MSE will be heavily biased due to the effect of large shocks and thereby is less reliable in measuring the performance of model in finance (Le Trung Thanh, 2012). Hence, another method to evaluate the out-of-sample forecasting performance of a model is in terms of the model's ability to produce interval forecasts (also known as prediction intervals) and is called Backtesting [1]. Jorian (2001) refers to these tests aptly as "reality checks". A brief summary about interval forecasts follows below.

4.1.1 Point vs interval forecasts

Forecasts are often expressed as single numbers, called *point forecasts*, which give no guidance as to their likely accuracy. In the majority of research studies conducted in the financial world, forecasting centers around producing and evaluating point forecasts. Despite of the fact that point forecasts are easy to compute, they are of limited importance since they describe only one possible outcome. Computing *interval forecasts* is an important part of the forecasting intended to indicate the likely uncertainty in the point forecasts. Simply put, they indicate the likely range of outcomes. Interval forecasts tend to be too narrow. The various reasons for this include: 1) parameter uncertainty, 2) non-normally distributed innovations, 3) identification of the wrong model, and 4) changes in the structure of the underlying model. Empirical evidence shows that out-of ample forecast errors tend to be larger than model-fitted residuals, implying that more future observations will fall outside the forecast intervals on average. Often this leads to poorer out-of-sample forecasts than what is expected from in-sample fit, particularly for interval forecasts calculated conditionally using a model fitted to past data (Chatfield, 1993).

Before proceeding further, we must define more carefully what is meant by an interval forecast. An interval forecast usually consists of an upper and a lower limit, between which the future value is expected to lie with a prescribed probability. These limits are called forecast limits (Wei 1990) or prediction bounds (Brockwell and Davis, 1991, p. 182), while the interval is sometimes called a confidence interval (e.g., Granger and Newbold, 1986) or a forecast region (Hyndman, 1995). In this thesis, we prefer the most widely-used term *forecast interval* (Abraham Ledolter, 1983; Bowerman and O'Connell; 1987, Chatfield, 1996a; Harvey, 1989), because the term *confidence interval* is usually applied to estimates of (fixed but unknown) parameters. In contrast, prediction interval is an estimate of an (unknown) future value, which can be regarded as a random variable at the time the forecast is made. Interval forecasts typically get wider as the prescribed probability increases, indicating increasing uncertainty about future values (Chatfield, 2009).

[1]Backtesting is referred to as testing a predictive model using historical data.

4.1.2 How to calculate interval forecasts?

An observed time series $\left(\ y_1,\ \ y_2,\ \ .\ \ .\ \ .\ \ ,\ \ y_T\ \right)$ is regarded as a finite realization of a stochastic process $\{y_t,\, t \in T\}$, where the index set t could, for example, be the positive integers. The point forecast of this random variable y_{t+h} is made conditional on the data up to and including time t for h steps ahead is denoted by $\hat{y}_{t+h|t}$. Note that it is essential to specify both the time at which the forecast is made and the lead time about which the forecast is made. The conditional forecast error [2] corresponding to $\hat{y}_{t+h|t}$ is also the random variable denoted by:

$$e_{t+h|t} = y_{t+h} - \hat{y}_{t+h|t}$$

The conditional forecast error is simply the difference between observed and forecast value. Similarly, let the variance of this forecast error, which is simply the mean of the conditional forecast errors, is given by:

$$\sigma^2\left(e_{t+h|t}\right) = E\left(e^2_{t+h|t}\right)$$

It should be noted here that we are not interested in the variance of the forecast but in the variance of the forecast error (the particular value of the point forecast has zero variance). After having all the measures, we are now in the position to calculate the forecast interval. A general formula for $100\,(1-\alpha)\,\%$ interval forecast for y_{t+h} is as follows:

$$\hat{y}_{t+h|t} \pm\ z_{\alpha/2}\sqrt{\sigma^2\left(e_{t+h|t}\right)}, \tag{4.1.1}$$

where $z_{\alpha/2}$ is the α-quantile of the standard normal distribution. 4.1.1 implicitly assumes that the forecast is unbiased and errors are normally distributed.[3] As described

[2]It is important to differentiate between the (out-of-sample) conditional forecast errors, the fitted residuals (within-sample "forecast" errors) and the model innovations. The out-of-sample observed errors are the true forecasting errors, whereas the within-sample observed "forecasting" errors are the residuals from the fitted model, i.e., the difference between observed and fitted values. These fitted residuals are not the same as the true model innovations because they depend on parameter estimates (and perhaps also on estimated starting values). They are not true forecasting errors if the parameters have been estimated using a part of the data but not the whole data. However, if one finds the "true" model and the latter does not change, then it is reasonable to expect the true forecast errors to have properties similar to the fitted residuals.

[3]The normality assumption may be true asymptotically, but it can be shown that one-step-ahead conditional forecast-error distribution will not in general be normal, even for linear model with normally distributed innovations, when model parameters have to be estimated from the data set used to compute forecasts. This also applies to h-step-ahead forecast errors (Phillips, 1979).

earlier, the assumption of normality of the forecast errors could be one of the reasons for the narrowness of interval forecasts. As a remedy to this problem, different error distributions are assumed in the financial literature including the fat-tailed ones, e.g., Harvey (1989) suggests replacing $z_{\alpha/2}$ in (4.1.1) by the α-quantile of the Student's t -distribution with an appropriate number of degrees of freedom. Similarly, Christoffersen (1998) evaluated these interval forecasts and reported that GARCH model with t-distribution performs best. In this regard, the study of Kuester et. al (2006) is worth mentioning here: in this study, more than six different distributions were applied in order to evaluate the model performance using VaR (Value-at-Risk) prediction. It was concluded that heavy tailed GARCH models show the best performance overall. VaR estimates have been mentioned as a special case of interval forecasts, when the intervals are one-sided (Kupiec, 1995 and Lopez, 1997). By one-sided or open interval we mean that the lower or the upper bound of interval forecasts tends to infinity.

4.2 Forecasting scheme

After a brief introduction of interval forecasts, this section describes the forecasting scheme used to construct interval forecasts with our proposed methodology.

4.2.1 Splitting of data set

In order to evaluate any model using the out-of-sample evaluation method, the most fundamental issue is how to split the sample between the fit and test periods. The former is used to identify and estimate a model (or a method), while the latter is reserved for assessing the model forecasting accuracy. The final point of time in the fit period (the point from which the forecasts are generated) is called forecasting origin, whereas the number of time periods between the origin and the time being forecast is called the lead time or the forecasting horizon. The splitting of sample is guided by several considerations (not discussed in detail here). However, keeping in mind the fact that a much larger number of forecasts is needed to examine the distribution of the forecast errors, we reserve the 40% ($t = 1, 2, ..., 1268$), which corresponds to roughly five years of trading data as a fit period, while the remaining 60% ($t = 1, 2, ..., 2000$) was reserved for the test period. The total sample size was 3268 which corresponds to 13 years of daily financial data.

4.2.2 Rolling window technique

After splitting the data set into the fit and test periods, the next obvious question to address is whether the forecasting origin should be kept constant (*Fixed-origin evaluation*) or whether it should be changed with respect to time (*Rolling-origin* *evaluation*)? However, the outcomes of an empirical forecast comparison exercise can depend on whether the model coefficients are continuously updated or are held fixed at in-sample values, especially when there are non-consistencies. Models that are robust to location shifts will have a relative advantage for fixed coefficients (see Eitrheim et. al, 1999). Continuously updating of estimation window may blunt this edge, consistent with the success of re-estimating as new information occurs (see, e.g., Phillips, 1994, 1995, 1996, and Swanson and White, 1997). In the present analysis, we apply the second one, i.e., we successively update the forecasting origin and produce forecast from each new origin. However, this succession does not change the size of the window ($w = 1268$) , which is roughly equal to five years of trading data. We do it by discarding the oldest observation at each update and adding the next available observation to the window, hence, the widow size does not change. Following this rolling-origin evaluation, we allow the corresponding parameters of the estimation equation to update each time with an increment of one day since we are dealing with daily data. Armstrong and Grohman (1972) were among the first to explicitly describe this procedure. Armstrong (1985, p. 343) also provided a schematic illustration of the rolling-origin procedure.

4.2.3 Forecasting horizon

A common objection to much long-range forecasting is that it is virtually impossible to predict with accuracy what will happen several steps ahead into the future. This is because the uncertainty increases (due to the accumulation of forecasting error) when a forecast horizon increases i.e., when a forecast is made for a period more than one steps ahead (Enders, 2010). However, at the very least, the forecast and a measure of its accuracy enable the researcher to know the risks in pursuing a selected strategy and, with this knowledge, to choose an appropriate strategy from those available.

In this study, one through five ($h = 1, 2, 3, 4, 5$) steps ahead interval forecasts were generated recursively; hence, for each forecasting horizon (h) 2000 forecasts were generated. Since we are using daily data five-step-ahead forecast corresponds to one-weak-ahead forecast.

4.2.4 Interval forecasts from SVX model

After describing the forecasting scheme in detail, we proceed by addressing the question of how interval forecasts were generated using our proposed methodology. Following the notation from the Methodology chapter, interval forecasts were constructed using the generally adopted formula; however, the notation is bit different as explained below. For example, assuming that forecast errors are uncorrelated and normally distributed, a 95% interval forecast for the next observation in a time series y_{t+h} is:

$$\hat{y}_{t+h|t} \pm \, z_{\alpha/2}\sqrt{s^2_{t+h|t}}, \tag{4.2.1}$$

where $\hat{y}_{t+h|t}$ is the h-step-ahead forecasts for the mean model of the returns, i.e., from ARMA(0,0) and ARMA(2,0). Since we assume that returns follow a constant mean process, i.e., the forecast for tomorrow's return is simply the mean of the returns until today when based on BIC, the AR(2) model was selected to forecast the mean returns; for this reason, $\hat{y}_{t+h|t}$ comes from these two models using the forecasting scheme described above. $z_{\alpha/2}$ is the percentile of the standard normal distribution and $s_{t+h|t}$ is the square root of the diagonal elements from the forecasted covariance matrices $S_{t+h|t}$. It should be noted that forecasts for these covariance matrices were generated using 2.2.1. In practice, it is common to calculate 90% and 95% interval forecasts, although any percentage may be used. In this study, we constructed 95%, 97.5%, 99%, and 99.5% interval forecasts (i.e., using $\alpha = 0.05$, 0.025, 0.01 and 0.005). This choice enables us to investigate the accuracy of the forecasts over the extremes of the distribution. We used central intervals so that the 95% interval is formed by 2.5^{th} and 97.5^{th} percentiles of the respective distribution. For example, following the standard normal distribution, the upper and the lower limit of interval forecasts were calculated using the following relationship:

$$[L\,(p)\,,\,U\,(p)] = \left[F^{-1}\left(\frac{1-p}{2}\right),\,F^{-1}\left(\frac{1+p}{2}\right)\right],$$

where $F(.)$ is the unconditional cumulative distribution function (*cdf*) of y_t.

Since three distributional assumptions were used to specify the conditional distribution of returns, interval forecasts were also constructed using these three distribution assumptions. Therefore, the percentile of the respective distribution was used when calculating the interval forecasts, for example, in the case of Student's t and skewed Student's t-distributions, $z_{\alpha/2}$ in 4.2.1 was replaced with $t_{\nu,\alpha/2}$ and $t_{\nu,\varsigma,\alpha/2}$ respectively. The ν stands for degrees of freedom parameter which measures tail-thickness, while the shape parameterς measures the skewness of the distribution.

4.3 Evaluation of interval forecasts

As mentioned earlier, one of the ways to evaluate the performance of a model is by means of its ability to produce interval forecasts. In this section, we evaluate our proposed model on this basis. A variety of different testing methods have been proposed the evaluation of interval forecasts. In this regard, one of the basic tests was proposed by Kupiec (1995) and is known as POF (*proportion-of-failures*) test.

In terms of interval forecasts, this test examines the frequency of losses and gains of returns that exceeds the corresponding interval forecasts. This *proportion-of-failure* should be in line with the selected confidence level. Using an example of daily data, if interval forecasts are computed at 99% confidence for one year (250 trading days), we would expect on average 2.5 of the observed returns to fall outside the interval bounds during this period. The point in time for which the observed value of return is falling outside the interval forecasts, is termed as "violation" or "exception". If the number of violations is smaller than the selected confidence level would indicate, the model overestimates the interval forecasts. On the contrary, too many violations signal an underestimation of interval forecasts. Naturally, it is rarely the case that we observe the exact amount of violations suggested by the confidence level. According to Dowd (2006), the confidence level [4] (i.e, the critical value) for any test should be selected to balance type-I and type-II errors. It is common to choose any arbitrary level and apply this in all of the following tests. A level of this magnitude implies that the model will be rejected only if the evidence against it is fairly strong. In the POF-test, we would then examine whether the amount of violations is reasonable compared to the expected amount.

Kupiec's POF-test is hampered by two shortcomings. First, the test is statistically weak with sample sizes, which has has already been recognized by Kupiec himself. Secondly, POF-test considers only the frequency of losses and not the time when they occur. As a result, it may fail to reject a model that produces clustered exceptions. Thus, model backtesting should not rely solely on tests of unconditional coverage (Campbell, 2005).

In this study, evaluation of interval forecasts is conducted by means of the likelihood ratio test of correct conditional coverage as proposed by Christoffersen (1998). By observing a sample path, $\{y_t\}_{t=t+h}^{T}$ of a time series y_t and the corresponding sequence of out-of-sample interval forecasts, $\left\{\left(L_{t+h|t}\left(p\right), U_{t+h|t}\left(p\right)\right)\right\}_{t=1}^{T}$ with the coverage probability p. Christoffersen (1998) shows that correctly calibrated conditional interval forecasts will provide a hit sequence I_t for $(t = 1, 2, ..., T)$, with value 1 if the realization is contained in the forecast interval, and 0 otherwise, i.e., distributed i.i.d. Bernoulli $\left(I_t \mid \Psi_{t-1} \overset{iid}{\sim} Ber\left(p\right)\right)$ with p.[5] Following Christoffersen (1998), the indicator variable I_t, for a given interval forecast at time t, is defined as follows:

[4]Note that the confidence level of the backtest is not in any way related to the confidence level used in the actual interval forecasts calculation

[5]Bernoulli trial is an experiment where a certain action is repeated many times. Each time the process has two possible outcomes, either success or failure. The probabilities of the outcomes are the same in every trial, i.e., the repeated actions must be independent of each other.

$$I_t = \begin{cases} 1, & if\ y_{t+h} \in \left[L_{t+h|t}(p), U_{t+h|t}(p)\right] \\ 0, & if\ y_{t+h} \notin \left[L_{t+h|t}(p), U_{t+h|t}(p)\right] \end{cases}.$$

The sequence of interval forecasts is efficient with respect to the information set Ψ_{t-1}, if $E[I_t \mid \Psi_{t-1}] = p$, for all t. The information set Ψ_{t-1}, simply consists of the past realizations of indicator variable $\{I_{t-1}, I_{t-2}, ..., I_t\}$. The standard requirement for evaluating the interval forecasts is simply compare the nominal coverage, $\Sigma_{t=1}^{T} I_t/T$ with the empirical coverage, p (Baillie and Bollerslev, 1992). The likelihood ratio test statistic for the correct conditional coverage LR_{cc} combines a test of unconditional coverage, LR_{uc}, with a test of independence LR_{ind} , both described below.

4.3.1 Test of unconditional coverage

A sequence of interval forecasts is said to have correct unconditional coverage if $E[I_t] = p$, for all t. Denoting the nominal coverage with p, n_1 and n_0 the realizations inside and outside the forecast interval respectively and $\hat{\pi} = n_1/(n_0 + n_1)$ is the maximum likelihood estimate of π (showing the sample proportion of successes), the null and the alternative hypotheses are formulated as follow:

Null hypothesis H_0; $E[I_t] = p$, i.e., the proportion of success is equal to the nominal coverage.

The likelihood under the null hypothesis is simply:

$$\mathcal{L}(p; I_1, I_2, ..., I_T) = (1-p)^{n_0} p^{n_1}.$$

Alternate Hypothesis H_a; $E[I_t] \neq p$ i.e., the proportion of success is not equal to the nominal coverage. The likelihood under the alternative is:

$$\mathcal{L}(\pi; I_1, I_2, ..., I_T) = (1-\pi)^{n_0} \pi^{n_1}.$$

The test for unconditional coverage can be formulated in terms of the standard likelihood ratio test:

$$LR_{uc} = -2log\left[\mathcal{L}(p; I_1, I_2, ..., I_T) / \mathcal{L}(\hat{\pi}; I_1, I_2, ..., I_T)\right] \overset{asy}{\sim} \chi_1^2.$$

In addition to an acceptable amount of violations, another equally important aspect is to make sure that these violations are serially independent, i.e., spread evenly over time. As reported by Christoffersen (1998), this test does not have the power against the alternative that the zeros and ones are clustered in the time-dependent fashion, and only the total number of ones plays a role. Therefore, a simple test for correct unconditional coverage is insufficient in the presence of higher-order moments dynamics (conditional heteroskedasticity, for example). In order to overcome this limitation, Christoffersen proposed a following test for independence (LR_{ind}) and a joint test for independence and correct coverage LR_{cc}.

4.3.2 Test of independence

The LR test for independence assumes a binary first-order Markov chain for the indicator variable, $\{I_t\}$, with a transition probability matrix given by:

$$\Pi_1 = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ \\ 1-\pi_{11} & \pi_{11} \end{bmatrix},$$

where $\pi_{ij} = Pr\left(I_t = j \mid I_t = i\right)$. Under independence $\pi_{ij} = \pi_j,\ i,\ j = 0,\ 1$, where $\pi_j = Pr\left(I_t = j\right)$, and the null hypothesis states that a violation today has no effect on the violation tomorrow. Simply put, a violation today should not depend on whether or not a violation occurred on the previous day. The approximate likelihood function for this process is:

$$\mathcal{L}\left(\Pi_1; I_2, I_3, ..., I_T \mid I_1\right) = \left(1-\pi_{01}\right)^{n_{00}} \pi_{01}^{n_{01}} \left(1-\pi_{11}\right)^{n_{10}} \pi_{11}^{n_{11}},$$

where n_{ij} represents the number of transitions from state i to state j, that is,

$$\pi_{ij} = \Sigma_{t=2}^{T} I\left(I_t = i \mid I_t = j\right).$$

It is then easy to maximize the log-likelihood function and solve for the parameters, which are simply ratios of the counts of the appropriate cells:

$$\hat{\Pi}_1 = \begin{bmatrix} \frac{n_{00}}{n_{00}+n_{01}} & \frac{n_{01}}{n_{00}+n_{01}} \\ \\ \frac{n_{10}}{n_{10}+n_{11}} & \frac{n_{11}}{n_{10}+n_{11}} \end{bmatrix}.$$

To test the hypothesis that the sequence is independent, the transition matrix for the first-order Markov chain model on the sequence is:

$$\Pi_2 = \begin{bmatrix} 1-\pi_2 & \pi_2 \\ \\ 1-\pi_2 & \pi_2 \end{bmatrix}$$

which corresponds to independence. The likelihood under the null hypothesis thus becomes:

$$\mathcal{L}\left(\Pi_2; I_2, I_3, ..., I_T \mid I_1\right) = \left(1-\pi_2\right)^{(n_{00}+n_{10})} \pi_2^{(n_{01}+n_{11})}$$

and the maximum likelihood estimate is $\hat{\Pi}_2 = \hat{\pi}_2 = (n_{01} + n_{11}) / (n_{00} + n_{10} + n_{01} + n_{11})$. The LR test of independence is:

$$LR_{ind} = -2log \left[\mathcal{L}(\Pi_2; I_2, I_3, ..., I_T \mid I_1) / \mathcal{L}(\Pi_1; I_2, I_3, ..., I_T \mid I_1)\right] \overset{asy}{\sim} \chi^2_1.$$

LR_{ind} is asymptotical chi-square distributed with one (since working with binary sequence) degree of freedom under the null hypothesis of independently distributed indicator variable values.

It is worth mentioning here that Christoffersen's independence test is a useful back test in studying the independence of violations unfortunately, it is unable to capture dependence in all forms because it considers only the dependence of observations between two successive days. It is possible that likelihood of violation today does not depend on whether a violation occurred yesterday but whether the violation occurred, for instance, a week ago.[6]

4.3.3 Test of conditional coverage

While the unconditional coverage ratio test does not check for independence, the independence test does not account for the correct coverage ratio. Although the two parameters are set to be equal, i.e., $\pi_{00} = \pi_{10}$, they are jointly unconstrained. Therefore, correct conditional coverage test is designed to test jointly for independence and the correct unconditional coverage. The above-mentioned two tests are linearly combined, resulting in a test for correct conditional coverage. The joint likelihood LR_{cc} is obtained as the sum of the two likelihood ratio LR tests and is asymptotically χ^2 distributed with two degrees of freedom. In this test, the null of unconditional coverage is tested against the alternative of the independence. Consequently, we need to find the distribution of the following test statistic:

$$LR_{cc} = -2log \left[\mathcal{L}(p; I_2, I_3, ..., I_T \mid I_1) / \mathcal{L}\left(\hat{\Pi}_1; I_2, I_3, ..., I_T \mid I_1\right)\right]$$

and have the following result in the form of theorem.

[6] Campbell (2005) in the empirical part of his study provides some evidence that Christoffersen's test is perhaps inadequate method in capturing dependence between exceptions. Haas (2001) argues that the interval forecast test by Christoffersen is too weak to produce feasible results. Therefore, he introduced an improved test for independence and coverage, using the ideas by Christoffersen and Kupiec. Haas proposes a mixed Kupiec-test, which measures the time between exceptions instead of observing only whether an exception today depends on the outcome of the previous day. Thus, the test is potentially able to capture more general forms of dependence.

Proposition 4.1 The distribution of the LR test of conditional coverage is asymptotically χ^2 with degrees of freedom $s\,(s-1)$, that is,

$$LR_{cc} = -2log\left[\mathcal{L}\left(p; I_2, I_3, ..., I_T \mid I_1\right)/\mathcal{L}\left(\hat{\Pi}_1; I_2, I_3, ..., I_T \mid I_1\right)\right] \overset{asy}{\sim} \chi^2_2.$$

Proof. See Appendix 4.7.1.

It should be noted that conditioning on the first observation in testing for unconditional coverage results $\hat{\pi} = \hat{\pi}_2 = \widehat{\Pi}_2$, which in turns implies that when the first observation is ignored. After ignoring the first observation the three LR tests are have the following numerical relationship:

$$LR_{cc} = LR_{uc} + LR_{ind}.$$

The above relationship provides a means to check in which regard the indicator sequence I_t fails the correct conditional coverage property $I_t \mid \Psi_{t-1} \overset{iid}{\sim} Ber\,(p)$. The likelihood ratio frame work proposed by Christoffersen enables one to jointly test randomness and correct coverage, while retaining the individual hypothesis as subcomponents.

Christoffersen's framework allows to examine whether the reason for not passing the test is caused by an inaccurate coverage, clustered exceptions, or even both. Campbell (2005) reported that in some cases it is possible that the model passes the joint test while still failing either the independence test or the unconditional coverage test. Therefore, it is advisable to run separate tests even when the joint test yields a positive result.

4.4 Comparison with other multivariate GARCH models

4.4.1 Dynamic Conditional Correlation (DCC) model

Among the different approaches to model the heteroskedastic time-varying covariance matrices, one is through correlation models. The first model of this type was proposed by Bollerslev (1990) and is known as the constant conditional correlation (CCC)-GARCH model. In this model, the conditional correlations are assumed to be constant and only the conditional covariances which are proportional to the product of the corresponding standard deviations were found to be time-varying. It has been reported in many studies that the assumption of constant correlation is very restrictive in a sense that correlation between assets and financial markets changes over time.

The generalization of the CCC- GARCH model was proposed by Christodoulakis and Satchell (2002), Engle (2002) and Tse & Tsui (2002) by making the conditional correlation matrix time-dependent. In such a case, the model is called a dynamic conditional correlation (DCC)-GARCH model. The model proposed by Christodoulakis and Satchell (2002) is a bivariate in nature, however, the DCC models of Engle (2002) and Tse & Tsui (2002) are multivariate and can be easily applied when dealing with high dimensional data sets (Bauwens et al. 2006). The CCC-GARCH model of Bollerslev (1990) is defined as:

$$H_t = D_t\, R\, D_t,$$

where D_t and R are $k \times k$ diagonal and square matrices, respectively, i.e.,

$$D_t = diag\left(h_{1t}^{1/2}......h_{kt}^{1/2}\right).$$

h_{it} are assumed to follow a univariate GARCH process,

$$h_{i,t} = \omega_i + \sum_{p=1}^{P_i} \alpha_{ip}\varepsilon_{i,t-p}^2 + \sum_{q=1}^{Q_i} \beta_{iq} h_{i,t-q}, \qquad i = 1, ...k.$$

with usual GARCH restrictions for non-negativity and stationary being imposed, such as non-negativity of variances and $\sum_{p=1}^{P_i} \alpha_{ip} + \sum_{q=1}^{Q_i} \beta_{iq} < 1$. The subscripts are present on the individual P and Q for each series to indicate that the chosen lag lengths do not need to be the same. Furthermore, the specification of the univariate GARCH model is not limited to the standard GARCH(p,q) model but can include any GARCH process with normally distributed errors that satisfies appropriate stationary conditions and non-negativity constraints. R is a correlation matrix. A simple estimate of R is the unconditional correlation matrix of the standardized returns $z_t = D_t^{-1} r_t$.

$$R = \rho_{ij}$$

is a symmetric positive definite. For this reason, the time-varying conditional covariances are proportional to the square root of the product of the corresponding two conditional variances, while leaving the conditional correlations time-invariant:

$$h_{ijt} = \rho_{ij}\left(h_{iit}\, h_{jjt}\right)^{1/2} \quad j = 1, ..., k,\; i = j+1, ..., k.$$

A generalized versions of CCC-GARCH model have been proposed by Engle (2002) and Tse and Tsui (2002) by considering the correlation matrix R as time-dependent.

$$H_t = D_t\, R_t\, D_t.$$

DCC models proposed by these authors are useful for modeling high-dimensional data set. Since their models are based on the decomposition of the conditional covariance matrix into conditional standard deviations and correlations, a smaller number of parameters is required in comparison to those which are resulted from a direct generalization of univariate GARCH models such as VEC and BEKK models. The conditional correlation between two random variables $r_{1,t}$ and $r_{2,t}$ with mean zero is defined as:

$$\rho_{12,t} = \frac{E_{t-1}(r_{1,t}\, r_{2,t})}{\sqrt{E_{t-1}(r_{1,t}^2)E(r_{2,t}^2)}}. \tag{4.4.1}$$

This correlation is defined by using the information known up to and including time $t-1$ and the correlation stated in the above definition must lie within the interval -1 and 1. To specify the relation between conditional correlations and conditional variances, it is convenient to write the returns as a conditional standard deviation times a standardized disturbance:

$$h_{i,t} = E_{t-1}(r_{i,t}^2), \qquad r_{i,t} = \sqrt{h_{i,t}}\, z_{i,t} \qquad i = 1, 2, \tag{4.4.2}$$

$$z_t = D_t^{-1}\, r_t, \tag{4.4.3}$$

where z_t is a standardized disturbance with a zero mean and unit variance for each series. Substitution of the relationship described in (4.4.2) in (4.4.1) yield the following result.

$$\rho_{12,t} = \frac{E_{t-1}(z_{1,t}\, z_{2,t})}{\sqrt{E_{t-1}(z_{1,t}^2)E(z_{2,t}^2)}} = E_{t-1}(z_{1,t}\, z_{2,t}). \tag{4.4.4}$$

Thus, the conditional correlation is also the conditional covariance between the standardized disturbances.

Several estimators have been proposed in the literature for conditional correlations such as rolling correlation estimator, exponential smoother, etc. In this study, the following dynamic correlation structure proposed by Engle (2002) is used:

$$R_t = Q_t^{*-1} Q_t\, Q_t^{*-1}$$

$$Q_t = \left(1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n\right)\bar{Q} + \sum_{m=1}^{M} a_m \left(z_{t-m}\, z_{t-m}^{'}\right) + \sum_{n=1}^{N} b_n Q_{t-n}. \tag{4.4.5}$$

In 4.4.5 a_m and b_n are non-negative parameters satisfying $a_m + b_n \leq 1$. $\bar{Q}$ is the unconditional covariance matrix of the standardized residuals and

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11}} & 0 & 0 & . & . & . & 0 \\ 0 & \sqrt{q_{22}} & 0 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & \sqrt{q_{kk}} \end{bmatrix},$$

so that Q_t^* is a diagonal matrix composed of a square root of the diagonal elements of Q_t. The typical element of R_t will be of the form $\rho_{ij} = \frac{q_{ij}}{\sqrt{q_{ii}q_{jj}}}$. Hence, if and Q_0 and Q_{t-1} are well-defined correlation matrices, i.e., positive definite with unit diagonal elements, R_t will also be a well-defined correlation matrix. This process ensures positive definiteness but does not generally produce valid correlation matrices (Silvennoinen & Teräsvirta, 2008). The valid correlation matrices can be obtained by rescaling R_t as follows:

$$R_t = (I \odot Q_t)^{-1/2} Q_t (I \odot Q_t)^{-1/2}.$$

Compared to the CCC model, the weaknesses of DCC-GARCH model are: the advantage of numerically simple estimation is lost due to the inversion of correlation matrix at each point in time, a few extra parameters are needed to estimate R_t, and all $K(K-1)/2$ correlation processes are restricted to follow the same dynamics.

4.4.1.1 Estimation

This section deals with the estimation of the DCC-GARCH model using two different distributions for the standardized errors; the multivariate Gaussian and the multivariate Student's t-distribution.

Multivariate Gaussian distributed errors

When the standardized errors, z_t, are multivariate Gaussian-distributed, the joint distribution of $z_1, ..., z_k$ is:

$$f(\epsilon_t) = \prod_{t=1}^{T} \frac{1}{(2\pi)^{n/2}} exp\left\{-\frac{1}{2} z_t z_t^{'}\right\}.$$

since $E[z_t] = 0$ and $E\left[z_t z_t^{'}\right] = I$. Here, $t = 1, ..., T$ is the time period used to estimate the model.

Using the rule of linear transformation of variables (see Anderson, 1984: page 13), the likelihood function for $\varepsilon_t = H^{1/2} z_t$ is:

$$\mathcal{L}(\theta) = \prod_{t=1}^{T} \frac{1}{(2\pi)^{n/2} \mid H_t \mid^{1/2}} exp\left\{-\frac{1}{2}\varepsilon^{'} H_t^{-1} \varepsilon_t\right\}, \quad (4.4.6)$$

where θ denotes the parameters of the model. Let the parameters, θ, be divided in two groups; $(\phi, \psi) = (\phi_1, \phi_2, ..., \phi_k, \psi)$, where $\phi_i = (\omega, \alpha_{1i}, ..., \alpha_{p_i i}, \beta_{1i}, ..., \beta_{q_i i})$ correspond to the parameters of the univariate GARCH model for the i^{th} asset return series, $i = 1, 2, ..., k$. $\psi = (a, b)$ are the parameters of the correlation structure in 4.4.5.

By taking the logarithm of equation (4.4.6) and substituting $H_t = D_t R_t D_t$, we get the following log-likelihood:

$$\begin{aligned}
ln\left(\mathcal{L}\left(\theta\right)\right) &= -\frac{1}{2}\sum_{t=1}^{T}\Big(kln(2\pi) + ln\left(\mid H_t \mid\right) + \varepsilon_t^{'} H_t^{-1}\varepsilon_t\Big) \\
&= -\frac{1}{2}\sum_{t=1}^{T}\Big(kln(2\pi) + ln\left(\mid D_t R_t D_t \mid\right) + \varepsilon_t^{'} D_t^{-1} R_t^{-1} D_t^{-1}\varepsilon_t\Big) \qquad (4.4.7) \\
&= -\frac{1}{2}\sum_{t=1}^{T}\Big(kln(2\pi) + 2ln\left(\mid D_t \mid\right) + ln\left(\mid R_t \mid\right) + \varepsilon_t^{'} D_t^{-1} R_t^{-1} D_t^{-1}\varepsilon_t\Big)\,.
\end{aligned}$$

The estimation of this correctly specified log-likelihood function is difficult. Due to this, DCC-model was designed to allow for the following two stage estimation. In the first stage the parameter ϕ of the univariate GARCH model is estimated for each asset series. The likelihood used in the first stage results in the replacement of R_t with the identity matrix I_k. In the second stage, the set of parameter ψ is estimated using the correctly specified log-likelihood in 4.4.7 , conditioned on parameter ϕ.

Step one

In the first stage, R_t is replaced with the identity matrix I_k in 4.4.7, which leaves us with the following quasi-likelihood function:

$$\begin{aligned}
ln\left(\mathcal{L}_1\left(\phi\right)\right) &= -\frac{1}{2}\sum_{t=1}^{T}\Big(kln\left(2\pi\right) + 2ln\left(\mid D_t \mid\right) + 2ln\left(\mid I_k \mid\right) + \varepsilon_t^{'} D_t^{-1} I_k D_t^{-1}\varepsilon_t\Big) \\
&= -\frac{1}{2}\sum_{t=1}^{T}\Big(kln\left(2\pi\right) + 2ln\left(\mid D_t \mid\right) + \varepsilon_t^{'} D_t^{-1} I_k D_t^{-1}\varepsilon_t\Big) \\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(kln\left(2\pi\right) + \sum_{i=1}^{K}\left(ln(h_{it}) + \frac{\varepsilon_{it}^2}{h_{it}}\right)\right) \qquad (4.4.8) \\
&= \sum_{i=1}^{n}\left(kln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left(ln(h_{it}) + \frac{\varepsilon_{it}^2}{h_{it}}\right)\right),
\end{aligned}$$

which is simply the sum of the log-likelihoods of the individual GARCH models for k assets, meaning that the parameters of the different univariate models may be separately determined. In this first step, the parameter set $\phi = \phi_1, ..., \phi_k$ is estimated.

When ϕ is estimated, the conditional variance h_{it} is also estimated for each asset $i = 1, ..., k$, and $z_t = D_t^{-1}\varepsilon_t$ and $\bar{Q} = E\left[z_t z_t^{'}\right]$ can be estimated. After the first step, only the parameters a and b are unknown. These parameters are estimated in the second step.

Step two

In the second step, $\psi = (a,\, b)$ is estimated using the correctly specified log-likelihood in 4.4.7, given the estimated parameters from step one. Conditioning on the parameters estimated in the first stage, the second stage quasi-likelihood function is then:

$$\begin{aligned} ln\left(\mathcal{L}_2\left(\psi\right)\right) &= -\frac{1}{2}\sum_{t=1}^{T}\left(kln\left(2\pi\right) + 2ln\left(\mid D_t \mid\right) + ln\left(\mid R_t \mid\right) + \varepsilon_t^{'} D_t^{-1} R_t^{-1} D_t^{-1}\varepsilon_t\right) \\ &= -\frac{1}{2}\sum_{t=1}^{T}\left(kln\left(2\pi\right) + 2ln\left(\mid D_t \mid\right) + ln\left(\mid R_t \mid\right) + z_t^{'} R_t^{-1} z_t\right). \qquad (4.4.9) \end{aligned}$$

Since D_t is constant when conditioning on the parameters from step one, we can exclude the constant term and maximize:

$$ln\left(\mathcal{L}_2^{*}\left(\psi\right)\right) = -\frac{1}{2}\sum_{t=1}^{T}\left(ln\left(\mid R_t \mid\right) + z_t^{'} R_t^{-1} z_t\right).$$

Since one of the objectives of this formulation is to allow the model to be estimated more easily even when the covariance matrix is very large, it can be shown under certain conditions that the pseudo-maximum-likelihood method yields consistent and asymptotically normal estimators (Engle & Sheppard, 2002). The estimated parameters from step two are consistent, but not efficient. Newey and McFadden (1994) have provided a proof for the asymptotic distribution for the above mentioned two stage parameters using generalized method of moments (GMM). The proofs for consistency and asymptotic normality of the parameter estimates of the two stage DCC estimator closely follow the results presented for GMM.

Multivariate Student's *t*-distributed errors

There are many candidates for the multivariate generalization of the univariate Student's t-distribution. In this thesis, the most commonly used distribution is considered. When the standardized errors, z_t, are multivariate Student's t-distributed, the joint density of $z_1, ..., z_T$ is:

$$f\left(z_t \mid \nu\right) = \prod_{t=1}^{T} \frac{\Gamma\left(\frac{\nu+k}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\left[\pi\left(\nu-2\right)\right]^{n/2}}\left[1 + \frac{z_t^{'} z_t}{\nu-2}\right]^{-\frac{k+\nu}{2}},$$

where $\Gamma\left(.\right)$ is the Gamma function. Again, by using the transformation rule, the likelihood function of $\varepsilon_t = H_t^{1/2} z_t$ is:

$$\mathcal{L}(\theta) = \prod_{t=1}^{T} \frac{\Gamma\left(\frac{\nu+k}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\left[\pi(\nu-2)\right]^{k/2} \mid H_t \mid^{1/2}} \left[1 + \frac{\varepsilon_t' H_t^{-1} \varepsilon_t}{\nu - 2}\right]^{-\frac{k+\nu}{2}},$$

where θ denotes the parameters of the model. The log-likelihood is obtained by taking the logarithm and substituting $H_t = D_t R_t D_t$:

$$\begin{aligned} ln\left(\mathcal{L}(\theta)\right) &= \sum_{t=1}^{T}\left(ln\left[\Gamma\left(\frac{\nu+k}{2}\right)\right] - ln\left[\Gamma\left(\frac{\nu}{2}\right)\right] - \frac{n}{2} ln\left[\pi(\nu-2)\right] - \frac{1}{2} ln\left[\mid D_t R_t D_t \mid\right] \right. \\ &\left. -\frac{\nu+k}{2} ln\left[1 + \frac{\varepsilon_t' D_t^{-1} R_t^{-1} D_t^{-1} \varepsilon_t}{\nu-2}\right]\right). \qquad (4.4.10) \end{aligned}$$

Like in Gaussian case, the parameter set θ is divided in two groups (ϕ, ψ); parameters of the univariate GARCH models (ϕ) and correlation parameters (ψ) . This time the set of correlation parameters contains one more parameter - the degrees of freedom, i.e., ν for Student's t-distribution. The optimization of 4.4.10 is difficult. Hence, also in this case the parameters are obtained in two steps. In the first step, the parameter ϕ is estimated assuming that the standardized errors are Gaussian-distributed, while the set of parameters ψ is estimated in the second step using the correctly specified log-likelihood in 4.4.10, given the parameter ϕ. The total of parameters in DCC-MGARCH model with Student's t-distributed errors is $2m+3$, including $2m$ parameters from the univariate GARCH, 2 parameters from the correlation dynamic process, and the remaining one is the degrees of freedom of the Student's t-distribution. All these parameters are estimated using the same two-step procedure as in the Gaussian distribution case. However, the log-likelihood function is different from Gaussian distribution.

Step one

Among others, Jensen and Lunde (2001) and Ventor and Jongh (2002) have shown that the change of error distribution does not virtually affect the parameters. Hence, the parameters $\phi = \phi_1, \phi_2, ..., \phi_k$ of the univariate GARCH models are estimated using the pseudo-maximum-likelihood, assuming the errors are Gaussian-distributed.

Assuming that errors are Gaussian-distributed, the first stage quasi-likelihood is the same as in 4.4.8:

$$ln\left(\mathcal{L}_1(\phi)\right) = \sum_{i=1}^{n}\left(nln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left(ln(h_{it}) + \frac{\varepsilon_{it}^2}{h_{it}}\right)\right).$$

The parameter set $\phi_i, i = 1, ..., k$ is estimated considering univariate GARCH model with the assumption that errors follow Gaussian distribution. In the second step, parameters that remain to be estimated are correlation parameters a, b and an additional parameter for Student's t distribution ν.

Step two
The parameters $\psi = (a, b, \nu)$ are estimated in the second step using the correctly specified log-likelihood in 4.4.10, given the estimated parameters in step one. The second stage quasi-likelihood function is:

$$\begin{aligned} ln\left(\mathcal{L}_2\left(\psi\right)\right) &= \sum_{t=1}^{T}\Bigg(ln\left[\Gamma\left(\frac{\nu+k}{2}\right)\right] - ln\left[\Gamma\left(\frac{\nu}{2}\right)\right] - \frac{n}{2}ln\left[\pi\left(\nu-2\right)\right] - \frac{1}{2}ln\left[\mid D_t R_t D_t \mid\right] \\ &\quad -\frac{\nu+k}{2}ln\left[1+\frac{\varepsilon_t' D_t^{-1} R_t^{-1} D_t^{-1}\varepsilon_t}{\nu-2}\right]\Bigg) \\ &= \sum_{t=1}^{T}\Bigg(ln\left[\Gamma\left(\frac{\nu+k}{2}\right)\right] - ln\left[\Gamma\left(\frac{\nu}{2}\right)\right] - \frac{n}{2}ln\left[\pi\left(\nu-2\right)\right] - \frac{1}{2}ln\left[\mid R_t \mid\right] \\ &\quad -ln\left[\mid D_t \mid\right] - \frac{\nu+k}{2}ln\left[1+\frac{z_t' R_t^{-1} z_t}{\nu-2}\right]\Bigg). \end{aligned}$$

Since D_t is constant when conditioning the parameters from step one, we can exclude the constant term and maximize the following:

$$\begin{aligned} ln\left(\mathcal{L}^*{}_2\left(\psi\right)\right) &= \sum_{t=1}^{T}\Bigg(ln\left[\Gamma\left(\frac{\nu+k}{2}\right)\right] - ln\left[\Gamma\left(\frac{\nu}{2}\right)\right] - \frac{n}{2}ln\left[\pi\left(\nu-2\right)\right] - \frac{1}{2}ln\left[\mid R_t \mid\right] \\ &\quad -\frac{\nu+k}{2}ln\left[1+\frac{z_t' R_t^{-1} z_t}{\nu-2}\right]\Bigg). \end{aligned} \tag{4.4.11}$$

4.4.1.2 Forecasting

After the parameters of the DCC model are estimated, one might be interested in forecasting the conditional covariance matrix H_{t+h} at time $t+h$ when the history up to time t is known. h-step-ahead forecasts using DCC model can be generated using the following relationship:

$$H_{t+h} = D_{t+h} R_{t+h} D_{t+h}.$$

Like estimation, the forecasting of the covariance matrix with DCC model can be split in the following two steps:

Step one: forecasting the conditional variances in D_{t+h}
Step one consists of forecasting the diagonal matrix of time varying standard deviation

from the univariate GARCH(1,1) model separately for each of the k assets. The h-step-ahead forecast of a standard GARCH(1,1) model is given by:

$$E\left[h_{i,t+h} \mid \Omega_t\right] = h_{i,t+h|t} = \sum_{i=1}^{h-2} \omega\left(\alpha+\beta\right)^i + \left(\alpha+\beta\right)^{h-1} E\left[h_{i,t+1} \mid \Omega_t\right], \tag{4.4.12}$$

where

$$E\left[h_{i,t+1} \mid \Omega_t\right] = \omega + \alpha_1 \varepsilon_{i,t}^2 + \beta_1 h_{i,t}$$

.

The memory will decline with exponential rate of $(\alpha+\beta)$. Compared with empirical studies, the GARCH(1,1) model has been criticized to have too short memory, especially with high frequency data (Peters, 2004).
The forecast for the conditional variance is.

$$E\left[D_{t+h} \mid \Omega_t\right] = diag\left(\sqrt{E\left[h_{1,t+h} \mid \Omega_t\right]} \quad , \quad . \quad . \quad . \quad , \sqrt{E\left[h_{k,t+h} \mid \Omega_t\right]} \right).$$

Step two: forecasting the conditional correlation matrix R_{t+h}
This step consists of forecasting the conditional correlation matrix of the standardized disturbances. The elements in the conditional correlation matrix, R_{t+h}, are not themselves forecasts but the ratios of the conditional covariance forecasts to the square root of the product of the conditional variance forecasts, i.e., $\hat{\rho}_{ij} = \frac{\hat{q}_{ij}}{\hat{q}_{ii}\hat{q}_{jj}}$, where $\hat{q}_{ij}$, $\hat{q}_{ii}$, and $\hat{q}_{jj}$ are the forecast elements in Q_{t+h}. Thus unbiased forecasts are not easily computed. Recall from 4.4.5 that the structure of the conditional correlation matrix is the following non-linear GARCH-like process:

$$Q_t = (1-a-b)\,\bar{Q} + a\left(z_{t-1}\, z_{t-1}^{'}\right) + bQ_{t-1}.$$

Under the assumption that $\overline{R} \approx \overline{Q}$ and $E\left[R_{t+i} \mid \Omega_t\right] \approx E\left[Q_{t+i} \mid \Omega_t\right]$ for $i = 1, ..., k$. A similar approach as in GARCH(1,1) case can be used to derive the formula for $E\left[R_{t+h} \mid \Omega_t\right]$, since it is also known that $E\left[z_{t+h-1}\, z_{t+h-1}^{'} \mid \Omega_t\right] = E\left[R_{t+k-1} \mid \Omega_t\right]$. Therefore, the formula to forecast R_{t+h} for $h > 1$ is:

$$
\begin{aligned}
E\left[R_{t+h} \mid \Omega_t\right] &\approx E\left[Q_{t+h} \mid \Omega_t\right] \\
&= (1-a-b)\,\bar{Q} + aE\left[R_{t+h-1} \mid \Omega_t\right] + bE\left[Q_{t+h-1} \mid \Omega_t\right] \\
&\approx (1-a-b)\,\bar{R} + (a+b)\,E\left[R_{t+k-1} \mid \Omega_t\right] \\
&\approx (1-a-b)\,\bar{R} + (a+b)\left[(1-a-b)\,\bar{R} + (a+b)\,E\left[R_{t+h-2} \mid \Omega_t\right]\right] \\
&= (1-a-b)\,\bar{R} + (1-a-b)\,\bar{R}\,(a+b) + (a+b)\,E\left[R_{t+h-2} \mid \Omega_t\right] \\
&\approx \ldots \\
&\approx \sum_{i=0}^{h-2} (1-a-b)\,\overline{R}\,(a+b)^i + (a+b)^{h-1} E\left[R_{t+1} \mid \Omega_t\right] \\
&= \left(1-(a+b)^{h-1}\right)\bar{R} + (a+b)^{h-1} E\left[R_{t+1} \mid \Omega_t\right], \qquad (4.4.13)
\end{aligned}
$$

where $E\left[R_{t+1} \mid \Omega_t\right] \approx \widehat{Q}_t^{*-1}\hat{Q}_{t+1}\widehat{Q}_{t+1}^{*-1}$, $\hat{Q}_{t+1} = (1-a-b)\,\bar{Q} + a\left(z_t z_t^{\prime}\right) + bQ_t$ and $\bar{R} = \bar{Q}^*\bar{Q}\bar{Q}^*$, where $\bar{Q}^*$ is a diagonal matrix with the square root of the diagonal elements of $\bar{Q}$ on the diagonal.

By using the notation $\hat{H}_{t+k} = E\left[H_{t+k} \mid \Omega_t\right]$, $\hat{R}_{t+k} = E\left[R_{t+k} \mid \Omega_t\right]$ and $\hat{D}_{t+k} = E\left[D_{t+k} \mid \Omega_t\right]$, we can finally calculate $\hat{H}_{t+k} = \hat{D}_{t+k}\hat{R}_{t+k}\hat{D}_{t+k}$. Even the forecast of the conditional correlation matrix will in the long run converge to the unconditional correlation matrix of the standardized residuals. Engle and Sheppard (2002) reported that R_{t+1} will decay with the ratio of $(a+b)$ for each further step ahead.

4.4.2 Generalized Orthogonal GARCH (GO-GARCH) model

This class of multivariate GARCH (MGARCH) model is constructed as a linear combination of the univariate GARCH models. Each of these univariate GARCH models is not necessarily a standard GARCH model but could be EGARCH model of Nelson (1991), the APARCH model of Ding et al. (1993), FIGARCH of Baillie et al. (1996) or could be any of the existing GARCH model not listed here.

Since GO-GARCH model was proposed by van der Weide. R (2002) as the generalization of the Orthogonal GARCH (O-GARCH) model of Alexander & Chibumba (1997), it is important to first describe the OGARCH model in order to have a better understanding of GO-GARCH model.

Orthogonal GARCH (O-GARCH) model

The basic idea behind orthogonal GARCH model is that the observed data are assumed to be generated by an orthogonal transformation of k (or a small number of) linear combination of uncorrelated components. Each of these components follows a GARCH process. The matrix of linear transformation is an orthogonal matrix of eigenvectors of the population unconditional covariance matrix of the standardized re-

turns. In the orthogonal GARCH model of Kariya (1988) and Alexander & Chibumba (1997), the $k \times k$ time varying covariance matrix is generated by $m \leq k$ univariate GARCH models (where m indicates the number of components).
The O-GARCH(1,1,m) model is defined as follows:

$$V^{-1/2}\varepsilon_t \;\; = \Lambda_m f_t = P\, L^{1/2} f_t$$

$$\Sigma_t = E_{t-1}\left(f_t\, f_t^{'}\right) = diag\left(\sigma^2_{f1t},\, \ldots \sigma^2_{fmt}\right)$$

$$H_t = V^{1/2} P\, L^{1/2}\, \Sigma_t\, P\, L^{1/2}\, V^{1/2},$$

where $V = diag\,(v_1, v_2, ..., v_k)$, v_i is the variance of ε_{it} , L and P are $m \times m$ and $k \times m$ matrices of the largest eigenvalues and the associated eigenvectors of the unconditional correlation matrix, respectively, so that:

$$\Lambda_m = P_m diag\left(l_1^{1/2}.....l_m^{1/2}\right),$$

where $l_1 \geq\, \ldots\, \geq l_m > 0$. The vector $f_t = (f_{1t},\, \ldots f_{mt})^{'}$ is a random process described by a GARCH(1,1) model.

$$\sigma^2_{fit} = (1 - \alpha_i - \beta_i) + \alpha_i\, f^2_{i,t-1} + \beta_i \sigma^2_{fi,t-1}, \quad i = 1,\, \ldots, m. \tag{4.4.14}$$

The parameters of the model are V, Λ_m and the parameters of the GARCH factors are $\alpha_i^{\prime}$s and $\beta_i^{\prime}$s. The number of parameters is $K\,(K+5)\,/2$ if $m = K$. In practice, V and Λ_m are replaced by their sample counterparts, and m is chosen by principal component analysis, which is applied to the standardized residuals. It should be noted that if the chosen m is smaller than k , the conditional covariance matrix has a reduced rank, which may be a problem for applications and diagnostics tests where the inverse of H_t is used.

Generalized Orthogonal GARCH (GO-GARCH) model

As mentioned earlier, the GO-GARCH model was proposed by van der Weide. R (2002) as the generalization of the O-GARCH model. The basic idea behind this model is the same (observed process is a linear combination of uncorrelated components) except that the orthogonality condition imposed in O-GARCH model was relaxed by arguing that the matrix $\Lambda_m = P\, L^{1/2}$ in the following relation is square and invertible, but not necessarily orthogonal:

$$V^{-1/2}\varepsilon_t = \Lambda_m f_t.$$

The parametrization of the $\mathit{\Lambda}_m$ was done through singular value decomposition (van der Weide. R (2002)) i.e., $\mathit{\Lambda}_m = P\,L^{1/2}U_0$, where matrix U is orthogonal and P and L are the eigenvalues and the eigenvectors respectively which can be extracted by exploiting the unconditional information i.e., from the sample covariance matrix of the observed process. However, conditional information is required to estimate U_0. The O-GARCH model is then a particular case of the GO-GARCH model where $U = I_k$.

According to [Weide, R. van der (2002.Lemma 3)], every k- dimensional orthogonal matrix U with $det\,(U) = 1$ can be represented as a product of $\begin{pmatrix} k \\ 2 \end{pmatrix} = [k(k-1)/2]$ rotation matrices:

$$U = \prod_{i<j} R_{ij}\left(\theta_{ij}\right) \quad -\pi \leq \theta_{ij} \leq \pi, \quad i,j = 1,...,k.$$

where $R_{ij}\left(\theta_{ij}\right)$ performs a rotation in the plane spanned by e_i and e_j over an angle θ_{ij}.

For proof, see Vilenkin (1968).

The implied conditional correlations $\{R_t\}$ of the observed process can be computed as:

$$R_t = D_t^{-1} V_t D_t^{-1} \quad D_t = \left(V_t \circ I\right)^{1/2},$$

where $\{V_t\} = \left\{ZH_tZ'\right\}$ denotes the conditional covariances of the observed process and $\circ$ denotes the Hadamard product.

4.4.2.1 Estimation

Multivariate Gaussian distributed errors

If the errors follow multivariate Gaussian distribution then by exploiting the conditional information the parameters that need to be estimated include the vector θ of rotation coefficients and the parameters for each univariate GARCH(1,1) specifications. The log-likelihood for the GO-GARCH model can be represented as:

$$\begin{aligned} ln\left(\mathcal{L}_{\theta,\alpha,\beta}\right) &= -\frac{1}{2}\sum_t klog\left(2\pi\right) + log \mid V_t \mid + \varepsilon_t' V_t^{-1} \varepsilon_t \\ &= -\frac{1}{2}\sum_t klog\left(2\pi\right) + log \mid \mathit{\Lambda}_m \Sigma_t \mathit{\Lambda}_m' \mid + \varepsilon_t' \left(\mathit{\Lambda}_m \Sigma_t \mathit{\Lambda}_m'\right)^{-1} \varepsilon_t \\ &= -\frac{1}{2}\sum_t klog\left(2\pi\right) + log \mid \mathit{\Lambda}_m \mathit{\Lambda}_m' \mid + log \mid \Sigma_t \mid + u_t' \Sigma_t^{-1} u_t. \quad (4.4.15) \end{aligned}$$

Identity matrix is taken for the initial value of H_0, which is equal to the implied unconditional covariance of u_t. Van der Weide. R (2002) argued that even in high dimensional data sets where the covariance matrices are very large it should not be a problem to maximize the log likelihood over the $[k(k-1)/2]+2k$ parameters.

It should be noted that in order to avoid convergence problems of estimation algorithms, Van der Weide. R (2002) proposed a kind of two-step estimation. By exploiting the unconditional information first, the number of parameters for Λ that are estimated through maximum likelihood is $[k(k-1)/2]$ instead of k^2(see, Van der Weide. R (2002: Lemma 2)).

Multivariate Student's t-distributed errors

Student t-density could be the natural alternative of the Gaussian density with an extra scalar parameter, the degrees of freedom (denoted by ν). When this parameter tends to infinity, the Student's t-density tends to Gaussian density. This parameter determines the existence of the moments, e.g., if $\nu = 2$, only the first order moment exists, whereas the second order moment does not. Therefore, it is convenient, although not necessary, to assume that $\nu > 2$, which makes it easy to interpret H_t as a conditional covariance matrix.

Under the assumption that the errors follow Student's t-distribution, the log likelihood for the GO-GARCH model takes the following form:

$$\begin{aligned} ln\left(\mathcal{L}_{\theta,\alpha,\beta,\nu}\right) &= \sum_{t=1}^{T}\Bigg(ln\left[\Gamma\left(\frac{\nu+k}{2}\right)\right] - ln\left[\Gamma\left(\frac{\nu}{2}\right)\right] - \frac{n}{2}ln\left[\pi\left(\nu-2\right)\right] - \frac{1}{2}ln\left[\mid V_t \mid\right] \\ &\quad -\frac{\nu+k}{2}ln\left[1+\frac{\varepsilon_t^{'}\left(V_t\right)^{-1}\varepsilon_t}{\nu-2}\right]\Bigg) \\ &= \sum_{t=1}^{T}\Bigg(ln\left[\Gamma\left(\frac{\nu+k}{2}\right)\right] - ln\left[\Gamma\left(\frac{\nu}{2}\right)\right] - \frac{n}{2}ln\left[\pi\left(\nu-2\right)\right] \\ &\quad -\frac{1}{2}ln\left[\mid \Lambda_m\Sigma_t\Lambda_m^{'} \mid\right] - \frac{\nu+k}{2}ln\left[1+\frac{\varepsilon_t^{'}\left(\Lambda_m\Sigma_t\Lambda_m^{'}\right)^{-1}\varepsilon_t}{\nu-2}\right]\Bigg) \\ &= \sum_{t=1}^{T}\Bigg(ln\left[\Gamma\left(\frac{\nu+k}{2}\right)\right] - ln\left[\Gamma\left(\frac{\nu}{2}\right)\right] - \frac{n}{2}ln\left[\pi\left(\nu-2\right)\right] - \frac{1}{2}ln\left[\mid \Lambda_m\Lambda_m^{'} \mid\right] \\ &\quad -\frac{1}{2}ln\left[\mid \Sigma_t \mid\right] - \frac{\nu+k}{2}ln\left[1+\frac{u_t^{'}\Sigma^{-1}u_t}{\nu-2}\right]\Bigg), \end{aligned} \tag{4.4.16}$$

where $\Gamma(.)$ is the Gamma function.

Multivariate skew Student's t-distributed errors

As for the multivariate Student's t-distribution, there are also many candidates for the multivariate skew Student's t-distribution. Here, we used the skewed Student's t-distribution described by Azzalini and Capitanio (2003). When the errors ε_t are assumed to follow a multivariate skewed Student's t-distribution, the joint distribution of $\varepsilon_1, ..., \varepsilon_t$ is:

$$f(\varepsilon_t \mid \nu, \varsigma) = \prod_{t=1}^{T} 2t_d(\varepsilon_t; \nu, \varsigma)\, T_1 \left\{ \delta^T D^{-1}(\varepsilon_t - \xi) \left[\frac{\nu + k}{Q_{\varepsilon_t} + \nu} \right]^{1/2}; \nu + k \right\}, \tag{4.4.17}$$

where D is the diagonal matrix with the square root of the diagonal elements of Ω on the diagonal,

$$Q_{\varepsilon_t} = (\varepsilon_t - \xi)^T \Omega^{-1} (\varepsilon_t - \xi),$$

$$t_d(\varepsilon_t; \nu, \varsigma) = \frac{\Gamma\left(\frac{\nu+k}{2}\right)}{|\, \Omega \,|^{1/2} (\pi\nu)^{1/2}\, \Gamma(\nu/2)} \left[1 + \frac{Q_{\varepsilon_t}}{\nu} \right]^{-(\nu+n)/2},$$

where $T_1(\,.\,; \nu + k)$ denotes the scalar Student's t-distribution with $\nu + k$ degrees of freedom, and $\Gamma(\,.\,)$ is the Gamma function. The above joint density is well-defined if $\nu > 2$.

We shall denote the skewed Student's t-distribution as:

$$\mathbf{Y} \sim St_d(\xi, \Omega, \delta, \nu).$$

Define:

$$\varsigma = D^{-1}\delta. \tag{4.4.18}$$

Aas and Dimakos (2005) showed that Azzalini's skewed Student's t-distribution may be standardized to have a mean vector of value 0 and covariance matrix I_k by letting:

$$\Omega = \begin{cases} \frac{\nu-2}{\nu} \left[I_k + \frac{1}{\varsigma^T\varsigma} \left(-1 + \frac{\pi\Gamma\left(\frac{\nu}{2}\right)^2 \left(\nu(\nu-2)\varsigma^T\varsigma\right)}{2\varsigma^T\varsigma \left[\pi\Gamma\left(\frac{\nu}{2}\right)^2 - (\nu-2)\Gamma\left(\frac{\nu-1}{2}\right)^2\right]} (1 + K) \right) \varsigma\varsigma^T \right] & for\, \varsigma \neq 0 \\ \frac{\nu-2}{\nu} I_k, & for\, \varsigma = 0 \end{cases}. \tag{4.4.19}$$

Here

$$K = \sqrt{1 + \frac{4\nu(\nu-2)\left[\pi\Gamma\left(\frac{\nu}{2}\right)^2 - (\nu-2)\Gamma\left(\frac{\nu-1}{2}\right)^2\right]\varsigma^T\varsigma}{\pi\Gamma\left(\frac{\nu}{2}\right)^2 \left(\nu - (\nu-2)\varsigma^T\varsigma\right)^2}}$$

and

$$\xi = -\sqrt{\frac{\nu}{\pi}}\frac{\Gamma\left(\frac{\nu-1}{2}\right)}{\Gamma\left(\nu/2\right)}\frac{\Omega\varsigma}{\sqrt{1+\varsigma^T\Omega\varsigma}}. \tag{4.4.20}$$

By using the transformation rule, the likelihood function of $\varepsilon_t = H_t^{1/2} z_t$ is:

$$L\left(\theta \mid F_t\right) = \prod_{t=1}^{T} 2t_d\left(H_t^{1/2}\varepsilon_t;\nu,\varsigma\right) T_1\left\{\delta^T D^{-1}\left(H_t^{1/2}\varepsilon_t - \xi\right)\left[\frac{\nu+k}{Q_{\varepsilon_t}+\nu}\right]^{1/2};\nu+k\right\}\frac{1}{\mid H_t\mid^{1/2}},$$

where

$$Q_{\varepsilon_t} = \left(H_t^{-1/2}\varepsilon_t - \xi\right)^T \Omega^{-1}\left(H_t^{-1/2}\varepsilon_t - \xi\right)$$

and θ denotes the parameters of the model.
We get the log-likelihood by taking the logarithm and substituting $H_t = \Lambda_m \Sigma_t \Lambda_m^{'}$:

$$\begin{aligned} ln\left(L\left(\theta\right)\right) &= \sum_{t=1}^{T}\Bigg(ln\left(2\right) + ln\left[t_d\left(H_t^{-1/2}\varepsilon_t;\nu,\varsigma\right)\right] \\ &+ ln\left[T_1\left\{\delta^T D^{-1}\left(H_t^{-1/2}\varepsilon_t - \xi\right)\left[\frac{\nu+k}{Q_{\varepsilon_t}+\nu}\right]^{1/2};\nu+k\right\}\right] \\ &- \frac{1}{2}ln\left[\mid H_t\mid\right]\Bigg), \end{aligned} \tag{4.4.21}$$

where θ is divided in two groups; $(\phi,\psi) = (\phi_1,\phi_2,....,\phi_k,\psi)$, where ϕ is the vector of parameters of the univariate GARCH model for the i^{th} factor, $i = 1,2,...,k$, and ψ is the vector that consists of the parameters of the multivariate skewed Student's t-distribution.

Under the assumption that the errors follow the skewed Student's t-distribution, the log likelihood for the GO-GARCH model takes the following form:

$$\begin{aligned} ln\left(L\left(\theta\right)\right) &= \sum_{t=1}^{T}\Bigg(ln\left(2\right) + ln\left[t_d\left(\left(\Lambda_m\Sigma_t\Lambda_m^{'}\right)^{-1/2}\varepsilon_t;\nu,\varsigma\right)\right] \\ &+ ln\left[T_1\left\{\delta^T D^{-1}\left(\left(\Lambda_m\Sigma_t\Lambda_m^{'}\right)^{-1/2}\varepsilon_t - \xi\right)\left[\frac{\nu+k}{Q_{\varepsilon_t}+\nu}\right]^{1/2};\nu+k\right\}\right] \\ &- \frac{1}{2}ln\left[\mid \Lambda_m\Sigma_t\Lambda_m^{'}\mid\right]\Bigg) \end{aligned}$$

$$
\begin{aligned}
ln\left(L\left(\theta\right)\right) &= \sum_{t=1}^{T}\Bigg(ln\left(2\right)+ln\left[\Gamma\left(\frac{\nu+k}{2}\right)\right]-\frac{1}{2}ln\left[\mid\Omega\right]-\frac{n}{2}ln\left[\pi\nu\right]-ln\left[\Gamma\left(\frac{\nu}{2}\right)\right]\\
&- \frac{\nu+k}{2}ln\left[1+\frac{Q_{\varepsilon_t}}{\nu}\right]\\
&+ \left[lnT_1\left\{\delta^T D^{-1}\left(\left(\Lambda_m\Sigma_t\Lambda_m^{'}\right)^{-1/2}\varepsilon_t-\xi\right)\left[\frac{\nu+k}{Q_{\varepsilon_t}+\nu}\right]^{1/2};\nu+k\right\}\right]\\
&- \frac{1}{2}ln\left[\mid\Sigma_t\mid\right]-ln\left[\mid\Lambda_m\mid\right]\Bigg).
\end{aligned}
$$

We exclude the constant term and maximize:

$$
\begin{aligned}
ln\left(L\left(\theta\right)\right) &= \sum_{t=1}^{T}\Bigg(ln\left[\Gamma\left(\frac{\nu+k}{2}\right)\right]-\frac{1}{2}ln\left[\mid\Omega\right]-\frac{n}{2}ln\left[\pi\nu\right]-ln\left[\Gamma\left(\frac{\nu}{2}\right)\right]\\
&- \frac{\nu+k}{2}ln\left[1+\frac{Q_{\varepsilon_t}}{\nu}\right]\\
&+ \left[lnT_1\left\{\delta^T D^{-1}\left(\left(\Lambda_m\Sigma_t\Lambda_m^{'}\right)^{-1/2}\varepsilon_t-\xi\right)\left[\frac{\nu+k}{Q_{\varepsilon_t}+\nu}\right]^{1/2};\nu+k\right\}\right]\\
&- \frac{1}{2}ln\left[\mid\Sigma_t\mid\right]-ln\left[\mid\Lambda_m\mid\right]\Bigg),
\end{aligned}
\tag{4.4.22}
$$

inserting 4.4.19 and 4.4.20 for Ω and ξ respectively.

Estimation problems

In multivariate data, the choice of starting values is often extremely important. When the number of parameters that have to estimated is large, the likelihood function becomes flat, and there is a great danger of reaching a local optimum. To make sure that a global maximum has already been found, one should run the estimation with many different starting values. One way to choose the starting values is to make a grid of the possible values the parameters may take and choose the starting values as the combination of values that yields the highest likelihood.

Another convergence problem can occur if there are outliers in the data. Then the gradient algorithm used for the maximization may hit a boundary. To deal with this problem, one should try to remove the outliers. If there is still a problem with convergence after removing the outliers, one should try to change the starting values.

4.4.2.2 Forecasting

Forecasts from GO-GARCH model can be generated using the following relation:

$$H_{t+h} = V^{1/2} \Lambda \, \Sigma_{t+h} \, \Lambda \, V^{1/2}, \tag{4.4.23}$$

where, as stated earlier $\Sigma_t = E_{t-1}\left(f_t \, f_t^{'}\right) = diag\left(\sigma^2_{f1t}, \ldots \sigma^2_{fmt}\right)$, means that we need to forecast the conditional variances of the factors and then plug these forecasted variances in the above relation to get the forecast of the conditional covariance matrix. It should be noted that the multi-step-ahead forecasts for conditional variances are obtained using the same relation as described in the forecasting section of DCC-GARCH model when forecasting D_{t+h}.

4.5 Results

4.5.1 Estimation Results

Before presenting the forecasting results from our proposed model and the two competitive models, we would like to present the estimation results for these two competitive models. It should be noted that for our proposed model, these estimates were already presented in the last chapter. Hence, here we report only the estimates for the two competing models.

Table 4.1 shows the total number of parameters needed to estimate the conditional mean and the conditional covariance for all models considered in this study while assuming different error distributions. We did not estimate DCC using skewed t-distribution; the reason for this will be presented later in this chapter. The fourth and the sixth columns of the table show the number of parameters required to estimate the conditional mean and the conditional covariance, respectively; the last column of the table shows their total. From this table, one can easily notice that our proposed model overcomes the curse of dimensionality problem faced by the other two competitive models. In case of our proposed model, the number of parameters need to estimate SVX model reduces to even one-fourth as compared to the other two models. Even if we assume that the returns follow ARMA(0,0) mean process, this number still reduces more.

It should be noted that in Tables 4.2 and 4.3 we reported the estimates of the ARMA and the GARCH models only. Besides these, both DCC and GO-GARCH models involve estimation of symmetric correlation matrix and the linear mapping matrix, respectively. These matrices contain $K\,(K-1)\,/2$ elements. For the data set used for empirical purposes in this study, these matrices contain 406 elements in addition to the parameters shown in Tables 4.2 and 4.3. Since these matrices are too large to show here and hence, we have included them on the CD supplemented to this dissertation.

Table 4.1: Total number of parameters with different error distributions

Model	Distribution	Mean Model	# Parameters	Variance Model	# Parameters	Total Parameters
DCC	**Gaussian**	ARMA(0,0)	29	GARCH(1,1) + DCC(1,1) + Corr.	87 + 2 + 406 = 495	524
		ARMA(2,0)	87	GARCH(1,1) + DCC(1,1) + Corr.	87 + 2 + 406 = 495	582
	Student's t	ARMA(0,0)	29	GARCH(1,1) + DCC(1,1) + Corr.	87 + 3 + 406 = 496	525
		ARMA(2,0)	87	GARCH(1,1) + DCC(1,1) + Corr.	87 + 3 + 406 = 496	583
GO-GARCH	**Gaussian**	ARMA(0,0)	29	GARCH(1,1) + Corr.	87 + 406 = 493	522
		ARMA(2,0)	87	GARCH(1,1) + Corr.	87 + 406 = 493	580
	Student's t	ARMA(0,0)	29	GARCH(1,1) + Corr.	87 + 1 + 406 = 494	523
		ARMA(2,0)	87	GARCH(1,1) + Corr.	87 + 1 + 406 = 494	581
	Skewed t	ARMA(0,0)	29	GARCH(1,1) + Corr.	87 + 2 + 406 = 495	524
		ARMA(2,0)	87	GARCH(1,1) + Corr.	87 + 2 + 406 = 495	582
SVX	**Gaussian**	ARMA(0,0)	29	ARMA(0,0) + ARMA(1,1)	29 + 9 = 38	67
		ARMA(2,0)	87	ARMA(0,0) + ARMA(1,1)	29 + 9 = 38	125
	Student's t	ARMA(0,0)	29	ARMA(0,0) + ARMA(1,1)	29 + 9 = 38	67
		ARMA(2,0)	87	ARMA(0,0) + ARMA(1,1)	29 + 9 = 38	125
	Skewed t	ARMA(0,0)	29	ARMA(0,0) + ARMA(1,1)	29 + 9 = 38	67
		ARMA(2,0)	87	ARMA(0,0) + ARMA(1,1)	29 + 9 = 38	125

Note: In GO-GARCH model, the GARCH parameters represents the factor while in SVX model, the singular vectors $U's$ and the singular values $\lambda's$ are modeled using ARMA(0,0) and ARMA(1,1) models respectively.

Table 4.2 and 4.3 show the estimates of the DCC-GARCH and GO-GARCH models with different distributional assumptions for errors when the mean returns follow ARMA(0,0) (means those returns follow a constant mean process) and ARMA(2,0) processes, respectively. The estimates of ARMA(0,0) are shown against $\mu_1, \mu_2, ..., \mu_{29}$ while the GARCH estimates are presented against ω_i, α_i and β_i. In case of GO-GARCH model, ω_i, α_i, and β_i show the GARCH estimates for factors $f_1, ..., f_{29.}$

For the DCC model, it can be seen that the estimates of the first stage, i.e., GARCH estimates are the same for Gaussian and Student's t-distributions. This is because in the first step of estimation under both distributional assumptions we assumed that errors follow Gaussian distribution. However, the estimated parameters for the dynamic conditional correlation matrix (R_t) are a bit different. For example, in the case of Gaussian distributed errors, the parameters a and b are 0.004 and 0.957, respectively, whereas in the case of Student's t-distributed errors, these estimates are $a = 0.002$ and $b = 0.968$, i.e., a is bit smaller, while b is a bit larger. Student's t-distribution has one more parameter known as shape parameter (ν) which was estimated as 8.212. The estimate of this parameter is shown in Table 4.2. The statistical significance of this shape parameter lead us to conclude that the distribution of returns are not normally distributed.

One more important feature regarding DCC model is that the mean filtering process does not affect the estimates of the mean μ_t. It can be seen in both tables that the estimates of conditional means are the same for ARMA(0,0) and ARMA(2,0). The reason for this could be that the AR(2) estimates (ϕ_1 & ϕ_2) are too

Table 4.2: Estimates of multivariate GARCH models with different error distributions with mean model = ARMA(0,0)

	DCC-GARCH		**GO-GARCH**		
Conditional distribution	**Gaussian**	**t**	**Gaussian**	**t**	**Skewed t**
μ_1	0.060 (0.021)	0.060 (0.021)	0.068 (0.028)	0.066 (0.022)	0.056 (0.033)
ω_1	0.038 (0.011)	0.038 (0.011)	0.004 (0.009)	0.012 (0.009)	0.036 (0.010)
α_1	0.080 (0.013)	0.080 (0.013)	0.015 (0.009)	0.050 (0.011)	0.082 (0.016)
β_1	0.902 (0.016)	0.902 (0.016)	0.908 (0.011)	0.904 (0.013)	0.902 (0.014)
μ_2	0.039 (0.028)	0.039 (0.028)	0.026 (0.033)	0.007 (0.025)	0.039 (0.027)
ω_2	0.043 (0.041)	0.043 (0.041)	0.004 (0.011)	0.003 (0.006)	0.016 (0.009)
α_2	0.051 (0.036)	0.051 (0.036)	0.028 (0.009)	0.018 (0.010)	0.056 (0.019)
β_2	0.936 (0.046)	0.936 (0.046)	0.940 (0.012)	0.947 (0.009)	0.941 (0.015)
μ_3	0.050 (0.022)	0.050 (0.022)	-0.001 (0.036)	0.009 (0.025)	0.041 (0.022)
ω_3	0.010 (0.005)	0.010 (0.005)	0.004 (0.003)	0.010 (0.005)	0.020 (0.005)
α_3	0.050 (0.010)	0.050 (0.010)	0.052 (0.007)	0.045 (0.011)	0.055 (0.009)
β_3	0.949 (0.009)	0.949 (0.009)	0.944 (0.007)	0.941 (0.009)	0.942 (0.010)
μ_4	0.059 (0.016)	0.059 (0.016)	0.000 (0.015)	0.028 (0.016)	0.046 (0.024)
ω_4	0.026 (0.008)	0.026 (0.008)	0.006 (0.005)	0.007 (0.006)	0.025 (0.022)
α_4	0.121 (0.023)	0.121 (0.023)	0.047 (0.013)	0.022 (0.017)	0.119 (0.087)
β_4	0.863 (0.023)	0.863 (0.023)	0.821 (0.021)	0.870 (0.017)	0.867 (0.121)
μ_5	0.027 (0.020)	0.027 (0.020)	0.018 (0.024)	0.017(0.020)	0.026 (0.018)
ω_5	0.014 (0.005)	0.014 (0.005)	0.014 (0.004)	0.008 (0.004)	0.008 (0.004)
α_5	0.044 (0.009)	0.044 (0.009)	0.051 (0.006)	0.026 (0.007)	0.042 (0.008)
β_5	0.948 (0.010)	0.948 (0.010)	0.935 (0.010)	0.965 (0.008)	0.954 (0.008)
μ_6	0.088 (0.022)	0.088 (0.022)	0.046 (0.029)	0.093 (0.023)	0.080 (0.018)
ω_6	0.043 (0.012)	0.043 (0.012)	0.007 (0.009)	0.005 (0.010)	0.034 (0.010)
α_6	0.086 (0.013)	0.086 (0.013)	0.022 (0.010)	0.019 (0.011)	0.081 (0.011)
β_6	0.896 (0.014)	0.896 (0.014)	0.970 (0.011)	0.974 (0.013)	0.902 (0.012)
μ_7	0.066 (0.026)	0.066 (0.026)	0.021 (0.048)	0.017 (0.029)	0.057 (0.031)
ω_7	0.018 (0.009)	0.018 (0.009)	0.083 (0.006)	0.012 (0.007)	0.019 (0.007)
α_7	0.077 (0.020)	0.077 (0.020)	0.178 (0.009)	0.053 (0.012)	0.074 (0.011)
β_7	0.922 (0.020)	0.922 (0.020)	0.927 (0.009)	0.928 (0.010)	0.925 (0.010)
μ_8	0.044 (0.018)	0.044 (0.018)	0.032 (0.021)	0.037 (0.016)	0.046 (0.016)
ω_8	0.026 (0.036)	0.026 (0.036)	0.005 (0.007)	0.004 (0.010)	0.027 (0.010)
α_8	0.055 (0.053)	0.055 (0.053)	0.022 (0.011)	0.019 (0.018)	0.074 (0.019)
β_8	0.925 (0.080)	0.925 (0.080)	0.923 (0.015)	0.925 (0.022)	0.927 (0.023)
μ_9	0.011 (0.027)	0.011 (0.027)	0.001 (0.028)	0.004 (0.023)	0.023 (0.025)
ω_9	0.041 (0.034)	0.041 (0.034)	0.022 (0.010)	0.003 (0.011)	0.036 (0.011)
α_9	0.065 (0.032)	0.065 (0.032)	0.102 (0.010)	0.017 (0.012)	0.068 (0.015)
β_9	0.920 (0.043)	0.920 (0.043)	0.901 (0.012)	0.909 (0.014)	0.917 (0.015)
μ_{10}	0.060 (0.023)	0.060 (0.023)	0.030 (0.028)	0.038 (0.021)	0.039 (0.023)
ω_{10}	0.076 (0.032)	0.076 (0.032)	0.002 (0.014)	0.009 (0.008)	0.022 (0.009)
α_{10}	0.120 (0.043)	0.120 (0.043)	0.032 (0.018)	0.033 (0.012)	0.060 (0.013)
β_{10}	0.850 (0.048)	0.850 (0.048)	0.865 (0.020)	0.854 (0.012)	0.861 (0.014)
μ_{11}	0.046 (0.021)	0.046 (0.021)	0.009 (0.029)	0.029 (0.023)	0.049 (0.023)
ω_{11}	0.016 (0.007)	0.016 (0.007)	0.002 (0.004)	0.011 (0.005)	0.014 (0.009)
α_{11}	0.067 (0.018)	0.067 (0.018)	0.023 (0.018)	0.037 (0.010)	0.062 (0.044)
β_{11}	0.927 (0.019)	0.927 (0.019)	0.928 (0.020)	0.925 (0.011)	0.933 (0.036)
μ_{12}	0.057 (0.018)	0.057 (0.018)	0.020 (0.022)	0.029 (0.017)	0.044 (0.017)
ω_{12}	0.019 (0.012)	0.019 (0.012)	0.073 (0.005)	0.017 (0.004)	0.013 (0.004)
α_{12}	0.075 (0.029)	0.075 (0.029)	0.150 (0.011)	0.082 (0.010)	0.056 (0.011)
β_{12}	0.914 (0.033)	0.914 (0.033)	0.918 (0.012)	0.903 (0.011)	0.916 (0.012)
μ_{13}	0.021 (0.027)	0.021 (0.027)	-0.002 (0.034)	0.036 (0.023)	0.035 (0.020)
ω_{13}	0.046 (0.075)	0.046 (0.075)	0.003 (0.014)	0.007 (0.019)	0.063 (0.023)
α_{13}	0.044 (0.046)	0.044 (0.046)	0.022 (0.009)	0.030 (0.016)	0.074 (0.019)
β_{13}	0.941 (0.069)	0.941 (0.069)	0.957 (0.013)	0.959 (0.020)	0.947 (0.023)
μ_{14}	0.042 (0.021)	0.042 (0.021)	0.021 (0.028)	0.027 (0.023)	0.041 (0.018)
ω_{14}	0.030 (0.011)	0.030 (0.011)	0.016 (0.008)	0.007 (0.008)	0.027 (0.009)
α_{14}	0.082 (0.019)	0.082 (0.019)	0.089 (0.012)	0.035 (0.014)	0.086 (0.015)
β_{14}	0.904 (0.022)	0.904 (0.022)	0.893 (0.013)	0.907 (0.016)	0.903 (0.017)
μ_{15}	0.094 (0.028)	0.094 (0.028)	0.036 (0.035)	0.029 (0.027)	0.080 (0.018)
ω_{15}	0.063 (0.022)	0.063 (0.022)	0.009 (0.013)	0.002 (0.009)	0.025 (0.009)
α_{15}	0.096 (0.030)	0.096 (0.030)	0.021 (0.012)	0.022 (0.010)	0.055 (0.010)
β_{15}	0.890 (0.028)	0.890 (0.028)	0.896 (0.013)	0.997 (0.011)	0.894 (0.011)
μ_{16}	0.045 (0.032)	0.045 (0.032)	0.000 (0.031)	0.020 (0.033)	0.043 (0.052)
ω_{16}	0.039 (0.015)	0.039 (0.015)	0.009 (0.042)	0.004 (0.008)	0.022 (0.010)
α_{16}	0.049 (0.010)	0.049 (0.010)	0.049 (0.006)	0.029 (0.007)	0.044 (0.009)
β_{16}	0.944 (0.010)	0.944 (0.010)	0.940 (0.007)	0.946 (0.007)	0.951 (0.010)

Note. Standard Errors are given in parenthesis.

Table 4.1 Continued.....

	DCC-GARCH		**GO-GARCH**		
Conditional distribution	**Gaussian**	**t**	**Gaussian**	**t**	**Skewed t**
μ_{17}	0.027 (0.040)	0.027 (0.040)	-0.010 (0.045)	0.040 (0.032)	0.038 (0.040)
ω_{17}	0.042 (0.011)	0.042 (0.011)	0.014 (0.009)	0.003 (0.007)	0.014 (0.009)
α_{17}	0.022 (0.002)	0.022 (0.002)	0.077 (0.003)	0.018 (0.005)	0.028 (0.005)
β_{17}	0.970 (0.002)	0.970 (0.002)	0.961 (0.004)	0.977 (0.005)	0.969 (0.006)
μ_{18}	0.083 (0.027)	0.083 (0.027)	0.026 (0.035)	0.013 (0.028)	0.067 (0.094)
ω_{18}	0.028 (0.020)	0.028 (0.020)	0.002 (0.008)	0.003 (0.012)	0.036 (0.012)
α_{18}	0.065 0.(026)	0.065 0.(026)	0.025 (0.009)	0.019 (0.013)	0.076 (0.020)
β_{18}	0.927 (0.031)	0.927 (0.031)	0.937 (0.010)	0.929 (0.014)	0.925 (0.019)
μ_{19}	0.124 (0.033)	0.124 (0.033)	0.042 (0.033)	0.056 (0.024)	0.077 (0.025)
ω_{19}	0.036 (0.020)	0.036 (0.020)	0.007 (0.008)	0.007 (0.009)	0.029 (0.008)
α_{19}	0.110 (0.034)	0.110 (0.034)	0.039 (0.013)	0.047 (0.011)	0.066 (0.012)
β_{19}	0.890 (0.020)	0.890 (0.020)	0.954 (0.012)	0.943 (0.012)	0.923 (0.013)
μ_{20}	0.107 (0.028)	0.107 (0.028)	0.032 (0.034)	0.063 (0.029)	0.086 (0.028)
ω_{20}	0.062 (0.022)	0.062 (0.022)	0.010 (0.015)	0.005 (0.015)	0.049 (0.015)
α_{20}	0.084 (0.021)	0.084 (0.021)	0.024 (0.010)	0.026 (0.012)	0.071 (0.012)
β_{20}	0.900 (0.024)	0.900 (0.024)	0.907 (0.012)	0.906 (0.004)	0.915 (0.014)
μ_{21}	0.065 (0.020)	0.065 (0.020)	0.042 (0.026)	0.057 (0.021)	0.060 (0.019)
ω_{21}	0.012 (0.005)	0.012 (0.005)	0.012 (0.003)	0.003 (0.007)	0.010 (0.004)
α_{21}	0.049 (0.008)	0.049 (0.008)	0.049 (0.006)	0.032 (0.007)	0.045 (0.007)
β_{21}	0.946 (0.008)	0.946 (0.008)	0.942 (0.006)	0.946 (0.006)	0.951 (0.007)
μ_{22}	0.089 (0.025)	0.089 (0.025)	0.027 (0.043)	0.051 (0.029)	0.068 (0.140)
ω_{22}	0.024 (0.012)	0.024 (0.012)	0.006 (0.007)	0.002 (0.012)	0.016 (0.019)
α_{22}	0.087 (0.023)	0.087 (0.023)	0.018 (0.011)	0.019 (0.009)	0.072 (0.061)
β_{22}	0.911 (0.022)	0.911 (0.022)	0.916 (0.011)	0.918 (0.008)	0.927 (0.050)
μ_{23}	0.058 (0.023)	0.058 (0.023)	0.036 (0.026)	0.055 (0.021)	0.061 (0.021)
ω_{23}	0.058 (0.023)	0.058 (0.023)	0.005 (0.013)	0.004 (0.011)	0.025 (0.009)
α_{23}	0.062 (0.022)	0.062 (0.022)	0.021 (0.010)	0.019 (0.012)	0.058 (0.011)
β_{23}	0.910 (0.033)	0.910 (0.033)	0.907 (0.015)	0.906 (0.009)	0.903 (0.013)
μ_{24}	0.065 (0.034)	0.065 (0.034)	0.021 (0.043)	0.021 (0.032)	0.073 (0.029)
ω_{24}	0.032 (0.018)	0.032 (0.018)	0.004 (0.009)	0.004 (0.007)	0.027 (0.011)
α_{24}	0.054 (0.016)	0.054 (0.016)	0.020 (0.007)	0.021 (0.008)	0.049 (0.008)
β_{24}	0.940 (0.018)	0.940 (0.018)	0.943 (0.008)	0.931 (0.023)	0.944 (0.009)
μ_{25}	0.104 (0.031)	0.104 (0.031)	0.052 (0.037)	0.084 (0.027)	0.083 (0.025)
ω_{25}	0.084 (0.043)	0.084 (0.043)	0.007 (0.018)	0.009 (0.014)	0.062 (0.022)
α_{25}	0.077 (0.029)	0.077 (0.029)	0.020 (0.012)	0.047 (0.018)	0.050 (0.014)
β_{25}	0.903 (0.036)	0.903 (0.036)	0.902 (0.015)	0.904 (0.019)	0.903 (0.019)
μ_{26}	0.080 (0.031)	0.080 (0.031)	0.052 (0.037)	0.069 (0.031)	0.084 (0.032)
ω_{26}	0.057 (0.074)	0.057 (0.074)	0.004 (0.020)	0.005 (0.012)	0.042 (0.020)
α_{26}	0.048 (0.036)	0.048 (0.036)	0.022 (0.009)	0.029 (0.015)	0.057 (0.013)
β_{26}	0.938 (0.054)	0.938 (0.054)	0.938 (0.013)	0.937 (0.000)	0.934 (0.016)
μ_{27}	0.064 (0.024)	0.064 (0.024)	0.024 (0.032)	0.036 (0.025)	0.060 (0.026)
ω_{27}	0.042 (0.017)	0.042 (0.017)	0.017 (0.010)	0.013 (0.010)	0.025 (0.008)
α_{27}	0.080 (0.022)	0.080 (0.022)	0.084 (0.011)	0.077 (0.011)	0.061 (0.010)
β_{27}	0.907 (0.024)	0.907 (0.024)	0.898 (0.012)	0.906 (0.005)	0.929 (0.012)
μ_{28}	0.105 (0.030)	0.105 (0.030)	0.059 (0.032)	0.050 (0.024)	0.086 (0.023)
ω_{28}	0.042 (0.036)	0.042 (0.036)	0.006 (0.011)	0.006 (0.010)	0.011 (0.005)
α_{28}	0.046 (0.030)	0.046 (0.030)	0.032 (0.008)	0.035 (0.005)	0.034 (0.005)
β_{28}	0.941 (0.038)	0.941 (0.038)	0.961 (0.010)	0.959 (0.013)	0.963 (0.005)
μ_{29}	0.075 (0.024)	0.075 (0.024)	0.028(0.034)	0.028 (0.024)	0.057 (0.028)
ω_{29}	0.041 (0.039)	0.041 (0.039)	0.012 (0.012)	0.019 (0.013)	0.038 (0.017)
α_{29}	0.067 (0.038)	0.067 (0.038)	0.032 (0.011)	0.062 (0.016)	0.083 (0.018)
β_{29}	0.919 (0.050)	0.919 (0.050)	0.916 (0.014)	0.917 (0.018)	0.906 (0.024)
LL	-150314.00	-145494.80	-151161.6	-146542.80	-146250.30
a	0.004 (0.001)	0.002 (0.001)	———	———	———
b	0.957 (0.003)	0.968 (0.003)	———	———	———
ν	———	8.212 (0.211)	———	7.718 (0.321)	7.8091 (0.4492)
ζ	———	———	———	———	0.521 (0.0278)

Note. Standard Errors are given in parenthesis. The starting value of Q_t is set to $Q_0 = \bar{Q}$.

Table 4.3: Estimates of multivariate GARCH models with different error distributions with mean model = ARMA(2,0)

	DCC-GARCH		GO-GARCH		
Conditional distribution	**Gaussian**	**t**	**Gaussian**	**t**	**Skewed t**
μ_1	0.060 (0.020)	0.060 (0.020)	0.043 (0.029)	0.081 (0.022)	0.062 (0.024)
$\phi_{1,1}$	-0.061 (0.019)	-0.061 (0.019)	-0.137 (0.017)	-0.010 (0.0136)	-0.075 (0.023)
$\phi_{1,2}$	-0.024 (0.019)	-0.024 (0.019)	-0.108 (0.006)	-0.033 (0.0134)	-0.032 (0.020)
ω_1	0.037 (0.011)	0.037 (0.011)	0.015 (0.009)	0.037 (0.009)	0.035 (0.009)
α_1	0.079 (0.013)	0.079 (0.013)	0.076 (0.009)	0.078 (0.009)	0.079 (0.012)
β_1	0.904 (0.016)	0.904 (0.016)	0.909 (0.011)	0.905 (0.011)	0.905 (0.014)
μ_2	0.040 (0.026)	0.040 (0.026)	0.029 (0.0349	0.0127 (0.025)	0.041 (0.026)
$\phi_{2,1}$	-0.044 (0.020)	-0.044 (0.020)	-0.076 (0.018)	-0.062 (0.013)	-0.043 (0.040)
$\phi_{2,2}$	-0.003 (0.021)	-0.003 (0.021)	-0.049 (0.005)	-0.0078 (0.013)	-0.003 (0.017)
ω_2	0.043 (0.045)	0.043 (0.045)	0.016 (0.011)	0.043 (0.011)	0.016 (0.007)
α_2	0.052 (0.040)	0.052 (0.040)	0.091 (0.009)	0.051 (0.009)	0.057 (0.012)
β_2	0.935 (0.051)	0.935 (0.051)	0.938 (0.012)	0.936 (0.012)	0.941 (0.012)
μ_3	0.049 (0.022)	0.049 (0.022)	-0.001 (0.0356)	0.010 (0.025)	0.042 (0.022)
$\phi_{3,1}$	0.014 (0.019)	0.014 (0.019)	-0.010 (0.018)	-0.040 (0.012)	-0.006 (0.016)
$\phi_{3,2}$	-0.014 (0.020)	-0.014 (0.020)	0.008 (0.002)	0.009 (0.012)	-0.015 (0.025)
ω_3	0.010 (0.005)	0.010 (0.005)	0.007 (0.003)	0.010 (0.003)	0.012 (0.006)
α_3	0.051 (0.011)	0.051 (0.011)	0.029 (0.006)	0.049 (0.006)	0.057 (0.011)
β_3	0.947 (0.011)	0.947 (0.011)	0.946 (0.006)	0.958 (0.006)	0.941 (0.011)
μ_4	0.058 (0.015)	0.058 (0.015)	0.032 (0.021)	0.032 (0.016)	0.047 (0.015)
$\phi_{4,1}$	-0.003 (0.020)	-0.003 (0.020)	-0.044 (0.017)	-0.040 (0.012)	-0.004 (0.018)
$\phi_{4,2}$	-0.028 (0.020)	-0.028 (0.020)	-0.085 (0.004)	-0.055 (0.013)	-0.035 (0.018)
ω_4	0.026 (0.008)	0.026 (0.008)	0.006 (0.005)	0.026 (0.005)	0.025 (0.006)
α_4	0.120 (0.023)	0.120 (0.023)	0.016 (0.014)	0.119 (0.014)	0.118 (0.016)
β_4	0.863 (0.023)	0.863 (0.023)	0.877 (0.015)	0.864 (0.015)	0.868 (0.017)
μ_5	0.027 (0.019)	0.027 (0.019)	0.020 (0.024)	0.021 (0.020)	0.027 (0.017)
$\phi_{5,1}$	-0.027 (0.020)	-0.027 (0.020)	-0.075 (0.018)	-0.0623 (0.014)	-0.031 (0.025)
$\phi_{5,2}$	-0.022 (0.019)	-0.022 (0.019)	-0.055 (0.005)	-0.035 (0.014)	-0.026 (0.022)
ω_5	0.014 (0.006)	0.014 (0.006)	0.012 (0.008)	0.014 (0.004)	0.008 (0.005)
α_5	0.0447 (0.010)	0.0447 (0.010)	0.057 (0.006)	0.043 (0.006)	0.043 (0.014)
β_5	0.947 (0.012)	0.947 (0.012)	0.932 (0.007)	0.948 (0.007)	0.953 (0.013)
μ_6	0.087 (0.021)	0.087 (0.021)	0.054 (0.030)	0.103 (0.023)	0.085 (0.063)
$\phi_{6,1}$	-0.032 (0.018)	-0.032 (0.018)	-0.098 (0.017)	-0.074 (0.014)	-0.051 (0.043)
$\phi_{6,2}$	-0.013 (0.019)	-0.013 (0.019)	-0.075 (0.005)	-0.006 (0.014)	-0.028 (0.050)
ω_6	0.042 (0.012)	0.042 (0.012)	0.010 (0.009)	0.043 (0.009)	0.039 (0.011)
α_6	0.085 (0.013)	0.085 (0.013)	0.024 (0.010)	0.084 (0.010)	0.079 (0.012)
β_6	0.896 (0.015)	0.896 (0.015)	0.894 (0.011)	0.897 (0.011)	0.904 (0.015)
μ_7	0.065 (0.025)	0.065 (0.025)	0.023 (0.049)	0.027 (0.030)	0.059 (0.025)
$\phi_{7,1}$	-0.042 (0.019)	-0.042 (0.019)	-0.091 (0.018)	-0.057 (0.011)	-0.049 (0.0189
$\phi_{7,2}$	-0.001 (0.020)	-0.001 (0.020)	-0.028 (0.005)	-0.008 (0.011)	-0.008 (0.017)
ω_7	0.018 (0.009)	0.018 (0.009)	0.073 (0.006)	0.019 (0.006)	0.018 (0.007)
α_7	0.072 (0.020)	0.072 (0.020)	0.144 (0.009)	0.078 (0.009)	0.075 (0.010)
β_7	0.922 (0.019)	0.922 (0.019)	0.919 (0.009)	0.921 (0.009)	0.925 (0.010)
μ_8	0.044 (0.016)	0.044 (0.016)	0.038 (0.021)	0.045 (0.016)	0.051 (0.017)
$\phi_{8,1}$	-0.067 (0.020)	-0.067 (0.020)	-0.094 (0.018)	-0.0656 (0.018)	-0.059 (0.018)
$\phi_{8,2}$	-0.041 (0.020)	-0.041 (0.020)	-0.068 (0.005)	-0.0534 (0.013)	-0.043 (0.018)
ω_8	0.024 (0.033)	0.024 (0.033)	0.004 (0.007)	0.024 (0.007)	0.026 (0.010)
α_8	0.052 (0.050)	0.052 (0.050)	0.015 (0.010)	0.051 (0.010)	0.070 (0.018)
β_8	0.928 (0.074)	0.928 (0.074)	0.929 (0.014)	0.930 (0.014)	0.912 (0.022)
μ_9	0.012 (0.025)	0.012 (0.025)	0.002 (0.029)	0.007 (0.023)	0.024 (0.020)
$\phi_{9,1}$	-0.033 (0.027)	-0.033 (0.027)	-0.034 (0.017)	-0.022 (0.013)	-0.019 (0.018)
$\phi_{9,2}$	-0.054 (0.021)	-0.054 (0.021)	-0.084 (0.003)	-0.040 (0.013)	-0.026 (0.017)
ω_9	0.042 (0.038)	0.042 (0.038)	0.008 (0.020)	0.045 (0.012)	0.037 (0.012)
α_9	0.067 (0.037)	0.067 (0.037)	0.020 (0.012)	0.069 (0.012)	0.069 (0.012)
β_9	0.917 (0.049)	0.917 (0.049)	0.927 (0.014)	0.914 (0.014)	0.916 (0.015)
μ_{10}	0.060 (0.023)	0.060 (0.023)	0.030 (0.028)	0.041 (0.021)	0.040 (0.019)
$\phi_{10,1}$	0.011 (0.022)	0.011 (0.022)	-0.006 (0.018)	-0.033 (0.013)	-0.012 (0.017)
$\phi_{10,2}$	0.010 (0.022)	0.010 (0.022)	0.009 (0.001)	-0.013 (0.012)	0.007 (0.016)
ω_{10}	0.089 (0.036)	0.089 (0.036)	0.012 (0.014)	0.075 (0.014)	0.023 (0.011)
α_{10}	0.140 (0.050)	0.140 (0.050)	0.032 (0.018)	0.121 (0.018)	0.065 (0.016)
β_{10}	0.828 (0.053)	0.828 (0.053)	0.836 (0.020)	0.851 (0.020)	0.926 (0.019)
μ_{11}	0.046 (0.021)	0.046 (0.021)	0.010 (0.029)	0.030 (0.023)	0.049 (0.027)
$\phi_{11,1}$	-0.002 (0.019)	-0.002 (0.019)	-0.033 (0.018)	-0.028 (0.013)	0.000 (0.024)
$\phi_{11,2}$	-0.012 (0.019)	-0.012 (0.019)	-0.054 (0.003)	-0.029 (0.013)	-0.019 (0.038)
ω_{11}	0.016 (0.008)	0.016 (0.008)	0.009 (0.005)	0.017 (0.005)	0.014 (0.005)
α_{11}	0.067 (0.018)	0.067 (0.018)	0.021 (0.009)	0.068 (0.009)	0.062 (0.011)
β_{11}	0.927 (0.019)	0.927 (0.019)	0.928 (0.010)	0.926 (0.010)	0.933 (0.011)
μ_{12}	0.057 (0.017)	0.057 (0.017)	0.021 (0.023)	0.030 (0.017)	0.044 (0.113)
ϕ_1	-0.010 (0.020)	-0.010 (0.020)	-0.041 (0.018)	-0.030 (0.013)	-0.015 (0.189)
ϕ_2	-0.005 (0.021)	-0.005 (0.021)	-0.015 (0.004)	0.024 (0.0127)	0.014 (0.130)
ω_{12}	0.019 (0.012)	0.019 (0.012)	0.009 (0.005)	0.020 (0.005)	0.013 (0.006)
α_{12}	0.075 (0.029)	0.075 (0.029)	0.037 (0.012)	0.076 (0.011)	0.056 (0.017)
β_{12}	0.914 (0.033)	0.914 (0.033)	0.915 (0.012)	0.912 (0.012)	0.937 (0.035)
μ_{13}	0.020 (0.027)	0.020 (0.027)	-0.002 (0.034)	0.036 (0.023)	0.035 (0.023)
$\phi_{13,1}$	0.010 (0.022)	0.010 (0.022)	0.000 (0.000)	-0.012 (0.012)	0.003 (0.018)
$\phi_{13,2}$	-0.002 (0.020)	-0.002 (0.020)	-0.0201(0.000)	0.008 (0.011)	0.012 (0.017)
ω_{13}	0.045 (0.067)	0.045 (0.067)	0.078 (0.014)	0.047 (0.014)	0.062 (0.023)
α_{13}	0.044 (0.040)	0.044 (0.040)	0.177 (0.009)	0.045 (0.010)	0.074 (0.019)
β_{13}	0.941 (0.062)	0.941 (0.062)	0.950 (0.012)	0.912 (0.012)	0.905 (0.024)
μ_{14}	0.042 (0.021)	0.042 (0.021)	0.023 (0.028)	0.030 (0.023)	0.043 (0.019)
$\phi_{14,1}$	-0.021 (0.020)	-0.021 (0.020)	-0.046 (0.018)	-0.064 (0.013)	-0.022 (0.018)
$\phi_{14,2}$	-0.022 (0.019)	-0.022 (0.019)	-0.059 (0.004)	-0.038 (0.013)	-0.023 (0.018)
ω_{14}	0.031 (0.012)	0.031 (0.012)	0.002 (0.008)	0.047 (0.016)	0.027 (0.009)
α_{14}	0.085 (0.020)	0.085 (0.020)	0.023 (0.012)	0.045 (0.009)	0.088 (0.016)
β_{14}	0.901 (0.023)	0.901 (0.023)	0.907 (0.014)	0.940 (0.012)	0.901 (0.017)
μ_{15}	0.093 (0.026)	0.093 (0.026)	0.039 (0.035)	0.035 (0.027)	0.084 (0.029)
$\phi_{15,1}$	-0.039 (0.020)	-0.039 (0.020)	-0.032 (0.018)	-0.044 (0.013)	-0.028 (0.013)
$\phi_{15,2}$	-0.026 (0.019)	-0.026 (0.019)	-0.050 (0.003)	-0.044 (0.013)	-0.025 (0.017)
ω_{15}	0.063 (0.022)	0.063 (0.022)	0.004 (0.013)	0.032 (0.008)	0.025 (0.010)
α_{15}	0.097 (0.030)	0.097 (0.030)	0.028 (0.020)	0.087 (0.012)	0.055 (0.013)
β_{15}	0.889 (0.029)	0.889 (0.029)	0.890 (0.013)	0.899 (0.014)	0.938 (0.014)
μ_{16}	0.045 (0.031)	0.045 (0.031)	0.003 (0.042)	0.021 (0.033)	0.044 (0.030)
$\phi_{16,1}$	-0.016 (0.019)	-0.016 (0.019)	-0.046 (0.018)	-0.017 (0.013)	-0.007 (0.018)
$\phi_{16,2}$	-0.013 (0.020)	-0.013 (0.020)	-0.046 (0.004)	-0.014 (0.013)	-0.013 (0.018)
ω_{16}	0.039 (0.015)	0.039 (0.015)	0.002 (0.009)	0.063 (0.013)	0.022 (0.009)
α_{16}	0.050 (0.010)	0.050 (0.010)	0.026 (0.006)	0.096 (0.012)	0.045 (0.008)
β_{16}	0.942 (0.011)	0.942 (0.011)	0.947 (0.007)	0.890 (0.013)	0.950 (0.009)

Note. Standard Errors are given in parenthesis.

Table 4.2 Continued.....

	DCC-GARCH		**GO-GARCH**		
Conditional distribution	**Gaussian**	**t**	**Gaussian**	**t**	**Skewed t**
μ_{17}	0.026 (0.039)	0.026 (0.039)	-0.012 (0.045)	0.045 (0.032)	0.039 (0.035)
$\phi_{17,1}$	-0.021 (0.020)	-0.021 (0.020)	-0.030 (0.018)	-0.026 (0.012)	-0.022 (0.018)
$\phi_{17,2}$	-0.033 (0.025)	-0.033 (0.025)	-0.053 (0.003)	-0.033 (0.020)	-0.021 (0.017)
ω_{17}	0.045 (0.012)	0.045 (0.012)	0.022 (0.008)	0.044 (0.008)	0.016 (0.008)
α_{17}	0.024 (0.003)	0.024 (0.003)	0.102 (0.003)	0.023 (0.003)	0.031 (0.006)
β_{17}	0.968 (0.002)	0.968 (0.002)	0.966 (0.004)	0.968 (0.004)	0.9657 0.0065
μ_{18}	0.083 (0.026)	0.083 (0.026)	0.027 (0.035)	0.015 (0.028)	0.067 (0.040)
$\phi_{18,1}$	-0.001 (0.020)	-0.001 (0.020)	0.0004 (0.000)	0.002 (0.013)	0.004 (0.040)
$\phi_{18,2}$	-0.029 (0.019)	-0.029 (0.019)	-0.032 (-0.000)	-0.019 (0.013)	-0.014 (0.026)
ω_{18}	0.030 (0.022)	0.030 (0.022)	0.007 (0.008)	0.029 (0.008)	0.037 (0.067)
α_{18}	0.070 (0.029)	0.070 (0.029)	0.036 (0.009)	0.065 (0.009)	0.080 (0.080)
β_{18}	0.922 (0.034)	0.922 (0.034)	0.927 (0.010)	0.927 (0.010)	0.911 (0.090)
μ_{19}	0.123 (0.032)	0.123 (0.032)	0.047 (0.033)	0.065 (0.024)	0.085 (0.024)
$\phi_{19,1}$	-0.063 (0.021)	-0.063 (0.021)	-0.082 (0.018)	-0.084 (0.0123)	-0.059 (0.018)
$\phi_{19,2}$	-0.0548 (0.024)	-0.0548 (0.024)	-0.039 (0.005)	-0.026 (0.012)	-0.035 (0.019)
ω_{19}	0.038 (0.020)	0.038 (0.020)	0.003 (0.009)	0.035 (0.009)	0.030 (0.009)
α_{19}	0.113 (0.034)	0.113 (0.034)	0.023 (0.013)	0.114 (0.013)	0.068 (0.012)
β_{19}	0.886 (0.021)	0.886 (0.021)	0.897 (0.011)	0.889 (0.011)	0.921 (0.013)
μ_{20}	0.107 (0.028)	0.107 (0.028)	0.032 (0.036)	0.063 (0.029)	0.089 (0.030)
$\phi_{20,1}$	-0.027 (0.020)	-0.027 (0.020)	0.0015 (0.000)	-0.026 (0.014)	-0.030 (0.017)
$\phi_{20,2}$	0.011 (0.020)	0.011 (0.020)	0.011 (-0.000)	0.015 (0.014)	0.005 (0.019)
ω_{20}	0.062 (0.022)	0.062 (0.022)	0.005 (0.015)	0.062 (0.015)	0.049 (0.016)
α_{20}	0.084 (0.021)	0.084 (0.021)	0.020 (0.011)	0.082 (0.010)	0.072 (0.014)
β_{20}	0.901 (0.023)	0.901 (0.023)	0.907 (0.012)	0.902 (0.012)	0.915 (0.015)
μ_{21}	0.065 (0.019)	0.065 (0.019)	0.045 (0.026)	0.061 (0.021)	0.0640 (0.021)
$\phi_{21,1}$	-0.029 (0.020)	-0.029 (0.020)	-0.028 (0.018)	-0.027 (0.014)	-0.022 (0.018)
$\phi_{21,2}$	-0.040 (0.019)	-0.040 (0.019)	-0.048 (0.003)	-0.013 (0.013)	-0.030 (0.018)
ω_{21}	0.012 (0.005)	0.012 (0.005)	0.002 (0.003)	0.012 (0.003)	0.010 (0.004)
α_{21}	0.048 (0.008)	0.048 (0.008)	0.025 (0.005)	0.049 (0.006)	0.045 (0.006)
β_{21}	0.945 (0.008)	0.945 (0.008)	0.947 (0.006)	0.946 (0.006)	0.951 (0.006)
μ_{22}	0.089 (0.025)	0.089 (0.025)	0.029 (0.043)	0.056 (0.028)	0.072 (0.024)
$\phi_{22,1}$	-0.039 (0.019)	-0.039 (0.019)	-0.079 (0.018)	-0.090 (0.012)	-0.052 (0.018)
$\phi_{22,2}$	-0.008 (0.019)	-0.008 (0.019)	-0.038 (0.005)	-0.012 (0.012)	-0.014 (0.018)
ω_{22}	0.023 (0.012)	0.023 (0.012)	0.014 (0.006)	0.024 (0.006)	0.015 (0.006)
α_{22}	0.085 (0.024)	0.085 (0.024)	0.080 (0.010)	0.084 (0.011)	0.069 (0.011)
β_{22}	0.913 (0.023)	0.913 (0.023)	0.908 (0.011)	0.913 (0.011)	0.930 (0.010)
μ_{23}	0.058 (0.022)	0.058 (0.022)	0.040 (0.026)	0.060 (0.021)	0.064 (0.032)
$\phi_{23,1}$	-0.040 (0.019)	-0.040 (0.019)	-0.066 (0.018)	-0.065 (0.013)	-0.049 (0.027)
$\phi_{23,2}$	-0.014 (0.020)	-0.014 (0.020)	-0.042 (0.004)	-0.0002 (0.013)	-0.012 (0.063)
ω_{23}	0.059 (0.028)	0.059 (0.028)	0.004 (0.013)	0.061 (0.013)	0.025 (0.029)
α_{23}	0.062 (0.022)	0.062 (0.022)	0.020 (0.010)	0.063 (0.010)	0.058 (0.040)
β_{23}	0.909 (0.033)	0.909 (0.033)	0.907 (0.015)	0.907 (0.015)	0.931 (0.038)
μ_{24}	0.064 (0.033)	0.064 (0.033)	0.023 (0.044)	0.025 (0.032)	0.074 (0.044)
$\phi_{24,1}$	-0.034 (0.019)	-0.034 (0.019)	-0.050 (0.018)	-0.036 (0.012)	-0.032 (0.015)
$\phi_{24,2}$	-0.014 (0.021)	-0.014 (0.021)	-0.033 (0.004)	0.016 (0.012)	-0.003 (0.018)
ω_{24}	0.032 (0.019)	0.032 (0.019)	0.007 (0.009)	0.031 (0.009)	0.027 (0.009)
α_{24}	0.055 (0.017)	0.055 (0.017)	0.022 (0.007)	0.052 (0.007)	0.051 (0.009)
β_{24}	0.938 (0.019)	0.938 (0.019)	0.937 (0.008)	0.941 (0.008)	0.943 (0.009)
μ_{25}	0.103 (0.031)	0.103 (0.031)	0.054 (0.039)	0.089 (0.027)	0.085 (0.036)
$\phi_{25,1}$	-0.008 (0.021)	-0.008 (0.021)	-0.022 (0.018)	-0.031 (0.012)	-0.010 (0.017)
$\phi_{25,2}$	0.015 (0.022)	0.015 (0.022)	-0.017 (0.003)	-0.010 (0.012)	0.002 (0.019)
ω_{25}	0.083 (0.043)	0.083 (0.043)	0.002 (0.018)	0.082 (0.018)	0.061 (0.023)
α_{25}	0.076 (0.029)	0.076 (0.029)	0.032 (0.013)	0.074 (0.013)	0.054 (0.015)
β_{25}	0.904 (0.036)	0.904 (0.036)	0.906 (0.015)	0.906 (0.015)	0.928 (0.019)
μ_{26}	0.079 (0.032)	0.079 (0.032)	0.053 (0.037)	0.072 (0.031)	0.083 (0.032)
$\phi_{26,1}$	0.029 (0.021)	0.029 (0.021)	0.002 (0.0003)	0.004 (0.014)	0.025 (0.018)
$\phi_{26,2}$	-0.017 (0.021)	-0.017 (0.021)	-0.022 (-0.000)	-0.025 (0.014)	-0.022 (0.019)
ω_{26}	0.061 (0.093)	0.061 (0.093)	0.006 (0.021)	0.061 (0.021)	0.045 (0.023)
α_{26}	0.050 (0.045)	0.050 (0.045)	0.048 (0.009)	0.050 (0.009)	0.059 (0.015)
β_{26}	0.935 (0.068)	0.935 (0.068)	0.947 (0.014)	0.935 (0.014)	0.932 (0.019)
μ_{27}	0.064 (0.024)	0.064 (0.024)	0.025 (0.032)	0.038 (0.025)	0.060 (0.024)
$\phi_{27,1}$	-0.016 (0.018)	-0.016 (0.018)	-0.046 (0.018)	-0.023 (0.013)	-0.015 (0.018)
$\phi_{27,2}$	0.010 (0.020)	0.010 (0.020)	-0.023 (0.004)	-0.003 (0.013)	0.000 (0.018)
ω_{27}	0.042 (0.017)	0.042 (0.017)	0.009 (0.010)	0.043 (0.010)	0.026 80.009)
α_{27}	0.079 (0.021)	0.079 (0.021)	0.048 (0.011)	0.079 (0.011)	0.062 (0.011)
β_{27}	0.908 (0.024)	0.908 (0.024)	0.914 (0.012)	0.906 (0.012)	0.929 (0.012)
μ_{28}	0.105 (0.029)	0.105 (0.029)	0.063 (0.033)	0.057 (0.024)	0.089 (0.039)
$\phi_{28,1}$	-0.025 (0.019)	-0.025 (0.019)	-0.023 (0.018)	-0.0112 (0.012)	-0.016 (0.019)
$\phi_{28,2}$	-0.033 (0.023)	-0.033 (0.023)	-0.035 (0.003)	-0.040 (0.013)	-0.044 (0.041)
ω_{28}	0.043 (0.043)	0.043 (0.043)	0.011 (0.010)	0.043 (0.011)	0.011 (0.008)
α_{28}	0.047 (0.035)	0.047 (0.035)	0.047 (0.008)	0.046 (0.008)	0.034 (0.007)
β_{28}	0.940 (0.045)	0.940 (0.045)	0.942 (0.011)	0.941 (0.011)	0.964 (0.009)
μ_{29}	0.075 (0.022)	0.075 (0.022)	0.034 (0.035)	0.034 (0.024)	0.061 (0.016)
$\phi_{29,1}$	-0.081 (0.022)	-0.081 (0.022)	-0.132 (0.018)	-0.062 (0.012)	-0.072 (0.056)
$\phi_{29,2}$	-0.015 (0.020)	-0.015 (0.020)	-0.061 (0.006)	-0.008 (0.012)	-0.010 (0.032)
ω_{29}	0.042 (0.038)	0.042 (0.038)	0.004 (0.013)	0.046 (0.013)	0.037 (0.037)
α_{29}	0.069 (0.039)	0.069 (0.039)	0.052 (0.012)	0.072 (0.012)	0.083 (0.103)
β_{29}	0.918 (0.050)	0.918 (0.050)	0.924 (0.015)	0.913 (0.015)	0.907 (0.093)
LL	-150391.30	-145514.90	-151160.60	146631.10	146390.38
a	0.004 (0.001)	0.003 (0.001)	———	———	———
b	0.987 (0.003)	0.985 (0.004)	———	———	———
ν	———	8.150 (0.204)	———	8.373 (1.121)	8.3730 (1.244)
ζ	———	———	———	———	0.541 (0.084)

Note. Standard Errors are given in parenthesis. The starting value of Q_t is set to $Q_0 = \bar{Q}$.

small to affect the conditional mean of the returns. Due to this fact, the estimates for the univariate GARCH models are also very close and do not depend on the mean filtering process. This fact has already been documented by Engle and Sheppard (2002).

As far as the estimates of GO-GARCH model with different error distributions are concerned, these estimates vary with respect to the assumed distribution. It can be seen from Table 4.2 and Table 4.3 that estimates of conditional mean for a few of the returns series were reported as negative, which was not true in the case of DCC model. This implies that the GO-GARCH model has the capacity to model the dynamics of the conditional mean more correctly as compared to the DCC model, in which all of these estimates are reported as positive. These negative estimates correspond to those stocks which were reported to have negative mean returns in Table 3.2.

4.5.2 Forecasting Results

In this section, we present the forecasting results and evaluate them using the already mentioned tests. As we know from the previous chapter, three rounds of SVD were required to capture the entire volatility clustering from the data set; hence, the forecasts of the covariance matrices are obtained by combining the results of these three rounds (for details, see Section 2.2). Before describing the results of this chapter, another important issue regarding the presentation of the results when dealing with such a high dimensional data set has to be discussed.

Since we are dealing with a large-dimensional data set, we preferred to use the Box-and-Whisker plot (usually simply called Box-plot only) of the p-values of the corresponding test statistics. In all of the following figures, the horizontal axis of each of the sub-plots shows the forecast horizons, while the p-values are plotted on the vertical axis. In each Box-and-Whisker plot, the horizontal line (as indicated by red color) shows the median of the distribution of the p-values,and the areas above and below the median correspond to the third and the first quartiles of the distribution respectively i.e., Q_3 and Q_1. Similarly, the horizontal bars outside of the box in the middle are called whiskers (hence the name of the Box-and-Whisker plot) and they represent the variability outside the upper and lower quartiles (Hubert and Vandervieren, 2008). Sometimes, these whiskers represent the maximum and the minimum of the data (McGill et al. 1978).[7]It should be noted that in the following figures, these whiskers represent the minimum and maximum of the p-values.

[7]The whiskers mark those values which are minimum and maximum unless they exceed 1.5 * IQR. The IQR is the inter-quartile range, the distance between Q_1 and Q_3. If there are observations which are outside 1.5×IQR or even 3×IQR, they are considered as mild and extreme outliers, respectively.

As mentioned earlier, Campbell (2005) has reported that in some cases it is possible that the model passes the joint test while still failing either the independence test or the unconditional coverage test. Following his argument, all the tests were conducted separately for all the models and as well as for all stocks under study. This helps us to know the reason for failing the joint test of coverage. The p-values of these test statistics along with the percent violation for all the significance levels are presented in the Appendices 4.7.2 to 4.7.17. Since $\alpha = 0.01$ is used to conduct all these tests so the bold type entries of tables are not significant at 1% level.

4.5.3 Results of unconditional coverage test

Figure 4.5.1 shows the box plots of the p-values of the unconditional coverage test statistics. As described in the forecasting scheme, two models ARMA(0,0) and ARMA(2,0) - with three distributional assumptions (Gaussian, Student's t and skewed Student's t) were used when estimating these models. In all following figures, the first three panels show the box-plots with ARMA(0,0) as the mean filtering model, while the last three are reserved for ARMA(2,0) as the mean model. The results reveal that when using ARMA(0,0) or ARMA(2,0) as the mean filtering model, the performance of all models remains the same. For the nominal coverage of 95 % and 97.5%, the DCC-GARCH and GOGARCH models perform better than the SVX model. At these nominal coverages, the DCC model outperforms the other two models. However, the performance of all considered models becomes poorer as the level of coverage increases, i.e., for 99% and 99.5% none of the models pass the unconditional coverage test. The reason for poor the performance of SVX model could be the choice of wrong distributional assumption.

Similarly, Figure 4.5.2 shows the results of the unconditional coverage test using the Student's t-distribution. Under the assumption that returns follow ARMA(0,0) process, the GO-GARCH model is the only one that outperforms the other two competing models at higher significance levels, i.e., $\alpha = 0.01 \,\&\, 0.005$. Under t-distributed errors, the DCC-GARCH model almost fails for all intervals (95%, 97.5%, 995, and 99.5%) with the exception of a few stocks for which the model passes the test with very small p-values as the median values for these box-plots are nearly equal to 0; however, the GOGARCH and SVX models are performing better than DCC-GARCH, especially for 99.5% interval. Among GOGARCH and SVX models, the latter is outperforming the former at a nominal coverage of 99.5%.

Figure 4.5.1: Box-and-Whisker plots of p-values of $\mathbf{LR_{uc}}$ test statistic using Gaussian distributed errors

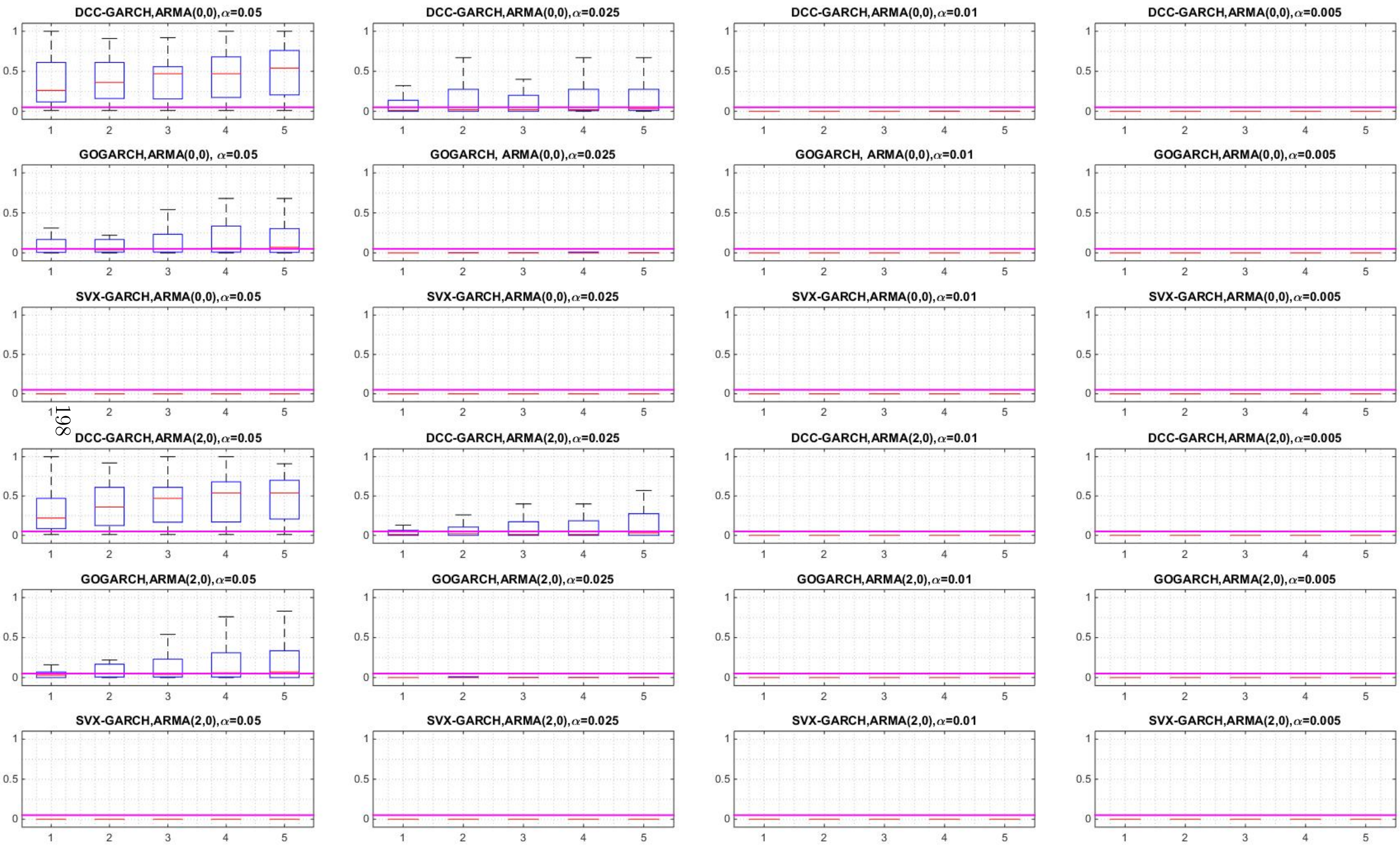


Figure 4.5.2: Box-and-Whisker plots of p-values of $\mathbf{LR_{uc}}$ test statistic using Student's t-distributed errors

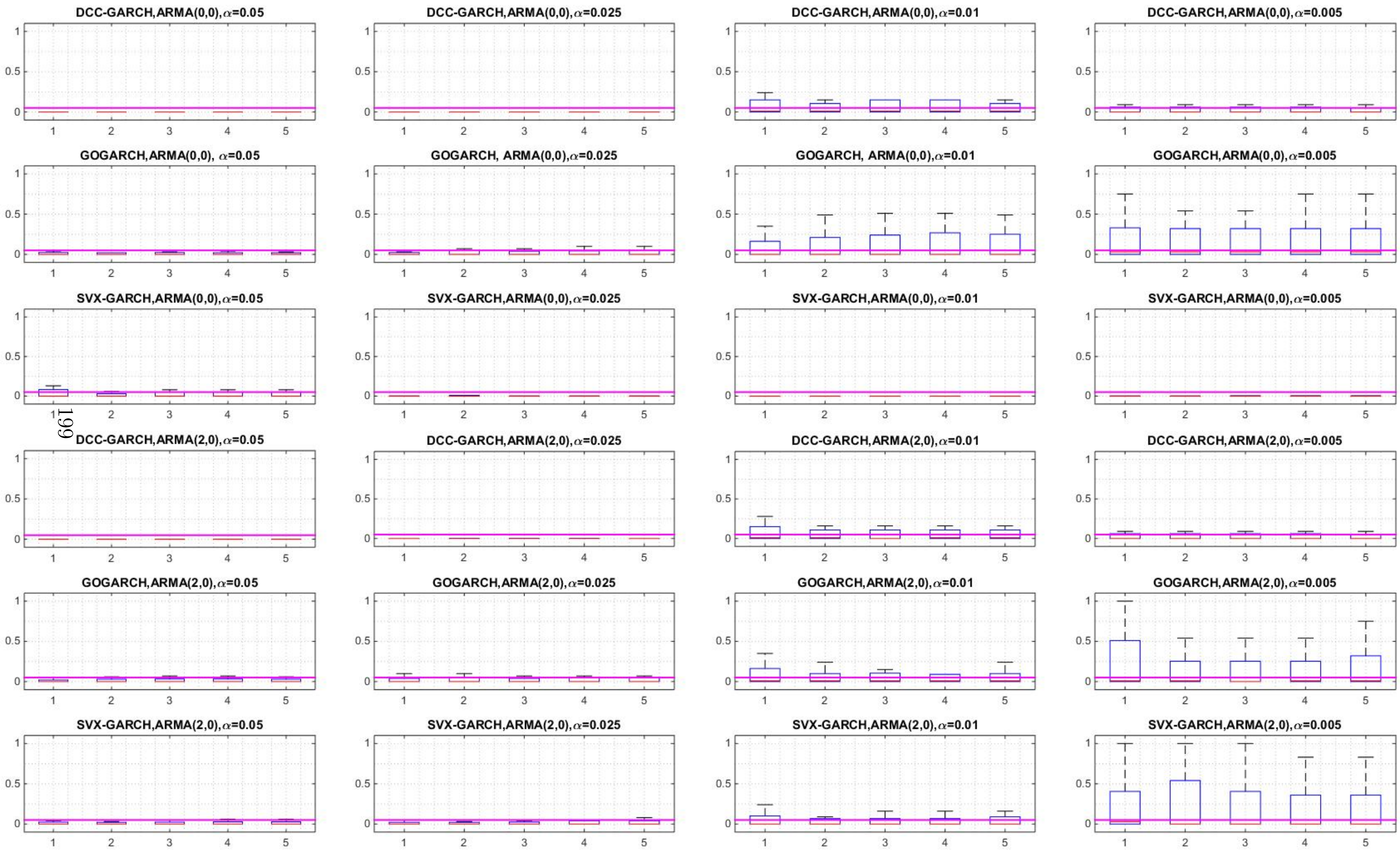
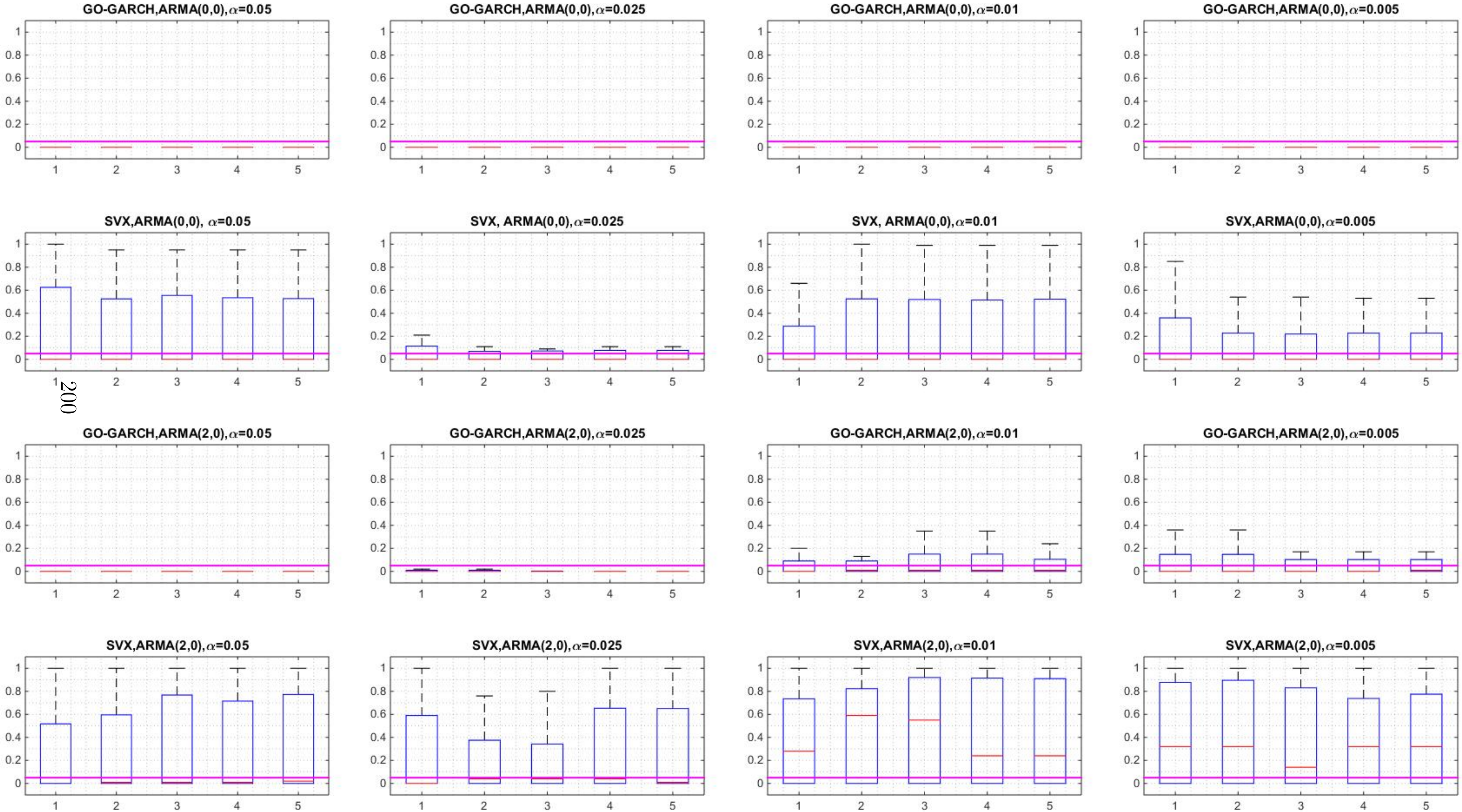


Figure 4.5.3: Box-and-Whisker plots of p-values of $\mathbf{LR_{uc}}$test statistic using skewed Student's t-distributed errors



The last figure regarding the unconditional coverage test contains eight less subplots as compared to the previous two figures i.e., the results for DCC-GARCH model with Skewed Student's t-distributed errors are not there. The reasons for not considering this model are: 1) The estimation cost is very high as it took nearly 25 days to estimate the DCC-GARCH model with Student's t-distributed errors, and for skewed Student's t-distribution it could take even more time to estimate the coefficients, 2) The results of DCC-GARCH with Student's t-distribution are not good and one can expect even worse results for a skewed t distribution. However, from the presented plots one can easily see that the GOGARCH model with ARMA(0,0) totally fails the unconditional coverage test for all intervals considered in this study, while our proposed model (SVX) passes the test at all intervals. Similarly, when using ARMA(2,0) as a mean model, the GOGARCH model hardly passed the test only for 99% and 99.5% intervals, whereas the SVX model performs consistently better than the GOGARCH model. It can also be seen that when the α-level increases, the SVX model performance is improving for all forecast horizons.

4.5.4 Results of independence test

The following three figures show the results of the tests conducted in order to check the independence property, i.e., that all the violations do not occur around the same time. Similarly to the results of the unconditional coverage test presented earlier, DCC-GARCH performs equally well for all forecast horizons and for all intervals. As argued by Engle (2000), while introducing the DCC-GARCH model, that returns can be either zero mean or the residuals from a filtered time series; changing the mean model does not affect the model performance.[8] However, the same is not true for GOGARCH model as the median varies when using ARMA(0,0) and ARMA(2,0) models, although the model passes the independence test for all intervals and for all forecasting horizons. So far as the performance of SVX model is concerned, it fails the independence test when the mean model is ARMA(0,0) but passes the test at all intervals when the mean model is ARMA(2,0). The reason for this could possibly be the incorrect specification for model error distribution as was noticed from the unconditional coverage test results.

The results of the independence test for all models with other two distributions (Student's t-distribution and its skewed version) are displayed in Figures 4.5.5 and 4.5.6, respectively. The explanation for DCC and GOGARCH models is the same as presented in the previous paragraph. However, it is worth mentioning the performance of SVX model when using ARMA(2,0) as a mean model, which passes the test

[8]Since the standard errors of the model do not depend on the choice of the filtration (ARMA, demeaning), the cross partial derivatives of the log-likelihood with respect to mean and variance parameters have zero expectation value when using the normal likelihood.

Figure 4.5.4: Box-and-Whisker plots of p-values of $\mathbf{LR_{ind}}$ test statistic using Gaussian distributed errors

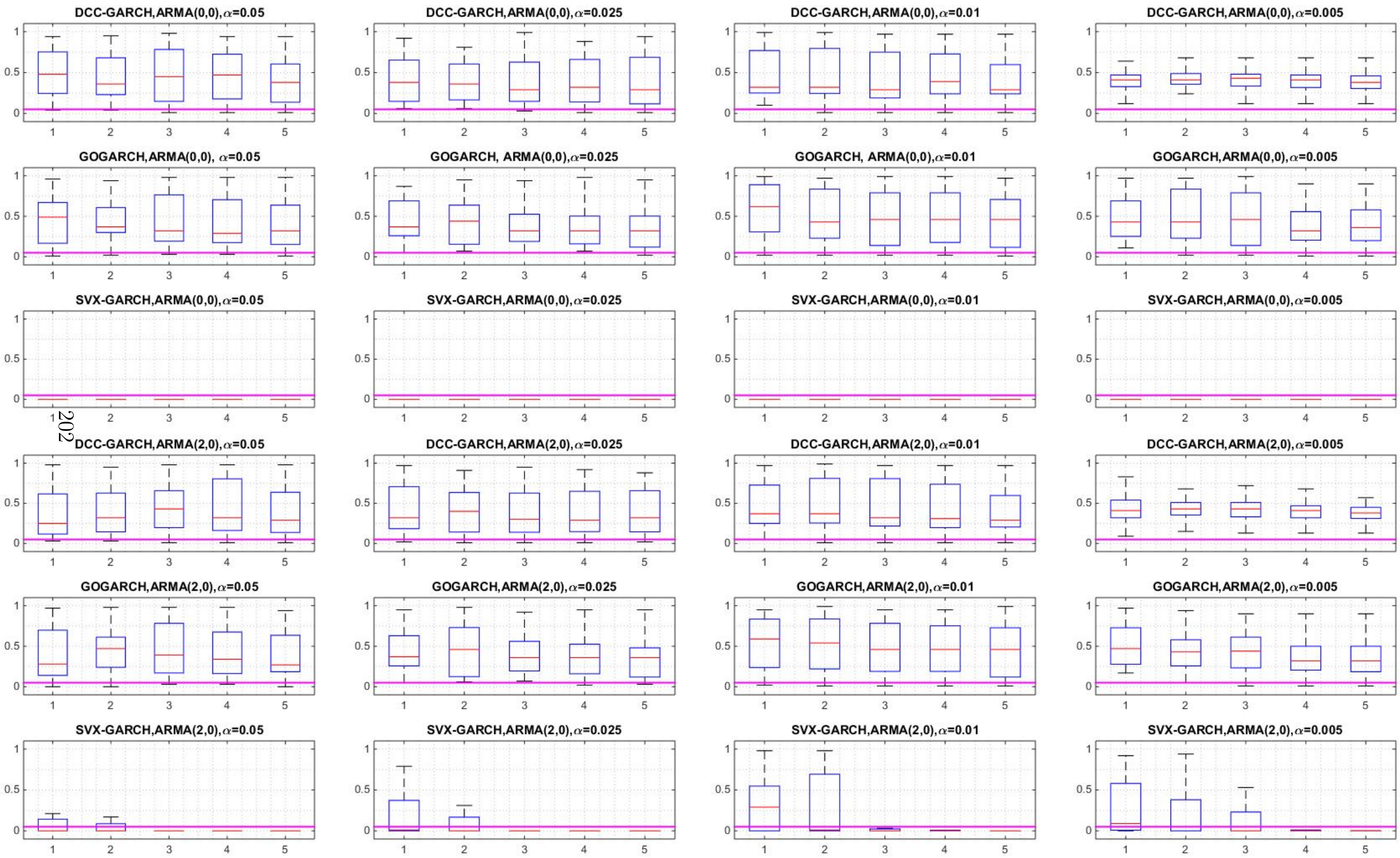


Figure 4.5.5: Box-and-Whisker plots of p-values of $\mathbf{LR_{ind}}$ test statistic using Student's t-distributed errors

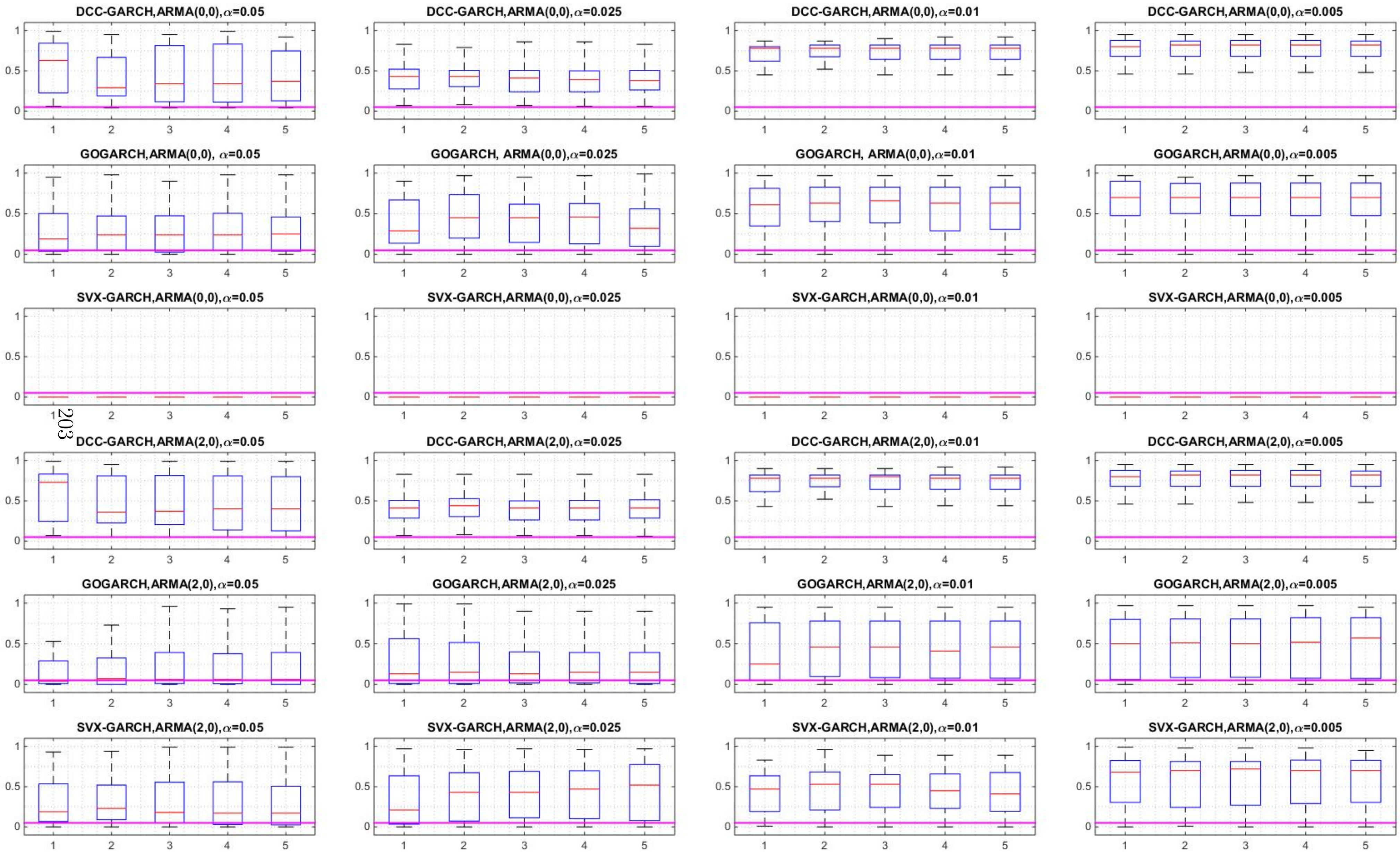
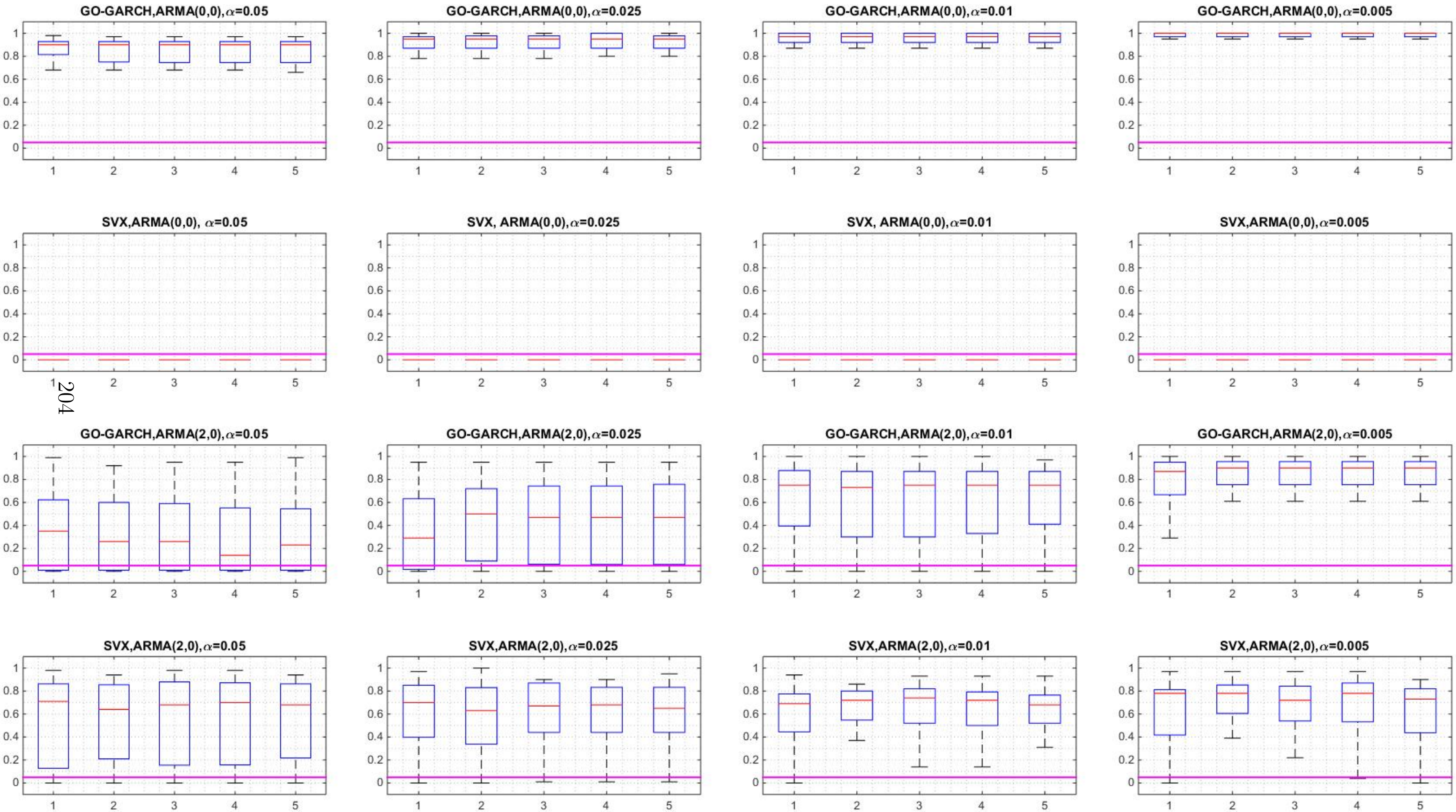


Figure 4.5.6: Box-and-Whisker plots of p-values of $\mathbf{LR_{ind}}$ test statistic using skewed Student's t-distributed errors



at all intervals and for all forecasting horizons as shown by the last panel of the figures. It can be seen that even when using the skewed Student's t-distribution, all the box-plots have their median values above the 0.65, which shows that the results strongly support the null hypothesis. The right choice of the distributional assumption could also be the reason for such performance of our model.

Despite the good outcome of the independence test, no instant conclusions should be drawn from the results of this test. Nevertheless, it is fair to say that for the most part, all the models seem to avoid at least the most severe type of independence, namely multiple exceptions occurring on consecutive days, especially when using ARMA(2,0), and the assumption of fat-tailed distribution of errors..

4.5.5 Results of conditional coverage test

Now that we have separately presented the results of the test of coverage and independence, we present the results for conditional coverage test, which jointly test for unconditional coverage and independence. Like the previous tests, the results of this test are presented in the following three consecutive figures based on the distributional assumptions of error terms. From the first three panels of the Figure 4.5.7 it can be easily seen that none of the models is passing the conditional coverage test for nominal coverage of 99% and 99.5% when using Gaussian distributed errors. The is because these models do not pass the test of unconditional coverage for these levels as shown by the first three panels of the Figure 4.5.1. Likewise, the reason for all the p-values in the box-plots being equal to zero in the last two panels (i.e., for GOGARCH and SVX models) is because these models also do not pass their respective unconditional coverage tests. The exception is observed for DCC-GARCH model, which is due to the fact that the model passed both the test of unconditional coverage and the test of independence as shown in earlier figures. This also supports our decision of performing these tests individually in order to know the reason for not passing the conditional coverage test.

Now lets look at Figure 4.5.8, which shows the results of the test while assuming that errors follow Student's t-distribution. Among the three competing models only interval forecasts from SVX could pass the conditional coverage test for all intervals. The interval forecasts from the other two competing models can only pass the test at nominal coverage of 99% and 99.5%.

Considering the variations in the p-values (which are represented by the height of the box), our proposed model ranks first, followed by the GOGARCH and the DCC-GARCH models, as largest p-values show a high probability of accepting the null of conditional coverage. The last figure of this section shows the results of the

Figure 4.5.7: Box-plots of p-values of $\mathbf{LR_{cc}}$ test statistic using Gaussian distributed errors

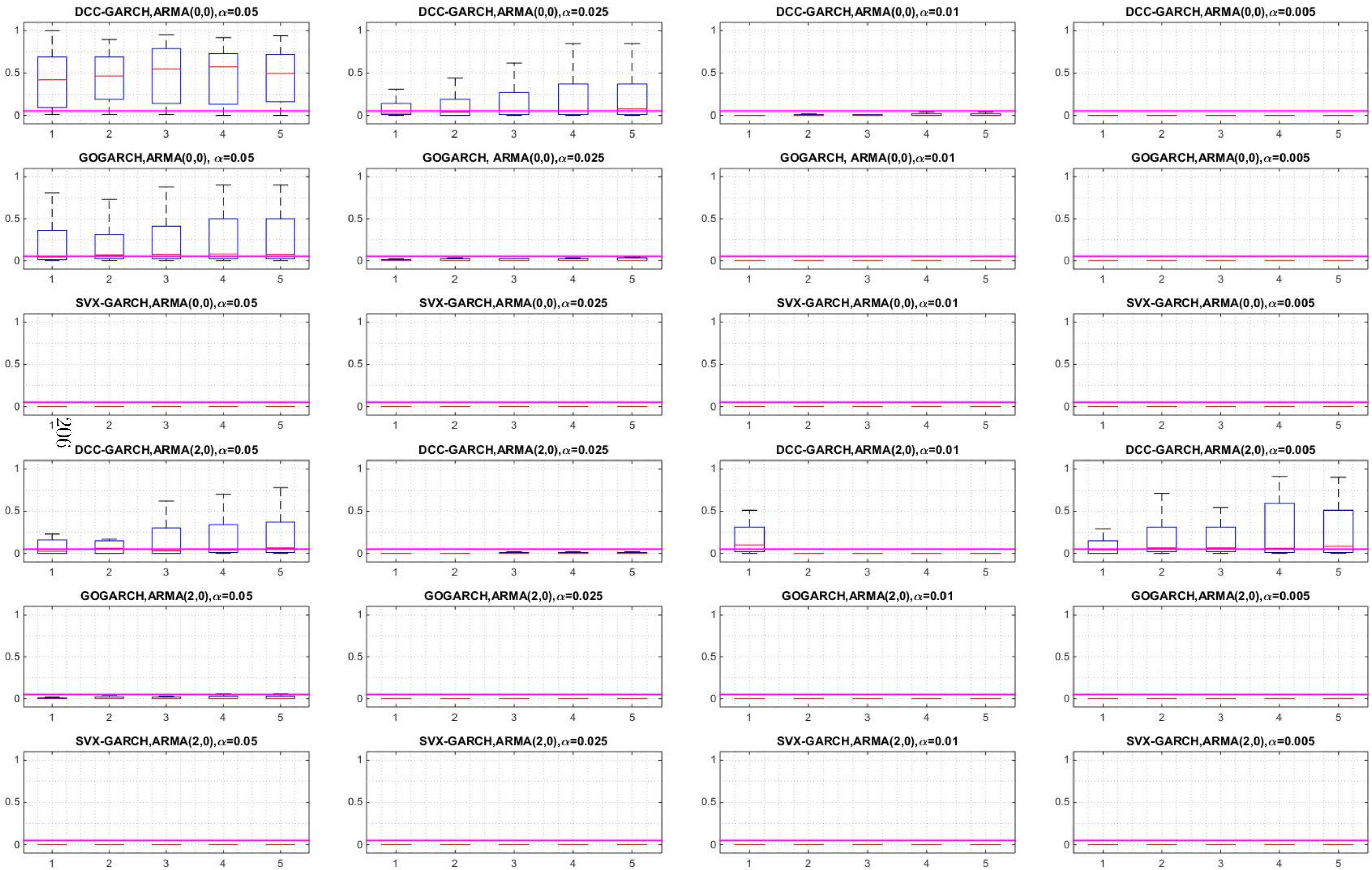


Figure 4.5.8: Box-plots of p-values of $\mathbf{LR_{cc}}$ test statistic using Student's t-distributed errors

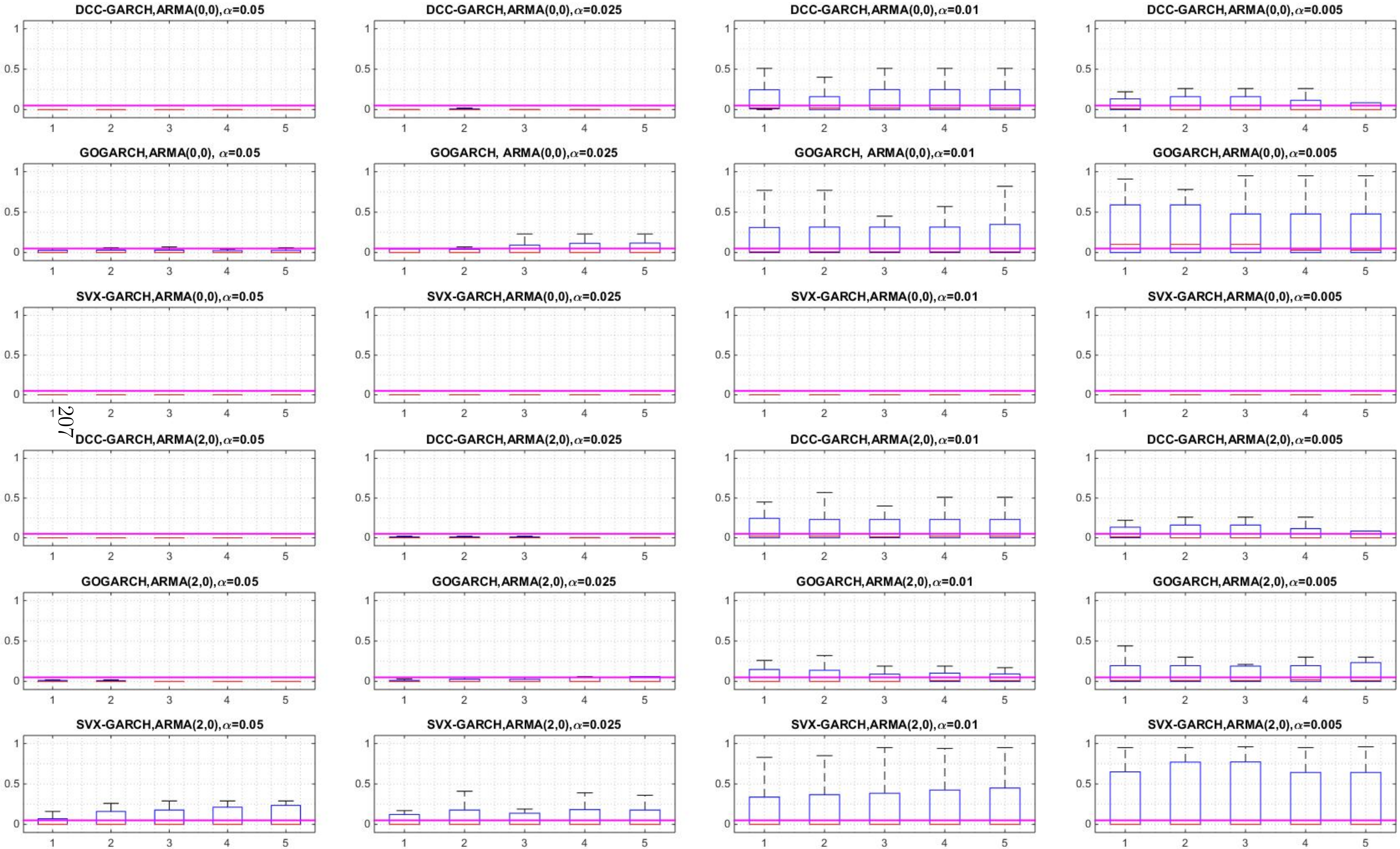
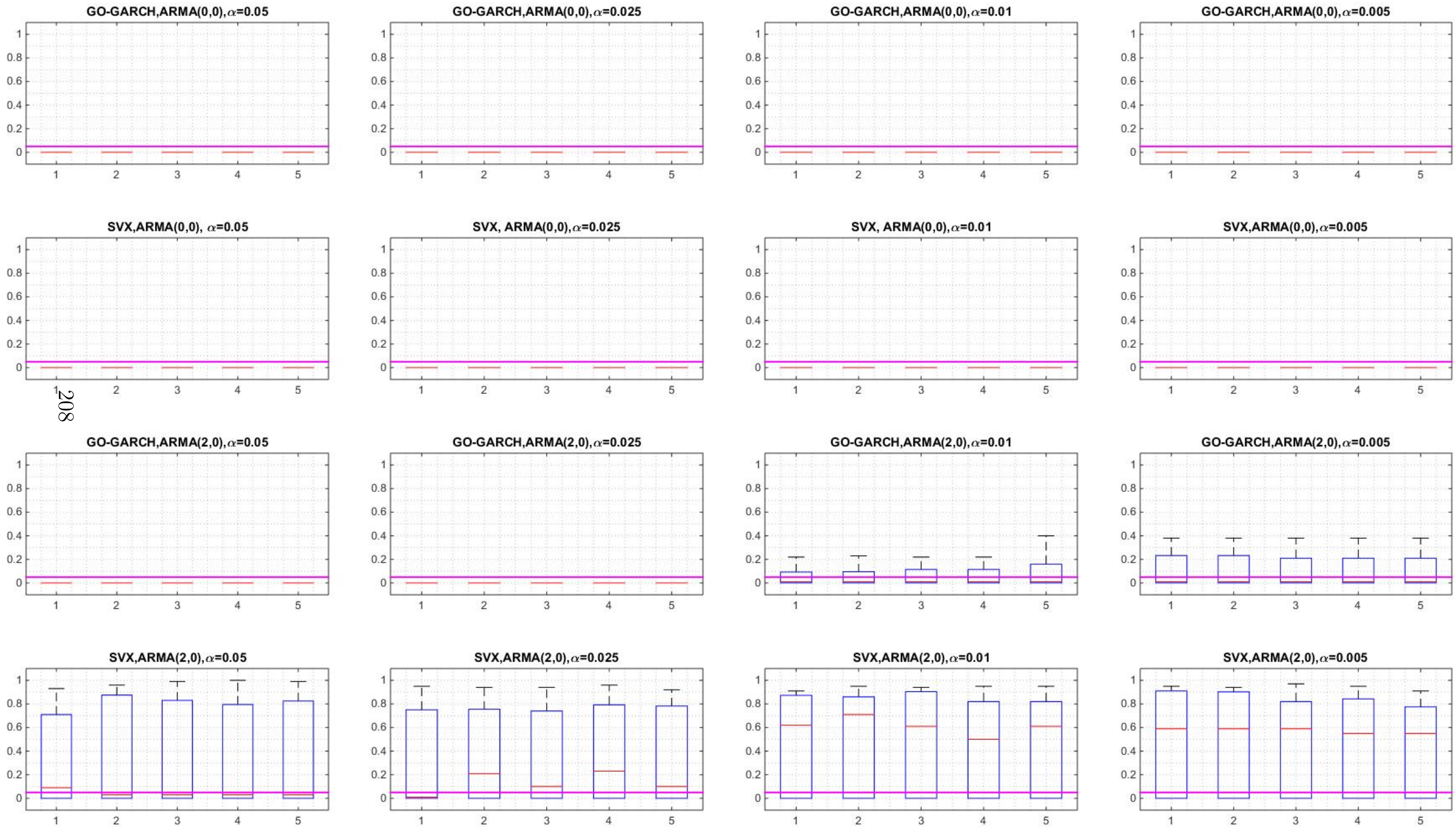


Figure 4.5.9: Box-plots of p-values of $\mathbf{LR_{cc}}$ test statistic using Skewed Student's t-distributed errors



conditional coverage tests applied to GOGARCH and SVX models. Between the two models, the SVX model outperforms the GOGARCH model at almost all levels of nominal coverage. It can be seen from the very last panel of Figure 4.5.9 that the p-values of SVX model are greater than those of the GOGARCH model.

4.6 Conclusion

In this chapter, we compared the out-of-sample forecasting performance of our proposed methodology with the alternative multivariate volatility models for the Dow 30 data set. The two most widely-used non-linear models, namely DCC-GARCH and GOGARCH models, were contrasted as competitors. Fat-tailed behavior of the financial time series has already been well documented in the financial literature, e.g., the conditional distribution of the stock returns can be better approximated by using the distributions that have fatter tails than normal such as Student's t- and skewed Student's t-distributions. Following this fact, all the models considered in this study were estimated using three distributions (i.e., Gaussian, Student's t- and its skewed version). As described in the beginning of this chapter, standard measures which are usually used to evaluate out-of-sample forecasting performance of models such as MSE and RMSE are heavily biased due to the effects of large forecasting errors. Because of this, we evaluated the forecasting performance of the models in terms of their ability to produce interval forecasts (which is also known as *Backtesting*). While evaluating these models, we considered intervals with a nominal coverage of 95%, 97.5%, 99% and 99.5%. Roughly speaking, this choice enables us to investigate the accuracy of the forecasts over the extremes of the distribution. The contextual definition of *extremes* of distribution differs in the literature: some studies consider more than 90% as the extreme of the distribution, while in other studies quantile above the 95.5% of the distribution is considered as extremes of the distribution.

A good model should produce interval forecasts with two equally important properties. First, it should produce the "correct" amount of violations (exceptions) indicated by the confidence level used in the construction of interval forecasts. If too many or too few violations are observed, the model is rejected because the former condition leads us to conclude that the model over-estimates the risk whereas in latter situation (having too few violations) we conclude that the model under-estimates the risk. This property is checked with an unconditional coverage test. It can be concluded from the results that under the assumption of normality all of the considered models fail to produce a correct amount of violations as the confidence level increases. However, our proposed model outperforms the other two models when the forecasting errors are assumed to follow Student's t-distribution and skewed Student's t-distribution when ARMA(2,0) was used as a mean-filtering model. We also observe that all models fail to capture some aspects of the underlying data generating process as indicated by the highly significant tests for correct unconditional coverage at 99% and 99.5% intervals. In particular, all models appear to generate interval forecasts with the actual coverage (π) greater than the nominal coverage (p = 0.01 and 0.005).

Second desirable property is that a good model produce violations which are spread evenly over time and do not occur in clusters. This property was checked with the likelihood ratio test of independence. It can be concluded that under normality assumption and using ARMA(0,0) and ARMA(2,0) as mean filtration model, DCC-GARCH and GOGARCH both perform well for all forecasting horizons and for all confidence levels as well. Our proposed model fails in independence test when the mean model was ARMA(0,0), but its performance improved when ARMA(2,0) was used as a mean filtration model. Similarly, all models perform equally well when the errors of the model were assumed to follow some fat-tailed distributions such as Student's t-distribution and skewed Student's t-distribution.

Lastly, we presented the results of the test that jointly examines the unconditional coverage and independence properties. This test provides an opportunity to detect interval forecasts which are deficient in one way or another. As concluded by Christoffersen (1998), the rejection of unconditional coverage leads to the rejection of conditional coverage; hence, we saw that in spite of passing the independence test at all confidence levels, all considered models could not pass the conditional coverage test. However, SVX model is the only one which passes the conditional coverage test at all intervals when the errors were assumed to follow student's t-distribution. Similarly, with the skewed student's t-distribution, SVX model outperforms the GOGARCH model in all forecast horizons and for all intervals as well. Finally, the interval forecasts from the SVX with Student's t- and skewed student's t-distribution show the best performance. The unconditional test is passed at all coverage levels in case of our proposed model. The only exception to this is ARMA(0,0), where the conditional coverage is rejected at all significance levels.

The observation made by Chatfield (1993) that out-of-sample interval forecasts tend to be narrow in practice holds true in this application. This result may be attributed either to an underestimate of the standard errors used in the calculation of interval forecasts or to an inappropriate error distribution. As reported by Boero and Marrocu (2005), it is possible that some models do better than others in predicting the tails of the distribution, but worse in predicting the other aspects. This statement justifies the use of our proposed model (SVX) in predicting the tails of the distribution.

4.7 Appendix

4.7.1 Proof of Proposition 4.1

Conditional on the first observation, the likelihood function for a first-order Markov chain with s states is $L = \prod_{i,j}^{s} \pi_{ij}^{n_{ij}}$. Consider testing the null hypothesis that $\pi_{ij} = \pi_j$. The ML estimates under the alternative are $\hat{\pi}_{ij} = {}^{n_{ij}}/_{n_i}$, with $n_i = \sum_{j=1}^{s} n_{ij}$. We want to find the distribution of $-2log\left(\lambda\right)$, where $\lambda = L_0\left(\pi_j\right)/L\left(\hat{\pi}_{ij}\right)$. Bertlett (1951) shows that the transition counts, n_{ij}, are asymptotically normally distributed so that

$$L \sim c \mid A \mid^{1/2} exp\left(-\frac{1}{2}\left[n-\mu\right]' A \left[n-\mu\right]\right),$$

where $[n-\mu]$ is the vector of linearly independent variables $n_{ij} - \mu_{ij}$, with μ_{ij} being the expected value of n_{ij}. Using the result in the expression for λ provides.

$$\lambda \sim \frac{c \mid A^0 \mid^{1/2} exp\left(-\frac{1}{2}\left[n-\mu^0\right]' A^0 \left[n-\mu^0\right]\right)}{c \mid \hat{A} \mid^{1/2} exp\left(-\frac{1}{2}\left[n-\hat{\mu}\right]' \hat{A} \left[n-\hat{\mu}\right]\right)},$$

where the parameters have been replaced by their ML estimates. This can then be written as

$$-2log\left(\lambda\right) \sim log\left(\frac{\mid \hat{A} \mid}{\mid A^0 \mid}\right) + \left[n-\mu^0\right]' A^0 \left[n-\mu^0\right] + \left[n-\hat{\mu}\right]' \hat{A} \left[n-\hat{\mu}\right].$$

Under the null, $\hat{\pi}_{ij}$ converges to $\pi_{ij} = \pi_j$, thus, $\mid \hat{A} \mid$ converges to $\mid A^0 \mid$, and we get

$$-2log\left(\lambda\right) \sim \left[n-\mu^0\right]' A^0 \left[n-\mu^0\right] + \left[n-\hat{\mu}\right]' \hat{A} \left[n-\hat{\mu}\right].$$

It can be shown that $\hat{\mu}_{ij} = n_{ij}$, so that the second term in this expression will vanish, and what is left is $-2log\left(\lambda\right) \sim \left[n-\mu^0\right]' A^0 \left[n-\mu^0\right]$.
The typical element in $[n-\mu^0]$is $w_{ij} = n_{ij} - \pi_i n_j$. Note that there are $s-1$ independent restrictions of the form $\sum_{i=1}^{s} n_{ij} = n_j = \sum_{t=1}^{s} n_{jt}$, and in addition, $\sum_{i}^{s} n_{ij} = n$. Thus, there are only $s^2 - s = s(s-1)$ independent variables in the quadratic form, and we get

$$-2log\left(\lambda\right) \sim \chi^2\left(s\left(s-1\right)\right).$$

In the binary case, $s = 2$, and we get a χ_2^2 distribution.

4.7.2 Interval Forecast Evaluation of DCC Model with Gaussian Distributed Errors, Mean Model = ARMA(0,0)

Table 4.4: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

	h = 1															
Stocks	**100(α) = 5**				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	5.65	**0.19**	**0.87**	**0.42**	3.55	0.00	**0.37**	0.01	2.25	0.00	**0.10**	0.00	1.50	0.00	**0.34**	0.00
MSFT	5.25	**0.61**	**0.83**	**0.86**	3.65	0.00	**0.66**	0.01	2.20	0.00	**0.16**	0.00	1.70	0.00	**0.28**	0.00
GE	6.35	0.01	**0.48**	**0.02**	3.90	0.00	**0.28**	0.00	2.15	0.00	**0.94**	0.00	1.50	0.00	**0.47**	0.00
JNJ	5.20	**0.68**	**0.49**	**0.72**	3.70	0.00	**0.87**	0.01	2.30	0.00	**0.95**	0.00	1.50	0.00	**0.47**	0.00
WMT	5.50	**0.31**	**0.05**	**0.09**	3.70	0.00	**0.07**	0.00	2.25	0.00	**0.10**	0.00	1.85	0.00	**0.19**	0.00
CVX	5.05	**0.91**	**0.59**	**0.86**	3.05	**0.13**	0.92	**0.31**	1.55	**0.02**	**0.32**	**0.04**	0.90	0.02	**0.57**	0.06
JPM	5.40	**0.41**	**0.37**	**0.48**	3.60	0.00	**0.06**	0.00	2.10	0.00	**0.29**	0.00	1.65	0.00	**0.12**	0.00
PG	5.80	**0.11**	**0.61**	**0.24**	4.00	0.00	**0.14**	0.00	2.50	0.00	**0.53**	0.00	1.90	0.00	**0.20**	0.00
PFE	5.35	**0.47**	**0.90**	**0.77**	2.85	**0.32**	**0.32**	**0.37**	1.50	0.04	**0.34**	**0.07**	1.20	0.00	**0.45**	0.00
IBM	5.35	**0.47**	**0.74**	**0.73**	3.40	0.01	**0.32**	**0.03**	2.10	0.00	**0.90**	0.00	1.50	0.00	**0.47**	0.00
T	5.55	**0.26**	**0.94**	**0.54**	2.95	**0.21**	**0.13**	**0.14**	1.65	0.01	**0.29**	**0.02**	1.25	0.00	**0.43**	0.00
KO	5.60	**0.22**	**0.13**	**0.15**	3.75	0.00	**0.60**	0.00	2.60	0.00	**0.59**	0.00	1.95	0.00	**0.79**	0.00
MRK	3.80	0.01	**0.09**	0.01	2.75	**0.48**	**0.08**	**0.17**	2.05	0.00	**0.27**	0.00	1.60	0.00	**0.11**	0.00
VZ	5.20	**0.68**	**0.27**	**0.50**	3.40	0.01	**0.65**	**0.04**	1.85	0.00	**0.72**	0.00	1.30	0.00	**0.35**	0.00
DIS	5.65	**0.19**	**0.87**	**0.42**	3.60	0.00	**0.69**	0.01	2.00	0.00	**0.83**	0.00	1.35	0.00	**0.39**	0.00
INTC	4.75	**0.61**	**0.48**	**0.69**	3.50	0.01	**0.73**	**0.02**	1.75	0.00	**0.64**	0.01	1.20	0.00	**0.29**	0.00
CSCO	5.00	**1.00**	**0.09**	**0.23**	3.65	0.00	**0.43**	0.01	1.90	0.00	**0.22**	0.00	1.50	0.00	**0.34**	0.00
HD	5.55	**0.26**	**0.94**	**0.54**	3.35	0.02	**0.27**	**0.04**	2.15	0.00	**0.32**	0.00	1.50	0.00	**0.47**	0.00
UTX	6.25	0.01	**0.47**	**0.04**	4.10	0.00	**0.72**	0.00	2.00	0.00	**0.25**	0.00	1.45	0.00	**0.36**	0.00
BA	6.05	**0.04**	**0.17**	**0.04**	3.60	0.00	**0.40**	0.01	2.30	0.00	**0.95**	0.00	1.70	0.00	**0.28**	0.00
MCD	4.40	**0.21**	**0.57**	**0.39**	2.75	**0.48**	**0.65**	**0.70**	1.90	0.00	**0.75**	0.00	1.45	0.00	**0.44**	0.00
AXP	5.55	**0.26**	**0.04**	**0.07**	3.25	**0.04**	**0.38**	**0.08**	1.85	0.00	**0.24**	0.00	1.25	0.00	**0.43**	0.00
MMM	5.90	**0.07**	**0.13**	**0.06**	4.00	0.00	**0.14**	0.00	2.55	0.00	**0.19**	0.00	1.75	0.00	**0.64**	0.00
GS	5.65	**0.19**	**0.30**	**0.25**	3.20	**0.05**	**0.40**	**0.11**	1.80	0.00	**0.25**	0.00	1.40	0.00	**0.37**	0.00
UNH	5.55	**0.26**	**0.61**	**0.47**	3.00	**0.16**	**0.88**	**0.37**	1.90	0.00	**0.75**	0.00	1.40	0.00	**0.41**	0.00
CAT	6.10	**0.03**	**0.34**	**0.06**	3.55	0.00	**0.15**	0.01	2.25	0.00	**0.99**	0.00	1.40	0.00	**0.37**	0.00
DD	5.10	**0.83**	**0.27**	**0.54**	3.40	0.01	**0.32**	**0.03**	1.75	0.00	**0.26**	0.01	1.15	0.00	**0.46**	0.00
NKE	4.25	**0.12**	**0.47**	**0.22**	2.75	**0.48**	**0.65**	**0.70**	2.10	0.00	**0.90**	0.00	1.90	0.00	**0.75**	0.00
TRV	5.20	**0.68**	**0.79**	**0.89**	3.00	**0.16**	**0.14**	**0.13**	2.10	0.00	**0.29**	0.00	1.60	0.00	**0.54**	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.5: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	**h = 2**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	5.30	**0.54**	**0.87**	**0.82**	3.00	**0.16**	**0.40**	**0.27**	2.20	0.00	**0.97**	0.00	1.45	0.00	**0.36**	0.00
MSFT	5.20	**0.68**	**0.85**	**0.90**	3.65	0.00	**0.66**	0.01	2.20	0.00	**0.16**	0.00	1.55	0.00	**0.32**	0.00
GE	6.05	**0.04**	**0.32**	**0.07**	3.70	0.00	**0.20**	0.00	2.15	0.00	**0.94**	0.00	1.50	0.00	**0.47**	0.00
JNJ	5.10	**0.83**	**0.43**	**0.71**	3.55	0.00	**0.72**	**0.02**	2.20	0.00	**0.97**	0.00	1.50	0.00	**0.47**	0.00
WMT	5.35	**0.47**	**0.34**	**0.49**	3.60	0.00	**0.06**	0.00	2.15	0.00	**0.32**	0.00	1.75	0.00	**0.15**	0.00
CVX	5.25	**0.61**	**0.48**	**0.68**	2.85	**0.32**	**0.07**	**0.12**	1.35	**0.13**	**0.39**	**0.23**	0.90	**0.02**	**0.57**	0.06
JPM	5.50	**0.31**	**0.23**	**0.29**	3.55	0.00	**0.15**	0.01	2.15	0.00	**0.32**	0.00	1.65	0.00	**0.57**	0.00
PG	5.70	**0.16**	**0.32**	**0.23**	4.10	0.00	**0.07**	0.00	2.65	0.00	**0.23**	0.00	2.10	0.00	**0.29**	0.00
PFE	5.20	**0.68**	**0.79**	**0.89**	2.75	**0.48**	**0.27**	**0.42**	1.60	0.01	**0.31**	**0.03**	1.20	0.00	**0.45**	0.00
IBM	5.40	**0.41**	**0.71**	**0.67**	3.35	**0.02**	**0.34**	**0.04**	2.05	0.00	**0.86**	0.00	1.40	0.00	**0.41**	0.00
T	5.60	**0.22**	**0.76**	**0.46**	2.80	**0.40**	**0.09**	**0.17**	1.65	0.01	**0.57**	**0.02**	1.20	0.00	**0.45**	0.00
KO	5.85	**0.09**	**0.95**	**0.23**	3.80	0.00	**0.57**	0.00	2.50	0.00	**0.81**	0.00	1.85	0.00	**0.72**	0.00
MRK	3.75	0.01	**0.08**	0.01	2.65	**0.67**	**0.23**	**0.44**	2.15	0.00	**0.08**	0.00	1.60	0.00	**0.11**	0.00
VZ	5.25	**0.61**	**0.15**	**0.31**	3.35	**0.02**	**0.27**	**0.04**	1.65	0.01	**0.57**	**0.02**	1.30	0.00	**0.35**	0.00
DIS	5.55	**0.26**	**0.61**	**0.47**	3.55	0.00	**0.72**	**0.02**	1.95	0.00	**0.79**	0.00	1.30	0.00	**0.41**	0.00
INTC	4.75	**0.61**	**0.48**	**0.69**	3.30	**0.03**	**0.59**	**0.08**	1.60	0.01	**0.54**	**0.04**	1.10	0.00	**0.24**	0.00
CSCO	4.90	**0.84**	**0.07**	**0.19**	3.60	0.00	**0.17**	0.00	1.90	0.00	**0.22**	0.00	1.50	0.00	**0.34**	0.00
HD	5.45	**0.36**	**0.67**	**0.60**	3.10	**0.10**	**0.46**	**0.19**	2.10	0.00	**0.29**	0.00	1.50	0.00	**0.47**	0.00
UTX	6.00	**0.05**	**0.36**	**0.09**	4.20	0.00	**0.44**	0.00	2.05	0.00	**0.27**	0.00	1.45	0.00	**0.36**	0.00
BA	5.70	**0.16**	**0.17**	**0.15**	3.60	0.00	**0.40**	0.01	2.25	0.00	**0.99**	0.00	1.80	0.00	**0.68**	0.00
MCD	4.35	**0.17**	**0.66**	**0.36**	2.70	**0.57**	**0.68**	**0.78**	1.80	0.00	**0.68**	0.00	1.40	0.00	**0.41**	0.00
AXP	5.55	**0.26**	**0.04**	**0.07**	3.10	**0.10**	**0.45**	**0.19**	1.75	0.00	**0.26**	0.01	1.20	0.00	**0.45**	0.00
MMM	5.85	**0.09**	**0.06**	**0.04**	3.95	0.00	**0.13**	0.00	2.50	0.00	**0.17**	0.00	1.85	0.00	**0.72**	0.00
GS	5.35	**0.47**	**0.34**	**0.49**	3.40	0.01	**0.65**	**0.04**	1.75	0.00	**0.26**	0.01	1.45	0.00	**0.36**	0.00
UNH	5.20	**0.68**	**0.85**	**0.90**	2.90	**0.26**	**0.81**	**0.52**	1.75	0.00	**0.64**	0.01	1.40	0.00	**0.41**	0.00
CAT	5.85	**0.09**	**0.23**	**0.11**	3.60	0.00	**0.17**	0.00	2.15	0.00	**0.17**	0.00	1.40	0.00	**0.37**	0.00
DD	5.05	**0.91**	**0.29**	**0.57**	3.30	**0.03**	**0.36**	**0.06**	1.80	0.00	**0.25**	0.00	1.15	0.00	**0.46**	0.00
NKE	4.35	**0.17**	**0.27**	**0.22**	2.65	**0.67**	**0.71**	**0.85**	2.10	0.00	**0.90**	0.00	1.80	0.00	**0.68**	0.00
TRV	5.30	**0.54**	**0.55**	**0.69**	2.85	**0.32**	**0.10**	**0.16**	2.15	0.00	0.01	0.00	1.60	0.00	**0.54**	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.6: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	5.25	**0.61**	**0.83**	**0.86**	3.00	**0.16**	**0.40**	**0.27**	2.20	0.00	**0.97**	0.00	1.40	0.00	**0.37**	0.00
MSFT	5.35	**0.47**	**0.59**	**0.67**	3.60	0.00	**0.69**	0.01	2.15	0.00	**0.17**	0.00	1.60	0.00	**0.31**	0.00
GE	5.90	**0.07**	**0.43**	**0.14**	3.55	0.00	**0.15**	0.01	2.30	0.00	**0.11**	0.00	1.45	0.00	**0.44**	0.00
JNJ	4.95	**0.92**	**0.61**	**0.88**	3.40	0.01	**0.65**	**0.04**	2.10	0.00	**0.90**	0.00	1.55	0.00	**0.51**	0.00
WMT	5.35	**0.47**	**0.59**	**0.67**	3.60	0.00	**0.06**	0.00	2.20	0.00	**0.09**	0.00	1.70	0.00	**0.13**	0.00
CVX	5.30	**0.54**	**0.45**	**0.62**	2.85	**0.32**	**0.07**	**0.12**	1.35	**0.13**	**0.39**	**0.23**	0.90	**0.02**	**0.57**	0.06
JPM	5.45	**0.36**	**0.05**	**0.09**	3.50	0.01	**0.14**	0.01	2.10	0.00	**0.29**	0.00	1.60	0.00	**0.54**	0.00
PG	5.70	**0.16**	**0.04**	**0.04**	4.25	0.00	**0.10**	0.00	2.90	0.00	**0.11**	0.00	2.35	0.00	**0.12**	0.00
PFE	5.15	**0.76**	**0.76**	**0.91**	2.70	**0.57**	**0.25**	**0.44**	1.60	0.01	**0.54**	**0.04**	1.25	0.00	**0.43**	0.00
IBM	5.45	**0.36**	**0.67**	**0.60**	3.45	0.01	**0.30**	**0.02**	2.05	0.00	**0.86**	0.00	1.40	0.00	**0.41**	0.00
T	5.45	**0.36**	**0.98**	**0.66**	2.80	**0.40**	**0.29**	**0.40**	1.60	0.01	**0.54**	**0.04**	1.15	0.00	**0.46**	0.00
KO	5.60	**0.22**	**0.31**	**0.28**	3.80	0.00	**0.57**	0.00	2.45	0.00	**0.85**	0.00	1.70	0.00	**0.61**	0.00
MRK	3.75	0.01	**0.08**	0.01	2.65	**0.67**	**0.23**	**0.44**	2.10	0.00	**0.07**	0.00	1.45	0.00	**0.07**	0.00
VZ	5.35	**0.47**	**0.08**	**0.17**	3.35	**0.02**	**0.27**	**0.04**	1.60	0.01	**0.54**	**0.04**	1.25	0.00	**0.32**	0.00
DIS	5.35	**0.47**	**0.90**	**0.77**	3.55	0.00	**0.26**	0.01	1.90	0.00	**0.75**	0.00	1.25	0.00	**0.43**	0.00
INTC	4.70	**0.54**	**0.78**	**0.79**	3.30	**0.03**	**0.59**	**0.08**	1.60	0.01	**0.54**	**0.04**	1.10	0.00	**0.24**	0.00
CSCO	4.80	**0.68**	**0.05**	**0.14**	3.60	0.00	**0.06**	0.00	1.85	0.00	**0.24**	0.00	1.50	0.00	**0.34**	0.00
HD	5.30	**0.54**	**0.78**	**0.80**	3.10	**0.10**	**0.17**	**0.10**	2.10	0.00	**0.29**	0.00	1.50	0.00	**0.47**	0.00
UTX	6.10	**0.03**	**0.31**	**0.05**	4.10	0.00	**0.83**	0.00	1.80	0.00	**0.25**	0.00	1.45	0.00	**0.36**	0.00
BA	5.70	**0.16**	**0.17**	**0.15**	3.40	0.01	**0.83**	**0.05**	2.20	0.00	**0.97**	0.00	1.75	0.00	**0.64**	0.00
MCD	4.30	**0.14**	**0.87**	**0.34**	2.80	**0.40**	**0.62**	**0.62**	1.90	0.00	**0.75**	0.00	1.45	0.00	**0.44**	0.00
AXP	5.60	**0.22**	**0.04**	**0.05**	3.10	0.10	**0.45**	**0.19**	1.90	0.00	**0.22**	0.00	1.25	0.00	**0.43**	0.00
MMM	5.95	**0.06**	0.01	0.01	4.00	0.00	**0.05**	0.00	2.55	0.00	**0.19**	0.00	1.95	0.00	**0.79**	0.00
GS	5.30	**0.54**	**0.32**	**0.50**	3.15	**0.07**	**0.99**	**0.20**	1.75	0.00	**0.26**	0.01	1.30	0.00	**0.41**	0.00
UNH	4.85	**0.76**	**0.89**	**0.95**	3.05	**0.13**	**0.92**	**0.31**	1.70	0.00	**0.61**	0.01	1.30	0.00	**0.35**	0.00
CAT	5.85	**0.09**	**0.23**	**0.11**	3.55	0.00	**0.15**	0.01	2.05	0.00	**0.19**	0.00	1.40	0.00	**0.37**	0.00
DD	4.90	**0.84**	**0.36**	**0.64**	3.20	**0.05**	**0.40**	**0.11**	1.70	0.00	**0.28**	0.01	1.20	0.00	**0.45**	0.00
NKE	4.25	**0.12**	**0.83**	**0.28**	2.65	**0.67**	**0.71**	**0.85**	2.10	0.00	**0.90**	0.00	1.80	0.00	**0.68**	0.00
TRV	5.20	**0.68**	**0.79**	**0.89**	2.85	**0.32**	**0.03**	**0.05**	2.05	0.00	0.01	0.00	1.70	0.00	**0.13**	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.7: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	5.10	**0.83**	**0.72**	**0.92**	2.90	**0.26**	**0.34**	**0.34**	2.15	0.00	**0.94**	0.00	1.35	0.00	**0.39**	0.00
MSFT	5.35	**0.47**	**0.59**	**0.67**	3.55	0.00	**0.72**	**0.02**	2.15	0.00	**0.17**	0.00	1.60	0.00	**0.31**	0.00
GE	5.80	**0.11**	**0.37**	**0.19**	3.50	0.01	**0.14**	0.01	2.30	0.00	**0.11**	0.00	1.40	0.00	**0.41**	0.00
JNJ	5.00	**1.00**	**0.65**	**0.90**	3.35	**0.02**	**0.62**	**0.06**	2.00	0.00	**0.83**	0.00	1.50	0.00	**0.47**	0.00
WMT	5.35	**0.47**	**0.34**	**0.49**	3.55	0.00	**0.05**	0.00	2.15	0.00	**0.08**	0.00	1.70	0.00	**0.13**	0.00
CVX	5.25	**0.61**	**0.48**	**0.68**	2.80	**0.40**	0.07	**0.14**	1.35	**0.13**	**0.39**	**0.23**	0.85	**0.04**	**0.59**	**0.11**
JPM	5.15	**0.76**	**0.02**	**0.07**	3.50	0.01	**0.14**	0.01	2.10	0.00	**0.29**	0.00	1.60	0.00	**0.54**	0.00
PG	5.65	**0.19**	**0.01**	**0.02**	4.15	0.00	**0.08**	0.00	2.80	0.00	**0.29**	0.00	2.40	0.00	**0.14**	0.00
PFE	5.20	**0.68**	**0.49**	**0.72**	2.80	**0.40**	**0.29**	**0.40**	1.55	**0.02**	**0.51**	**0.06**	1.25	0.00	**0.43**	0.00
IBM	5.40	**0.41**	**0.94**	**0.71**	3.60	0.00	**0.69**	0.01	2.05	0.00	**0.86**	0.00	1.40	0.00	**0.41**	0.00
T	5.40	**0.41**	**0.94**	**0.71**	2.75	**0.48**	**0.27**	**0.42**	1.60	0.01	**0.54**	**0.04**	1.15	0.00	**0.46**	0.00
KO	5.35	**0.47**	**0.19**	**0.33**	3.80	0.00	**0.57**	0.00	2.30	0.00	**0.95**	0.00	1.60	0.00	**0.54**	0.00
MRK	3.80	0.01	**0.09**	0.01	2.65	**0.67**	**0.23**	**0.44**	2.05	0.00	**0.06**	0.00	1.45	0.00	**0.07**	0.00
VZ	5.20	**0.68**	**0.27**	**0.50**	3.35	**0.02**	**0.27**	**0.04**	1.60	0.01	**0.54**	**0.04**	1.20	0.00	**0.29**	0.00
DIS	5.20	**0.68**	**0.85**	**0.90**	3.45	0.01	**0.30**	**0.02**	1.80	0.00	**0.68**	0.00	1.30	0.00	**0.41**	0.00
INTC	4.65	**0.47**	**0.74**	**0.73**	3.30	**0.03**	**0.59**	**0.08**	1.60	0.01	**0.54**	**0.04**	1.10	0.00	**0.24**	0.00
CSCO	4.75	**0.61**	**0.05**	**0.12**	3.50	0.01	**0.04**	0.00	1.75	0.00	**0.26**	0.01	1.50	0.00	**0.34**	0.00
HD	5.20	**0.68**	**0.85**	**0.90**	3.10	**0.10**	**0.17**	**0.10**	2.10	0.00	**0.29**	0.00	1.50	0.00	**0.47**	0.00
UTX	6.00	**0.05**	**0.36**	**0.09**	4.05	0.00	**0.87**	0.00	1.90	0.00	**0.75**	0.00	1.45	0.00	**0.36**	0.00
BA	5.60	**0.22**	**0.14**	**0.16**	3.40	0.01	**0.83**	**0.05**	2.20	0.00	**0.97**	0.00	1.70	0.00	**0.61**	0.00
MCD	4.20	**0.09**	**0.80**	**0.24**	2.75	**0.48**	**0.65**	**0.70**	1.85	0.00	**0.72**	0.00	1.40	0.00	**0.41**	0.00
AXP	5.35	**0.47**	**0.06**	**0.13**	3.05	**0.13**	**0.48**	**0.24**	1.80	0.00	**0.25**	0.00	1.20	0.00	**0.45**	0.00
MMM	5.95	**0.06**	**0.01**	0.00	3.85	0.00	0.01	0.00	2.60	0.00	**0.21**	0.00	1.95	0.00	**0.79**	0.00
GS	5.25	**0.61**	**0.52**	**0.71**	3.00	**0.16**	**0.88**	**0.37**	1.70	0.00	**0.28**	0.01	1.35	0.00	**0.39**	0.00
UNH	4.80	**0.68**	**0.85**	**0.90**	3.00	**0.16**	**0.88**	**0.37**	1.60	0.01	**0.54**	**0.04**	1.25	0.00	**0.32**	0.00
CAT	5.80	**0.11**	**0.21**	**0.12**	3.45	0.01	**0.32**	**0.02**	2.00	0.00	**0.20**	0.00	1.40	0.00	**0.37**	0.00
DD	4.85	**0.76**	**0.38**	**0.65**	3.00	**0.16**	**0.50**	**0.30**	1.65	0.01	**0.29**	**0.02**	1.20	0.00	**0.45**	0.00
NKE	4.25	**0.12**	**0.47**	**0.22**	2.65	**0.67**	**0.71**	**0.85**	2.00	0.00	**0.83**	0.00	1.80	0.00	**0.68**	0.00
TRV	5.10	**0.83**	**0.72**	**0.92**	2.85	**0.32**	**0.03**	**0.05**	2.05	0.00	0.01	0.00	1.65	0.00	**0.12**	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.8: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	h = 5															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	5.15	**0.76**	**0.46**	**0.72**	2.90	**0.26**	**0.34**	**0.34**	2.15	0.00	**0.94**	0.00	1.40	0.00	**0.41**	0.00
MSFT	5.35	**0.47**	**0.59**	**0.67**	3.50	0.01	**0.76**	**0.02**	2.15	0.00	**0.17**	0.00	1.60	0.00	**0.31**	0.00
GE	5.75	**0.13**	**0.35**	**0.21**	3.45	0.01	**0.12**	0.01	2.20	0.00	**0.35**	0.00	1.35	0.00	**0.38**	0.00
JNJ	4.85	**0.76**	**0.55**	**0.80**	3.35	**0.02**	**0.27**	**0.04**	2.00	0.00	**0.25**	0.00	1.50	0.00	**0.47**	0.00
WMT	5.25	**0.61**	**0.29**	**0.50**	3.50	0.01	**0.04**	0.00	2.10	0.00	**0.07**	0.00	1.70	0.00	**0.13**	0.00
CVX	5.25	**0.61**	**0.48**	**0.68**	2.80	**0.40**	**0.07**	**0.14**	1.20	**0.38**	**0.45**	**0.51**	0.85	**0.04**	**0.59**	**0.11**
JPM	5.05	**0.91**	**0.02**	**0.05**	3.40	0.01	**0.11**	0.01	2.10	0.00	**0.29**	0.00	1.60	0.00	**0.54**	0.00
PG	5.70	**0.16**	0.01	0.01	4.10	0.00	**0.07**	0.00	2.90	0.00	**0.11**	0.00	2.45	0.00	**0.15**	0.00
PFE	5.15	**0.76**	**0.25**	**0.49**	2.85	**0.32**	**0.32**	**0.37**	1.55	**0.02**	**0.51**	0.06	1.30	0.00	**0.41**	0.00
IBM	5.35	**0.47**	**0.59**	**0.67**	3.55	0.00	**0.72**	**0.02**	2.00	0.00	**0.83**	0.00	1.45	0.00	**0.44**	0.00
T	5.40	**0.41**	**0.94**	**0.71**	2.70	**0.57**	**0.25**	**0.44**	1.55	**0.02**	**0.51**	**0.06**	1.15	0.00	**0.46**	0.00
KO	5.35	**0.47**	**0.19**	**0.33**	3.75	0.00	**0.60**	0.00	2.20	0.00	**0.97**	0.00	1.60	0.00	**0.54**	0.00
MRK	3.75	0.01	**0.08**	0.01	2.60	**0.77**	**0.21**	**0.43**	2.00	0.00	**0.05**	0.00	1.30	0.00	**0.05**	0.00
VZ	5.15	**0.76**	0.25	**0.49**	3.15	**0.07**	**0.19**	**0.08**	1.60	0.01	**0.54**	**0.04**	1.20	0.00	**0.29**	0.00
DIS	5.15	**0.76**	**0.89**	**0.94**	3.50	0.01	**0.28**	0.01	1.80	0.00	**0.25**	0.00	1.25	0.00	**0.43**	0.00
INTC	4.70	**0.54**	**0.45**	**0.62**	3.30	**0.03**	**0.59**	**0.08**	1.60	0.01	**0.54**	**0.04**	1.10	0.00	**0.24**	0.00
CSCO	4.75	**0.61**	**0.05**	**0.12**	3.50	0.01	**0.04**	0.00	1.75	0.00	**0.26**	0.01	1.50	0.00	**0.34**	0.00
HD	5.15	**0.76**	**0.89**	**0.94**	3.15	**0.07**	**0.19**	**0.08**	2.10	0.00	**0.29**	0.00	1.35	0.00	**0.38**	0.00
UTX	5.95	**0.06**	**0.38**	**0.11**	3.95	0.00	**0.94**	0.00	1.80	0.00	**0.68**	0.00	1.45	0.00	**0.36**	0.00
BA	5.55	**0.26**	**0.13**	**0.17**	3.35	**0.02**	**0.86**	**0.07**	2.20	0.00	**0.97**	0.00	1.65	0.00	**0.57**	0.00
MCD	4.20	**0.09**	**0.80**	**0.24**	2.70	**0.57**	**0.68**	**0.78**	1.80	0.00	**0.68**	0.00	1.35	0.00	**0.38**	0.00
AXP	5.30	**0.54**	**0.07**	**0.15**	3.05	**0.13**	**0.48**	**0.24**	1.80	0.00	**0.25**	0.00	1.15	0.00	**0.46**	0.00
MMM	5.95	**0.06**	0.01	0.00	3.90	0.00	**0.01**	0.00	2.60	0.00	**0.21**	0.00	2.05	0.00	**0.06**	0.00
GS	5.15	**0.76**	**0.46**	**0.72**	3.00	**0.16**	**0.88**	**0.37**	1.70	0.00	**0.28**	0.01	1.35	0.00	**0.39**	0.00
UNH	4.70	**0.54**	**0.78**	**0.79**	3.00	**0.16**	**0.88**	**0.37**	1.65	0.01	**0.57**	**0.02**	1.25	0.00	**0.32**	0.00
CAT	5.60	**0.22**	**0.14**	**0.16**	3.40	0.01	**0.29**	**0.03**	1.95	0.00	**0.21**	0.00	1.40	0.00	**0.37**	0.00
DD	4.85	**0.76**	**0.38**	**0.65**	2.95	**0.21**	**0.53**	**0.37**	1.65	0.01	**0.29**	**0.02**	1.20	0.00	**0.45**	0.00
NKE	4.25	**0.12**	**0.83**	**0.28**	2.65	**0.67**	**0.71**	**0.85**	1.95	0.00	**0.79**	0.00	1.80	0.00	**0.68**	0.00
TRV	5.00	**1.00**	**0.65**	**0.90**	2.85	**0.32**	**0.03**	**0.05**	2.05	0.00	0.01	0.00	1.65	0.00	**0.12**	0.00

Note. Bold type entries are not significant at the 1% level.

4.7.3 Interval Forecast Evaluation of DCC Model with Gaussian Distributed Errors, Mean Model = ARMA(2,0)

Table 4.9: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	**h = 1**															
	$100(\alpha) = 5$				**$100(\alpha) = 2.5$**				**$100(\alpha) = 1$**				**$100(\alpha) = 0.5$**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	6.10	**0.03**	**0.83**	**0.09**	3.50	0.01	**0.14**	0.01	2.20	0.00	**0.09**	0.00	1.30	0.00	**0.41**	**0.31**
MSFT	5.35	**0.47**	**0.18**	**0.31**	3.70	0.00	**0.87**	0.01	2.30	0.00	**0.14**	0.00	1.75	0.00	**0.26**	0.01
GE	6.10	**0.03**	**0.19**	**0.04**	3.80	0.00	**0.24**	0.00	2.25	0.00	**0.10**	0.00	1.60	0.00	**0.54**	**0.04**
JNJ	5.60	**0.22**	**0.03**	**0.04**	3.65	0.00	**0.83**	0.01	2.35	0.00	**0.92**	0.00	1.60	0.00	**0.54**	**0.04**
WMT	5.35	**0.47**	**0.08**	**0.17**	3.70	0.00	**0.02**	0.00	2.25	0.00	**0.37**	0.00	1.90	0.00	**0.20**	0.00
CVX	5.50	**0.31**	**0.98**	**0.60**	3.05	**0.13**	**0.43**	**0.23**	1.60	0.01	**0.31**	**0.03**	0.95	0.01	**0.55**	**0.81**
JPM	5.45	**0.36**	**0.11**	**0.18**	3.50	0.01	**0.04**	0.00	2.25	0.00	**0.37**	0.00	1.60	0.00	**0.54**	**0.04**
PG	6.10	**0.03**	**0.83**	**0.09**	4.15	0.00	**0.19**	0.00	2.50	0.00	**0.53**	0.00	2.15	0.00	**0.32**	0.00
PFE	5.25	**0.61**	**0.81**	**0.85**	3.05	**0.13**	**0.16**	**0.11**	1.60	0.01	**0.31**	**0.03**	1.25	0.01	**0.43**	**0.40**
IBM	5.50	**0.31**	**0.64**	**0.54**	3.50	0.01	**0.28**	0.01	2.10	0.00	**0.90**	0.00	1.50	0.00	**0.47**	**0.09**
T	5.65	**0.19**	**0.30**	**0.25**	3.10	**0.10**	**0.46**	**0.19**	1.65	0.01	**0.29**	**0.02**	1.30	0.01	**0.41**	**0.31**
KO	5.70	**0.16**	**0.11**	**0.10**	4.00	0.00	**0.45**	0.00	2.65	0.00	**0.71**	0.00	2.00	0.00	**0.83**	0.00
MRK	3.80	0.01	**0.09**	0.01	2.75	**0.48**	**0.27**	**0.42**	2.00	0.00	**0.05**	0.00	1.55	0.00	**0.09**	**0.02**
VZ	5.35	**0.47**	**0.34**	**0.49**	3.35	**0.02**	**0.27**	**0.04**	1.80	0.00	**0.68**	0.00	1.25	0.00	**0.32**	**0.34**
DIS	5.85	**0.09**	**0.95**	**0.23**	3.60	0.00	**0.69**	0.01	2.10	0.00	**0.90**	0.00	1.40	0.00	**0.37**	**0.16**
INTC	5.00	**1.00**	**0.19**	**0.42**	3.60	0.00	**0.80**	0.01	1.75	0.00	**0.64**	0.01	1.25	0.00	**0.32**	**0.34**
CSCO	5.00	**1.00**	**0.09**	**0.23**	3.55	0.00	**0.76**	**0.02**	1.80	0.00	**0.25**	0.00	1.55	0.00	**0.32**	**0.04**
HD	5.60	**0.22**	**0.58**	**0.41**	3.35	**0.02**	**0.27**	**0.04**	2.15	0.00	**0.32**	0.00	1.75	0.00	**0.64**	0.01
UTX	6.25	0.01	**0.25**	**0.02**	4.20	0.00	**0.80**	0.00	2.20	0.00	**0.35**	0.00	1.40	0.00	**0.37**	**0.16**
BA	5.85	**0.09**	**0.12**	**0.07**	3.40	0.01	**0.29**	**0.03**	2.25	0.00	**0.15**	0.00	1.70	0.00	**0.28**	0.01
MCD	4.30	**0.14**	**0.50**	**0.27**	2.70	**0.57**	**0.68**	**0.78**	1.90	0.00	**0.75**	0.00	1.60	0.00	**0.54**	**0.04**
AXP	5.45	**0.36**	**0.05**	**0.10**	3.25	**0.04**	**0.38**	**0.08**	1.90	0.00	**0.75**	0.00	1.30	0.00	**0.41**	**0.31**
MMM	5.90	**0.07**	**0.13**	**0.06**	3.85	0.00	**0.10**	0.00	2.50	0.00	**0.17**	0.00	1.45	0.00	**0.36**	**0.11**
GS	5.70	**0.16**	**0.32**	**0.23**	3.20	**0.05**	**0.97**	**0.16**	1.80	0.00	**0.25**	0.00	1.40	0.00	**0.37**	**0.16**
UNH	5.55	**0.26**	**0.61**	**0.47**	2.90	**0.26**	**0.81**	**0.52**	1.85	0.00	**0.72**	0.00	1.40	0.00	**0.41**	**0.17**
CAT	6.20	**0.02**	**0.23**	**0.03**	3.60	0.00	**0.17**	0.00	2.20	0.00	**0.97**	0.00	1.45	0.00	**0.36**	**0.11**
DD	5.05	**0.91**	**0.59**	**0.86**	3.40	0.01	**0.32**	**0.03**	1.75	0.00	**0.26**	0.01	1.20	0.00	**0.45**	**0.51**
NKE	4.40	**0.21**	**0.13**	**0.15**	2.80	**0.40**	**0.62**	**0.62**	2.10	0.00	**0.90**	0.00	1.80	0.00	**0.68**	0.00
TRV	5.30	**0.54**	**0.87**	**0.82**	3.25	**0.04**	**0.08**	**0.03**	2.30	0.00	**0.40**	0.00	1.65	0.00	**0.29**	**0.02**

Note. Bold type entries are not significant at the 1% level.

Table 4.10: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	h = 2															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	5.45	**0.36**	**0.39**	**0.46**	3.15	**0.07**	**0.49**	**0.16**	2.15	0.00	**0.94**	0.00	1.55	0.00	**0.51**	0.00
MSFT	5.30	**0.54**	**0.32**	**0.50**	3.75	0.00	**0.91**	0.00	2.30	0.00	**0.95**	0.00	1.60	0.00	**0.31**	0.00
GE	5.95	**0.06**	**0.27**	**0.09**	3.70	0.00	**0.20**	0.00	2.25	0.00	**0.37**	0.00	1.55	0.00	**0.51**	0.00
JNJ	5.20	**0.68**	**0.13**	**0.30**	3.50	0.01	**0.76**	**0.02**	2.10	0.00	**0.90**	0.00	1.45	0.00	**0.44**	0.00
WMT	5.30	**0.54**	**0.32**	**0.50**	3.70	0.00	**0.02**	0.00	2.10	0.00	**0.29**	0.00	1.75	0.00	**0.15**	0.00
CVX	5.05	**0.91**	**0.59**	**0.86**	2.80	**0.40**	**0.07**	**0.14**	1.35	**0.13**	**0.39**	**0.23**	0.95	0.01	**0.55**	0.03
JPM	5.35	**0.47**	**0.34**	**0.49**	3.65	0.00	**0.18**	0.00	2.20	0.00	**0.35**	0.00	1.65	0.00	**0.57**	0.00
PG	5.85	**0.09**	**0.40**	**0.16**	4.30	0.00	**0.11**	0.00	2.75	0.00	**0.27**	0.00	2.20	0.00	**0.09**	0.00
PFE	4.90	**0.84**	**0.93**	**0.98**	2.80	**0.40**	**0.29**	**0.40**	1.50	**0.04**	**0.34**	**0.07**	1.20	0.00	**0.45**	0.00
IBM	5.35	**0.47**	**0.42**	**0.56**	3.45	0.01	**0.30**	**0.02**	2.15	0.00	**0.94**	0.00	1.40	0.00	**0.41**	0.00
T	5.70	**0.16**	**0.84**	**0.36**	2.90	**0.26**	**0.11**	**0.15**	1.65	0.01	**0.29**	**0.02**	1.15	0.00	**0.46**	0.00
KO	5.50	**0.31**	**0.15**	**0.21**	3.85	0.00	**0.54**	0.00	2.55	0.00	**0.78**	0.00	1.90	0.00	**0.75**	0.00
MRK	3.85	0.01	**0.10**	0.01	2.75	**0.48**	**0.08**	**0.17**	1.95	0.00	**0.23**	0.00	1.55	0.00	**0.09**	0.00
VZ	5.25	**0.61**	**0.03**	**0.08**	3.45	0.01	**0.12**	0.01	1.65	0.01	**0.57**	**0.02**	1.40	0.00	**0.41**	0.00
DIS	5.75	**0.13**	**0.80**	**0.31**	3.60	0.00	**0.69**	0.01	1.90	0.00	**0.75**	0.00	1.25	0.00	**0.43**	0.00
INTC	4.75	**0.61**	**0.25**	**0.46**	3.35	**0.02**	**0.62**	**0.06**	1.65	0.01	**0.57**	**0.02**	1.10	0.00	**0.24**	0.00
CSCO	4.95	**0.92**	**0.08**	**0.21**	3.35	**0.02**	**0.62**	**0.06**	1.90	0.00	**0.22**	0.00	1.50	0.00	**0.34**	0.00
HD	5.35	**0.47**	**0.74**	**0.73**	3.20	**0.05**	**0.52**	**0.13**	2.10	0.00	**0.90**	0.00	1.55	0.00	**0.51**	0.00
UTX	6.20	**0.02**	**0.27**	**0.03**	4.20	0.00	**0.80**	0.00	2.05	0.00	**0.27**	0.00	1.45	0.00	**0.36**	0.00
BA	5.80	**0.11**	**0.21**	**0.12**	3.60	0.00	**0.40**	0.01	2.25	0.00	**0.99**	0.00	1.75	0.00	**0.64**	0.00
MCD	4.40	**0.21**	**0.95**	**0.46**	2.70	**0.57**	**0.68**	**0.78**	1.80	0.00	**0.68**	0.00	1.55	0.00	**0.51**	0.00
AXP	5.50	**0.31**	**0.05**	**0.08**	3.25	**0.04**	**0.38**	**0.08**	1.90	0.00	**0.22**	0.00	1.30	0.00	**0.41**	0.00
MMM	5.95	**0.06**	**0.03**	**0.02**	3.95	0.00	**0.01**	0.00	2.50	0.00	**0.17**	0.00	1.60	0.00	**0.54**	0.00
GS	5.45	**0.36**	**0.11**	**0.18**	3.20	**0.05**	**0.52**	**0.13**	1.75	0.00	**0.26**	0.01	1.40	0.00	**0.37**	0.00
UNH	5.20	**0.68**	**0.85**	**0.90**	2.95	**0.21**	**0.84**	**0.44**	1.80	0.00	**0.68**	0.00	1.40	0.00	**0.41**	0.00
CAT	6.00	**0.05**	**0.29**	**0.08**	3.55	0.00	**0.15**	0.01	2.10	0.00	**0.18**	0.00	1.45	0.00	**0.36**	0.00
DD	5.15	**0.76**	**0.53**	**0.78**	3.20	**0.05**	**0.40**	**0.11**	1.85	0.00	**0.24**	0.00	1.15	0.00	**0.46**	0.00
NKE	4.35	**0.17**	**0.27**	**0.22**	2.80	**0.40**	**0.73**	**0.66**	2.10	0.00	**0.90**	0.00	1.80	0.00	**0.68**	0.00
TRV	5.35	**0.47**	**0.90**	**0.77**	3.15	**0.07**	**0.19**	**0.08**	2.15	0.00	0.01	0.00	1.75	0.00	**0.15**	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.11: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	5.50	**0.31**	**0.42**	**0.43**	2.95	**0.21**	**0.37**	**0.30**	2.05	0.00	**0.86**	0.00	1.55	0.00	**0.51**	0.00
MSFT	5.30	**0.54**	**0.55**	**0.69**	3.60	0.00	**0.80**	0.01	2.20	0.00	**0.97**	0.00	1.60	0.00	**0.31**	0.00
GE	5.90	**0.07**	**0.43**	**0.14**	3.60	0.00	**0.17**	0.00	2.35	0.00	**0.03**	0.00	1.55	0.00	**0.51**	0.00
JNJ	5.00	**1.00**	**0.65**	**0.90**	3.50	0.01	**0.73**	**0.02**	2.05	0.00	**0.86**	0.00	1.50	0.00	**0.47**	0.00
WMT	5.30	**0.54**	**0.55**	**0.69**	3.55	0.00	**0.05**	0.00	2.10	0.00	**0.07**	0.00	1.70	0.00	**0.13**	0.00
CVX	5.25	**0.61**	**0.48**	**0.68**	2.70	**0.57**	**0.08**	**0.19**	1.30	0.20	**0.41**	**0.31**	0.85	**0.04**	**0.59**	0.11
JPM	5.35	**0.47**	**0.04**	**0.09**	3.60	0.00	**0.17**	0.00	2.15	0.00	**0.32**	0.00	1.60	0.00	**0.54**	0.00
PG	5.65	**0.19**	0.01	**0.02**	4.30	0.00	**0.11**	0.00	2.80	0.00	**0.09**	0.00	2.40	0.00	**0.14**	0.00
PFE	5.05	**0.91**	**0.68**	**0.91**	2.90	**0.26**	**0.34**	**0.34**	1.45	**0.06**	**0.36**	**0.11**	1.25	0.00	**0.43**	0.00
IBM	5.35	**0.47**	**0.74**	**0.73**	3.45	0.01	**0.30**	**0.02**	2.05	0.00	**0.86**	0.00	1.40	0.00	**0.41**	0.00
T	5.45	**0.36**	**0.98**	**0.66**	2.80	**0.40**	**0.29**	**0.40**	1.55	0.02	**0.32**	**0.04**	1.25	0.00	**0.43**	0.00
KO	5.70	**0.16**	**0.52**	**0.30**	3.90	0.00	**0.51**	0.00	2.40	0.00	**0.88**	0.00	1.80	0.00	**0.68**	0.00
MRK	3.85	0.01	**0.10**	0.01	2.60	**0.77**	**0.21**	**0.43**	2.10	0.00	**0.07**	0.00	1.35	0.00	**0.05**	0.00
VZ	5.30	**0.54**	**0.16**	**0.31**	3.40	0.01	**0.11**	0.01	1.65	0.01	**0.57**	**0.02**	1.20	0.00	**0.29**	0.00
DIS	5.30	**0.54**	**0.87**	**0.82**	3.70	0.00	**0.21**	0.00	1.95	0.00	**0.79**	0.00	1.25	0.00	**0.43**	0.00
INTC	4.75	**0.61**	**0.48**	**0.69**	3.40	0.01	**0.65**	**0.04**	1.60	0.01	**0.54**	**0.04**	1.15	0.00	**0.27**	0.00
CSCO	4.80	**0.68**	**0.05**	**0.14**	3.55	0.00	**0.05**	0.00	1.80	0.00	**0.25**	0.00	1.50	0.00	**0.34**	0.00
HD	5.25	**0.61**	**0.81**	**0.85**	3.10	0.10	**0.17**	**0.10**	2.15	0.00	**0.32**	0.00	1.50	0.00	**0.47**	0.00
UTX	6.10	**0.03**	**0.31**	**0.05**	4.25	0.00	**0.73**	0.00	2.00	0.00	**0.25**	0.00	1.50	0.00	**0.34**	0.00
BA	5.80	**0.11**	**0.21**	**0.12**	3.45	0.01	**0.32**	**0.02**	2.30	0.00	**0.95**	0.00	1.75	0.00	**0.64**	0.00
MCD	4.40	**0.21**	**0.95**	**0.46**	2.80	**0.40**	**0.62**	**0.62**	1.85	0.00	**0.72**	0.00	1.55	0.00	**0.51**	0.00
AXP	5.55	**0.26**	**0.04**	**0.07**	3.25	**0.04**	**0.38**	**0.08**	1.80	0.00	**0.25**	0.00	1.30	0.00	**0.41**	0.00
MMM	5.80	**0.11**	**0.02**	**0.02**	3.90	0.00	**0.01**	0.00	2.60	0.00	**0.21**	0.00	1.85	0.00	**0.72**	0.00
GS	5.35	**0.47**	**0.34**	**0.49**	3.10	**0.10**	**0.95**	**0.25**	1.75	0.00	**0.26**	0.01	1.35	0.00	**0.39**	0.00
UNH	4.95	**0.92**	**0.61**	**0.88**	3.00	**0.16**	**0.88**	**0.37**	1.70	0.00	**0.61**	0.01	1.35	0.00	**0.38**	0.00
CAT	5.85	**0.09**	**0.23**	**0.11**	3.55	0.00	**0.15**	0.01	2.05	0.00	**0.19**	0.00	1.45	0.00	**0.36**	0.00
DD	5.05	**0.91**	**0.29**	**0.57**	3.00	**0.16**	**0.50**	**0.30**	1.90	0.00	**0.22**	0.00	1.25	0.00	**0.43**	0.00
NKE	4.35	**0.17**	**0.27**	**0.22**	2.70	**0.57**	**0.68**	**0.78**	2.10	0.00	**0.90**	0.00	1.80	0.00	**0.68**	0.00
TRV	5.30	**0.54**	**0.87**	**0.82**	3.00	**0.16**	**0.04**	**0.05**	2.15	0.00	0.01	0.00	1.80	0.00	0.00	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.12: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	5.20	**0.68**	**0.79**	**0.89**	2.90	**0.26**	**0.34**	**0.34**	2.05	0.00	**0.86**	0.00	1.40	0.00	**0.41**	0.00
MSFT	5.45	**0.36**	**0.65**	**0.59**	3.60	0.00	**0.80**	0.01	2.20	0.00	**0.97**	0.00	1.55	0.00	**0.32**	0.00
GE	5.80	**0.11**	**0.37**	**0.19**	3.55	0.00	**0.15**	0.01	2.30	0.00	**0.02**	0.00	1.50	0.00	**0.47**	0.00
JNJ	4.90	**0.84**	**0.93**	**0.98**	3.40	0.01	**0.65**	**0.04**	1.95	0.00	**0.79**	0.00	1.50	0.00	**0.47**	0.00
WMT	5.30	**0.54**	**0.32**	**0.50**	3.55	0.00	**0.05**	0.00	2.15	0.00	**0.08**	0.00	1.70	0.00	**0.13**	0.00
CVX	5.25	**0.61**	**0.48**	**0.68**	2.75	**0.48**	**0.08**	**0.16**	1.20	**0.38**	**0.45**	**0.51**	0.90	**0.02**	**0.57**	**0.06**
JPM	5.15	**0.76**	**0.02**	**0.07**	3.35	**0.02**	**0.10**	**0.02**	2.20	0.00	**0.35**	0.00	1.60	0.00	**0.54**	0.00
PG	5.85	**0.09**	0.01	0.01	4.15	0.00	**0.08**	0.00	2.85	0.00	**0.03**	0.00	2.45	0.00	**0.15**	0.00
PFE	5.20	**0.68**	**0.27**	**0.50**	2.85	**0.32**	**0.32**	**0.37**	1.55	**0.02**	**0.32**	**0.04**	1.25	0.00	**0.43**	0.00
IBM	5.40	**0.41**	**0.94**	**0.71**	3.55	0.00	**0.26**	0.01	1.95	0.00	**0.79**	0.00	1.40	0.00	**0.41**	0.00
T	5.45	**0.36**	**0.98**	**0.66**	2.80	**0.40**	**0.29**	**0.40**	1.60	0.01	**0.31**	**0.03**	1.20	0.00	**0.45**	0.00
KO	5.50	**0.31**	**0.35**	**0.39**	3.85	0.00	**0.54**	0.00	2.35	0.00	**0.92**	0.00	1.60	0.00	**0.54**	0.00
MRK	3.85	0.01	**0.10**	0.01	2.55	**0.88**	**0.19**	**0.42**	2.15	0.00	**0.08**	0.00	1.25	0.00	**0.04**	0.00
VZ	5.20	**0.68**	**0.27**	**0.50**	3.40	0.01	**0.29**	**0.03**	1.65	0.01	**0.57**	**0.02**	1.25	0.00	**0.32**	0.00
DIS	5.20	**0.68**	**0.85**	**0.90**	3.55	0.00	**0.26**	0.01	1.85	0.00	**0.24**	0.00	1.20	0.00	**0.45**	0.00
INTC	4.70	**0.54**	**0.45**	**0.62**	3.40	0.01	**0.65**	**0.04**	1.60	0.01	**0.54**	**0.04**	1.15	0.00	**0.27**	0.00
CSCO	4.85	**0.76**	**0.06**	**0.17**	3.50	0.01	**0.04**	0.00	1.80	0.00	**0.25**	0.00	1.50	0.00	**0.34**	0.00
HD	5.15	**0.76**	**0.89**	**0.94**	3.10	**0.10**	**0.17**	**0.10**	2.10	0.00	**0.29**	0.00	1.50	0.00	**0.47**	0.00
UTX	6.15	**0.02**	**0.29**	**0.04**	4.15	0.00	**0.80**	0.00	1.90	0.00	**0.20**	0.00	1.50	0.00	**0.34**	0.00
BA	5.70	**0.16**	**0.17**	**0.15**	3.50	0.01	**0.34**	**0.02**	2.20	0.00	**0.97**	0.00	1.65	0.00	**0.57**	0.00
MCD	4.35	**0.17**	**0.91**	**0.40**	2.75	**0.48**	**0.65**	**0.70**	1.85	0.00	**0.72**	0.00	1.45	0.00	**0.44**	0.00
AXP	5.40	**0.41**	**0.06**	**0.11**	3.25	**0.04**	**0.38**	**0.08**	1.75	0.00	**0.26**	0.01	1.30	0.00	**0.41**	0.00
MMM	5.80	**0.11**	**0.02**	**0.02**	3.95	0.00	0.01	0.00	2.55	0.00	**0.19**	0.00	1.85	0.00	**0.72**	0.00
GS	5.25	**0.61**	**0.29**	**0.50**	3.05	**0.13**	**0.92**	**0.31**	1.70	0.00	**0.28**	0.01	1.40	0.00	**0.37**	0.00
UNH	4.85	**0.76**	**0.89**	**0.95**	3.00	**0.16**	**0.88**	**0.37**	1.60	0.01	**0.54**	**0.04**	1.25	0.00	**0.32**	0.00
CAT	5.60	**0.22**	**0.14**	**0.16**	3.55	0.00	**0.15**	0.01	2.05	0.00	**0.19**	0.00	1.45	0.00	**0.36**	0.00
DD	5.00	**1.00**	**0.31**	**0.60**	3.00	**0.16**	**0.50**	**0.30**	1.80	0.00	**0.25**	0.00	1.25	0.00	**0.43**	0.00
NKE	4.35	**0.17**	**0.27**	**0.22**	2.75	**0.48**	**0.65**	**0.70**	2.00	0.00	**0.83**	0.00	1.80	0.00	**0.68**	0.00
TRV	5.20	**0.68**	**0.49**	**0.72**	3.00	**0.16**	**0.14**	**0.13**	2.05	0.00	**0.01**	0.00	1.80	0.00	0.00	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.13: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	5.25	**0.61**	**0.29**	**0.50**	2.85	**0.32**	**0.32**	**0.37**	2.05	0.00	**0.86**	0.00	1.35	0.00	**0.38**	0.00
MSFT	5.30	**0.54**	**0.78**	**0.80**	3.60	0.00	**0.80**	0.01	2.20	0.00	**0.97**	0.00	1.55	0.00	**0.32**	0.00
GE	5.75	**0.13**	**0.35**	**0.21**	3.55	0.00	**0.15**	0.01	2.30	0.00	**0.02**	0.00	1.45	0.00	**0.44**	0.00
JNJ	4.80	**0.68**	**0.85**	**0.90**	3.35	**0.02**	**0.62**	**0.06**	1.95	0.00	**0.23**	0.00	1.50	0.00	**0.47**	0.00
WMT	5.25	**0.61**	**0.52**	**0.71**	3.45	0.01	**0.04**	0.00	2.10	0.00	**0.07**	0.00	1.70	0.00	**0.13**	0.00
CVX	5.15	**0.76**	**0.53**	**0.78**	2.75	**0.48**	**0.08**	**0.16**	1.10	**0.66**	**0.48**	**0.71**	0.90	**0.02**	**0.57**	**0.06**
JPM	5.05	**0.91**	**0.10**	**0.25**	3.30	**0.03**	**0.09**	**0.02**	2.15	0.00	**0.32**	0.00	1.65	0.00	**0.57**	0.00
PG	5.75	**0.13**	0.01	0.01	4.10	0.00	**0.07**	0.00	3.00	0.00	0.01	0.00	2.45	0.00	**0.15**	0.00
PFE	5.10	**0.83**	**0.23**	**0.47**	2.85	**0.32**	**0.32**	**0.37**	1.55	**0.02**	**0.32**	**0.04**	1.20	0.00	**0.45**	0.00
IBM	5.35	**0.47**	**0.59**	**0.67**	3.55	0.00	**0.72**	**0.02**	2.00	0.00	**0.83**	0.00	1.35	0.00	**0.38**	0.00
T	5.45	**0.36**	**0.98**	**0.66**	2.75	**0.48**	**0.27**	**0.42**	1.55	**0.02**	**0.32**	**0.04**	1.20	0.00	**0.45**	0.00
KO	5.40	**0.41**	**0.18**	**0.29**	3.85	0.00	**0.54**	0.00	2.20	0.00	**0.97**	0.00	1.60	0.00	**0.54**	0.00
MRK	3.80	0.01	**0.09**	0.01	2.60	**0.77**	**0.21**	**0.43**	2.00	0.00	**0.05**	0.00	1.20	0.00	**0.03**	0.00
VZ	5.20	**0.68**	**0.27**	**0.50**	3.20	**0.05**	**0.21**	**0.07**	1.65	0.01	**0.57**	0.02	1.20	0.00	**0.29**	0.00
DIS	5.15	**0.76**	**0.89**	**0.94**	3.60	0.00	**0.69**	0.01	1.85	0.00	**0.24**	0.00	1.20	0.00	**0.45**	0.00
INTC	4.60	**0.41**	**0.39**	**0.49**	3.35	**0.02**	**0.62**	**0.06**	1.60	0.01	**0.54**	**0.04**	1.15	0.00	**0.27**	0.00
CSCO	4.85	**0.76**	**0.06**	**0.17**	3.45	0.01	**0.04**	0.00	1.85	0.00	**0.24**	0.00	1.50	0.00	**0.34**	0.00
HD	5.15	**0.76**	**0.89**	**0.94**	3.10	**0.10**	**0.17**	**0.10**	2.10	0.00	**0.29**	0.00	1.45	0.00	**0.44**	0.00
UTX	6.15	**0.02**	**0.29**	**0.04**	4.05	0.00	**0.87**	0.00	1.85	0.00	**0.19**	0.00	1.50	0.00	**0.34**	0.00
BA	5.60	**0.22**	**0.14**	**0.16**	3.50	0.01	**0.34**	**0.02**	2.20	0.00	**0.97**	0.00	1.60	0.00	**0.54**	0.00
MCD	4.25	**0.12**	**0.83**	**0.28**	2.70	**0.57**	**0.68**	**0.78**	1.80	0.00	**0.68**	0.00	1.40	0.00	**0.41**	0.00
AXP	5.35	**0.47**	**0.06**	**0.13**	3.10	**0.10**	**0.45**	**0.19**	1.70	0.00	**0.28**	0.01	1.20	0.00	**0.45**	0.00
MMM	5.80	**0.11**	0.01	0.01	4.00	0.00	**0.02**	0.00	2.55	0.00	**0.19**	0.00	1.90	0.00	**0.04**	0.00
GS	5.20	**0.68**	**0.27**	**0.50**	3.00	**0.16**	**0.88**	**0.37**	1.70	0.00	**0.28**	0.01	1.40	0.00	**0.37**	0.00
UNH	4.80	**0.68**	**0.85**	**0.90**	2.95	**0.21**	**0.84**	**0.44**	1.60	0.01	**0.54**	**0.04**	1.25	0.00	**0.32**	0.00
CAT	5.55	**0.26**	**0.13**	**0.17**	3.40	0.01	**0.29**	**0.03**	1.95	0.00	**0.21**	0.00	1.40	0.00	**0.37**	0.00
DD	4.80	**0.68**	**0.40**	**0.65**	2.90	**0.26**	**0.56**	**0.45**	1.80	0.00	**0.25**	0.00	1.25	0.00	**0.43**	0.00
NKE	4.35	**0.17**	**0.27**	**0.22**	2.75	**0.48**	**0.65**	**0.70**	2.00	0.00	**0.83**	0.00	1.80	0.00	**0.68**	0.00
TRV	5.10	**0.83**	**0.43**	**0.71**	2.95	**0.21**	**0.13**	**0.14**	2.05	0.00	0.01	0.00	1.80	0.00	0.00	0.00

Note. Bold type entries are not significant at the 1% level.

4.7.4 Interval Forecast Evaluation of DCC Model with Student's t-distributed Errors, Mean Model = ARMA(0,0)

Table 4.14: Interval forecast evaluation using Student's t-distribution, mean model = ARMA(0,0)

Stocks	h = 1															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	2.45	0.00	**0.15**	0.00	1.00	0.00	**0.52**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00
MSFT	3.05	0.00	**0.92**	0.00	1.75	0.02	**0.26**	**0.04**	0.90	**0.65**	**0.57**	**0.77**	0.75	**0.24**	**0.63**	**0.45**
GE	3.40	0.00	**0.11**	0.00	1.45	0.00	**0.44**	0.00	0.40	0.00	**0.80**	0.01	0.30	0.00	**0.85**	0.00
JNJ	3.00	0.00	**0.88**	0.00	1.35	0.00	**0.38**	0.00	0.40	0.00	**0.80**	0.01	0.30	0.00	**0.85**	0.00
WMT	2.65	0.00	**0.06**	0.00	1.45	0.00	**0.07**	0.00	0.75	**0.24**	**0.63**	**0.45**	0.45	0.01	**0.78**	**0.02**
CVX	1.70	0.00	**0.28**	0.00	0.55	0.00	**0.73**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00
JPM	2.65	0.00	**0.23**	0.00	1.40	0.00	**0.41**	0.00	0.60	**0.05**	**0.70**	**0.14**	0.60	**0.05**	**0.70**	**0.14**
PG	3.15	0.00	**0.19**	0.00	1.70	**0.02**	**0.61**	**0.05**	0.85	**0.49**	**0.59**	**0.68**	0.80	**0.35**	**0.61**	**0.57**
PFE	2.50	0.00	**0.81**	0.00	1.25	0.00	**0.43**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
IBM	2.95	0.00	**0.84**	0.00	1.50	0.00	**0.47**	0.01	0.70	**0.15**	**0.09**	**0.08**	0.60	**0.05**	**0.06**	**0.03**
T	2.10	0.00	**0.29**	0.00	1.00	0.00	**0.52**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00
KO	3.40	0.00	**0.83**	0.00	1.30	0.00	**0.35**	0.00	0.60	**0.05**	**0.06**	**0.03**	0.35	0.00	**0.82**	0.00
MRK	3.25	0.00	**0.23**	0.00	2.00	**0.14**	**0.83**	**0.33**	0.90	**0.65**	**0.15**	**0.32**	0.75	**0.24**	**0.10**	**0.13**
VZ	1.95	0.00	**0.79**	0.00	0.90	0.00	**0.15**	0.00	0.30	0.00	**0.85**	0.00	0.20	0.00	**0.90**	0.00
DIS	3.20	0.00	**0.97**	0.00	1.10	0.00	**0.48**	0.00	0.40	0.00	**0.80**	0.01	0.40	0.00	**0.80**	0.01
INTC	2.60	0.00	**0.21**	0.00	0.95	0.00	**0.17**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	0.01	**0.78**	**0.02**
CSCO	3.35	0.00	**0.86**	0.00	1.75	**0.02**	**0.26**	**0.04**	1.00	**1.00**	**0.52**	**0.82**	0.85	**0.49**	**0.59**	**0.68**
HD	2.60	0.00	**0.21**	0.00	1.10	0.00	**0.24**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00
UTX	2.60	0.00	**0.21**	0.00	1.15	0.00	**0.46**	0.00	0.45	0.01	**0.78**	**0.02**	0.25	0.00	**0.87**	0.00
BA	2.85	0.00	**0.77**	0.00	1.40	0.00	**0.37**	0.00	0.40	0.00	**0.80**	0.01	0.20	0.00	**0.90**	0.00
MCD	2.25	0.00	**0.99**	0.00	1.10	0.00	**0.24**	0.00	0.40	0.00	**0.80**	0.01	0.40	0.00	**0.80**	0.01
AXP	2.45	0.00	**0.85**	0.00	1.00	0.00	**0.52**	0.00	0.40	0.00	**0.80**	0.01	0.20	0.00	**0.90**	0.00
MMM	3.70	0.01	**0.46**	0.02	1.70	0.02	**0.28**	**0.03**	0.65	**0.09**	**0.68**	**0.22**	0.65	**0.09**	**0.68**	**0.22**
GS	2.45	0.00	**0.85**	0.00	1.25	0.00	**0.43**	0.00	0.50	0.01	**0.75**	**0.04**	0.40	0.00	**0.80**	0.01
UNH	2.85	0.00	**0.32**	0.00	1.45	0.00	**0.44**	0.00	0.70	**0.15**	**0.66**	**0.33**	0.65	**0.09**	**0.68**	**0.22**
CAT	3.10	0.00	**0.46**	0.00	1.45	0.00	**0.36**	0.00	0.50	0.01	**0.75**	**0.04**	0.40	0.00	**0.80**	0.01
DD	2.55	0.00	**0.78**	0.00	1.00	0.00	**0.52**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
NKE	2.60	0.00	**0.74**	0.00	1.95	**0.10**	**0.79**	**0.25**	1.20	**0.38**	**0.45**	**0.51**	1.15	**0.51**	**0.46**	**0.62**
TRV	2.65	0.00	**0.63**	0.00	1.40	0.00	**0.41**	0.00	0.45	0.01	**0.78**	**0.02**	0.30	0.00	**0.85**	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.15: Interval forecast evaluation using Student's t-distribution, mean model = ARMA(0,0)

Stocks	**h = 2**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	2.35	0.00	**0.92**	0.00	1.05	0.00	**0.50**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00
MSFT	3.20	0.00	**0.40**	0.00	1.60	0.01	**0.31**	0.01	0.95	**0.82**	**0.55**	**0.81**	0.75	**0.24**	**0.63**	**0.45**
GE	3.15	0.00	**0.06**	0.00	1.35	0.00	**0.38**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
JNJ	3.15	0.00	**0.49**	0.00	1.30	0.00	**0.35**	0.00	0.45	0.01	**0.78**	**0.02**	0.30	0.00	**0.85**	0.00
WMT	2.60	0.00	**0.21**	0.00	1.50	0.00	**0.08**	0.00	0.60	**0.05**	**0.70**	**0.14**	0.45	0.01	**0.78**	**0.02**
CVX	1.65	0.00	**0.29**	0.00	0.50	0.00	**0.75**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00
JPM	2.65	0.00	**0.06**	0.00	1.50	0.00	**0.47**	0.01	0.60	**0.05**	**0.70**	**0.14**	0.60	**0.05**	**0.70**	**0.14**
PG	3.35	0.00	**0.27**	0.00	1.70	**0.02**	**0.61**	**0.05**	0.85	**0.49**	**0.59**	**0.68**	0.80	**0.35**	**0.61**	**0.57**
PFE	2.50	0.00	**0.53**	0.00	1.30	0.00	**0.41**	0.00	0.35	0.00	**0.82**	0.00	0.30	0.00	**0.85**	0.00
IBM	2.90	0.00	**0.81**	0.00	1.45	0.00	**0.44**	0.00	0.70	**0.15**	**0.09**	**0.08**	0.60	**0.05**	**0.06**	**0.03**
T	1.95	0.00	**0.23**	0.00	0.85	0.00	**0.59**	0.00	0.30	0.00	**0.85**	0.00	0.25	0.00	**0.87**	0.00
KO	3.35	0.00	**0.86**	0.00	1.10	0.00	**0.24**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.35	0.00	**0.82**	0.00
MRK	3.40	0.00	**0.29**	0.00	1.95	**0.10**	**0.79**	**0.25**	0.90	**0.65**	**0.15**	**0.32**	0.85	**0.49**	**0.13**	**0.26**
VZ	2.05	0.00	**0.86**	0.00	0.90	0.00	**0.15**	0.00	0.30	0.00	**0.85**	0.00	0.20	0.00	**0.90**	0.00
DIS	3.10	0.00	**0.45**	0.00	1.10	0.00	**0.48**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
INTC	2.55	0.00	**0.19**	0.00	1.00	0.00	**0.19**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	0.01	**0.78**	**0.02**
CSCO	3.30	0.00	**0.90**	0.00	1.65	0.01	**0.29**	**0.02**	1.00	**1.00**	**0.52**	**0.82**	0.90	**0.65**	**0.57**	**0.77**
HD	2.55	0.00	**0.19**	0.00	1.10	0.00	**0.24**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00
UTX	2.60	0.00	**0.21**	0.00	1.25	0.00	**0.43**	0.00	0.45	0.01	**0.78**	**0.02**	0.30	0.00	**0.85**	0.00
BA	2.80	0.00	**0.62**	0.00	1.50	0.00	**0.47**	0.01	0.45	0.01	**0.78**	**0.02**	0.25	0.00	**0.87**	0.00
MCD	2.30	0.00	**0.95**	0.00	1.25	0.00	**0.32**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
AXP	2.45	0.00	**0.12**	0.00	1.00	0.00	**0.52**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00
MMM	3.90	0.02	**0.04**	0.01	1.80	0.04	**0.25**	**0.06**	0.65	**0.09**	**0.68**	**0.22**	0.65	**0.09**	**0.68**	**0.22**
GS	2.50	0.00	**0.11**	0.00	1.25	0.00	**0.43**	0.00	0.50	0.01	**0.75**	**0.04**	0.45	0.01	**0.78**	**0.02**
UNH	2.80	0.00	**0.29**	0.00	1.45	0.00	**0.44**	0.00	0.70	**0.15**	**0.66**	**0.33**	0.65	**0.09**	**0.68**	**0.22**
CAT	3.25	0.00	**0.23**	0.00	1.45	0.00	**0.36**	0.00	0.50	0.01	**0.75**	**0.04**	0.30	0.00	**0.85**	0.00
DD	2.50	0.00	**0.81**	0.00	1.00	0.00	**0.52**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
NKE	2.60	0.00	**0.59**	0.00	2.00	**0.14**	**0.83**	**0.33**	1.25	**0.28**	**0.43**	**0.40**	1.15	**0.51**	**0.46**	**0.62**
TRV	2.45	0.00	**0.04**	0.00	1.50	0.00	**0.47**	0.01	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.16: Interval forecast evaluation using Student's t-distribution, mean model = ARMA(0,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	2.40	0.00	**0.88**	0.00	1.05	0.00	**0.50**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.00	**0.92**	0.00
MSFT	3.20	0.00	**0.40**	0.00	1.60	0.01	**0.31**	0.01	0.95	**0.82**	**0.55**	**0.81**	0.75	**0.24**	**0.63**	**0.45**
GE	3.00	0.00	**0.04**	0.00	1.45	0.00	**0.07**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
JNJ	3.05	0.00	**0.92**	0.00	1.30	0.00	**0.35**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
WMT	2.60	0.00	**0.06**	0.00	1.45	0.00	**0.07**	0.00	0.70	**0.15**	**0.66**	**0.33**	0.45	0.01	**0.78**	**0.02**
CVX	1.65	0.00	**0.29**	0.00	0.50	0.00	**0.75**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00
JPM	2.60	0.00	**0.06**	0.00	1.45	0.00	**0.44**	0.00	0.60	**0.05**	**0.70**	**0.14**	0.60	**0.05**	**0.70**	**0.14**
PG	3.50	0.00	**0.34**	0.00	1.80	**0.04**	**0.17**	**0.04**	0.85	**0.49**	**0.59**	**0.68**	0.75	**0.24**	**0.63**	**0.45**
PFE	2.65	0.00	**0.06**	0.00	1.30	0.00	**0.41**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
IBM	2.85	0.00	**0.77**	0.00	1.40	0.00	**0.41**	0.00	0.70	**0.15**	0.09	**0.08**	0.60	**0.05**	**0.06**	**0.03**
T	1.95	0.00	**0.23**	0.00	0.90	0.00	**0.57**	0.00	0.30	0.00	**0.85**	0.00	0.20	0.00	**0.90**	0.00
KO	3.40	0.00	**0.83**	0.00	1.10	0.00	**0.24**	0.00	0.60	**0.05**	**0.06**	**0.03**	0.40	0.00	**0.80**	0.01
MRK	3.45	0.00	**0.32**	0.00	2.05	**0.18**	**0.86**	**0.41**	0.95	**0.82**	**0.17**	**0.39**	0.85	**0.49**	**0.13**	**0.26**
VZ	2.05	0.00	**0.86**	0.00	0.90	0.00	**0.15**	0.00	0.25	0.00	**0.87**	0.00	0.20	0.00	**0.90**	0.00
DIS	3.05	0.00	**0.48**	0.00	1.00	0.00	**0.52**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
INTC	2.60	0.00	**0.21**	0.00	1.00	0.00	**0.19**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	0.01	**0.78**	**0.02**
CSCO	3.40	0.00	**0.83**	0.00	1.65	0.01	**0.29**	**0.02**	0.95	**0.82**	**0.55**	**0.81**	0.85	**0.49**	**0.59**	**0.68**
HD	2.55	0.00	**0.19**	0.00	1.05	0.00	**0.22**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00
UTX	2.60	0.00	**0.06**	0.00	1.20	0.00	**0.45**	0.00	0.45	0.01	**0.78**	**0.02**	0.30	0.00	**0.85**	0.00
BA	2.85	0.00	**0.77**	0.00	1.35	0.00	**0.39**	0.00	0.40	0.00	**0.80**	0.01	0.20	0.00	**0.90**	0.00
MCD	2.15	0.00	**0.94**	0.00	1.25	0.00	**0.32**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
AXP	2.50	0.00	**0.81**	0.00	1.00	0.00	**0.52**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00
MMM	3.75	0.01	**0.08**	0.01	1.85	**0.05**	**0.24**	**0.07**	0.65	**0.09**	**0.68**	**0.22**	0.65	0.09	**0.68**	**0.22**
GS	2.35	0.00	**0.13**	0.00	1.20	0.00	**0.45**	0.00	0.50	0.01	**0.75**	**0.04**	0.45	0.01	**0.78**	**0.02**
UNH	2.90	0.00	**0.34**	0.00	1.45	0.00	**0.44**	0.00	0.70	**0.15**	**0.66**	**0.33**	0.65	**0.09**	**0.68**	**0.22**
CAT	3.10	0.00	**0.95**	0.00	1.45	0.00	**0.36**	0.00	0.50	0.01	**0.75**	**0.04**	0.35	0.00	**0.82**	0.00
DD	2.50	0.00	**0.81**	0.00	1.00	0.00	**0.52**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
NKE	2.65	0.00	**0.63**	0.00	2.00	**0.14**	**0.83**	**0.33**	1.20	**0.38**	**0.45**	**0.51**	1.10	**0.66**	**0.48**	**0.71**
TRV	2.50	0.00	**0.04**	0.00	1.50	0.00	**0.47**	0.01	0.35	0.00	**0.82**	0.00	0.30	0.00	**0.85**	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.17: Interval forecast evaluation using Student's t-distribution, mean model = ARMA(0,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	2.35	0.00	**0.92**	0.00	1.05	0.00	**0.50**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00
MSFT	3.20	0.00	**0.40**	0.00	1.60	0.01	**0.31**	0.01	0.95	**0.82**	**0.55**	**0.81**	0.75	**0.24**	**0.63**	**0.45**
GE	3.00	0.00	**0.04**	0.00	1.40	0.00	**0.06**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
JNJ	2.95	0.00	**0.84**	0.00	1.35	0.00	**0.38**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
WMT	2.60	0.00	**0.06**	0.00	1.45	0.00	**0.07**	0.00	0.70	**0.15**	**0.66**	**0.33**	0.45	0.01	**0.78**	**0.02**
CVX	1.55	0.00	**0.32**	0.00	0.50	0.00	**0.75**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00
JPM	2.65	0.00	**0.06**	0.00	1.45	0.00	**0.44**	0.00	0.60	**0.05**	**0.70**	**0.14**	0.55	**0.03**	**0.73**	**0.08**
PG	3.50	0.00	**0.34**	0.00	1.85	**0.05**	**0.19**	**0.06**	0.85	**0.49**	**0.59**	**0.68**	0.75	**0.24**	**0.63**	**0.45**
PFE	2.80	0.00	**0.09**	0.00	1.35	0.00	**0.39**	0.00	0.45	0.01	**0.78**	**0.02**	0.40	0.00	**0.80**	0.01
IBM	2.80	0.00	**0.73**	0.00	1.40	0.00	**0.41**	0.00	0.70	**0.15**	**0.09**	**0.08**	0.60	**0.05**	**0.06**	**0.03**
T	1.90	0.00	**0.20**	0.00	0.90	0.00	**0.57**	0.00	0.30	0.00	**0.85**	0.00	0.20	0.00	**0.90**	0.00
KO	3.45	0.00	**0.79**	0.00	1.10	0.00	**0.24**	0.00	0.60	**0.05**	**0.06**	**0.03**	0.45	0.01	**0.78**	**0.02**
MRK	3.70	0.01	**0.20**	0.01	2.05	**0.18**	**0.86**	**0.41**	1.00	**1.00**	**0.19**	**0.43**	0.85	**0.49**	**0.13**	**0.26**
VZ	2.05	0.00	**0.86**	0.00	0.90	0.00	**0.15**	0.00	0.25	0.00	**0.87**	0.00	0.20	0.00	**0.90**	0.00
DIS	3.10	0.00	**0.45**	0.00	1.05	0.00	**0.50**	0.00	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00
INTC	2.50	0.00	**0.17**	0.00	1.00	0.00	**0.19**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	0.01	**0.78**	**0.02**
CSCO	3.40	0.00	**0.83**	0.00	1.65	0.01	**0.29**	**0.02**	0.95	**0.82**	**0.55**	**0.81**	0.85	**0.49**	**0.59**	**0.68**
HD	2.35	0.00	**0.12**	0.00	1.05	0.00	**0.22**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00
UTX	2.60	0.00	**0.06**	0.00	1.20	0.00	**0.45**	0.00	0.45	0.01	**0.78**	**0.02**	0.30	0.00	**0.85**	0.00
BA	2.85	0.00	**0.77**	0.00	1.40	0.00	**0.37**	0.00	0.40	0.00	**0.80**	0.01	0.20	0.00	**0.90**	0.00
MCD	2.05	0.00	**0.86**	0.00	1.25	0.00	**0.32**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
AXP	2.45	0.00	**0.85**	0.00	0.95	0.00	**0.55**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00
MMM	3.80	0.01	**0.09**	0.01	1.85	**0.05**	**0.24**	**0.07**	0.65	**0.09**	**0.68**	**0.22**	0.65	**0.09**	**0.68**	**0.22**
GS	2.40	0.00	**0.12**	0.00	1.20	0.00	**0.45**	0.00	0.50	0.01	**0.75**	**0.04**	0.45	0.01	**0.78**	**0.02**
UNH	2.90	0.00	**0.34**	0.00	1.45	0.00	**0.44**	0.00	0.70	**0.15**	**0.66**	**0.33**	0.65	**0.09**	**0.68**	**0.22**
CAT	3.15	0.00	**0.99**	0.00	1.45	0.00	**0.36**	0.00	0.50	0.01	**0.75**	**0.04**	0.35	0.00	**0.82**	0.00
DD	2.45	0.00	**0.85**	0.00	1.00	0.00	**0.52**	0.00	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00
NKE	2.60	0.00	**0.74**	0.00	2.00	**0.14**	**0.83**	**0.33**	1.20	**0.38**	**0.45**	**0.51**	1.10	**0.66**	**0.48**	**0.71**
TRV	2.50	0.00	**0.04**	0.00	1.50	0.00	**0.47**	0.01	0.35	0.00	**0.82**	0.00	0.30	0.00	**0.85**	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.18: Interval forecast evaluation using Student's t-distribution, mean model = ARMA(0,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	2.35	0.00	**0.92**	0.00	1.00	0.00	**0.52**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00
MSFT	3.20	0.00	**0.40**	0.00	1.60	0.01	**0.31**	0.01	0.95	**0.82**	**0.55**	**0.81**	0.85	**0.49**	**0.59**	**0.68**
GE	3.00	0.00	**0.04**	0.00	1.40	0.00	**0.06**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
JNJ	3.00	0.00	**0.40**	0.00	1.35	0.00	**0.38**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
WMT	2.60	0.00	**0.06**	0.00	1.45	0.00	**0.07**	0.00	0.70	**0.15**	**0.66**	**0.33**	0.45	0.01	**0.78**	**0.02**
CVX	1.50	0.00	**0.34**	0.00	0.50	0.00	**0.75**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00
JPM	2.60	0.00	**0.06**	0.00	1.50	0.00	**0.47**	0.01	0.60	**0.05**	**0.70**	**0.14**	0.50	0.01	**0.75**	**0.04**
PG	3.55	0.00	**0.37**	0.01	1.80	**0.04**	**0.17**	**0.04**	0.85	**0.49**	**0.59**	**0.68**	0.80	**0.35**	**0.61**	**0.57**
PFE	2.95	0.00	**0.13**	0.00	1.35	0.00	**0.39**	0.00	0.45	0.01	**0.78**	**0.02**	0.45	0.01	**0.78**	**0.02**
IBM	2.80	0.00	**0.73**	0.00	1.35	0.00	**0.38**	0.00	0.65	**0.09**	**0.07**	**0.05**	0.55	**0.03**	**0.05**	0.01
T	1.90	0.00	**0.20**	0.00	0.85	0.00	**0.59**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00
KO	3.45	0.00	**0.79**	0.00	1.10	0.00	**0.24**	0.00	0.60	**0.05**	**0.06**	**0.03**	0.45	0.01	**0.78**	**0.02**
MRK	3.65	0.00	**0.18**	0.01	2.05	0.18	**0.27**	**0.23**	1.10	**0.66**	**0.24**	**0.46**	0.85	**0.49**	**0.13**	**0.26**
VZ	2.05	0.00	**0.86**	0.00	0.90	0.00	**0.15**	0.00	0.25	0.00	**0.87**	0.00	0.20	0.00	**0.90**	0.00
DIS	3.10	0.00	**0.45**	0.00	1.05	0.00	**0.50**	0.00	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00
INTC	2.55	0.00	**0.19**	0.00	1.00	0.00	**0.19**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	0.01	**0.78**	**0.02**
CSCO	3.40	0.00	**0.65**	0.00	1.65	0.01	**0.29**	**0.02**	0.90	**0.65**	**0.57**	**0.77**	0.85	**0.49**	**0.59**	**0.68**
HD	2.35	0.00	**0.12**	0.00	1.05	0.00	**0.22**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00
UTX	2.60	0.00	**0.06**	0.00	1.20	0.00	**0.45**	0.00	0.45	0.01	**0.78**	**0.02**	0.25	0.00	**0.87**	0.00
BA	2.85	0.00	**0.77**	0.00	1.40	0.00	**0.37**	0.00	0.40	0.00	**0.80**	0.01	0.20	0.00	**0.90**	0.00
MCD	2.05	0.00	**0.86**	0.00	1.25	0.00	**0.32**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
AXP	2.45	0.00	**0.85**	0.00	0.95	0.00	**0.55**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
MMM	3.80	0.01	**0.09**	0.01	1.90	**0.07**	**0.75**	**0.19**	0.65	**0.09**	**0.68**	**0.22**	0.65	**0.09**	**0.68**	**0.22**
GS	2.35	0.00	**0.13**	0.00	1.20	0.00	**0.45**	0.00	0.50	0.01	**0.75**	**0.04**	0.45	0.01	**0.78**	**0.02**
UNH	2.90	0.00	**0.34**	0.00	1.45	0.00	**0.44**	0.00	0.70	**0.15**	**0.66**	**0.33**	0.65	**0.09**	**0.68**	**0.22**
CAT	3.20	0.00	**0.52**	0.00	1.45	0.00	**0.36**	0.00	0.50	0.01	**0.75**	**0.04**	0.35	0.00	**0.82**	0.00
DD	2.45	0.00	**0.85**	0.00	1.00	0.00	**0.52**	0.00	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00
NKE	2.60	0.00	**0.74**	0.00	2.00	**0.14**	**0.83**	**0.33**	1.20	**0.38**	**0.45**	**0.51**	1.10	**0.66**	**0.48**	**0.71**
TRV	2.45	0.00	**0.04**	0.00	1.45	0.00	**0.44**	0.00	0.35	0.00	**0.82**	0.00	0.30	0.00	**0.85**	0.00

Note. Bold type entries are not significant at the 1% level.

4.7.5 Interval Forecast Evaluation of DCC Model with Student's t-distributed Errors, Mean Model = ARMA(2,0)

Table 4.19: Interval forecast evaluation using Student's t-distribution, mean model = ARMA(2,0)

Stocks	h = 1															
	$100(\alpha) = 5$				$100(\alpha) = 2.5$				$100(\alpha) = 1$				$100(\alpha) = 0.5$			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	2.55	0.00	**0.19**	0.00	1.00	0.00	**0.52**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00
MSFT	3.10	0.00	**0.95**	0.00	1.75	**0.02**	**0.26**	**0.04**	0.95	**0.82**	**0.55**	**0.81**	0.75	**0.24**	**0.63**	**0.45**
GE	3.40	0.00	**0.11**	0.00	1.50	0.00	**0.47**	0.01	0.35	0.00	**0.82**	0.00	0.30	0.00	**0.85**	0.00
JNJ	2.95	0.00	**0.84**	0.00	1.35	0.00	**0.38**	0.00	0.40	0.00	**0.80**	0.01	0.30	0.00	**0.85**	0.00
WMT	2.70	0.00	**0.07**	0.00	1.45	0.00	**0.07**	0.00	0.75	**0.24**	**0.63**	**0.45**	0.45	0.01	**0.78**	**0.02**
CVX	1.70	0.00	**0.28**	0.00	0.55	0.00	**0.73**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00
JPM	2.85	0.00	**0.32**	0.00	1.40	0.00	**0.41**	0.00	0.60	**0.05**	**0.70**	**0.14**	0.60	**0.05**	**0.70**	**0.14**
PG	3.25	0.00	**0.23**	0.00	1.70	**0.02**	**0.61**	**0.05**	0.90	**0.65**	**0.57**	**0.77**	0.80	**0.35**	**0.61**	**0.57**
PFE	2.45	0.00	**0.85**	0.00	1.30	0.00	**0.41**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
IBM	3.00	0.00	**0.88**	0.00	1.50	0.00	**0.47**	0.01	0.70	**0.15**	**0.09**	**0.08**	0.60	**0.05**	**0.06**	**0.03**
T	2.00	0.00	**0.25**	0.00	0.90	0.00	**0.57**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00
KO	3.45	0.00	**0.79**	0.00	1.40	0.00	**0.41**	0.00	0.60	**0.05**	**0.06**	**0.03**	0.35	0.00	**0.82**	0.00
MRK	3.30	0.00	**0.25**	0.00	2.00	**0.14**	**0.83**	**0.33**	0.90	**0.65**	**0.15**	**0.32**	0.75	**0.24**	**0.10**	**0.13**
VZ	2.00	0.00	**0.83**	0.00	0.90	0.00	**0.15**	0.00	0.20	0.00	**0.90**	0.00	0.20	0.00	**0.90**	0.00
DIS	3.15	0.00	**0.99**	0.00	1.15	0.00	**0.46**	0.00	0.40	0.00	**0.80**	0.01	0.40	0.00	**0.80**	0.01
INTC	2.60	0.00	**0.21**	0.00	0.95	0.00	**0.17**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	0.01	**0.78**	**0.02**
CSCO	3.40	0.00	**0.83**	0.00	1.80	**0.04**	**0.25**	**0.06**	1.00	**1.00**	**0.52**	**0.82**	0.85	**0.49**	**0.59**	**0.68**
HD	2.65	0.00	**0.23**	0.00	1.05	0.00	**0.22**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00
UTX	2.60	0.00	**0.21**	0.00	1.10	0.00	**0.48**	0.00	0.45	0.01	**0.78**	**0.02**	0.25	0.00	**0.87**	0.00
BA	2.85	0.00	**0.77**	0.00	1.40	0.00	**0.37**	0.00	0.40	0.00	**0.80**	0.01	0.20	0.00	**0.90**	0.00
MCD	2.30	0.00	**0.95**	0.00	1.15	0.00	**0.27**	0.00	0.40	0.00	**0.80**	0.01	0.40	0.00	**0.80**	0.01
AXP	2.55	0.00	**0.78**	0.00	1.05	0.00	**0.50**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00
MMM	3.65	0.00	**0.43**	0.01	1.65	0.01	**0.29**	**0.02**	0.65	**0.09**	**0.68**	**0.22**	0.65	**0.09**	**0.68**	**0.22**
GS	2.45	0.00	**0.85**	0.00	1.30	0.00	**0.41**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.40	0.00	**0.80**	0.01
UNH	2.80	0.00	**0.29**	0.00	1.50	0.00	**0.47**	0.01	0.70	**0.15**	**0.66**	**0.33**	0.65	0.09	**0.68**	**0.22**
CAT	3.10	0.00	**0.46**	0.00	1.45	0.00	**0.36**	0.00	0.50	0.01	**0.75**	**0.04**	0.40	0.00	**0.80**	0.01
DD	2.50	0.00	**0.81**	0.00	0.95	0.00	**0.55**	0.00	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00
NKE	2.60	0.00	**0.74**	0.00	1.95	**0.10**	**0.79**	**0.25**	1.25	**0.28**	**0.43**	**0.40**	1.15	**0.51**	**0.46**	**0.62**
TRV	2.80	0.00	**0.73**	0.00	1.50	0.00	**0.47**	0.01	0.40	0.00	**0.80**	0.01	0.30	0.00	**0.85**	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.20: Interval forecast evaluation using Student's t-distribution, mean model = ARMA(2,0)

Stocks	**h = 2**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	2.40	0.00	**0.88**	0.00	1.05	0.00	**0.50**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.00	**0.92**	0.00
MSFT	3.25	0.00	**0.38**	0.00	1.60	0.01	**0.31**	0.01	0.95	**0.82**	**0.55**	**0.81**	0.75	**0.24**	**0.63**	**0.45**
GE	3.10	0.00	**0.05**	0.00	1.35	0.00	**0.38**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
JNJ	3.15	0.00	**0.49**	0.00	1.30	0.00	**0.35**	0.00	0.40	0.00	**0.80**	0.01	0.30	0.00	**0.85**	0.00
WMT	2.60	0.00	**0.21**	0.00	1.50	0.00	**0.08**	0.00	0.65	**0.09**	**0.68**	**0.23**	0.45	0.01	**0.78**	**0.02**
CVX	1.65	0.00	**0.29**	0.00	0.50	0.00	**0.75**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00
JPM	2.75	0.00	**0.27**	0.00	1.50	0.00	**0.47**	0.01	0.65	**0.09**	**0.68**	**0.23**	0.60	**0.05**	**0.70**	**0.14**
PG	3.35	0.00	**0.27**	0.00	1.75	**0.02**	**0.64**	**0.07**	0.80	**0.35**	**0.61**	**0.57**	0.80	**0.35**	**0.61**	**0.57**
PFE	2.45	0.00	**0.49**	0.00	1.30	0.00	**0.41**	0.00	0.40	0.00	**0.80**	0.01	0.30	0.00	**0.85**	0.00
IBM	3.00	0.00	**0.88**	0.00	1.45	0.00	**0.44**	0.00	0.70	**0.16**	**0.09**	**0.08**	0.60	**0.05**	**0.06**	**0.03**
T	1.90	0.00	**0.21**	0.00	0.90	0.00	**0.57**	0.00	0.30	0.00	**0.85**	0.00	0.25	0.00	**0.87**	0.00
KO	3.40	0.00	**0.83**	0.00	1.20	0.00	**0.29**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.35	0.00	**0.82**	0.00
MRK	3.45	0.00	**0.32**	0.00	2.00	**0.14**	**0.83**	**0.33**	0.90	**0.65**	**0.15**	**0.32**	0.85	**0.49**	**0.13**	**0.26**
VZ	2.10	0.00	**0.90**	0.00	0.90	0.00	**0.15**	0.00	0.25	0.00	**0.87**	0.00	0.20	0.00	**0.90**	0.00
DIS	3.10	0.00	**0.45**	0.00	1.10	0.00	**0.48**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
INTC	2.55	0.00	**0.19**	0.00	1.00	0.00	**0.19**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	0.01	**0.78**	**0.02**
CSCO	3.30	0.00	**0.36**	0.00	1.65	0.01	**0.29**	**0.02**	1.00	**1.00**	**0.52**	**0.82**	0.90	**0.65**	**0.57**	**0.77**
HD	2.55	0.00	**0.19**	0.00	1.10	0.00	**0.24**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00
UTX	2.65	0.00	**0.23**	0.00	1.20	0.00	**0.44**	0.00	0.45	0.01	**0.78**	**0.02**	0.30	0.00	**0.85**	0.00
BA	2.80	0.00	**0.62**	0.00	1.50	0.00	**0.47**	0.01	0.40	0.00	**0.80**	0.01	0.25	0.00	**0.87**	0.00
MCD	2.30	0.00	**0.95**	0.00	1.25	0.00	**0.32**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
AXP	2.50	0.00	**0.81**	0.00	1.00	0.00	**0.52**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00
MMM	3.85	0.01	**0.10**	0.01	1.80	**0.04**	**0.25**	**0.06**	0.65	**0.09**	**0.68**	**0.23**	0.65	0.09	**0.68**	**0.22**
GS	2.45	0.00	**0.85**	0.00	1.25	0.00	**0.43**	0.00	0.50	0.01	**0.75**	**0.04**	0.45	0.01	**0.78**	**0.02**
UNH	2.80	0.00	**0.29**	0.00	1.45	0.00	**0.44**	0.00	0.70	**0.16**	**0.66**	**0.33**	0.65	**0.09**	**0.68**	**0.22**
CAT	3.25	0.00	**0.23**	0.00	1.45	0.00	**0.36**	0.00	0.50	0.01	**0.75**	**0.04**	0.30	0.00	**0.85**	0.00
DD	2.50	0.00	**0.81**	0.00	0.95	0.00	**0.55**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
NKE	2.75	0.00	**0.70**	0.00	2.00	**0.14**	**0.83**	**0.33**	1.25	**0.28**	**0.43**	**0.40**	1.15	**0.51**	**0.46**	**0.62**
TRV	2.55	0.00	**0.05**	0.00	1.75	**0.02**	**0.64**	**0.07**	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00

Note. Bold type entries are not significant at the 1% level.

Table 4.21: Interval forecast evaluation using Student's t-distribution, mean model = ARMA(2,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	2.40	0.00	**0.88**	0.00	1.05	0.00	**0.50**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.00	**0.92**	0.00
MSFT	3.35	0.00	**0.34**	0.00	1.60	0.01	**0.31**	0.01	0.95	**0.83**	**0.55**	**0.81**	0.75	**0.24**	**0.63**	**0.45**
GE	3.10	0.00	**0.05**	0.00	1.45	0.00	**0.07**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
JNJ	3.05	0.00	**0.92**	0.00	1.40	0.00	**0.41**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
WMT	2.70	0.00	**0.07**	0.00	1.45	0.00	**0.07**	0.00	0.65	**0.09**	**0.68**	**0.23**	0.45	0.01	**0.78**	**0.02**
CVX	1.60	0.00	**0.31**	0.00	0.50	0.00	**0.75**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00
JPM	2.70	0.00	**0.07**	0.00	1.50	0.00	**0.47**	0.01	0.60	**0.05**	**0.70**	**0.14**	0.60	**0.05**	**0.70**	**0.14**
PG	3.55	0.00	**0.37**	0.01	1.85	**0.05**	**0.19**	**0.06**	0.85	**0.49**	**0.59**	**0.68**	0.75	**0.24**	**0.63**	**0.45**
PFE	2.65	0.00	**0.23**	0.00	1.30	0.00	**0.41**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
IBM	2.85	0.00	**0.77**	0.00	1.40	0.00	**0.41**	0.00	0.70	**0.16**	**0.09**	**0.08**	0.60	**0.05**	**0.06**	**0.03**
T	1.95	0.00	**0.23**	0.00	0.90	0.00	**0.57**	0.00	0.30	0.00	**0.85**	0.00	0.20	0.00	**0.90**	0.00
KO	3.45	0.00	**0.79**	0.00	1.15	0.00	**0.27**	0.00	0.60	**0.05**	**0.06**	**0.03**	0.40	0.00	**0.80**	0.01
MRK	3.45	0.00	**0.32**	0.00	2.05	**0.19**	**0.86**	**0.41**	0.95	**0.83**	**0.17**	**0.39**	0.85	**0.49**	**0.13**	**0.26**
VZ	2.05	0.00	**0.86**	0.00	0.90	0.00	**0.15**	0.00	0.25	0.00	**0.87**	0.00	0.20	0.00	**0.90**	0.00
DIS	3.05	0.00	**0.48**	0.00	1.10	0.00	**0.48**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
INTC	2.60	0.00	**0.21**	0.00	1.00	0.00	**0.19**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	0.01	**0.78**	**0.02**
CSCO	3.40	0.00	**0.83**	0.00	1.65	0.01	**0.29**	**0.02**	0.95	**0.83**	**0.55**	**0.81**	0.85	**0.49**	**0.59**	**0.68**
HD	2.55	0.00	**0.19**	0.00	1.05	0.00	**0.22**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00
UTX	2.65	0.00	**0.06**	0.00	1.15	0.00	**0.46**	0.00	0.40	0.00	**0.80**	0.01	0.30	0.00	**0.85**	0.00
BA	2.80	0.00	**0.62**	0.00	1.40	0.00	**0.37**	0.00	0.40	0.00	**0.80**	0.01	0.20	0.00	**0.90**	0.00
MCD	2.25	0.00	**0.99**	0.00	1.20	0.00	**0.29**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
AXP	2.45	0.00	**0.85**	0.00	1.05	0.00	**0.50**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00
MMM	3.80	0.01	**0.52**	**0.03**	1.85	**0.05**	**0.24**	**0.08**	0.65	**0.09**	**0.68**	**0.23**	0.65	**0.09**	**0.68**	**0.22**
GS	2.40	0.00	**0.12**	0.00	1.20	0.00	**0.44**	0.00	0.50	0.01	**0.75**	**0.04**	0.45	0.01	**0.78**	**0.02**
UNH	2.85	0.00	**0.32**	0.00	1.45	0.00	**0.44**	0.00	0.70	**0.16**	**0.66**	**0.33**	0.65	**0.09**	**0.68**	**0.22**
CAT	3.15	0.00	**0.99**	0.00	1.40	0.00	**0.37**	0.00	0.50	0.01	**0.75**	**0.04**	0.35	0.00	**0.82**	0.00
DD	2.50	0.00	**0.81**	0.00	0.95	0.00	**0.55**	0.00	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00
NKE	2.65	0.00	**0.63**	0.00	2.00	**0.14**	**0.83**	**0.33**	1.25	**0.28**	**0.43**	**0.40**	1.10	**0.66**	**0.48**	**0.71**
TRV	2.60	0.00	**0.06**	0.00	1.75	**0.02**	**0.64**	**0.07**	0.35	0.00	**0.82**	0.00	0.30	0.00	**0.85**	0.00

Table 4.22: Interval forecast evaluation using Student's t-distribution, mean model = ARMA(2,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	2.45	0.00	**0.85**	0.00	1.00	0.00	**0.52**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00
MSFT	3.30	0.00	**0.36**	0.00	1.60	0.01	**0.31**	0.01	0.95	**0.83**	**0.55**	**0.81**	0.75	**0.24**	**0.63**	**0.45**
GE	3.10	0.00	**0.05**	0.00	1.45	0.00	**0.07**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
JNJ	2.95	0.00	**0.85**	0.00	1.40	0.00	**0.41**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
WMT	2.70	0.00	**0.07**	0.00	1.45	0.00	**0.07**	0.00	0.65	**0.09**	**0.68**	**0.23**	0.45	0.01	**0.78**	**0.02**
CVX	1.60	0.00	**0.31**	0.00	0.50	0.00	**0.75**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00
JPM	2.65	0.00	**0.06**	0.00	1.45	0.00	**0.44**	0.00	0.60	**0.05**	**0.70**	**0.14**	0.55	**0.03**	**0.73**	**0.08**
PG	3.61	0.00	**0.40**	0.01	1.90	**0.08**	**0.21**	**0.09**	0.85	**0.49**	**0.59**	**0.68**	0.75	**0.24**	**0.63**	**0.45**
PFE	2.80	0.00	**0.09**	0.00	1.35	0.00	**0.39**	0.00	0.45	0.01	**0.78**	**0.02**	0.40	0.00	**0.80**	0.01
IBM	2.80	0.00	**0.74**	0.00	1.40	0.00	**0.41**	0.00	0.70	**0.16**	**0.09**	**0.08**	0.60	**0.05**	**0.06**	**0.03**
T	1.90	0.00	**0.21**	0.00	0.90	0.00	**0.57**	0.00	0.30	0.00	**0.85**	0.00	0.20	0.00	**0.90**	0.00
KO	3.46	0.00	**0.79**	0.00	1.15	0.00	**0.27**	0.00	0.60	**0.05**	**0.06**	**0.03**	0.45	0.01	**0.78**	**0.02**
MRK	3.71	0.01	**0.20**	0.01	2.10	**0.24**	**0.90**	**0.50**	1.00	**0.99**	**0.19**	**0.43**	0.85	**0.49**	**0.13**	**0.26**
VZ	2.05	0.00	**0.86**	0.00	0.90	0.00	**0.15**	0.00	0.25	0.00	**0.87**	0.00	0.20	0.00	**0.90**	0.00
DIS	3.10	0.00	**0.45**	0.00	1.10	0.00	**0.48**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
INTC	2.55	0.00	**0.19**	0.00	1.00	0.00	**0.19**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	0.01	**0.78**	**0.02**
CSCO	3.41	0.00	**0.83**	0.00	1.65	0.01	**0.29**	0.02	0.95	**0.83**	**0.55**	**0.81**	0.85	**0.49**	**0.59**	**0.68**
HD	2.40	0.00	**0.14**	0.00	1.05	0.00	**0.22**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00
UTX	2.65	0.00	**0.06**	0.00	1.15	0.00	**0.46**	0.00	0.40	0.00	**0.80**	0.01	0.30	0.00	**0.85**	0.00
BA	2.80	0.00	**0.62**	0.00	1.40	0.00	**0.37**	0.00	0.40	0.00	**0.80**	0.01	0.20	0.00	**0.90**	0.00
MCD	2.15	0.00	**0.94**	0.00	1.20	0.00	**0.29**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
AXP	2.50	0.00	**0.81**	0.00	1.05	0.00	**0.50**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00
MMM	3.86	0.01	**0.55**	**0.04**	1.85	**0.05**	**0.24**	**0.08**	0.65	**0.09**	**0.68**	**0.23**	0.65	**0.09**	**0.68**	**0.22**
GS	2.35	0.00	**0.13**	0.00	1.20	0.00	**0.44**	0.00	0.50	0.01	**0.75**	**0.04**	0.45	0.01	**0.78**	**0.02**
UNH	2.85	0.00	**0.32**	0.00	1.45	0.00	**0.44**	0.00	0.70	**0.16**	**0.66**	**0.33**	0.65	**0.09**	**0.68**	**0.22**
CAT	3.15	0.00	**0.99**	0.00	1.40	0.00	**0.37**	0.00	0.50	0.01	**0.75**	**0.04**	0.35	0.00	**0.82**	0.00
DD	2.50	0.00	**0.81**	0.00	1.00	0.00	**0.52**	0.00	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00
NKE	2.65	0.00	**0.63**	0.00	2.00	**0.14**	**0.83**	**0.33**	1.20	**0.38**	**0.44**	**0.51**	1.10	**0.66**	**0.48**	**0.71**
TRV	2.60	0.00	**0.06**	0.00	1.60	0.01	**0.54**	**0.02**	0.35	0.00	**0.82**	0.00	0.30	0.00	**0.85**	0.00

Table 4.23: Interval forecast evaluation using Student's t-distribution, mean model = ARMA(2,0)

Stocks	h = 5															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	2.40	0.00	**0.88**	0.00	0.95	0.00	**0.55**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00
MSFT	3.31	0.00	**0.36**	0.00	1.60	0.01	**0.31**	0.01	0.95	**0.83**	**0.55**	**0.81**	0.85	**0.49**	**0.59**	**0.68**
GE	3.06	0.00	**0.05**	0.00	1.40	0.00	**0.06**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
JNJ	3.01	0.00	**0.40**	0.00	1.40	0.00	**0.41**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
WMT	2.66	0.00	**0.06**	0.00	1.45	0.00	**0.07**	0.00	0.65	**0.09**	**0.68**	**0.23**	0.45	0.01	**0.78**	**0.02**
CVX	1.55	0.00	**0.32**	0.00	0.45	0.00	**0.78**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00
JPM	2.61	0.00	**0.06**	0.00	1.45	0.00	**0.44**	0.00	0.60	**0.05**	**0.70**	**0.14**	0.50	0.01	**0.75**	**0.04**
PG	3.66	0.00	**0.43**	0.01	1.85	**0.05**	**0.19**	**0.06**	0.85	**0.50**	**0.59**	**0.69**	0.80	**0.35**	**0.61**	**0.57**
PFE	2.86	0.00	**0.10**	0.00	1.35	0.00	**0.39**	0.00	0.45	0.01	**0.78**	**0.02**	0.45	0.01	**0.78**	**0.02**
IBM	2.81	0.00	**0.74**	0.00	1.40	0.00	**0.41**	0.00	0.70	**0.16**	**0.09**	**0.08**	0.55	**0.03**	**0.05**	0.01
T	1.90	0.00	**0.21**	0.00	0.90	0.00	**0.57**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00
KO	3.46	0.00	**0.79**	0.00	1.15	0.00	**0.27**	0.00	0.60	**0.05**	**0.06**	**0.03**	0.45	0.01	**0.78**	**0.02**
MRK	3.71	0.01	**0.20**	0.01	2.15	**0.31**	**0.32**	**0.37**	1.15	**0.50**	**0.27**	**0.43**	0.85	**0.49**	**0.13**	**0.26**
VZ	2.00	0.00	**0.83**	0.00	0.90	0.00	**0.15**	0.00	0.20	0.00	**0.90**	0.00	0.20	0.00	**0.90**	0.00
DIS	3.11	0.00	**0.45**	0.00	1.10	0.00	**0.48**	0.00	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00
INTC	2.61	0.00	**0.21**	0.00	1.00	0.00	**0.19**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	0.01	**0.78**	**0.02**
CSCO	3.36	0.00	**0.86**	0.00	1.65	0.01	**0.29**	**0.02**	0.90	**0.66**	**0.57**	**0.77**	0.85	**0.49**	**0.59**	**0.68**
HD	2.35	0.00	**0.12**	0.00	1.05	0.00	**0.22**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00
UTX	2.61	0.00	**0.06**	0.00	1.20	0.00	**0.44**	0.00	0.45	0.01	**0.78**	**0.02**	0.25	0.00	**0.87**	0.00
BA	2.81	0.00	**0.62**	0.00	1.40	0.00	**0.37**	0.00	0.40	0.00	**0.80**	0.01	0.20	0.00	**0.90**	0.00
MCD	2.15	0.00	**0.94**	0.00	1.25	0.00	**0.32**	0.00	0.40	0.00	**0.80**	0.01	0.35	0.00	**0.82**	0.00
AXP	2.45	0.00	**0.84**	0.00	1.00	0.00	**0.52**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00
MMM	3.81	0.01	**0.52**	0.03	1.90	**0.08**	**0.75**	**0.20**	0.65	**0.09**	**0.68**	**0.23**	0.65	**0.09**	**0.68**	**0.22**
GS	2.35	0.00	**0.13**	0.00	1.20	0.00	**0.44**	0.00	0.50	0.01	**0.75**	**0.04**	0.45	0.01	**0.78**	**0.02**
UNH	2.91	0.00	**0.34**	0.00	1.45	0.00	**0.44**	0.00	0.70	**0.16**	**0.66**	**0.33**	0.65	0.09	**0.68**	**0.22**
CAT	3.16	0.00	**0.99**	0.00	1.45	0.00	**0.35**	0.00	0.50	0.01	**0.75**	**0.04**	0.35	0.00	**0.82**	0.00
DD	2.45	0.00	**0.84**	0.00	1.00	0.00	**0.52**	0.00	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00
NKE	2.66	0.00	**0.63**	0.00	2.00	**0.14**	**0.83**	**0.33**	1.20	**0.38**	**0.44**	**0.51**	1.10	**0.66**	**0.48**	**0.71**
TRV	2.61	0.00	**0.06**	0.00	1.55	0.00	**0.51**	0.01	0.35	0.00	**0.82**	0.00	0.30	0.00	**0.85**	0.00

4.7.6 Interval Forecast Evaluation of GO-GARCH with Gaussian Distributed Errors, Mean Model = ARMA(0,0)

Table 4.24: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	h = 1															
	$100(\alpha) = 5$				$100(\alpha) = 2.5$				$100(\alpha) = 1$				$100(\alpha) = 0.5$			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	5.60	**0.22**	**0.91**	**0.48**	3.55	0.00	**0.37**	0.01	2.15	0.00	**0.94**	0.00	1.60	0.00	**0.31**	0.00
MSFT	6.25	0.01	**0.07**	0.01	4.45	0.00	**0.26**	0.00	2.50	0.00	**0.11**	0.00	1.90	0.00	**0.22**	0.00
GE	6.80	0.00	**0.55**	0.00	4.35	0.00	**0.27**	0.00	2.75	0.00	**0.02**	0.00	1.85	0.00	**0.19**	0.00
JNJ	5.65	**0.19**	**0.16**	**0.16**	4.30	0.00	**0.69**	0.00	2.60	0.00	**0.74**	0.00	1.80	0.00	**0.68**	0.00
WMT	5.80	**0.11**	**0.01**	0.01	4.00	0.00	**0.05**	0.00	2.45	0.00	**0.15**	0.00	1.95	0.00	**0.23**	0.00
CVX	4.70	**0.54**	**0.83**	**0.81**	2.80	**0.40**	**0.73**	**0.66**	1.35	**0.13**	**0.39**	**0.23**	0.90	**0.02**	**0.57**	**0.06**
JPM	5.75	**0.13**	**0.04**	**0.04**	3.60	0.00	**0.06**	0.00	2.15	0.00	**0.32**	0.00	1.75	0.00	**0.15**	0.00
PG	6.70	0.00	**0.48**	0.00	4.45	0.00	**0.60**	0.00	2.65	0.00	**0.63**	0.00	2.20	0.00	**0.97**	0.00
PFE	6.65	0.00	**0.96**	0.01	3.90	0.00	**0.12**	0.00	2.15	0.00	**0.94**	0.00	1.55	0.00	**0.32**	0.00
IBM	6.05	**0.04**	**0.17**	**0.04**	4.05	0.00	**0.69**	0.00	2.40	0.00	**0.46**	0.00	1.90	0.00	**0.75**	0.00
T	5.70	**0.16**	**0.84**	**0.36**	3.15	**0.07**	**0.19**	**0.08**	1.70	0.00	**0.61**	0.01	1.25	0.00	**0.43**	0.00
KO	6.25	0.01	**0.66**	**0.04**	4.25	0.00	**0.73**	0.00	3.05	0.00	**0.92**	0.00	2.00	0.00	**0.83**	0.00
MRK	6.00	**0.05**	**0.49**	**0.11**	4.30	0.00	**0.50**	0.00	2.75	0.00	**0.27**	0.00	2.20	0.00	**0.97**	0.00
VZ	5.55	**0.26**	**0.73**	**0.51**	3.35	**0.02**	**0.27**	**0.04**	1.80	0.00	**0.68**	0.00	1.20	0.00	**0.29**	0.00
DIS	6.55	0.00	**0.61**	0.01	4.30	0.00	**0.87**	0.00	2.35	0.00	**0.92**	0.00	1.70	0.00	**0.61**	0.00
INTC	5.50	**0.31**	**0.42**	**0.43**	3.80	0.00	**0.52**	0.00	2.05	0.00	**0.27**	0.00	1.50	0.00	**0.47**	0.00
CSCO	5.95	**0.06**	**0.07**	**0.03**	4.35	0.00	**0.27**	0.00	2.80	0.00	**0.62**	0.00	1.95	0.00	**0.21**	0.00
HD	5.95	**0.06**	**0.66**	**0.15**	3.65	0.00	**0.43**	0.01	2.30	0.00	**0.40**	0.00	1.70	0.00	**0.61**	0.00
UTX	6.40	0.01	**0.65**	**0.02**	4.20	0.00	**0.80**	0.00	2.05	0.00	**0.86**	0.00	1.50	0.00	**0.47**	0.00
BA	6.15	**0.02**	**0.21**	**0.03**	3.70	0.00	**0.46**	0.00	2.40	0.00	**0.88**	0.00	1.75	0.00	**0.26**	0.00
MCD	4.60	**0.41**	**0.70**	**0.66**	2.95	**0.21**	**0.84**	**0.44**	2.05	0.00	**0.86**	0.00	1.75	0.00	**0.64**	0.00
AXP	6.25	0.01	**0.04**	0.01	3.50	0.01	**0.28**	0.01	2.20	0.00	**0.97**	0.00	1.55	0.00	**0.32**	0.00
MMM	7.05	0.00	**0.32**	0.00	5.00	0.00	**0.37**	0.00	3.30	0.00	**0.25**	0.00	2.30	0.00	**0.11**	0.00
GS	6.05	**0.04**	**0.32**	**0.07**	3.80	0.00	**0.19**	0.00	1.90	0.00	**0.22**	0.00	1.50	0.00	**0.34**	0.00
UNH	7.20	0.00	**0.64**	0.00	4.00	0.00	**0.33**	0.00	2.25	0.00	**0.99**	0.00	1.85	0.00	**0.72**	0.00
CAT	6.85	0.00	**0.23**	0.00	4.65	0.00	**0.42**	0.00	2.80	0.00	**0.62**	0.00	2.00	0.00	**0.20**	0.00
DD	5.30	**0.54**	**0.78**	**0.80**	3.55	0.00	**0.26**	0.01	1.90	0.00	**0.75**	0.00	1.35	0.00	**0.39**	0.00
NKE	5.95	**0.06**	**0.14**	**0.06**	3.50	0.01	**0.76**	**0.02**	2.35	0.00	**0.92**	0.00	2.10	0.00	**0.90**	0.00
TRV	5.85	**0.09**	**0.65**	**0.21**	3.85	0.00	**0.26**	0.00	2.35	0.00	**0.43**	0.00	1.90	0.00	**0.75**	0.00

Table 4.25: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	**h = 2**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	5.65	**0.19**	**0.87**	**0.42**	3.20	**0.05**	**0.52**	**0.13**	2.20	0.00	**0.97**	0.00	2.20	0.00	**0.97**	0.00
MSFT	6.15	**0.02**	**0.21**	**0.03**	4.60	0.00	**0.51**	0.00	2.50	0.00	**0.11**	0.00	2.50	0.00	**0.11**	0.00
GE	6.80	0.00	**0.55**	0.00	4.20	0.00	**0.21**	0.00	2.80	0.00	**0.02**	0.00	2.80	0.00	**0.02**	0.00
JNJ	5.70	**0.16**	**0.02**	**0.02**	4.15	0.00	**0.76**	0.00	2.55	0.00	**0.78**	0.00	2.55	0.00	**0.78**	0.00
WMT	5.60	**0.22**	**0.14**	**0.16**	4.05	0.00	**0.16**	0.00	2.35	0.00	**0.43**	0.00	2.35	0.00	**0.43**	0.00
CVX	4.80	**0.68**	**0.76**	**0.88**	2.50	**1.00**	**0.11**	**0.28**	1.30	**0.20**	**0.41**	**0.31**	1.30	0.00	**0.41**	0.00
JPM	5.80	**0.11**	**0.37**	**0.19**	3.65	0.00	**0.07**	0.00	2.15	0.00	**0.32**	0.00	2.15	0.00	**0.32**	0.00
PG	6.70	0.00	**0.72**	0.00	4.45	0.00	**0.60**	0.00	2.90	0.00	**0.11**	0.00	2.90	0.00	**0.11**	0.00
PFE	6.40	0.01	**0.94**	**0.02**	3.80	0.00	**0.09**	0.00	2.00	0.00	**0.83**	0.00	2.00	0.00	**0.83**	0.00
IBM	6.20	**0.02**	**0.39**	**0.04**	4.05	0.00	**0.69**	0.00	2.30	0.00	**0.40**	0.00	2.30	0.00	**0.40**	0.00
T	5.75	**0.13**	**0.88**	**0.32**	3.00	**0.16**	**0.14**	**0.13**	1.70	0.00	**0.13**	0.01	1.70	0.00	**0.13**	0.00
KO	6.80	0.00	**0.35**	0.00	4.25	0.00	**0.73**	0.00	3.20	0.00	**0.97**	0.00	3.20	0.00	**0.97**	0.00
MRK	6.05	**0.04**	**0.32**	**0.07**	4.20	0.00	**0.44**	0.00	2.95	0.00	**0.37**	0.00	2.95	0.00	**0.37**	0.00
VZ	5.35	**0.47**	**0.18**	**0.31**	3.35	**0.02**	**0.62**	**0.06**	1.80	0.00	**0.68**	0.00	1.80	0.00	**0.68**	0.00
DIS	6.40	0.01	**0.51**	**0.02**	4.40	0.00	**0.95**	0.00	2.45	0.00	**0.85**	0.00	2.45	0.00	**0.85**	0.00
INTC	5.30	**0.54**	**0.32**	**0.50**	3.75	0.00	**0.49**	0.00	2.00	0.00	**0.25**	0.00	2.00	0.00	**0.25**	0.00
CSCO	5.90	**0.07**	**0.06**	**0.03**	4.30	0.00	**0.11**	0.00	2.65	0.00	**0.71**	0.00	2.65	0.00	**0.71**	0.00
HD	5.90	**0.07**	**0.69**	**0.18**	3.45	0.01	**0.69**	**0.03**	2.25	0.00	**0.10**	0.00	2.25	0.00	**0.10**	0.00
UTX	6.25	0.01	**0.47**	**0.04**	4.20	0.00	**0.44**	0.00	1.95	0.00	**0.79**	0.00	1.95	0.00	**0.79**	0.00
BA	5.95	**0.06**	**0.46**	**0.13**	3.80	0.00	**0.52**	0.00	2.15	0.00	**0.94**	0.00	2.15	0.00	**0.94**	0.00
MCD	4.90	**0.84**	**0.58**	**0.84**	3.10	0.10	**0.95**	**0.25**	1.95	0.00	**0.79**	0.00	1.95	0.00	**0.79**	0.00
AXP	6.10	**0.03**	**0.31**	**0.05**	3.50	0.01	**0.28**	0.01	2.00	0.00	**0.20**	0.00	2.00	0.00	**0.20**	0.00
MMM	7.10	0.00	**0.21**	0.00	4.95	0.00	**0.35**	0.00	3.05	0.00	**0.43**	0.00	3.05	0.00	**0.43**	0.00
GS	6.05	**0.04**	**0.32**	**0.07**	4.10	0.00	**0.40**	0.00	1.85	0.00	**0.24**	0.00	1.85	0.00	**0.24**	0.00
UNH	7.10	0.00	**0.27**	0.00	3.90	0.00	**0.28**	0.00	2.30	0.00	**0.95**	0.00	2.30	0.00	**0.95**	0.00
CAT	6.80	0.00	**0.35**	0.00	4.60	0.00	**0.08**	0.00	2.90	0.00	**0.56**	0.00	2.90	0.00	**0.56**	0.00
DD	5.20	**0.68**	**0.50**	**0.73**	3.65	0.00	**0.23**	0.00	2.05	0.00	**0.86**	0.00	2.05	0.00	**0.86**	0.00
NKE	6.10	**0.03**	**0.34**	**0.06**	3.55	0.00	**0.76**	**0.02**	2.35	0.00	**0.92**	0.00	2.35	0.00	**0.92**	0.00
TRV	5.95	**0.06**	**0.72**	**0.15**	3.95	0.00	**0.13**	0.00	2.30	0.00	**0.02**	0.00	2.30	0.00	**0.02**	0.00

Table 4.26: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	5.45	**0.36**	**0.98**	**0.66**	3.20	**0.05**	**0.52**	**0.13**	2.15	0.00	**0.94**	0.00	2.15	0.00	**0.94**	0.00
MSFT	6.15	**0.02**	**0.21**	**0.03**	4.55	0.00	**0.54**	0.00	2.55	0.00	**0.10**	0.00	2.55	0.00	**0.10**	0.00
GE	6.70	0.00	**0.72**	0.00	4.15	0.00	**0.19**	0.00	2.80	0.00	**0.02**	0.00	2.80	0.00	**0.02**	0.00
JNJ	5.40	**0.41**	**0.20**	**0.31**	3.95	0.00	**0.94**	0.00	2.50	0.00	**0.53**	0.00	2.50	0.00	**0.53**	0.00
WMT	5.65	**0.19**	**0.16**	**0.16**	3.90	0.00	**0.12**	0.00	2.30	0.00	**0.11**	0.00	2.30	0.00	**0.11**	0.00
CVX	4.80	**0.68**	**0.76**	**0.88**	2.50	**1.00**	**0.11**	**0.28**	1.25	**0.28**	**0.43**	**0.40**	1.25	0.00	**0.43**	0.00
JPM	5.75	**0.13**	**0.10**	**0.08**	3.65	0.00	**0.07**	0.00	2.20	0.00	**0.35**	0.00	2.20	0.00	**0.35**	0.00
PG	6.65	0.00	**0.28**	0.00	4.60	0.00	**0.39**	0.00	3.00	0.00	**0.14**	0.00	3.00	0.00	**0.14**	0.00
PFE	6.40	0.01	**0.94**	**0.02**	3.95	0.00	**0.31**	0.00	2.05	0.00	**0.86**	0.00	2.05	0.00	**0.86**	0.00
IBM	6.15	**0.02**	**0.87**	**0.07**	3.95	0.00	**0.62**	0.00	2.30	0.00	**0.40**	0.00	2.30	0.00	**0.40**	0.00
T	5.65	**0.19**	**0.80**	**0.41**	2.85	**0.32**	**0.32**	**0.37**	1.70	0.00	**0.13**	0.01	1.70	0.00	**0.13**	0.00
KO	6.40	0.01	**0.32**	0.01	4.25	0.00	**0.73**	0.00	3.10	0.00	**0.95**	0.00	3.10	0.00	**0.95**	0.00
MRK	6.00	**0.05**	**0.29**	**0.08**	4.40	0.00	**0.05**	0.00	3.00	0.00	**0.14**	0.00	3.00	0.00	**0.14**	0.00
VZ	5.25	**0.61**	**0.52**	**0.71**	3.20	**0.05**	**0.52**	**0.13**	1.60	0.01	**0.54**	**0.04**	1.60	0.00	**0.54**	0.00
DIS	6.50	0.00	**0.37**	0.01	4.35	0.00	**0.91**	0.00	2.55	0.00	**0.78**	0.00	2.55	0.00	**0.78**	0.00
INTC	5.30	**0.54**	**0.32**	**0.50**	3.70	0.00	**0.46**	0.00	1.95	0.00	**0.23**	0.00	1.95	0.00	**0.23**	0.00
CSCO	5.90	**0.07**	**0.03**	**0.02**	4.25	0.00	**0.10**	0.00	2.80	0.00	**0.62**	0.00	2.80	0.00	**0.62**	0.00
HD	5.70	**0.16**	**0.83**	**0.36**	3.45	0.01	**0.32**	**0.02**	2.20	0.00	**0.09**	0.00	2.20	0.00	**0.09**	0.00
UTX	6.20	**0.02**	**0.27**	**0.03**	4.20	0.00	**0.44**	0.00	1.95	0.00	**0.79**	0.00	1.95	0.00	**0.79**	0.00
BA	6.05	**0.04**	**0.32**	**0.07**	3.75	0.00	**0.49**	0.00	2.25	0.00	**0.99**	0.00	2.25	0.00	**0.99**	0.00
MCD	4.70	**0.54**	**0.78**	**0.79**	3.05	**0.13**	**0.92**	**0.31**	1.95	0.00	**0.79**	0.00	1.95	0.00	**0.79**	0.00
AXP	6.00	**0.05**	**0.17**	**0.05**	3.45	0.01	**0.30**	**0.02**	2.10	0.00	**0.18**	0.00	2.10	0.00	**0.18**	0.00
MMM	7.20	0.00	**0.25**	0.00	5.00	0.00	**0.19**	0.00	3.10	0.00	**0.46**	0.00	3.10	0.00	**0.46**	0.00
GS	6.00	**0.05**	**0.16**	**0.05**	3.90	0.00	**0.51**	0.00	1.85	0.00	**0.24**	0.00	1.85	0.00	**0.24**	0.00
UNH	7.15	0.00	**0.44**	0.00	3.95	0.00	**0.31**	0.00	2.55	0.00	**0.56**	0.00	2.55	0.00	**0.56**	0.00
CAT	6.85	0.00	**0.13**	0.00	4.65	0.00	**0.09**	0.00	2.85	0.00	**0.77**	0.00	2.85	0.00	**0.77**	0.00
DD	5.30	**0.54**	**0.78**	**0.80**	3.60	0.00	**0.24**	0.01	2.05	0.00	**0.86**	0.00	2.05	0.00	**0.86**	0.00
NKE	5.90	**0.07**	**0.13**	**0.06**	3.55	0.00	**0.72**	**0.02**	2.35	0.00	**0.92**	0.00	2.35	0.00	**0.92**	0.00
TRV	6.10	**0.03**	**0.56**	**0.08**	3.85	0.00	**0.26**	0.00	2.30	0.00	**0.02**	0.00	2.30	0.00	**0.02**	0.00

Table 4.27: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	5.45	**0.36**	**0.98**	**0.66**	3.20	**0.05**	**0.52**	**0.13**	2.15	0.00	**0.94**	0.00	2.15	0.00	**0.94**	0.00
MSFT	6.15	**0.02**	**0.21**	**0.03**	4.55	0.00	**0.54**	0.00	2.55	0.00	**0.10**	0.00	2.55	0.00	**0.10**	0.00
GE	6.70	0.00	**0.72**	0.00	4.15	0.00	**0.19**	0.00	2.80	0.00	**0.02**	0.00	2.80	0.00	**0.02**	0.00
JNJ	5.40	**0.41**	**0.20**	**0.31**	3.95	0.00	**0.94**	0.00	2.50	0.00	**0.53**	0.00	2.50	0.00	**0.53**	0.00
WMT	5.65	**0.19**	**0.16**	**0.16**	3.90	0.00	**0.12**	0.00	2.30	0.00	**0.11**	0.00	2.30	0.00	**0.11**	0.00
CVX	4.80	**0.68**	**0.76**	**0.88**	2.50	**1.00**	**0.11**	**0.28**	1.25	**0.28**	**0.43**	0.40	1.25	0.00	**0.43**	0.00
JPM	5.75	**0.13**	**0.10**	**0.08**	3.65	0.00	**0.07**	0.00	2.20	0.00	**0.35**	0.00	2.20	0.00	**0.35**	0.00
PG	6.65	0.00	**0.28**	0.00	4.60	0.00	**0.39**	0.00	3.00	0.00	**0.14**	0.00	3.00	0.00	**0.14**	0.00
PFE	6.40	0.01	**0.94**	**0.02**	3.95	0.00	**0.31**	0.00	2.05	0.00	**0.86**	0.00	2.05	0.00	**0.86**	0.00
IBM	6.15	**0.02**	**0.87**	**0.07**	3.95	0.00	**0.62**	0.00	2.30	0.00	**0.40**	0.00	2.30	0.00	**0.40**	0.00
T	5.65	**0.19**	**0.80**	**0.41**	2.85	**0.32**	**0.32**	**0.37**	1.70	0.00	**0.13**	0.01	1.70	0.00	**0.13**	0.00
KO	6.40	0.01	**0.32**	0.01	4.25	0.00	**0.73**	0.00	3.10	0.00	**0.95**	0.00	3.10	0.00	**0.95**	0.00
MRK	6.00	**0.05**	**0.29**	**0.08**	4.40	0.00	**0.05**	0.00	3.00	0.00	**0.14**	0.00	3.00	0.00	**0.14**	0.00
VZ	5.25	**0.61**	**0.52**	**0.71**	3.20	**0.05**	**0.52**	**0.13**	1.60	0.01	**0.54**	0.04	1.60	0.00	**0.54**	0.00
DIS	6.50	0.00	**0.37**	0.01	4.35	0.00	**0.91**	0.00	2.55	0.00	**0.78**	0.00	2.55	0.00	**0.78**	0.00
INTC	5.30	**0.54**	**0.32**	**0.50**	3.70	0.00	**0.46**	0.00	1.95	0.00	**0.23**	0.00	1.95	0.00	**0.23**	0.00
CSCO	5.90	**0.07**	**0.03**	**0.02**	4.25	0.00	**0.10**	0.00	2.80	0.00	**0.62**	0.00	2.80	0.00	**0.62**	0.00
HD	5.70	**0.16**	**0.83**	**0.36**	3.45	0.01	**0.32**	**0.02**	2.20	0.00	**0.09**	0.00	2.20	0.00	**0.09**	0.00
UTX	6.20	**0.02**	**0.27**	**0.03**	4.20	0.00	**0.44**	0.00	1.95	0.00	**0.79**	0.00	1.95	0.00	**0.79**	0.00
BA	6.05	**0.04**	**0.32**	**0.07**	3.75	0.00	**0.49**	0.00	2.25	0.00	**0.99**	0.00	2.25	0.00	**0.99**	0.00
MCD	4.70	**0.54**	**0.78**	**0.79**	3.05	**0.13**	**0.92**	**0.31**	1.95	0.00	**0.79**	0.00	1.95	0.00	**0.79**	0.00
AXP	6.00	**0.05**	**0.17**	**0.05**	3.45	0.01	**0.30**	**0.02**	2.10	0.00	**0.18**	0.00	2.10	0.00	**0.18**	0.00
MMM	7.20	0.00	**0.25**	0.00	5.00	0.00	**0.19**	0.00	3.10	0.00	**0.46**	0.00	3.10	0.00	**0.46**	0.00
GS	6.00	**0.05**	**0.16**	**0.05**	3.90	0.00	**0.51**	0.00	1.85	0.00	**0.24**	0.00	1.85	0.00	**0.24**	0.00
UNH	7.15	0.00	**0.44**	0.00	3.95	0.00	**0.31**	0.00	2.55	0.00	**0.56**	0.00	2.55	0.00	**0.56**	0.00
CAT	6.85	0.00	**0.13**	0.00	4.65	0.00	**0.09**	0.00	2.85	0.00	**0.77**	0.00	2.85	0.00	**0.77**	0.00
DD	5.30	**0.54**	**0.78**	**0.80**	3.60	0.00	**0.24**	0.01	2.05	0.00	**0.86**	0.00	2.05	0.00	**0.86**	0.00
NKE	5.90	**0.07**	**0.13**	**0.06**	3.55	0.00	**0.72**	**0.02**	2.35	0.00	**0.92**	0.00	2.35	0.00	**0.92**	0.00
TRV	6.10	**0.03**	**0.56**	**0.08**	3.85	0.00	**0.26**	0.00	2.30	0.00	**0.02**	0.00	2.30	0.00	**0.02**	0.00

Table 4.28: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	5.20	**0.68**	**0.85**	**0.90**	3.05	**0.13**	**0.43**	**0.23**	2.10	0.00	**0.90**	0.00	1.50	0.00	**0.47**	0.00
MSFT	6.15	**0.02**	**0.21**	**0.03**	4.55	0.00	**0.54**	0.00	2.45	0.00	**0.12**	0.00	1.95	0.00	**0.21**	0.00
GE	6.65	0.00	**0.68**	0.00	4.10	0.00	**0.17**	0.00	2.65	0.00	**0.01**	0.00	2.00	0.00	**0.05**	0.00
JNJ	5.30	**0.54**	**0.32**	**0.50**	4.00	0.00	**0.91**	0.00	2.40	0.00	**0.46**	0.00	1.75	0.00	**0.15**	0.00
WMT	5.55	**0.26**	**0.13**	**0.17**	3.90	0.00	**0.12**	0.00	2.30	0.00	**0.11**	0.00	1.80	0.00	**0.17**	0.00
CVX	4.70	**0.54**	**0.45**	**0.62**	2.50	**1.00**	**0.11**	**0.28**	1.15	**0.51**	**0.46**	**0.62**	0.80	0.08	**0.61**	0.19
JPM	5.60	**0.22**	**0.07**	**0.09**	3.65	0.00	**0.07**	0.00	2.20	0.00	**0.35**	0.00	1.80	0.00	**0.17**	0.00
PG	6.55	0.00	**0.24**	0.00	4.75	0.00	**0.12**	0.00	3.35	0.00	**0.27**	0.00	2.20	0.00	**0.09**	0.00
PFE	6.25	0.01	**0.66**	**0.04**	3.85	0.00	**0.10**	0.00	2.10	0.00	**0.07**	0.00	1.65	0.00	**0.57**	0.00
IBM	6.05	**0.04**	**0.32**	**0.07**	3.80	0.00	**0.95**	0.00	2.25	0.00	**0.37**	0.00	1.80	0.00	**0.68**	0.00
T	5.60	**0.22**	**0.76**	**0.46**	2.85	**0.32**	**0.32**	**0.37**	1.65	0.01	**0.12**	0.01	1.20	0.00	**0.45**	0.00
KO	6.30	0.01	**0.27**	**0.02**	4.15	0.00	**0.38**	0.00	3.05	0.00	**0.48**	0.00	1.95	0.00	**0.79**	0.00
MRK	6.50	0.00	**0.01**	0.00	4.50	0.00	**0.02**	0.00	3.15	0.00	**0.06**	0.00	2.65	0.00	**0.23**	0.00
VZ	5.20	**0.68**	**0.49**	**0.72**	3.10	**0.10**	**0.46**	**0.19**	1.55	**0.02**	**0.51**	**0.06**	1.25	0.00	**0.32**	0.00
DIS	6.55	0.00	**0.40**	0.01	4.35	0.00	**0.91**	0.00	2.50	0.00	**0.81**	0.00	1.75	0.00	**0.26**	0.00
INTC	5.35	**0.47**	**0.34**	**0.49**	3.70	0.00	**0.46**	0.00	1.90	0.00	**0.04**	0.00	1.45	0.00	**0.44**	0.00
CSCO	5.90	**0.07**	**0.03**	**0.02**	4.25	0.00	**0.04**	0.00	2.70	0.00	**0.68**	0.00	2.05	0.00	**0.86**	0.00
HD	5.50	**0.31**	**0.98**	**0.60**	3.35	**0.02**	**0.27**	**0.04**	2.20	0.00	**0.09**	0.00	1.45	0.00	**0.44**	0.00
UTX	6.20	**0.02**	**0.27**	**0.03**	4.10	0.00	**0.38**	0.00	1.95	0.00	**0.79**	0.00	1.50	0.00	**0.34**	0.00
BA	5.95	**0.06**	**0.27**	**0.09**	3.75	0.00	**0.49**	0.00	2.20	0.00	**0.97**	0.00	1.80	0.00	**0.68**	0.00
MCD	4.50	**0.30**	**0.63**	**0.52**	3.00	**0.16**	**0.88**	**0.37**	1.95	0.00	**0.79**	0.00	1.55	0.00	**0.51**	0.00
AXP	5.80	**0.11**	**0.09**	**0.07**	3.55	0.00	**0.26**	0.01	2.10	0.00	**0.18**	0.00	1.45	0.00	**0.36**	0.00
MMM	7.15	0.00	**0.23**	0.00	4.95	0.00	**0.17**	0.00	3.15	0.00	**0.49**	0.00	2.40	0.00	**0.14**	0.00
GS	6.00	**0.05**	**0.16**	**0.05**	3.80	0.00	**0.57**	0.00	1.80	0.00	**0.25**	0.00	1.55	0.00	**0.32**	0.00
UNH	7.05	0.00	**0.49**	0.00	4.10	0.00	**0.38**	0.00	2.55	0.00	**0.56**	0.00	1.90	0.00	**0.75**	0.00
CAT	6.85	0.00	**0.13**	0.00	4.70	0.00	**0.10**	0.00	2.75	0.00	**0.65**	0.00	1.85	0.00	**0.24**	0.00
DD	5.20	**0.68**	**0.85**	**0.90**	3.40	0.01	**0.32**	**0.03**	2.05	0.00	**0.86**	0.00	1.25	0.00	**0.43**	0.00
NKE	5.85	**0.09**	**0.12**	**0.07**	3.55	0.00	**0.72**	**0.02**	2.35	0.00	**0.92**	0.00	2.10	0.00	**0.90**	0.00
TRV	5.90	**0.07**	**0.68**	**0.18**	3.65	0.00	**0.18**	0.00	2.20	0.00	**0.02**	0.00	2.00	0.00	0.01	0.00

4.7.7 Interval Forecast Evaluation of GO-GARCH with Gaussian Distributed Errors, Mean Model = ARMA(2,0)

Table 4.29: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	h = 1															
	$100(\alpha) = 5$				$100(\alpha) = 2.5$				$100(\alpha) = 1$				$100(\alpha) = 0.5$			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	6.10	**0.03**	**0.83**	**0.09**	3.60	0.00	**0.17**	0.00	2.20	0.00	**0.09**	0.00	1.45	0.00	**0.36**	0.00
MSFT	6.30	0.01	**0.15**	0.01	4.50	0.00	**0.57**	0.00	2.60	0.00	**0.74**	0.00	2.05	0.00	**0.19**	0.00
GE	6.75	0.00	**0.52**	0.00	4.35	0.00	**0.27**	0.00	2.80	0.00	**0.02**	0.00	1.95	0.00	**0.23**	0.00
JNJ	6.20	**0.02**	**0.00**	0.00	4.40	0.00	**0.63**	0.00	2.70	0.00	**0.66**	0.00	1.90	0.00	**0.75**	0.00
WMT	5.70	**0.16**	**0.04**	**0.04**	3.95	0.00	**0.05**	0.00	2.40	0.00	**0.46**	0.00	1.95	0.00	**0.23**	0.00
CVX	5.05	**0.91**	**0.59**	**0.86**	2.85	**0.32**	**0.77**	**0.59**	1.45	**0.06**	**0.36**	**0.11**	0.95	0.01	**0.55**	0.03
JPM	6.10	**0.03**	**0.10**	**0.02**	3.65	0.00	**0.07**	0.00	2.35	0.00	**0.43**	0.00	1.85	0.00	**0.19**	0.00
PG	6.80	0.00	**0.79**	0.00	4.50	0.00	**0.63**	0.00	2.70	0.00	**0.66**	0.00	2.20	0.00	**0.97**	0.00
PFE	6.75	0.00	**0.97**	0.00	3.90	0.00	**0.28**	0.00	2.10	0.00	**0.90**	0.00	1.65	0.00	**0.29**	0.00
IBM	6.10	**0.03**	**0.19**	**0.04**	4.00	0.00	**0.65**	0.00	2.35	0.00	**0.43**	0.00	1.90	0.00	**0.75**	0.00
T	5.95	**0.06**	**0.14**	**0.06**	3.15	**0.07**	**0.19**	**0.08**	1.75	0.00	**0.15**	0.00	1.40	0.00	**0.37**	0.00
KO	6.55	0.00	**0.32**	0.01	4.35	0.00	**0.29**	0.00	3.00	0.00	**0.88**	0.00	1.85	0.00	**0.72**	0.00
MRK	5.95	**0.06**	**0.46**	**0.13**	4.25	0.00	**0.47**	0.00	2.75	0.00	**0.27**	0.00	2.30	0.00	**0.95**	0.00
VZ	5.45	**0.36**	**0.65**	**0.59**	3.30	**0.03**	**0.25**	**0.05**	1.80	0.00	**0.68**	0.00	1.10	0.00	**0.24**	0.00
DIS	6.65	0.00	**0.28**	0.00	4.35	0.00	**0.91**	0.00	2.40	0.00	**0.88**	0.00	1.65	0.00	**0.57**	0.00
INTC	5.50	**0.31**	**0.23**	**0.29**	3.85	0.00	**0.26**	0.00	2.00	0.00	**0.25**	0.00	1.50	0.00	**0.47**	0.00
CSCO	6.00	**0.05**	**0.04**	**0.02**	4.25	0.00	**0.23**	0.00	2.85	0.00	**0.59**	0.00	1.95	0.00	**0.79**	0.00
HD	5.90	**0.07**	**0.69**	**0.18**	3.60	0.00	**0.40**	0.01	2.30	0.00	**0.40**	0.00	1.80	0.00	**0.17**	0.00
UTX	6.30	0.01	**0.72**	**0.03**	4.40	0.00	**0.95**	0.00	1.90	0.00	**0.75**	0.00	1.60	0.00	**0.54**	0.00
BA	5.95	**0.06**	**0.14**	**0.06**	3.85	0.00	**0.55**	0.00	2.45	0.00	**0.85**	0.00	1.65	0.00	**0.29**	0.00
MCD	4.55	**0.35**	**0.18**	**0.26**	2.95	**0.21**	**0.84**	**0.44**	2.00	0.00	**0.83**	0.00	1.70	0.00	**0.61**	0.00
AXP	6.15	**0.02**	**0.13**	**0.02**	3.55	0.00	**0.26**	0.01	2.05	0.00	**0.86**	0.00	1.50	0.00	**0.34**	0.00
MMM	7.05	0.00	**0.72**	0.00	5.00	0.00	**0.37**	0.00	3.00	0.00	**0.14**	0.00	2.45	0.00	**0.49**	0.00
GS	6.10	**0.03**	**0.19**	**0.04**	3.75	0.00	**0.49**	0.00	2.00	0.00	**0.20**	0.00	1.45	0.00	**0.36**	0.00
UNH	7.25	0.00	**0.86**	0.00	3.90	0.00	**0.28**	0.00	2.30	0.00	**0.95**	0.00	1.75	0.00	**0.64**	0.00
CAT	6.75	0.00	**0.19**	0.00	4.60	0.00	**0.39**	0.00	2.60	0.00	**0.10**	0.00	1.90	0.00	**0.22**	0.00
DD	5.30	**0.54**	**0.78**	**0.80**	3.50	0.01	**0.28**	0.01	2.00	0.00	**0.83**	0.00	1.30	0.00	**0.41**	0.00
NKE	5.90	**0.07**	**0.13**	**0.06**	3.55	0.00	**0.76**	**0.02**	2.45	0.00	**0.85**	0.00	2.15	0.00	**0.94**	0.00
TRV	5.95	**0.06**	**0.66**	**0.15**	3.80	0.00	**0.09**	0.00	2.55	0.00	**0.19**	0.00	2.00	0.00	**0.83**	0.00

Table 4.30: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

	h = 2															
Stocks	**100(α) = 5**				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	5.60	**0.22**	**0.48**	**0.37**	3.25	**0.04**	**0.55**	**0.10**	2.20	0.00	**0.97**	0.00	1.65	0.00	**0.57**	0.00
MSFT	6.15	**0.02**	**0.21**	**0.03**	4.65	0.00	**0.87**	0.00	2.50	0.00	**0.81**	0.00	1.95	0.00	**0.21**	0.00
GE	6.80	0.00	**0.55**	0.00	4.30	0.00	**0.11**	0.00	2.60	0.00	**0.01**	0.00	2.00	0.00	**0.05**	0.00
JNJ	5.75	**0.13**	**0.00**	0.00	4.10	0.00	**0.72**	0.00	2.60	0.00	**0.74**	0.00	1.85	0.00	**0.72**	0.00
WMT	5.65	**0.19**	**0.01**	**0.02**	4.05	0.00	**0.06**	0.00	2.30	0.00	**0.40**	0.00	1.85	0.00	**0.19**	0.00
CVX	4.80	**0.68**	**0.76**	**0.88**	2.70	**0.57**	**0.08**	**0.19**	1.30	**0.20**	**0.41**	**0.31**	0.90	**0.02**	**0.57**	**0.06**
JPM	5.70	**0.16**	**0.55**	**0.31**	3.75	0.00	**0.08**	0.00	2.20	0.00	**0.35**	0.00	1.85	0.00	**0.19**	0.00
PG	6.55	0.00	**0.88**	0.01	4.45	0.00	**0.32**	0.00	2.85	0.00	**0.10**	0.00	2.25	0.00	**0.37**	0.00
PFE	6.30	0.01	**0.98**	**0.04**	3.70	0.00	**0.46**	0.00	2.05	0.00	**0.86**	0.00	1.55	0.00	**0.32**	0.00
IBM	6.30	0.01	**0.27**	**0.02**	3.95	0.00	**0.94**	0.00	2.30	0.00	**0.40**	0.00	1.90	0.00	**0.75**	0.00
T	5.80	**0.11**	**0.61**	**0.24**	2.95	**0.21**	**0.13**	**0.14**	1.65	0.01	**0.12**	0.01	1.15	0.00	**0.46**	0.00
KO	6.45	0.00	**0.90**	**0.02**	4.35	0.00	**0.29**	0.00	3.10	0.00	**0.95**	0.00	1.90	0.00	**0.75**	0.00
MRK	6.00	**0.05**	**0.49**	**0.11**	4.30	0.00	**0.50**	0.00	2.95	0.00	**0.13**	0.00	2.40	0.00	**0.46**	0.00
VZ	5.35	**0.47**	**0.18**	**0.31**	3.30	**0.03**	**0.09**	**0.02**	1.60	0.01	**0.54**	0.04	1.35	0.00	**0.38**	0.00
DIS	6.50	0.00	**0.58**	0.01	4.45	0.00	**0.98**	0.00	2.50	0.00	**0.81**	0.00	1.50	0.00	**0.34**	0.00
INTC	5.60	**0.22**	**0.48**	**0.37**	3.70	0.00	**0.46**	0.00	2.00	0.00	**0.25**	0.00	1.50	0.00	**0.47**	0.00
CSCO	5.90	**0.07**	**0.13**	**0.06**	4.30	0.00	**0.11**	0.00	2.55	0.00	**0.78**	0.00	2.00	0.00	**0.20**	0.00
HD	6.00	**0.05**	**0.62**	**0.12**	3.40	0.01	**0.65**	**0.04**	2.35	0.00	**0.12**	0.00	1.60	0.00	**0.54**	0.00
UTX	6.20	**0.02**	**0.27**	**0.03**	4.15	0.00	**0.76**	0.00	1.95	0.00	**0.79**	0.00	1.55	0.00	**0.51**	0.00
BA	6.05	**0.04**	**0.32**	**0.07**	3.95	0.00	**0.62**	0.00	2.30	0.00	**0.95**	0.00	1.75	0.00	**0.64**	0.00
MCD	4.85	**0.76**	**0.89**	**0.95**	2.95	**0.21**	**0.84**	**0.44**	2.00	0.00	**0.83**	0.00	1.70	0.00	**0.61**	0.00
AXP	5.95	**0.06**	**0.38**	**0.11**	3.40	0.01	**0.32**	**0.03**	1.90	0.00	**0.22**	0.00	1.45	0.00	**0.36**	0.00
MMM	7.15	0.00	**0.37**	0.00	5.00	0.00	**0.37**	0.00	3.00	0.00	**0.40**	0.00	2.40	0.00	**0.14**	0.00
GS	6.00	**0.05**	**0.16**	**0.05**	3.90	0.00	**0.58**	0.00	1.90	0.00	**0.22**	0.00	1.55	0.00	**0.32**	0.00
UNH	7.10	0.00	**0.47**	0.00	3.90	0.00	**0.28**	0.00	2.25	0.00	**0.99**	0.00	1.85	0.00	**0.72**	0.00
CAT	6.90	0.00	**0.14**	**0.00**	4.60	0.00	**0.08**	0.00	2.75	0.00	**0.65**	0.00	1.90	0.00	**0.22**	0.00
DD	5.40	**0.41**	**0.94**	**0.71**	3.50	0.01	**0.76**	**0.02**	2.05	0.00	**0.86**	0.00	1.25	0.00	**0.43**	0.00
NKE	5.90	**0.07**	**0.25**	**0.10**	3.60	0.00	**0.80**	0.01	2.30	0.00	**0.95**	0.00	2.15	0.00	**0.94**	0.00
TRV	5.85	0.09	**0.40**	**0.16**	3.80	0.00	**0.52**	0.00	2.40	0.00	**0.03**	0.00	2.05	0.00	**0.27**	0.00

Table 4.31: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	5.75	**0.13**	**0.58**	**0.27**	3.00	**0.16**	**0.50**	**0.30**	2.10	0.00	**0.90**	0.00	1.65	0.00	**0.57**	0.00
MSFT	6.05	**0.04**	**0.52**	**0.09**	4.60	0.00	**0.51**	0.00	2.50	0.00	**0.11**	0.00	1.95	0.00	**0.21**	0.00
GE	6.75	0.00	**0.52**	0.00	4.20	0.00	**0.21**	0.00	2.60	0.00	**0.01**	0.00	2.10	0.00	**0.07**	0.00
JNJ	5.55	**0.26**	**0.25**	**0.28**	4.00	0.00	**0.91**	0.00	2.50	0.00	**0.53**	0.00	1.80	0.00	**0.68**	0.00
WMT	5.65	**0.19**	**0.03**	**0.04**	3.95	0.00	**0.13**	0.00	2.30	0.00	**0.11**	0.00	1.80	0.00	**0.17**	0.00
CVX	4.75	**0.61**	**0.80**	**0.85**	2.55	**0.88**	**0.10**	**0.26**	1.15	**0.51**	**0.46**	**0.62**	0.85	**0.04**	**0.59**	**0.11**
JPM	5.60	**0.22**	**0.07**	**0.09**	3.70	0.00	**0.07**	0.00	2.25	0.00	**0.37**	0.00	1.85	0.00	**0.19**	0.00
PG	6.70	0.00	**0.17**	0.00	4.65	0.00	**0.21**	0.00	3.15	0.00	**0.19**	0.00	2.25	0.00	**0.02**	0.00
PFE	6.35	0.01	**0.98**	**0.03**	4.05	0.00	**0.36**	0.00	2.10	0.00	**0.90**	0.00	1.60	0.00	**0.31**	0.00
IBM	6.20	**0.02**	**0.39**	**0.04**	4.00	0.00	**0.65**	0.00	2.25	0.00	**0.37**	0.00	1.80	0.00	**0.68**	0.00
T	5.55	**0.26**	**0.94**	**0.54**	2.90	**0.26**	**0.34**	**0.34**	1.65	0.01	**0.12**	0.01	1.15	0.00	**0.46**	0.00
KO	6.50	0.00	**0.37**	0.01	4.20	0.00	**0.36**	0.00	3.10	0.00	**0.45**	0.00	1.95	0.00	**0.79**	0.00
MRK	6.05	**0.04**	**0.09**	**0.03**	4.45	0.00	**0.15**	0.00	2.95	0.00	**0.37**	0.00	2.55	0.00	**0.56**	0.00
VZ	5.20	**0.68**	**0.79**	**0.89**	3.35	**0.02**	**0.62**	**0.06**	1.60	0.01	**0.54**	**0.04**	1.25	0.00	**0.32**	0.00
DIS	6.45	0.00	**0.55**	0.01	4.35	0.00	**0.66**	0.00	2.55	0.00	**0.78**	0.00	1.55	0.00	**0.32**	0.00
INTC	5.45	**0.36**	**0.39**	**0.46**	3.70	0.00	**0.46**	0.00	1.90	0.00	**0.20**	0.00	1.45	0.00	**0.44**	0.00
CSCO	5.90	**0.07**	**0.03**	**0.02**	4.25	0.00	**0.10**	0.00	2.75	0.00	**0.65**	0.00	2.05	0.00	**0.86**	0.00
HD	5.75	**0.13**	**0.80**	**0.31**	3.45	0.01	**0.32**	**0.02**	2.25	0.00	**0.10**	0.00	1.45	0.00	**0.44**	0.00
UTX	6.10	**0.03**	**0.31**	**0.05**	4.25	0.00	**0.47**	0.00	1.95	0.00	**0.79**	0.00	1.60	0.00	**0.54**	0.00
BA	5.90	**0.07**	**0.25**	**0.10**	3.70	0.00	**0.46**	0.00	2.30	0.00	**0.95**	0.00	1.80	0.00	**0.68**	0.00
MCD	4.70	**0.54**	**0.78**	**0.79**	3.05	**0.13**	**0.92**	**0.31**	1.95	0.00	**0.79**	0.00	1.55	0.00	**0.51**	0.00
AXP	6.20	**0.02**	**0.27**	**0.03**	3.60	0.00	**0.69**	0.01	2.05	0.00	**0.19**	0.00	1.55	0.00	**0.32**	0.00
MMM	7.15	0.00	**0.37**	0.00	5.00	0.00	**0.37**	0.00	3.10	0.00	**0.46**	0.00	2.35	0.00	**0.12**	0.00
GS	6.05	**0.04**	**0.17**	**0.04**	3.85	0.00	**0.54**	0.00	1.85	0.00	**0.24**	0.00	1.55	0.00	**0.32**	0.00
UNH	7.10	0.00	**0.47**	0.00	3.90	0.00	**0.28**	0.00	2.55	0.00	**0.56**	0.00	1.85	0.00	**0.72**	0.00
CAT	6.75	0.00	**0.10**	0.00	4.70	0.00	**0.10**	0.00	2.70	0.00	**0.68**	0.00	1.85	0.00	**0.24**	0.00
DD	5.20	**0.68**	**0.85**	**0.90**	3.40	0.01	**0.32**	**0.03**	2.05	0.00	**0.86**	0.00	1.30	0.00	**0.41**	0.00
NKE	5.75	**0.13**	**0.10**	**0.08**	3.60	0.00	**0.80**	0.01	2.35	0.00	**0.92**	0.00	2.10	0.00	**0.90**	0.00
TRV	6.05	**0.04**	**0.79**	**0.11**	3.80	0.00	**0.09**	0.00	2.25	0.00	**0.02**	0.00	2.05	0.00	0.01	0.00

Table 4.32: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	5.45	**0.36**	**0.65**	**0.59**	3.05	**0.13**	**0.43**	**0.23**	2.10	0.00	**0.90**	0.00	1.50	0.00	**0.47**	0.00
MSFT	6.10	**0.03**	**0.34**	**0.06**	4.55	0.00	**0.54**	0.00	2.50	0.00	**0.11**	0.00	1.95	0.00	**0.21**	0.00
GE	6.70	0.00	**0.48**	0.00	4.10	0.00	**0.17**	0.00	2.60	0.00	0.01	0.00	2.05	0.00	**0.06**	0.00
JNJ	5.50	**0.31**	**0.42**	**0.43**	4.00	0.00	**0.91**	0.00	2.50	0.00	**0.53**	0.00	1.80	0.00	**0.17**	0.00
WMT	5.65	**0.19**	**0.03**	**0.04**	3.90	0.00	**0.12**	0.00	2.30	0.00	**0.11**	0.00	1.80	0.00	**0.17**	0.00
CVX	4.80	**0.68**	**0.76**	**0.88**	2.50	**1.00**	**0.11**	**0.28**	1.05	**0.82**	**0.50**	**0.78**	0.85	**0.04**	**0.59**	**0.11**
JPM	5.60	**0.22**	**0.07**	**0.09**	3.65	0.00	**0.07**	0.00	2.20	0.00	**0.35**	0.00	1.85	0.00	**0.19**	0.00
PG	6.65	0.00	**0.28**	0.00	4.65	0.00	**0.21**	0.00	3.20	0.00	**0.21**	0.00	2.10	0.00	**0.07**	0.00
PFE	6.25	0.01	**0.66**	**0.04**	3.95	0.00	**0.13**	0.00	2.10	0.00	**0.90**	0.00	1.60	0.00	**0.31**	0.00
IBM	6.10	**0.03**	**0.34**	**0.06**	4.00	0.00	**0.65**	0.00	2.30	0.00	**0.40**	0.00	1.80	0.00	**0.68**	0.00
T	5.50	**0.31**	**0.98**	**0.60**	2.85	**0.32**	**0.32**	**0.37**	1.65	0.01	**0.12**	0.01	1.20	0.00	**0.45**	0.00
KO	6.50	0.00	**0.37**	0.01	4.20	0.00	**0.36**	0.00	3.10	0.00	**0.45**	0.00	1.95	0.00	**0.79**	0.00
MRK	6.30	0.01	**0.08**	0.01	4.45	0.00	**0.02**	0.00	3.00	0.00	**0.40**	0.00	2.60	0.00	**0.21**	0.00
VZ	5.15	**0.76**	**0.76**	**0.91**	3.20	**0.05**	**0.52**	**0.13**	1.60	0.01	**0.54**	**0.04**	1.25	0.00	**0.32**	0.00
DIS	6.45	0.00	**0.55**	0.01	4.40	0.00	**0.95**	0.00	2.60	0.00	**0.74**	0.00	1.75	0.00	**0.26**	0.00
INTC	5.35	**0.47**	**0.34**	**0.49**	3.70	0.00	**0.46**	0.00	1.85	0.00	**0.19**	0.00	1.45	0.00	**0.44**	0.00
CSCO	5.90	**0.07**	**0.03**	**0.02**	4.25	0.00	**0.10**	0.00	2.80	0.00	**0.62**	0.00	2.05	0.00	**0.86**	0.00
HD	5.50	**0.31**	**0.98**	**0.60**	3.40	0.01	**0.29**	**0.03**	2.20	0.00	**0.09**	0.00	1.45	0.00	**0.44**	0.00
UTX	6.15	**0.02**	**0.29**	**0.04**	4.15	0.00	**0.41**	0.00	1.95	0.00	**0.79**	0.00	1.55	0.00	**0.32**	0.00
BA	5.90	**0.07**	**0.25**	**0.10**	3.65	0.00	**0.43**	0.01	2.30	0.00	**0.95**	0.00	1.80	0.00	**0.68**	0.00
MCD	4.65	**0.47**	**0.74**	**0.73**	3.05	**0.13**	**0.92**	**0.31**	1.95	0.00	**0.79**	0.00	1.50	0.00	**0.47**	0.00
AXP	6.10	**0.03**	**0.14**	**0.03**	3.60	0.00	**0.24**	0.01	2.05	0.00	**0.19**	0.00	1.50	0.00	**0.34**	0.00
MMM	7.10	0.00	**0.34**	0.00	5.00	0.00	**0.37**	0.00	3.10	0.00	**0.46**	0.00	2.40	0.00	**0.14**	0.00
GS	5.95	**0.06**	**0.14**	**0.06**	3.85	0.00	**0.54**	0.00	1.85	0.00	**0.24**	0.00	1.55	0.00	**0.32**	0.00
UNH	6.95	0.00	**0.55**	0.00	4.15	0.00	**0.41**	0.00	2.55	0.00	**0.56**	0.00	1.80	0.00	**0.68**	0.00
CAT	7.00	0.00	**0.17**	0.00	4.70	0.00	**0.10**	0.00	2.70	0.00	**0.68**	0.00	1.80	0.00	**0.25**	0.00
DD	5.20	**0.68**	**0.85**	**0.90**	3.30	**0.03**	**0.36**	**0.06**	2.00	0.00	**0.83**	0.00	1.30	0.00	**0.41**	0.00
NKE	5.75	**0.13**	**0.10**	**0.08**	3.60	0.00	**0.80**	0.01	2.35	0.00	**0.92**	0.00	2.10	0.00	**0.90**	0.00
TRV	5.95	**0.06**	**0.72**	**0.15**	3.60	0.00	**0.17**	0.00	2.25	0.00	**0.02**	0.00	2.00	0.00	0.01	0.00

Table 4.33: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	5.30	**0.54**	**0.87**	**0.82**	2.95	**0.21**	**0.37**	**0.30**	2.05	0.00	**0.86**	0.00	1.45	0.00	**0.44**	0.00
MSFT	6.10	**0.03**	**0.19**	**0.04**	4.55	0.00	**0.54**	0.00	2.45	0.00	**0.12**	0.00	1.95	0.00	**0.21**	0.00
GE	6.70	0.00	**0.48**	0.00	4.05	0.00	**0.16**	0.00	2.60	0.00	0.01	0.00	2.05	0.00	**0.06**	0.00
JNJ	5.50	**0.31**	**0.23**	**0.29**	4.05	0.00	**0.69**	0.00	2.45	0.00	**0.49**	0.00	1.75	0.00	**0.15**	0.00
WMT	5.65	**0.19**	**0.03**	**0.04**	3.90	0.00	**0.12**	0.00	2.30	0.00	**0.11**	0.00	1.80	0.00	**0.17**	0.00
CVX	4.75	**0.61**	**0.80**	**0.85**	2.45	**0.89**	**0.12**	**0.29**	1.05	**0.82**	**0.50**	**0.78**	0.85	**0.04**	**0.59**	**0.11**
JPM	5.60	**0.22**	**0.07**	**0.09**	3.65	0.00	**0.07**	0.00	2.25	0.00	**0.10**	0.00	1.75	0.00	**0.15**	0.00
PG	6.60	0.00	**0.26**	0.00	4.70	0.00	**0.10**	0.00	3.35	0.00	**0.27**	0.00	2.20	0.00	**0.09**	0.00
PFE	6.20	**0.02**	**0.62**	**0.05**	3.90	0.00	**0.12**	0.00	2.10	0.00	**0.07**	0.00	1.60	0.00	**0.31**	0.00
IBM	5.95	**0.06**	**0.27**	**0.09**	3.75	0.00	**0.91**	0.00	2.30	0.00	**0.40**	0.00	1.80	0.00	**0.68**	0.00
T	5.55	**0.26**	**0.73**	**0.51**	2.90	**0.26**	**0.34**	**0.34**	1.65	0.01	**0.12**	0.01	1.20	0.00	**0.45**	0.00
KO	6.50	0.00	**0.22**	0.01	4.20	0.00	**0.36**	0.00	3.00	0.00	**0.50**	0.00	2.00	0.00	**0.83**	0.00
MRK	6.50	0.00	0.00	0.00	4.55	0.00	**0.03**	0.00	3.25	0.00	**0.23**	0.00	2.55	0.00	**0.19**	0.00
VZ	5.10	**0.83**	**0.43**	**0.71**	3.25	**0.04**	**0.23**	**0.06**	1.50	**0.04**	**0.47**	**0.09**	1.25	0.00	**0.32**	0.00
DIS	6.55	0.00	**0.40**	0.01	4.40	0.00	**0.95**	0.00	2.55	0.00	**0.78**	0.00	1.75	0.00	**0.26**	0.00
INTC	5.30	**0.54**	**0.32**	**0.50**	3.70	0.00	**0.46**	0.00	1.85	0.00	**0.19**	0.00	1.45	0.00	**0.44**	0.00
CSCO	5.85	**0.09**	**0.02**	**0.02**	4.15	0.00	**0.08**	0.00	2.70	0.00	**0.68**	0.00	2.10	0.00	**0.90**	0.00
HD	5.40	**0.41**	**0.94**	**0.71**	3.40	0.01	**0.29**	**0.03**	2.20	0.00	**0.09**	0.00	1.45	0.00	**0.44**	0.00
UTX	6.20	**0.02**	**0.27**	**0.03**	4.15	0.00	**0.41**	0.00	1.95	0.00	**0.79**	0.00	1.50	0.00	**0.34**	0.00
BA	5.90	**0.07**	**0.25**	**0.10**	3.65	0.00	**0.43**	0.01	2.25	0.00	**0.99**	0.00	1.80	0.00	**0.68**	0.00
MCD	4.60	**0.41**	**0.70**	**0.66**	3.00	**0.16**	**0.88**	**0.37**	1.95	0.00	**0.79**	0.00	1.50	0.00	**0.47**	0.00
AXP	5.80	**0.11**	**0.23**	**0.13**	3.55	0.00	**0.26**	0.01	2.05	0.00	**0.19**	0.00	1.55	0.00	**0.32**	0.00
MMM	7.15	0.00	**0.37**	0.00	5.00	0.00	**0.37**	0.00	3.10	0.00	**0.46**	0.00	2.40	0.00	**0.14**	0.00
GS	5.95	**0.06**	**0.14**	**0.06**	3.85	0.00	**0.54**	0.00	1.85	0.00	**0.24**	0.00	1.55	0.00	**0.32**	0.00
UNH	6.95	0.00	**0.55**	0.00	4.20	0.00	**0.44**	0.00	2.45	0.00	**0.49**	0.00	1.90	0.00	**0.75**	0.00
CAT	7.00	0.00	**0.17**	0.00	4.70	0.00	**0.10**	0.00	2.65	0.00	**0.71**	0.00	1.80	0.00	**0.25**	0.00
DD	5.20	**0.68**	**0.85**	**0.90**	3.30	**0.03**	**0.36**	**0.06**	2.00	0.00	**0.83**	0.00	1.30	0.00	**0.41**	0.00
NKE	5.70	**0.16**	**0.09**	**0.08**	3.60	0.00	**0.80**	0.01	2.35	0.00	**0.92**	0.00	2.10	0.00	**0.90**	0.00
TRV	5.90	**0.07**	**0.68**	**0.18**	3.55	0.00	**0.15**	0.01	2.20	0.00	**0.02**	0.00	1.95	0.00	0.01	0.00

4.7.8 Interval Forecast Evaluation of GO-GARCH with Students't Distributed Errors, Mean Model = ARMA(0,0)

Table 4.34: Interval forecast evaluation using Students't distribution, mean model = ARMA(0,0)

	h = 1															
Stocks	**100(α) = 5**				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	2.35	0.00	**0.12**	0.00	0.90	0.00	**0.15**	0.00	0.40	0.00	**0.80**	**0.01**	0.35	**0.32**	**0.82**	**0.59**
MSFT	4.05	**0.04**	**0.69**	**0.12**	2.55	**0.88**	**0.78**	**0.95**	1.00	**1.00**	**0.52**	**0.82**	0.85	**0.04**	**0.59**	**0.11**
GE	6.10	0.03	**0.02**	0.01	3.35	**0.02**	**0.10**	**0.02**	1.30	**0.20**	**0.41**	**0.31**	1.05	0.00	**0.50**	0.01
JNJ	0.45	0.00	**0.03**	0.00	0.20	0.00	**0.90**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	0.00	0.00
WMT	1.70	0.00	**0.02**	0.00	0.80	0.00	0.01	0.00	0.35	0.00	**0.02**	0.00	0.10	0.00	**0.95**	0.01
CVX	2.80	0.00	**0.29**	0.00	1.00	0.00	0.01	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	**0.75**	**0.78**	**0.91**
JPM	11.35	0.00	0.00	0.00	6.80	0.00	0.00	0.00	3.60	0.00	0.00	0.00	3.00	0.00	0.00	0.00
PG	1.10	0.00	**0.24**	0.00	0.55	0.00	**0.73**	0.00	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.01
PFE	2.15	0.00	**0.32**	0.00	1.15	0.00	**0.27**	0.00	0.40	0.00	**0.80**	0.01	0.25	**0.08**	**0.87**	**0.21**
IBM	2.00	0.00	**0.05**	0.00	1.20	0.00	**0.29**	0.00	0.45	0.01	**0.78**	**0.02**	0.40	**0.51**	**0.80**	**0.78**
T	1.35	0.00	**0.39**	0.00	0.55	0.00	**0.73**	0.00	0.30	0.00	**0.85**	0.00	0.20	**0.03**	**0.90**	**0.10**
KO	0.70	0.00	**0.66**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.01	**0.92**	**0.03**
MRK	2.85	0.00	0.00	0.00	1.65	0.01	**0.57**	**0.03**	0.80	**0.35**	**0.61**	**0.57**	0.60	**0.54**	**0.70**	**0.77**
VZ	1.60	0.00	**0.11**	0.00	0.50	0.00	**0.75**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.01	**0.92**	**0.03**
DIS	4.00	**0.03**	**0.91**	**0.11**	1.70	**0.02**	**0.61**	**0.05**	0.90	**0.65**	**0.57**	**0.77**	0.60	**0.54**	**0.70**	**0.77**
INTC	5.95	**0.06**	**0.46**	**0.13**	3.15	**0.07**	**0.49**	**0.16**	1.10	**0.66**	**0.48**	**0.71**	0.80	**0.08**	**0.61**	**0.19**
CSCO	5.85	**0.09**	**0.95**	**0.23**	3.30	**0.03**	**0.25**	**0.05**	1.65	0.01	**0.57**	**0.02**	1.50	0.00	**0.47**	0.00
HD	4.65	**0.47**	**0.04**	**0.09**	2.10	**0.24**	**0.29**	**0.29**	0.70	**0.15**	**0.66**	**0.33**	0.35	**0.32**	**0.82**	**0.59**
UTX	2.90	0.00	**0.81**	0.00	1.25	0.00	**0.32**	0.00	0.30	0.00	**0.85**	0.00	0.30	**0.17**	**0.85**	**0.38**
BA	6.45	0.00	**0.11**	0.00	3.40	0.01	**0.65**	**0.04**	1.15	**0.51**	**0.03**	0.07	0.95	0.01	**0.17**	**0.02**
MCD	1.05	0.00	**0.50**	0.00	0.45	0.00	**0.78**	0.00	0.30	0.00	**0.85**	0.00	0.20	**0.03**	**0.90**	**0.10**
AXP	9.85	0.00	0.01	0.00	5.90	0.00	0.00	0.00	2.65	0.00	0.00	0.00	2.20	0.00	**0.97**	0.00
MMM	1.85	0.00	**0.19**	0.00	1.05	0.00	**0.50**	0.00	0.65	**0.09**	**0.68**	**0.22**	0.60	**0.54**	**0.70**	**0.77**
GS	10.20	0.00	**0.15**	0.00	5.90	0.00	**0.25**	0.00	2.85	0.00	**0.59**	0.00	2.20	0.00	**0.97**	0.00
UNH	8.85	0.00	0.00	0.00	4.45	0.00	**0.06**	0.00	2.15	0.00	**0.08**	0.00	1.70	0.00	**0.13**	0.00
CAT	9.05	0.00	**0.67**	0.00	5.25	0.00	**0.29**	0.00	2.50	0.00	**0.17**	0.00	2.05	0.00	**0.27**	0.00
DD	4.50	**0.30**	**0.24**	**0.29**	1.95	**0.10**	**0.21**	**0.12**	0.70	**0.15**	**0.66**	**0.33**	0.65	**0.36**	**0.68**	**0.61**
NKE	3.90	**0.02**	**0.51**	**0.05**	2.10	**0.24**	**0.18**	**0.20**	1.30	**0.20**	**0.41**	**0.31**	1.10	0.00	**0.48**	0.00
TRV	3.90	**0.02**	**0.12**	**0.02**	1.90	**0.07**	**0.04**	**0.02**	0.70	**0.15**	**0.09**	**0.08**	0.65	**0.36**	**0.07**	**0.13**

Table 4.35: Interval forecast evaluation using Students't distribution, mean model = ARMA(0,0)

Stocks	**h = 2**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	2.30	0.00	**0.11**	0.00	0.85	0.00	**0.59**	0.00	0.35	0.00	**0.82**	0.00	0.35	**0.32**	**0.82**	**0.59**
MSFT	3.90	**0.02**	**0.28**	**0.04**	2.35	**0.67**	**0.92**	**0.91**	1.00	**1.00**	**0.52**	**0.82**	0.85	**0.04**	**0.59**	**0.11**
GE	6.15	**0.02**	**0.05**	0.01	3.40	0.01	**0.29**	**0.03**	1.30	**0.20**	**0.41**	**0.31**	1.00	0.01	**0.52**	**0.02**
JNJ	0.40	0.00	**0.02**	0.00	0.20	0.00	**0.90**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	0.00	0.00
WMT	1.65	0.00	**0.02**	0.00	0.75	0.00	0.00	0.00	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.01
CVX	2.95	0.00	**0.37**	0.00	1.00	0.00	**0.52**	0.00	0.45	0.01	**0.78**	**0.02**	0.40	**0.51**	**0.80**	**0.78**
JPM	10.75	0.00	0.00	0.00	6.70	0.00	0.00	0.00	3.55	0.00	0.00	0.00	3.25	0.00	0.00	0.00
PG	1.10	0.00	**0.24**	0.00	0.50	0.00	**0.75**	0.00	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.01
PFE	2.15	0.00	**0.32**	0.00	1.05	0.00	**0.22**	0.00	0.40	0.00	**0.80**	0.01	0.30	**0.17**	**0.85**	**0.38**
IBM	1.90	0.00	**0.04**	0.00	1.20	0.00	**0.29**	0.00	0.45	0.01	**0.78**	**0.02**	0.35	**0.32**	**0.82**	**0.59**
T	1.35	0.00	**0.39**	0.00	0.55	0.00	**0.73**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
KO	0.80	0.00	**0.61**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.01	**0.92**	**0.03**
MRK	2.85	0.00	0.00	0.00	1.65	0.01	**0.57**	**0.03**	0.75	**0.24**	**0.63**	**0.45**	0.65	**0.36**	**0.68**	**0.61**
VZ	1.55	0.00	**0.51**	0.00	0.55	0.00	**0.73**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.01	**0.92**	**0.03**
DIS	3.90	**0.02**	**0.98**	**0.06**	1.55	0.00	**0.51**	0.01	0.90	**0.65**	**0.57**	**0.77**	0.65	**0.36**	**0.68**	**0.61**
INTC	5.85	**0.09**	**0.40**	**0.16**	3.05	**0.13**	**0.43**	**0.23**	0.95	**0.82**	**0.55**	**0.81**	0.75	**0.14**	**0.63**	**0.30**
CSCO	5.55	**0.26**	**0.94**	**0.54**	3.20	**0.05**	**0.21**	**0.07**	1.60	0.01	**0.54**	**0.04**	1.55	0.00	**0.51**	0.00
HD	4.55	**0.35**	**0.18**	**0.26**	2.05	**0.18**	**0.27**	**0.23**	0.80	**0.35**	**0.61**	**0.57**	0.35	**0.32**	**0.82**	**0.59**
UTX	2.90	0.00	**0.81**	0.00	1.20	0.00	**0.45**	0.00	0.40	0.00	**0.80**	0.01	0.25	**0.08**	**0.87**	**0.21**
BA	6.20	**0.02**	**0.23**	**0.03**	3.20	0.05	**0.97**	0.16	1.10	**0.66**	**0.48**	**0.71**	0.80	**0.08**	**0.61**	**0.19**
MCD	1.15	0.00	**0.46**	0.00	0.45	0.00	**0.78**	0.00	0.30	0.00	**0.85**	0.00	0.20	**0.03**	**0.90**	**0.10**
AXP	9.70	0.00	**0.03**	0.00	5.90	0.00	0.00	0.00	2.70	0.00	**0.07**	0.00	2.20	0.00	**0.02**	0.00
MMM	1.70	0.00	**0.13**	0.00	1.00	0.00	**0.52**	0.00	0.65	**0.09**	**0.68**	**0.22**	0.60	**0.54**	**0.70**	**0.77**
GS	10.45	0.00	**0.15**	0.00	5.90	0.00	**0.13**	0.00	2.75	0.00	**0.27**	0.00	2.40	0.00	**0.88**	0.00
UNH	8.45	0.00	0.00	0.00	4.45	0.00	**0.06**	0.00	2.15	0.00	**0.08**	0.00	1.75	0.00	**0.02**	0.00
CAT	9.10	0.00	**0.70**	0.00	5.10	0.00	**0.43**	0.00	2.55	0.00	**0.19**	0.00	1.95	0.00	**0.23**	0.00
DD	4.45	**0.25**	**0.26**	**0.27**	2.05	**0.18**	**0.86**	**0.41**	0.70	**0.15**	**0.66**	**0.33**	0.60	**0.54**	**0.70**	**0.77**
NKE	3.80	0.01	**0.57**	**0.03**	2.15	**0.31**	**0.17**	**0.23**	1.35	**0.13**	**0.39**	**0.23**	1.10	0.00	**0.48**	0.00
TRV	3.90	**0.02**	**0.12**	**0.02**	1.90	**0.07**	0.01	0.00	0.85	**0.49**	0.00	0.00	0.65	**0.36**	**0.07**	**0.13**

Table 4.36: Interval forecast evaluation using Students't distribution, mean model = ARMA(0,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	2.10	0.00	**0.90**	0.00	0.85	0.00	**0.59**	0.00	0.35	0.00	**0.82**	0.00	0.35	**0.32**	**0.82**	**0.59**
MSFT	3.80	0.01	**0.52**	**0.03**	2.30	**0.56**	**0.95**	**0.85**	1.00	**1.00**	**0.52**	**0.82**	0.85	**0.04**	**0.59**	**0.11**
GE	6.10	**0.03**	0.00	0.00	3.25	**0.04**	**0.55**	**0.10**	1.30	**0.20**	**0.41**	**0.31**	1.00	0.01	**0.52**	**0.02**
JNJ	0.40	0.00	0.02	0.00	0.20	0.00	**0.90**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	0.00	0.00
WMT	1.60	0.00	**0.11**	0.00	0.70	0.00	**0.09**	0.00	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.01
CVX	2.80	0.00	**0.29**	0.00	1.00	0.00	**0.52**	0.00	0.45	0.01	**0.78**	**0.02**	0.35	**0.32**	**0.82**	**0.59**
JPM	10.90	0.00	0.00	0.00	6.70	0.00	0.00	0.00	3.60	0.00	0.00	0.00	3.20	0.00	0.00	0.00
PG	1.10	0.00	**0.24**	0.00	0.55	0.00	**0.05**	0.00	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.01
PFE	2.10	0.00	**0.29**	0.00	1.10	0.00	**0.24**	0.00	0.40	0.00	**0.80**	0.01	0.25	**0.08**	**0.87**	**0.21**
IBM	1.90	0.00	**0.04**	0.00	1.20	0.00	**0.29**	0.00	0.45	0.01	**0.78**	**0.02**	0.40	**0.51**	**0.80**	**0.78**
T	1.30	0.00	**0.41**	0.00	0.55	0.00	**0.73**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
KO	0.70	0.00	**0.66**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.01	**0.92**	**0.03**
MRK	2.80	0.00	0.00	0.00	1.65	0.01	**0.57**	**0.03**	0.75	**0.24**	**0.63**	**0.45**	0.60	**0.54**	**0.70**	**0.77**
VZ	1.70	0.00	**0.02**	0.00	0.60	0.00	**0.70**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.01	**0.92**	**0.03**
DIS	3.85	0.01	**0.26**	**0.03**	1.60	0.01	**0.54**	**0.02**	0.75	**0.24**	**0.63**	**0.45**	0.70	**0.23**	**0.66**	**0.44**
INTC	5.85	**0.09**	**0.40**	**0.16**	3.00	**0.16**	**0.40**	**0.27**	1.00	**1.00**	**0.52**	**0.82**	0.75	**0.14**	**0.63**	**0.30**
CSCO	5.65	**0.19**	**0.87**	**0.42**	3.15	**0.07**	**0.49**	**0.16**	1.60	0.01	**0.54**	**0.04**	1.50	0.00	**0.47**	0.00
HD	4.60	**0.41**	**0.03**	**0.07**	2.05	**0.18**	**0.27**	**0.23**	0.85	**0.49**	**0.13**	**0.26**	0.40	**0.51**	**0.80**	**0.78**
UTX	2.75	0.00	**0.65**	0.00	1.20	0.00	**0.45**	0.00	0.40	0.00	**0.80**	0.01	0.25	**0.08**	**0.87**	**0.21**
BA	6.15	**0.02**	**0.21**	**0.03**	3.30	**0.03**	**0.90**	**0.09**	1.15	**0.51**	0.27	**0.43**	0.85	**0.04**	**0.59**	**0.11**
MCD	1.15	0.00	**0.46**	0.00	0.45	0.00	**0.78**	0.00	0.30	0.00	**0.85**	0.00	0.20	**0.03**	**0.90**	**0.10**
AXP	9.75	0.00	0.01	0.00	5.70	0.00	0.00	0.00	2.90	0.00	**0.81**	0.00	2.15	0.00	**0.32**	0.00
MMM	1.70	0.00	**0.13**	0.00	0.95	0.00	**0.55**	0.00	0.65	**0.09**	**0.68**	**0.22**	0.60	**0.54**	**0.70**	**0.77**
GS	10.10	0.00	**0.12**	0.00	5.95	0.00	**0.14**	0.00	2.85	0.00	**0.32**	0.00	2.20	0.00	**0.97**	0.00
UNH	8.35	0.00	0.00	0.00	4.45	0.00	**0.06**	0.00	2.15	0.00	**0.08**	0.00	1.75	0.00	**0.02**	0.00
CAT	8.85	0.00	**0.72**	0.00	5.25	0.00	**0.15**	0.00	2.50	0.00	**0.17**	0.00	1.90	0.00	**0.20**	0.00
DD	4.35	**0.17**	**0.29**	**0.23**	2.05	**0.18**	**0.19**	**0.18**	0.70	**0.15**	**0.66**	**0.33**	0.50	**1.00**	**0.75**	**0.95**
NKE	3.95	**0.03**	**0.62**	**0.07**	2.15	**0.31**	**0.17**	**0.23**	1.25	**0.28**	**0.43**	**0.40**	1.10	0.00	**0.48**	0.00
TRV	3.95	**0.03**	**0.05**	0.01	1.80	**0.04**	0.00	0.00	0.80	**0.35**	0.00	0.00	0.60	**0.54**	**0.06**	**0.14**

Table 4.37: Interval forecast evaluation using Students't distribution, mean model = ARMA(0,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	2.10	0.00	**0.90**	0.00	0.85	0.00	**0.59**	0.00	0.35	0.00	**0.82**	0.00	0.35	**0.32**	**0.82**	**0.59**
MSFT	3.75	0.01	**0.49**	**0.02**	2.30	**0.56**	**0.95**	**0.85**	1.00	**1.00**	**0.52**	**0.82**	0.85	**0.04**	**0.59**	**0.11**
GE	6.05	**0.04**	0.01	0.00	3.25	**0.04**	**0.55**	**0.10**	1.30	**0.20**	**0.41**	**0.31**	1.00	0.01	**0.52**	**0.02**
JNJ	0.40	0.00	**0.02**	0.00	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	0.00	0.00
WMT	1.60	0.00	**0.11**	0.00	0.70	0.00	**0.09**	0.00	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.01
CVX	2.75	0.00	**0.27**	0.00	0.95	0.00	**0.55**	0.00	0.45	0.01	**0.78**	**0.02**	0.35	**0.32**	**0.82**	**0.59**
JPM	10.90	0.00	0.00	0.00	6.75	0.00	0.00	0.00	3.60	0.00	0.00	0.00	3.25	0.00	0.00	0.00
PG	1.10	0.00	**0.24**	0.00	0.55	0.00	**0.05**	0.00	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.01
PFE	2.00	0.00	**0.25**	0.00	1.10	0.00	**0.24**	0.00	0.40	0.00	**0.80**	0.01	0.25	**0.08**	**0.87**	**0.21**
IBM	1.90	0.00	**0.04**	0.00	1.20	0.00	**0.29**	0.00	0.45	0.01	**0.78**	**0.02**	0.40	**0.51**	**0.80**	**0.78**
T	1.25	0.00	**0.43**	0.00	0.55	0.00	**0.73**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
KO	0.70	0.00	**0.66**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.01	**0.92**	**0.03**
MRK	2.80	0.00	0.00	0.00	1.65	0.01	**0.57**	**0.03**	0.75	**0.24**	**0.63**	**0.45**	0.60	**0.54**	**0.70**	**0.77**
VZ	1.60	0.00	**0.11**	0.00	0.50	0.00	**0.75**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.01	**0.92**	**0.03**
DIS	3.85	0.01	**0.55**	**0.04**	1.55	0.00	**0.51**	0.01	0.80	**0.35**	**0.61**	**0.57**	0.70	**0.23**	**0.66**	**0.44**
INTC	5.75	**0.13**	**0.35**	**0.21**	2.90	**0.26**	**0.34**	**0.34**	1.00	**1.00**	**0.52**	**0.82**	0.75	**0.14**	**0.63**	**0.30**
CSCO	5.50	**0.31**	**0.98**	**0.60**	3.10	**0.10**	**0.46**	**0.19**	1.65	0.01	**0.57**	**0.02**	1.50	0.00	**0.47**	0.00
HD	4.55	**0.35**	**0.07**	**0.13**	2.00	**0.14**	**0.25**	**0.17**	0.85	**0.49**	**0.13**	**0.26**	0.45	**0.75**	**0.78**	**0.91**
UTX	2.65	0.00	**0.71**	0.00	1.15	0.00	**0.46**	0.00	0.40	0.00	**0.80**	0.01	0.25	**0.08**	**0.87**	**0.21**
BA	6.15	**0.02**	**0.21**	**0.03**	3.20	**0.05**	**0.97**	**0.16**	1.15	**0.51**	**0.27**	**0.43**	0.80	**0.08**	**0.61**	**0.19**
MCD	1.15	0.00	**0.46**	0.00	0.45	0.00	**0.78**	0.00	0.30	0.00	**0.85**	0.00	0.20	**0.03**	**0.90**	**0.10**
AXP	9.80	0.00	0.01	0.00	5.75	0.00	0.00	0.00	2.80	0.00	**0.29**	0.00	2.15	0.00	**0.32**	0.00
MMM	1.65	0.00	**0.12**	0.00	0.95	0.00	**0.55**	0.00	0.65	**0.09**	**0.68**	**0.22**	0.60	**0.54**	**0.70**	**0.77**
GS	10.15	0.00	**0.13**	0.00	5.80	0.00	**0.11**	0.00	2.80	0.00	**0.29**	0.00	2.20	0.00	**0.97**	0.00
UNH	8.35	0.00	0.00	0.00	4.40	0.00	**0.13**	0.00	2.05	0.00	**0.06**	0.00	1.75	0.00	**0.02**	0.00
CAT	8.80	0.00	**0.68**	0.00	5.20	0.00	**0.13**	0.00	2.35	0.00	**0.12**	0.00	1.90	0.00	**0.20**	0.00
DD	4.35	**0.17**	**0.29**	**0.23**	2.05	**0.18**	**0.19**	**0.18**	0.70	**0.15**	**0.66**	**0.33**	0.50	**1.00**	**0.75**	**0.95**
NKE	3.90	**0.02**	**0.58**	**0.06**	2.15	**0.31**	**0.17**	**0.23**	1.20	**0.38**	**0.45**	**0.51**	1.10	0.00	**0.48**	0.00
TRV	3.95	**0.03**	**0.05**	0.01	1.70	**0.02**	0.00	0.00	0.80	**0.35**	0.00	0.00	0.65	**0.36**	0.00	0.00

Table 4.38: Interval forecast evaluation using Students't distribution, mean model = ARMA(0,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	2.15	0.00	**0.08**	0.00	0.85	0.00	**0.59**	0.00	0.40	0.00	**0.80**	0.01	0.35	**0.32**	**0.82**	**0.59**
MSFT	3.70	0.01	**0.46**	**0.02**	2.30	**0.56**	**0.95**	**0.85**	1.00	**1.00**	**0.52**	**0.82**	0.85	**0.04**	**0.59**	**0.11**
GE	6.10	**0.03**	0.01	0.00	3.25	**0.04**	**0.55**	**0.10**	1.25	**0.28**	**0.43**	**0.40**	1.00	0.01	**0.52**	**0.02**
JNJ	0.40	0.00	**0.02**	0.00	0.15	0.00	**0.92**	0.00	0.03	0.00	**0.97**	0.00	0.00	0.00	0.00	0.00
WMT	1.60	0.00	**0.11**	0.00	0.65	0.00	**0.07**	0.00	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.01
CVX	2.75	0.00	**0.27**	0.00	0.95	0.00	**0.55**	0.00	0.45	0.01	**0.78**	**0.02**	0.35	**0.32**	**0.82**	**0.59**
JPM	10.85	0.00	0.00	0.00	6.70	0.00	0.00	0.00	3.60	0.00	0.00	0.00	3.20	0.00	0.00	0.00
PG	1.10	0.00	**0.24**	0.00	0.55	0.00	**0.05**	0.00	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.01
PFE	2.00	0.00	**0.25**	0.00	1.05	0.00	**0.22**	0.00	0.40	0.00	**0.80**	0.01	0.25	**0.08**	**0.87**	**0.21**
IBM	1.90	0.00	**0.04**	0.00	1.25	0.00	**0.04**	0.00	0.45	0.01	**0.78**	**0.02**	0.40	**0.51**	**0.80**	**0.78**
T	1.25	0.00	**0.43**	0.00	0.55	0.00	**0.73**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
KO	0.70	0.00	**0.66**	0.00	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.01	**0.92**	**0.03**
MRK	2.75	0.00	0.00	0.00	1.60	0.01	**0.54**	**0.02**	0.75	**0.24**	**0.63**	**0.45**	0.60	**0.54**	**0.70**	**0.77**
VZ	1.60	0.00	**0.11**	0.00	0.55	0.00	**0.05**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.01	**0.92**	**0.03**
DIS	3.90	**0.02**	**0.58**	**0.06**	1.55	0.00	**0.51**	0.01	0.75	**0.24**	**0.63**	**0.45**	0.70	**0.23**	**0.66**	**0.44**
INTC	5.75	**0.13**	**0.35**	**0.21**	2.85	**0.32**	**0.32**	**0.37**	1.00	**1.00**	**0.52**	**0.82**	0.75	**0.14**	**0.63**	**0.30**
CSCO	5.50	**0.31**	**0.98**	**0.60**	3.10	**0.10**	**0.46**	**0.19**	1.60	0.01	**0.54**	**0.04**	1.50	0.00	**0.47**	0.00
HD	4.40	**0.21**	**0.05**	**0.07**	2.00	**0.14**	**0.25**	**0.17**	0.85	**0.49**	**0.13**	**0.26**	0.45	**0.75**	**0.78**	**0.91**
UTX	2.65	0.00	**0.71**	0.00	1.10	0.00	**0.48**	0.00	0.35	0.00	**0.82**	0.00	0.25	**0.08**	**0.87**	**0.21**
BA	6.10	**0.03**	**0.34**	**0.06**	3.15	**0.07**	**0.99**	**0.20**	1.10	**0.66**	**0.24**	**0.46**	0.85	**0.04**	**0.59**	**0.11**
MCD	1.15	0.00	**0.46**	0.00	0.40	0.00	**0.80**	0.00	0.30	0.00	**0.85**	0.00	0.20	**0.03**	**0.90**	**0.10**
AXP	9.75	0.00	0.01	0.00	5.75	0.00	0.00	0.00	2.85	0.00	**0.32**	0.00	2.15	0.00	**0.32**	0.00
MMM	1.70	0.00	**0.13**	0.00	0.95	0.00	**0.55**	0.00	0.65	**0.09**	**0.68**	**0.22**	0.60	**0.54**	**0.70**	**0.77**
GS	10.05	0.00	**0.25**	0.00	5.80	0.00	**0.11**	0.00	2.75	0.00	**0.27**	0.00	2.20	0.00	**0.97**	0.00
UNH	8.30	0.00	0.00	0.00	4.40	0.00	**0.13**	0.00	2.05	0.00	**0.06**	0.00	1.75	0.00	**0.02**	0.00
CAT	8.70	0.00	**0.97**	0.00	5.15	0.00	**0.25**	0.00	2.35	0.00	**0.12**	0.00	1.85	0.00	**0.19**	0.00
DD	4.35	**0.17**	**0.29**	**0.23**	2.05	**0.18**	**0.19**	**0.18**	0.70	**0.15**	**0.66**	**0.33**	0.50	**1.00**	**0.75**	**0.95**
NKE	3.85	0.01	**0.55**	**0.04**	2.15	**0.31**	**0.17**	**0.23**	1.20	**0.38**	**0.45**	**0.51**	1.10	0.00	**0.48**	0.00
TRV	3.90	**0.02**	**0.04**	0.01	1.75	**0.02**	0.00	0.00	0.80	**0.35**	0.00	0.00	0.65	**0.36**	0.00	0.00

4.7.9 Interval Forecast Evaluation of GO-GARCH with Students't Distributed Errors, Mean Model = ARMA(2,0)

Table 4.39: Interval forecast evaluation using Students't distribution, mean model = ARMA(2,0)

Stocks	h = 1															
	$100(\alpha) = 5$				$100(\alpha) = 2.5$				$100(\alpha) = 1$				$100(\alpha) = 0.5$			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	2.90	0.00	**0.03**	0.00	1.35	0.00	0.00	0.00	0.45	0.01	0.00	0.00	0.45	**0.75**	0.00	0.00
MSFT	4.35	**0.17**	**0.53**	**0.33**	2.35	**0.67**	**0.43**	**0.67**	1.10	**0.66**	**0.48**	**0.71**	1.05	0.00	**0.50**	0.01
GE	6.50	0.00	0.00	0.00	3.60	0.00	0.00	0.00	1.90	0.00	**0.04**	0.00	1.65	0.00	**0.02**	0.00
JNJ	0.65	0.00	**0.07**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.01
WMT	1.75	0.00	0.00	0.00	0.85	0.00	**0.13**	0.00	0.40	0.00	**0.02**	0.00	0.05	0.00	**0.97**	0.00
CVX	3.65	0.00	**0.18**	0.01	1.25	0.00	**0.04**	0.00	0.70	**0.15**	0.00	0.00	0.55	**0.75**	0.00	0.00
JPM	11.75	0.00	0.00	0.00	7.30	0.00	0.00	0.00	4.35	0.00	0.00	0.00	3.75	0.00	0.00	0.00
PG	1.35	0.00	**0.05**	0.00	0.75	0.00	**0.63**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.01	**0.92**	**0.03**
PFE	2.50	0.00	**0.04**	0.00	1.30	0.00	**0.35**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.50	**1.00**	**0.75**	**0.95**
IBM	2.20	0.00	**0.09**	0.00	1.15	0.00	**0.03**	0.00	0.45	0.01	**0.78**	**0.02**	0.40	**0.51**	**0.80**	**0.78**
T	1.80	0.00	**0.03**	0.00	0.85	0.00	**0.13**	0.00	0.45	0.01	**0.78**	**0.02**	0.40	**0.51**	**0.80**	**0.78**
KO	1.10	0.00	**0.02**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00	0.20	**0.03**	**0.90**	**0.10**
MRK	3.65	0.00	0.01	0.00	1.95	**0.10**	0.01	0.01	0.80	**0.35**	**0.61**	**0.57**	0.80	**0.08**	**0.61**	**0.19**
VZ	1.80	0.00	**0.17**	0.00	0.75	0.00	**0.63**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.01	**0.92**	**0.03**
DIS	4.50	**0.30**	**0.02**	**0.05**	2.50	**1.00**	**0.17**	**0.39**	1.00	**1.00**	**0.52**	**0.82**	1.00	0.01	**0.52**	**0.02**
INTC	6.25	0.01	**0.75**	**0.04**	3.35	**0.02**	**0.10**	**0.02**	1.45	**0.06**	**0.44**	**0.12**	1.15	0.00	**0.46**	0.00
CSCO	6.00	**0.05**	**0.29**	**0.08**	3.55	0.00	**0.15**	0.01	1.90	0.00	**0.20**	0.00	1.70	0.00	**0.28**	0.00
HD	5.25	**0.61**	0.00	0.00	2.50	**1.00**	**0.17**	**0.39**	0.95	**0.82**	**0.17**	**0.39**	0.70	**0.23**	**0.66**	**0.44**
UTX	3.55	0.00	0.01	0.00	1.65	0.01	**0.57**	**0.03**	0.50	0.01	**0.75**	**0.04**	0.40	**0.51**	**0.80**	**0.78**
BA	6.60	0.00	**0.04**	0.00	3.45	0.01	**0.12**	0.01	1.30	**0.20**	**0.05**	**0.06**	1.15	0.00	**0.03**	0.00
MCD	1.15	0.00	**0.46**	0.00	0.60	0.00	**0.70**	0.00	0.25	0.00	**0.87**	0.00	0.25	**0.08**	**0.87**	**0.21**
AXP	10.95	0.00	0.00	0.00	7.00	0.00	0.00	0.00	3.45	0.00	**0.12**	0.00	2.85	0.00	**0.32**	0.00
MMM	2.25	0.00	**0.99**	0.00	1.30	0.00	**0.35**	0.00	0.85	**0.49**	**0.13**	**0.26**	0.60	**0.54**	**0.70**	**0.77**
GS	11.25	0.00	0.01	0.00	6.55	0.00	0.01	0.00	2.85	0.00	**0.03**	0.00	2.60	0.00	**0.06**	0.00
UNH	9.55	0.00	0.00	0.00	5.35	0.00	0.00	0.00	2.50	0.00	**0.17**	0.00	2.20	0.00	**0.35**	0.00
CAT	9.20	0.00	**0.29**	0.00	5.60	0.00	**0.03**	0.00	2.70	0.00	**0.25**	0.00	2.15	0.00	**0.08**	0.00
DD	4.95	**0.92**	**0.96**	**0.99**	2.55	**0.88**	**0.56**	**0.83**	0.85	**0.49**	**0.13**	**0.26**	0.60	**0.54**	**0.06**	**0.14**
NKE	4.20	**0.09**	**0.44**	**0.18**	2.25	**0.47**	**0.99**	**0.77**	1.35	**0.13**	**0.39**	**0.23**	1.30	0.00	**0.41**	0.00
TRV	5.55	**0.26**	0.01	**0.02**	3.00	**0.16**	0.01	0.01	1.45	**0.06**	0.00	0.00	1.25	0.00	0.00	0.00

Table 4.40: Interval forecast evaluation using Students't distribution, mean model = ARMA(2,0)

Stocks	**h = 2**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	2.85	0.00	**0.03**	0.00	1.45	0.00	0.01	0.00	0.50	0.01	**0.75**	**0.04**	0.40	**0.51**	**0.80**	**0.78**
MSFT	4.45	**0.25**	**0.32**	**0.31**	2.25	**0.47**	**0.37**	**0.52**	1.15	**0.51**	**0.46**	**0.62**	1.10	0.00	**0.48**	0.00
GE	6.15	**0.02**	0.01	0.00	3.70	0.00	0.00	0.00	1.85	0.00	**0.03**	0.00	1.55	0.00	**0.51**	0.00
JNJ	0.65	0.00	**0.07**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.01
WMT	1.70	0.00	0.00	0.00	0.75	0.00	**0.10**	0.00	0.35	0.00	**0.82**	0.00	0.05	0.00	**0.97**	0.00
CVX	3.50	0.00	**0.14**	0.00	1.30	0.00	**0.05**	0.00	0.65	**0.09**	0.00	0.00	0.55	**0.75**	0.00	0.00
JPM	11.80	0.00	0.00	0.00	7.35	0.00	0.00	0.00	4.50	0.00	0.00	0.00	3.80	0.00	0.00	0.00
PG	1.35	0.00	**0.05**	0.00	0.90	0.00	**0.15**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.01
PFE	2.60	0.00	**0.06**	0.00	1.25	0.00	**0.32**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.45	**0.75**	**0.78**	**0.91**
IBM	2.15	0.00	**0.08**	0.00	1.20	0.00	**0.03**	0.00	0.45	0.01	**0.78**	**0.02**	0.40	**0.51**	**0.80**	**0.78**
T	1.75	0.00	**0.15**	0.00	0.85	0.00	**0.13**	0.00	0.45	0.01	**0.78**	**0.02**	0.35	**0.32**	**0.82**	**0.59**
KO	0.90	0.00	**0.15**	0.00	0.40	0.00	**0.80**	0.00	0.20	0.00	**0.90**	0.00	0.20	**0.03**	**0.90**	**0.10**
MRK	3.70	0.01	0.00	0.00	1.95	**0.10**	0.01	0.01	0.85	**0.49**	**0.59**	**0.68**	0.80	**0.08**	**0.61**	**0.19**
VZ	1.95	0.00	**0.04**	0.00	0.80	0.00	**0.61**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.01	**0.92**	**0.03**
DIS	4.35	**0.17**	0.00	0.01	2.50	**1.00**	**0.17**	**0.39**	1.05	**0.82**	**0.50**	**0.78**	1.00	0.01	**0.52**	**0.02**
INTC	6.40	0.01	**0.51**	**0.02**	3.30	**0.03**	**0.25**	**0.05**	1.55	**0.02**	**0.09**	**0.02**	1.15	0.00	**0.46**	0.00
CSCO	5.95	**0.06**	**0.27**	**0.09**	3.55	0.00	**0.76**	**0.02**	1.80	0.00	**0.68**	0.00	1.70	0.00	**0.28**	0.00
HD	5.15	**0.76**	0.00	0.00	2.35	**0.67**	**0.12**	**0.28**	0.90	**0.65**	**0.15**	**0.32**	0.75	**0.14**	**0.63**	**0.30**
UTX	3.45	0.00	**0.12**	0.00	1.65	0.01	**0.29**	**0.02**	0.50	0.01	**0.75**	**0.04**	0.40	**0.51**	**0.80**	**0.78**
BA	6.45	0.00	**0.34**	0.01	3.30	**0.03**	**0.59**	**0.08**	1.35	**0.13**	**0.38**	**0.22**	1.00	0.01	**0.19**	0.01
MCD	1.15	0.00	**0.46**	0.00	0.60	0.00	**0.70**	0.00	0.25	0.00	**0.87**	0.00	0.25	**0.08**	**0.87**	**0.21**
AXP	11.15	0.00	0.00	0.00	6.80	0.00	0.00	0.00	3.95	0.00	0.00	0.00	3.05	0.00	0.00	0.00
MMM	2.35	0.00	**0.92**	0.00	1.30	0.00	**0.35**	0.00	0.85	**0.49**	**0.13**	**0.26**	0.70	**0.23**	**0.09**	**0.11**
GS	11.30	0.00	0.00	0.00	6.45	0.00	0.01	0.00	2.85	0.00	**0.03**	0.00	2.65	0.00	0.01	0.00
UNH	9.10	0.00	0.00	0.00	5.10	0.00	0.00	0.00	2.45	0.00	**0.15**	0.00	2.35	0.00	**0.12**	0.00
CAT	9.25	0.00	**0.62**	0.00	5.40	0.00	**0.04**	0.00	2.70	0.00	**0.25**	0.00	2.10	0.00	**0.07**	0.00
DD	4.85	**0.76**	**0.73**	**0.90**	2.45	**0.89**	**0.49**	**0.78**	0.75	**0.24**	**0.10**	**0.13**	0.60	**0.54**	**0.06**	**0.14**
NKE	4.10	**0.06**	**0.72**	**0.15**	2.25	**0.47**	**0.99**	**0.77**	1.40	**0.09**	**0.37**	**0.16**	1.25	0.00	**0.43**	0.00
TRV	5.65	**0.19**	0.01	**0.02**	2.90	**0.26**	0.01	0.01	1.40	**0.09**	0.00	0.00	1.20	0.00	0.00	0.00

Table 4.41: Interval forecast evaluation using Students't distribution, mean model = ARMA(2,0)

Stocks	h = 3															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	3.00	0.00	0.01	0.00	1.45	0.00	0.01	0.00	0.50	0.01	**0.75**	**0.04**	0.45	**0.75**	**0.78**	**0.91**
MSFT	4.45	**0.25**	**0.60**	**0.45**	2.25	**0.47**	**0.37**	**0.52**	1.15	**0.51**	**0.46**	**0.62**	1.05	0.00	**0.50**	0.01
GE	6.15	**0.02**	0.01	0.00	3.55	0.00	0.00	0.00	1.85	0.00	**0.03**	0.00	1.60	0.00	**0.11**	0.00
JNJ	0.60	0.00	**0.06**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.01
WMT	1.75	0.00	0.00	0.00	0.85	0.00	**0.13**	0.00	0.35	0.00	**0.82**	0.00	0.05	0.00	**0.97**	0.00
CVX	3.50	0.00	**0.14**	0.00	1.30	0.00	**0.05**	0.00	0.70	**0.15**	0.00	0.00	0.50	**1.00**	0.00	0.00
JPM	11.85	0.00	0.00	0.00	7.30	0.00	0.00	0.00	4.40	0.00	0.00	0.00	3.90	0.00	0.00	0.00
PG	1.40	0.00	**0.06**	0.00	0.90	0.00	**0.15**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.01
PFE	2.50	0.00	**0.17**	0.00	1.30	0.00	**0.35**	0.00	0.50	0.01	**0.75**	**0.04**	0.40	**0.51**	**0.80**	**0.78**
IBM	2.15	0.00	**0.08**	0.00	1.15	0.00	**0.03**	0.00	0.45	0.01	**0.78**	**0.02**	0.40	**0.51**	**0.80**	**0.78**
T	1.70	0.00	**0.02**	0.00	0.80	0.00	**0.12**	0.00	0.45	0.01	**0.78**	**0.02**	0.35	**0.32**	**0.82**	**0.59**
KO	0.90	0.00	**0.15**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00	0.20	**0.03**	**0.90**	**0.10**
MRK	3.65	0.00	0.01	0.00	1.90	0.07	0.01	0.00	0.90	**0.65**	**0.57**	**0.77**	0.80	**0.08**	**0.61**	**0.19**
VZ	1.85	0.00	**0.19**	0.00	0.75	0.00	**0.63**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.01	**0.92**	**0.03**
DIS	4.50	**0.30**	0.01	**0.02**	2.35	**0.67**	**0.03**	**0.08**	1.00	**1.00**	**0.52**	**0.82**	0.90	**0.02**	**0.57**	**0.06**
INTC	6.35	0.01	**0.98**	**0.03**	3.30	**0.03**	**0.25**	**0.05**	1.55	**0.02**	**0.09**	**0.02**	1.15	0.00	**0.46**	0.00
CSCO	5.80	**0.11**	**0.37**	**0.19**	3.40	0.01	**0.65**	**0.04**	1.80	0.00	**0.68**	0.00	1.70	0.00	**0.28**	0.00
HD	5.10	**0.83**	0.00	0.00	2.45	**0.89**	**0.15**	**0.36**	0.90	**0.65**	**0.15**	**0.32**	0.80	**0.08**	**0.61**	**0.19**
UTX	3.40	0.00	**0.65**	0.00	1.65	0.01	**0.29**	**0.02**	0.55	**0.03**	**0.73**	**0.08**	0.40	**0.51**	**0.80**	**0.78**
BA	6.70	0.00	0.01	0.00	3.45	0.01	**0.12**	0.01	1.40	**0.09**	**0.06**	**0.04**	1.05	0.00	**0.22**	0.00
MCD	1.15	0.00	**0.46**	0.00	0.65	0.00	**0.68**	0.00	0.25	0.00	**0.87**	0.00	0.25	**0.08**	**0.87**	**0.21**
AXP	11.15	0.00	0.00	0.00	6.75	0.00	0.00	0.00	3.95	0.00	0.00	0.00	3.05	0.00	0.00	0.00
MMM	2.30	0.00	**0.95**	0.00	1.30	0.00	**0.35**	0.00	0.85	**0.49**	**0.13**	**0.26**	0.70	**0.23**	**0.09**	**0.11**
GS	11.10	0.00	0.01	0.00	6.25	0.00	**0.03**	0.00	2.95	0.00	**0.04**	0.00	2.60	0.00	0.01	0.00
UNH	9.15	0.00	0.00	0.00	5.15	0.00	0.00	0.00	2.45	0.00	**0.15**	0.00	2.25	0.00	**0.37**	0.00
CAT	9.15	0.00	**0.74**	0.00	5.45	0.00	**0.02**	0.00	2.65	0.00	**0.23**	0.00	2.15	0.00	**0.08**	0.00
DD	4.95	**0.92**	**0.96**	**0.99**	2.45	**0.89**	**0.49**	**0.78**	0.80	**0.35**	**0.12**	**0.19**	0.60	**0.54**	**0.06**	**0.14**
NKE	4.15	**0.07**	**0.08**	**0.04**	2.25	**0.47**	**0.99**	**0.77**	1.45	**0.06**	**0.44**	**0.12**	1.20	0.00	**0.45**	0.00
TRV	5.75	**0.13**	0.00	0.00	3.00	**0.16**	0.01	0.01	1.40	**0.09**	0.00	0.00	1.20	0.00	0.00	0.00

Table 4.42: Interval forecast evaluation using Students't distribution, mean model = ARMA(2,0)

Stocks	h = 4															
	100(α) = 5				100(α) = 2.5				100(α) = 1				100(α) = 0.5			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	3.00	0.00	0.01	0.00	1.45	0.00	0.01	0.00	0.50	0.01	**0.75**	**0.04**	0.45	**0.75**	**0.78**	**0.91**
MSFT	4.40	**0.21**	**0.57**	**0.39**	2.25	**0.47**	**0.37**	**0.52**	1.15	**0.51**	**0.46**	**0.62**	1.00	0.01	**0.52**	**0.02**
GE	6.15	**0.02**	0.01	0.00	3.50	0.01	0.00	0.00	1.85	0.00	**0.03**	0.00	1.65	0.00	**0.02**	0.00
JNJ	0.60	0.00	**0.06**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.01
WMT	1.75	0.00	0.00	0.00	0.80	0.00	**0.12**	0.00	0.35	0.00	**0.82**	0.00	0.05	0.00	**0.97**	0.00
CVX	3.50	0.00	**0.14**	0.00	1.30	0.00	**0.05**	0.00	0.65	**0.09**	0.00	0.00	0.50	**1.00**	0.00	0.00
JPM	11.80	0.00	0.00	0.00	7.30	0.00	0.00	0.00	4.35	0.00	0.00	0.00	3.90	0.00	0.00	0.00
PG	1.40	0.00	**0.06**	0.00	0.90	0.00	**0.15**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.01	**0.92**	**0.03**
PFE	2.50	0.00	**0.17**	0.00	1.30	0.00	**0.35**	0.00	0.50	0.01	**0.75**	**0.04**	0.40	**0.51**	**0.80**	**0.78**
IBM	2.15	0.00	**0.08**	0.00	1.15	0.00	**0.03**	0.00	0.45	0.01	**0.78**	**0.02**	0.35	**0.32**	**0.82**	**0.59**
T	1.70	0.00	**0.02**	0.00	0.80	0.00	**0.12**	0.00	0.45	0.01	**0.78**	**0.02**	0.35	**0.32**	**0.82**	**0.59**
KO	0.90	0.00	**0.15**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00	0.20	**0.03**	**0.90**	**0.10**
MRK	3.60	0.00	0.00	0.00	1.90	**0.07**	0.01	0.00	0.90	**0.65**	**0.57**	**0.77**	0.80	**0.08**	**0.61**	**0.19**
VZ	1.85	0.00	**0.19**	0.00	0.75	0.00	**0.63**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.01	**0.92**	**0.03**
DIS	4.50	**0.30**	0.01	**0.02**	2.30	**0.56**	**0.02**	**0.06**	1.00	**1.00**	**0.52**	**0.82**	0.95	0.01	**0.55**	**0.03**
INTC	6.30	0.01	**0.98**	**0.04**	3.25	**0.04**	**0.23**	**0.06**	1.50	**0.04**	**0.08**	**0.02**	1.20	0.00	**0.45**	0.00
CSCO	5.75	**0.13**	**0.35**	**0.21**	3.35	**0.02**	**0.62**	**0.06**	1.75	0.00	**0.26**	0.01	1.70	0.00	**0.28**	0.00
HD	5.15	**0.76**	0.00	0.00	2.45	**0.89**	**0.15**	**0.36**	0.90	**0.65**	**0.15**	**0.32**	0.75	**0.14**	**0.63**	**0.30**
UTX	3.30	0.00	**0.59**	0.00	1.65	0.01	**0.29**	**0.02**	0.55	**0.03**	**0.73**	**0.08**	0.40	**0.51**	**0.80**	**0.78**
BA	6.60	0.00	**0.02**	0.00	3.35	**0.02**	**0.27**	**0.04**	1.40	**0.09**	**0.06**	**0.04**	1.00	0.01	**0.19**	0.01
MCD	1.15	0.00	**0.46**	0.00	0.60	0.00	**0.70**	0.00	0.25	0.00	**0.87**	0.00	0.25	**0.08**	**0.87**	**0.21**
AXP	11.10	0.00	0.00	0.00	6.80	0.00	0.00	0.00	3.95	0.00	0.00	0.00	3.05	0.00	0.01	0.00
MMM	2.30	0.00	**0.95**	0.00	1.30	0.00	**0.35**	0.00	0.85	**0.49**	**0.13**	**0.26**	0.70	**0.23**	**0.09**	**0.11**
GS	11.20	0.00	0.01	0.00	6.25	0.00	**0.03**	0.00	2.95	0.00	**0.04**	0.00	2.60	0.00	0.01	0.00
UNH	9.20	0.00	0.00	0.00	5.10	0.00	0.01	0.00	2.45	0.00	**0.15**	0.00	2.25	0.00	**0.37**	0.00
CAT	9.15	0.00	**0.74**	0.00	5.30	0.00	**0.08**	0.00	2.55	0.00	**0.19**	0.00	2.15	0.00	**0.08**	0.00
DD	4.90	**0.84**	**0.93**	**0.98**	2.40	**0.78**	**0.46**	**0.73**	0.80	**0.35**	**0.12**	**0.19**	0.60	**0.54**	**0.06**	**0.14**
NKE	4.15	**0.07**	**0.08**	**0.04**	2.25	**0.47**	**0.99**	**0.77**	1.40	**0.09**	**0.41**	**0.17**	1.20	0.00	**0.45**	0.00
TRV	5.75	**0.13**	0.00	0.00	2.95	**0.21**	0.01	0.01	1.40	**0.09**	0.00	0.00	1.25	0.00	0.00	0.00

Table 4.43: Interval forecast evaluation using Students't distribution, mean model = ARMA(2,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	3.00	0.00	0.01	0.00	1.45	0.00	0.01	0.00	0.50	0.01	**0.75**	**0.04**	0.45	**0.75**	**0.78**	**0.91**
MSFT	4.35	**0.17**	**0.53**	**0.33**	2.25	**0.47**	**0.37**	**0.52**	1.15	**0.51**	**0.46**	**0.62**	1.05	0.00	**0.50**	0.01
GE	6.20	**0.02**	0.01	0.00	3.50	0.01	0.00	0.00	1.85	0.00	**0.03**	0.00	1.65	0.00	**0.02**	0.00
JNJ	0.60	0.00	**0.06**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.01
WMT	1.75	0.00	0.00	0.00	0.80	0.00	**0.12**	0.00	0.35	0.00	**0.82**	0.00	0.10	0.00	**0.95**	0.01
CVX	3.45	0.00	**0.12**	0.00	1.30	0.00	**0.05**	0.00	0.65	**0.09**	0.00	0.00	0.50	**1.00**	0.00	0.00
JPM	11.80	0.00	0.00	0.00	7.25	0.00	0.00	0.00	4.35	0.00	0.00	0.00	3.90	0.00	0.00	0.00
PG	1.40	0.00	**0.06**	0.00	0.90	0.00	**0.15**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.01	**0.92**	**0.03**
PFE	2.50	0.00	**0.17**	0.00	1.20	0.00	**0.29**	0.00	0.50	0.01	**0.75**	**0.04**	0.40	**0.51**	**0.80**	**0.78**
IBM	2.15	0.00	**0.08**	0.00	1.15	0.00	**0.03**	0.00	0.45	0.01	**0.78**	**0.02**	0.35	**0.32**	**0.82**	**0.59**
T	1.70	0.00	**0.02**	0.00	0.85	0.00	**0.13**	0.00	0.45	0.01	**0.78**	**0.02**	0.35	**0.32**	**0.82**	**0.59**
KO	0.90	0.00	**0.15**	0.00	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00	0.20	**0.03**	**0.90**	**0.10**
MRK	3.60	0.00	0.00	0.00	1.90	**0.07**	0.01	0.00	0.90	**0.65**	**0.57**	**0.77**	0.80	**0.08**	**0.61**	**0.19**
VZ	1.80	0.00	**0.17**	0.00	0.75	0.00	**0.63**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.01	**0.92**	**0.03**
DIS	4.50	**0.30**	0.01	**0.02**	2.30	**0.56**	**0.02**	**0.06**	1.00	**1.00**	**0.52**	**0.82**	0.90	**0.02**	**0.57**	**0.06**
INTC	6.20	**0.02**	**0.91**	**0.06**	3.25	**0.04**	**0.23**	**0.06**	1.50	**0.04**	**0.08**	**0.02**	1.20	0.00	**0.45**	0.00
CSCO	5.80	**0.11**	**0.37**	**0.19**	3.35	**0.02**	**0.62**	**0.06**	1.75	0.00	**0.26**	0.01	1.65	0.00	**0.29**	0.00
HD	5.15	**0.76**	0.00	0.00	2.45	**0.89**	**0.15**	**0.36**	0.85	**0.49**	**0.59**	**0.68**	0.75	**0.14**	**0.63**	**0.30**
UTX	3.25	0.00	**0.55**	0.00	1.60	0.01	**0.31**	0.01	0.55	**0.03**	**0.73**	**0.08**	0.40	**0.51**	**0.80**	**0.78**
BA	6.60	0.00	0.01	0.00	3.25	**0.04**	**0.23**	**0.06**	1.40	**0.09**	**0.06**	**0.04**	1.00	0.01	**0.19**	0.01
MCD	1.15	0.00	**0.46**	0.00	0.60	0.00	**0.70**	0.00	0.25	0.00	**0.87**	0.00	0.25	**0.08**	**0.87**	**0.21**
AXP	11.15	0.00	0.00	0.00	6.80	0.00	0.00	0.00	3.95	0.00	0.00	0.00	3.10	0.00	0.00	0.00
MMM	2.30	0.00	**0.95**	0.00	1.30	0.00	**0.35**	0.00	0.75	**0.24**	**0.10**	**0.13**	0.65	**0.36**	**0.68**	**0.61**
GS	11.20	0.00	0.00	0.00	6.20	0.00	0.01	0.00	2.95	0.00	**0.04**	0.00	2.60	0.00	0.01	0.00
UNH	9.15	0.00	0.00	0.00	5.05	0.00	0.01	0.00	2.40	0.00	**0.14**	0.00	2.25	0.00	**0.37**	0.00
CAT	9.10	0.00	**0.91**	0.00	5.25	0.00	**0.07**	0.00	2.50	0.00	**0.17**	0.00	2.15	0.00	**0.08**	0.00
DD	4.90	**0.84**	**0.93**	**0.98**	2.40	**0.78**	**0.46**	**0.73**	0.85	**0.49**	**0.13**	**0.26**	0.55	**0.75**	**0.05**	**0.14**
NKE	4.10	**0.06**	**0.07**	**0.03**	2.25	**0.47**	**0.99**	**0.77**	1.40	**0.09**	**0.41**	**0.17**	1.20	0.00	**0.45**	0.00
TRV	5.75	**0.13**	0.00	0.00	2.95	**0.21**	0.01	0.01	1.35	**0.13**	0.00	0.00	1.20	0.00	0.00	0.00

4.7.10 Interval Forecast Evaluation of GO-GARCH with Skewed Student's t-distributed Errors, Mean Model = ARMA(0,0)

Table 4.44: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(0,0)

Stocks	h = 1															
	$100(\alpha) = 5$				$100(\alpha) = 2.5$				$100(\alpha) = 1$				$100(\alpha) = 0.5$			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MSFT	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00
GE	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
JNJ	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
WMT	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
CVX	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
JPM	0.50	0.00	**0.04**	0.00	0.25	0.00	**0.01**	0.00	0.15	0.00	0.00	0.00	0.00	0.00	**1.00**	0.00
PG	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
PFE	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
IBM	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
T	0.00	0.00	**0.98**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
KO	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MRK	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00
VZ	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
DIS	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
INTC	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
CSCO	0.50	0.00	**0.75**	0.00	0.45	0.00	**0.78**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.00	**0.92**	0.00
HD	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
UTX	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
BA	0.20	0.00	**0.90**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MCD	0.15	0.00	**0.92**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
AXP	0.40	0.00	**0.80**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00
MMM	0.30	0.00	**0.85**	0.00	0.30	0.00	**0.85**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00
GS	0.55	0.00	**0.73**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00	0.00	0.00	**1.00**	0.00
UNH	0.60	0.00	**0.70**	0.00	0.25	0.00	**0.87**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.945**	0.00
CAT	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
DD	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
NKE	0.65	0.00	**0.68**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00
TRV	0.40	0.00	**0.02**	0.00	0.20	0.00	0.00	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00

Table 4.45: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(0,0)

	h = 2															
Stocks	**100(α) = 5**				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MSFT	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00
GE	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
JNJ	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
WMT	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
CVX	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
JPM	0.55	0.00	0.00	0.00	0.30	0.00	0.01	0.00	0.10	0.00	0.00	0.00	0.00	0.00	**1.00**	0.00
PG	0.30	0.00	0.01	0.00	0.20	0.00	**0.90**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
PFE	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
IBM	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
T	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
KO	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MRK	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00
VZ	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
DIS	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
INTC	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
CSCO	0.50	0.00	**0.75**	0.00	0.45	0.00	**0.78**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.00	**0.92**	0.00
HD	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
UTX	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
BA	0.20	0.00	**0.90**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MCD	0.15	0.00	**0.92**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
AXP	0.50	0.00	**0.75**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00
MMM	0.30	0.00	**0.85**	0.00	0.30	0.00	**0.85**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00
GS	0.55	0.00	**0.73**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00	0.00	0.00	**1.00**	0.00
UNH	0.60	0.00	**0.70**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00
CAT	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
DD	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
NKE	0.65	0.00	**0.68**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00
TRV	0.40	0.00	**0.02**	0.00	0.25	0.00	0.01	0.00	0.15	0.00	0.00	0.00	0.05	0.00	**0.97**	0.00

Table 4.46: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(0,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MSFT	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00
GE	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
JNJ	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
WMT	0.20	0.00	**0.90**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
CVX	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
JPM	0.55	0.00	0.00	0.00	0.30	0.00	0.01	0.00	0.10	0.00	**0.00**	0.00	0.00	0.00	**1.00**	0.00
PG	0.30	0.00	0.01	0.00	0.20	0.00	**0.90**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
PFE	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
IBM	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
T	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
KO	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MRK	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00
VZ	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
DIS	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
INTC	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
CSCO	0.50	0.00	**0.75**	0.00	0.45	0.00	**0.78**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.00	**0.92**	0.00
HD	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
UTX	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
BA	0.20	0.00	**0.90**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MCD	0.15	0.00	**0.92**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
AXP	0.55	0.00	**0.73**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00
MMM	0.30	0.00	**0.85**	0.00	0.30	0.00	**0.85**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00
GS	0.55	0.00	**0.73**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00
UNH	0.60	0.00	**0.70**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00
CAT	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
DD	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
NKE	0.65	0.00	**0.68**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00
TRV	0.40	0.00	**0.02**	0.00	0.30	0.00	0.01	0.00	0.15	0.00	0.00	0.00	0.05	0.00	**0.97**	0.00

Table 4.47: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(0,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MSFT	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00
GE	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
JNJ	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
WMT	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
CVX	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
JPM	0.55	0.00	0.00	0.00	0.30	0.00	0.01	0.00	0.15	0.00	0.00	0.00	0.00	0.00	**1.00**	0.00
PG	0.30	0.00	0.01	0.00	0.20	0.00	**0.90**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
PFE	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
IBM	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
T	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
KO	0.20	0.00	**0.90**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MRK	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00
VZ	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
DIS	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
INTC	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
CSCO	0.50	0.00	**0.75**	0.00	0.40	0.00	**0.80**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.00	**0.92**	0.00
HD	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
UTX	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
BA	0.20	0.00	**0.90**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MCD	0.15	0.00	**0.92**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
AXP	0.60	0.00	**0.70**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00
MMM	0.30	0.00	**0.85**	0.00	0.30	0.00	**0.85**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00
GS	0.55	0.00	**0.73**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00
UNH	0.55	0.00	**0.73**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00
CAT	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
DD	0.15	0.00	**0.92**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
NKE	0.65	0.00	**0.68**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00
TRV	0.40	0.00	**0.02**	0.00	0.30	0.00	0.01	0.00	0.15	0.00	0.00	0.00	0.05	0.00	**0.97**	0.00

Table 4.48: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(0,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MSFT	0.30	0.00	**0.85**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00
GE	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
JNJ	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
WMT	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
CVX	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
JPM	0.55	0.00	0.00	0.00	0.30	0.00	0.01	0.00	0.15	0.00	**0.00**	0.00	0.00	0.00	**1.00**	0.00
PG	0.30	0.00	0.01	0.00	0.20	0.00	**0.90**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
PFE	0.10	0.00	**0.95**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
IBM	0.35	0.00	**0.82**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
T	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
KO	0.20	0.00	**0.90**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MRK	0.35	0.00	**0.82**	0.00	0.35	0.00	**0.82**	0.00	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00
VZ	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
DIS	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
INTC	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
CSCO	0.50	0.00	**0.75**	0.00	0.40	0.00	**0.80**	0.00	0.20	0.00	**0.90**	0.00	0.15	0.00	**0.92**	0.00
HD	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
UTX	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
BA	0.20	0.00	**0.90**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
MCD	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
AXP	0.60	0.00	**0.70**	0.00	0.15	0.00	**0.92**	0.00	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00
MMM	0.30	0.00	**0.85**	0.00	0.30	0.00	**0.85**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.00
GS	0.55	0.00	**0.73**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00
UNH	0.55	0.00	**0.73**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00
CAT	0.15	0.00	**0.92**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
DD	0.15	0.00	**0.92**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
NKE	0.70	0.00	**0.66**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00
TRV	0.40	0.00	**0.02**	0.00	0.30	0.00	0.01	0.00	0.15	0.00	0.00	0.00	0.05	0.00	**0.97**	0.00

4.7.11 Interval Forecast Evaluation of GO-GARCH with Skewed Students't Distributed Errors, Mean Model = ARMA(2,0)

Table 4.49: Interval forecast evaluation using skewed Students't distribution, mean model = ARMA(2,0)

Stocks	h = 1															
	$100(\alpha) = 5$				$100(\alpha) = 2.5$				$100(\alpha) = 1$				$100(\alpha) = 0.5$			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	1.45	0.00	0.01	0.00	0.70	0.00	**0.09**	0.00	0.40	0.00	**0.80**	0.01	0.05	0.00	**0.97**	0.00
MSFT	2.60	0.00	**0.74**	0.00	1.60	0.01	**0.31**	0.01	0.85	**0.49**	**0.59**	**0.68**	0.25	**0.08**	**0.87**	**0.21**
GE	4.10	0.06	0.00	0.00	1.90	**0.07**	**0.04**	**0.02**	1.10	**0.66**	**0.24**	**0.46**	0.30	**0.17**	**0.85**	**0.38**
JNJ	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
WMT	1.05	0.00	0.00	0.00	0.30	0.00	0.01	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
CVX	1.65	0.00	**0.02**	0.00	0.85	0.00	0.01	0.00	0.45	0.01	**0.78**	**0.02**	0.10	0.00	**0.95**	0.01
JPM	8.15	0.00	0.00	0.00	4.90	0.00	0.00	0.00	2.70	0.00	0.00	0.00	1.55	0.00	0.00	0.00
PG	0.80	0.00	**0.61**	0.00	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
PFE	1.30	0.00	**0.35**	0.00	0.50	0.00	**0.75**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
IBM	1.40	0.00	**0.41**	0.00	0.60	0.00	**0.70**	0.00	0.20	0.00	**0.90**	0.00	0.10	0.00	**0.95**	0.01
T	0.90	0.00	**0.57**	0.00	0.40	0.00	**0.80**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**0.90**	0.00
KO	0.45	0.00	**0.78**	0.00	0.30	0.00	**0.85**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
MRK	2.10	0.00	0.00	0.00	1.20	0.00	**0.45**	0.00	0.50	0.01	**0.75**	**0.04**	0.35	**0.32**	**0.82**	**0.59**
VZ	0.80	0.00	**0.12**	0.00	0.40	0.00	**0.02**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**0.05**	0.00
DIS	2.25	0.00	**0.99**	0.00	1.15	0.00	**0.46**	0.00	0.65	**0.09**	**0.68**	**0.22**	0.25	**0.08**	**0.87**	**0.21**
INTC	3.20	0.00	**0.52**	0.00	1.65	0.01	**0.29**	**0.02**	0.65	**0.09**	**0.68**	**0.22**	0.45	**0.75**	**0.78**	**0.91**
CSCO	3.95	**0.03**	**0.62**	**0.07**	2.10	**0.24**	**0.29**	**0.29**	1.30	**0.20**	**0.41**	**0.31**	0.75	**0.14**	**0.63**	**0.30**
HD	2.60	0.00	**0.06**	0.00	1.20	0.00	**0.29**	0.00	0.65	**0.09**	**0.07**	**0.05**	0.10	0.00	**0.95**	0.01
UTX	1.70	0.00	**0.61**	0.00	0.80	0.00	**0.61**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
BA	4.10	**0.06**	0.01	0.00	1.75	**0.02**	0.00	0.00	0.80	**0.35**	**0.61**	**0.57**	0.30	**0.17**	**0.85**	**0.38**
MCD	0.75	0.00	**0.63**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00	0.00	0.00	**0.91**	0.00
AXP	6.85	0.00	0.00	0.00	4.20	0.00	0.00	0.00	2.10	0.00	**0.07**	0.00	0.85	**0.04**	**0.13**	**0.04**
MMM	1.15	0.00	**0.46**	0.00	0.80	0.00	**0.61**	0.00	0.50	0.01	**0.75**	**0.04**	0.25	**0.08**	**0.87**	**0.21**
GS	7.25	0.00	0.00	0.00	3.75	0.00	**0.22**	0.00	2.20	0.00	**0.35**	0.00	1.20	0.00	**0.29**	0.00
UNH	5.50	**0.31**	**0.05**	**0.09**	3.45	0.01	0.01	0.00	1.65	0.01	**0.57**	**0.02**	1.05	0.00	**0.22**	0.00
CAT	6.25	0.01	**0.25**	**0.02**	3.40	0.01	**0.03**	0.01	1.75	0.00	**0.64**	0.01	0.35	**0.32**	**0.82**	**0.59**
DD	2.60	0.00	**0.74**	0.00	1.30	0.00	**0.41**	0.00	0.45	0.01	**0.78**	**0.02**	0.15	0.01	**0.92**	**0.03**
NKE	2.40	0.00	**0.88**	0.00	1.55	0.00	**0.51**	0.01	1.05	**0.82**	**0.22**	**0.46**	0.65	**0.36**	**0.68**	**0.61**
TRV	2.90	0.00	0.01	0.00	1.60	0.01	0.00	0.00	0.70	**0.15**	0.00	0.00	0.50	**1.00**	0.00	0.00

Table 4.50: Interval forecast evaluation using skewed Students't distribution, mean model = ARMA(2,0)

Stocks	**h = 2**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	1.60	0.00	0.01	0.00	0.65	0.00	**0.68**	0.00	0.45	0.01	**0.78**	**0.02**	0.10	0.00	**0.95**	0.01
MSFT	2.60	0.00	**0.74**	0.00	1.60	0.01	**0.31**	0.01	0.85	**0.49**	**0.59**	**0.68**	0.35	**0.32**	**0.82**	**0.59**
GE	4.00	**0.03**	0.00	0.00	1.85	0.05	**0.19**	**0.06**	1.10	**0.66**	**0.24**	**0.46**	0.25	**0.08**	**0.87**	**0.21**
JNJ	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
WMT	1.00	0.00	0.00	0.00	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
CVX	1.75	0.00	0.00	0.00	0.80	0.00	**0.61**	0.00	0.40	0.00	**0.80**	0.01	0.10	0.00	**0.95**	0.01
JPM	8.15	0.00	0.00	0.00	4.85	0.00	0.00	0.00	2.70	0.00	0.00	0.00	1.55	0.00	0.00	0.00
PG	0.90	0.00	**0.15**	0.00	0.25	0.00	**0.87**	0.00	0.15	0.00	**0.92**	0.00	0.00	0.00	**1.00**	0.00
PFE	1.30	0.00	**0.35**	0.00	0.60	0.00	**0.70**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
IBM	1.40	0.00	**0.41**	0.00	0.60	0.00	**0.70**	0.00	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00
T	0.85	0.00	**0.59**	0.00	0.45	0.00	**0.78**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
KO	0.45	0.00	**0.78**	0.00	0.30	0.00	**0.85**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
MRK	1.95	0.00	0.01	0.00	1.20	0.00	**0.45**	0.00	0.50	0.01	**0.75**	**0.04**	0.35	**0.32**	**0.82**	**0.59**
VZ	0.85	0.00	**0.13**	0.00	0.45	0.00	**0.03**	0.00	0.15	0.00	**0.92**	0.00	0.00	0.00	**0.98**	0.00
DIS	2.40	0.00	**0.88**	0.00	1.05	0.00	**0.50**	0.00	0.60	**0.05**	**0.70**	**0.14**	0.20	**0.03**	**0.90**	**0.10**
INTC	3.40	0.00	**0.65**	0.00	1.70	**0.02**	**0.28**	**0.03**	0.65	**0.09**	**0.68**	**0.22**	0.45	**0.75**	**0.78**	**0.91**
CSCO	3.85	0.01	**0.55**	**0.04**	2.15	**0.31**	**0.94**	**0.59**	1.35	**0.13**	**0.39**	**0.23**	0.80	**0.08**	**0.61**	**0.19**
HD	2.55	0.00	**0.19**	0.00	1.15	0.00	**0.27**	0.00	0.65	**0.09**	**0.07**	**0.05**	0.10	0.00	**0.95**	0.01
UTX	1.65	0.00	**0.29**	0.00	0.80	0.00	**0.61**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
BA	3.85	0.01	**0.26**	**0.03**	1.65	0.01	**0.02**	0.00	0.80	**0.35**	**0.61**	**0.57**	0.30	**0.17**	**0.85**	**0.38**
MCD	0.75	0.00	**0.63**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00	0.00	0.00	**0.91**	0.00
AXP	6.80	0.00	0.00	0.00	4.25	0.00	0.00	0.00	2.20	0.00	**0.02**	0.00	0.75	**0.14**	**0.63**	**0.30**
MMM	1.20	0.00	**0.45**	0.00	0.80	0.00	**0.61**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.25	**0.08**	**0.87**	**0.21**
GS	7.00	0.00	0.00	0.00	3.80	0.00	**0.09**	0.00	2.15	0.00	**0.32**	0.00	1.20	0.00	**0.29**	0.00
UNH	5.60	**0.22**	0.01	**0.02**	3.40	0.01	0.00	0.00	1.60	0.01	**0.11**	0.01	1.00	0.01	**0.19**	0.01
CAT	6.15	**0.02**	**0.21**	**0.03**	3.30	**0.03**	**0.09**	**0.02**	1.60	0.01	**0.54**	**0.04**	0.30	**0.17**	**0.85**	**0.38**
DD	2.65	0.00	**0.63**	0.00	1.30	0.00	**0.41**	0.00	0.45	0.01	**0.78**	**0.02**	0.15	0.01	**0.92**	**0.03**
NKE	2.55	0.00	**0.56**	0.00	1.60	0.01	**0.54**	**0.02**	1.05	**0.82**	**0.22**	**0.46**	0.65	**0.36**	**0.68**	**0.61**
TRV	3.00	0.00	0.01	0.00	1.50	0.00	0.01	0.00	0.75	**0.24**	0.00	0.01	0.50	**1.00**	0.00	0.00

Table 4.51: Interval forecast evaluation using skewed Students't distribution, mean model = ARMA(2,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	1.55	0.00	0.01	0.00	0.65	0.00	**0.68**	0.00	0.45	0.01	**0.78**	**0.02**	0.10	0.00	**0.95**	0.01
MSFT	2.60	0.00	**0.74**	0.00	1.60	0.01	**0.31**	0.01	0.85	**0.49**	**0.59**	**0.68**	0.35	**0.32**	**0.82**	**0.59**
GE	4.05	**0.04**	0.01	0.00	1.90	**0.07**	**0.04**	**0.02**	1.10	**0.66**	**0.24**	**0.46**	0.25	**0.08**	**0.87**	**0.21**
JNJ	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
WMT	1.00	0.00	0.00	0.00	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
CVX	1.75	0.00	0.00	0.00	0.80	0.00	**0.61**	0.00	0.40	0.00	**0.80**	0.01	0.10	0.00	**0.95**	0.01
JPM	8.10	0.00	0.00	0.00	4.85	0.00	0.00	0.00	2.65	0.00	0.00	0.00	1.60	0.00	0.00	0.00
PG	0.90	0.00	**0.15**	0.00	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
PFE	1.30	0.00	**0.35**	0.00	0.55	0.00	**0.73**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
IBM	1.40	0.00	**0.41**	0.00	0.60	0.00	**0.70**	0.00	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00
T	0.85	0.00	**0.59**	0.00	0.45	0.00	**0.78**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
KO	0.45	0.00	**0.78**	0.00	0.30	0.00	**0.85**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
MRK	1.95	0.00	0.01	0.00	1.20	0.00	**0.45**	0.00	0.50	0.01	**0.75**	**0.04**	0.35	**0.32**	**0.82**	**0.59**
VZ	0.85	0.00	**0.13**	0.00	0.45	0.00	**0.03**	0.00	0.15	0.00	**0.92**	0.00	0.00	0.00	**0.97**	0.00
DIS	2.30	0.00	**0.95**	0.00	1.05	0.00	**0.50**	0.00	0.70	**0.15**	**0.66**	**0.33**	0.20	**0.03**	**0.90**	**0.10**
INTC	3.30	0.00	**0.59**	0.00	1.60	0.01	**0.31**	0.01	0.65	**0.09**	**0.68**	**0.22**	0.45	**0.75**	**0.78**	**0.91**
CSCO	3.85	0.01	**0.55**	**0.04**	2.05	**0.18**	**0.86**	**0.41**	1.25	**0.28**	**0.43**	**0.40**	0.80	**0.08**	**0.61**	**0.19**
HD	2.60	0.00	**0.06**	0.00	1.20	0.00	**0.29**	0.00	0.70	**0.15**	**0.09**	**0.08**	0.10	0.00	**0.95**	0.01
UTX	1.65	0.00	**0.29**	0.00	0.85	0.00	**0.59**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
BA	3.85	0.01	**0.26**	0.03	1.60	0.01	0.01	0.00	0.80	**0.35**	**0.61**	**0.57**	0.30	**0.17**	**0.85**	**0.38**
MCD	0.75	0.00	**0.63**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00	0.00	0.00	**0.92**	0.00
AXP	6.80	0.00	0.00	0.00	4.30	0.00	0.00	0.00	2.30	0.00	**0.02**	0.00	0.80	**0.08**	**0.61**	**0.19**
MMM	1.20	0.00	**0.45**	0.00	0.80	0.00	**0.61**	0.00	0.50	0.01	**0.75**	**0.04**	0.25	**0.08**	**0.87**	**0.21**
GS	6.90	0.00	0.00	0.00	3.70	0.00	**0.07**	0.00	2.15	0.00	**0.32**	0.00	1.20	0.00	**0.29**	0.00
UNH	5.40	**0.41**	**0.02**	**0.04**	3.40	0.01	0.00	0.00	1.65	0.01	**0.12**	0.01	1.00	0.01	**0.19**	0.01
CAT	6.00	**0.05**	**0.16**	**0.05**	3.25	**0.04**	**0.08**	**0.03**	1.60	0.01	**0.54**	**0.04**	0.30	**0.17**	**0.85**	**0.38**
DD	2.65	0.00	**0.63**	0.00	1.30	0.00	**0.41**	0.00	0.50	0.01	**0.75**	**0.04**	0.20	**0.03**	**0.90**	**0.10**
NKE	2.55	0.00	**0.56**	0.00	1.50	0.00	**0.47**	0.01	1.05	**0.82**	**0.22**	**0.46**	0.65	**0.36**	**0.68**	**0.61**
TRV	3.00	0.00	0.01	0.00	1.50	0.00	0.01	0.00	0.75	**0.24**	0.00	0.01	0.50	**1.00**	0.00	0.00

Table 4.52: Interval forecast evaluation using skewed Students't distribution, mean model = ARMA(2,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	1.55	0.00	0.01	0.00	0.65	0.00	**0.68**	0.00	0.45	0.01	**0.78**	**0.02**	0.10	0.00	**0.95**	0.01
MSFT	2.55	0.00	**0.10**	0.00	1.55	0.00	**0.32**	0.01	0.85	**0.49**	**0.59**	**0.68**	0.35	**0.32**	**0.82**	**0.59**
GE	3.95	**0.03**	0.00	0.00	1.95	**0.10**	0.01	0.01	1.15	**0.51**	**0.27**	**0.43**	0.25	**0.08**	**0.87**	**0.21**
JNJ	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**	0.00
WMT	1.00	0.00	0.00	0.00	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
CVX	1.75	0.00	0.00	0.00	0.80	0.00	**0.61**	0.00	0.40	0.00	**0.80**	0.01	0.10	0.00	**0.95**	0.01
JPM	8.10	0.00	0.00	0.00	4.90	0.00	0.00	0.00	2.65	0.00	0.00	0.00	1.60	0.00	0.00	0.00
PG	0.85	0.00	**0.13**	0.00	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
PFE	1.30	0.00	**0.35**	0.00	0.55	0.00	**0.73**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
IBM	1.40	0.00	**0.41**	0.00	0.55	0.00	**0.73**	0.00	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00
T	0.85	0.00	**0.59**	0.00	0.45	0.00	**0.78**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
KO	0.45	0.00	**0.78**	0.00	0.30	0.00	**0.85**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
MRK	2.05	0.00	0.01	0.00	1.20	0.00	**0.45**	0.00	0.50	0.01	**0.75**	**0.04**	0.35	**0.32**	**0.82**	**0.59**
VZ	0.85	0.00	**0.13**	0.00	0.45	0.00	**0.03**	0.00	0.15	0.00	**0.92**	0.00	0.00	0.00	**0.97**	0.00
DIS	2.30	0.00	**0.95**	0.00	1.10	0.00	**0.48**	0.00	0.70	**0.15**	**0.66**	**0.33**	0.20	**0.03**	**0.90**	**0.10**
INTC	3.25	0.00	**0.55**	0.00	1.60	0.01	**0.31**	0.01	0.65	**0.09**	**0.68**	**0.22**	0.45	**0.75**	**0.78**	**0.91**
CSCO	3.85	0.01	**0.55**	**0.04**	2.05	**0.18**	**0.86**	**0.41**	1.25	**0.28**	**0.43**	**0.40**	0.80	**0.08**	**0.61**	**0.19**
HD	2.60	0.00	**0.06**	0.00	1.30	0.00	**0.35**	0.00	0.70	**0.15**	**0.09**	**0.08**	0.10	0.00	**0.95**	0.01
UTX	1.60	0.00	**0.31**	0.00	0.80	0.00	**0.61**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
BA	3.85	0.01	**0.26**	**0.03**	1.65	0.01	**0.02**	0.00	0.80	**0.35**	**0.61**	**0.57**	0.30	**0.17**	**0.85**	**0.38**
MCD	0.75	0.00	**0.63**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00	0.00	0.00	**0.93**	0.00
AXP	6.80	0.00	0.00	0.00	4.30	0.00	0.00	0.00	2.35	0.00	**0.03**	0.00	0.80	**0.08**	**0.61**	**0.19**
MMM	1.25	0.00	**0.43**	0.00	0.80	0.00	**0.61**	0.00	0.50	0.01	**0.75**	**0.04**	0.25	**0.08**	**0.87**	**0.21**
GS	6.90	0.00	0.00	0.00	3.65	0.00	**0.07**	0.00	2.20	0.00	**0.35**	0.00	1.20	0.00	**0.29**	0.00
UNH	5.35	**0.47**	0.01	**0.04**	3.35	**0.02**	0.00	0.00	1.60	0.01	**0.11**	0.01	1.00	0.01	**0.19**	0.01
CAT	5.95	**0.06**	**0.14**	**0.06**	3.20	**0.05**	**0.07**	**0.03**	1.60	0.01	**0.54**	**0.04**	0.30	**0.17**	**0.85**	**0.38**
DD	2.70	0.00	**0.66**	0.00	1.30	0.00	**0.41**	0.00	0.50	0.01	**0.75**	**0.04**	0.20	**0.03**	**0.90**	**0.10**
NKE	2.55	0.00	**0.56**	0.00	1.50	0.00	**0.47**	0.01	1.05	**0.82**	**0.22**	**0.46**	0.65	**0.36**	**0.68**	**0.61**
TRV	3.00	0.00	0.01	0.00	1.50	0.00	0.01	0.00	0.75	**0.24**	0.00	0.01	0.50	**1.00**	0.00	0.00

Table 4.53: Interval forecast evaluation using skewed Students't distribution, mean model = ARMA(2,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	1.55	0.00	0.01	0.00	0.65	0.00	**0.68**	0.00	0.45	0.01	**0.78**	**0.02**	0.15	0.01	**0.92**	**0.03**
MSFT	2.55	0.00	**0.10**	0.00	1.55	0.00	**0.32**	0.01	0.85	**0.49**	**0.59**	**0.68**	0.35	**0.32**	**0.82**	**0.59**
GE	3.85	0.01	0.00	0.00	1.95	**0.10**	0.01	0.01	1.15	**0.51**	**0.27**	**0.43**	0.25	**0.08**	**0.87**	**0.21**
JNJ	0.15	0.00	**0.92**	0.00	0.10	0.00	**0.95**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
WMT	0.90	0.00	0.00	0.00	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00	0.00	0.00	**1.00**	0.00
CVX	1.75	0.00	0.00	0.00	0.80	0.00	**0.61**	0.00	0.40	0.00	**0.80**	0.01	0.10	0.00	**0.95**	0.01
JPM	8.10	0.00	0.00	0.00	5.00	0.00	0.00	0.00	2.65	0.00	0.00	0.00	1.65	0.00	0.00	0.00
PG	0.85	0.00	**0.13**	0.00	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
PFE	1.30	0.00	**0.35**	0.00	0.55	0.00	**0.73**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
IBM	1.40	0.00	**0.41**	0.00	0.50	0.00	**0.75**	0.00	0.25	0.00	**0.87**	0.00	0.05	0.00	**0.97**	0.00
T	0.85	0.00	**0.59**	0.00	0.45	0.00	**0.78**	0.00	0.10	0.00	**0.95**	0.00	0.00	0.00	**1.00**	0.00
KO	0.45	0.00	**0.78**	0.00	0.30	0.00	**0.85**	0.00	0.05	0.00	**0.97**	0.00	0.05	0.00	**0.97**	0.00
MRK	2.00	0.00	0.01	0.00	1.20	0.00	**0.45**	0.00	0.50	0.01	**0.75**	**0.04**	0.35	**0.32**	**0.82**	**0.59**
VZ	0.80	0.00	**0.12**	0.00	0.45	0.00	**0.03**	0.00	0.15	0.00	**0.92**	0.00	0.00	0.00	**0.98**	0.00
DIS	2.25	0.00	**0.99**	0.00	1.10	0.00	**0.48**	0.00	0.70	**0.15**	**0.66**	**0.33**	0.20	**0.03**	**0.90**	**0.10**
INTC	3.30	0.00	**0.59**	0.00	1.60	0.01	**0.31**	0.01	0.65	**0.09**	**0.68**	**0.22**	0.45	**0.75**	**0.78**	**0.91**
CSCO	3.80	0.01	**0.52**	0.03	2.05	**0.18**	**0.86**	**0.41**	1.25	**0.28**	**0.43**	**0.40**	0.80	**0.08**	**0.61**	**0.19**
HD	2.65	0.00	**0.06**	0.00	1.25	0.00	**0.32**	0.00	0.60	**0.05**	**0.70**	**0.14**	0.10	0.00	**0.95**	0.01
UTX	1.60	0.00	**0.31**	0.00	0.85	0.00	**0.59**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
BA	3.85	0.01	**0.26**	**0.03**	1.65	0.01	**0.02**	0.00	0.80	**0.35**	**0.61**	**0.57**	0.30	**0.17**	**0.85**	**0.38**
MCD	0.75	0.00	**0.63**	0.00	0.35	0.00	**0.82**	0.00	0.25	0.00	**0.87**	0.00	0.00	0.00	**0.95**	0.00
AXP	6.75	0.00	0.00	0.00	4.20	0.00	0.00	0.00	2.40	0.00	**0.03**	0.00	0.80	**0.08**	**0.61**	**0.19**
MMM	1.25	0.00	**0.43**	0.00	0.80	0.00	**0.61**	0.00	0.50	0.01	**0.75**	**0.04**	0.25	**0.08**	**0.87**	**0.21**
GS	6.80	0.00	0.00	0.00	3.65	0.00	**0.07**	0.00	2.20	0.00	**0.35**	0.00	1.20	0.00	**0.29**	0.00
UNH	5.35	**0.47**	0.01	**0.04**	3.35	**0.02**	0.00	0.00	1.60	0.01	**0.11**	0.01	1.00	0.01	**0.19**	0.01
CAT	5.85	**0.09**	**0.23**	**0.11**	3.20	**0.05**	**0.07**	**0.03**	1.55	**0.02**	**0.51**	**0.06**	0.30	**0.17**	**0.85**	**0.38**
DD	2.70	0.00	**0.66**	0.00	1.30	0.00	**0.41**	0.00	0.50	0.01	**0.75**	**0.04**	0.20	**0.03**	**0.90**	**0.10**
NKE	2.50	0.00	**0.53**	0.00	1.50	0.00	**0.47**	0.01	1.00	**1.00**	**0.19**	**0.43**	0.65	**0.36**	**0.68**	**0.61**
TRV	3.05	0.00	0.01	0.00	1.50	0.00	0.01	0.00	0.75	**0.24**	0.00	0.01	0.50	**1.00**	0.00	0.00

4.7.12 Interval Forecast Evaluation of SVX with Gaussian Distributed Errors, Mean Model = ARMA(0,0)

Table 4.54: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	h = 1															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	11.75	0.00	0.00	0.00	8.85	0.00	0.00	0.00	6.00	0.00	0.00	0.00	3.50	0.00	0.00	0.00
MSFT	13.90	0.00	0.00	0.00	11.00	0.00	0.00	0.00	9.25	0.00	0.00	0.00	6.05	0.00	0.00	0.00
GE	9.05	0.00	0.00	0.00	7.55	0.00	0.00	0.00	6.40	0.00	0.00	0.00	4.45	0.00	0.00	0.00
JNJ	9.40	0.00	0.00	0.00	7.30	0.00	0.00	0.00	5.45	0.00	0.00	0.00	3.55	0.00	0.00	0.00
WMT	6.90	0.00	0.00	0.00	5.05	0.00	0.00	0.00	3.35	0.00	0.00	0.00	2.25	0.00	0.00	0.00
CVX	7.45	0.00	0.00	0.00	5.10	0.00	0.00	0.00	3.55	0.00	0.00	0.00	1.95	0.00	0.00	0.00
JPM	8.30	0.00	0.00	0.00	5.00	0.00	0.00	0.00	3.20	0.00	0.00	0.00	2.30	0.00	0.00	0.00
PG	9.95	0.00	0.00	0.00	8.10	0.00	0.00	0.00	6.10	0.00	0.00	0.00	4.10	0.00	0.00	0.00
PFE	8.10	0.00	0.00	0.00	6.05	0.00	0.00	0.00	4.70	0.00	0.00	0.00	3.20	0.00	0.00	0.00
IBM	11.35	0.00	0.00	0.00	8.80	0.00	0.00	0.00	7.35	0.00	0.00	0.00	4.95	0.00	0.00	0.00
T	6.75	0.00	0.00	0.00	5.25	0.00	0.00	0.00	3.60	0.00	0.00	0.00	2.15	0.00	0.00	0.00
KO	6.85	0.00	0.00	0.00	5.10	0.00	0.00	0.00	4.05	0.00	0.00	0.00	2.50	0.00	0.00	0.00
MRK	16.20	0.00	0.00	0.00	13.20	0.00	0.00	0.00	9.95	0.00	0.00	0.00	6.80	0.00	0.00	0.00
VZ	5.90	**0.07**	0.00	0.00	4.15	0.00	0.00	0.00	2.95	0.00	0.00	0.00	2.05	0.00	0.00	0.00
DIS	11.05	0.00	0.00	0.00	9.05	0.00	0.00	0.00	7.55	0.00	0.00	0.00	4.90	0.00	0.00	0.00
INTC	10.80	0.00	0.00	0.00	8.45	0.00	0.00	0.00	6.45	0.00	0.00	0.00	4.40	0.00	0.00	0.00
CSCO	10.65	0.00	0.00	0.00	8.65	0.00	0.00	0.00	6.80	0.00	0.00	0.00	4.70	0.00	0.00	0.00
HD	15.25	0.00	0.00	0.00	12.65	0.00	0.00	0.00	9.85	0.00	0.00	0.00	7.80	0.00	0.00	0.00
UTX	4.75	**0.61**	0.00	0.00	3.30	**0.03**	0.00	0.00	2.40	0.00	0.00	0.00	1.50	0.00	0.00	0.00
BA	8.95	0.00	0.00	0.00	6.55	0.00	0.00	0.00	4.85	0.00	0.00	0.00	3.25	0.00	0.00	0.00
MCD	5.65	**0.19**	0.00	0.00	4.10	0.00	0.00	0.00	2.40	0.00	0.00	0.00	1.75	0.00	0.00	0.00
AXP	8.35	0.00	0.00	0.00	5.00	0.00	0.00	0.00	3.20	0.00	0.00	0.00	2.55	0.00	0.00	0.00
MMM	6.65	0.00	0.00	0.00	4.75	0.00	0.00	0.00	3.05	0.00	0.00	0.00	2.10	0.00	0.00	0.00
GS	6.85	0.00	0.00	0.00	4.90	0.00	0.00	0.00	2.40	0.00	0.00	0.00	1.35	0.00	0.00	0.00
UNH	16.45	0.00	0.00	0.00	12.80	0.00	0.00	0.00	10.05	0.00	0.00	0.00	6.75	0.00	0.00	0.00
CAT	5.55	**0.26**	0.00	0.00	4.20	0.00	0.00	0.00	2.70	0.00	**0.07**	0.00	1.45	0.00	0.01	0.00
DD	7.00	0.00	0.00	0.00	5.60	0.00	0.00	0.00	4.25	0.00	0.00	0.00	2.80	0.00	0.00	0.00
NKE	16.10	0.00	0.00	0.00	12.95	0.00	0.00	0.00	10.60	0.00	0.00	0.00	7.55	0.00	0.00	0.00
TRV	8.05	0.00	0.00	0.00	6.20	0.00	0.00	0.00	4.65	0.00	0.00	0.00	2.80	0.00	0.00	0.00

Table 4.55: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	**h = 2**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	11.16	0.00	0.00	0.00	8.25	0.00	0.00	0.00	6.15	0.00	0.00	0.00	3.60	0.00	0.00	0.00
MSFT	13.86	0.00	0.00	0.00	11.11	0.00	0.00	0.00	8.90	0.00	0.00	0.00	6.15	0.00	0.00	0.00
GE	9.00	0.00	0.00	0.00	7.45	0.00	0.00	0.00	6.10	0.00	0.00	0.00	4.25	0.00	0.00	0.00
JNJ	9.45	0.00	0.00	0.00	7.25	0.00	0.00	0.00	5.35	0.00	0.00	0.00	3.70	0.00	0.00	0.00
WMT	6.86	0.00	0.00	0.00	5.01	0.00	0.00	0.00	3.35	0.00	0.00	0.00	2.30	0.00	0.00	0.00
CVX	7.60	0.00	0.00	0.00	5.20	0.00	0.00	0.00	3.30	0.00	0.00	0.00	1.80	0.00	0.00	0.00
JPM	8.23	0.00	0.00	0.00	5.11	0.00	0.00	0.00	3.26	0.00	0.00	0.00	2.15	0.00	0.00	0.00
PG	10.16	0.00	0.00	0.00	8.50	0.00	0.00	0.00	6.05	0.00	0.00	0.00	4.25	0.00	0.00	0.00
PFE	8.75	0.00	0.00	0.00	6.30	0.00	0.00	0.00	4.60	0.00	0.00	0.00	3.15	0.00	0.00	0.00
IBM	11.31	0.00	0.00	0.00	9.00	0.00	0.00	0.00	7.55	0.00	0.00	0.00	4.95	0.00	0.00	0.00
T	6.85	0.00	0.00	0.00	5.35	0.00	0.00	0.00	3.65	0.00	0.00	0.00	2.25	0.00	0.00	0.00
KO	6.90	0.00	0.00	0.00	5.30	0.00	0.00	0.00	4.00	0.00	0.00	0.00	2.35	0.00	0.00	0.00
MRK	16.01	0.00	0.00	0.00	13.11	0.00	0.00	0.00	9.90	0.00	0.00	0.00	6.80	0.00	0.00	0.00
VZ	5.65	**0.19**	0.00	0.00	4.15	0.00	0.00	0.00	3.00	0.00	0.00	0.00	1.90	0.00	0.00	0.00
DIS	11.26	0.00	0.00	0.00	8.95	0.00	0.00	0.00	7.40	0.00	0.00	0.00	5.25	0.00	0.00	0.00
INTC	10.51	0.00	0.00	0.00	8.15	0.00	0.00	0.00	6.25	0.00	0.00	0.00	4.25	0.00	0.00	0.00
CSCO	10.46	0.00	0.00	0.00	8.75	0.00	0.00	0.00	6.90	0.00	0.00	0.00	4.65	0.00	0.00	0.00
HD	14.96	0.00	0.00	0.00	12.51	0.00	0.00	0.00	10.01	0.00	0.00	0.00	7.75	0.00	0.00	0.00
UTX	4.50	**0.30**	0.00	0.00	3.10	0.10	0.00	0.00	2.25	0.00	0.00	0.00	1.45	0.00	0.00	0.00
BA	8.70	0.00	0.00	0.00	6.30	0.00	0.00	0.00	4.60	0.00	0.00	0.00	3.30	0.00	0.00	0.00
MCD	5.50	**0.31**	0.00	0.00	3.85	0.00	0.00	0.00	2.70	0.00	0.00	0.00	1.65	0.00	0.00	0.00
AXP	8.16	0.00	0.00	0.00	5.41	0.00	0.00	0.00	3.36	0.00	0.00	0.00	2.25	0.00	0.00	0.00
MMM	6.70	0.00	0.00	0.00	4.60	0.00	0.00	0.00	3.30	0.00	0.00	0.00	1.95	0.00	0.00	0.00
GS	6.76	0.00	0.00	0.00	4.80	0.00	0.00	0.00	2.02	0.00	0.00	0.00	1.30	0.00	0.00	0.00
UNH	16.45	0.00	0.00	0.00	12.90	0.00	0.00	0.00	10.10	0.00	0.00	0.00	6.60	0.00	0.00	0.00
CAT	5.70	**0.16**	0.00	0.00	4.25	0.00	0.04	0.00	2.85	0.00	0.10	0.00	1.50	0.00	0.08	0.00
DD	7.20	0.00	0.00	0.00	5.75	0.00	0.00	0.00	4.20	0.00	0.00	0.00	2.75	0.00	0.00	0.00
NKE	15.91	0.00	0.00	0.00	13.11	0.00	0.00	0.00	10.76	0.00	0.00	0.00	7.55	0.00	0.00	0.00
TRV	8.20	0.00	0.00	0.00	5.90	0.00	0.00	0.00	4.80	0.00	0.00	0.00	3.10	0.00	0.00	0.00

Table 4.56: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	11.36	0.00	0.00	0.00	8.66	0.00	0.00	0.00	6.16	0.00	0.00	0.00	4.00	0.00	0.00	0.00
MSFT	14.31	0.00	0.00	0.00	11.06	0.00	0.00	0.00	9.16	0.00	0.00	0.00	6.41	0.00	0.00	0.00
GE	9.30	0.00	0.00	0.00	7.60	0.00	0.00	0.00	6.15	0.00	0.00	0.00	4.25	0.00	0.00	0.00
JNJ	9.15	0.00	0.00	0.00	7.00	0.00	0.00	0.00	5.75	0.00	0.00	0.00	3.45	0.00	0.00	0.00
WMT	6.86	0.00	0.00	0.00	5.06	0.00	0.00	0.00	3.26	0.00	0.00	0.00	2.36	0.00	0.00	0.00
CVX	8.11	0.00	0.00	0.00	5.56	0.00	0.00	0.00	3.65	0.00	0.00	0.00	1.70	0.00	0.00	0.00
JPM	8.11	0.00	0.00	0.00	5.91	0.00	0.00	0.00	2.86	0.00	0.00	0.00	2.01	0.00	0.00	0.00
PG	10.31	0.00	0.00	0.00	8.41	0.00	0.00	0.00	6.36	0.00	0.00	0.00	4.25	0.00	0.00	0.00
PFE	8.96	0.00	0.00	0.00	6.46	0.00	0.00	0.00	4.65	0.00	0.00	0.00	3.15	0.00	0.00	0.00
IBM	11.41	0.00	0.00	0.00	8.91	0.00	0.00	0.00	7.41	0.00	0.00	0.00	5.01	0.00	0.00	0.00
T	6.86	0.00	0.00	0.00	5.46	0.00	0.00	0.00	3.95	0.00	0.00	0.00	2.05	0.00	0.00	0.00
KO	7.15	0.00	0.00	0.00	5.30	0.00	0.00	0.00	4.10	0.00	0.00	0.00	2.35	0.00	0.00	0.00
MRK	15.57	0.00	0.00	0.00	12.96	0.00	0.00	0.00	9.91	0.00	0.00	0.00	6.81	0.00	0.00	0.00
VZ	6.00	**0.05**	0.00	0.00	4.30	0.00	0.00	0.00	2.90	0.00	0.00	0.00	1.90	0.00	0.00	0.00
DIS	11.26	0.00	0.00	0.00	9.21	0.00	0.00	0.00	7.21	0.00	0.00	0.00	5.06	0.00	0.00	0.00
INTC	10.61	0.00	0.00	0.00	8.16	0.00	0.00	0.00	6.41	0.00	0.00	0.00	4.45	0.00	0.00	0.00
CSCO	10.71	0.00	0.00	0.00	8.61	0.00	0.00	0.00	6.76	0.00	0.00	0.00	4.70	0.00	0.00	0.00
HD	15.02	0.00	0.00	0.00	12.46	0.00	0.00	0.00	10.11	0.00	0.00	0.00	7.71	0.00	0.00	0.00
UTX	4.65	**0.47**	0.00	0.00	3.30	0.03	0.00	0.00	2.30	0.00	0.00	0.00	1.30	0.00	0.00	0.00
BA	8.66	0.00	0.00	0.00	6.26	0.00	0.00	0.00	4.70	0.00	0.00	0.00	3.35	0.00	0.00	0.00
MCD	5.80	**0.11**	0.00	0.00	3.90	0.00	0.00	0.00	2.60	0.00	0.00	0.00	1.70	0.00	0.00	0.00
AXP	8.31	0.00	0.00	0.00	5.02	0.00	0.00	0.00	3.36	0.00	0.00	0.00	2.31	0.00	0.00	0.00
MMM	6.61	0.00	0.00	0.00	4.85	0.00	0.00	0.00	3.10	0.00	0.00	0.00	2.10	0.00	0.00	0.00
GS	6.76	0.00	0.00	0.00	4.66	0.00	0.00	0.00	2.01	0.00	0.00	0.00	1.35	0.00	0.00	0.00
UNH	16.50	0.00	0.00	0.00	12.85	0.00	0.00	0.00	10.05	0.00	0.00	0.00	6.80	0.00	0.00	0.00
CAT	5.81	**0.11**	0.00	0.00	4.15	0.00	0.08	0.00	2.75	0.00	0.08	0.00	1.45	0.00	0.01	0.00
DD	7.16	0.00	0.00	0.00	5.71	0.00	0.00	0.00	4.30	0.00	0.00	0.00	2.90	0.00	0.00	0.00
NKE	16.02	0.00	0.00	0.00	13.21	0.00	0.00	0.00	10.66	0.00	0.00	0.00	7.51	0.00	0.00	0.00
TRV	8.00	0.00	0.00	0.00	6.10	0.00	0.00	0.00	4.90	0.00	0.00	0.00	3.25	0.00	0.00	0.00

Table 4.57: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

	h = 4															
Stocks	**100(α) = 5**				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	11.37	0.00	0.00	0.00	8.16	0.00	0.00	0.00	6.26	0.00	0.00	0.00	3.86	0.00	0.00	0.00
MSFT	14.37	0.00	0.00	0.00	11.07	0.00	0.00	0.00	9.16	0.00	0.00	0.00	6.41	0.00	0.00	0.00
GE	9.30	0.00	0.00	0.00	7.60	0.00	0.00	0.00	6.10	0.00	0.00	0.00	4.30	0.00	0.00	0.00
JNJ	8.85	0.00	0.00	0.00	7.00	0.00	0.00	0.00	5.65	0.00	0.00	0.00	3.50	0.00	0.00	0.00
WMT	6.92	0.00	0.00	0.00	2.15	0.00	0.00	0.00	3.21	0.00	0.00	0.00	2.36	0.00	0.00	0.00
CVX	8.16	0.00	0.00	0.00	5.41	0.00	0.00	0.00	3.71	0.00	0.00	0.00	1.80	0.00	0.00	0.00
JPM	8.12	0.00	0.00	0.00	5.02	0.00	0.00	0.00	2.56	0.00	0.00	0.00	2.06	0.00	0.00	0.00
PG	10.37	0.00	0.00	0.00	8.26	0.00	0.00	0.00	6.46	0.00	0.00	0.00	4.36	0.00	0.00	0.00
PFE	8.81	0.00	0.00	0.00	6.36	0.00	0.00	0.00	4.71	0.00	0.00	0.00	3.15	0.00	0.00	0.00
IBM	11.47	0.00	0.00	0.00	8.91	0.00	0.00	0.00	7.41	0.00	0.00	0.00	5.01	0.00	0.00	0.00
T	7.01	0.00	0.00	0.00	5.46	0.00	0.00	0.00	3.81	0.00	0.00	0.00	2.15	0.00	0.00	0.00
KO	7.10	0.00	0.00	0.00	5.35	0.00	0.00	0.00	4.10	0.00	0.00	0.00	2.25	0.00	0.00	0.00
MRK	15.57	0.00	0.00	0.00	12.92	0.00	0.00	0.00	9.96	0.00	0.00	0.00	6.81	0.00	0.00	0.00
VZ	6.05	**0.04**	0.00	0.00	4.35	0.00	0.00	0.00	2.85	0.00	0.00	0.00	1.90	0.00	0.00	0.00
DIS	11.42	0.00	0.00	0.00	9.16	0.00	0.00	0.00	7.26	0.00	0.00	0.00	5.06	0.00	0.00	0.00
INTC	10.62	0.00	0.00	0.00	8.16	0.00	0.00	0.00	6.41	0.00	0.00	0.00	4.46	0.00	0.00	0.00
CSCO	10.67	0.00	0.00	0.00	8.61	0.00	0.00	0.00	6.81	0.00	0.00	0.00	4.76	0.00	0.00	0.00
HD	15.02	0.00	0.00	0.00	12.47	0.00	0.00	0.00	10.17	0.00	0.00	0.00	7.76	0.00	0.00	0.00
UTX	4.70	**0.54**	0.00	0.00	3.30	0.03	0.00	0.00	2.25	0.00	0.00	0.00	1.30	0.00	0.00	0.00
BA	8.71	0.00	0.00	0.00	6.31	0.00	0.00	0.00	4.71	0.00	0.00	0.00	3.36	0.00	0.00	0.00
MCD	5.85	**0.09**	0.00	0.00	3.90	0.00	0.00	0.00	2.60	0.00	0.00	0.00	1.70	0.00	0.00	0.00
AXP	8.47	0.00	0.00	0.00	5.52	0.00	0.00	0.00	3.37	0.00	0.00	0.00	2.31	0.00	0.00	0.00
MMM	6.66	0.00	0.00	0.00	4.86	0.00	0.00	0.00	3.15	0.00	0.00	0.00	2.15	0.00	0.00	0.00
GS	6.82	0.00	0.00	0.00	4.66	0.00	0.00	0.00	2.05	0.00	0.00	0.00	1.32	0.00	0.00	0.00
UNH	16.55	0.00	0.00	0.00	12.85	0.00	0.00	0.00	10.05	0.00	0.00	0.00	6.75	0.00	0.00	0.00
CAT	5.81	**0.10**	0.00	0.00	4.21	0.00	**0.04**	0.00	2.75	0.00	0.08	0.00	1.45	0.00	0.01	0.00
DD	7.16	0.00	0.00	0.00	5.71	0.00	0.00	0.00	4.31	0.00	0.00	0.00	2.85	0.00	0.00	0.00
NKE	15.97	0.00	0.00	0.00	13.22	0.00	0.00	0.00	10.62	0.00	0.00	0.00	7.51	0.00	0.00	0.00
TRV	8.20	0.00	0.00	0.00	5.95	0.00	0.00	0.00	4.80	0.00	0.00	0.00	3.25	0.00	0.00	0.00

Table 4.58: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(0,0)

Stocks	h = 5															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	11.32	0.00	0.00	0.00	8.17	0.00	0.00	0.00	6.31	0.00	0.00	0.00	3.86	0.00	0.00	0.00
MSFT	14.48	0.00	0.00	0.00	11.07	0.00	0.00	0.00	9.17	0.00	0.00	0.00	6.36	0.00	0.00	0.00
GE	9.20	0.00	0.00	0.00	7.55	0.00	0.00	0.00	6.15	0.00	0.00	0.00	4.30	0.00	0.00	0.00
JNJ	9.05	0.00	0.00	0.00	7.05	0.00	0.00	0.00	5.60	0.00	0.00	0.00	3.50	0.00	0.00	0.00
WMT	6.93	0.00	0.00	0.00	2.07	0.00	0.00	0.00	3.32	0.00	0.00	0.00	2.36	0.00	0.00	0.00
CVX	8.12	0.00	0.00	0.00	5.36	0.00	0.00	0.00	3.71	0.00	0.00	0.00	1.80	0.00	0.00	0.00
JPM	8.18	0.00	0.00	0.00	5.07	0.00	0.00	0.00	2.52	0.00	0.00	0.00	2.07	0.00	0.00	0.00
PG	10.17	0.00	0.00	0.00	8.17	0.00	0.00	0.00	6.46	0.00	0.00	0.00	4.31	0.00	0.00	0.00
PFE	8.82	0.00	0.00	0.00	6.31	0.00	0.00	0.00	4.71	0.00	0.00	0.00	3.16	0.00	0.00	0.00
IBM	11.42	0.00	0.00	0.00	8.92	0.00	0.00	0.00	7.41	0.00	0.00	0.00	5.01	0.00	0.00	0.00
T	6.86	0.00	0.00	0.00	5.36	0.00	0.00	0.00	3.76	0.00	0.00	0.00	2.10	0.00	0.00	0.00
KO	7.10	0.00	0.00	0.00	5.30	0.00	0.00	0.00	4.10	0.00	0.00	0.00	2.30	0.00	0.00	0.00
MRK	15.58	0.00	0.00	0.00	12.98	0.00	0.00	0.00	9.97	0.00	0.00	0.00	6.81	0.00	0.00	0.00
VZ	6.05	**0.04**	0.00	0.00	4.40	0.00	0.00	0.00	2.85	0.00	0.00	0.00	1.90	0.00	0.00	0.00
DIS	11.37	0.00	0.00	0.00	9.27	0.00	0.00	0.00	7.31	0.00	0.00	0.00	5.06	0.00	0.00	0.00
INTC	10.62	0.00	0.00	0.00	8.17	0.00	0.00	0.00	6.41	0.00	0.00	0.00	4.46	0.00	0.00	0.00
CSCO	10.77	0.00	0.00	0.00	8.72	0.00	0.00	0.00	6.86	0.00	0.00	0.00	4.81	0.00	0.00	0.00
HD	15.08	0.00	0.00	0.00	12.47	0.00	0.00	0.00	10.07	0.00	0.00	0.00	7.72	0.00	0.00	0.00
UTX	4.60	**0.41**	0.00	0.00	3.30	**0.03**	0.00	0.00	2.25	0.00	0.00	0.00	1.35	0.00	0.00	0.00
BA	8.72	0.00	0.00	0.00	6.31	0.00	0.00	0.00	4.71	0.00	0.00	0.00	3.36	0.00	0.00	0.00
MCD	5.80	**0.11**	0.00	0.00	3.85	0.00	0.00	0.00	2.60	0.00	0.00	0.00	1.70	0.00	0.00	0.00
AXP	8.33	0.00	0.00	0.00	5.47	0.00	0.00	0.00	3.42	0.00	0.00	0.00	2.31	0.00	0.00	0.00
MMM	6.66	0.00	0.00	0.00	4.86	0.00	0.00	0.00	3.21	0.00	0.00	0.00	2.15	0.00	0.00	0.00
GS	6.82	0.00	0.00	0.00	4.67	0.00	0.00	0.00	2.00	0.00	0.00	0.00	1.30	0.00	0.00	0.00
UNH	16.50	0.00	0.00	0.00	12.85	0.00	0.00	0.00	10.10	0.00	0.00	0.00	6.75	0.00	0.00	0.00
CAT	5.81	**0.10**	0.00	0.00	4.21	0.00	0.01	0.00	2.76	0.00	0.08	0.00	1.45	0.00	0.01	0.00
DD	7.26	0.00	0.00	0.00	5.71	0.00	0.00	0.00	4.31	0.00	0.00	0.00	2.86	0.00	0.00	0.00
NKE	15.98	0.00	0.00	0.00	13.23	0.00	0.00	0.00	10.67	0.00	0.00	0.00	7.52	0.00	0.00	0.00
TRV	8.25	0.00	0.00	0.00	6.00	0.00	0.00	0.00	4.80	0.00	0.00	0.00	3.25	0.00	0.00	0.00

4.7.13 Interval Forecast Evaluation of SVX with Gaussian Distributed Errors, Mean Model = ARMA(2,0)

Table 4.59: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	h = 1															
	$100(\alpha) = 5$				$100(\alpha) = 2.5$				$100(\alpha) = 1$				$100(\alpha) = 0.5$			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	14.00	0.00	**0.61**	0.00	9.95	0.00	**0.77**	0.00	3.90	0.00	**0.28**	0.00	3.70	0.00	**0.07**	0.00
MSFT	15.95	0.00	0.00	0.00	12.25	0.00	0.00	0.00	4.45	0.00	0.00	0.00	5.60	0.00	0.01	0.00
GE	8.10	0.00	0.00	0.00	5.55	0.00	0.01	0.00	4.40	0.00	0.00	0.00	2.45	0.00	**0.45**	0.00
JNJ	8.35	0.00	**0.06**	0.00	7.25	0.00	**0.32**	0.00	5.45	0.00	**0.68**	0.00	3.55	0.00	**0.61**	0.00
WMT	7.20	0.00	0.00	0.00	3.55	0.00	0.00	0.00	2.50	0.00	**0.34**	0.00	1.60	0.00	0.01	0.00
CVX	8.75	0.00	**0.21**	0.00	5.55	0.00	**0.94**	0.00	2.10	0.00	**0.29**	0.00	1.70	0.00	0.00	0.00
JPM	7.70	0.00	0.00	0.00	5.30	0.00	0.00	0.00	2.90	0.00	0.00	0.00	1.35	0.00	0.00	0.00
PG	10.00	0.00	**0.06**	0.00	7.20	0.00	0.01	0.00	2.85	0.00	0.01	0.00	2.75	0.00	**0.02**	0.00
PFE	8.30	0.00	**0.36**	0.00	5.70	0.00	0.02	0.00	2.15	0.00	**0.94**	0.00	2.20	0.00	**0.09**	0.00
IBM	11.20	0.00	0.00	0.00	8.85	0.00	0.00	0.00	3.65	0.00	0.01	0.00	3.95	0.00	**0.05**	0.00
T	6.50	0.00	0.00	0.00	4.65	0.00	0.00	0.00	1.60	**0.02**	**0.54**	**0.04**	1.25	0.00	**0.04**	0.00
KO	6.80	0.00	**0.09**	0.00	5.05	0.00	**0.58**	0.00	4.05	0.00	**0.79**	0.00	2.50	0.00	**0.85**	0.00
MRK	16.95	0.00	0.00	0.00	13.10	0.00	0.00	0.00	4.75	0.00	0.00	0.00	6.25	0.00	0.00	0.00
VZ	5.80	**0.11**	0.00	0.00	4.15	0.00	0.00	0.00	2.95	0.00	0.00	0.00	2.00	0.00	0.00	0.00
DIS	10.45	0.00	0.00	0.00	8.15	0.00	0.01	0.00	2.95	0.00	**0.13**	0.00	3.80	0.00	**0.03**	0.00
INTC	11.25	0.00	0.10	0.00	8.45	0.00	**0.50**	0.00	3.25	0.00	**0.94**	0.00	3.55	0.00	**0.37**	0.00
CSCO	11.50	0.00	**0.44**	0.00	8.50	0.00	**0.04**	0.00	2.85	0.00	**0.32**	0.00	3.30	0.00	**0.90**	0.00
HD	15.75	0.00	0.00	0.00	12.85	0.00	0.00	0.00	5.05	0.00	0.00	0.00	6.45	0.00	0.00	0.00
UTX	4.75	**0.61**	**0.79**	**0.88**	3.20	**0.05**	**0.72**	**0.78**	1.35	**0.59**	**0.85**	**0.85**	0.75	**0.90**	**0.92**	**0.91**
BA	8.80	0.00	**0.68**	0.00	6.75	0.00	**0.33**	0.00	2.80	0.00	**0.29**	0.00	2.05	0.00	**0.86**	0.00
MCD	5.60	**0.22**	0.01	0.00	4.05	0.00	0.00	0.00	2.40	0.00	0.00	0.00	1.75	0.00	0.00	0.00
AXP	8.85	0.00	0.00	0.00	5.75	0.00	0.00	0.00	2.55	0.00	**0.18**	0.00	1.90	0.00	**0.25**	0.00
MMM	5.55	**0.26**	**0.14**	0.01	3.70	0.00	**0.63**	0.01	1.45	**0.06**	**0.44**	**0.12**	0.90	**0.02**	**0.57**	**0.06**
GS	8.50	0.00	0.01	0.00	4.40	0.00	**0.06**	0.00	2.45	0.00	**0.49**	0.00	1.30	0.00	**0.40**	0.00
UNH	16.40	0.00	**0.89**	0.00	12.75	0.00	**0.89**	0.00	10.05	0.00	**0.98**	0.00	6.70	0.00	**0.91**	0.00
CAT	5.20	**0.68**	**0.85**	0.03	3.45	**0.02**	**0.79**	**0.03**	0.95	**0.82**	**0.32**	**0.04**	0.55	**0.75**	**0.73**	0.90
DD	6.30	0.01	**0.15**	0.00	4.45	0.00	**0.98**	0.00	1.65	**0.02**	**0.57**	**0.02**	1.90	0.00	**0.20**	0.00
NKE	16.80	0.00	0.00	0.00	13.30	0.00	0.00	0.00	5.30	0.00	**0.16**	0.00	6.05	0.00	**0.52**	0.00
TRV	6.00	**0.30**	0.00	0.00	5.80	0.00	0.00	0.00	1.60	**0.25**	0.00	0.00	2.89	0.00	0.00	0.00

Table 4.60: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	**h = 2**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	14.01	0.00	**0.11**	0.00	10.06	0.00	**0.16**	0.00	6.75	0.00	**0.33**	0.00	3.95	0.00	**0.05**	0.00
MSFT	15.71	0.00	0.00	0.00	12.61	0.00	0.00	0.00	9.45	0.00	0.00	0.00	5.75	0.00	**0.02**	0.00
GE	8.00	0.00	0.00	0.00	5.45	0.00	0.01	0.00	4.10	0.00	0.00	0.00	2.25	0.00	**0.47**	0.00
JNJ	9.40	0.00	**0.08**	0.00	7.20	0.00	**0.31**	0.00	5.35	0.00	**0.57**	0.00	3.70	0.00	**0.68**	0.00
WMT	7.26	0.00	0.00	0.00	3.76	0.00	0.01	0.00	2.35	0.00	0.01	0.00	1.70	0.00	**0.04**	0.00
CVX	8.35	0.00	**0.99**	0.00	5.85	0.00	**0.95**	0.00	3.65	0.00	**0.84**	0.00	1.75	0.00	**0.02**	0.00
JPM	7.71	0.00	0.00	0.00	5.81	0.00	0.00	0.00	2.55	0.00	0.00	0.00	1.25	0.00	0.00	0.00
PG	10.06	0.00	0.00	0.00	7.50	0.00	0.00	0.00	5.75	0.00	0.00	0.00	3.25	0.00	0.00	0.00
PFE	8.65	0.00	**0.02**	0.00	6.30	0.00	**0.02**	0.00	4.15	0.00	**0.08**	0.00	2.65	0.00	0.01	0.00
IBM	11.41	0.00	0.00	0.00	9.55	0.00	0.00	0.00	6.90	0.00	0.00	0.00	4.85	0.00	0.01	0.00
T	6.85	0.00	0.00	0.00	5.10	0.00	0.01	0.00	3.40	0.00	**0.03**	0.00	1.45	0.00	0.01	0.00
KO	6.90	0.00	**0.12**	0.00	5.30	0.00	**0.27**	0.00	4.00	0.00	**0.81**	0.00	2.30	0.00	**0.83**	0.00
MRK	16.91	0.00	0.00	0.00	13.26	0.00	0.00	0.00	9.90	0.00	0.00	0.00	6.90	0.00	0.00	0.00
VZ	5.70	**0.16**	0.00	0.00	4.10	0.00	0.00	0.00	3.00	0.00	0.00	0.00	1.90	0.00	0.00	0.00
DIS	11.76	0.00	0.00	0.00	8.85	0.00	0.00	0.00	6.25	0.00	0.01	0.00	3.95	0.00	**0.05**	0.00
INTC	11.81	0.00	0.00	0.00	8.95	0.00	**0.19**	0.00	6.30	0.00	**0.70**	0.00	3.95	0.00	**0.13**	0.00
CSCO	11.71	0.00	**0.17**	0.00	8.90	0.00	**0.06**	0.00	6.30	0.00	0.00	0.00	3.60	0.00	**0.69**	0.00
HD	15.71	0.00	0.00	0.00	13.06	0.00	0.00	0.00	10.81	0.00	0.00	0.00	7.35	0.00	0.00	0.00
UTX	4.50	**0.30**	**0.69**	**0.82**	3.10	**0.10**	**0.72**	**0.81**	1.45	**0.51**	**0.89**	**0.87**	0.70	**0.92**	**0.94**	**0.90**
BA	9.10	0.00	**0.52**	0.00	6.70	0.00	**0.48**	0.00	4.90	0.00	**0.69**	0.00	2.60	0.00	**0.06**	0.00
MCD	5.50	**0.31**	0.00	0.00	3.80	0.00	0.00	0.00	2.60	0.00	0.00	0.00	1.65	0.00	0.00	0.00
AXP	8.41	0.00	0.00	0.00	5.46	0.00	0.00	0.00	2.40	0.00	0.00	0.00	1.45	0.00	0.00	0.00
MMM	5.95	**0.06**	**0.38**	0.00	4.40	0.00	**0.28**	0.00	2.70	0.00	**0.68**	0.00	1.30	0.00	**0.41**	0.00
GS	8.31	0.00	0.00	0.00	4.15	0.00	**0.05**	0.00	2.05	0.00	**0.17**	0.00	1.35	0.00	**0.27**	0.00
UNH	16.50	0.00	**0.88**	0.00	12.85	0.00	**0.90**	0.00	10.05	0.00	**0.98**	0.00	6.60	0.00	**0.92**	0.00
CAT	5.50	**0.31**	**0.35**	0.00	3.70	0.00	**0.63**	0.00	2.40	0.00	**0.88**	0.00	1.05	0.00	**0.50**	0.01
DD	6.35	0.01	0.00	0.00	5.20	0.00	**0.06**	0.00	3.70	0.00	**0.87**	0.00	2.25	0.00	**0.37**	0.00
NKE	17.01	0.00	0.00	0.00	13.91	0.00	0.00	0.00	11.01	0.00	0.00	0.00	6.90	0.00	**0.08**	0.00
TRV	6.10	**0.07**	0.00	0.00	2.90	**0.49**	0.00	0.00	1.70	**0.10**	0.00	0.00	0.85	**0.15**	0.00	0.00

Table 4.61: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	h = 3															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	13.56	0.00	0.00	0.00	10.11	0.00	0.00	0.00	6.91	0.00	0.00	0.00	4.00	0.00	0.00	0.00
MSFT	15.22	0.00	0.00	0.00	12.61	0.00	0.00	0.00	10.01	0.00	0.00	0.00	6.71	0.00	0.00	0.00
GE	7.25	0.00	0.00	0.00	5.55	0.00	**0.02**	0.00	4.15	0.00	0.00	0.00	2.25	0.00	**0.47**	0.00
JNJ	9.15	0.00	0.00	0.00	6.95	0.00	0.00	0.00	5.70	0.00	**0.59**	0.00	3.45	0.00	**0.72**	0.00
WMT	7.27	0.00	0.00	0.00	3.66	0.00	0.00	0.00	2.56	0.00	0.00	0.00	1.91	0.00	0.00	0.00
CVX	8.46	0.00	0.01	0.00	5.76	0.00	**0.95**	0.00	3.70	0.00	**0.02**	0.00	1.80	0.00	0.00	0.00
JPM	7.17	0.00	0.00	0.00	5.46	0.00	0.00	0.00	2.21	0.00	0.00	0.00	1.61	0.00	0.00	0.00
PG	10.66	0.00	0.00	0.00	8.06	0.00	0.00	0.00	6.31	0.00	0.00	0.00	3.85	0.00	0.00	0.00
PFE	8.71	0.00	0.00	0.00	6.31	0.00	**0.02**	0.00	4.25	0.00	0.00	0.00	2.75	0.00	0.00	0.00
IBM	11.91	0.00	0.00	0.00	10.01	0.00	0.00	0.00	7.36	0.00	0.00	0.00	5.11	0.00	0.00	0.00
T	6.76	0.00	0.00	0.00	5.31	0.00	0.00	0.00	3.90	0.00	0.00	0.00	2.05	0.00	0.00	0.00
KO	7.10	0.00	**0.15**	0.00	5.30	0.00	**0.27**	0.00	4.10	0.00	**0.85**	0.00	2.35	0.00	**0.85**	0.00
MRK	17.37	0.00	0.00	0.00	13.36	0.00	0.00	0.00	10.16	0.00	0.00	0.00	7.31	0.00	0.00	0.00
VZ	6.00	**0.05**	0.00	0.00	4.25	0.00	0.00	0.00	2.90	0.00	0.00	0.00	1.90	0.00	0.00	0.00
DIS	11.71	0.00	0.00	0.00	9.46	0.00	0.00	0.00	6.81	0.00	0.00	0.00	4.85	0.00	0.00	0.00
INTC	11.76	0.00	0.00	0.00	9.26	0.00	0.00	0.00	7.16	0.00	0.00	0.00	4.45	0.00	**0.06**	0.00
CSCO	11.76	0.00	0.00	0.00	9.36	0.00	0.00	0.00	6.71	0.00	0.00	0.00	4.30	0.00	0.00	0.00
HD	15.82	0.00	0.00	0.00	13.51	0.00	0.00	0.00	11.26	0.00	0.00	0.00	7.51	0.00	0.00	0.00
UTX	4.50	**0.30**	**0.69**	**0.82**	3.10	**0.10**	**0.72**	**0.81**	1.45	**0.51**	**0.89**	**0.87**	0.70	**0.92**	**0.94**	**0.90**
BA	9.16	0.00	0.00	0.00	6.61	0.00	**0.14**	0.00	4.95	0.00	**0.03**	0.00	2.80	0.00	0.00	0.00
MCD	5.80	**0.11**	0.00	0.00	3.90	0.00	0.00	0.00	2.55	0.00	0.00	0.00	1.70	0.00	0.00	0.00
AXP	8.86	0.00	0.00	0.00	5.21	0.00	0.00	0.00	2.31	0.00	0.00	0.00	1.36	0.00	0.00	0.00
MMM	6.26	0.01	**0.07**	0.00	4.65	0.00	**0.42**	0.00	3.10	0.00	**0.17**	0.00	1.65	0.00	**0.29**	0.00
GS	8.91	0.00	0.00	0.00	4.10	0.00	**0.05**	0.00	2.16	0.00	0.00	0.00	1.60	0.00	0.00	0.00
UNH	16.50	0.00	**0.82**	0.00	12.85	0.00	**0.88**	0.00	10.10	0.00	**0.91**	0.00	6.80	0.00	**0.92**	0.00
CAT	5.86	0.09	**0.72**	0.00	3.95	0.00	**0.94**	0.00	2.65	0.00	**0.71**	0.00	1.15	0.00	**0.46**	0.00
DD	7.11	0.00	0.00	0.00	5.66	0.00	0.00	0.00	4.05	0.00	0.00	0.00	2.50	0.00	**0.53**	0.00
NKE	17.42	0.00	0.00	0.00	14.06	0.00	0.00	0.00	10.91	0.00	0.00	0.00	7.36	0.00	0.00	0.00
TRV	6.00	**0.59**	0.00	0.00	2.10	**0.30**	0.00	0.00	1.90	**0.07**	0.00	0.00	0.80	**0.35**	0.00	0.00

Table 4.62: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	13.47	0.00	0.00	0.00	9.96	0.00	0.00	0.00	7.16	0.00	0.00	0.00	4.01	0.00	0.00	0.00
MSFT	15.57	0.00	0.00	0.00	12.57	0.00	0.00	0.00	10.12	0.00	0.00	0.00	7.21	0.00	0.00	0.00
GE	7.25	0.00	0.00	0.00	5.50	0.00	**0.03**	0.00	4.10	0.00	0.00	0.00	2.30	0.00	**0.35**	0.00
JNJ	8.85	0.00	0.00	0.00	6.90	0.00	0.00	0.00	5.50	0.00	**0.57**	0.00	3.50	0.00	**0.85**	0.00
WMT	7.23	0.00	0.00	0.00	3.67	0.00	0.00	0.00	2.50	0.00	0.00	0.00	1.62	0.00	0.00	0.00
CVX	8.66	0.00	0.00	0.00	5.96	0.00	**0.95**	0.00	3.81	0.00	0.00	0.00	2.05	0.00	0.00	0.00
JPM	7.17	0.00	0.00	0.00	5.37	0.00	0.00	0.00	2.72	0.00	0.00	0.00	1.91	0.00	0.00	0.00
PG	11.22	0.00	0.00	0.00	8.46	0.00	0.00	0.00	6.46	0.00	0.00	0.00	4.06	0.00	0.00	0.00
PFE	9.11	0.00	0.00	0.00	6.76	0.00	**0.02**	0.00	4.66	0.00	0.00	0.00	2.95	0.00	0.00	0.00
IBM	11.92	0.00	0.00	0.00	10.07	0.00	0.00	0.00	7.76	0.00	0.00	0.00	5.51	0.00	0.00	0.00
T	7.31	0.00	0.00	0.00	5.61	0.00	0.00	0.00	4.26	0.00	0.00	0.00	2.10	0.00	0.00	0.00
KO	7.10	0.00	**0.15**	0.00	5.30	0.00	**0.27**	0.00	4.10	0.00	0.88	0.00	2.25	0.00	**0.89**	0.00
MRK	18.03	0.00	0.00	0.00	13.47	0.00	0.00	0.00	10.57	0.00	0.00	0.00	7.56	0.00	0.00	0.00
VZ	6.10	**0.03**	0.00	0.00	4.35	0.00	0.00	0.00	2.85	0.00	0.00	0.00	1.90	0.00	0.00	0.00
DIS	11.87	0.00	0.00	0.00	9.71	0.00	0.00	0.00	7.36	0.00	0.00	0.00	5.31	0.00	0.00	0.00
INTC	12.07	0.00	0.00	0.00	9.46	0.00	0.00	0.00	7.31	0.00	0.00	0.00	4.91	0.00	0.00	0.00
CSCO	11.92	0.00	0.00	0.00	9.61	0.00	0.00	0.00	6.96	0.00	0.00	0.00	4.56	0.00	0.00	0.00
HD	16.07	0.00	0.00	0.00	13.87	0.00	0.00	0.00	11.52	0.00	0.00	0.00	7.76	0.00	0.00	0.00
UTX	4.55	**0.35**	**0.79**	**0.81**	3.30	0.03	**0.78**	**0.80**	1.35	**0.73**	**0.94**	**0.92**	0.72	**0.90**	**0.96**	**0.93**
BA	9.61	0.00	0.00	0.00	7.01	0.00	0.00	0.00	4.96	0.00	**0.03**	0.00	3.15	0.00	0.00	0.00
MCD	5.85	**0.09**	0.00	0.00	3.90	0.00	0.00	0.00	2.55	0.00	0.00	0.00	1.70	0.00	0.00	0.00
AXP	8.42	0.00	0.00	0.00	5.52	0.00	0.00	0.00	2.67	0.00	0.00	0.00	1.66	0.00	0.00	0.00
MMM	6.86	0.00	0.00	0.00	4.91	0.00	**0.32**	0.00	3.36	0.00	**0.03**	0.00	1.90	0.00	**0.04**	0.00
GS	8.42	0.00	0.00	0.00	4.00	0.00	**0.05**	0.00	2.71	0.00	0.00	0.00	1.91	0.00	0.00	0.00
UNH	16.55	0.00	**0.87**	0.00	12.85	0.00	**0.89**	0.00	10.05	0.00	**0.97**	0.00	6.75	0.00	**0.91**	0.00
CAT	6.26	**0.02**	**0.42**	0.00	4.11	0.00	**0.73**	0.00	2.75	0.00	**0.70**	0.00	1.35	0.00	**0.39**	0.00
DD	7.56	0.00	0.00	0.00	5.81	0.00	0.00	0.00	4.31	0.00	0.00	0.00	2.85	0.00	0.00	0.00
NKE	17.43	0.00	0.00	0.00	14.57	0.00	0.00	0.00	10.97	0.00	0.00	0.00	7.66	0.00	0.00	0.00
TRV	6.20	**0.65**	0.00	0.00	2.90	**0.31**	0.00	0.00	1.75	**0.10**	0.00	0.00	0.85	**0.30**	0.00	0.00

Table 4.63: Interval forecast evaluation using Gaussian distribution, mean model = ARMA(2,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	13.18	0.00	0.00	0.00	9.92	0.00	0.00	0.00	7.11	0.00	0.00	0.00	4.26	0.00	0.00	0.00
MSFT	15.38	0.00	0.00	0.00	12.37	0.00	0.00	0.00	10.12	0.00	0.00	0.00	7.36	0.00	0.00	0.00
GE	7.20	0.00	0.00	0.00	5.50	0.00	**0.02**	0.00	4.10	0.00	0.00	0.00	2.30	0.00	**0.35**	0.00
JNJ	9.05	0.00	0.00	0.00	6.95	0.00	0.00	0.00	5.55	0.00	**0.56**	0.00	3.50	0.00	**0.87**	0.00
WMT	7.43	0.00	0.00	0.00	3.83	0.00	0.00	0.00	2.52	0.00	0.00	0.00	1.56	0.00	0.00	0.00
CVX	8.72	0.00	0.00	0.00	6.21	0.00	**0.95**	0.00	4.06	0.00	0.00	0.00	2.05	0.00	0.00	0.00
JPM	7.48	0.00	0.00	0.00	5.68	0.00	0.00	0.00	2.77	0.00	0.00	0.00	1.37	0.00	0.00	0.00
PG	11.32	0.00	0.00	0.00	8.82	0.00	0.00	0.00	6.56	0.00	0.00	0.00	4.46	0.00	0.00	0.00
PFE	9.37	0.00	0.00	0.00	7.01	0.00	**0.02**	0.00	5.01	0.00	0.00	0.00	3.21	0.00	0.00	0.00
IBM	12.07	0.00	0.00	0.00	10.32	0.00	0.00	0.00	7.97	0.00	0.00	0.00	5.76	0.00	0.00	0.00
T	7.62	0.00	0.00	0.00	5.86	0.00	0.00	0.00	4.31	0.00	0.00	0.00	2.45	0.00	0.00	0.00
KO	7.05	0.00	**0.15**	0.00	5.30	0.00	**0.27**	0.00	4.10	0.00	**0.85**	0.00	2.30	0.00	**0.90**	0.00
MRK	18.09	0.00	0.00	0.00	13.73	0.00	0.00	0.00	10.82	0.00	0.00	0.00	7.92	0.00	0.00	0.00
VZ	6.05	**0.04**	0.00	0.00	4.35	0.00	0.00	0.00	2.85	0.00	0.00	0.00	1.90	0.00	0.00	0.00
DIS	12.07	0.00	0.00	0.00	9.82	0.00	0.00	0.00	7.82	0.00	0.00	0.00	5.56	0.00	0.00	0.00
INTC	12.12	0.00	0.00	0.00	9.32	0.00	0.00	0.00	7.31	0.00	0.00	0.00	5.01	0.00	0.00	0.00
CSCO	12.12	0.00	0.00	0.00	9.72	0.00	0.00	0.00	7.31	0.00	0.00	0.00	4.81	0.00	0.00	0.00
HD	15.98	0.00	0.00	0.00	13.88	0.00	0.00	0.00	11.52	0.00	0.00	0.00	8.07	0.00	0.00	0.00
UTX	4.55	**0.35**	**0.78**	**0.81**	3.25	**0.04**	**0.70**	**0.82**	1.35	**0.73**	**0.91**	**0.90**	0.70	**0.92**	**0.97**	**0.95**
BA	9.67	0.00	0.00	0.00	7.31	0.00	0.00	0.00	5.16	0.00	0.00	0.00	3.51	0.00	0.00	0.00
MCD	5.80	**0.11**	0.00	0.00	3.85	0.00	0.00	0.00	2.60	0.00	0.00	0.00	1.70	0.00	0.00	0.00
AXP	8.73	0.00	0.00	0.00	5.88	0.00	0.00	0.00	1.92	0.00	0.00	0.00	1.27	0.00	0.00	0.00
MMM	7.16	0.00	0.00	0.00	5.61	0.00	0.00	0.00	3.51	0.00	**0.05**	0.00	2.05	0.00	0.01	0.00
GS	8.42	0.00	0.00	0.00	4.12	0.00	**0.05**	0.00	2.91	0.00	0.00	0.00	4.06	0.00	0.00	0.00
UNH	16.55	0.00	**0.85**	0.00	12.85	0.00	**0.84**	0.00	10.10	0.00	**0.97**	0.00	6.75	0.00	**0.93**	0.00
CAT	6.46	0.00	**0.02**	0.00	4.46	0.00	**0.15**	0.00	2.96	0.00	**0.85**	0.00	1.55	0.00	**0.51**	0.00
DD	7.92	0.00	0.00	0.00	6.01	0.00	0.00	0.00	4.56	0.00	0.00	0.00	2.96	0.00	0.00	0.00
NKE	17.54	0.00	0.00	0.00	14.73	0.00	0.00	0.00	11.62	0.00	0.00	0.00	8.07	0.00	0.00	0.00
TRV	6.25	**0.60**	0.00	0.00	2.85	**0.45**	0.00	0.00	1.80	**0.15**	0.00	0.00	0.85	**0.30**	0.00	0.00

4.7.14 Interval Forecast Evaluation of SVX with Students't Distributed Errors, Mean Model = ARMA(0,0)

Table 4.64: Interval forecast evaluation using Students't distribution, mean model = ARMA(0,0)

Stocks	h = 1															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	7.10	0.00	0.00	0.00	4.15	0.00	0.00	0.00	2.35	0.00	0.00	0.00	1.25	0.00	0.00	0.00
MSFT	11.20	0.00	0.00	0.00	7.80	0.00	0.00	0.00	5.15	0.00	0.00	0.00	3.75	0.00	0.00	0.00
GE	6.80	0.00	0.00	0.00	4.60	0.00	0.00	0.00	3.60	0.00	0.00	0.00	2.25	0.00	0.00	0.00
JNJ	5.75	**0.13**	0.00	0.00	3.70	0.00	0.00	0.00	2.65	0.00	0.00	0.00	2.10	0.00	0.00	0.00
WMT	7.35	0.00	0.00	0.00	4.75	0.00	0.00	0.00	3.80	0.00	0.00	0.00	1.25	0.00	0.00	0.00
CVX	4.25	**0.12**	0.00	0.00	2.50	**1.00**	0.00	0.00	1.45	**0.06**	0.00	0.00	0.75	**0.14**	0.00	0.00
JPM	7.30	0.00	0.00	0.00	5.30	0.00	0.00	0.00	3.55	0.00	0.00	0.00	2.25	0.00	0.00	0.00
PG	8.85	0.00	0.00	0.00	6.15	0.00	0.00	0.00	3.85	0.00	0.00	0.00	2.55	0.00	0.00	0.00
PFE	5.25	**0.61**	0.00	0.00	3.60	0.00	0.00	0.00	2.40	0.00	0.00	0.00	1.25	0.00	0.00	0.00
IBM	9.95	0.00	0.00	0.00	7.35	0.00	0.00	0.00	4.30	0.00	0.00	0.00	3.00	0.00	0.00	0.00
T	4.70	**0.54**	0.00	0.00	3.05	**0.13**	0.00	0.00	1.70	0.00	0.00	0.00	0.95	0.01	**0.17**	**0.02**
KO	5.60	**0.22**	0.00	0.00	3.90	0.00	0.00	0.00	2.10	0.00	0.00	0.00	0.70	**0.23**	0.00	0.00
MRK	13.95	0.00	0.00	0.00	9.35	0.00	0.00	0.00	6.25	0.00	0.00	0.00	4.55	0.00	0.00	0.00
VZ	3.50	0.00	0.00	0.00	2.10	**0.24**	0.00	0.00	1.20	**0.38**	0.00	0.00	0.80	**0.08**	0.00	0.00
DIS	8.70	0.00	0.00	0.00	6.30	0.00	0.00	0.00	3.85	0.00	0.00	0.00	2.45	0.00	0.00	0.00
INTC	9.25	0.00	0.00	0.00	6.35	0.00	0.00	0.00	4.00	0.00	0.00	0.00	2.95	0.00	0.00	0.00
CSCO	9.90	0.00	0.00	0.00	7.05	0.00	0.00	0.00	4.65	0.00	0.00	0.00	2.95	0.00	0.00	0.00
HD	12.00	0.00	0.00	0.00	8.40	0.00	0.00	0.00	6.10	0.00	0.00	0.00	3.80	0.00	0.00	0.00
UTX	2.70	0.00	0.00	0.00	1.60	0.01	0.00	0.00	0.90	**0.65**	0.00	0.00	0.30	**0.67**	0.00	0.00
BA	7.50	0.00	0.00	0.00	4.90	0.00	0.00	0.00	2.95	0.00	0.00	0.00	1.90	0.00	0.00	0.00
MCD	3.10	0.00	0.00	0.00	1.80	**0.04**	0.00	0.00	1.05	**0.82**	0.00	0.00	0.60	**0.54**	0.00	0.00
AXP	7.85	0.00	0.00	0.00	8.95	0.00	0.00	0.00	6.90	0.00	0.00	0.00	4.45	0.00	0.00	0.00
MMM	5.90	**0.07**	0.00	0.00	3.70	0.00	0.00	0.00	1.95	0.00	0.00	0.00	1.15	0.00	**0.27**	0.00
GS	7.40	0.00	0.00	0.00	4.85	0.00	0.00	0.00	3.55	0.00	0.00	0.00	2.10	0.00	0.00	0.00
UNH	11.15	0.00	0.00	0.00	7.55	0.00	0.00	0.00	4.60	0.00	0.00	0.00	3.30	0.00	0.00	0.00
CAT	4.75	**0.61**	0.00	0.01	2.60	**0.77**	**0.06**	**0.15**	1.25	**0.28**	**0.04**	**0.07**	0.90	**0.02**	**0.57**	0.00
DD	6.25	0.01	0.00	0.00	4.45	0.00	0.00	0.00	2.60	0.00	0.00	0.00	1.75	0.00	0.00	0.00
NKE	12.90	0.00	0.00	0.00	9.30	0.00	0.00	0.00	6.15	0.00	0.00	0.00	3.90	0.00	0.00	0.00
TRV	5.00	**1.00**	0.00	0.00	3.35	**0.02**	0.00	0.00	1.75	0.00	0.00	0.00	1.35	0.00	0.00	0.00

Table 4.65: Interval forecast evaluation using Students't distribution, mean model = ARMA(0,0)

Stocks	h = 2															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	6.85	0.00	0.00	0.00	4.25	0.00	0.00	0.00	2.25	0.00	0.00	0.00	1.40	0.00	0.00	0.00
MSFT	11.26	0.00	0.00	0.00	8.15	0.00	0.00	0.00	4.95	0.00	0.00	0.00	3.70	0.00	0.00	0.00
GE	6.60	0.00	0.00	0.00	4.55	0.00	0.00	0.00	3.45	0.00	0.00	0.00	2.30	0.00	0.00	0.00
JNJ	6.15	**0.02**	0.00	0.00	3.85	0.00	0.00	0.00	2.60	0.00	0.00	0.00	2.05	0.00	0.00	0.00
WMT	7.71	0.00	0.00	0.00	4.31	0.00	0.00	0.00	3.85	0.00	0.00	0.00	1.10	0.00	0.00	0.00
CVX	3.90	**0.09**	0.00	0.00	2.30	**0.89**	0.00	0.00	1.25	**0.28**	0.00	0.00	0.80	**0.13**	0.00	0.00
JPM	6.76	0.00	0.00	0.00	4.90	0.00	0.00	0.00	3.30	0.00	0.00	0.00	2.00	0.00	0.00	0.00
PG	9.00	0.00	0.00	0.00	5.95	0.00	0.00	0.00	3.95	0.00	0.00	0.00	2.50	0.00	0.00	0.00
PFE	4.90	**0.84**	0.00	0.00	3.25	**0.04**	0.00	0.00	2.05	0.00	0.00	0.00	1.15	0.00	0.00	0.00
IBM	9.80	0.00	0.00	0.00	7.45	0.00	0.00	0.00	4.45	0.00	0.00	0.00	3.05	0.00	0.00	0.00
T	4.50	**0.30**	0.00	0.00	2.85	**0.32**	0.00	0.00	1.70	0.00	0.00	0.00	0.90	**0.02**	**0.15**	**0.03**
KO	5.60	**0.22**	0.00	0.00	4.00	0.00	0.00	0.00	2.00	0.00	0.00	0.00	0.75	**0.14**	0.00	0.00
MRK	13.76	0.00	0.00	0.00	9.25	0.00	0.00	0.00	6.20	0.00	0.00	0.00	4.55	0.00	0.00	0.00
VZ	3.40	0.00	0.00	0.00	2.05	**0.18**	0.00	0.00	1.15	**0.51**	0.00	0.00	0.85	**0.04**	0.00	0.00
DIS	8.70	0.00	0.00	0.00	6.30	0.00	0.00	0.00	3.70	0.00	0.00	0.00	2.60	0.00	0.00	0.00
INTC	9.05	0.00	0.00	0.00	6.25	0.00	0.00	0.00	4.10	0.00	0.00	0.00	2.95	0.00	0.00	0.00
CSCO	9.95	0.00	0.00	0.00	7.05	0.00	0.00	0.00	4.70	0.00	0.00	0.00	3.05	0.00	0.00	0.00
HD	11.86	0.00	0.00	0.00	8.65	0.00	0.00	0.00	6.10	0.00	0.00	0.00	3.85	0.00	0.00	0.00
UTX	2.60	0.00	0.00	0.00	1.60	0.01	0.00	0.00	0.90	**0.65**	0.00	0.00	0.45	**0.75**	0.00	0.00
BA	6.90	0.00	0.00	0.00	4.60	0.00	0.00	0.00	3.10	0.00	0.00	0.00	1.75	0.00	0.00	0.00
MCD	3.20	0.00	0.00	0.00	1.80	**0.04**	0.00	0.00	1.05	**0.82**	0.00	0.00	0.60	**0.54**	0.00	0.00
AXP	11.01	0.00	0.00	0.00	8.71	0.00	0.00	0.00	6.21	0.00	0.00	0.00	4.25	0.00	0.00	0.00
MMM	5.95	**0.06**	0.00	0.00	3.55	0.00	0.00	0.00	2.00	0.00	0.00	0.00	1.20	0.00	**0.03**	0.00
GS	7.46	0.00	0.00	0.00	5.01	0.00	0.00	0.00	3.50	0.00	0.00	0.00	2.20	0.00	0.00	0.00
UNH	11.05	0.00	0.00	0.00	7.40	0.00	0.00	0.00	4.50	0.00	0.00	0.00	3.35	0.00	0.00	0.00
CAT	4.45	**0.25**	**0.06**	**0.09**	2.55	**0.88**	**0.19**	**0.42**	1.20	**0.38**	**0.03**	**0.07**	0.95	**0.01**	**0.55**	0.00
DD	6.25	0.01	0.00	0.00	4.50	0.00	0.00	0.00	2.60	0.00	0.00	0.00	1.75	0.00	0.00	0.00
NKE	13.11	0.00	0.00	0.00	9.50	0.00	0.00	0.00	6.10	0.00	0.00	0.00	4.05	0.00	0.00	0.00
TRV	5.20	**0.68**	0.00	0.00	3.40	0.01	0.00	0.00	1.80	0.00	0.00	0.00	1.30	0.00	0.00	0.00

Table 4.66: Interval forecast evaluation using Students't distribution, mean model = ARMA(0,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	6.91	0.00	0.00	0.00	4.35	0.00	0.00	0.00	2.15	0.00	0.00	0.00	1.45	0.00	0.00	0.00
MSFT	11.16	0.00	0.00	0.00	8.16	0.00	0.00	0.00	5.01	0.00	0.00	0.00	3.70	0.00	0.00	0.00
GE	6.70	0.00	0.00	0.00	4.65	0.00	0.00	0.00	3.45	0.00	0.00	0.00	2.15	0.00	0.00	0.00
JNJ	6.00	**0.05**	0.00	0.00	3.80	0.00	0.00	0.00	2.65	0.00	0.00	0.00	2.05	0.00	0.00	0.00
WMT	7.76	0.00	0.00	0.00	4.35	0.00	0.00	0.00	3.91	0.00	0.00	0.00	1.10	0.00	0.00	0.00
CVX	3.86	**0.08**	0.00	0.00	2.35	**0.90**	0.00	0.00	1.25	**0.28**	0.00	0.00	0.80	**0.13**	0.00	0.00
JPM	6.81	0.00	0.00	0.00	4.86	0.00	0.00	0.00	3.30	0.00	0.00	0.00	2.00	0.00	0.00	0.00
PG	9.06	0.00	0.00	0.00	6.36	0.00	0.00	0.00	3.75	0.00	0.00	0.00	2.60	0.00	0.00	0.00
PFE	5.01	**0.99**	0.00	0.00	3.25	**0.04**	0.00	0.00	2.00	0.00	0.00	0.00	1.15	0.00	0.00	0.00
IBM	9.81	0.00	0.00	0.00	7.31	0.00	0.00	0.00	4.40	0.00	0.00	0.00	3.00	0.00	0.00	0.00
T	4.51	**0.30**	0.00	0.00	2.85	**0.32**	0.00	0.00	1.70	0.00	0.00	0.00	0.90	**0.02**	**0.15**	**0.03**
KO	5.70	**0.16**	0.00	0.00	3.85	0.00	0.00	0.00	2.10	0.00	0.00	0.00	0.75	**0.14**	0.00	0.00
MRK	13.71	0.00	0.00	0.00	9.26	0.00	0.00	0.00	6.21	0.00	0.00	0.00	4.45	0.00	0.00	0.00
VZ	3.35	0.00	0.00	0.00	2.15	**0.31**	0.00	0.00	1.20	**0.38**	0.00	0.00	0.80	**0.08**	0.00	0.00
DIS	8.91	0.00	0.00	0.00	6.26	0.00	0.00	0.00	3.65	0.00	0.00	0.00	2.55	0.00	0.00	0.00
INTC	9.21	0.00	0.00	0.00	6.36	0.00	0.00	0.00	4.10	0.00	0.00	0.00	2.95	0.00	0.00	0.00
CSCO	9.86	0.00	0.00	0.00	7.06	0.00	0.00	0.00	4.65	0.00	0.00	0.00	3.05	0.00	0.00	0.00
HD	11.96	0.00	0.00	0.00	8.56	0.00	0.00	0.00	6.16	0.00	0.00	0.00	3.85	0.00	0.00	0.00
UTX	2.70	0.00	0.00	0.00	1.50	0.00	0.00	0.00	0.90	**0.65**	0.00	0.00	0.45	**0.75**	0.00	0.00
BA	7.06	0.00	0.00	0.00	4.65	0.00	0.00	0.00	3.00	0.00	0.00	0.00	1.80	0.00	0.00	0.00
MCD	3.10	0.00	0.00	0.00	1.70	**0.02**	0.00	0.00	1.05	**0.82**	0.00	0.00	0.60	**0.54**	0.00	0.00
AXP	11.27	0.00	0.00	0.00	8.81	0.00	0.00	0.00	6.26	0.00	0.00	0.00	4.25	0.00	0.00	0.00
MMM	5.91	**0.07**	0.00	0.00	3.55	0.00	0.00	0.00	1.95	0.00	0.00	0.00	1.20	0.00	**0.03**	0.00
GS	7.46	0.00	0.00	0.00	5.06	0.00	0.00	0.00	3.46	0.00	0.00	0.00	2.20	0.00	0.00	0.00
UNH	11.15	0.00	0.00	0.00	7.55	0.00	0.00	0.00	4.55	0.00	0.00	0.00	3.30	0.00	0.00	0.00
CAT	4.60	**0.41**	0.00	0.01	2.55	**0.88**	**0.05**	**0.14**	1.15	**0.51**	**0.27**	**0.43**	0.90	**0.02**	**0.57**	0.00
DD	6.21	**0.02**	0.00	0.00	4.50	0.00	0.00	0.00	2.55	0.00	0.00	0.00	1.75	0.00	0.00	0.00
NKE	13.31	0.00	0.00	0.00	9.46	0.00	0.00	0.00	6.16	0.00	0.00	0.00	4.10	0.00	0.00	0.00
TRV	5.25	**0.61**	0.00	0.00	3.50	0.01	0.00	0.00	1.90	0.00	0.00	0.00	1.30	0.00	0.00	0.00

Table 4.67: Interval forecast evaluation using Students't distribution, mean model = ARMA(0,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	6.91	0.00	0.00	0.00	4.36	0.00	0.00	0.00	2.20	0.00	0.00	0.00	1.45	0.00	0.00	0.00
MSFT	11.17	0.00	0.00	0.00	8.16	0.00	0.00	0.00	5.01	0.00	0.00	0.00	3.71	0.00	0.00	0.00
GE	6.75	0.00	0.00	0.00	4.60	0.00	0.00	0.00	3.45	0.00	0.00	0.00	2.20	0.00	0.00	0.00
JNJ	6.00	**0.05**	0.00	0.00	3.80	0.00	0.00	0.00	2.65	0.00	0.00	0.00	2.05	0.00	0.00	0.00
WMT	7.77	0.00	0.00	0.00	4.31	0.00	0.00	0.00	3.91	0.00	0.00	0.00	1.12	0.00	0.00	0.00
CVX	3.91	0.08	0.00	0.00	2.35	**0.90**	0.00	0.00	1.25	**0.28**	0.00	0.00	0.80	**0.13**	0.00	0.00
JPM	6.76	0.00	0.00	0.00	4.86	0.00	0.00	0.00	3.31	0.00	0.00	0.00	2.00	0.00	0.00	0.00
PG	9.06	0.00	0.00	0.00	6.36	0.00	0.00	0.00	3.76	0.00	0.00	0.00	2.60	0.00	0.00	0.00
PFE	5.01	**0.98**	0.00	0.00	3.26	**0.04**	0.00	0.00	2.00	0.00	0.00	0.00	1.15	0.00	0.00	0.00
IBM	9.81	0.00	0.00	0.00	7.31	0.00	0.00	0.00	4.41	0.00	0.00	0.00	3.00	0.00	0.00	0.00
T	4.51	**0.31**	0.00	0.00	2.86	**0.32**	0.00	0.00	1.70	0.00	0.00	0.00	0.90	**0.02**	**0.15**	**0.03**
KO	5.65	**0.19**	0.00	0.00	3.85	0.00	0.00	0.00	2.10	0.00	0.00	0.00	0.75	**0.14**	0.00	0.00
MRK	13.77	0.00	0.00	0.00	9.26	0.00	0.00	0.00	6.21	0.00	0.00	0.00	4.46	0.00	0.00	0.00
VZ	3.30	0.00	0.00	0.00	2.15	**0.31**	0.00	0.00	1.25	**0.28**	0.00	0.00	0.80	**0.08**	0.00	0.00
DIS	8.86	0.00	0.00	0.00	6.31	0.00	0.00	0.00	3.71	0.00	0.00	0.00	2.55	0.00	0.00	0.00
INTC	9.21	0.00	0.00	0.00	6.36	0.00	0.00	0.00	4.11	0.00	0.00	0.00	2.95	0.00	0.00	0.00
CSCO	9.86	0.00	0.00	0.00	7.06	0.00	0.00	0.00	4.66	0.00	0.00	0.00	3.00	0.00	0.00	0.00
HD	11.97	0.00	0.00	0.00	8.56	0.00	0.00	0.00	6.16	0.00	0.00	0.00	3.91	0.00	0.00	0.00
UTX	2.70	0.00	0.00	0.00	1.55	0.00	0.00	0.00	0.90	**0.65**	0.00	0.00	0.45	**0.75**	0.00	0.00
BA	7.06	0.00	0.00	0.00	4.66	0.00	0.00	0.00	3.00	0.00	0.00	0.00	1.80	0.00	0.00	0.00
MCD	3.05	0.00	0.00	0.00	1.70	**0.02**	0.00	0.00	1.05	**0.82**	0.00	0.00	0.60	**0.54**	0.00	0.00
AXP	11.27	0.00	0.00	0.00	8.82	0.00	0.00	0.00	6.26	0.00	0.00	0.00	4.26	0.00	0.00	0.00
MMM	5.91	**0.07**	0.00	0.00	3.56	0.00	0.00	0.00	1.95	0.00	0.00	0.00	1.20	0.00	**0.03**	0.00
GS	7.46	0.00	0.00	0.00	5.06	0.00	0.00	0.00	3.46	0.00	0.00	0.00	2.20	0.00	0.00	0.00
UNH	11.15	0.00	0.00	0.00	7.55	0.00	0.00	0.00	4.55	0.00	0.00	0.00	3.30	0.00	0.00	0.00
CAT	4.61	**0.42**	0.00	0.01	2.55	**0.88**	**0.05**	**0.14**	1.15	**0.50**	**0.27**	**0.43**	0.90	**0.02**	**0.57**	0.00
DD	6.21	**0.02**	0.00	0.00	4.51	0.00	0.00	0.00	2.55	0.00	0.00	0.00	1.75	0.00	0.00	0.00
NKE	13.32	0.00	0.00	0.00	9.46	0.00	0.00	0.00	6.16	0.00	0.00	0.00	4.11	0.00	0.00	0.00
TRV	5.25	**0.61**	0.00	0.00	3.50	0.01	0.00	0.00	1.90	0.00	0.00	0.00	1.30	0.00	0.00	0.00

Table 4.68: Interval forecast evaluation using Students't distribution, mean model = ARMA(0,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	6.91	0.00	0.00	0.00	4.36	0.00	0.00	0.00	2.20	0.00	0.00	0.00	1.45	0.00	0.00	0.00
MSFT	11.22	0.00	0.00	0.00	8.17	0.00	0.00	0.00	5.01	0.00	0.00	0.00	3.71	0.00	0.00	0.00
GE	6.75	0.00	0.00	0.00	4.60	0.00	0.00	0.00	3.45	0.00	0.00	0.00	2.20	0.00	0.00	0.00
JNJ	6.00	**0.05**	0.00	0.00	3.80	0.00	0.00	0.00	2.65	0.00	0.00	0.00	2.05	0.00	0.00	0.00
WMT	7.77	0.00	0.00	0.00	4.31	0.00	0.00	0.00	3.91	0.00	0.00	0.00	1.10	0.00	0.00	0.00
CVX	3.86	**0.08**	0.00	0.00	2.36	**0.91**	0.00	0.00	1.25	**0.28**	0.00	0.00	0.80	**0.08**	0.00	0.00
JPM	6.77	0.00	0.00	0.00	4.86	0.00	0.00	0.00	3.31	0.00	0.00	0.00	2.00	0.00	0.00	0.00
PG	9.02	0.00	0.00	0.00	6.36	0.00	0.00	0.00	3.76	0.00	0.00	0.00	2.56	0.00	0.00	0.00
PFE	5.01	**0.98**	0.00	0.00	3.26	**0.04**	0.00	0.00	2.01	0.00	0.00	0.00	1.15	0.00	0.00	0.00
IBM	9.82	0.00	0.00	0.00	7.31	0.00	0.00	0.00	4.41	0.00	0.00	0.00	3.01	0.00	0.00	0.00
T	4.51	**0.31**	0.00	0.00	2.86	**0.32**	0.00	0.00	1.70	0.00	0.00	0.00	0.90	**0.02**	**0.15**	**0.03**
KO	5.65	**0.19**	0.00	0.00	3.85	0.00	0.00	0.00	2.10	0.00	0.00	0.00	0.75	**0.14**	0.00	0.00
MRK	13.78	0.00	0.00	0.00	9.27	0.00	0.00	0.00	6.26	0.00	0.00	0.00	4.51	0.00	0.00	0.00
VZ	3.35	0.00	0.00	0.00	2.15	**0.31**	0.00	0.00	1.20	**0.38**	0.00	0.00	0.80	**0.08**	0.00	0.00
DIS	8.87	0.00	0.00	0.00	6.36	0.00	0.00	0.00	3.71	0.00	0.00	0.00	2.61	0.00	0.00	0.00
INTC	9.22	0.00	0.00	0.00	6.36	0.00	0.00	0.00	4.11	0.00	0.00	0.00	2.96	0.00	0.00	0.00
CSCO	9.87	0.00	0.00	0.00	7.06	0.00	0.00	0.00	4.71	0.00	0.00	0.00	3.01	0.00	0.00	0.00
HD	11.97	0.00	0.00	0.00	8.57	0.00	0.00	0.00	6.16	0.00	0.00	0.00	3.91	0.00	0.00	0.00
UTX	2.70	0.00	0.00	0.00	1.55	0.00	0.00	0.00	0.90	**0.65**	0.00	0.00	0.45	**0.75**	0.00	0.00
BA	7.06	0.00	0.00	0.00	4.66	0.00	0.00	0.00	3.01	0.00	0.00	0.00	1.80	0.00	0.00	0.00
MCD	3.10	0.00	0.00	0.00	1.70	**0.02**	0.00	0.00	1.05	**0.82**	0.00	0.00	0.60	**0.54**	0.00	0.00
AXP	11.28	0.00	0.00	0.00	8.82	0.00	0.00	0.00	6.27	0.00	0.00	0.00	4.26	0.00	0.00	0.00
MMM	5.91	**0.07**	0.00	0.00	3.56	0.00	0.00	0.00	1.95	0.00	0.00	0.00	1.20	0.00	**0.03**	0.00
GS	7.47	0.00	0.00	0.00	5.06	0.00	0.00	0.00	3.46	0.00	0.00	0.00	2.20	0.00	0.00	0.00
UNH	11.15	0.00	0.00	0.00	7.55	0.00	0.00	0.00	4.55	0.00	0.00	0.00	3.30	0.00	0.00	0.00
CAT	4.61	**0.42**	0.00	0.01	2.56	**0.87**	**0.05**	**0.14**	1.15	**0.50**	**0.27**	**0.43**	0.90	**0.02**	**0.57**	0.00
DD	6.21	**0.02**	0.00	0.00	4.51	0.00	0.00	0.00	2.56	0.00	0.00	0.00	1.75	0.00	0.00	0.00
NKE	13.33	0.00	0.00	0.00	9.47	0.00	0.00	0.00	6.16	0.00	0.00	0.00	4.11	0.00	0.00	0.00
TRV	5.25	**0.61**	0.00	0.00	3.50	0.01	0.00	0.00	1.90	0.00	0.00	0.00	1.30	0.00	0.00	0.00

4.7.15 Interval Forecast Evaluation of SVX with Students't Distributed Errors, Mean Model = ARMA(2,0)

Table 4.69: Interval forecast evaluation using Students't distribution, mean model = ARMA(2,0)

Stocks	**h = 1**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	8.00	0.00	**0.51**	0.00	4.15	0.00	**0.76**	0.00	1.50	**0.04**	**0.47**	**0.09**	0.60	**0.54**	**0.70**	**0.77**
MSFT	10.45	0.00	**0.04**	0.00	6.60	0.00	0.01	0.00	3.35	0.00	**0.62**	0.00	1.60	0.00	**0.31**	0.00
GE	7.10	0.00	**0.58**	**0.06**	3.55	**0.02**	**0.19**	**0.07**	2.70	0.00	**0.12**	0.00	1.75	0.00	**0.20**	0.00
JNJ	6.45	0.00	**0.08**	0.00	4.40	0.00	**0.09**	0.00	2.80	0.00	**0.02**	0.00	2.20	0.00	**0.75**	0.00
WMT	7.11	0.00	**0.68**	0.00	4.60	0.00	**0.52**	0.00	3.85	0.00	**0.52**	0.00	1.00	0.01	**0.21**	0.00
CVX	4.00	**0.03**	**0.83**	**0.66**	1.85	**0.05**	**0.72**	**0.64**	1.10	**0.66**	**0.72**	**0.86**	0.35	**0.83**	**0.82**	**0.95**
JPM	5.85	**0.09**	**0.89**	**0.82**	3.65	0.00	**0.85**	**0.83**	1.75	0.00	**0.83**	**0.95**	0.90	**0.04**	**0.84**	**0.91**
PG	7.45	0.00	**0.07**	0.00	3.90	0.00	**0.04**	0.00	1.80	0.00	**0.17**	0.00	1.20	0.00	**0.29**	0.00
PFE	5.65	**0.19**	**0.93**	**0.83**	2.70	**0.57**	**0.68**	**0.78**	1.30	**0.20**	**0.69**	**0.83**	0.55	**0.98**	**0.74**	**0.95**
IBM	8.30	0.00	**0.08**	0.00	4.75	0.00	**0.02**	0.00	2.40	0.00	**0.46**	0.00	1.35	**0.13**	**0.05**	0.00
T	3.70	0.01	**0.46**	**0.16**	1.55	0.00	**0.51**	**0.17**	0.90	**0.65**	**0.53**	**0.32**	0.20	**0.03**	**0.59**	**0.10**
KO	6.20	**0.02**	**0.12**	0.60	4.35	0.00	**0.64**	0.00	2.45	0.00	**0.72**	0.00	0.80	**0.08**	**0.85**	**0.80**
MRK	13.15	0.00	0.00	0.00	7.70	0.00	0.00	0.00	4.45	0.00	0.01	0.00	2.65	0.00	0.00	0.00
VZ	5.65	**0.30**	**0.59**	0.00	2.20	**0.38**	**0.97**	**0.88**	1.40	**0.09**	**0.53**	**0.31**	0.95	**0.20**	**0.63**	**0.35**
DIS	6.10	**0.03**	**0.34**	**0.06**	3.75	0.00	**0.91**	0.00	1.70	0.00	**0.61**	0.01	0.70	**0.23**	**0.66**	**0.44**
INTC	8.15	0.00	**0.43**	0.00	4.65	0.00	**0.74**	0.00	2.15	0.00	**0.32**	0.00	1.25	**0.28**	**0.43**	0.00
CSCO	8.45	0.00	**0.19**	0.00	4.75	0.00	**0.02**	0.00	2.60	0.00	**0.21**	0.00	1.70	0.00	**0.61**	0.00
HD	10.75	0.00	0.00	0.00	6.65	0.00	0.01	0.00	3.00	0.00	**0.14**	0.00	1.45	0.00	0.01	0.00
UTX	2.95	0.00	0.00	0.00	1.75	**0.02**	**0.02**	**0.04**	0.95	**0.82**	**0.16**	**0.50**	0.50	**1.00**	**0.95**	**0.95**
BA	6.20	**0.02**	**0.06**	0.01	3.10	**0.10**	**0.17**	**0.10**	1.35	**0.13**	**0.38**	**0.22**	0.55	**0.75**	**0.73**	**0.90**
MCD	5.50	0.00	0.01	0.00	2.00	**0.14**	**0.07**	**0.13**	1.15	**0.51**	**0.29**	**0.39**	0.65	**0.36**	**0.07**	**0.13**
AXP	9.35	0.00	0.00	0.00	5.95	0.00	0.00	0.00	3.90	0.00	0.01	0.00	1.85	0.00	**0.72**	0.00
MMM	3.90	**0.02**	**0.58**	**0.06**	1.75	**0.02**	**0.26**	**0.04**	0.75	**0.24**	**0.63**	**0.45**	0.50	**1.00**	**0.75**	**0.95**
GS	6.05	**0.04**	**0.17**	**0.24**	3.35	**0.02**	**0.62**	**0.36**	1.50	**0.04**	**0.63**	**0.57**	0.65	**0.84**	**0.68**	**0.61**
UNH	7.05	0.00	**0.61**	0.00	7.90	0.00	**0.47**	**0.54**	1.05	**0.95**	**0.47**	**0.89**	0.70	**0.85**	**0.91**	**0.92**
CAT	3.10	0.00	**0.45**	0.00	1.30	0.00	**0.35**	0.00	0.40	0.00	**0.80**	0.01	0.15	0.01	**0.92**	**0.03**
DD	4.00	**0.03**	**0.65**	**0.10**	1.95	**0.10**	**0.21**	**0.12**	0.55	**0.03**	**0.73**	**0.08**	0.20	**0.03**	**0.90**	**0.10**
NKE	11.50	0.00	**0.11**	0.00	6.80	0.00	**0.21**	0.00	3.50	0.00	**0.73**	0.00	2.25	0.00	**0.99**	0.00
TRV	5.35	**0.47**	**0.11**	**0.29**	2.75	**0.56**	**0.17**	**0.55**	0.90	**0.95**	**0.65**	**0.89**	1.40	0.00	**0.68**	**0.92**

Table 4.70: Interval forecast evaluation using Students't distribution, mean model = ARMA(2,0)

Stocks	h = 2															
	100(α) = 5				100(α) = 2.5				100(α) = 1				100(α) = 0.5			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	8.15	0.00	**0.06**	0.00	4.05	0.00	**0.43**	0.00	1.45	**0.06**	**0.36**	**0.11**	0.60	**0.54**	**0.70**	**0.77**
MSFT	10.31	0.00	**0.12**	0.00	6.65	0.00	**0.28**	0.00	3.35	0.00	**0.10**	0.00	1.55	0.00	**0.32**	0.00
GE	7.15	0.00	**0.39**	**0.05**	3.95	0.00	**0.37**	0.00	2.55	0.00	**0.08**	0.00	1.60	0.00	**0.15**	0.00
JNJ	6.55	0.00	**0.10**	0.00	4.30	0.00	**0.11**	0.00	2.90	0.00	**0.06**	0.00	2.25	0.00	**0.79**	0.00
WMT	7.21	0.00	**0.65**	0.00	4.86	0.00	**0.50**	0.00	3.80	0.00	**0.53**	0.00	1.05	0.00	**0.16**	0.00
CVX	3.70	**0.02**	**0.87**	**0.66**	1.70	**0.02**	**0.75**	**0.65**	0.90	**0.65**	**0.76**	**0.85**	0.30	**0.82**	**0.85**	**0.94**
JPM	4.90	**0.76**	**0.91**	**0.83**	3.10	**0.13**	**0.86**	**0.84**	1.75	0.00	**0.86**	**0.93**	0.80	**0.14**	**0.82**	**0.93**
PG	7.25	0.00	**0.09**	0.00	4.35	0.00	0.00	0.00	1.95	0.00	**0.23**	0.00	1.30	0.00	**0.05**	0.00
PFE	5.10	**0.91**	**0.94**	**0.85**	2.25	**0.38**	**0.68**	**0.77**	1.00	**0.82**	**0.65**	**0.81**	0.50	**1.00**	**0.76**	**0.91**
IBM	8.45	0.00	**0.12**	0.00	4.75	0.00	**0.02**	0.00	2.55	0.00	**0.56**	0.00	1.40	0.00	**0.06**	0.00
T	3.60	0.00	**0.40**	**0.16**	1.40	0.00	**0.52**	**0.20**	0.65	**0.09**	**0.57**	**0.35**	0.20	**0.03**	**0.59**	**0.10**
KO	6.20	**0.02**	**0.30**	**0.46**	4.20	0.00	**0.77**	0.00	2.40	0.00	**0.78**	0.00	0.85	**0.05**	**0.81**	**0.71**
MRK	13.21	0.00	0.00	0.00	8.05	0.00	0.00	0.00	4.45	0.00	0.00	0.00	2.50	0.00	0.01	0.00
VZ	3.80	**0.02**	**0.49**	0.00	2.25	**0.47**	**0.96**	**0.39**	1.45	**0.06**	**0.24**	**0.29**	0.95	0.01	**0.68**	**0.40**
DIS	6.35	0.01	**0.29**	**0.02**	3.90	0.00	**0.51**	0.00	1.65	0.01	**0.57**	**0.02**	0.60	**0.54**	**0.70**	**0.77**
INTC	8.30	0.00	**0.23**	0.00	4.65	0.00	**0.42**	0.00	2.20	0.00	**0.35**	0.00	1.15	0.00	**0.27**	0.00
CSCO	8.55	0.00	**0.23**	0.00	4.65	0.00	**0.04**	0.00	2.65	0.00	**0.23**	0.00	1.75	0.00	**0.64**	0.00
HD	10.91	0.00	0.00	0.00	6.65	0.00	0.00	0.00	3.00	0.00	**0.14**	0.00	1.45	0.00	0.01	0.00
UTX	3.10	0.00	0.00	0.00	1.70	**0.02**	**0.08**	**0.59**	0.95	**0.82**	**0.23**	**0.53**	0.55	**0.75**	**0.98**	**0.93**
BA	5.95	**0.06**	**0.14**	**0.06**	3.30	**0.03**	**0.59**	**0.08**	1.30	**0.20**	**0.41**	**0.31**	0.55	**0.75**	**0.73**	**0.90**
MCD	3.50	0.00	0.00	0.00	1.90	**0.07**	**0.05**	**0.17**	1.20	**0.38**	**0.15**	**0.42**	0.65	**0.36**	**0.07**	**0.13**
AXP	8.65	0.00	0.00	0.00	5.35	0.00	**0.01**	0.00	3.25	0.00	0.00	0.00	1.35	0.00	**0.79**	0.00
MMM	4.10	**0.06**	**0.83**	**0.16**	1.75	**0.02**	**0.26**	**0.04**	0.85	**0.49**	**0.59**	**0.68**	0.50	**1.00**	**0.75**	**0.95**
GS	6.40	0.01	**0.18**	**0.25**	3.45	0.01	**0.67**	0.40	1.65	0.01	**0.69**	**0.55**	0.60	**0.75**	**0.70**	**0.77**
UNH	7.00	0.00	**0.66**	0.00	2.80	**0.45**	**0.66**	**0.65**	1.15	**0.80**	**0.96**	**0.94**	0.60	**0.90**	**0.96**	**0.93**
CAT	3.05	0.00	**0.48**	0.00	1.35	0.00	**0.38**	0.00	0.30	0.00	**0.85**	0.00	0.10	0.00	**0.95**	0.01
DD	3.95	**0.03**	**0.94**	**0.08**	2.05	**0.19**	**0.86**	**0.41**	0.55	**0.03**	**0.73**	**0.08**	0.20	**0.03**	**0.90**	**0.10**
NKE	11.61	0.00	**0.09**	0.00	7.05	0.00	**0.50**	0.00	3.50	0.00	**0.76**	0.00	2.35	0.00	**0.92**	0.00
TRV	5.60	**0.22**	**0.24**	**0.26**	2.90	**0.20**	**0.74**	**0.67**	1.00	**1.00**	**0.76**	**0.90**	0.40	**0.90**	**0.79**	**0.93**

Table 4.71: Interval forecast evaluation using Students't distribution, mean model = ARMA(2,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	8.16	0.00	**0.11**	0.00	4.25	0.00	**0.33**	0.00	1.45	**0.06**	**0.36**	**0.11**	0.55	**0.75**	**0.73**	**0.89**
MSFT	10.61	0.00	**0.09**	0.00	6.66	0.00	**0.16**	0.00	3.35	0.00	**0.10**	0.00	1.60	0.00	**0.31**	0.00
GE	7.20	0.00	**0.57**	**0.05**	4.00	0.00	**0.69**	0.00	2.50	0.00	**0.18**	0.00	1.55	0.00	**0.21**	0.00
JNJ	6.55	0.00	**0.15**	0.00	4.30	0.00	**0.36**	0.00	2.85	0.00	**0.55**	0.00	2.25	0.00	**0.77**	0.00
WMT	7.96	0.00	**0.78**	0.00	4.46	0.00	**0.53**	0.00	3.26	0.00	**0.59**	0.00	1.05	0.00	**0.16**	0.00
CVX	3.85	**0.03**	**0.99**	**0.65**	1.70	**0.02**	**0.82**	**0.66**	0.90	**0.65**	**0.76**	**0.88**	0.35	**0.83**	**0.82**	**0.96**
JPM	5.11	**0.91**	**0.94**	**0.86**	3.10	**0.13**	**0.87**	**0.88**	1.80	0.00	**0.87**	**0.95**	0.80	**0.14**	**0.86**	**0.96**
PG	7.31	0.00	**0.03**	0.00	4.25	0.00	0.01	0.00	2.00	0.00	**0.25**	0.00	1.25	0.00	**0.04**	0.00
PFE	5.11	**0.91**	**0.94**	**0.87**	2.25	**0.39**	**0.69**	**0.79**	1.05	**0.99**	**0.64**	**0.82**	0.50	**1.00**	**0.76**	**0.91**
IBM	8.51	0.00	**0.13**	0.00	4.75	0.00	**0.12**	0.00	2.55	0.00	**0.56**	0.00	1.45	0.00	**0.07**	0.00
T	3.60	0.00	**0.40**	**0.17**	1.45	0.00	**0.54**	**0.19**	0.70	**0.16**	**0.54**	**0.38**	0.20	**0.03**	**0.60**	**0.10**
KO	6.00	**0.05**	**0.44**	**0.48**	4.25	0.00	**0.68**	0.00	2.45	0.00	**0.77**	0.00	0.85	**0.05**	**0.81**	**0.71**
MRK	13.26	0.00	0.00	0.00	7.86	0.00	0.00	0.00	4.45	0.00	0.00	0.00	2.70	0.00	0.00	0.00
VZ	3.85	0.01	**0.46**	0.00	2.45	**0.89**	**0.97**	**0.50**	1.40	**0.09**	**0.53**	**0.32**	1.00	0.01	**0.72**	**0.42**
DIS	6.46	0.00	**0.05**	0.00	3.95	0.00	**0.48**	0.00	1.75	0.00	**0.64**	0.01	0.60	**0.54**	**0.70**	**0.77**
INTC	8.41	0.00	**0.42**	0.00	4.65	0.00	**0.42**	0.00	2.20	0.00	**0.35**	0.00	1.20	0.00	**0.29**	0.00
CSCO	8.46	0.00	**0.04**	0.00	4.70	0.00	**0.04**	0.00	2.70	0.00	**0.25**	0.00	1.75	0.00	**0.64**	0.00
HD	11.11	0.00	0.00	0.00	6.81	0.00	0.00	0.00	3.15	0.00	**0.19**	0.00	1.55	0.00	0.01	0.00
UTX	3.15	0.00	0.00	0.00	1.65	**0.02**	**0.09**	**0.02**	1.00	**1.00**	**0.16**	**0.63**	0.60	**0.78**	**0.96**	**0.93**
BA	6.21	**0.02**	**0.06**	0.01	3.25	**0.04**	**0.55**	**0.10**	1.50	**0.04**	**0.47**	**0.09**	0.65	**0.36**	**0.68**	**0.60**
MCD	3.45	0.00	0.01	0.00	2.10	**0.24**	**0.04**	**0.12**	1.35	**0.13**	**0.28**	**0.40**	0.70	**0.23**	**0.09**	**0.14**
AXP	8.71	0.00	0.00	0.00	5.41	0.00	0.00	0.00	3.25	0.00	0.00	0.00	1.45	0.00	**0.77**	0.00
MMM	4.15	**0.08**	**0.80**	**0.20**	1.80	**0.04**	**0.25**	**0.06**	0.75	**0.24**	**0.63**	**0.45**	0.50	**1.00**	**0.75**	**0.95**
GS	6.41	0.01	**0.18**	**0.29**	3.45	0.01	**0.79**	**0.39**	1.70	0.00	**0.68**	**0.55**	0.60	**0.75**	**0.67**	**0.78**
UNH	7.05	0.00	**0.85**	0.00	2.90	**0.20**	**0.70**	**0.73**	1.05	**0.95**	**0.47**	**0.89**	0.65	**0.85**	**0.98**	**0.95**
CAT	3.05	0.00	**0.48**	0.00	1.35	0.00	**0.38**	0.00	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.01
DD	4.15	**0.08**	**0.80**	**0.20**	2.05	**0.19**	**0.86**	**0.41**	0.55	**0.03**	**0.73**	**0.08**	0.25	**0.08**	**0.87**	**0.21**
NKE	11.56	0.00	**0.08**	0.00	7.01	0.00	**0.47**	0.00	3.55	0.00	**0.37**	0.00	2.30	0.00	**0.95**	0.00
TRV	5.55	**0.26**	**0.30**	**0.33**	2.85	**0.33**	**0.77**	**0.75**	1.10	**0.85**	**0.69**	**0.79**	0.45	**0.93**	**0.81**	**0.96**

Table 4.72: Interval forecast evaluation using Students't distribution, mean model = ARMA(2,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	$\mathbf{LR_{uc}}$	$\mathbf{LR_{ind}}$	$\mathbf{LR_{cc}}$	% Viol.	$\mathbf{LR_{uc}}$	$\mathbf{LR_{ind}}$	$\mathbf{LR_{cc}}$	% Viol.	$\mathbf{LR_{uc}}$	$\mathbf{LR_{ind}}$	$\mathbf{LR_{cc}}$	% Viol.	$\mathbf{LR_{uc}}$	$\mathbf{LR_{ind}}$	$\mathbf{LR_{cc}}$
XOM	8.16	0.00	**0.10**	0.00	4.16	0.00	**0.11**	0.00	1.45	**0.06**	**0.36**	**0.11**	0.65	**0.36**	**0.68**	**0.60**
MSFT	10.82	0.00	**0.02**	0.00	6.66	0.00	**0.16**	0.00	3.41	0.00	**0.11**	0.00	1.65	0.00	**0.29**	0.00
GE	7.20	0.00	**0.53**	**0.06**	4.00	0.00	**0.69**	0.00	2.50	0.00	**0.18**	0.00	1.55	0.00	**0.21**	0.00
JNJ	6.55	0.00	**0.12**	0.00	4.35	0.00	**0.55**	0.00	2.80	0.00	**0.45**	0.00	2.25	0.00	**0.78**	0.00
WMT	7.92	0.00	**0.74**	0.00	4.46	0.00	**0.57**	0.00	3.26	0.00	**0.59**	0.00	1.05	0.00	**0.16**	0.00
CVX	3.86	**0.05**	**0.99**	**0.65**	1.70	**0.02**	**0.86**	**0.64**	0.90	**0.65**	**0.78**	**0.86**	0.35	**0.83**	**0.82**	**0.95**
JPM	5.06	**0.98**	**0.98**	**0.82**	3.10	**0.13**	**0.85**	**0.83**	1.80	0.00	**0.88**	**0.94**	0.80	**0.14**	**0.82**	**0.93**
PG	7.31	0.00	**0.03**	0.00	4.31	0.00	0.01	0.00	2.00	0.00	**0.25**	0.00	1.25	0.00	**0.04**	0.00
PFE	5.11	0.90	**0.94**	**0.85**	2.25	**0.39**	**0.79**	**0.77**	1.05	**0.99**	**0.65**	**0.81**	0.50	**1.00**	**0.76**	**0.91**
IBM	8.61	0.00	**0.09**	0.00	4.76	0.00	**0.12**	0.00	2.55	0.00	**0.56**	0.00	1.45	0.00	**0.07**	0.00
T	3.61	0.00	**0.40**	**0.17**	1.45	0.00	**0.44**	**0.19**	0.70	**0.16**	**0.55**	**0.38**	0.20	**0.03**	**0.69**	**0.10**
KO	6.00	**0.05**	**0.44**	**0.48**	4.25	0.00	**0.68**	0.00	2.45	0.00	**0.77**	0.00	0.85	**0.05**	**0.81**	**0.71**
MRK	13.32	0.00	**0.00**	0.00	7.81	0.00	0.00	0.00	4.46	0.00	0.00	0.00	2.70	0.00	0.00	0.00
VZ	3.85	0.01	**0.50**	0.00	2.45	**0.89**	**0.96**	**0.52**	1.40	**0.09**	**0.57**	**0.38**	1.00	0.01	**0.78**	**0.45**
DIS	6.41	0.01	**0.05**	0.00	3.96	0.00	**0.48**	0.00	1.75	0.00	**0.64**	0.01	0.60	**0.53**	**0.70**	**0.77**
INTC	8.31	0.00	**0.36**	0.00	4.76	0.00	**0.48**	0.00	2.25	0.00	**0.10**	0.00	1.20	0.00	**0.29**	0.00
CSCO	8.56	0.00	**0.03**	0.00	4.76	0.00	**0.05**	0.00	2.75	0.00	**0.27**	0.00	1.75	0.00	**0.64**	0.00
HD	11.07	0.00	0.00	0.00	6.96	0.00	0.00	0.00	3.25	0.00	**0.08**	0.00	1.65	0.00	**0.02**	0.00
UTX	3.15	0.00	0.00	0.00	1.70	**0.02**	**0.08**	**0.02**	1.00	**1.00**	**0.19**	**0.60**	0.60	**0.78**	**0.95**	**0.92**
BA	6.21	**0.02**	**0.12**	**0.02**	3.25	**0.04**	**0.55**	**0.10**	1.45	**0.06**	**0.44**	**0.12**	0.65	**0.36**	**0.68**	**0.60**
MCD	3.40	0.00	0.01	0.00	2.10	**0.24**	**0.07**	**0.18**	1.35	**0.13**	**0.25**	**0.56**	0.65	**0.36**	**0.07**	**0.20**
AXP	8.66	0.00	0.00	0.00	5.41	0.00	0.00	0.00	3.25	0.00	0.00	0.00	1.45	0.00	**0.77**	0.00
MMM	4.21	**0.10**	**0.76**	**0.24**	1.80	**0.04**	**0.25**	**0.06**	0.80	**0.36**	**0.61**	**0.57**	0.50	**0.99**	**0.75**	**0.95**
GS	6.41	0.01	**0.17**	**0.24**	3.46	0.01	**0.79**	**0.39**	1.70	0.00	**0.68**	**0.56**	0.60	**0.75**	**0.70**	**0.77**
UNH	7.05	0.00	**0.85**	0.00	2.80	**0.45**	**0.75**	**0.67**	1.05	**0.95**	**0.66**	**0.91**	0.60	**0.90**	**0.98**	**0.96**
CAT	3.05	0.00	**0.48**	0.00	1.35	0.00	**0.38**	0.00	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.01
DD	4.26	**0.12**	**0.84**	**0.29**	2.10	**0.24**	**0.90**	**0.50**	0.55	**0.03**	**0.73**	**0.08**	0.25	**0.08**	**0.87**	**0.21**
NKE	11.62	0.00	**0.14**	0.00	7.01	0.00	**0.47**	0.00	3.56	0.00	**0.37**	0.00	2.30	0.00	**0.95**	0.00
TRV	5.55	**0.26**	**0.38**	**0.41**	2.85	**0.33**	**0.75**	**0.78**	1.10	**0.85**	**0.84**	**0.89**	0.45	**0.93**	**0.79**	**0.95**

Table 4.73: Interval forecast evaluation using Students't distribution, mean model = ARMA(2,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	8.17	0.00	**0.06**	0.00	4.26	0.00	**0.73**	0.00	1.50	**0.04**	**0.34**	**0.07**	0.65	**0.36**	**0.68**	**0.60**
MSFT	10.87	0.00	0.01	0.00	6.81	0.00	**0.21**	0.00	3.56	0.00	**0.15**	0.00	1.75	0.00	**0.26**	0.00
GE	7.20	0.00	**0.58**	**0.07**	4.00	0.00	**0.69**	0.00	2.50	0.00	**0.18**	0.00	1.55	0.00	**0.21**	0.00
JNJ	6.60	0.00	**0.12**	0.00	4.30	0.00	**0.52**	0.00	2.80	0.00	**0.40**	0.00	2.25	0.00	**0.52**	0.00
WMT	7.93	0.00	**0.75**	0.00	4.74	0.00	**0.57**	0.00	3.26	0.00	**0.59**	0.00	1.06	0.00	**0.16**	0.00
CVX	3.86	**0.05**	**0.99**	**0.65**	1.70	**0.02**	**0.86**	**0.65**	0.90	**0.66**	**0.86**	**0.86**	0.35	**0.83**	**0.82**	**0.96**
JPM	5.11	**0.90**	**0.94**	**0.82**	3.11	**0.12**	**0.85**	**0.85**	1.80	0.00	**0.87**	**0.95**	0.80	**0.14**	**0.86**	**0.95**
PG	7.26	0.00	**0.05**	0.00	4.26	0.00	0.01	0.00	2.05	0.00	**0.27**	0.00	1.30	0.00	**0.05**	0.00
PFE	5.11	**0.90**	**0.94**	**0.85**	2.25	**0.39**	**0.80**	**0.77**	1.05	**0.99**	**0.69**	**0.83**	0.50	**1.00**	**0.76**	**0.91**
IBM	8.57	0.00	**0.09**	0.00	4.86	0.00	**0.06**	0.00	2.56	0.00	**0.56**	0.00	1.45	0.00	**0.07**	0.00
T	3.61	0.00	**0.40**	**0.17**	1.45	0.00	**0.54**	**0.17**	0.70	**0.16**	**0.55**	**0.38**	0.20	**0.03**	**0.70**	**0.10**
KO	5.95	**0.06**	**0.49**	**0.59**	4.25	0.00	**0.64**	0.00	2.45	0.00	**0.79**	0.00	0.80	**0.08**	**0.88**	**0.79**
MRK	13.53	0.00	0.00	0.00	7.97	0.00	0.00	0.00	4.56	0.00	0.00	0.00	2.81	0.00	0.00	0.00
VZ	3.85	0.01	**0.52**	0.00	2.45	**0.89**	**0.97**	**0.68**	1.40	**0.09**	**0.57**	**0.45**	1.00	0.01	**0.78**	**0.44**
DIS	6.56	0.00	0.01	0.00	4.01	0.00	**0.90**	0.00	1.75	0.00	**0.65**	0.01	0.60	**0.53**	**0.70**	**0.77**
INTC	8.42	0.00	**0.18**	0.00	4.76	0.00	**0.49**	0.00	2.30	0.00	**0.11**	0.00	1.25	0.00	**0.32**	0.00
CSCO	8.67	0.00	**0.03**	0.00	4.96	0.00	**0.08**	0.00	2.91	0.00	**0.34**	0.00	1.80	0.00	**0.68**	0.00
HD	11.22	0.00	0.00	0.00	6.96	0.00	0.00	0.00	3.16	0.00	**0.06**	0.00	1.65	0.00	**0.02**	0.00
UTX	3.15	0.00	0.00	0.00	1.70	**0.02**	**0.08**	**0.02**	1.00	**1.00**	**0.28**	**0.62**	0.60	**0.54**	**0.91**	**0.94**
BA	6.26	0.01	**0.14**	0.01	3.26	**0.04**	**0.55**	**0.10**	1.40	0.09	**0.41**	**0.17**	0.65	**0.36**	**0.68**	**0.60**
MCD	3.45	0.00	0.01	0.00	2.00	**0.14**	**0.08**	**0.20**	1.35	**0.13**	**0.30**	**0.56**	0.65	**0.36**	**0.09**	**0.18**
AXP	8.67	0.00	0.00	0.00	5.41	0.00	0.00	0.00	3.26	0.00	0.00	0.00	1.45	0.00	**0.77**	0.00
MMM	4.26	**0.12**	**0.73**	**0.28**	1.90	**0.08**	**0.22**	**0.10**	0.75	**0.24**	**0.63**	**0.45**	0.50	**0.99**	**0.75**	**0.95**
GS	6.41	0.01	**0.19**	**0.24**	3.46	0.01	**0.68**	**0.36**	1.70	0.00	**0.67**	**0.57**	0.60	**0.75**	**0.70**	**0.77**
UNH	7.10	0.00	**0.85**	0.00	2.80	**0.45**	**0.79**	**0.68**	1.05	**0.95**	**0.89**	**0.94**	0.65	**0.85**	**0.88**	**0.93**
CAT	3.01	0.00	**0.50**	0.00	1.40	0.00	**0.41**	0.00	0.25	0.00	**0.87**	0.00	0.10	0.00	**0.95**	0.01
DD	4.31	**0.15**	**0.50**	**0.28**	2.10	**0.25**	**0.90**	**0.51**	0.55	**0.03**	**0.73**	**0.08**	0.30	**0.17**	**0.85**	**0.39**
NKE	11.82	0.00	**0.06**	0.00	7.06	0.00	**0.50**	0.00	3.61	0.00	**0.17**	0.00	2.35	0.00	**0.92**	0.00
TRV	5.55	**0.26**	**0.37**	**0.52**	2.85	**0.33**	**0.76**	**0.79**	1.10	**0.85**	**0.84**	**0.89**	0.45	**0.93**	**0.79**	**0.95**

4.7.16 Interval Forecast Evaluation of SVX with Skewed Student's t-distributed Errors, Mean Model = ARMA(0,0)

Table 4.74: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(0,0)

Stocks	h = 1															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	6.95	0.00	0.00	0.00	4.65	0.00	0.00	0.00	2.70	0.00	0.00	0.00	1.65	0.00	0.00	0.00
MSFT	8.90	0.00	0.00	0.00	6.30	0.00	0.00	0.00	4.40	0.00	0.00	0.00	2.45	0.00	0.00	0.00
GE	5.53	**0.27**	0.00	0.00	3.02	**0.69**	0.00	0.00	1.79	0.00	0.00	0.00	1.12	0.00	0.00	0.00
JNJ	5.28	**0.58**	0.00	0.00	3.21	**0.05**	0.00	0.00	1.04	**0.96**	0.00	0.00	1.20	0.00	0.00	0.00
WMT	7.56	0.00	0.00	0.00	4.15	0.00	0.00	0.00	3.70	0.00	0.00	0.00	1.15	0.01	0.00	0.00
CVX	6.10	**0.03**	0.00	0.00	3.45	0.01	0.00	0.00	1.75	0.00	0.00	0.00	0.65	**0.36**	0.00	0.00
JPM	6.90	0.00	0.00	0.00	4.35	0.00	0.00	0.00	2.85	0.00	0.00	0.00	1.05	**0.02**	0.00	0.00
PG	8.15	0.00	0.00	0.00	5.75	0.00	0.00	0.00	4.10	0.00	0.00	0.00	2.55	0.00	0.00	0.00
PFE	5.15	**0.76**	0.00	0.00	2.95	**0.21**	0.00	0.00	1.45	**0.06**	0.00	0.00	0.65	**0.36**	0.00	0.00
IBM	7.85	0.00	0.00	0.00	5.75	0.00	0.00	0.00	3.85	0.00	0.00	0.00	2.10	0.00	0.00	0.00
T	4.35	**0.17**	0.00	0.00	2.30	**0.56**	0.00	0.00	1.00	**1.00**	0.00	0.00	0.45	**0.75**	0.00	0.00
KO	5.00	**1.00**	0.00	0.00	2.89	**0.73**	0.00	0.00	1.12	**0.64**	0.00	0.00	0.60	**0.85**	0.00	0.00
MRK	11.06	0.00	0.00	0.00	7.90	0.00	0.00	0.00	5.80	0.00	0.00	0.00	3.70	0.00	0.00	0.00
VZ	5.18	**0.83**	0.00	0.00	2.02	**0.11**	0.00	0.00	1.00	**1.00**	0.00	0.00	0.60	**0.85**	0.00	0.00
DIS	7.30	0.00	0.00	0.00	5.30	0.00	0.00	0.00	3.55	0.00	0.00	0.00	2.25	0.00	0.00	0.00
INTC	7.10	0.00	0.00	0.00	4.75	0.00	0.00	0.00	3.55	0.00	0.00	0.00	1.95	0.00	0.00	0.00
CSCO	8.55	0.00	0.00	0.00	6.55	0.00	0.00	0.00	4.70	0.00	0.00	0.00	2.80	0.00	0.00	0.00
HD	9.40	0.00	0.00	0.00	7.45	0.00	0.00	0.00	5.20	0.00	0.00	0.00	2.90	0.00	0.00	0.00
UTX	2.22	0.00	0.00	0.00	1.53	0.00	0.00	0.00	0.72	**0.17**	0.00	0.00	0.15	**0.02**	0.00	0.00
BA	4.95	**0.92**	0.00	0.00	3.80	0.00	0.00	0.00	2.25	0.00	0.00	0.00	1.05	0.00	0.00	0.00
MCD	5.12	**0.83**	0.00	0.00	1.67	0.00	0.00	0.00	0.88	**0.64**	0.00	0.00	0.50	**1.00**	0.00	0.00
AXP	7.58	0.00	0.00	0.00	5.05	0.00	0.00	0.00	2.01	0.00	0.00	0.00	1.45	0.00	0.00	0.00
MMM	4.35	**0.17**	0.00	0.00	2.50	**1.00**	0.00	0.01	1.65	0.01	0.00	0.00	0.90	**0.02**	**0.57**	**0.06**
GS	6.40	0.01	0.00	0.00	4.20	0.00	0.00	0.00	2.25	0.00	0.00	0.00	1.00	0.01	0.00	0.00
UNH	5.11	**0.84**	0.00	0.00	6.10	0.00	0.00	0.00	2.89	0.00	0.00	0.00	0.58	**0.97**	0.00	0.00
CAT	3.60	0.00	0.02	0.00	2.00	**0.14**	**0.05**	**0.05**	1.10	**0.66**	**0.24**	**0.46**	0.65	**0.36**	**0.68**	**0.61**
DD	4.95	**0.92**	0.00	0.00	3.40	0.01	0.00	0.00	2.20	0.00	0.00	0.00	1.35	0.00	0.00	0.00
NKE	11.01	0.00	0.00	0.00	8.35	0.00	0.00	0.00	5.80	0.00	0.00	0.00	3.10	0.00	0.00	0.00
TRV	4.67	**0.50**	0.00	0.00	2.80	**0.13**	0.00	0.00	0.95	**0.95**	0.00	0.00	0.81	**0.45**	0.00	0.00

Table 4.75: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(0,0)

Stocks	**h = 2**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	6.31	0.01	0.00	0.00	4.25	0.00	0.00	0.00	2.85	0.00	0.00	0.00	1.55	0.00	0.00	0.00
MSFT	9.01	0.00	0.00	0.00	6.56	0.00	0.00	0.00	4.50	0.00	0.00	0.00	2.75	0.00	0.00	0.00
GE	5.50	**0.30**	0.00	0.00	3.00	**0.70**	0.00	0.00	1.75	0.00	0.00	0.00	1.10	0.00	0.00	0.00
JNJ	5.26	**0.59**	0.00	0.00	3.20	**0.06**	0.00	0.00	1.02	**0.98**	0.00	0.00	1.20	0.00	0.00	0.00
WMT	6.91	0.00	0.00	0.00	4.75	0.00	0.00	0.00	3.55	0.00	0.00	0.00	1.20	0.00	0.00	0.00
CVX	6.15	0.01	0.00	0.00	3.10	**0.24**	0.00	0.00	0.85	**0.49**	0.00	0.00	0.30	**0.17**	0.00	0.00
JPM	7.06	0.00	0.00	0.00	4.35	0.00	0.00	0.00	2.80	0.00	0.00	0.00	1.55	0.00	0.00	0.00
PG	6.56	0.00	0.00	0.00	4.85	0.00	0.00	0.00	3.10	0.00	0.00	0.00	1.80	0.00	0.00	0.00
PFE	3.15	0.00	0.00	0.00	1.45	0.00	0.00	0.00	1.00	**1.00**	0.00	0.00	0.45	**0.75**	0.00	0.00
IBM	7.86	0.00	0.00	0.00	6.01	0.00	0.00	0.00	3.80	0.00	0.00	0.00	2.30	0.00	0.00	0.00
T	2.55	0.00	0.00	0.00	1.05	0.00	0.00	0.00	0.75	**0.24**	0.00	0.00	0.15	0.01	0.00	0.00
KO	5.05	**0.95**	0.00	0.00	2.87	**0.71**	0.00	0.00	1.13	**0.63**	0.00	0.00	0.59	**0.84**	0.00	0.00
MRK	11.21	0.00	0.00	0.00	8.11	0.00	0.00	0.00	5.76	0.00	0.00	0.00	3.80	0.00	0.00	0.00
VZ	5.20	**0.65**	0.00	0.00	2.01	**0.10**	0.00	0.00	1.01	**0.99**	0.00	0.00	0.61	**0.86**	0.00	0.00
DIS	6.76	0.00	0.00	0.00	4.90	0.00	0.00	0.00	3.30	0.00	0.00	0.00	2.00	0.00	0.00	0.00
INTC	7.36	0.00	0.00	0.00	4.90	0.00	0.00	0.00	3.75	0.00	0.00	0.00	2.20	0.00	0.00	0.00
CSCO	7.81	0.00	0.00	0.00	5.76	0.00	0.00	0.00	3.90	0.00	0.00	0.00	2.30	0.00	0.00	0.00
HD	9.36	0.00	0.00	0.00	7.36	0.00	0.00	0.00	5.21	0.00	0.00	0.00	2.80	0.00	0.00	0.00
UTX	2.25	0.00	0.00	0.00	1.53	0.00	0.00	0.00	0.75	**0.20**	0.00	0.00	0.18	**0.04**	0.00	0.00
BA	5.36	**0.47**	0.00	0.00	3.75	0.00	0.00	0.00	2.65	0.00	0.00	0.00	1.25	0.00	0.00	0.00
MCD	5.14	**0.81**	0.00	0.00	1.69	0.00	0.00	0.00	0.91	**0.67**	0.00	0.00	0.53	**0.97**	0.00	0.00
AXP	6.86	0.00	0.00	0.00	4.65	0.00	0.00	0.00	2.05	0.00	0.00	0.00	1.20	0.00	0.00	0.00
MMM	4.40	**0.21**	0.00	0.00	2.65	**0.67**	0.00	0.00	1.65	0.01	0.00	0.00	0.85	**0.04**	**0.59**	**0.11**
GS	4.70	**0.54**	0.00	0.00	2.65	**0.67**	0.00	0.00	1.30	**0.20**	0.00	0.00	0.80	0.08	0.00	0.00
UNH	5.10	**0.90**	0.00	0.00	6.11	0.00	0.00	0.00	2.91	0.00	0.00	0.00	0.62	**0.88**	0.00	0.00
CAT	2.90	0.00	0.01	0.00	1.70	**0.02**	**0.02**	0.00	1.05	**0.82**	**0.22**	**0.46**	0.60	**0.54**	**0.70**	**0.77**
DD	4.90	**0.84**	0.00	0.00	3.35	**0.02**	0.00	0.00	2.35	0.00	0.00	0.00	1.35	0.00	0.00	0.00
NKE	10.16	0.00	0.00	0.00	7.56	0.00	0.00	0.00	5.31	0.00	0.00	0.00	2.85	0.00	0.00	0.00
TRV	4.67	**0.52**	0.00	0.00	2.78	**0.11**	0.00	0.00	0.95	**0.95**	0.00	0.00	0.85	**0.40**	0.00	0.00

Table 4.76: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(0,0)

	h = 3															
Stocks	**100(α) = 5**				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	6.51	0.00	0.00	0.00	4.26	0.00	0.00	0.00	2.95	0.00	0.00	0.00	1.60	0.00	0.00	0.00
MSFT	8.96	0.00	0.00	0.00	6.51	0.00	0.00	0.00	4.56	0.00	0.00	0.00	2.75	0.00	0.00	0.00
GE	5.55	**0.25**	0.00	0.00	3.00	**0.70**	0.00	0.00	1.78	0.00	0.00	0.00	1.10	0.00	0.00	0.00
JNJ	5.24	**0.62**	0.00	0.00	3.21	**0.07**	0.00	0.00	1.13	**0.63**	0.00	0.00	1.20	0.00	0.00	0.00
WMT	6.91	0.00	0.00	0.00	4.76	0.00	0.00	0.00	3.55	0.00	0.00	0.00	1.20	0.00	0.00	0.00
CVX	6.61	0.00	0.00	0.00	3.10	**0.24**	0.00	0.00	0.85	**0.49**	0.00	0.00	0.30	**0.17**	0.00	0.00
JPM	7.16	0.00	0.00	0.00	4.36	0.00	0.00	0.00	2.80	0.00	0.00	0.00	1.55	0.00	0.00	0.00
PG	6.71	0.00	0.00	0.00	4.96	0.00	0.00	0.00	3.15	0.00	0.00	0.00	2.10	0.00	0.00	0.00
PFE	3.20	0.00	0.00	0.00	1.45	0.00	0.00	0.00	1.00	**0.99**	0.00	0.00	0.45	**0.75**	0.00	0.00
IBM	7.86	0.00	0.00	0.00	6.01	0.00	0.00	0.00	3.81	0.00	0.00	0.00	2.30	0.00	0.00	0.00
T	2.55	0.00	0.00	0.00	1.10	0.00	0.00	0.00	0.75	**0.24**	0.00	0.00	0.15	0.01	0.00	0.00
KO	5.05	**0.95**	0.00	0.00	2.89	**0.71**	0.00	0.00	1.15	**0.61**	0.00	0.00	0.66	**0.70**	0.00	0.00
MRK	11.22	0.00	0.00	0.00	8.11	0.00	0.00	0.00	5.76	0.00	0.00	0.00	3.80	0.00	0.00	0.00
VZ	5.23	**0.68**	0.00	0.00	2.05	**0.08**	0.00	0.00	1.03	**0.97**	0.00	0.00	0.62	**0.88**	0.00	0.00
DIS	6.81	0.00	0.00	0.00	4.86	0.00	0.00	0.00	3.30	0.00	0.00	0.00	2.00	0.00	0.00	0.00
INTC	7.36	0.00	0.00	0.00	4.91	0.00	0.00	0.00	3.76	0.00	0.00	0.00	2.20	0.00	0.00	0.00
CSCO	7.91	0.00	0.00	0.00	5.81	0.00	0.00	0.00	3.96	0.00	0.00	0.00	2.40	0.00	0.00	0.00
HD	9.36	0.00	0.00	0.00	7.31	0.00	0.00	0.00	5.21	0.00	0.00	0.00	2.80	0.00	0.00	0.00
UTX	2.28	0.00	0.00	0.00	1.59	0.00	0.00	0.00	0.78	**0.23**	0.00	0.00	0.13	**0.03**	0.00	0.00
BA	5.31	**0.53**	0.00	0.00	3.76	0.00	0.00	0.00	2.65	0.00	0.00	0.00	1.25	0.00	0.00	0.00
MCD	5.18	**0.78**	0.00	0.00	1.74	0.00	0.00	0.00	0.88	**0.65**	0.00	0.00	0.55	**0.95**	0.00	0.00
AXP	6.96	0.00	0.00	0.00	4.66	0.00	0.00	0.00	2.70	0.00	0.00	0.00	1.25	0.00	0.00	0.00
MMM	4.36	**0.18**	0.00	0.00	2.65	**0.66**	0.00	0.00	1.60	0.01	0.00	0.00	0.85	**0.04**	**0.59**	**0.11**
GS	4.71	**0.54**	0.00	0.00	2.70	**0.56**	0.00	0.00	1.30	**0.20**	0.00	0.00	0.80	**0.08**	0.00	0.00
UNH	5.18	**0.78**	0.00	0.00	6.15	0.00	0.00	0.00	2.93	0.00	0.00	0.00	0.64	**0.86**	0.00	0.00
CAT	2.90	0.00	0.01	0.00	1.70	**0.02**	**0.02**	0.00	1.05	**0.82**	**0.22**	**0.46**	0.60	**0.54**	**0.70**	**0.77**
DD	4.91	**0.85**	0.00	0.00	3.36	**0.02**	0.00	0.00	2.35	0.00	0.00	0.00	1.35	0.00	0.00	0.00
NKE	10.27	0.00	0.00	0.00	7.61	0.00	0.00	0.00	5.41	0.00	0.00	0.00	2.85	0.00	0.00	0.00
TRV	4.71	**0.60**	0.00	0.00	2.85	**0.09**	0.00	0.00	0.97	**0.97**	0.00	0.00	0.88	**0.37**	0.00	0.00

Table 4.77: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(0,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	6.51	0.00	0.00	0.00	4.26	0.00	0.00	0.00	2.96	0.00	0.00	0.00	1.60	0.00	0.00	0.00
MSFT	8.97	0.00	0.00	0.00	6.51	0.00	0.00	0.00	4.56	0.00	0.00	0.00	2.75	0.00	0.00	0.00
GE	5.50	**0.30**	0.00	0.00	3.00	**0.70**	0.00	0.00	1.75	0.00	0.00	0.00	1.10	0.00	0.00	0.00
JNJ	5.26	**0.60**	0.00	0.00	3.20	**0.07**	0.00	0.00	1.02	**0.98**	0.00	0.00	1.20	0.00	0.00	0.00
WMT	6.91	0.00	0.00	0.00	4.76	0.00	0.00	0.00	3.56	0.00	0.00	0.00	1.20	0.00	0.00	0.00
CVX	6.61	0.00	0.00	0.00	3.10	**0.24**	0.00	0.00	0.85	**0.49**	0.00	0.00	0.30	**0.17**	0.00	0.00
JPM	7.16	0.00	0.00	0.00	4.36	0.00	0.00	0.00	2.81	0.00	0.00	0.00	1.55	0.00	0.00	0.00
PG	6.71	0.00	0.00	0.00	4.96	0.00	0.00	0.00	3.11	0.00	0.00	0.00	2.10	0.00	0.00	0.00
PFE	3.21	0.00	0.00	0.00	1.45	0.00	0.00	0.00	1.00	**0.99**	0.00	0.00	0.45	**0.75**	0.00	0.00
IBM	7.87	0.00	0.00	0.00	6.01	0.00	0.00	0.00	3.81	0.00	0.00	0.00	2.30	0.00	0.00	0.00
T	2.56	0.00	0.00	0.00	1.10	0.00	0.00	0.00	0.75	**0.24**	0.00	0.00	0.15	0.01	0.00	0.00
KO	5.05	**0.95**	0.00	0.00	2.87	**0.73**	0.00	0.00	1.13	**0.59**	0.00	0.00	0.65	**0.70**	0.00	0.00
MRK	11.22	0.00	0.00	0.00	8.12	0.00	0.00	0.00	5.76	0.00	0.00	0.00	3.81	0.00	0.00	0.00
VZ	5.20	**0.65**	0.00	0.00	2.01	**0.10**	0.00	0.00	1.01	**0.99**	0.00	0.00	0.61	**0.89**	0.00	0.00
DIS	6.76	0.00	0.00	0.00	4.86	0.00	0.00	0.00	3.31	0.00	0.00	0.00	2.00	0.00	0.00	0.00
INTC	7.36	0.00	0.00	0.00	4.91	0.00	0.00	0.00	3.76	0.00	0.00	0.00	2.20	0.00	0.00	0.00
CSCO	7.92	0.00	0.00	0.00	5.81	0.00	0.00	0.00	3.96	0.00	0.00	0.00	2.40	0.00	0.00	0.00
HD	9.37	0.00	0.00	0.00	7.31	0.00	0.00	0.00	5.21	0.00	0.00	0.00	2.80	0.00	0.00	0.00
UTX	2.25	0.00	0.00	0.00	1.53	0.00	0.00	0.00	0.75	**0.25**	0.00	0.00	0.18	0.01	0.00	0.00
BA	5.31	**0.53**	0.00	0.00	3.76	0.00	0.00	0.00	2.66	0.00	0.00	0.00	1.25	0.00	0.00	0.00
MCD	5.14	**0.74**	0.00	0.00	1.69	0.00	0.00	0.00	0.91	**0.68**	0.00	0.00	0.53	**0.97**	0.00	0.00
AXP	6.91	0.00	0.00	0.00	4.66	0.00	0.00	0.00	2.71	0.00	0.00	0.00	1.25	0.00	0.00	0.00
MMM	4.36	**0.18**	0.00	0.00	2.66	**0.66**	0.00	0.00	1.60	0.01	0.00	0.00	0.85	**0.04**	**0.59**	**0.11**
GS	4.71	**0.55**	0.00	0.00	2.71	**0.56**	0.00	0.00	1.30	**0.19**	0.00	0.00	0.80	**0.08**	0.00	0.00
UNH	5.10	**0.90**	0.00	0.00	6.11	0.00	0.00	0.00	2.91	0.00	0.00	0.00	0.62	**0.88**	0.00	0.00
CAT	2.91	0.00	0.01	0.00	1.70	**0.02**	**0.02**	0.00	1.05	**0.82**	**0.22**	**0.46**	0.60	**0.53**	**0.70**	**0.77**
DD	4.91	**0.85**	0.00	0.00	3.36	**0.02**	0.00	0.00	2.35	0.00	0.00	0.00	1.35	0.00	0.00	0.00
NKE	10.22	0.00	0.00	0.00	7.62	0.00	0.00	0.00	5.41	0.00	0.00	0.00	2.85	0.00	0.00	0.00
TRV	4.50	**0.31**	0.00	0.00	2.78	**0.11**	0.00	0.00	0.95	**0.95**	0.00	0.00	0.85	**0.40**	0.00	0.00

Table 4.78: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(0,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	6.52	0.00	0.00	0.00	4.26	0.00	0.00	0.00	2.96	0.00	0.00	0.00	1.60	0.00	0.00	0.00
MSFT	8.97	0.00	0.00	0.00	6.52	0.00	0.00	0.00	4.56	0.00	0.00	0.00	2.76	0.00	0.00	0.00
GE	5.50	**0.30**	0.00	0.00	3.00	**0.70**	0.00	0.00	1.75	0.00	0.00	0.00	1.10	0.00	0.00	0.00
JNJ	5.26	**0.60**	0.00	0.00	3.20	**0.07**	0.00	0.00	1.02	**0.98**	0.00	0.00	1.20	0.00	0.00	0.00
WMT	6.92	0.00	0.00	0.00	4.76	0.00	0.00	0.00	3.56	0.00	0.00	0.00	1.20	0.00	0.00	0.00
CVX	6.61	0.00	0.00	0.00	3.11	**0.25**	0.00	0.00	0.85	**0.50**	0.00	0.00	0.30	**0.17**	0.00	0.00
JPM	7.17	0.00	0.00	0.00	4.36	0.00	0.00	0.00	2.81	0.00	0.00	0.00	1.55	0.00	0.00	0.00
PG	6.72	0.00	0.00	0.00	4.96	0.00	0.00	0.00	3.11	0.00	0.00	0.00	2.10	0.00	0.00	0.00
PFE	3.21	0.00	0.00	0.00	1.45	0.00	0.00	0.00	1.00	**0.99**	0.00	0.00	0.45	**0.75**	0.00	0.00
IBM	7.87	0.00	0.00	0.00	6.02	0.00	0.00	0.00	3.81	0.00	0.00	0.00	2.30	0.00	0.00	0.00
T	2.56	0.00	0.00	0.00	1.10	0.00	0.00	0.00	0.75	**0.24**	0.00	0.00	0.15	0.01	0.00	0.00
KO	5.05	**0.95**	0.00	0.00	2.87	**0.73**	0.00	0.00	1.13	**0.59**	0.00	0.00	0.60	**0.65**	0.00	0.00
MRK	11.23	0.00	0.00	0.00	8.12	0.00	0.00	0.00	5.76	0.00	0.00	0.00	3.81	0.00	0.00	0.00
VZ	5.20	**0.65**	0.00	0.00	2.01	**0.10**	0.00	0.00	1.01	**0.99**	0.00	0.00	0.61	**0.89**	0.00	0.00
DIS	6.77	0.00	0.00	0.00	4.86	0.00	0.00	0.00	3.31	0.00	0.00	0.00	2.00	0.00	0.00	0.00
INTC	7.37	0.00	0.00	0.00	4.91	0.00	0.00	0.00	3.76	0.00	0.00	0.00	2.20	0.00	0.00	0.00
CSCO	7.92	0.00	0.00	0.00	5.81	0.00	0.00	0.00	3.96	0.00	0.00	0.00	2.40	0.00	0.00	0.00
HD	9.37	0.00	0.00	0.00	7.32	0.00	0.00	0.00	5.21	0.00	0.00	0.00	2.81	0.00	0.00	0.00
UTX	2.25	0.00	0.00	0.00	1.53	0.00	0.00	0.00	0.75	**0.25**	0.00	0.00	0.18	0.01	0.00	0.00
BA	5.31	**0.52**	0.00	0.00	3.76	0.00	0.00	0.00	2.66	0.00	0.00	0.00	1.25	0.00	0.00	0.00
MCD	5.14	**0.74**	0.00	0.00	1.69	0.00	0.00	0.00	0.91	**0.68**	0.00	0.00	0.53	**0.97**	0.00	0.00
AXP	6.92	0.00	0.00	0.00	4.66	0.00	0.00	0.00	2.71	0.00	0.00	0.00	1.25	0.00	0.00	0.00
MMM	4.36	**0.18**	0.00	0.00	2.66	**0.66**	0.00	0.00	1.60	0.01	0.00	0.00	0.85	**0.04**	**0.59**	**0.11**
GS	4.71	**0.55**	0.00	0.00	2.71	**0.56**	0.00	0.00	1.30	**0.19**	0.00	0.00	0.80	**0.08**	0.00	0.00
UNH	5.10	**0.90**	0.00	0.00	6.11	0.00	0.00	0.00	2.91	0.00	0.00	0.00	0.62	**0.88**	0.00	0.00
CAT	2.91	0.00	0.01	0.00	1.70	**0.02**	**0.02**	0.00	1.05	**0.81**	**0.22**	**0.45**	0.60	**0.53**	**0.70**	**0.77**
DD	4.91	**0.86**	0.00	0.00	3.36	**0.02**	0.00	0.00	2.36	0.00	0.00	0.00	1.35	0.00	0.00	0.00
NKE	10.28	0.00	0.00	0.00	7.62	0.00	0.00	0.00	5.41	0.00	0.00	0.00	2.86	0.00	0.00	0.00
TRV	4.67	**0.52**	0.00	0.00	2.78	**0.11**	0.00	0.00	0.95	**0.95**	0.00	0.00	0.85	**0.40**	0.00	0.00

4.7.17 Interval Forecast Evaluation of SVX with Skewed Students't Distributed Errors, Mean Model = ARMA(2,0)

Table 4.79: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(2,0)

	h = 1															
Stocks	**100(α) = 5**				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	**% Viol.**	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	4.35	**0.17**	**0.53**	**0.33**	2.35	**0.67**	**0.92**	**0.91**	1.25	**0.28**	**0.32**	**0.34**	0.45	**0.75**	**0.78**	**0.91**
MSFT	9.50	0.00	**0.05**	0.00	6.65	0.00	**0.16**	0.00	4.05	0.00	**0.69**	0.00	2.10	0.00	**0.07**	0.00
GE	5.50	**0.30**	**0.87**	**0.89**	3.05	**0.65**	**0.91**	**0.90**	1.15	**0.95**	**0.79**	**0.91**	0.80	**0.75**	**0.80**	**0.92**
JNJ	5.15	**0.80**	**0.53**	**0.71**	3.15	**0.56**	**0.51**	**0.74**	1.10	**0.97**	**0.59**	**0.77**	0.75	**0.78**	**0.69**	**0.80**
WMT	6.02	**0.03**	**0.71**	**0.09**	3.05	**0.11**	**0.63**	**0.25**	1.50	**0.05**	**0.73**	**0.62**	0.60	**0.90**	**0.45**	**0.75**
CVX	5.00	**1.00**	**0.89**	**0.71**	2.08	**0.09**	**0.81**	**0.70**	0.83	**0.60**	**0.83**	**0.89**	0.50	**1.00**	**0.89**	**0.91**
JPM	5.50	**0.30**	**0.90**	**0.87**	2.62	**0.70**	**0.89**	**0.87**	1.00	**1.00**	**0.74**	**0.90**	0.60	**0.90**	**0.81**	**0.95**
PG	8.75	0.00	**0.93**	0.00	6.15	0.00	**0.59**	0.00	4.10	0.00	**0.40**	0.00	1.95	0.00	**0.21**	0.00
PFE	4.75	**0.57**	**0.88**	**0.89**	2.50	**1.00**	**0.85**	**0.89**	0.93	**0.67**	**0.76**	**0.88**	0.35	**0.82**	**0.86**	**0.93**
IBM	7.40	0.00	**0.03**	0.00	5.15	0.00	**0.25**	0.00	2.85	0.00	**0.77**	0.00	1.25	0.00	**0.04**	0.00
T	4.50	**0.49**	**0.72**	**0.55**	1.10	0.00	**0.67**	0.00	0.75	**0.58**	**0.68**	**0.45**	0.15	**0.02**	**0.73**	**0.45**
KO	4.80	**0.77**	**0.77**	**0.85**	2.45	**0.95**	**0.84**	**0.92**	1.25	**0.80**	**0.86**	**0.89**	0.61	**0.90**	**0.86**	**0.92**
MRK	13.91	0.00	0.00	0.00	8.95	0.00	0.00	0.00	5.90	0.00	0.00	0.00	3.30	0.00	0.00	0.00
VZ	5.39	**0.78**	**0.86**	**0.89**	2.10	**0.71**	**0.85**	**0.95**	1.10	**0.97**	**0.57**	**0.75**	0.65	**0.87**	**0.67**	**0.81**
DIS	5.85	**0.09**	**0.67**	**0.21**	3.65	0.00	**0.85**	0.01	1.75	0.00	**0.26**	0.01	0.90	**0.04**	**0.58**	**0.11**
INTC	8.00	0.00	**0.08**	0.00	4.65	0.00	**0.87**	0.00	2.40	0.00	**0.46**	0.00	1.05	0.00	**0.22**	0.00
CSCO	8.05	0.00	**0.24**	0.00	5.55	0.00	**0.25**	0.00	2.85	0.00	**0.03**	0.00	1.25	0.00	**0.32**	0.00
HD	9.70	0.00	**0.03**	0.00	6.55	0.00	**0.24**	0.00	4.10	0.00	**0.17**	0.00	1.75	0.00	**0.26**	0.00
UTX	2.00	0.00	**0.82**	0.00	1.50	0.00	**0.70**	0.00	0.71	**0.78**	**0.87**	**0.81**	0.39	**0.60**	**0.81**	**0.79**
BA	5.50	**0.31**	**0.98**	**0.60**	3.00	**0.16**	**0.40**	**0.27**	1.55	**0.02**	**0.51**	**0.06**	0.80	**0.08**	**0.61**	**0.19**
MCD	5.20	**0.50**	**0.84**	0.00	1.80	**0.05**	**0.87**	**0.60**	0.90	**0.80**	**0.83**	**0.89**	0.49	**0.98**	**0.81**	**0.92**
AXP	7.10	0.00	**0.05**	0.00	4.73	0.00	**0.00**	0.00	2.89	0.00	**0.02**	0.00	1.25	0.00	**0.79**	0.00
MMM	2.50	0.00	**0.81**	0.00	1.35	0.00	**0.39**	0.00	0.90	**0.65**	**0.57**	**0.77**	0.35	**0.32**	**0.82**	**0.59**
GS	5.05	**0.95**	**0.39**	**0.45**	2.25	**0.57**	**0.75**	**0.52**	0.87	**0.63**	**0.70**	**0.87**	0.44	**0.92**	**0.85**	**0.89**
UNH	5.08	**0.97**	**0.82**	**0.93**	6.95	**0.09**	**0.97**	**0.78**	2.85	0.00	**0.85**	**0.89**	0.43	**0.92**	**0.87**	**0.94**
CAT	2.10	0.00	**0.90**	0.00	1.15	0.00	**0.46**	0.00	0.55	**0.03**	**0.73**	**0.08**	0.00	0.00	**0.60**	0.00
DD	2.80	0.00	**0.09**	0.00	1.50	0.00	**0.47**	0.01	0.85	**0.49**	**0.59**	**0.68**	0.05	0.00	**0.97**	0.00
NKE	12.21	0.00	**0.14**	0.00	8.90	0.00	**0.75**	0.00	5.55	0.00	**0.94**	0.00	2.55	0.00	**0.78**	0.00
TRV	3.95	**0.05**	**0.62**	**0.15**	2.50	**1.00**	**0.71**	**0.65**	0.79	**0.72**	**0.72**	**0.69**	0.83	**0.50**	**0.78**	**0.79**

Table 4.80: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(2,0)

Stocks	**h = 2**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**	% Viol.	**LR_{uc}**	**LR_{ind}**	**LR_{cc}**
XOM	3.85	0.01	**0.26**	**0.03**	1.95	**0.10**	**0.21**	**0.12**	0.95	**0.82**	**0.55**	**0.81**	0.40	**0.51**	**0.80**	**0.78**
MSFT	9.65	0.00	0.00	0.00	6.60	0.00	**0.07**	0.00	3.90	0.00	**0.59**	0.00	2.20	0.00	**0.09**	0.00
GE	5.45	**0.25**	**0.88**	**0.93**	2.20	**0.76**	**0.89**	**0.89**	1.13	**0.97**	**0.82**	**0.95**	0.82	**0.69**	**0.78**	**0.90**
JNJ	5.10	**0.83**	**0.64**	**0.74**	2.15	**0.64**	**0.54**	**0.75**	1.15	**0.95**	**0.57**	**0.81**	0.72	**0.69**	**0.76**	**0.87**
WMT	6.00	**0.04**	**0.75**	**0.16**	3.10	**0.10**	**0.68**	**0.35**	1.47	**0.07**	**0.80**	**0.71**	0.59	**0.91**	**0.56**	**0.79**
CVX	4.97	**0.98**	**0.92**	**0.95**	2.07	**0.08**	**0.79**	**0.75**	0.87	**0.63**	**0.82**	**0.92**	0.49	**0.99**	**0.85**	**0.93**
JPM	5.45	**0.27**	**0.87**	**0.89**	2.60	**0.69**	**0.83**	**0.89**	1.00	**1.00**	**0.78**	**0.92**	0.58	**0.89**	**0.84**	**0.91**
PG	7.20	0.00	**0.64**	0.00	5.00	0.00	**1.00**	0.00	3.20	0.00	**0.40**	0.00	1.35	0.00	**0.39**	0.00
PFE	4.72	**0.54**	**0.89**	**0.90**	2.55	**0.95**	**0.87**	**0.92**	0.90	**0.66**	**0.80**	**0.89**	0.37	**0.83**	**0.86**	**0.94**
IBM	7.60	0.00	0.00	0.00	5.20	0.00	**0.13**	0.00	2.90	0.00	**0.81**	0.00	1.25	0.00	**0.04**	0.00
T	4.55	**0.50**	**0.78**	**0.63**	1.08	0.00	**0.63**	**0.21**	0.76	**0.59**	**0.75**	**0.58**	0.15	**0.02**	**0.75**	**0.46**
KO	4.85	**0.80**	**0.79**	**0.87**	2.50	**1.00**	**0.83**	**0.92**	1.21	**0.82**	**0.85**	**0.90**	0.55	**0.95**	**0.89**	**0.91**
MRK	13.96	0.00	0.00	0.00	8.90	0.00	0.00	0.00	5.95	0.00	0.00	0.00	3.30	0.00	0.00	0.00
VZ	4.85	**0.80**	**0.90**	**0.92**	2.08	**0.70**	**0.85**	**0.94**	1.00	**1.00**	**0.54**	**0.83**	0.63	**0.86**	**0.79**	**0.85**
DIS	4.90	**0.76**	**0.91**	**0.95**	3.10	**0.13**	**0.46**	**0.24**	1.75	0.00	**0.63**	0.01	0.80	**0.14**	**0.62**	**0.30**
INTC	8.20	0.00	**0.07**	0.00	4.90	0.00	**0.58**	0.00	2.65	0.00	**0.63**	0.00	1.10	0.00	**0.02**	0.00
CSCO	7.35	0.00	**0.49**	0.00	4.60	0.00	**0.19**	0.00	2.25	0.00	**0.37**	0.00	1.05	0.00	**0.22**	0.00
HD	9.55	0.00	**0.03**	0.00	6.05	0.00	**0.17**	0.00	4.00	0.00	**0.14**	0.00	1.85	0.00	**0.72**	0.00
UTX	2.11	0.00	**0.83**	0.00	1.65	0.01	**0.72**	**0.77**	0.77	**0.83**	**0.86**	**0.84**	0.45	**0.96**	**0.87**	**0.89**
BA	5.75	**0.13**	**0.49**	**0.25**	3.25	**0.04**	**0.55**	**0.10**	1.85	0.00	**0.72**	0.00	0.75	**0.14**	**0.63**	**0.30**
MCD	2.18	0.00	**0.85**	0.00	1.91	**0.08**	**0.85**	**0.66**	0.90	**0.85**	**0.85**	**0.91**	0.50	**1.00**	**0.81**	**0.94**
AXP	7.00	0.00	**0.06**	0.00	4.80	0.00	**0.07**	0.00	2.85	0.00	**0.02**	0.00	1.27	0.00	**0.71**	0.00
MMM	2.55	0.00	**0.78**	0.00	1.35	0.00	**0.38**	0.00	1.00	**1.00**	**0.52**	**0.82**	0.35	**0.32**	**0.82**	**0.59**
GS	5.03	**0.97**	**0.43**	**0.54**	2.20	**0.60**	**0.83**	**0.64**	0.87	**0.63**	**0.72**	**0.85**	0.40	**0.85**	**0.87**	**0.92**
UNH	5.01	**1.00**	**0.94**	**0.96**	2.02	**0.30**	**0.95**	**0.86**	0.88	**0.80**	**0.57**	**0.91**	0.55	**0.95**	**0.80**	**0.91**
CAT	1.85	0.00	**0.24**	0.00	0.75	0.00	**0.63**	0.00	0.45	0.01	**0.78**	**0.02**	0.00	0.00	**0.63**	0.00
DD	2.65	0.00	**0.23**	0.00	1.60	0.01	**0.54**	**0.02**	0.75	**0.24**	**0.63**	**0.45**	0.05	0.00	**0.97**	0.00
NKE	11.26	0.00	**0.15**	0.00	7.80	0.00	**0.58**	0.00	4.85	0.00	**0.72**	0.00	2.10	0.00	**0.90**	0.00
TRV	4.02	**0.07**	**0.67**	**0.35**	1.99	**0.20**	**0.74**	**0.69**	0.83	**0.81**	**0.77**	**0.73**	0.45	**0.96**	**0.88**	**0.89**

Table 4.81: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(2,0)

Stocks	**h = 3**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	3.85	0.01	**0.26**	**0.03**	2.10	**0.24**	**0.90**	**0.50**	1.00	**1.00**	**0.52**	**0.82**	0.35	**0.32**	**0.82**	**0.59**
MSFT	9.61	0.00	0.00	0.00	6.61	0.00	**0.08**	0.00	3.95	0.00	**0.31**	0.00	2.15	0.00	**0.08**	0.00
GE	5.45	**0.25**	**0.90**	**0.95**	2.10	**0.80**	**0.89**	**0.91**	1.17	**0.91**	**0.84**	**0.92**	0.90	**0.59**	**0.70**	**0.81**
JNJ	5.11	**0.76**	**0.53**	**0.71**	2.15	**0.64**	**0.51**	**0.74**	1.10	**0.97**	**0.59**	**0.77**	0.75	**0.78**	**0.69**	**0.80**
WMT	6.05	**0.02**	**0.72**	**0.18**	3.20	**0.08**	**0.72**	**0.31**	1.57	**0.05**	**0.81**	**0.54**	0.68	**0.79**	**0.65**	**0.74**
CVX	4.97	**0.98**	**0.89**	**0.91**	2.06	**0.07**	**0.81**	**0.74**	0.87	**0.63**	**0.82**	**0.92**	0.49	**0.99**	**0.85**	**0.93**
JPM	5.45	**0.27**	**0.87**	**0.89**	2.60	**0.69**	**0.83**	**0.89**	1.00	**1.00**	**0.78**	**0.92**	0.58	**0.89**	**0.84**	**0.91**
PG	7.36	0.00	**0.79**	0.00	5.26	0.00	**0.81**	0.00	3.40	0.00	**0.32**	0.00	1.50	0.00	**0.34**	0.00
PFE	4.89	**0.79**	**0.82**	**0.81**	2.68	**0.62**	**0.87**	**0.90**	1.00	**1.00**	**0.82**	**0.94**	0.45	**0.90**	**0.90**	**0.72**
IBM	7.61	0.00	0.00	0.00	5.26	0.00	**0.15**	0.00	2.90	0.00	**0.81**	0.00	1.25	0.00	**0.04**	0.00
T	4.75	**0.65**	**0.88**	**0.69**	1.00	0.00	**0.67**	0.00	0.83	**0.60**	**0.82**	**0.61**	0.10	0.01	**0.54**	**0.20**
KO	5.00	**1.00**	**0.89**	**0.95**	2.50	**1.00**	**0.90**	**0.94**	1.15	**0.95**	**0.87**	**0.94**	0.50	**1.00**	**0.90**	**0.95**
MRK	13.91	0.00	0.00	0.00	8.96	0.00	0.01	0.00	5.96	0.00	0.00	0.00	3.30	0.00	0.00	0.00
VZ	4.85	**0.80**	**0.91**	**0.92**	2.00	**0.25**	**0.88**	**0.89**	1.21	**0.82**	**0.69**	**0.72**	0.70	**0.81**	**0.81**	**0.82**
DIS	5.11	**0.91**	**0.94**	**0.99**	3.10	**0.12**	**0.46**	**0.24**	1.80	0.00	**0.66**	0.01	0.80	**0.14**	**0.62**	**0.30**
INTC	8.21	0.00	**0.07**	0.00	4.90	0.00	**0.58**	0.00	2.65	0.00	**0.63**	0.00	1.10	0.00	**0.02**	0.00
CSCO	7.31	0.00	**0.67**	0.00	4.65	0.00	**0.09**	0.00	2.35	0.00	**0.43**	0.00	1.05	0.00	**0.22**	0.00
HD	9.56	0.00	**0.03**	0.00	6.06	0.00	**0.18**	0.00	4.00	0.00	**0.14**	0.00	1.85	0.00	**0.72**	0.00
UTX	2.25	0.00	**0.84**	0.00	1.70	**0.03**	**0.87**	0.00	0.85	**0.87**	**0.88**	**0.90**	0.50	**1.00**	**0.89**	**0.97**
BA	5.76	**0.13**	**0.49**	**0.25**	3.25	**0.04**	**0.55**	**0.10**	1.85	0.00	**0.72**	0.00	0.75	**0.14**	**0.63**	**0.30**
MCD	2.50	0.00	**0.88**	0.00	2.20	**0.76**	**0.84**	**0.74**	1.00	**1.00**	**0.86**	**0.93**	0.54	**0.96**	**0.83**	**0.92**
AXP	6.85	0.00	**0.08**	0.00	4.53	0.00	**0.09**	0.00	2.71	0.00	**0.03**	0.00	1.10	0.00	**0.54**	0.00
MMM	2.50	0.00	**0.81**	0.00	1.35	0.00	**0.38**	0.00	1.00	**0.99**	**0.52**	**0.82**	0.35	**0.32**	**0.82**	**0.59**
GS	5.00	**1.00**	**0.54**	**0.68**	2.35	**0.75**	**0.87**	**0.72**	0.71	**0.55**	**0.74**	**0.81**	0.25	**0.10**	**0.81**	**0.81**
UNH	5.00	**1.00**	**0.98**	**0.97**	2.00	**0.25**	**0.89**	**0.85**	0.90	**0.81**	**0.87**	**0.93**	0.50	**1.00**	**0.90**	**0.93**
CAT	1.85	0.00	0.00	0.00	0.75	0.00	**0.63**	0.00	0.45	0.01	**0.78**	**0.02**	0.00	0.00	**0.72**	0.00
DD	2.65	0.00	**0.23**	0.00	1.60	0.01	**0.54**	**0.02**	0.75	**0.24**	**0.63**	**0.45**	0.05	0.00	**0.97**	0.00
NKE	11.36	0.00	**0.18**	0.00	7.91	0.00	**0.65**	0.00	4.90	0.00	**0.93**	0.00	2.10	0.00	**0.90**	0.00
TRV	4.25	**0.20**	**0.68**	**0.54**	1.99	**0.20**	**0.72**	**0.70**	0.83	**0.81**	**0.79**	**0.75**	0.31	**0.45**	**0.79**	**0.82**

Table 4.82: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(2,0)

Stocks	**h = 4**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}	% Viol.	LR_{uc}	LR_{ind}	LR_{cc}
XOM	3.86	0.01	**0.26**	**0.03**	2.10	**0.24**	**0.90**	**0.50**	1.00	**0.99**	**0.52**	**0.82**	0.35	**0.32**	**0.82**	**0.59**
MSFT	9.56	0.00	0.00	0.00	6.61	0.00	**0.08**	0.00	3.96	0.00	**0.31**	0.00	2.15	0.00	**0.08**	0.00
GE	5.43	**0.23**	**0.95**	**0.90**	2.10	**0.80**	**0.86**	**0.88**	1.20	**0.90**	**0.89**	**0.93**	0.90	**0.59**	**0.73**	**0.79**
JNJ	5.10	**0.83**	**0.67**	**0.71**	2.13	**0.72**	**0.55**	**0.80**	1.11	**0.96**	**0.73**	**0.81**	0.77	**0.72**	**0.75**	**0.81**
WMT	6.25	0.01	**0.78**	**0.19**	3.35	**0.06**	**0.81**	**0.45**	1.62	**0.04**	**0.79**	**0.50**	0.75	**0.50**	**0.54**	**0.55**
CVX	4.68	**0.73**	**0.70**	**0.81**	2.25	**0.60**	**0.74**	**0.84**	1.00	**1.00**	**0.85**	**0.94**	0.62	**0.70**	**0.78**	**0.91**
JPM	5.25	**0.61**	**0.91**	**0.94**	2.40	**0.90**	**0.85**	**0.91**	0.82	**0.59**	**0.80**	**0.87**	0.50	**1.00**	**0.88**	**0.93**
PG	7.36	0.00	**0.78**	0.00	5.26	0.00	**0.81**	0.00	3.41	0.00	**0.32**	0.00	1.45	0.00	**0.36**	0.00
PFE	5.00	**1.00**	**0.84**	**0.72**	2.45	**0.94**	**0.84**	**0.96**	1.18	**0.64**	**0.72**	**0.87**	0.57	**0.88**	**0.81**	**0.79**
IBM	7.61	0.00	0.00	0.00	5.26	0.00	**0.15**	0.00	2.90	0.00	**0.81**	0.00	1.25	0.00	**0.04**	0.00
T	4.81	**0.70**	**0.88**	**0.72**	1.13	0.00	**0.68**	0.00	0.71	**0.55**	**0.74**	**0.51**	0.22	**0.09**	**0.62**	**0.38**
KO	5.05	**0.95**	**0.89**	**0.92**	2.50	**1.00**	**0.83**	**0.93**	1.00	**1.00**	**0.75**	**0.82**	0.50	**1.00**	**0.87**	**0.90**
MRK	13.92	0.00	0.00	0.00	8.96	0.00	0.01	0.00	5.96	0.00	0.00	0.00	3.30	0.00	**0.00**	0.00
VZ	4.90	**0.82**	**0.89**	**0.92**	2.02	**0.66**	**0.88**	**0.90**	1.16	**0.87**	**0.65**	**0.71**	0.68	**0.79**	**0.89**	**0.78**
DIS	5.06	**0.98**	**0.98**	**1.00**	3.10	**0.12**	**0.46**	**0.23**	1.80	0.00	**0.66**	0.01	0.80	**0.14**	**0.62**	**0.30**
INTC	8.21	0.00	**0.07**	0.00	4.91	0.00	**0.58**	0.00	2.65	0.00	**0.63**	0.00	1.10	0.00	**0.02**	0.00
CSCO	7.36	0.00	**0.49**	0.00	4.66	0.00	**0.09**	0.00	2.35	0.00	**0.43**	0.00	1.05	0.00	**0.22**	0.00
HD	9.56	0.00	**0.03**	0.00	6.06	0.00	**0.18**	0.00	4.01	0.00	**0.14**	0.00	1.85	0.00	**0.72**	0.00
UTX	2.10	0.00	**0.82**	0.00	1.67	**0.02**	**0.85**	**0.79**	0.70	**0.69**	**0.76**	**0.80**	0.50	**1.00**	**0.92**	**0.95**
BA	5.76	**0.13**	**0.49**	**0.24**	3.25	**0.04**	**0.55**	**0.10**	1.85	0.00	**0.72**	0.00	0.75	**0.14**	**0.63**	**0.30**
MCD	2.50	0.00	**0.87**	0.00	2.00	**0.65**	**0.87**	**0.76**	1.00	**1.00**	**0.89**	**0.93**	0.55	**0.95**	**0.85**	**0.89**
AXP	7.10	0.00	**0.09**	0.00	4.89	0.00	**0.09**	0.00	3.01	0.00	**0.04**	0.00	1.36	0.00	**0.51**	0.00
MMM	2.50	0.00	**0.81**	0.00	1.35	0.00	**0.38**	0.00	1.00	**0.99**	**0.52**	**0.82**	0.35	**0.32**	**0.82**	**0.59**
GS	5.11	**0.71**	**0.74**	**0.79**	2.55	**0.64**	**0.81**	**0.62**	0.49	**0.09**	**0.44**	**0.10**	0.38	**0.71**	**0.87**	**0.84**
UNH	4.98	**0.99**	**0.98**	**0.97**	2.10	**0.80**	**0.82**	**0.75**	1.00	**1.00**	**0.86**	**0.95**	0.49	**0.99**	**0.87**	**0.90**
CAT	1.85	0.00	0.00	0.00	0.75	0.00	**0.63**	0.00	0.45	0.01	**0.78**	**0.02**	0.00	0.00	**0.79**	0.00
DD	2.65	0.00	**0.23**	0.00	1.60	0.00	**0.54**	**0.02**	0.75	**0.24**	**0.63**	**0.45**	0.05	0.00	**0.97**	0.00
NKE	11.37	0.00	**0.18**	0.00	7.91	0.00	**0.65**	0.00	4.91	0.00	**0.93**	0.00	2.10	0.00	**0.90**	0.00
TRV	4.50	**0.41**	**0.56**	**0.61**	1.75	**0.08**	**0.77**	**0.67**	0.83	**0.81**	**0.74**	**0.72**	0.30	**0.44**	**0.82**	**0.85**

Table 4.83: Interval forecast evaluation using skewed Student's t-distribution, mean model = ARMA(2,0)

Stocks	**h = 5**															
	100(α) = 5				**100(α) = 2.5**				**100(α) = 1**				**100(α) = 0.5**			
	% Viol.	$\mathbf{LR_{uc}}$	$\mathbf{LR_{ind}}$	$\mathbf{LR_{cc}}$	% Viol.	$\mathbf{LR_{uc}}$	$\mathbf{LR_{ind}}$	$\mathbf{LR_{cc}}$	% Viol.	$\mathbf{LR_{uc}}$	$\mathbf{LR_{ind}}$	$\mathbf{LR_{cc}}$	% Viol.	$\mathbf{LR_{uc}}$	$\mathbf{LR_{ind}}$	$\mathbf{LR_{cc}}$
XOM	3.86	**0.02**	**0.26**	**0.03**	2.10	0.25	**0.90**	0.51	1.00	**0.99**	**0.52**	**0.82**	0.35	**0.32**	**0.82**	**0.59**
MSFT	9.57	0.00	0.00	0.00	6.61	0.00	**0.08**	0.00	3.96	0.00	**0.31**	0.00	2.15	0.00	**0.08**	0.00
GE	5.50	**0.30**	**0.91**	**0.87**	2.11	0.80	**0.88**	0.91	1.00	**1.00**	**0.82**	**0.95**	0.88	**0.56**	**0.79**	**0.77**
JNJ	5.12	**0.85**	**0.63**	**0.81**	2.10	0.80	**0.54**	0.82	1.13	**0.94**	**0.75**	**0.82**	0.70	**0.68**	**0.89**	**0.79**
WMT	6.07	**0.03**	**0.80**	**0.22**	3.50	0.01	**0.79**	0.55	1.85	0.01	**0.83**	**0.61**	0.89	**0.05**	**0.43**	**0.50**
CVX	4.75	**0.61**	**0.68**	**0.73**	2.50	1.00	**0.81**	0.89	1.11	**0.73**	**0.72**	**0.84**	0.50	**1.00**	**0.85**	**0.69**
JPM	5.15	**0.76**	**0.89**	**0.90**	2.30	0.70	**0.95**	0.85	0.75	**0.60**	**0.81**	**0.93**	0.45	**0.95**	**0.78**	**0.91**
PG	7.36	0.00	**0.78**	0.00	5.26	0.00	**0.81**	0.00	3.41	0.00	**0.32**	0.00	1.45	0.00	**0.35**	0.00
PFE	5.01	**0.98**	**0.88**	**0.80**	2.71	0.56	**0.83**	0.92	1.55	**0.05**	**0.65**	**0.28**	0.30	**0.62**	**0.85**	**0.74**
IBM	7.62	0.00	0.00	0.00	5.26	0.00	**0.15**	0.00	2.91	0.00	**0.81**	0.00	1.25	0.00	**0.04**	0.00
T	4.71	**0.55**	**0.85**	**0.63**	1.51	0.00	**0.59**	0.00	1.00	**1.00**	**0.75**	**0.82**	0.39	**0.72**	**0.78**	**0.88**
KO	5.00	**1.00**	**0.89**	**0.93**	2.48	0.98	**0.90**	0.91	1.05	**0.95**	**0.73**	**0.80**	0.48	**0.97**	**0.73**	**0.72**
MRK	13.93	0.00	0.00	0.00	8.97	0.00	0.01	0.00	5.96	0.00	0.00	0.00	3.31	0.00	0.00	0.00
VZ	4.88	**0.81**	**0.86**	**0.89**	2.00	0.65	**0.88**	0.79	1.10	**0.90**	**0.63**	**0.73**	0.55	**0.95**	**0.70**	**0.83**
DIS	5.11	**0.90**	**0.94**	**0.99**	3.11	0.12	**0.46**	0.23	1.80	0.00	**0.66**	0.01	0.80	**0.14**	**0.62**	**0.29**
INTC	8.22	0.00	**0.07**	0.00	4.91	0.00	**0.58**	0.00	2.66	0.00	**0.63**	0.00	1.10	0.00	**0.02**	0.00
CSCO	7.31	0.00	**0.67**	0.00	4.66	0.00	**0.09**	0.00	2.35	0.00	**0.43**	0.00	1.05	0.00	**0.22**	0.00
HD	9.57	0.00	**0.03**	0.00	6.06	0.00	**0.18**	0.00	4.01	0.00	**0.14**	0.00	1.85	0.00	**0.72**	0.00
UTX	2.00	0.00	**0.84**	0.00	1.50	0.00	**0.84**	0.00	0.70	**0.69**	**0.75**	**0.81**	0.35	**0.56**	**0.69**	**0.70**
BA	5.76	**0.13**	**0.48**	**0.24**	3.26	0.04	**0.55**	0.10	1.85	0.00	**0.72**	0.00	0.75	**0.14**	**0.63**	**0.30**
MCD	2.50	0.00	**0.87**	0.00	2.00	0.65	**0.88**	0.77	1.00	**1.00**	**0.86**	**0.95**	0.54	**0.94**	**0.75**	**0.80**
AXP	7.01	0.00	**0.07**	0.00	5.00	0.00	**0.06**	0.00	3.22	0.00	**0.03**	0.00	1.54	0.00	**0.44**	0.00
MMM	2.51	0.00	**0.81**	0.00	1.35	0.00	**0.38**	0.00	1.00	**0.99**	**0.52**	**0.82**	0.35	**0.32**	**0.82**	**0.59**
GS	5.00	**1.00**	**0.85**	**0.90**	2.70	0.57	**0.68**	0.78	0.65	**0.36**	**0.68**	**0.61**	0.45	**0.95**	**0.78**	**0.91**
UNH	5.00	**1.00**	**0.92**	**0.95**	2.00	0.65	**0.83**	0.78	0.85	**0.85**	**0.57**	**0.77**	0.50	**1.00**	**0.80**	**0.83**
CAT	1.85	0.00	**0.24**	0.00	0.75	0.00	**0.63**	0.00	0.45	0.01	**0.78**	**0.02**	0.00	0.00	**0.90**	0.00
DD	2.66	0.00	**0.23**	0.00	1.60	0.01	**0.54**	0.02	0.75	**0.24**	**0.63**	**0.45**	0.05	0.00	**0.83**	0.00
NKE	11.37	0.00	**0.18**	0.00	7.92	0.00	**0.65**	0.00	4.91	0.00	**0.93**	0.00	2.10	0.00	**0.90**	0.00
TRV	4.49	**0.40**	**0.57**	**0.60**	1.72	0.06	**0.72**	0.70	0.90	**0.85**	**0.76**	**0.81**	0.31	**0.43**	**0.70**	**0.55**

Chapter 5

Conclusions

5.1 Summary and contributions of this thesis

During the last two decades or so, the focus in volatility modeling has partly shifted from univariate to multivariate modeling due to the co-movements in the financial returns and financial markets. For this reason, multivariate volatility models have been taken into consideration in order to capture the dynamics of these co-movements. Numerous multivariate volatility models with different pros and cons have been proposed in the literature. As mentioned in the financial literature, the existing multivariate models are usually plagued by the curse of dimensionality, and fail to produce positive (semi) definite covariance matrices when modeling the conditional covariance matrices of high dimensional data sets. Due to these limitations, the model becomes difficult to estimate and is not useful in terms of prediction, which, in turn, makes the model inadequate for real-world applications. Among others, dimension reduction techniques can help to overcome these problems to a great extent and are thought of as a means for an efficient handling of large data sets in the sense that even hundreds of asset returns can be modeled simultaneously.

In this study, an attempt has been made to overcome the drawbacks of existing multivariate GARCH models by proposing a new parametrization to capture the dynamics of the second order moments when working with a large-dimensional data set. Based on the argument that co-volatility in financial markets is a function of some latent variables (called factors) and our proposition to model the dynamics of these factors, we can conclude that our proposed model resembles Stochastic Volatility (SV) models. The use of Singular Value Decomposition (SVD) to obtain the factors driving the volatility makes our approach innovative and different from the existing multivariate GARCH models in finance. Besides useful properties such as orthogonality and orthonormality, another valuable aspect is that it allows us to handle the rank-deficient matrices efficiently in terms of solving the least squares problems. These properties of SVD provide us with a solid foundation to distinguish the proposed model from the already existing ones, especially when considering the O-GARCH and the GO-GARCH models, which are based on eigenvalue decomposition. On one hand, the recursive use of SVD allows us to perform the extraction of volatility in a sequential manner, while, on other hand, it helps us to find out for which components of the data set there is a common factor that drives the volatility of these components. Because of the sequential extraction of volatility clustering from the data set we called our proposed model Sequential Volatility Extraction (**SVX**) model.

The results from the small scale simulation study (i.e., using a three-dimensional data set) encourages us to apply SVX model on a large scale data set (i.e., using 29-dimensional data from the Dow 30). The results from the empirical applications showed that for 29 Dow Jones stocks, we need at most three volatility factors that drive the volatility of these stocks. Furthermore, it was also found that our model performs well in terms of estimation as the number of parameters needed to estimate SVX model is much lower than that of the two competing MGARCH models, i.e., DCC-GARCH and GO-GARCH, which require large number of parameters to estimate. On the basis of such a small number of estimated parameters, we can conclude that our model overcomes the first limitation (i.e., the curse of dimensionality) of the existing multivariate volatility models.

Similarly, the backtesting results of out-of-sample interval forecasts lead us to conclude that SVX model is adequate in terms of forecasting and that it can also be used to model the tails (extremes) of the distribution ($\alpha = 0.01 \,\&\, 0.005$), provided that the return's distribution is correctly specified. Furthermore, based on the theorem regarding SVD, we can also conclude that the resulting covariance matrices from our proposed model are (semi) positive definite by construction as the singular values, which play the role of variances, are always greater than or equal to zero.

5.2 Possible directions for future research

Based on our proposed research methodology and its application in terms of estimation and forecasting, there are some topics that could be researched further in the future.

The first and most important point regarding future research is to investigate the performance of our proposed model by applying it to different data sets. These applications will help to strengthen the capability of our proposed model in order to capture the volatility clustering from the data sets. The data sets used for empirical purposes consist of stocks that are traded on the same stock exchange; however, one could use the data of stocks of different countries for empirical purposes. It would strengthen the adequacy of our approach in terms of its applications to real-world data.

As pointed out in the section describing empirical applications when modeling the dynamics of singular values, the second sequence of singular values is not showing the behavior of a normal distribution after applying the proposed transformations. In future research, we will also investigate how to handle this issue.

Based on the backtesting results, to check to what extent the proposed methodology helps to model the tail behavior of the asset returns?

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Eidesstattliche Versicherung

(Siehe Promotionsordnung vom 12.07.11, § 8, Abs. 2 Pkt. .5.)

Hiermit erkläre ich an Eidesstatt, dass die Dissertation von mir selbstständig, ohne unerlaubte Beihilfe angefertigt ist.

Ahmed, Naeem
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Name, Vorname

München, 27.10.2015
....

Ort, Datum

....

Unterschrift Doktorand/in