Essays on International Trade and Firm Dynamics

Inaugural-Dissertation

zur Erlangung des Grades

Doctor oeconomiae publicae (Dr. oec. publ.)

an der Ludwig-Maximilians-Universität München

2014

vorgelegt von

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Promotionsabschlussberatung:	13. Mai 2015

Datum der mündlichen Prüfung: 07. Mai 2015
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To my crazy little sister for coloring my life.

To my big brother who taught me to stand up against the bad boys in school.

To my caring parents.

Acknowledgements

First and foremost, I would like to thank my supervisor Gabriel Felbermayr for his inspiration, his advice, and the collaboration on chapter one. Gabriel is an amazing core of growth. Young researchers around him thrive almost by osmosis and benefit greatly from his constantly expanding network. His passion is contagious and his helpfulness truly remarkable. I am also very grateful to Carsten Eckel who joined my thesis committee as my second supervisor. His famous laughter, his constant support, and his sharp insights in economic discussions create a fresh and productive atmosphere. Together, Gabriel and Carsten provide a bright and stimulating environment for research in international trade in Munich. It was a great joy to write my dissertation within these opportune conditions.

I was very fortunate to stay at the UC San Diego for one semester as visiting research student with Marc-Andreas Mündler. I am grateful for his support and hospitality. He offered me the opportunity to present my work in his internal seminar and provided me with helpful comments and suggestions.

My thanks are also extended to Manuela Beckstein and Silke Englmaier for their continuous support with administrative issues. I am grateful to my students who kindly feigned interest in the things I taught and awarded me the economic teaching price 2014.

Being a doctoral student was not only stimulating academically, it also turned out to be a profoundly fulfilling and transformative period in my personal life. I was lucky to share this period with many inspiring and cheerful companions. I am very happy to have met Benjamin Böhninghausen, Sabrina Garland, Quinn Hamill, Alice Lauster, Linda Lee, Markus Michalek, Vlasta Paulic, Mira Sattelberger, Sebastian Stoll and Piers Trepper.

I am grateful to my family and all my old friends for their constant encouragement and support in every way, shape, and form.

Gilbert Spiegel

Contents

Preface

Globalization is presumably the most significant economic development of the past half century.

World's leading politicians agreed on a common framework for international commerce and finance at the Bretton Woods Conference. The World Bank and the International Monetary Fund came into existence. The General Agreement on Tariffs and Trade (GATT) was signed, pursuing the "substantial reduction of tariffs and other trade barriers and the elimination of preferences, on a reciprocal and mutually advantageous basis". GATT led to the foundation of the World Trade Organization and generated enormous economic integration. Today, the world map is covered with economic unions (CSME, EU), customs and monetary unions (CEMAC, UEMOA), common markets (EEA, EFTA, CES), customs unions (CAN, EAC, ECU, MERCOSUR,...), and free trade areas (AFTA, CISFTA, COMESA, NAFTA,...). The Trans-Pacific Partnership (TPP) comprising twelve countries throughout the Asia-Pacific region and the Transatlantic Trade and Investment Partnership (TTIP) between the EU and US are currently under negotiation. The OECD records an increase of global trade from 1 trillion USD in 1970 to 17 trillion USD in 2013.[1]

The world economy experiences a profound transformation. This transformation simultaneously affects firms, and is affected by their response itself. Firms are at the same time spectators and creators of globalization. Theoretical research increasingly emphasizes this mutual interdependence and explores global adjustments by studying firm-level decisions.

The theoretical literature has been influenced by a number of empirical findings. First, firms that participate in trade are larger, more productive, more capital intensive, more skill intensive, and pay higher wages than domestic firms within the same industry (e.g. Bernard and Jensen (1995, 1999), or Bernard et al. (2007a,b, 2009)). Second, Davis and Haltiwanger (1992) find substantial reallocations of re-

[1] Annual world trade in goods and services measured in 2005 constant USD, OECD (2014).

sources between firms within the same industry. Pavcnik (2002) and Trefler (2004) find evidence that globalization reallocates resources away from less efficient domestic firms towards more efficient exporters, and substantially changes the industry composition. Third, larger aggregate trade flows are not only achieved via this intensive margin, but also crucially depend on the extensive margin of the number of exporting firms (e.g. Eaton et al. (2004), or Bernard et al. (2011)). Fourth, firms respond to globalization along a number of dimensions including overall productivity, technology adoption, and markups of price over marginal cost (e.g. Pavcnik (2002), Bustos (2011), Lileeva and Trefler (2010), or De Loecker and Warzynski (2012)).

Models with heterogeneous firms provide a natural explanation for these features of disaggregated trade data. Trade liberalization leads to within-industry reallocations of resources. Less efficient firms exit and more efficient firms expand and enter export markets. The average industry efficiency rises. The increase in firm scale induced by export market entry enhances the return to complementary efficiency-enhancing investments, with the result that trade liberalization also raises firm efficiency. Models with heterogeneous firms are important for understanding the predominant margins along which an economy adjusts to globalization. They are important for exploring distributional implications, and for analyzing aggregate outcomes. It is only under strong conditions that aggregate outcomes (at the industry or country level) are sufficient statistics for overall welfare gains from trade. And even when these strong conditions hold heterogeneous and homogeneous firm models can have quite different distributional implications. Especially from a policy perspective it is important to identify potential winners and losers from globalization, to asses overall welfare changes, and to generate counterfactual predictions for trade related interventions.

This dissertation contributes to this broad area of research. I explore new margins of adjustment to trade. I explore distributional implications of trade in the presence of aggregate shocks that have only been studied in stationary environments. I explore the interaction of predominant forces shaping typical firm life cycles in a global economy that have until now only been studied separately.

The first chapter is joined work with Gabriel Felbermayr. We investigate the interaction of firm specific default probabilities and globalization. While heterogeneity in firm default risk is empirically obvious (e.g. substantial dispersion of credit default swap rates), trade theory consistently imposes the simplifying assumption of identical exit rates for all firms. Jointly with partial irreversibility of investments, this new type of heterogeneity generates heterogeneity in effective discount rates and, thus, in the cost of finance. As default probabilities are not perfectly observable (there are only noisy signals), the model entails firm dynamics from learning and belief updating. In line with evidence, the model predicts a negative correlation between firms' financing costs and their age. Over a firm's life cycle, per period net profits and the export participation probability grow. Exporters are less likely to default than purely domestic firms. Belief updating entails excessive financing of incumbents relative to entrants and too much exporting. Asymptotically, trade liberalization reduces overall general equilibrium exit rates, but it does not necessarily increase welfare. With multiple asymmetric export markets, firms gradually expand their market coverage and total sales. A confidence crisis modeled by belief reversion causes an over-proportional decrease in exports, thereby offering a novel interpretation of the over-proportional trade slump during the world-wide recession in 2008/2009.

The second chapter explores the dynamic response of a small open economy with heterogeneous firms and labor market frictions on trade and technology shocks. I study individual and aggregate firm dynamics, transitional wage rates, wage inequality, unemployment, and welfare.

There is direct job search. Firms compete for workers by publicly posting long-term contracts. Job seekers observe all offers (determine expected wages and probabilities of getting the job) and adjust their search accordingly. Convex vacancy costs make firms expand gradually and provide a natural rationalization for the empirical regularity that productivity distributions of exporters and non-exporters overlap substantially. Conditional on age (or size), more productive firms exhibit higher growth rates. Conditional on productivity, younger (or smaller) firms exhibit higher growth rates. Firms realize higher growth rates by both, posting more vacancies and filling each vacancy with a higher probability. Higher job-filling rates are realized by higher wage offers, creating wage dispersion across and within firms.

I calibrate the model to typical figures of an open economy and study its dynamic response to a trade liberalization and a positive technology shock. There are four predominant types (and durations) of aggregate adjustments along the transition path: Wage adjustments (immediate), firm adjustments (approx. 1.5 years), wage distribution adjustments (approx. 10 years), and firm distribution adjustments (approx. 100 years). While a trade liberalization generates overshooting wage averages, a positive technology shock entails monotonically increasing averages. Both scenarios imply significant transitory inequalities. Variances of the aggregate wage distribution overshoot substantially. While a trade liberalization pushes up unemployment, a positive technology shock decreases the number of jobless workers. The adjustment speed of unemployment after a positive technology shock is higher in less open economies. A trade liberalization increases welfare. A positive technology shock decreases welfare in more open economies and increases welfare in less open ones.

The third chapter explores drivers of gradual firm growth and decay in a global economy. I combine three separate approaches. First, there is supply uncertainty leading to concave improvement of production techniques on firm level. Second, there is demand uncertainty resulting in firm specific expansion paths. Third, there are knowledge spillovers which constantly intensify competition and diminish firms which do not improve their production technique sufficiently fast.

Each firm is assigned an unobservable productivity distribution upon its birth. This distribution generates a new productivity sample every period. Whenever a new productivity sample dominates the firms current productivity it switches technology and produces according to this new productivity. This generates firm growth and firm learning (from observing an ever-increasing sample history). Firm learning has no effect on firm productivity, but it makes firms more or less optimistic about their future productivity evolution, and via this channel influences market entry/exit decisions. Furthermore, each firm is assigned an unobservable per period demand shock probability for every country. Demand is either positive or - when hit by a shock - completely vanishes. Firms can learn about these market specific default probabilities by various means. As demand characteristics are positively correlated across countries, firm learning for a specific country also comprises observing demand signals in other countries. Firm specific demand signals entail firm specific learning and result in firm specific expansion paths. However, generally, more productive firms enter more markets. This generates a positive correlation of firm productivity and life expectancy. Finally, there are knowledge spillovers from incumbent firms to entrants. Start-ups draw their productivity type from a distribution that depends on the productivity type distribution of existing firms. This generates monotonically increasing average productivities and results in crowding out of old firms.

Each chapter is self-contained. Technical discussions are deferred to Appendices in the second part of this thesis. A comprehensive bibliography is provided at the end.

Chapter I

A Simple Theory of Trade, Finance, and Firm Dynamics*

I.1 Introduction

Recent theoretical work pioneered by Melitz (2003) has shed light on the role of productivity heterogeneity for the effect of international trade on firm behavior and aggregate outcomes. Given the presence of fixed costs, only more productive firms sort into exporting, and a reduction of trade costs increases aggregate productivity. Similar selection effects can be derived from firm-level differences in perceived product quality (Baldwin and Harrigan, 2011) or the degree of tradability of output (Bergin and Glick, 2009). The core prediction of these models, namely that more competitive firms are more likely to be exporters, enjoys massive empirical support (Bernard et al., 2007a). A smaller strand of theoretical work introduces heterogeneity regarding fixed market access costs into the Krugman (1980) framework while keeping marginal revenues constant across firms (e.g. Schmitt and Yu (2001), or Jorgenson and Schröder (2008)).

Unrecognized in the recent trade literature, firms also differ with respect to their exit probabilities, at least as *perceived* by financial markets.[1] Ashcraft and Santos (2009) study data on credit default swaps and document a remarkable degree of heterogeneity amongst firms with respect to their perceived risk of business discontinuation. The Melitz (2003) model does not capture this stylized fact, since at

*This chapter is based on joint work with Gabriel Felbermayr published in the Review of International Economics, Felbermayr and Spiegel (2014).

[1] Pflüger and Russek (2011) are the only exception known to us: they use a two-sector Melitz (2003) model where exit probabilities are assumed to be inversely related to firm-level productivity.

each period, all firms are equally likely to be hit by a death shock. Plant death is important for aggregate statistics: Bernard and Jensen (2007) show that plant deaths account for more than half of gross job destruction in U.S. manufacturing. To the best of our knowledge, we are the first conducting a thorough analysis of this relevant and ubiquitous source of firm heterogeneity and its implications.

The core purpose of this chapter is to explore the effect of heterogeneous default probabilities on individual firm life cycles and on aggregate outcomes. Apart from the introduction of this new type of heterogeneity, we leave everything else as standard as possible. This allows isolating and analyzing the two driving forces – the cleansing mechanism and the updating mechanism – in a well known and understood environment. In our model, we continue to assume that firms are uniquely identified by the single product they produce. Also, as Melitz (2003), we view business discontinuation as a discrete exogenous shock.[2] However, we allow firms to differ with respect to the probability of such death shocks. Upon developing a new product, firms trigger uncertain, publicly observable signals about the viability of their new product (i.e., their type), yielding beliefs that are correct in expectation and that are updated according to Bayes' law in case of firm survival. In the presence of partial irreversibility of investment, this assumption implies firm-level differences with respect to their cost of finance.[3] As in Melitz (2003), in our framework, firms are identical *ex ante*. The financial markets are risk neutral and perfectly competitive. However, the 'true' life expectancy of a firm is unknown to all agents (i.e., to producers, financial markets, consumers). At the beginning of each period, producers must invest a fixed cost which cannot be recovered at any stage and which depreciates at the end of the period. Assuming, without loss of generality, that funds are available at a zero baseline interest rate, a firm's effective financing cost is equal to its per-period exit probability. If a firm survives, at the end of the period, market participants update their believed exit rates downwards. So, as time elapses, the funding of fixed cost activities (such as exporting) becomes gradually cheaper.

Firms' marginal revenues remain constant over time, so that the model enjoys the tractability of Schmitt and Yu (2001). However, despite its simplicity, the setup generates additional insights that are not available in the Melitz (2003) framework.

[2] We are silent about the exact source of the shock. It may be due to the the sudden disappearance of demand, due to the emergence of a cheaper perfect substitute of the firm's variety, or due to a technology shock causing the immediate depreciation of the firm's assets.

[3] Impullitti, Irarrazabal and Oppromolla (2013) use a Melitz (2003) model with a stochastic evolution of productivity and irreversibility of investment. They provide a rich discussion of the empirical importance of sunk costs in trade related applications.

As only firms with sufficiently low exit hazards enter foreign markets, exporters are on average longer-lived than domestic firms. Trade liberalization allows those formerly domestic firms with lowest effective interest rates to take up exporting while domestic firms, facing high interest rates, are forced to exit. So, trade liberalization lowers the expected average survival time of exporters but increases that of domestic firms. Due to a composition effect, in the overall economy, expected average survival increases. Hence, liberalization leads to higher ex- post stability of firms in the long run, but effects differ between exporters and domestic firms.

The model also yields insights about firm and firm-generation dynamics. Recent literature studies the dynamic behavior of firms in open economies. The common objective is to explain the obvious stylized fact that firms are not typically born as exporters but evolve into exporting, and possibly out of it, over time. Dynamics may arise from the evolution of firm types. Impulliti, Irarrazabal and Oppromolla (2013) work with productivity shocks and irreversible investment in an otherwise standard Melitz (2003) model. Fajgelbaum (2011) stresses labor market frictions. Burstein and Melitz (2012) analyze the role of innovation. Alternatively, dynamics may also arise from learning about foreign markets or foreign customers. Nguyen (2012) studies the role of uncertainty about foreign market demand; Albornoz et al. (2012) offer a model of sequential exporting where firms gradually learn about foreign market profitability; Araujo et al. (2012) investigate the build up of trust between a producer and the foreign client in the absence of complete contracts.[4] In our model, uncertainty concerns the type of the producer or, equivalently, characteristics of the product, the 'true' economic life expectancy of a firm or product being unknown to all market participants. Dynamics are driven by two very simple mechanisms; the cleansing mechanism: *inferior firms are more likely to default*, and the updating mechanism: *trust in firms increases in firm age*.

The cleansing mechanism yields firm generation dynamics. As firms with high exit probability default more likely, the type distribution of firm generations evolves over time. Average exit probabilities of firm generations decrease with respect to their age, yielding decreasing average discount rates, increasing average net profits and an increasing fraction of exporters. The updating mechanism is driven by type uncertainty and the resulting Bayesian updating, yielding similar firm specific dynamics as the cleansing mechanism implies for firm generations. The older a firm, the lower the discount rate it is being assigned, yielding lower costs of finance,

[4] Aeberhardt et al. (2011) also study learning in the context of contract incompleteness.

increasing net profits and increasing probability of exporting. Besides, as firms anticipate these life cycle patterns, there are some firms that enter the domestic market realizing negative profits initially.[5] In contrast, on the export market such early entries do not occur as active firms can wait until belief updating pushes their discount rate below the threshold ensuring positive profits.

Even though belief updating is rational on the individual level of the firm, the joint analysis of the cleansing and updating mechanisms reveals that updating leads to misvaluation of firm generation averages. While the evolution of true average exit probabilities is solely driven by the cleansing mechanism, the evolution of perceived average exit probabilities is driven by both, the the cleansing mechanism *and* the updating mechanism. Thus, the older a firm generation gets, the further perceived and true magnitudes drift apart. Average discount rates of incumbents are inefficiently small, yielding excessive financing of incumbents relative to entrants (innovators). As incumbents and entrants compete for workforce, this yields insufficient entry of new firms. A corollary of this is that belief updating implies excessive exporting: If a firm enters the export market by a misjudgment of its type, it will, in expectation, default before accumulated profits balance exporting fixed costs, yielding a negative welfare effect.

The predictions of our model are consistent with a number of empirical stylized facts. First, firm survival and export status are positively correlated (Greenaway et al., 2008), the link between the two running through access to finance (Goerg and Spaliara, 2009). Second, over longer horizons of time, about 40% of total export growth occurs at the extensive margin (Bernard and Jensen, 2004). Third, over time, firms gradually expand the number of export markets that they serve (Lawless, 2009). Fourth, export activities are heavily persistent due to the existence of sunk costs (Das et al., 2007).[6]

We use the model to study a crisis of confidence, in which market participants revise their beliefs, i.e., they delete a portion of the updating history. Since type beliefs of exporters are on average farther away from true types, this revision leads to a

[5] Belief updating requires that the firm is active, i.e., producing, and therefore observed by market participants.

[6] An evident extension of our model could allow firms to accumulate tangible assets over their life cycle. But as this variation would only amplify the mechanisms at hand, via accelerating the reduction of financing costs in firm age, we omit it for the sake of simplicity. One could also think about evolution of true types, rather than fixing them to their initial value. However, again this would not change our results qualitatively as long as new types are positively correlated with old ones.

stronger decline in exports and, by trade balance, of imports relative to domestic sales. Credit conditions of large old firms (exporters) deteriorate more strongly than of small young ones. These observations are in line with the effect of the Lehman Brothers crash on September 15, 2008. This shock led to a tightening of credit restrictions, in particular of large firms, and to a collapse of trade. As documented in survey data from Germany and other countries, the ordering of perceived credit constraints of small, medium size, and large firms were reversed by that shock and has slowly returned to the pre-crisis pattern afterwards.[7]

The remainder of this chapter is structured as follows. Section I.2 describes the basic framework. Section I.3 derives our core results under the simplifying assumption that firms' expected life times are known with certainty after entry. Section I.4 extends the analysis to the more realistic case of uncertain default probabilities. Section I.5 concludes.

I.2 Setup

We analyze an infinitely repeated game of symmetric information. All transactions (costs, revenues, profits,...) are measured in units of the final good. To present our new mechanisms as clearly as possible, we chose a basic modeling framework that remains close to Melitz (2003).

Households

We consider $n+1$ symmetric countries. We relax symmetry in the third part of Section I.4. Each country is populated by a representative household of size L, who supplies labor inelastically, and who cares about the quantity of a final good C according to a linear utility function. Hence, per capita utility is $u = C/L$.

Production

In each country, there is a mass M of monopolistically competitive producers of differentiated intermediate inputs, indexed by ω. These inputs are assembled by a perfectly competitive final goods sector into the final good Y according to the CES

[7] Our model is too stylized to be used for a full quantitative analysis of the crisis. Rather, we wish to highlight a novel theoretical mechanism that may have played a role along more standard determinants such as the strong decline in demand.

production function:[8]

$$Y = \Big(\int q(\omega)^{\rho} d\omega\Big)^{1/\rho} = C + I, \rho \in (0,1). \tag{I.1}$$

The final good $Y = C + I$ can be either consumed by households or used as investment by firms. While the final good is freely tradable, differentiated inputs are subject to standard iceberg trade costs $\tau \geq 1$. Standard manipulation yields optimal input demands of final goods producers and associated expenditures:

$$q(\omega) = Q\Big(\frac{p(\omega)}{P}\Big)^{-\sigma} \text{ and } r(\omega) = R\Big(\frac{p(\omega)}{P}\Big)^{1-\sigma}, \tag{I.2}$$

with $\sigma = 1/(1-\rho) > 1$ and Dixit Stiglitz aggregates P, Q and R. The index P constitutes the associated price of the final output good, normalized to unity by choice of numeraire, Q constitutes the quantity index, and R is given by $R = PQ = Y$. Input goods are produced via a one-to-one technology, $q = \ell$, with labor ℓ being the only factor of production. As firms do not differ in productivity they charge identical prices, p_d on the domestic and p_x on the export markets:

$$p_d = \frac{w}{\rho}, \text{ and } p_x = \tau p_d, \tag{I.3}$$

where w denotes the wage rate. Thus, domestic per period *operating* profits and revenues are identical for all firms and are given by:

$$\pi_d = (p_d - w)q_d = \Big(\frac{wq_d}{\sigma - 1}\Big), \text{ and } r_d = p_d q_d = \sigma \pi_d, \tag{I.4}$$

with analogous expressions for exporters.

Heterogeneity

Firm heterogeneity is introduced via firm specific per period exit probabilities $\delta \in [0,1]$, distributed with pdf $g(\delta)$ and cdf $G(\delta)$. Per period exit probabilities are constant over firms' life time. In Section I.3 we assume that start-up investments reveal true types δ of firms, thereby deactivating the updating mechanism and isolating the dynamics generated by the cleansing mechanism. Then, from Section I.4 onwards, we drop this assumption and analyze the full dynamics triggered by the cleansing and the updating mechanisms. From Section I.4 onwards, the start-up investment triggers an uncertain signal of the firms exit probability that is correct in expectation. This uncertainty yields *perceived* types $\hat{\delta}$ that are updated according to Bayes' law as the firm grows older while *true* types δ do not change over time.[9]

[8]This expression admits external economies of scale; neutralizing them as Egger and Kreickemeier (2009) has no qualitative bearing on our results.

[9]For a detailed discussion of perceived types and Bayesian updating refer to section I.4

Financial Market

We consider a risk neutral, perfectly competitive financial market and normalize the interest rate required by households to zero. Thus, in case of revealed types, a firm δ is charged a per period rate of $\delta\alpha$ for a loan with nominal α, yielding zero expected profits for creditors.[10] Analogously, in case of type uncertainty, a firm of perceived type $\hat{\delta}$ is charged $\hat{\delta}\alpha$. As in Melitz (2003), firms only invest if investment costs are balanced by returns to investment in expectations.

Timing

Each period $t \in \mathbb{N}$ consists of three stages: $s = 1$: Inactive firms may turn active by sinking K units of the final output good into research and development. This effort yields a new variety of the differentiated input for sure, but the viability of the innovation δ is drawn from $g(\delta)$ and differs across firms. The market receives signals that reveal true firm types δ (Section I.3), or that yield certain beliefs of firm types $\hat{\delta}$ (Section I.4). $s = 2$: Active firms consider to either turn inactive or to sell on the domestic market (at market access costs f_d), or to additionally engage in exporting (at market access costs f_x). $s = 3$: Active firms may be forced to exit the market by idiosyncratic shocks, that arrive according to their per period exit probability δ, and turn inactive. Survivors remain active, generate profits and conduct loan rate repayments. In case of type uncertainty (Section I.4), beliefs are updated contingent on firm survival.

Aggregation

A long-run equilibrium is characterized by a mass M and a type distribution $h(\delta)$ of active firms and a mass M_x and a type distribution $h_x(\delta)$ of exporters in every country. As all active firms charge the same domestic price p_d and all exporters charge the same price p_x for their exports, we have:

$$1 = P = \Big(\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\Big)^{1/(1-\sigma)} = \Big(\int_0^1 p_d^{1-\sigma} M h(\delta) d\delta + n \int_0^1 p_x^{1-\sigma} M_x h_x(\delta) d\delta\Big)^{1/(1-\sigma)}$$

$$= (M p_d^{1-\sigma} + n M_x p_x^{1-\sigma})^{1/(1-\sigma)}, \qquad \text{(I.5)}$$

[10] Here we restrict our analysis to sunk fixed costs, that can not be recovered subsequent to firm default. One could additionally introduce a component that is not sunk. As additional insights are small – if more units of final good are needed for investment, aggregate consumption decreases, but idiosyncratic interest rates of firms are not affected – we simply assume sunkness of fixed costs for the purpose of technical simplicity.

by choice of numeraire. Analogously we get $Q = (Mq_d^\rho + nM_x q_x^\rho)^{1/\rho}$ and $R = Mr_d + nM_x r_x$.

I.3 Cleansing Mechanism

In this section we focus on the cleansing mechanism and its impact on firm generation dynamics. The updating mechanism is switched off by assuming perfect observability of firm types. Additionally, we assume $g(0) = 0$, i.e. no firm shall be able to survive all possible shocks. We denote expected values with respect to a certain distribution χ by $E_\chi(\cdot)$ and impose the technical assumption $E_g(1/\delta) \in (1, \infty)$.[11]

Zero Cut-Off Profit Conditions

Market access costs f_d and f_x are modeled as flow fixed costs which occur at the beginning of each period and which are sunk until the end of the period. So, in case of firm default they are lost and in case of firm survival firms repay them at the end of the period, and apply for new loans at the beginning of the next period. As the financial market is risk neutral and perfectly competitive an active firm of type δ faces per period loan rates of $\delta P f_d = \delta f_d$, plus $n\delta f_x$ in case of exporting. Thus, domestic entry occurs only if per period operating profits π_d dominate per period loan rates δf_d, yielding $\pi_d = \delta_d^* f_d$, with δ_d^* denoting the domestic cut-off type. Analogously we get $\pi_x = \delta_x^* f_x$, with the exporting cut-off type δ_x^*. As per period operating profits earned at each market do not depend on firm type we have:

$$\pi_d = \delta_d^* f_d \quad \text{and} \quad \pi_x = \delta_x^* f_x, \tag{I.6}$$

for all firms. Importantly, per period *net* profits do depend on firm types as loan rate repayments δf_d for domestic market entry and δf_x for foreign market entry are type-dependent. Thus, a firm of type $\delta \le \delta_d^*$ realizes per period net profits of:

$$\pi^n(\delta) = \begin{cases} \pi_d^n(\delta) = \pi_d - \delta f_d = (\delta_d^* - \delta) f_d & \text{if } \delta \in (\delta_x^*, \delta_d^*], \\ \pi_d^n(\delta) + n\pi_x^n(\delta) = (\delta_d^* - \delta) f_d + n(\delta_x^* - \delta) f_x & \text{if } \delta \in (0, \delta_x^*]. \end{cases} \tag{I.7}$$

Dividing domestic and exporting per period profits and applying (I.2) and (I.3), we get a one-to-one correspondence between cut-off types δ_x^* and δ_d^*:

$$\frac{\delta_x^* f_x}{\delta_d^* f_d} = \frac{\pi_x}{\pi_d} = \tau^{1-\sigma} \Rightarrow \delta_x^* = \tau^{1-\sigma} \frac{f_d}{f_x} \delta_d^*. \tag{I.8}$$

[11] The restriction $E_g(1/\delta) < \infty$ is equivalent to requiring that the density $g(\delta)$ converges faster than linearly towards zero as its argument δ converges against the boundary $\delta \to 0$. The restriction $E_g(1/\delta) > 1$ precludes convergence towards the degenerate density that assigns all probability to the outcome $\delta = 1$.

To ensure that all active firms serve their domestic market and only a subset of domestically active firms engages in exporting, we assume $f_x \geq f_d$.[12]

Free Entry Condition

As firm types are unobservable ex ante, firms are not able to offer banks the repayment of a fixed nominal in order to be granted the loan needed for carrying out the start-up investment K. If, for example, the firm turns out to be of the domestic cut-off type δ_d^*, it will realize zero per period net profits and hence will not be able to deduct any positive rate payments. Therefore, firms offer the repayment of a type dependent nominal $\alpha(\delta)$ that has to be less than their expected total net profits $\alpha(\delta) \leq \Sigma_{t=0}^{\infty}(1-\delta)^t \pi^n(\delta) = \pi^n(\delta)/\delta$. Banks accept only if they do not incur losses in expectation. Given that start-up investment costs K are sunk and that only a fraction $G(\delta_d^*)$ of new firms is able to enter the market, the above inequality can be rephrased as $E_g(\alpha(\delta)|\delta \leq \delta_d^*) \geq K/G(\delta_d^*)$. As banks face perfect competition, this inequality is binding. Free entry of firms drives down profits until nominal and expected total net profits coincide $\alpha(\delta) = \pi^n(\delta)/\delta$, leaving firms with zero profits and yielding:

$$E_g(\pi^n(\delta)/\delta|\delta \leq \delta_d^*) = K/G(\delta_d^*). \tag{I.9}$$

In the Appendix we prove that cut-off values δ_d^* and δ_x^* exist and are uniquely determined by (I.7), (I.8) and (I.9). Moreover, we also prove the following Proposition:

Proposition 1 (Trade Liberalization and Firm Churning)**.** *A reduction in variable trade costs τ lowers δ_d^* but increases δ_x^*. Trade liberalization yields lower average firm churning, while churning of exporters increases.*

Incumbent Distribution

In expectation, low-δ-firms drop out from the market later than high-δ-firms. Thus, the incumbent distribution $h(\delta)$ differs from the distribution of start-ups $g(\delta)$. Every period a certain measure M_e of g-distributed firms tries to enter the market (henceforth denoted as firm generation), yielding a certain measure $M_e g(\delta)$ of entrants per type δ. Let $i(\delta)$ denote the measure of incumbents of type δ, then firms of type δ accumulate until the measure of entrants $M_e g(\delta)$ coincides with the measure of defaulting firms $\delta i(\delta)$, yielding $i(\delta) = M_e g(\delta)/\delta$. Thus, the type distribution of

[12] A similar condition ensure the empirically relevant sorting pattern in the Melitz (2003) model.

incumbents is given by:[13]

$$h(\delta) = \begin{cases} \frac{g(\delta)/\delta}{\int_0^{\delta_d^*} g(\delta)/\delta \, d\delta} & \text{if } \delta \in (0, \delta_d^*], \\ 0 & \text{otherwise.} \end{cases} \tag{I.10}$$

Correspondingly the type distribution of exporters follows $h(\delta|\delta \leq \delta_x^*)$. As $h(\delta)$ shifts mass towards low values of δ, average turnover of firms entering the market $E_g(\delta|\delta \leq \delta_d^*)$ is higher than average market turnover $E_h(\delta)$. Summarizing, we obtain the next proposition.

Proposition 2 (Cleansing Mechanism)**.** *The older a firm generation, the lower its average exit probability.*

As loan rates, size of net profits and entry into exporting are determined by firms exit probabilities, we can directly infer

Proposition 3 (Firm Generation Effects)**.** *The older a firm generation, the lower its average loan rate, the higher its average net profit and the higher its fraction of exporters.*

With δ_d^*, δ_x^* and $h(\delta)$ characterized, now we close the model by determining firm masses and per period consumption.

Firm Masses

In steady state, firm entry balances firm exit, yielding $M_e = E_h(\delta)M/G(\delta_d^*)$. Using labor market clearing $L = Mq_d + nM_x\tau q_x$ and the relative mass of exporting firms $M_x = H(\delta_x^*)M$, with $H(\delta)$ denoting the cumulative density function of the incumbent distribution $h(\delta)$, we get:

$$M = wL/[(\sigma - 1)(f_d\delta_d^* + f_x nH(\delta_x^*)\delta_x^*)], \tag{I.11}$$

which is a first relation linking the two remaining unknown endogenous variables M and w. A detailed derivation of (I.11) is provided in the Appendix.

[13] As we imposed the assumption $E_g(1/\delta) < \infty$, the density $g(\delta)$ converges faster than linearly towards zero if δ becomes arbitrarily small. This ensures the existence of $h(\delta)$.

Consumption

We can determine the equilibrium wage rate w from $P = 1$, obtaining aggregate per period consumption:

$$C = Lw/P = Lw = L\rho(M + nH(\delta_x^*)M\tau^{1-\sigma})^{1/(\sigma-1)}, \tag{I.12}$$

a second relation linking M and w. As utility is linear in consumption, (I.12) constitutes a measure of welfare. Again, a detailed derivation of (I.12) is provided in the Appendix. From the measure of entering firms and the fixed costs they have to bear, we can directly determine the quantity of the final product spent for start-up investments and market entries every period:

$$I = (K + f_d G(\delta_d^*) + n f_x G(\delta_x^*))M_e. \tag{I.13}$$

From (I.2) and (I.3) we get $\tau p_x = \tau^{1-\sigma} q_d < q_d$. Thus, trade liberalization increases the number of available varieties in every country. Moreover, trade liberalization increases average productivity. Proposition 1 establishes that trade liberalization forces firms with low net profits out of the market ($\hat{\delta}$ decreases) shifting production towards more efficient firms. As per period net profits constitute the difference of per period profits (that are independent of firm type) and per period fixed costs (that decrease in length of firm life), trade liberalization raises $Y - I = C$ and we get:

Proposition 4 (Trade Liberalization and Welfare). *Trade liberalization increases welfare.*

I.4 Uncertain Firm Types and Updating

In this section, we discuss variations and applications of our simple baseline model from above. First, we introduce type uncertainty, leaving everything else unchanged (first subsection), then we discuss consequences of a confidence crisis (second subsection) and conclude with the analysis of the asymmetric country case (third subsection). Henceforth start-up investments trigger uncertain signals, yielding perceived types $\hat{\delta}_{t=0} \in [0, 1]$. When referring to the cross section of firms we drop the age indicating subscript and denote perceived types with $\hat{\delta}$. Perceived types $\hat{\delta}$ constitute expected values of their corresponding belief $\delta \sim b_{\hat{\delta}}(\delta)$, i.e. $E_{b_{\hat{\delta}}}(\delta) = \hat{\delta}$. Initial perceived types $\hat{\delta}_{t=0}$ are correct in expectation. Thus, perceived and true types are both distributed with the true type pdf g introduced in Section I.3 initially. Again, we impose the technical assumption $E_{b_{\hat{\delta}}}(1/\delta) \in (1, \infty)$ for all $\hat{\delta}$.[14] Turning to the perceived type evolution of individual firms, we can frame a very simple mechanism. Every period a firm survives, its perceived type is being updated according to Bayes' law until it is hit by a shock and forced to exit the market. As updating is only triggered by good news (firm survival), we get $\hat{\delta}_0 > \hat{\delta}_1 > \cdots > \hat{\delta}_t > \ldots$ for all periods a firm survives, with $\hat{\delta}_t$ denoting its perceived type in its t^{th} period subsequent foundation. To understand this mechanism more closely, consider a firm with a very poor start-up signal. Initially agents expect the firm to default with a high probability. The longer the firm survives, the less the agents will trust in the accuracy of its start-up signal and correct the perceived default probability downwards. In the limit, the firm could only survive for an infinite number of periods if its true type was $\delta = 0$. Thus, perceived type evolutions $(\hat{\delta}_a)_{a \leq t}$ constitute segments of length t of monotonically decreasing sequences that start at $\hat{\delta}_0$ and converge towards zero, $lim_{t \to \infty} \hat{\delta}_t = 0$. This updating mechanism is a direct implication of type uncertainty. It is solely driven by dropping the additional assumption of section I.3 that firms default probabilities are perfectly observable.[15]

Proposition 5 (Updating Mechanism)**.** *The older a firm, the lower its perceived exit probability.*

Symmetric Countries

Except from the type uncertainty introduced above, the setup from Section I.3 remains unchanged.

Zero Cut-Off Profit Conditions

As loans for market access costs are negotiated on a per period basis, firms face rate payments $\hat{\delta}_t f_d$ (plus $n\hat{\delta}_t f_x$ in case of exporting) that always reflect current firm

[14] The restriction $E_{b_{\hat{\delta}}}(1/\delta) < \infty$ is equivalent to requiring that the density $b_{\hat{\delta}}(\delta)$ converges faster than linearly towards zero as its argument δ converges against the boundary $\delta \to 0$. The restriction $E_{b_{\hat{\delta}}}(1/\delta) > 1$ precludes convergence towards the degenerate density that assigns all probability to the outcome $\delta = 1$.

[15] A signal is produced only at the moment of firm creation. If we suppress this signal we would loose firm heterogeneity within firm generations. All firms of the same generation would initially be assigned a perceived type matching the expected value of the true type distribution. This perceived type would simultaneously drop for all survivors of that generation and slowly converge towards zero as the age of the generation approaches infinity. Another possibility is to introduce more signals. Yet this would not qualitatively change the result of decreasing perceived types. No matter how many signals we introduce, firm survival will always constitute relevant information for the updating process and bias it downwards.

status $\hat{\delta}_t$. Thus, the older a firm the lower its rate payments. As firms anticipate this life cycle pattern, the entry decision arises from comparing present value of expected future profits with present value of expected future costs. Hence, some firms enter even though they are facing negative per period net profits initially. Consider a firm with initial perceived type $\hat{\delta}_0$, then present value of expected future profits from domestic activity equals $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi(\hat{\delta}_t)) = E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\pi_d + \Sigma_{t=t(\hat{\delta}_0)}^{\infty}(1-\delta)^t n\pi_x) = E_{b_{\hat{\delta}_0}}(1/\delta)\pi_d + E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}/\delta)n\pi_x$, with $t(\hat{\delta}_0)$ denoting the period of entry into exporting in case of survival. Let $\psi(\hat{\delta}_0)$ denote the weighted probability of survival until entry into exporting. It is defined by the condition satisfying $E_{b_{\hat{\delta}_0}}(\psi(\hat{\delta}_0)/\delta) = E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}/\delta)$. Then, the present value of expected future profits can be rewritten as $E_{b_{\hat{\delta}_0}}(1/\delta)(\pi_d + \psi(\hat{\delta}_0)n\pi_x)$. The present value of expected future costs from domestic entry equals $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\hat{\delta}_t f_d + \Sigma_{t=t(\hat{\delta}_0)}^{\infty}(1-\delta)^t n\hat{\delta}_t f_x) = E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\bar{\delta}(\hat{\delta}_0)f_d + \Sigma_{t=t(\hat{\delta}_0)}^{\infty}(1-\delta)^t n\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)})f_x) = E_{b_{\hat{\delta}_0}}(1/\delta)(\bar{\delta}(\hat{\delta}_0)f_d + \psi(\hat{\delta}_0)n\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)})f_x)$, with $\bar{\delta}(\hat{\delta})$ denoting the expected future average perceived type of a firm with perceived type $\hat{\delta}$.[16] For the cut-off value $\hat{\delta}_d^*$, present value of expected future profits and present value of expected future costs coincide, yielding $\pi_d + \psi(\hat{\delta}_d^*)n\pi_x = \bar{\delta}(\hat{\delta}_d^*)f_d + \psi(\hat{\delta}_d^*)n\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)})f_x$. Differently, in case of exporting, firms wait until their perceived type is low enough to realize positive per period net profits from exporting. As domestic and exporting per period operating profits do not depend on firm type we get:

$$\pi_d = \bar{\delta}(\hat{\delta}_d^*)f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))nf_x \quad \text{and} \quad \pi_x = \hat{\delta}_x^* f_x. \tag{I.14}$$

Even if the probability of exporting was zero $\psi(\hat{\delta}_d^*) = 0$, firms $\hat{\delta}_0 \in (\bar{\delta}(\hat{\delta}_d^*), \hat{\delta}_d^*]$ would realize negative net profits $(\bar{\delta}(\hat{\delta}_d^*) - \hat{\delta}_0)f_d < 0$ initially, speculating on positive net profits $(\bar{\delta}(\hat{\delta}_d^*) - \hat{\delta}_t)f_d > 0$ in future periods. The prospect of positive exporting profits lowers initial profits even further. A firm of age t and perceived type $\hat{\delta}_t$ realizes a per period net profit of:

$$\pi^n(\hat{\delta}_t) = \begin{cases} \pi_d^n(\hat{\delta}_t) = \pi_d - \hat{\delta}_t f_d & \text{if } \hat{\delta}_t \in (\hat{\delta}_x^*, \hat{\delta}_d^*], \\ \pi_d^n(\hat{\delta}_t) + n\pi_x^n(\hat{\delta}_t) = \pi_d - \hat{\delta}_t f_d + n(\pi_x - \hat{\delta}_t f_x) & \text{if } \hat{\delta}_t \in (0, \hat{\delta}_x^*]. \end{cases} \tag{I.15}$$

[16] As $\hat{\delta}_t$ decreases monotonically in t, the expected amount of cleared entry costs $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\hat{\delta}_t f_d) < E_{b_{\hat{\delta}_0}}(1/\delta)\hat{\delta}_0 f_d < \infty$ is finite by assumption $E_{b_{\hat{\delta}}}(1/\delta) \in (1,\infty)$. Thus, there exists a unique $\bar{\delta}(\hat{\delta}_0) \in (0, \hat{\delta}_0)$ fulfilling $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\hat{\delta}_t f_d) = E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t\bar{\delta}(\hat{\delta}_0)f_d)$. Existence and uniqueness of $\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)})$ holds analogously.

Dividing domestic and exporting per period operating profits and applying (I.2) and (I.3), we get a one-to-one correspondence between $\hat{\delta}_x^*$ and $\hat{\delta}_d^*$:

$$\frac{\hat{\delta}_x^* f_x}{\bar{\delta}(\hat{\delta}_d^*) f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))n f_x} = \frac{\pi_x}{\pi_d} = \frac{q_x}{q_d}\left(\frac{p_x - \tau w}{p_d - w}\right) = \tau^{1-\sigma}$$

$$\Rightarrow \hat{\delta}_x^* = \tau^{1-\sigma}\frac{\bar{\delta}(\hat{\delta}_d^*) f_d + \psi(\hat{\delta}_d^*)\bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}) n f_x}{(1 + \tau^{1-\sigma}\psi(\hat{\delta}_d^*) n) f_x}. \qquad \text{(I.16)}$$

Summarizing, the updating mechanism from proposition 5 yields:

Proposition 6 (Firm Specific Effects)**.** *Net profits of firms and ex-ante probability of exporting increase in firm age. Some firms face negative per period net profits from domestic activity initially, while entry into exporting occurs only in case of positive per period net profits.*

Free Entry Condition

In line with the known firm type case, firms offer the repayment of their signal dependent expected total net profits $E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \pi^n(\hat{\delta}_t))$ and risk neutral, perfectly competitive banks grant loans until expected profits coincide with expected costs:

$$E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \pi^n(\hat{\delta}_t))|\hat{\delta}_0 \leq \hat{\delta}_d^*) = K/G(\hat{\delta}_d^*). \qquad \text{(I.17)}$$

In the Appendix we prove that cut-off values $\hat{\delta}_d^*$ and $\hat{\delta}_x^*$ exist and are uniquely determined by (I.14), (I.15), (I.16) and (I.17). Moreover, we prove that lower iceberg trade costs τ lower $\hat{\delta}_d^*$ but increase $\hat{\delta}_x^*$, yielding identical trade liberalization effects on firm churning as in the known firm type case (Proposition 1).

Incumbent Distributions

First we determine the steady state distribution of true types and then, second, the steady state distribution of perceived types. Every period a certain measure M_e of firms with g-distributed true types tries to enter the market. As true types are not observable, even high-δ-firms may enter if their start-up signal is sufficiently good, i.e. if $\hat{\delta}_0 \leq \hat{\delta}_d^*$, yielding the modified distribution of entrants $j(\delta) = \int_0^{\hat{\delta}_d^*} b_{\hat{\delta}_0}(\delta) g(\hat{\delta}_0) d\hat{\delta}_0$. Let $i(\delta)$ denote the aggregate mass of incumbents of type δ. Then firms of true type δ accumulate until the measure of entrants, $M_e j(\delta)$, coincides with the measure of defaulting firms $\delta i(\delta)$ yielding $i(\delta) = M_e j(\delta)/\delta$. Thus, we get the true type distribution of incumbents:

$$h(\delta) = \frac{j(\delta)/\delta}{\int_0^1 j(\delta)/\delta \, d\delta}. \tag{I.18}$$

Perceived types of entrants are distributed with $g(\hat{\delta}_0|\hat{\delta}_0 \leq \hat{\delta}_d^*)$ and evolve according to the Bayesian updating process subsequently. Thus, if we fix a perceived type $\hat{\delta}$ and want to determine the density of incumbents for this perceived type, we have to consider two components: new entrants with perceived type $\hat{\delta}_0 = \hat{\delta}$ and older firms that started with a start-up perceived type $\hat{\delta}_0' > \hat{\delta}$ and happen to be assigned a current perceived type $\hat{\delta}$ by Bayesian updating. Let $\hat{\delta}_{-t} > \hat{\delta}$ denote the start-up perceived type that coincides with $\hat{\delta}$ after t periods of Bayesian updating. Then, the entry density of perceived type $\hat{\delta}_{-t}$ equals $M_e g(\hat{\delta}_{-t})$ and the probability that firms of this perceived type survive for t periods is given by $E_{b_{\hat{\delta}_{-t}}}((1-\delta)^t)$ yielding the perceived type density of incumbents $\hat{j}(\hat{\delta}) = \Sigma_{t=0}^{T(\hat{\delta})} E_{b_{\hat{\delta}_{-t}}}((1-\delta)^t) M_e g(\hat{\delta}_{-t})$.[17] Thus, we get the perceived type distribution of incumbents:

$$\hat{h}(\hat{\delta}) = \begin{cases} \frac{\hat{j}(\hat{\delta})}{\int_0^{\hat{\delta}_d^*} \hat{j}(\hat{\delta}) \, d\hat{\delta}} & \text{if } \hat{\delta} \in (0, \hat{\delta}_d^*], \\ 0 & \text{otherwise.} \end{cases} \tag{I.19}$$

Misvaluation of Active Firms

On individual firm level belief updating is rational. Per construction the perceived type $\hat{\delta}_t$ denotes the best approximation of the firms true type conditional on the start-up signal $\hat{\delta}_0$ and the information that the firm did not default for t periods. As financing is conducted at the firm level, the perfectly competitive financial market imposes the interest rate $\hat{\delta}_t$. However, this leads to misvaluation in aggregate terms. Consider a new-born firm generation. As start-up signals are correct in expectation, the true and the perceived average type of this firm generation coincides initially. Both decline with respect to generation age by excess exit of high-δ-types according to the cleansing mechanism. But as the decline of perceived types is amplified by the updating mechanism, the average perceived type is increasingly biased downwards the older the firm generation gets. So, incumbents face interest rates that are too small in expectation, and too many firms become exporters.[18] Still, the financial

[17] As $g(0) = 0$, only perceived types $\hat{\delta} > 0$ are possible. And as $\lim_{t\to\infty}(\hat{\delta}_d^*)_t = 0$, there always exists a finite t s.t. $(\hat{\delta}_d^*)_t < \hat{\delta}$. Thus, $T(\hat{\delta})$ is finite for all $\hat{\delta} > 0$.

[18] If a firm with true type $\delta > \hat{\delta}_x^*$ enters the export market by a misjudgement of its type $\hat{\delta}_t \leq \hat{\delta}_x^*$, it will (in expectation) default before sunk entry costs nf_x are balanced by accumulated per period net profits.

market does not incur losses in aggregate. Free entry and the financing of the start-up investment take those misvaluations into account. However, as start-ups and incumbents compete on the labor market, the relative prevalence of incumbents yields too little entry and thus decreases overall welfare. It might be a concern that this misvaluation is at odds with general equilibrium conditions. Yet all we need is the stationarity of true and perceived type distributions which is discussed in the previous paragraph. It is not necessary that they coincide. As the perceived type $\hat{\delta}_t$ denotes the best estimate of a firms' true type no agent has an incentive to deviate from this belief. Summarizing, the joint impact of the cleansing mechanism from Proposition 2 and the updating mechanism from Proposition 5 implies:

Proposition 7 (Firm Generation Effects)**.** *The older a firm generation, the further perceived and true average exit probabilities deviate, yielding inefficiently low interest rates for incumbents. Thus, the steady state exhibits excessive exporting and insufficient start-up investment.*

The misvaluation in aggregate figures is inherent to the model structure and results from perfectly rational valuations by individual firms.[19] The simplest way to deal with the resulting excessive prevalence of incumbents is to introduce a tax on operating profits that increases in firm age and to use the tax revenue to subsidize start-ups. Firm type generation effects from the known type case (Proposition 3) carry over to the uncertain firm type case.

Firm Masses

In steady state, firm entry balances firm exit, yielding $M_e = E_h(\delta)M/G(\hat{\delta}_d^*)$. Using labor market clearing $L = Mq_d + nM_x\tau q_x$ and the mass of exporting firms $M_x = \hat{H}(\hat{\delta}_x^*)M$, with $\hat{H}(\hat{\delta})$ denoting the cumulative density function of the incumbent distribution $\hat{h}(\hat{\delta})$, we get:

$$M = wL/[(\sigma - 1)(f_d\bar{\delta}(\hat{\delta}_d^*) + nf_x(\hat{H}(\hat{\delta}_x^*) - \psi(\hat{\delta}_d^*))\hat{\delta}_x^*))]. \tag{I.20}$$

A detailed derivation of (I.20) is provided in the Appendix.

[19] Contingent on revealed information, perceived types constitute best approximations of true types.

Consumption

We determine the equilibrium wage rate w from $P = 1$ and obtain aggregate per period consumption:

$$C = Lw/P = Lw = L\rho(M + n\hat{H}(\hat{\delta}_x^*)M\tau^{1-\sigma})^{1/(\sigma-1)}. \tag{I.21}$$

Again, a detailed derivation of (I.21) is provided in the Appendix. Apart from consumption, the final product is spent for start-up investments, market entry of new firms and for foreign market entry of incumbents that turn exporters by Bayesian updating. Let $(\hat{\delta}_x^*)_{-t}$ denote the start-up perceived type that coincides with $\hat{\delta}_x^*$ after t periods of updating, then (conditional on survival) all firms with start-up perceived types $\hat{\delta}_0 \in ((\hat{\delta}_x^*)_{-(t-1)}, (\hat{\delta}_x^*)_{-t}]$ will turn exporters in their t^{th} period. As the entry density of a perceived type $\hat{\delta}_0$ equals $M_e g(\hat{\delta}_0)$ and the probability that firms of this perceived type survive for t periods is given by $E_{b_{\hat{\delta}_0}}((1-\delta)^t)$, the measure of firms of age t that turn exporters by Bayesian updating every period equals $\int_{(\hat{\delta}_x^*)_{-(t-1)}}^{(\hat{\delta}_x^*)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t)M_e g(\hat{\delta}_0)d\hat{\delta}_0$. Adding all possible ages $t = 1, 2, \ldots, T(\hat{\delta}_x^*) < \infty$ we obtain:[20]

$$\begin{aligned} I &= (K + f_d G(\hat{\delta}_d^*) + n f_x G(\hat{\delta}_x^*))M_e \\ &\quad + n f_x \Sigma_{t=1}^{T(\hat{\delta}_x^*)} \int_{(\hat{\delta}_x^*)_{-(t-1)}}^{(\hat{\delta}_x^*)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t)M_e g(\hat{\delta}_0)d\hat{\delta}_0. \end{aligned} \tag{I.22}$$

Similar to the known type case, trade liberalization forces firms with low net profits out of the market shifting production towards firms with higher net profits. But as loan rates (that depend on perceived types) and real per period fixed costs (that depend on real types) differ systematically, this shift does not always improve average efficiency of the economy. As we prove in the Appendix the welfare result from Proposition 4 does not carry over to uncertain firm types.

Proposition 8 (Trade Liberalization and Welfare)**.** *In case of uncertain firm types, trade liberalization can have a negative welfare effect.*

The intuition for this result lies in the fact that belief-updating leads to excessive exporting, as explained above. Lower variable trade costs can exacerbate this inefficiency, which can lead to welfare losses from trade liberalization.

[20] $T(\hat{\delta}_x^*)$ denotes the number of periods of Bayesian updating a firm with highest possible start-up perceived exit probability $\hat{\delta}_d^*$ needs to turn exporter. As $\lim_{t\to\infty} \hat{\delta}_t = 0$ for all $\hat{\delta}$, $T(\hat{\delta}_x^*)$ has to be finite.

Crisis of Confidence

For many observers, the world-wide recession of 2008/09 has been particularly severe because it involved a massive reversal of beliefs on the stability of the financial system (Bacchetta et al., 2010). The relationship between output drop, falling demand, and the banking crisis epitomized by the collapse of the investment bank Lehman Brothers on September 15, 2008, is still a matter of academic debate. Our model is, of course, much too stylized to give a quantitative assessment of the crisis. However, it allows to shed light on the different effects of a belief revision on small as compared to large firms. It captures, admittedly in a a very stylized way, the facts that exports dropped much more than GDP in most countries and in the world (see Behrens et al. (2010) for a discussion) and that large firms saw their financing conditions deteriorate more strongly than small ones. This second fact has been documented using firm-level data for Germany by Rottmann and Wolmershäuser (2010), Costa et al. (2011) for Italy, and Kremp and Sevestre (2011) for France.[21] Costa also shows that exporting firms have been more severely affected than non-exporters.

Belief Revision

We consider a shock that triggers all agents to return to former beliefs, i.e. some firm survival information is deleted.[22] There are several natural ways to model a belief revision. A belief revision could prompt all agents to return to their beliefs a certain number of periods ago, it could prompt all agents to delete a certain fraction of firm survival histories, or in the extreme case prompt all agents to return to start-up perceived types of firms. These scenarios have in common that agents become suddenly less optimistic as to the survival of firms. As start-up beliefs constitute lower bounds for belief revisions, the shock does not force any firms to exit domestic markets. However, some firms stop exporting.

Proposition 9 (Crisis of Confidence)**.** *A belief revision forces some firms to exit foreign markets while leaving the number of domestic firms unaltered.*

[21] This finding relates to the change in the costs of funding; large firms still obtain credit at lower cost than small ones.

[22] Entry or exit information is excluded from the revision, as neither defaulted firms can be reanimated, nor new born firms can be eliminated by a change in belief.

This describes the immediate impact of a crisis in confidence. We compare firm sorting one period before and after the belief revision. Induced aggregate dynamics, or transition paths back to steady state are omitted.

Asymmetric Countries

By incorporating country heterogeneity with respect to fixed market entry costs, we generate multi-level growth into exporting. The older a firm the more export destinations it will serve. To avoid technical complications, we consider a continuum of countries $\iota \in [0,1]$, each being of zero measure.[23] A foreign firm faces fixed costs f_ι upon market entry in country ι. Countries are ordered according to the size of their entry costs, i.e. $\iota < \kappa$ yields $f_\iota < f_\kappa$. To circumvent the special case of all firms only serving the market of country $\iota = 0$, which arises due to our simplifying assumption of free tradability of final goods, we introduce an additional stage of production. The final goods produced by countries shall henceforth be referred to as country good. Those country goods are then used to produce the "new" final good without requiring other inputs according to the standard CES-production function. Both, country goods and final goods, are traded freely. This setup extension nests all previous results, as all countries produce identical amounts of country goods in the symmetric country case. Under this additional stage of production the (normalized) price index of the final good is given by:

$$1 = P = \Big(\int_0^1 P_\iota^{1-\sigma} d\iota \Big)^{1/(1-\sigma)}, \tag{I.23}$$

with

$$P_\iota = \Big(\int_0^1 \Big(\int_{\omega_{\kappa,\iota} \in \Omega_{\kappa,\iota}} p(\omega_{\kappa,\iota})^{1-\sigma} d\omega_{\kappa,\iota} \Big) d\kappa \Big)^{1/(1-\sigma)}, \tag{I.24}$$

denoting the price index of country ι, where $\Omega_{\kappa,\iota}$ denotes the set of intermediate goods imported from country κ. All payments, such as wage payments, loan rates or fixed costs, are still measured in units of final good. Whenever results are independent of country type, we suppress the country indicating subscript.

Uniform Wage Rate

Since individual countries are of zero measure, costs or profits a firm faces within one country are infinitesimal and hence negligible. Only costs or profits a firm faces

[23] The simplifying assumption of measure-zero countries does not alter the qualitative outcome. The key finding is that firms gradually expand their export markets. All we need for this result to hold is that higher exporting fix costs result in higher efficiencies of cut-off exporters. This link remains valid under quite general conditions.

within a positive measure of countries will influence its actions. Thus, all firms that conduct the start-up investment will enter domestic markets, as this entrance at infinitesimal entry costs entails a positive probability of entry into a positive measure of foreign countries, yielding positive expected profits. Hence, true and perceived types of entrants are distributed with probability density function g in all countries. Besides domestic entry fees, also domestic profits are infinitesimal and hence negligible. Thus, firm actions (the choice of export destinations and export prices) solely depend on perceived firm type and are independent of firm location. As neither the distribution, nor the action of firms depend on their location, the aggregate production of intermediate inputs by firms located in one country, is identical for all countries. Thus, by trade balance, all countries are compensated with identical amounts of the final good yielding identical wages in all countries.

Zero Cut-Off Profit Conditions

Firms enter a foreign market ι as soon as per period profit π_ι dominates per period costs $\hat{\delta}_t f_\iota$, yielding the first zero cut-off profit condition $\pi_\iota = \hat{\delta}^*_\iota f_\iota$, with $\hat{\delta}^*_\iota$ denoting the cut-off type for entry into market ι. Dividing per period profits, we get the second zero cut-off profit condition $\hat{\delta}^*_\iota = (f_\kappa / f_\iota)\hat{\delta}^*_\kappa$ for all $\iota, \kappa \in [0, 1]$. Thus, $f_\iota < f_\kappa$ yields $\hat{\delta}^*_\iota > \hat{\delta}^*_\kappa$, i.e. the higher the market entry costs the smaller the set of perceived firm types that enter. Let $\kappa(\hat{\delta}_t)$ denote the "last" country a firm of perceived type $\hat{\delta}_t$ exports to, i.e. the country with cut-off value $\hat{\delta}^*_\kappa = \hat{\delta}_t$. Then, a firm of perceived type $\hat{\delta}_t$ will export to all countries $\iota \in [0, \kappa(\hat{\delta}_t)]$. The lower the firms' perceived exit probability $\hat{\delta}_t$ the greater its measure of export destinations, until, for $\hat{\delta}_t \leq \hat{\delta}^*_{\iota=1}$ it exports to all countries.

Free Entry Condition

Free entry of firms ensures that expected future profits $E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)}(\pi_\kappa - \hat{\delta}_t f_\kappa) d\kappa))$ coincide with costs for the start-up investment K. As we prove in the Appendix, zero cutoff and free entry conditions determine cut-off values $\hat{\delta}^*_\iota$ uniquely. From the ordering of cut-off values ($\iota < \kappa \Rightarrow \hat{\delta}^*_\iota > \hat{\delta}^*_\kappa$) and the updating mechanism (Proposition 5) we find that firms enter more and more markets as they grow in age.

Proposition 10 (Firm Specific Effects)**.** *The measure of export destinations increases in firm age. In a crisis of confidence, firms exit markets with highest fixed costs first.*

Incumbent Distributions

As firms of all types enter, the true type distribution of incumbents equals $h(\delta) = (g(\delta)/\delta)/(\int_0^1 (g(\delta)/\delta)d\delta)$ and the perceived type distribution equals $\hat{h}(\hat{\delta}) = \hat{j}(\hat{\delta})/\int_0^1 \hat{j}(\hat{\delta})d\hat{\delta}$, with $\hat{j}(\hat{\delta}) = \Sigma_{t=0}^{T(\hat{\delta})} E_{b_{\hat{\delta}_{-t}}}((1-\delta)^t)M_e g(\hat{\delta}_{-t})$.

Firm Masses

All firms that conduct the start-up investment enter and firm exit occurs with respect to the true type distribution. Thus, the steady state correspondence of firm masses of entrants and incumbents equals $M_e = E_h(\delta)M$. Firm export status depends on perceived firm type. Thus, the mass of firms within a certain country that export to country κ equals $M_\kappa = \hat{H}(\hat{\delta}_\kappa^*)M$. Additionally, taking into account the labor market clearing condition, $L = \int_0^1 M_\kappa \tau q_\kappa d\kappa$, we obtain:[24]

$$M = wL/[(\sigma-1)\int_0^1 \hat{H}(\hat{\delta}_\kappa^*)\hat{\delta}_\kappa^* f_\kappa d\kappa]. \tag{I.25}$$

Consumption

Determining the equilibrium wage rate w from $P = 1$, we receive aggregate per period consumption:[25]

$$C = Lw/P = Lw = L(\rho/\tau)\Big(M\int_0^1 \hat{H}(\hat{\delta}_\iota^*)d\iota\Big)^{1/(\sigma-1)}. \tag{I.26}$$

In line with the symmetric country case, aggregate per period investment consists of the units of final product needed for start-up investment, KM_e, the units needed for direct market entry, $\int_0^1 G(\hat{\delta}_\iota^*)f_\iota d\iota M_e$, and the units needed for entry into market ι by Bayesian updating, $f_\iota \Sigma_{t=1}^{T(\hat{\delta}_\iota^*)} \int_{(\hat{\delta}_\iota^*)_{-(t-1)}}^{(\hat{\delta}_\iota^*)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t)M_e g(\hat{\delta}_0)d\hat{\delta}_0$. As the last term arises for all markets, we get:

$$\begin{aligned} I &= KM_e + \int_0^1 G(\hat{\delta}_\iota^*)f_\iota d\iota M_e \\ &+ \int_0^1 \Big(f_\iota \Sigma_{t=1}^{T(\hat{\delta}_\iota^*)} \int_{(\hat{\delta}_\iota^*)_{-(t-1)}}^{(\hat{\delta}_\iota^*)_{-t}} E_{b_{\hat{\delta}_0}}((1-\delta)^t)M_e g(\hat{\delta}_0)d\hat{\delta}_0\Big)d\iota, \end{aligned} \tag{I.27}$$

which completes the characterization of the general equilibrium under type uncertainty in an asymmetric country setting.

[24] Details of the derivation are in the Appendix.
[25] Again, see the Appendix for detailed derivations.

I.5 Conclusion

Newly created firms are uncertain as to the viability of their new product. Market expectations about the lifetime of an innovation determine the effective costs of finance for firms. So, if some fraction of firms' investment needs are irreversible, firms differing with respect to the perceived probability of death shocks face different financing possibilities. International trade interacts with this heterogeneity: firms with lower perceived default probabilities are more likely to be exporters, lower trade costs make the expected survival rates of domestic firms smaller but those of exporters larger; firm survival is longer in open compared to closed economies. All this facts are well supported by empirical evidence.

In contrast to firm-level heterogeneity in productivity or product quality, a firm's life expectancy cannot be easily inferred from its production process or its sales statistics. Rather, it is more likely that market participants only receive a noisy signal about the true type of a firm. Conditional on survival of the firm, market participants update their beliefs. This process has important further implications for firm behavior and aggregate outcomes. First, it implies that the financial conditions faced by firms improve over time. Second, due to this, firms will be gradually growing as they enter more and more markets. Third, the updating process leads to an excessive expansion of large incumbents to the expense of start-ups, so that the number of existing firms tends to be too small. Fourth, a sudden reversal of beliefs leads to reduction in economic activity, but the collapse of trade flows is larger than that of total income. Again, these facts square well with empirical facts.

The main advantage of the framework is its simplicity and generality. As long as firms are homogeneous with respect to variable components of revenue, aggregation is very simple. This allows an analytical characterization of firm dynamics without making assumptions on the form of distribution functions. It also makes further extensions of the model possible. One interesting avenue for further research would be to add a more complete description of financial frictions to the model or to allow for a second source of heterogeneity, possibly of the form used in Melitz (2003).

Chapter II

Labor Market Dynamics and Trade

II.1 Introduction

Recently, the attention directed towards trade and inequality is increasing strongly. Models with heterogeneous firms and frictional labor markets do a good job in explaining substantial empirical wage dispersions within narrowly defined skill classes, occupations, or industries. In most countries these within-group differences account for more than two thirds of the overall increase in wage inequality.[1]

Also, the analysis of transition dynamics is recently shifting into the focus of trade economists. Many questions can not be tackled by simply comparing pre- and post-shock steady states. Sometimes this approach even results in incorrect conclusions. Especially labor market frictions can generate grave distortions. E.g. Davidson and Matusz (2006) show that steady state benefits from an expansion of the high-wage-sector can be outbalanced by short-run costs. Kaas and Kircher (2013) show that a positive productivity shock pushes up unemployment initially.

This chapter combines both approaches. I explore the dynamic response of a small open economy with heterogeneous firms and labor market frictions on trade and technology shocks. I study individual and aggregate firm dynamics, transitional wage rates, wage inequality, unemployment, and welfare.

[1] See Heathcote, Perri and Violante (2010) for the US, Fuchs, Kruger and Sommer (2010), or Card, Heining and Kline (2013) for Germany, Blundell and Etheridge (2010) for the UK, Japelli and Pistaferri (2010) for Italy, Li and Xing (2012) for China, or Helpman, Itskhoki, Mündler and Redding (2014) for Brazil.

By incorporating direct labor market search into a simple small open economy, I am able to untangle firm decisions from the aggregate firm distribution. This independence constitutes the heart of this model. It provides tractability not only in steady state but also in the presence of aggregate shocks. It makes it feasible to derive all conclusions on individual firm dynamics analytically and to compute aggregate dynamics without the need to resort to approximation techniques, such as those of Krussel and Smith (1998), that have been applied in the heterogeneous-firm search model of Elsby and Michaels (2010) and Fujita and Nakajima (2009) to analyze aggregate labor market dynamics.

The small open economy setting untangles firm revenues from the aggregate firm distribution. Direct labor market search untangles firm costs.

The small open economy is similar to the Melitz (2003) framework. The key difference is that firms apply a concave production function to manufacture a homogeneous good. This good can either be sold domestically (at low fix costs) or it is exported and sold at a high world market price (and higher fix costs). Typical firm sorting follows: Least productive firms turn inactive, medium productive firms serve the domestic market, and firms with high productivities export. However, there is one crucial deviation: As prices and fix costs are exogenous, firm revenue does not depend on the aggregate firm distribution.

I integrate Kaas and Kircher's (2013) theory of direct labor market search. Firms compete for workers by publicly posting long-term contracts. Higher wages attract more applicants. This increases the job-filling rate for the firm, and decreases the probability of getting the job for the applicant. Job seekers observe all offers (determine expected wages and probabilities of getting the job) and adjust their search accordingly. Hence, expected payoffs of all vacancies coincide. This unique expected payoff constitutes the focal point for all general equilibrium feedback effects. It is the only general equilibrium object entering the firm's maximization problem. If it increases, firm costs increase. If it decreases, firm costs decrease. If it increases, values of start-ups increase. If it decreases, values of start-ups decrease. Hence, this unique expected payoff is pinned down by the free entry condition, and, consequently does not depend on the aggregate firm distribution.[2]

Neither firm revenue, nor firm costs depend on the aggregate firm distribution. They solely depend on firm productivity, firm size, and exogenous parameters. Hence, firm

[2]Here, I assume positive firm entry in every period. This is clearly fulfilled in any steady state with positive firm mass and positive firm default. However, it limits the size of shocks this framework can replicate without further adjustments.

policy functions behave like jump variables. Firms respond to aggregate shocks by immediately switching to new rules of optimal behavior and then keeping those new rules constant without slowly readjusting them as the system converges towards its new steady state. Results on firm dynamics are equally valid in and outside steady state. I confirm the findings of Felbermayr, Impulliti, and Prat (2014). However, while their more sophisticated model requires numerical methods to solve the general case, I derive all findings analytically. Convex vacancy costs make firms expand gradually. Conditional on age (or size), more productive firms exhibit higher growth rates. Conditional on productivity, younger (or smaller) firms exhibit higher growth rates. Firms realize higher growth rates by both, posting more vacancies and filling each vacancy with a higher probability. This stylized fact has recently been uncovered in US data by Davis, Fabermann, and Haltiwanger (2013). Higher job-filling rates are realized by higher wage offers, creating wage dispersion across and within firms. Random search postulates identical job-filling rates for all firms and vacancies. Moreover, directed search conforms mounting evidence that workers indeed direct their search and firms commit to wage contracts (e.g. Hall and Krueger, 2012). It avoids the counterfactual prediction of random search with individual bargaining that wages of existing employees fall when firms approach their optimal size. Furthermore, gradual firm growth provides a natural rationalization for the empirical regularity that productivity distributions of exporters and non-exporters overlap substantially.

I calibrate this model to typical figures of an open economy and explore its qualitative response to aggregate shocks. There are four predominant types (and durations) of aggregate adjustments along the transition path: Wage adjustments (immediate), firm adjustments (approx. 1.5 years), wage distribution adjustments (approx. 10 years), and firm distribution adjustments (approx. 100 years). If the system is hit by a shock, labor market tightness and new wages adjust immediately. Firm policy functions also respond immediately and prompt incumbents to gradually readjust their size and mode of activity. New wages adjust at once, however, old contracts predominate new contracts initially. As pre-shock matches dissolve, old contracts are gradually replaced by new ones, and wage distribution adjustments slowly converge. Firm distribution adjustments exhibit the lowest speed of convergence. There might be firms that would not enter the post-shock environment, but still find it optimal to stay active. There might be firms that would not grow to a certain size in the post-shock environment, but still find it optimal not to shrink below it. Those outliers are slowly eradicated by firm default.

Wage offers jump to a higher level after a positive technology shock. Average wages increase gradually while old contracts are successively replaced by new ones, and the temporary coexistence of both contract types results in a high transitory wage diffusion. The same mechanism occurs after a trade liberalization. However, here positive firm entry is violated for nine periods following the shock and I apply a modified approach to compute this passage.[3] Zero firm entry means that start-ups fail to compete for workers at the labor market. Hence, this violation is tantamount to overshooting wages. Computing the dynamic response of welfare and unemployment, the model exhibits an interesting interaction of technology shocks and openness. Adjustments in more open economies differ substantially from adjustments in less open economies. Welfare rises after a positive technology shock in less open economies and shrinks in more open economies. And while there are significant unemployment adjustments after the immediate response in more open economies, those adjustments are negligible in less open ones. These phenomena are driven by differing firm adjustments and firm distribution adjustments. A positive technology shock affects firm output via two channels. First, it increases firm output by increasing output per worker. Second, it decreases firm output by decreasing the average number of workers per firm (via increased competition on the labor market). Firm output is directly linked to exporting as only firms above a certain output level find it profitable to serve the world market. In less open economies, channel one predominates and the share of exporting firms rises. In more open economies, channel two predominates and the share of exporting firms drops. Exporting is positively connected to welfare: Consider two economies that produce an identical aggregate amount of goods. The first consists of many small firms that are not able to overcome the critical output level for exporting. The second economy consists of a few large firms that export. Then, neglecting fix costs, the welfare of country one equals roughly its aggregate output multiplied by the domestic price, the welfare of country two equals roughly its aggregate output multiplied by the world market price. Accordingly, the rising share of exporting firms in less open economies boosts welfare, and the falling share in more open economies reduces welfare. Exporting is also positively connected to unemployment: Workers are either employees or job seekers. Hence, the unemployment rate coincides with the sum of all workers that are queuing for jobs. Exporters pay higher wages, create longer lines of applicants, and push up unemployment. This counteracts the otherwise negative unemploy-

[3] The complete transition path consists of over 2000 periods. Computing the modified approach for 9 periods increases the run-time of the simulation from approximately 10 minutes to 4 hours.

ment response after a positive technology shock (via increased competition) in less open economies. Both forces oppose and almost neutralize each other. In more open economies the negative unemployment effect after a positive technology shock is amplified by the decreasing share of exporting firms. Here, both forces push in the same direction.

This model contributes to a large literature on trade, wage inequality and unemployment. One branch explores the relationship between trade and wage inequality in models with neoclassical labor markets (e.g. Burstein and Vogel (2009), Bustos (2009), or Yeaple (2005)). These models provide rationales for wage diffusion across different skill classes. However, they are silent about within-group inequality. Another branch introduces labor market frictions and assumes fair or efficiency wages (e.g. Amiti and Davis (2012), Davis and Harrigan (2007), or Egger and Kreickemeier (2009a,b)). These models capture within-group inequality. However, they fail to provide micro foundations for their assumptions. Yet another branch considers search and matching as natural explanation for labor market frictions. Most models consider random search (e.g. Cosar, Guner and Tybout (2011), Felbermayr, Prat and Schmerer (2011), or Helpman, Itskhoki and Redding (2010)). My model is most closely related to Felbermayr, Impulliti and Prat (2014), who introduce direct search in a Melitz (2003) environment. I depart from their setting by incorporating direct search into a small open economy framework. By doing so, I am able to untangle firm decisions from the aggregate firm distribution. This independence simplifies the model substantially. It makes it feasible to study firm dynamics analytically and to explore aggregate dynamics after a trade liberalization or a positive technology shock.

The remainder of this chapter is structured as follows. Section II.2 introduces the model. Section II.3 derives the general competitive search equilibrium. Section II.4 introduces a simplified search equilibrium and proves equivalence of both equilibrium concepts. Then, this simplified search equilibrium is applied to explore firm dynamics in section II.5, and aggregate dynamics in section II.6. Section II.7 concludes.

II.2 Setup

I consider an infinitely repeated game of symmetric information. All transactions (costs, revenues, profits,...) are measured in units of the final good. There is an endogenous mass of heterogeneous firms. Each firm employs a continuum of workers. Workers are homogeneous and their total mass is normalized to one. Both, firms and workers, discount future income with factor $\beta < 1$.

Goods Market

Upon entry, firms pay a set-up cost $K > 0$ and draw their productivity $z \in Z$ from a pdf $g(z)$. A firm with productivity z that employs ℓ workers produces $q = zA(\ell)$ units of a homogeneous good that can be sold domestically at the normalized price 1 or on the world market at price p. $A(\ell)$ denotes a strictly increasing and concave function. Selling domestically yields per period fixed costs $f_d > 0$. Selling on the world market yields both, per period fixed costs $f_x > f_d$ and standard iceberg type variable trade costs $\tau > 1$. Accordingly, I define firm (net) revenue via:

$$r_d(z,\ell) = zA(\ell) - f_d \tag{II.1}$$

in case the firm serves the domestic market, and

$$r_x(z,\ell) = pzA(\ell)/\tau - f_x \tag{II.2}$$

in case it exports.[4] I assume $p > \tau$. If $p \leq \tau$ there is no trade and the small open economy turns into an autarchy.

Labor Market

Firms die with exogenous per period probability $\delta > 0$. In case of firm default all workers are laid off into unemployment. Furthermore each existing firm-worker match separates with exogenous per period probability $\eta > 0$. Searching for new workers is costly. A firm that posts V vacancies incurs recruitment costs $C(V)$. Recruitment costs are strictly increasing and strictly convex. Recruiting firms offer contracts which specify a wage and separation path for all future periods $t \geq \hat{t}$:

$$B_{\hat{t}} = (w_{\hat{t},t}, \eta_{\hat{t},t})_{t \geq \hat{t}},$$

with $\eta_{\hat{t},t} \geq \eta$ for all $t \geq \hat{t}$. Unemployed workers search in the sub market (B,λ) promising the highest expected lifetime income, where a sub market is indexed by contract B and unemployment-vacancy ratio λ. A vacancy is matched with a worker with probability $m(\lambda)$ and a worker finds a job with probability $m(\lambda)/\lambda$. The matching function $m(\lambda)$ is differentiable, strictly increasing, strictly concave

[4] Including fix costs f_d and f_x in firm revenue simplifies the notation in the following.

and satisfies $m(0) = 0$ and $m(\lambda) \leq min(1, \lambda)$ for all $\lambda \in [0, \infty)$. As every firm hires a continuum of workers, firms know with certainty that posting V vacancies in sub market (B, λ) yields $m(\lambda)V$ new hires. There is no search on the job. Each worker is infinitely-lived, risk-neutral, and receives per period income $b \geq 0$ when unemployed.

Timing

Each period consists of four stages. $s = 1$: Inactive firms conduct the start-up investment K and draw their productivity z. Firms decide about market entry. $s = 2$: Firms default with exogenous probability $\delta > 0$ or turn inactive endogenously. Firm-worker match separations take place. $s = 3$: Firms hire new workers. $s = 4$: Firms produce and conduct payments.

II.3 Competitive Search Equilibrium

Optimal Job Search

Let $u_t(0)$ denote the utility of an unemployed worker in period t, and let $u_t(B)$ denote the utility of an employed worker under contract B. Every period t unemployed workers observe all offered contracts B_t and know that the probability to sign a contract B_t offered in sub market (λ, B_t) equals $m(\lambda)/\lambda$. Hence, the expected benefit of searching in sub market (λ, B_t) is given by:

$$\frac{m(\lambda)}{\lambda}\left\{u_t(B_t) - b - \beta u_{t+1}(0)\right\} \tag{II.3}$$

Suppose this value differs between two sub markets, then job seekers will immediately redirect their search to the sub market offering the higher expected benefit and by doing so drive up its worker job ratio λ, and simultaneously drive down the ratio of the other market. Hence, the expected benefit of job search has to coincide across all sub markets with $\lambda > 0$. Let ρ_t denote this unique expected benefit. Then (II.3) pins down the utility $u_t(B_t)$ firms have to offer in order to attract a worker queue of length λ:

$$u_t(B_t) = b + \beta u_{t+1}(0) + \frac{\lambda}{m(\lambda)}\rho_t. \tag{II.4}$$

The utility of unemployed worker solves the Bellman equation $u_t(0) = b + \rho_t + \beta u_{t+1}(0)$, i.e. $u_t(0) = (b + \rho_t)/(1 - \beta)$ in steady state. The size of ρ_t is pinned down by the free entry condition as discussed below.

Optimal Firm Growth

Let $\ell_{\hat{t},\hat{t}}$ and $B_{\hat{t}} = (w_{\hat{t},t}, \eta_{\hat{t},t})_{t\geq\hat{t}}$ denote those employees of a firm that were hired in period $\hat{t}$ and their contracts respectively. Then, in period t, this firm employs $\ell_{\hat{t},t} = \prod_{t'=\hat{t}}^{t}(1-\eta_{\hat{t},t'})\ell_{\hat{t},\hat{t}}$ workers that were hired in period $\hat{t}$. Suppose the firm is of age a. Then its accumulated employment stock from previous periods is given by $\ell_{t-1} = \sum_{\hat{t}=t-a}^{t-1} \ell_{\hat{t},t}$. This denotes its employment stock after separations have taken place (stage $s=2$) and before hiring is conducted (stage $s=3$). Firms solve the following maximization problem. In period t a firm takes as given its productivity z, its employment stock ℓ_{t-1} and the contracts signed with these workers $(B_{\hat{t}})_{\hat{t}=t-a}^{t-1}$. It chooses whether to stay active (in case it is not hit by an exogenous default shock) or to exit the market endogenously. In case it stays active, it chooses whether to serve the domestic market $\iota = d$ or to export $\iota = x$ and decides about the optimal number of vacancies V and the optimal contract B_t:

$$J_t\Big(z, \ell_{t-1}, (B_{\hat{t}})_{\hat{t}=t-a}^{t-1}\Big) = \max_{\iota,V,B_t}(1-\delta)\Bigg\{r_\iota\Big(z, \ell_{t-1}+\ell_{t,t}\Big) - C(V) - W + \beta J_{t+1}\Big(z, \ell_t, (B_{\hat{t}})_{\hat{t}=t-a}^{t}\Big)\Bigg\}, \tag{II.5}$$

$$\text{s.t.} \quad \ell_{t,t} = m(\lambda)V, \quad \ell_t = \sum_{\hat{t}=t-a}^{t} \ell_{\hat{t},t+1}, \quad \ell_{\hat{t},t+1} = \prod_{t'=\hat{t}}^{t+1}(1-\eta_{\hat{t},t'})\ell_{\hat{t},\hat{t}}, \tag{II.6}$$

$$W = \sum_{\hat{t}=t-a}^{t} w_{\hat{t},t}\ell_{\hat{t},t}, \quad u_t(B_t) = b + \beta u_{t+1}(0) + \frac{\lambda}{m(\lambda)}\rho_t. \tag{II.7}$$

Separation rates $\eta_{\hat{t},t'}$ and wages $w_{\hat{t},t}$ are specified in contract $B_{\hat{t}} = (w_{\hat{t},t}, \eta_{\hat{t},t})_{t\geq\hat{t}}$ and the second equation of (II.7) is enforced by optimal job search of unemployed workers (see (II.4)). It is no restriction to assume that the firm offers only one type of contract and searches in only one sub market. Obviously, only firms $(z, \ell_{t-1}, (B_{\hat{t}})_{\hat{t}=t-a}^{t-1})$ with positive value $J_t(z, \ell_{t-1}, (B_{\hat{t}})_{\hat{t}=t-a}^{t-1}) \geq 0$ will stay active, yielding a continuum of cut-off values $z_d(\ell_{t-1}, (B_{\hat{t}})_{\hat{t}=t-a}^{t-1})$ implicitly defined via:

$$J_t\Big(z_d, \ell_{t-1}, (B_{\hat{t}})_{\hat{t}=t-a}^{t-1}\Big) = 0. \tag{II.8}$$

Firms with productivity z below z_d will exit the market endogenously. Firms with productivity z above z_d will grow according to (II.5)-(II.7).

Remark 1. *Two firms with same age a and productivity z face the identical control problem (II.5) - (II.8) every period. Hence, this framework does not differentiate between them and firm type $(z, \ell_{t-1}, (B_{\hat{t}})_{\hat{t}=t-a}^{t-1})$ can likewise be expressed as (z, a).*

I will make us of this more concise firm type in the following.

Optimal Firm Entry

I close the model by formulating two additional identities. First, I pin down the size of ρ_t via the free entry condition for firms. Second, I derive the mass of entrants N_t from the resource constraint implied by the restriction of total labor force to the measure of one.

Assuming positive firm entry, the free entry condition is binding:

$$\int_z J_t(z, a = 0)\, dg(z) = K. \tag{II.9}$$

For start-ups ($a = 0$), it holds $\ell_{t-1} = 0$ and $(B_{\hat{t}})_{\hat{t}=t-a}^{t-1} = \emptyset$. The left hand side of (II.9) strictly decreases in ρ_t, the right hand side is constant in ρ_t. This yields a unique solution. The negative dependency of the left hand side on ρ_t can directly be inferred from (II.7). Raising ρ_t drives up the utility $u_t(B_t)$ firms have to offer to job seekers. Higher utility corresponds to higher expected wages and thus affects firm value negatively.

The resource feasibility implies that job seekers and employed workers add up to a measure of one. Equivalently, all workers holding contracts from previous periods and those queuing for a job (ignoring whether their application is successful or not) add up to one. Let $M_t(z, a)$ denote the measure of firms with productivity z and age a in period t, $\ell_{t-1}(z, a)$ each firm's accumulated employment stock from previous periods (see (II.6)) and $V_t(z, a)$, respectively $\lambda_t(z, a)$ its optimal number of vacancies respectively worker-job ratio. Then, firms $M_t(z, a)$ employ $\ell_{t-1}(z, a)M_t(z, a)$ workers hired in previous periods and additionally attract $\lambda_t(z, a)V_t(z, a)M_t(z, a)$ workers that are queuing for a job. Summing up all firm ages and productivities the resource constraint reads:

$$\sum_{a \geq 0} \int_z \Big\{ \ell_{t-1}(z, a) + \lambda_t(z, a)V_t(z, a) \Big\} M_t(z, a) dg(z) = 1 \tag{II.10}$$

Firm masses evolve according to:

$$M_t(z, a) = 0, \text{ if } z < z_d(a), \tag{II.11}$$
$$M_t(z, a) = (1 - \delta)N_t g(z), \text{ if } a = 0 \text{ and } z \geq z_d(a), \tag{II.12}$$
$$M_t(z, a) = (1 - \delta)M_{t-1}(z, a - 1), \text{ if } a \geq 1 \text{ and } z \geq z_d(a). \tag{II.13}$$

The parameter $z_d(a)$ denotes the domestic cut-off value defined in (II.8) and N_t denotes the measure of firms that conduct the start-up investment in period t. Thus, all firm masses $M_t(z, a \geq 1)$ in (II.10) are determined by firm masses of previous periods and only $M_t(z, a = 0) = (1 - \delta)N_t g(z)$ can adjust according to the resource constraint. This pins down the size of N_t.

General Equilibrium

The following equilibrium concept is equally valid in- and outside steady state. So it allows us to explore labor market dynamics after aggregate shocks.

Definition 2. *Given an initial firm distribution $M_{-1}(z, a)$, a competitive search equilibrium is a list:*

$$\Big\{\iota_t(z, a),\ V_t(z, a),\ B_t(z, a),\ \rho_t,\ M_t(z, a)\Big\}$$

that satisfies:

1. *Optimal firm decisions: $\iota_t(z, a)$, $V_t(z, a)$ and $B_t(z, a)$ solve (II.5)-(II.8).*
2. *Free entry: ρ_t evolves according to (II.9).*
3. *Labor market resource constraint: $M_t(z, a)$ solves (II.10)-(II.13).*

for every firm type (z, a) and every period $t \geq 0$.

This constitutes the general foundation of this model. Next, I prove that I can switch from this general foundation to a very simple and specific representation.

II.4 Simplified Search Equilibrium

I alter the competitive search approach by letting firms maximize a modified surplus (specified in (II.14)). As it turns out, this yields a very parsimonious equilibrium concept independent of job contracts and wage payments. I prove that this simplified search equilibrium is equivalent to the competitive search equilibrium and explore its structure.

Firm Surplus

Consider a firm with productivity z and an accumulated employment stock ℓ_{t-1}. Then, I define its modified surplus via:

$$\begin{aligned} S_t(z, \ell_{t-1}) \quad = \quad & \max_{\iota, V, \lambda, \eta_t} (1 - \delta)\Big\{r_\iota(z, \ell_{t-1} + \ell_{t,t}) - b(\ell_{t-1} + \ell_{t,t}) \\ & -\mu_t(\ell_{t-1} + \lambda V) - C(V) + \beta S_{t+1}(z, \ell_t)\Big\}, \end{aligned} \tag{II.14}$$

$$\text{s.t.} \quad \ell_{t,t} = m(\lambda)V, \quad \ell_t = (1-\eta_t)(\ell_{t-1}+\ell_{t,t}) \quad \text{and} \quad \eta_t \geq \eta, \tag{II.15}$$

if it is non-negative. If this expression is negative the firm turns inactive and its surplus equals zero. Firm surplus (II.14) can be considered as the firm's social value. It encompasses firm revenue net of opportunity cost of labor, and net of vacancy posting costs for all future periods. Opportunity cost of labor consists of two components. First, the opportunity cost of working at the firm $b(\ell_{t-1}+\ell_{t,t})$. Second, the opportunity cost of being allocated to the firm $\mu_t(\ell_{t-1}+\lambda V)$. While the first term is restricted to employed workers $\ell_{t-1}+\ell_{t,t}$, the second is evaluated at $\ell_{t-1}+\lambda V$. It additionally captures unemployed workers who are queuing for a job at the firm (and hence, are not able to search for another job). As the firm does not differentiate between different cohorts of employees, there is no reason to stick to the cohort specific separation rates from the competitive search setting. Hence, the firm chooses a uniform separation probability η_t for all its employees.

Again there is a one to one correspondence of firm type (z, ℓ_{t-1}) and firm type (z, a), with $a \geq 0$ denoting firm age (see Remark 1). I will use both characterizations interchangeably, depending on which fits the specific discussion better.

Optimal Job Search

Unemployed workers direct their search towards jobs which generate the highest additional surplus in expectation (taking into account the probability of getting the job $m(\lambda)/\lambda$). As each firm employs a continuum of workers this additional surplus is the marginal surplus of the firm with respect to its labor force. Suppose its value differs between two vacancies, then job seekers will immediately redirect their search activities towards the vacancy that offers the higher expected marginal surplus and by doing so drive up the corresponding worker job ratio. This decreases the vacancies' expected marginal surplus. Hence, its value has to coincide across all recruiting firms. The opportunity cost of being allocated to a certain firm is the cost of not being able to apply for other vacancies. The value of being able to apply for another vacancy is identical to the vacancies' expected marginal surplus. Thus, the opportunity cost μ_t and this unique expected marginal surplus coincide. Capturing the value of being able to search for a job, μ_t constitutes the dual object to the expected benefit ρ_t discussed in section II.3. As it turns out (proof of Proposition 11) their values indeed coincide in equilibrium. Accordingly, the value of μ_t in equilibrium is pinned down by the dual free entry condition:

$$\int_z S_t(z, a=0)dg(z) = K. \tag{II.16}$$

The left hand side decreases in μ_t, the right hand side is constant in μ_t. This yields a unique solution.

General Equilibrium

Again imposing resource feasibility (II.10) and firm distribution evolution (II.11)-(II.13), the modified equilibrium concept reads:

Definition 3. *Given an initial firm distribution $M_{-1}(z,a)$, a simplified search equilibrium is a list:*

$$\Big\{\iota_t(z,a),\, V_t(z,a),\, \lambda_t(z,a),\, \eta_t(z,a),\, \mu_t,\, M_t(z,a)\Big\}$$

that satisfies:

1. *Optimal firm decisions: $\iota_t(z,a)$, $V_t(z,a)$, $\lambda_t(z,a)$ and $\eta_t(z,a)$ solve (II.14)-(II.15).*
2. *Free entry: ρ_t evolves according to (II.16).*
3. *Labor market resource constraint: $M_t(z,a)$ solves (II.10)-(II.13).*

for every firm type (z,a) and every period $t \geq 0$.

Following proposition asserts the equivalence of the competitive search equilibrium concept and the simplified search equilibrium concept. There may be different competitive search equilibria (with different contract structures $B_t(z,a)$) giving rise to the same simplified search equilibrium. However, as firm dynamics and aggregate dynamics are pinned down by the simplified equilibrium those different competitive search equilibria belong to the same equivalence class concerning their outcomes within this framework.

Proposition 11. *Let $\{\iota_t(z,a),\, V_t(z,a),\, B_t(z,a),\, \rho_t,\, M_t(z,a)\}$ constitute a competitive search equilibrium allocation. Then this allocation also constitutes a simplified equilibrium $\{\iota_t(z,a),\, V_t(z,a),\, \lambda_t(z,a),\, \eta_t(z,a),\, \mu_t,\, M_t(z,a)\}$ with $\mu_t = \rho_t$ and $\lambda_t(z,a)$ and $\eta_t(z,a)$ pinned down by contracts $B_t(z,a)$. Likewise, every simplified equilibrium can be expressed as competitive search equilibrium with contracts $B_t(z,a)$ chosen in line with $\lambda_t(z,a)$ and $\eta_t(z,a)$.*

Proof: Appendix.

II.5 Firm Dynamics

Allowing vacancies to become negative and imposing zero firing costs $C(V) = 0$, perfect matching for separations $m(\lambda) = 1$ and $\lambda = 1$ whenever $V < 0$, then firms can separate from employees by choosing negative vacancies instead of increasing the natural separation rate η (see proof of Proposition 12). This restricts the identification of optimal firm behavior to the three policy functions $\iota_t(z, a)$, $V_t(z, a)$ and $\lambda_t(z, a)$.

Proposition 12 (Policy Functions)**.** *Consider a firm with productivity z and size $\ell_{-t} = \ell$. If the firm chooses $V(z, \ell) \geq 0$, it holds:*

1. $\lambda_z(z, \ell) > 0$ *and* $\lambda_\ell(z, \ell) < 0$,
2. $V_z(z, \ell) > 0$ *and* $V_\ell(z, \ell) < 0$,
3. $\iota(z, \ell) = x$ *if* $pzA(\ell) \geq (\tau - 1)(f_x - f_d)$, *otherwise* $\iota(z, \ell) = d$.

If the firm chooses $0 > V(z, \ell) > -\ell$, it holds:

1. $\lambda(z, \ell) = 1$,
2. $V(z, \ell) = (A')^{-1}((b + \mu)/z) - \ell < 0$,
3. $\iota(z, \ell) = x$ *if* $pzA(\ell) \geq (\tau - 1)(f_x - f_d)$, *otherwise* $\iota(z, \ell) = d$.

If the firm chooses $V(z, \ell) = -\ell$, it defaults endogenously and generates zero surplus $S(z, \ell) = 0$.

Proof: Appendix.

This proposition is equally valid in and outside steady state. If the economy is hit by an aggregate shock, firms immediately switch to new policy functions. Both regimes of optimal behavior (the policy functions before and after the shock) fulfill Proposition 12.

Corollary 4. *Suppose $V(z, \ell) \geq 0$. Then, conditional on size, more productive firms post more vacancies and exhibit higher job-filling rates, i.e. $V_z(z, \ell) > 0$ and $\lambda_z(z, \ell) > 0$. Conditional on productivity, older (and larger) firms post less vacancies and exhibit lower job-filling rates, i.e. $V_\ell(z, \ell) < 0$ and $\lambda_\ell(z, \ell) < 0$.*

Independence from the Aggregate Firm Distribution

One important restriction is the assumption of positive firm entry in every period. This is clearly fulfilled in any steady state with positive firm mass and positive firm default probability. Though, ensuring positive firm entry in every period along the transition path might limit the size of shocks we are able to analyze without further adjustments (depending on other model parameters). However, the assumption untangles firm decisions from the aggregate firm distribution. To gain intuition for this independence, consider the search problem of an unemployed worker. He can either direct his job search towards incumbents (firms of age $a > 0$), or towards start-ups (firms of age $a = 0$). Every recruiting firm has to offer the identical expected marginal surplus μ_t. Thus, the expected marginal surplus of incumbents is pinned down by the expected marginal surplus of start-ups. The expected surplus of start-ups in turn, has to fulfill the free entry condition (II.16). This pins down the size of μ_t independently of the aggregate firm distribution.

This is not true if the measure of start-ups equals zero. In this case, recruiting incumbents only compete with other recruiting incumbents. Hence, the incumbent offering the lowest expected marginal surplus that is still able to attract a positive measure of job seekers pins down the size of μ_t. The firm type of this least efficient recruiting incumbent depends on the distribution of existing firms.

If there is positive entry, μ_t solves (II.16) in every period. It constitutes a jump variable with respect to aggregate shocks. If the size of an exogenous parameter changes and affects firm surplus, μ_t responds immediately to keep the free entry condition satisfied. There is no time leg or slow adjustment process. Hence, also firm surplus $S_t(z, a)$ behaves like a jump variable. And consequently the same holds true for firm policy functions $\iota_t(z, a)$, $V_t(z, a)$, $\lambda_t(z, a)$. This means if an exogenous shock occurs, firms immediately switch to new rules of optimal behavior, and then keep those rules constant without slowly readjusting them as the system converges towards its new steady state.

II.6 Aggregate Dynamics

I consider two scenarios: A trade liberalization modeled via a sudden 1% reduction of variable trade costs τ, and a positive technology shock modeled via a 5% increase of the entrant productivity distribution $g(z)$. In neither scenario the shock is anticipated. It hits the system in its initial steady state and after the shock, parameters remain constant until the system reaches its new steady state. I explore resulting transition paths by implementing the simplified search equilibrium (Definition 3) in Matlab R2012a.

I infer salaries by assuming flat wage contracts. A flat wage contract $B_{\hat{t}} = (w_{\hat{t},t}, \eta_{\hat{t},t})_{t \geq \hat{t}}$ offers constant per period compensation $w_{\hat{t},t} = w$ as long as the match persists and zero compensation $w_{\hat{t},t} = 0$ subsequently. The separation probability is also constant and is chosen as small as possible, i.e. $\eta_{\hat{t},t} = \eta$. These contracts establish a one to one correspondence of wage rates w and sub markets λ, and provide a simple means to study the development of inequality along the transition path.

Calibration

I study the implications of this framework by calibrating it to typical figures of an open economy. The purpose of this numerical simulation is to reveal the qualitative response of the model, to explore predominant forces, and not to discuss the specific magnitudes of effects. I choose the functional forms in line with Kaas and Kircher (2013), i.e. firm production, search costs, and matching function are given by $A(\ell) = \ell^\alpha$, with $\alpha = 0.7$, by $C(V) = cV^2$, and by $m(\lambda) = 1/(1 + \kappa/\lambda)$ respectively. Also according to Kaas and Kircher (2013), I choose the target values for average worker job ratio $E[\lambda] = 1.4$ and for average job finding probability $E[m(\lambda)/\lambda] = 45\%$.[5] The remaining target values are taken from Felbermayr, Impullitti and Prat (2014). I calibrate the search cost parameter c, the matching function parameter κ, the unemployment benefit b and the monthly separation probability s, to match the target values for the labor market listed in the first panel of table II.1. The third target value ensures that average wages are approximately three times the size of the unemployment benefit b. The world market price p, initial set-up costs K, per period fixed costs of domestic activity f_d, and per period fixed costs for exporting f_x are chosen to match the target values of the second panel of table II.1. Firm size is measured in number of employees.

The model matches the labor market moments almost perfectly. It performs less accurate on firm distributional moments. This result corresponds to a rich labor market modeling and very stylized firm product markets in this framework. There are some more parameters which are not calibrated but pinned down directly. The discount factor $\beta = 0.9967$ matches an annual interest rate of 4%. The monthly

[5] $E[\cdot]$ denotes the expected value with respect to the firm distribution, i.e. $E[\lambda] = \frac{1}{M}\sum_{a=0}^{\infty}\int_{z_{min}}^{z_{max}} \lambda(z,a)M(z,a)dz$.

Labor Market	Parameter	Model	Target	Source
Avg. worker job ratio	$c = 0.02$	1.4	1.4	Kaas and Kircher
Avg. job finding rate	$\kappa = 0.90$	45%	45%	Kaas and Kircher
Unempl. benefit rel. to wage	$b = 0.02$	33%	35%	Felbermayr et. al
Unemployment rate	$s = 0.05$	8.3%	8.3%	Felbermayr et. al
Firms				
Share of exporting firms	$p = 3.75$	27%	28%	Felbermayr et. al
Avg. firm size	$K = 900$	32	36	Felbermayr et. al
Avg. size domestic firm	$f_d = 1$	16	15	Felbermayr et. al
Avg. size exporter	$f_x = 15$	75	89	Felbermayr et. al

Table II.1: Calibrated parameters.

firm default probability equals $\delta = 0.3\%$ from Felbermayr, Impullitti and Prat (2014). In line with Kaas and Kircher (2013) productivity levels of start-ups are uniformly distributed across 40 equidistant values between $z_{min} = 0.8$ and $z_{max} = 1$, with the minimum productivity level being smaller than the domestic cut-off value $z_{min} < z_d(a = 0)$ in all transition periods and steady states of both scenarios.

Response Types

There are four predominant types of dynamic responses to either shock.

1. *Immediate response:* Wage adjustments.
2. *Short-term response:* Firm adjustments (around 1.5 years).
3. *Medium-term response:* Wage distribution adjustments (around 10 years).
4. *Long-term response:* Firm distribution adjustments (around 100 years).

The marginal surplus μ_t (which can likewise be interpret as labor market tightness) constitutes a jump variable with respect to aggregate shocks. Hence, labor market tightness switches to its new value at once and then remains unchanged subsequently. This drives the immediate wage response. Incumbent firms switch to their new policy functions immediately, and then slowly adjust their size and mode of activity (inactive, domestic, or exporting) according to those new rules. Convex vacancy costs prevent them from directly jumping to their optimal new size and mode of activity. This generates the short-term response. New wages adjust immediately, however there are old contracts that substantially shape the aggregate wage distribution initially. As those old matches dissolve, the wage distribution adjustments diminish. The duration of this medium-term response is primarily driven by the value of the separation probability η and the value of the firm default probability δ. Firm distribution adjustments exhibit the lowest speed. There might be old firms of productivity levels that would not enter in the post-shock environment but still find it optimal to stay active. There might be firms that would not grow to a certain size in the post-shock environment but still find it optimal not to shrink below this size. Those outliers are slowly eradicated by firm default with probability δ, and the aggregate firm distribution converges.

Inequality

There is overshooting of average wages after a trade liberalization, and monotonic adjustment of average wages after a positive technology shock. In both scenarios there is overshooting of wage variance during the medium-term response of approximately 120 periods (or 10 years).

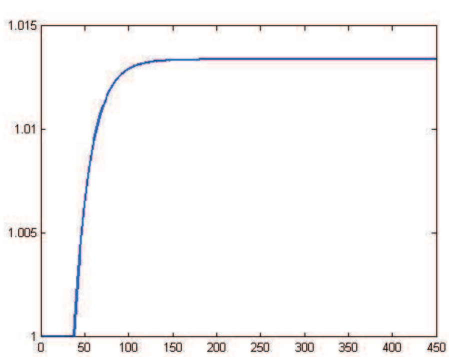

Figure II.1: Average wage impulse response to a permanent 5% increase of firm productivity.

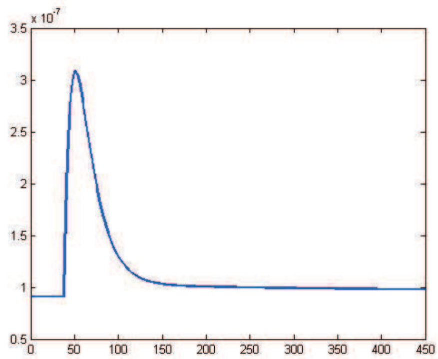

Figure II.2: Impulse response of wage variance to a permanent 5% increase of firm productivity.

The interpretation of figure II.1 and II.2 is straight forward. The positive technology shock intensifies firm competition and pushes the labor market tightness to a higher level. The average wage (of new contracts) adjusts immediately. It switches to a higher value and remains unchanged subsequently. Hence, the average wage (of all contracts) monotonically increases as new contracts are signed and old matches break. This results in an increased transitory wage variance that peaks around the time when the mass of new and old contracts coincide.

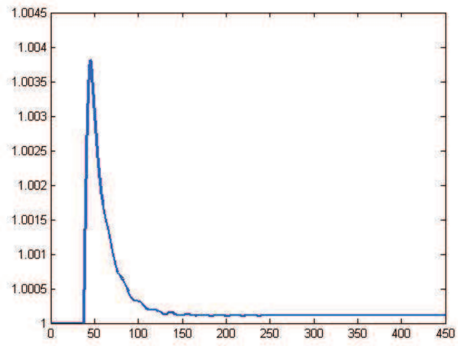

Figure II.3: Average wage impulse response to a permanent 1% reduction of variable trade cost τ.

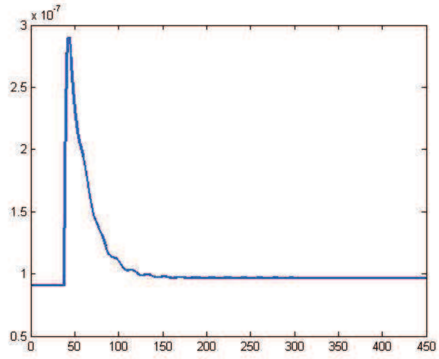

Figure II.4: Impulse response of wage variance to a permanent 1% reduction of variable trade cost τ.

The same mechanism underlies figure II.3 and II.4. However, there is one significant difference. While scenario one (the trade liberalization) increases the efficiency of incumbents and entrants alike, scenario two (the positive technology shock) only increases the efficiency of start-ups. In scenario one, a considerable fraction of incumbents responds with accelerated growth. In scenario two, all incumbents respond with reduced growth rates. It turns out that expanding incumbents in scenario one absorb the complete mass of workers initially. Positive firm entry is violated for 9 periods. Hence, the value of μ has to be pinned down by the alternative condition that the mass of absorbed workers coincides with the mass of available workers. The marginal surplus μ constitutes a variable cost component for workers in firm value (II.14). The higher μ the lower the mass of absorbed workers, the lower μ the higher the mass of absorbed workers, yielding an unique solution. However, this solution is not independent from the aggregate firm distribution. The value of μ depends on the type of the least efficient firm that is still able to attract a positive measure of job seekers. The more drastic the worker shortage, the higher this cutoff firm type. While there is zero firm entry, the aggregate firm mass slowly shrinks, the worker shortage lessens, the cutoff firm type decreases, and finally it reaches the level where firm entry turns positive again. Hence, μ jumps to a high level after the shock, then shrinks monotonically until it hits the threshold of positive firm entry, and then remains unchanged subsequently. The wage overshooting displayed in figure II.3 follows.[6]

[6] Solving this modified condition for 9 periods increases the run-time of the simulation from approximately 10 minutes to 4 hours.

Unemployment

There is no overshooting or undershooting of unemployment in either scenario. The unemployment rate is positively correlated with openness, and negatively correlated with productivity levels of entrants. A trade liberalization increases unemployment. A positive technology shock decreases unemployment. The first finding is not very surprising.[7] The second finding is standard. However, there is an interesting interaction of productivity shocks and openness: Unemployment responds stronger in more open economies. Figure II.5 shows the impulse response of the unemployment rate to a positive technology shock for different levels of openness, of an approximately 27% share of exporting firms (for $\tau = 1.4$ and $\tau = 1.407$), a 24% share (for $\tau = 1.414$) and a 21% share (for $\tau = 1.421$).

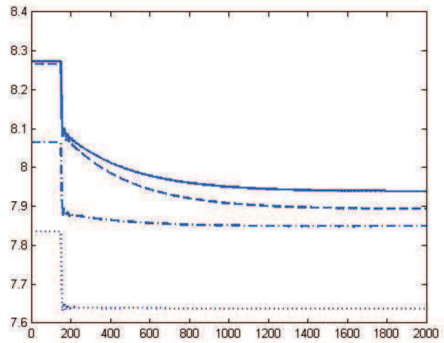

Figure II.5: Unemployment impulse response to a permanent 5% increase of firm productivity for $\tau = 1.4$ (solid line), $\tau = 1.407$ (dashed line), $\tau = 1.414$ (dash-dot line) and $\tau = 1.421$ (dotted line).

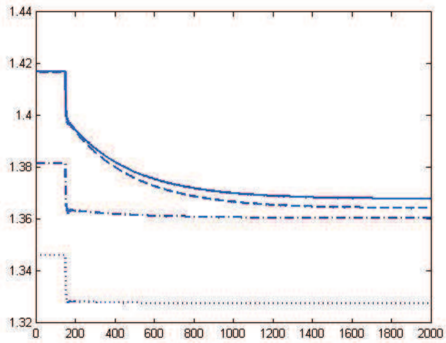

Figure II.6: Impulse response of average applicants per job-offer to a permanent 5% increase of firm productivity for $\tau = 1.4$ (solid line), $\tau = 1.407$ (dashed line), $\tau = 1.414$ (dash-dot line) and $\tau = 1.421$ (dotted line).

Workers are either employed, or they are applying for a job at a certain firm. Hence, unemployment is driven by the average number of applicants per job offer (figure II.6) and their average success in getting the job (figure II.7). A positive technology shock decreases unemployment by both, decreasing the average number of applicants per job, and increasing their probability of being successful. All curves exhibit a similar immediate response to the technology shock with respect to trade openness. However, their trend after this immediate response differs systematically. While there is a significant short-, and long-term response for $\tau = 1.4$ and $\tau = 1.407$, it is very small for $\tau = 1.414$, and almost non-existent for $\tau = 1.421$. The interpretation of the immediate response is straight forward. The technology shock pushes up μ,

[7] Helpman, Itskhoky and Redding (2010) show that the impact of trade on unemployment is ambiguous.

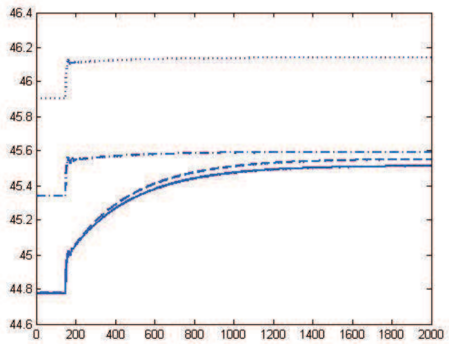

Figure II.7: Average job finding rate impulse response to a permanent 5% increase of firm productivity for $\tau = 1.4$ (solid line), $\tau = 1.407$ (dashed line), $\tau = 1.414$ (dash-dot line) and $\tau = 1.421$ (dotted line).

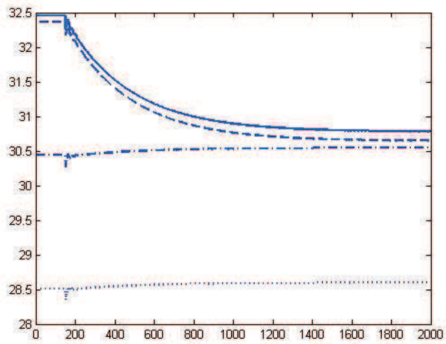

Figure II.8: Average firm size impulse response to a permanent 5% increase of firm productivity for $\tau = 1.4$ (solid line), $\tau = 1.407$ (dashed line), $\tau = 1.414$ (dash-dot line) and $\tau = 1.421$ (dotted line).

this increases the cost of having many applicants. The average number of applicants decreases, the job finding rate increases and unemployment drops. To understand the short- and long-term response we have to dig a little deeper. The short- and long-term response is driven by firm adjustments and firm distribution adjustments. Firms respond by adjusting their size and mode of activity (inactive, domestic, or exporting). Figure II.8 shows that for $\tau = 1.4$ and $\tau = 1.407$ firms adjust by decreasing their size. This yields lower average growth rates and entails lower average numbers of applicants per job. The negative short- and long-term response of unemployment follows. This channel is inactive for $\tau = 1.414$ and $\tau = 1.421$. The positive technology shock increases competition and prompts firm to shrink independent of τ. However, for $\tau = 1.414$ and $\tau = 1.421$ this firm size adjustment is neutralized by the firm sorting adjustment. The average size of domestic firms shrinks from 16.5 to 16.3, and from 16.6 to 16.4 for $\tau = 1.414$ and $\tau = 1.421$ respectively. The average size of exporters shrinks from 74.3 to 73.7, and from 74.1 to 73.5 for $\tau = 1.414$ and $\tau = 1.421$. However, the relative share of exporting firms increases from 24% to 25%, and from 20.5% to 21.5% for $\tau = 1.414$ and $\tau = 1.421$. This opposing effect neutralizes otherwise negative short- and long-term changes of average firm size, and diminish the short- and long-term response of unemployment.

Welfare

Welfare is measured in terms of aggregate output. Welfare is positively correlated with openness and increases after a trade liberalization. Surprisingly, welfare is not positively correlated with productivity in general. Figure II.9 shows a negative welfare response to a positive technology shock for more open economies ($\tau = 1.4$ and $\tau = 1.407$) and a positive response for less open economies ($\tau = 1.414$ and $\tau = 1.421$). Figure II.10 shows a similar result for the response of the relative share of exporting firms. The lines for $\tau = 1.4$ and $\tau = 1.407$ overlap. Their value is very similar, however not identical. The line for $\tau = 1.407$ lies below the line for $\tau = 1.4$ and tracks it with an approximately constant distance of 0.01.

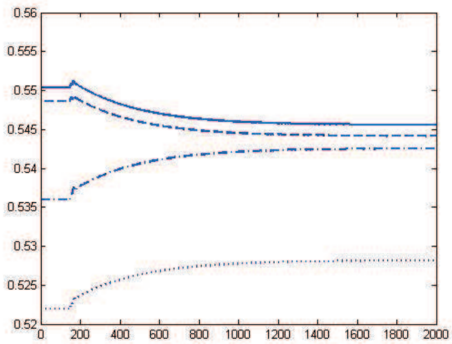

Figure II.9: Impulse response of aggregate output to a permanent 5% increase of firm productivity for $\tau = 1.4$ (solid line), $\tau = 1.407$ (dashed line), $\tau = 1.414$ (dash-dot line) and $\tau = 1.421$ (dotted line).

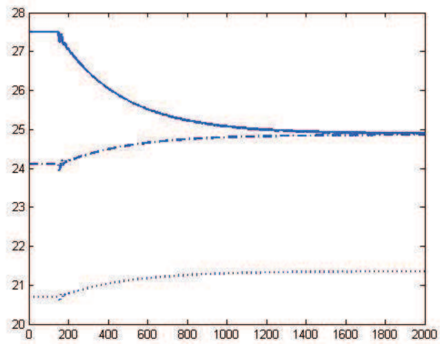

Figure II.10: Impulse response of share of exporters to a permanent 5% increase of firm productivity for $\tau = 1.4$ and $\tau = 1.407$ (solid line), $\tau = 1.414$ (dash-dot line) and $\tau = 1.421$ (dotted line).

Once a firm hits a certain output level, selling at the world market and paying exporting fix costs generates more profit than domestic activity. It becomes an exporter. This interaction of firm output and exporting constitutes the key driver of the welfare response in figure II.9. Consider two small open economies producing an identical aggregate amount of goods. The first economy consists of many small firms that are not able to overcome the critical output level for exporting. The second economy consists of few large enterprises that export. Then, neglecting fix costs, in the second case welfare will be approximately p times higher than in the first case (with p denoting the world market price). A positive technology shock affects firm output via two channels. First, it increases firm output via increasing output per worker. Second, it decreases firm output via decreasing the average number of workers per firm. This second effect results from intensified firm competition on the labor market after the positive technology shock. As it turns out, channel one predominates channel two for less open economies ($\tau = 1.414$ and $\tau = 1.421$). A positive technology shock increases average firm output, the share of exporters rises and pushes up welfare. For more open economies ($\tau = 1.4$ and $\tau = 1.407$) the impact of channel two prevails. A positive technology shock decreases average firm output, the share of exporters drops and welfare shrinks.

II.7 Conclusion

By incorporating direct labor market search into a simple small open economy, I am able to untangle firm decisions from the aggregate firm distribution. This independence provides tractability not only in steady state but also in the presence of aggregate shocks. It makes it feasible to derive all conclusions on firm dynamics analytically and to compute aggregate dynamics without the need to resort to approximation techniques. Firm policy functions solely depend on firm productivity, firm size, and exogenous parameters. Hence, firms respond to aggregate shocks by immediately switching to new rules of optimal behavior and then keeping those new rules constant without slowly readjusting them as the system converges towards its new steady state, and results on firm dynamics are equally valid in and outside steady state. Convex vacancy costs make firms expand gradually and provide a natural rationalization for the empirical regularity that productivity distributions of exporters and non-exporters overlap substantially. Conditional on age (or size), more productive firms exhibit higher growth rates. Conditional on productivity, younger (or smaller) firms exhibit higher growth rates. Firms realize higher growth rates by both posting more vacancies and filling each vacancy with a higher probability. Higher job-filling rates are realized by higher wage offers, creating wage dispersion across and within firms.

I calibrate the model to typical figures of an open economy and study its dynamic response to a trade liberalization and a positive technology shock. There are four predominant types (and durations) of aggregate adjustments along the transition path: Wage adjustments (immediate), firm adjustments (approx. 1.5 years), wage distribution adjustments (approx. 10 years), and firm distribution adjustments (approx. 100 years). While a trade liberalization generates overshooting wage averages, a positive technology shock entails monotonically increasing averages. Both scenarios imply significant transitory inequalities. Variances of the aggregate wage distribution overshoot substantially. While a trade liberalization pushes up unemployment, a positive technology shock decreases the number of jobless workers. The adjustment speed of unemployment after a positive technology shock is higher in less open economies. A trade liberalization increases welfare. A positive technology shock decreases welfare in more open economies and increases welfare in less open economies.

Chapter III

Firm Life Cycles in a Global Economy

III.1 Introduction

This chapter provides a coherent theory of gradual firm growth and decay in open economies.

Existing theories on firm dynamics can be divided into two groups: Theories exploring evolution of supply characteristics, and theories exploring evolution of demand characteristics. The first group is silent about firm specific expansion paths. The second group does not capture productivity evolution. Furthermore, both groups restrict on firm growth and neglect shrinkage. In contrast, Sampson (2014) puts forward a theory of firm shrinkage caused by aggregate technological progress. However, he neglects firm growth. Combining these separate approaches, this model replicates complete firm life cycles beginning with firm birth, followed by positive productivity growth and individual firm expansion, fading into stepwise contraction, and ending with either endogenous or exogenous death.

Typical firm aging patterns are driven by three core mechanisms: First, there is supply uncertainty leading to gradual improvement of production techniques on firm level. Second, there is demand uncertainty resulting in firm specific expansion paths. Third, there are knowledge spillovers which constantly intensify competition and lead to crowding out of old firms. While firms are young, expanding forces generated by improving production techniques outweigh contracting forces generated by increasing competition. Young firms grow and enter new markets. They learn about countries demand characteristics and adjust their expansion path accordingly.

As firms grow older, expected productivity improvements decrease and contracting forces take over. Market exit prevails. They are slowly driven out of all countries and by leaving their last destination they turn inactive and die.

Supply uncertainty generates firm learning and firm growth: Each firm is assigned an unobservable productivity distribution upon its birth. This distribution generates a new productivity sample every period. Whenever a new productivity sample dominates the firm's current productivity it switches technology and produces according to this new productivity. This generates productivity growth. As firm productivity distributions do not change along firm life cycles, expected growth rates are highest for young firms and converge towards zero as firm age converges towards infinity. Firms are heterogeneous with respect to their productivity distribution. Hence, they cannot learn about their distribution by observing competitors. However, they can learn about their distribution by observing their own productivity sample history. This learning has no effect on their current productivity, but it makes them more or less optimistic about their future productivity evolution, and via this channel market entry/exit decisions are influenced.

Demand uncertainty results in firm specific expansion paths: Each firm is assigned a per period demand shock probability for every country. Demand is either positive or - when hit by a shock - completely vanishes. Those market specific default probabilities do not change along firm life cycles. They are not observable, however, firms can learn about them by various means. First, market entry triggers a noisy signal of its associated default probability. Second, firms update their belief according to their survival histories. If a firm is hit by a demand shock in a certain country, it gets less optimistic. If it survives, it gets more optimistic. Third, there is cross-country learning. Demand characteristics of countries are positively correlated to a varying degree. Hence, a firm belief for a certain market is also affected by demand signals in other markets. Firm specific entry signals, firm specific survival histories and cross-country learning entail firm specific beliefs which result in firm specific expansion paths. However, more productive firms will enter more markets in expectation. This does not only result in a negative correlation of firm productivity and endogenous firm death, but also in negative correlation of firm productivity and exogenous firm death. Endogenous firm death corresponds to a firm that turns inactive voluntarily. Exogenous firm death corresponds to a firm that is hit by a demand shock in all its destinations simultaneously.

Technology diffusion constantly intensifies competition: There are knowledge spillovers from incumbent firms to entrants. Start-ups observe production techniques of existing firms and imperfectly apply them to their product. In line with Sampson (2014) I assume that entrants draw their productivity type from a distribution that depends on the productivity type distribution of incumbents. This generates monotonically improving entrant distributions of productivity distributions - their productivity type - and results in monotonically improving average productivities of incumbents.

This model contributes to a new and large literature on trade, firm heterogeneity, and firm dynamics. One branch features evolution of supply characteristics as drivers of firm dynamics. It explores firm dynamics that are generated by firm type evolution. For example, Arkolakis (2011) and Impullitti, Irarrazabal, and Opromolla (2013) consider dynamics extensions of Melitz (2003), in which firms experience exogenous random shocks to their productivity. Atkeson and Burstein (2010), and Burstein and Melitz (2012) model endogenous innovations in firm productivity. In Costantini and Melitz (2009) firms face both idiosyncratic uncertainty and sunk costs for both exporting and technology adoption. Liu (2012) considers firms that can adjust production capacities through capital investment over time. Another branch focuses on demand characteristics of firms. It explores dynamics that are generated by the environment of firms. Albornoz et al. (2012) and Akhmetova (2013) emphasize firm learning about uncertain demand. Eaton et al. (2012) and Chaney (2011) analyze how matches between buyers and sellers evolve over time and across markets. While supply uncertainty rationalizes positive productivity growth, it does not explain firm specific expansion paths. And while demand uncertainty rationalizes firm specific expansion paths, it is silent about productivity dynamics. Furthermore, both theories restrict on analyzing drivers for positive firm growth. Neither rationalizes firm shrinkage. Aging and contraction of firms can be rationalized by knowledge spillovers à la Sampson (2014). However, he limits his approach to a theory of firm shrinkage. Firms start their life cycles at maximum size and then slowly decay until they vanish. I depart from these existing models by exploring forces that generate firm supply evolution, firm demand evolution, and firm contraction within one simple unified framework.

Beside these recently developed theories, there also exists an older branch studying the interaction of trade and firm dynamics initiated by Vernons (1966) seminal article on product life cycles (e.g. Krugman (1979), or Grossman and Helpman (1991)). However, there is no gradual firm growth and shrinkage in these theories either. Firms in the innovating North are born (by inventing a new product), jump to their optimal size at once, produce, and finally turn inactive instantly (when their product is copied by the South). Firm life cycles in the South follow a similar pattern.

The remainder of this chapter is organized as follows. Section III.2 describes the basic framework. It introduces supply and demand uncertainty and technology diffusion. Section III.3 pins down firm profits, cut-off conditions, firm entry, and proves existence of the balanced growth path. Section III.4 explores firm dynamics and discusses resulting life cycle patterns. Section III.5 concludes.

III.2 Setup

I consider an infinitely repeated game of symmetric information. All transactions (costs, revenues, profits,...) are measured in units of the final good.

Countries

There are $n \in \mathbb{N}$ symmetric countries being located on a circle with constant space in between neighbors.[1] The corresponding metric on $\{1, \dots, n\}$ is given by

$$d(i,j) = \begin{cases} 0, & \text{if } |i-j| = 0 \\ |i-j|, & \text{if } |i-j| \leq n/2 \\ |(i+n)-j|, & \text{if } |i-j| > n/2 \text{ and } i < j \\ |i-(j+n)|, & \text{if } |i-j| > n/2 \text{ and } i > j. \end{cases}$$

The greater the distance $d(i,j)$ between two countries i and j, the higher the variable trade costs τ_{ij}. Let $\tau : \{0, \dots, n/2\} \to \mathbb{R}$ denote a monotonically increasing function with $\tau(0) = 1$, then I define $\tau_{ij} = \tau(d(i,j))$. This is the most general form of increasing iceberg-type trade costs ensuring symmetry, i.e. countries of identical distance face identical costs. Similarly, there is a demand correlation between countries that solely depends on the distance $d(i,j)$ as outlined below. Moreover, each country is populated by a representative household of size $L > 0$, who supplies labor inelastically, and who cares about the quantity of a final good C according to a linear utility function. Hence, per capita utility is $u = C/L$.

[1] Without loss of generality I assume n to be an even integer. This is avoids purely technical case distinctions that do not contribute to a deeper understanding of the economic mechanisms within this framework.

Final Good Sector

In each country there is an endogenous mass of producers of differentiated intermediate inputs. These inputs either enter the domestic final good sector, or they are exported and feed into a foreign final good sector. While the final good is freely tradable, shifting intermediate inputs to a foreign market is subject to iceberg-type trade costs $\tau_{ij} \geq 1$. Let us restrict our attention to one specific final good sector. Let $\omega \in \Omega$ denote the continuum of all possible intermediate inputs (or firms) in all countries, and let $q(\omega)$ denote the net quantity of the intermediate input ω that enters this specific final good sector. This quantity is either positive or zero. If the quantity is positive and the firm ω is located in the same country as the final good sector net and gross quantities coincide, otherwise net and gross quantities differ according to the variable trade cost that arise from shipping the firm's product. Each firm ω decides on a certain net quantity $q(\omega) \geq 0$. In aggregate, those decisions define a quantity mapping $q : \Omega \to \mathbb{R}$. Alternatively, we can regard this quantity mapping as an element of the space of mappings from Ω to $\mathbb{R}$, i.e. $q \in \Omega^{\mathbb{R}}$. The final good sector assembles all inputs according to a functional Γ on $\Omega^{\mathbb{R}}$ which transforms such quantity mappings $q \in \Omega^{\mathbb{R}}$ into the final good:

$$Y = \Gamma(q) = C + I. \tag{III.1}$$

The final good can be either consumed by households, C, or used as investment by firms, I. The final good sector is perfectly competitive, implying the identity of firm revenue and firm surplus. Existence of the general equilibrium (proof of Proposition 13) requires the restriction to functionals Γ which generate identical revenue functions $r_\omega(q(\omega)) = r(q(\omega))$ for all producers $\omega \in \Omega$. This means revenue solely depends on the amount of the net quantity $q(\omega)$ and is independent of the specific variety ω. Moreover, firm revenue $r(q(\omega))$ must exhibit decreasing returns to scale. This general setting incorporates various model structures. In case all quantity mappings are Lebesgue integrable and Ω is compact, the standard p-norm on $L^p(\Omega, \mathbb{R})$ would constitute an example for such a functional. Another more specific example would be the Melitz (2003) framework with Γ denoting the widely used CES aggregator function.

Supply Uncertainty

There is firm learning and evolution with respect to supply and demand characteristics. The supply-side heterogeneity is a noisy version of the standard Melitz (2003) heterogeneity. Each firm is assigned a distribution $g_\alpha(z)$ upon its creation that does not change during its entire life cycle. Every period the firm randomly draws a new productivity sample from this distribution. If this new productivity level dominates the old one, the firm switches technology and produces according to this new productivity. Hence, the current productivity z of firms is observable. However, their potential for improvement $g_\alpha(z)$ is not. It can only be noisily inferred from the firm's productivity sample history. The parameterized family of distributions $g_\alpha(z)$, $\alpha \in (0, \infty)$ is ordered such that $g_\alpha(z)$ first order stochastically dominates $g_{\alpha'}(z)$ whenever $\alpha > \alpha'$. Let $(z_s)_{t-a \leq s \leq t}$ denote the history of productivity draws of a firm of age a in period t, then its current productivity $z \in (0, \infty)$ equals the maximum of those draws and its perceived productivity potential $h(\alpha)$ denotes a distribution assigning every productivity potential α a probability that is consistent with its productivity sample history. This perceived productivity potential is updated every period according to Bayes law. While pricing decisions (affecting the current period) depend on the current productivity z only, market entry/exit decisions (affecting longer time horizons) also depend on the perceived productivity potential $h(\alpha)$. Those are the key objects capturing supply uncertainty: The productivity potential α is the firm's true type but is unobservable, hence all firm decisions are based on the firm's current productivity z, and its perceived productivity potential $h(\alpha)$.

Demand Uncertainty

The demand-side heterogeneity constitutes a very stylized version of demand uncertainty. Either there is positive demand for a firm's product in a certain country, or demand is hit by a default shock and completely vanishes. Upon its birth, a firm is assigned a per period default probability δ_j for each country $j \in \{1, \ldots, n\}$. Those default probabilities are unobservable and remain unchanged over the entire firm's life cycle. They are randomly generated by a publicly observable, joined distribution $v(\delta_1, \ldots, \delta_n)$. This distribution is identical for all firms in all countries. Moreover, it is symmetric with respect to countries. Thus, independent of firm origin, all firms have the identical ex-ante default probability for a certain market j and this ex-ante default probability for country j coincides with the ex-ante default probability for any other market $l \leq n$. Furthermore, $v(\delta_1, \ldots, \delta_n)$ introduces positive correlations among default probabilities for different countries $cor_v : \{0, \ldots, n/2\} \rightarrow [0, 1] \subset \mathbb{R}$, $d(j, l) \mapsto cor_v(d(j, l)) \in [0, 1]$ which solely depend on country-pair distance $d(j,l)$.[2] Admittedly, restricting country demand correlations on $d(j,l)$ is limiting, however, it is the most general form that ensures country symmetry. Without symmetry the model would loose its simplicity and tractability. Besides, note that I do not impose any monotonicity assumption on $cor_v(d(j,l))$, i.e. a country could exhibit very similar demand characteristics to a distant country, while deviating substantially from its neighbor. Upon entering country $j \in \{1,\dots,n\}$, a firm triggers a publicly observable noisy signal $\zeta_j \in (0,1)$ of its country specific per period default probability $\delta_j \in (0,1)$. This signal is correct in expectation. The corresponding perceived default probability distribution $k_{j,t}(\delta)$ in period t is inferred via Bayesian updating based on signals ζ_i and survival histories $(\chi_{i,s})_{i\leq n, t-a\leq s<t}$, again with a denoting firm age. It assigns probabilities to all possible default probabilities δ_j that are the most likely given the firm's demand signals. If $\chi_{i,s}=0$, the firm has been active in market i in period s, if $\chi_{i,s}=1$, it has been inactive. If the firm suffers a demand shock, $\chi_{i,s}=2$ is triggered. To understand updating more closely restrict the model to one country and consider a firm with a very poor start-up signal. Initially agents expect the firm to default with a high probability. The longer the firm survives, the less the agents will trust in the accuracy of its start-up signal and correct the perceived default probability downwards. In the limit, the firm could only survive for an infinite number of periods if its true type was $\delta = 0$. Thus, in the hypothetical one-country case, evolutions of expected values of perceived default probabilities $(E_{k_{i,s}}[\delta])_{t-a\leq s\leq t}$ constitute segments of length a of monotonically decreasing sequences that start at $E_{k_{i,t-a}}[\delta]$ and converge towards zero, $lim_{t\to\infty} E_{k_{i,t}}[\delta]=0$. This result is a direct implication of demand uncertainty. It is solely driven by the unobservability of firms market specific default probability. For a detailed discussion refer to Felbermayr and Spiegel (2013). Returning to the multi-country case, the monotonicity of perceived type evolutions is not guaranteed anymore. It is violated by the introduction of demand correlations among countries: If a firm receives a negative demand signal in a country that is positively correlated to country j, it will turn less optimistic about $k_j(\delta)$. Hence, perceived types will fluctuate up and down in the course of a firm's live and exhibit an downward drift whenever there are no demand shocks.

Technology Diffusion

Apart from firm supply and demand uncertainty, there is one more source of dynamics. There are knowledge spillovers from incumbents to entrants. Start-ups observe

[2] With a slight abuse of notation, I use $cor_v(d(j,l))$ and $cor_v(\delta_j,\delta_l)$ interchangeably.

production techniques of existing firms and imperfectly apply them to their product. In line with Sampson (2014), I assume that entrants draw their productivity type from a distribution that depends on the productivity distribution of incumbents. Let $m^e_{t-1}(\alpha)$, $m^i_{t-1}(\alpha)$ denote the probability density function (pdf) of entrant, respectively incumbent productivity potentials in period $t-1$, then the pdf of entrants in period t is given by:

$$m^e_t(\alpha) = (1-\vartheta)m^e_{t-1}(\alpha) + \vartheta m^i_{t-1}(\alpha).$$

The parameter $\vartheta \in [0,1]$ denotes the speed of technology diffusion: If $\vartheta = 0$, there is no technology diffusion. New firms draw their productivity α from a constant distribution $m^e_t(\alpha) = m^e(\alpha)$ in all periods t. This corresponds to the standard Melitz (2003) setting. If $\vartheta = 1$, technology diffusion is very high. The distribution of start-ups completely resembles the distribution of incumbents $m^e_t(\alpha) = m^i_{t-1}(\alpha)$. For any $\vartheta > 0$ the general equilibrium does not constitute a steady state, but a balanced growth path. While the distribution of productivity potentials $(m^i_t(\alpha))_{t\geq 0}$ represents a traveling wave with an increasing lower bound, the minimum current productivity z_{min} does not necessarily increase. This depends on the precise specification of $g_\alpha(z)$. However, as $g_\alpha(z)$ first order stochastically dominates $g_{\alpha'}(z)$ whenever $\alpha > \alpha'$, the incumbent distribution of current productivities also pushes more and more weight to higher values of z, and its average $\bar{z}_t$ increases monotonically with respect to t.

Timing of Actions and Flow of Information

Each period t consists of five stages. Bayesian updating processes new firm information whenever this new information is generated, during all stages. $s = 1$: Inactive firms may turn active by sinking the start-up investment K and are assigned their type $(\alpha, (\delta_i)_{i\leq n})$ which is randomly generated by ex-ante type distributions $m^e_t(\alpha)$ and $v(\delta_1, \ldots, \delta_n)$ respectively. While the firm type is not revealed, ex-ante type distributions are publicly observable. $s = 2$: Active firms receive a new productivity draw z_t randomly generated by $g_\alpha(z)$ and switch current productivity in case $z_t > z$. Perceived productivity potentials $h(\alpha)$ are updated. $s = 3$: Active firms may decide to enter, to re-enter, or to exit markets consecutively. So informations gathered by entering, can be used to assess further entry-/exit-decisions within the same stage. Upon entering a market j, firms incur entry costs F and trigger a market specific demand shock probability signal ζ_j. Re-entry also generates fix costs F but does not trigger a new signal ζ_j. Perceived default probabilities $(k_i(\delta))_{i\leq n}$ are updated. $s = 4$: Active firms produce and generate profits. $s = 5$: Demand shocks are evaluated. A firm $(\alpha, (\delta_i)_{i\leq n})$ that serves market j is hit by a demand shock with probability δ_j. Perceived default probabilities $(k_i(\delta))_{i\leq n}$ are updated again.

III.3 Balanced Growth Path

Firm Learning

Firm learning depends on three categories of data: The firm's true type, its observable type, and its information set. Firm dynamics are driven by the firm's true type, but as it is unobservable decisions are based on firm's observable type which in turn is derived from the signals the firm triggered in previous periods, its information set. The true type of a firm:

$$\big(\alpha, (\delta_i)_{i\leq n}\big), \tag{III.2}$$

is given by the firm's true productivity potential α (respectively $g_\alpha(z)$) and its true demand shock probability in all markets $(\delta_i)_{i\leq n}$. It remains unchanged in time. As it is unknown decisions in period t are based on the firm's observable type:

$$\big(z, h(\alpha), (k_i(\delta))_{i\leq n}\big)_t, \tag{III.3}$$

consisting of its current productivity z, its perceived productivity potential $h(\alpha)$, and its perceived default probabilities $(k_i(\delta))_{i\leq n}$. The firm's observable type is inferred via Bayesian updating based on the firm's information set:

$$\big((z_s)_{t-a\leq s<t}, (\zeta_i)_{i\leq n}, (\chi_{i,s})_{i\leq n, t-a\leq s<t}\big), \tag{III.4}$$

comprising all its productivity samples $(z_s)_{t-a\leq s<t}$, its market entry signals $(\zeta_i)_{i\leq n}$, and its complete survival history $(\chi_{i,s})_{i\leq n, t-a\leq s<t}$. The parameters a and n denote firm age and number of countries respectively.

Entry, Exit, Re-Entry and Firm Death

Entering or re-entering markets incurs initial fixed costs $F > 0$. The market entry signal ζ_i is only triggered in case the firm enters country i for the first time, re-entry does not trigger a new signal. In line with Melitz (2003), firms may invest whenever investment costs (K or F) are balanced by investment returns in expectation. There are two possibilities that a firm leaves a market. Either it decides to leave the market endogenously, or it is hit by a demand shock. Either way, it can re-enter upon paying the fixed payment F. Only if a firm is not active in any market it is dead and can not re-enter. As all δ_i are strictly positive, the probability of suffering a demand shock in all markets simultaneously is strictly positive too. Hence, no firm survives for an infinite number of periods even if the speed of technology diffusion equals zero. If there is positive technology diffusion some firms will be hit by such a default shock and exit, and some will be diminished by monotonically increasing productivities of competitors and finally exit voluntarily. I refer to the first option as exogenous firm death, and to the second as endogenous firm death.

Per Period Operating Profits

Let us first discuss per period operating profits of firms in a stationary environment without technology diffusion, i.e. $\vartheta = 0$, and consider the general case, $\vartheta > 0$, in step two. A firm $(z, h, k) = (z, h(\alpha), (k_i(\delta))_{i \leq n})$ needs $\ell = q/z$ units of labor in order to produce q units of output. Hence, a firm (z, h, k) located in country i realizes:

$$\pi_{ij}(z, h, k) = \max_{q \in \mathbb{R}_+} \left\{ r(q) - w\tau_{ij}\frac{q}{z} - E_{k_j}[\delta]f \right\} \tag{III.5}$$

per period operating profit from serving market j. The first term denotes firm revenue. The second term denotes labor costs: Given variable trade costs τ_{ij}, the net quantity q corresponds to a gross quantity $\tau_{ij}q$, which in turn results in labor demand $\ell = \tau_{ij}q/z$. The factor w denotes the wage rate in country i, and by symmetry, in all countries $l \leq n$. The last term, the component $E_{k_j}[\delta]f$ is generated by periodically arising flow fix costs f: In order to serve market j the firm has to invest f upfront. This investment is recovered at the end of the period if the firm is not hit by a demand shock on market j, otherwise it is lost, yielding an expected loss of $E_{k_j}[\delta]f$. Let us discuss some properties of $\pi_{ij}(z, h, k)$: Finite labor supply L and smooth firm revenue $r(q)$ ensures finiteness of per period operating profits for all firms. Hence, assuming $z > 0$ identity (III.5) constitutes a well defined function. Moreover, as z directly affects $\pi_{ij}(z, h, k)$ solely via labor demand $\ell = \tau_{ij}q/z$, a firm's operating profit increases in its current productivity. Next we consider the general case $\vartheta > 0$. If there is positive technology diffusion the general equilibrium does not constitute a steady state but a balanced growth path. Hence, both the revenue function $r_t(q)$ and the wage rate w_t depend on t. Thus, a firm (z, h, k) located in country i that delivers a net quantity q to market j in period t realizes operating profits:

$$\pi_{ij,t}(z, h, k) = \max_{q \in \mathbb{R}_+} \left\{ r_t(q) - w_t\tau_{ij}\frac{q}{z} - E_{k_j}[\delta]f \right\}. \tag{III.6}$$

For each period t the firm's operating profit (III.6) behaves like the firm's operating profit in a stationary environment (III.5). The properties discussed above remain valid. However, the formula gained an additional dimension that was invisible before: Per period profits do not only change with respect to firm type, but also with respect to period t. Given positive technology diffusion, average current productivities of incumbents $(\bar{z}_s)_{s\geq t}$ constitute a monotonically increasing sequence. As labor supply L is constant, this increases firm competition on the labor market and leads to crowding out of firms that were able to generate positive profits in previous periods. Hence, all else equal (especially assuming the current productivity z to be constant) operating profits $\pi_{ij,t}(z,h,k)$ decrease in t. With the help of country specific per period operating profits, we can now pin down total per period operating profits. Let $\Lambda = P\{1,\ldots,n\}$ denote the power set of $\{1,\ldots,n\}$, and let $\lambda_t \in \Lambda$ denote the subset of markets a firm (z,h,k) serves in period t, then its total operating profit is given by:

$$\pi_{i\lambda_t}(z,h,k) = \sum_{j\in\lambda_t} \pi_{ij,t}(z,h,k). \tag{III.7}$$

From the discussion of $\pi_{ij,t}(z,h,k)$ follows that $\pi_{i\lambda_t}(z,h,k)$ is well defined, it increases in z, and decreases in t.

Total Firm Profits

Total profits do not only depend on current productivities z but also on the future productivity evolution. The future productivity evolution is inferred from the firm's perceived potential h. Let $\psi_s(z'|z,h)$ denote the perceived switching probability of a firm (z,h) to switch to productivity z' in s periods, then the firm's expected operating profit in s periods is given by $E_{\psi_s}[\pi_{ij,s}]$.[3] Hence, the firm expects to generate:

$$\Pi_{ij,t}\big(z,h,k\big) = \sum_{s\geq t} E_{k_j,\psi_s}\big[(1-\delta)^{s-t}\pi_{ij,s}(z,h,k)\big] - F \tag{III.8}$$

total profits from selling its product to market j. F denotes initial market entry costs. Again let $\lambda_t \in \Lambda$ denote the markets the firm serves in period t. Then, total profits of this firm do not merely constitute a sum of market specific profits

[3] I call ψ_s *perceived* switching probability because it depends on the *perceived* productivity potential $h(\alpha)$ and therefore deviates from the true switching probability which would depend on the firm's true type α. As firms switch technology only if the new productivity sample dominates their current productivity, it holds $\psi_s(z'|z,h) = 0$ whenever $z' < z$. The longer the firm's productivity sample history $(z_s)_{t-a\leq s\leq t}$ the higher the probability for a high productivity draw. Hence, the expected current productivity in period s dominates the expected current productivity in period s', $E_{\psi_s}[z] \geq E_{\psi_{s'}}[z]$, whenever $s > s'$.

$\sum_{j\in\lambda_t}\Pi_{ij,t}(z,h,k)$. The reason is that serving a new market changes the firm's profit and decreases its death probability at the same time. Consider a purely domestic firm. If it suffers a demand shock it is dead. Had it been active in another country it would have survived and could re-enter the home market in the next period. Hence, entry in a new country does not only affect profits generated within this new country, but also potentially positive profits in other markets that would be lost in case of exogenous firm death. Total profits of a firm (z,h,k) serving markets $\lambda_t \in \Lambda$ are given by:

$$\Pi_{i\lambda_t}\big(z,h,k\big) = \sum_{s\geq t}\sum_{\lambda_t\in\Lambda} E_{k,\psi_s}\big[\rho_{\lambda_s}(z,h,k)\pi_{i\lambda_s}(z,h,k) - |\lambda_s\backslash\lambda_{s-1}|F\big], \tag{III.9}$$

with $\rho_{\lambda_s}(z,h,k)$ denoting the probability of serving markets λ_s in period $s\geq t$ and $|\lambda_s\backslash\lambda_{s-1}|$ denoting the cardinality of the set $\lambda_s\backslash\lambda_{s-1}$, i.e. the number of markets the firm would enter in case of λ_s. In this notation the firm's default probability is given by $\rho_{\lambda_s=\emptyset}(z,h,k)$ with $\emptyset\in\Lambda$ denoting the empty set. An explicit derivation of those probabilities is provided in the Appendix within the proof of Proposition 13.

Cut Off Conditions

Let $\Lambda_{-j} = \{\lambda\in\Lambda|\lambda\cap\{j\}=\emptyset\}$, $\Lambda_{+j} = \{\lambda\in\Lambda|j\in\lambda\}\subset\Lambda$ denote the set of all subsets of $\{1,\dots,n\}$ that *do not*, respectively *do* contain j. Consider a firm that serves markets $\lambda_{t-1}\in\Lambda_{-j}$ in period $t-1$. Then, this firm is not active on market j. It enters market j in period t if total profits given entry, $\lambda_t\in\Lambda_{+j}$, dominate total profits given non-entry, $\lambda_t\in\Lambda_{-j}$:

$$\Pi_{i\lambda_t\in\Lambda_{+j}}\big(z,h,k\big) \geq \Pi_{i\lambda_t\in\Lambda_{-j}}\big(z,h,k\big). \tag{III.10}$$

Analogously, a firm that is active in market j, $\lambda_{t-1}\in\Lambda_{+j}$, endogenously exits the market in period t if total profits given exit dominate total profits given non-exit:

$$\Pi_{i\lambda_t\in\Lambda_{-j}}\big(z,h,k\big) \geq \Pi_{i\lambda_t\in\Lambda_{+j}}\big(z,h,k\big). \tag{III.11}$$

As I verify in the proof of Proposition 13 these conditions yield unique current productivity cut-off values for entry, $z^*_{ij,t}(\lambda_{t-1},h,k)$, and exit, $z^{**}_{ij,t}(\lambda_{t-1},h,k)$.

Firm Entry and General Equilibrium

The mass of entrants M^e_t in period t is pinned down by the free entry condition:[4]

$$E_{m^e_t}[\Pi_{i\emptyset}(z,h,k)] = K. \tag{III.12}$$

[4]By symmetry the mass M^e_t coincides in all countries. Therefore I neglect country indices.

The left hand side denotes total profits an inactive firm ($\lambda_{t-1} = \emptyset$) expects to generate by turning active. $E_{m_t^e}[\cdot]$ denotes the expected value with respect to the ex-ante productivity distribution $m_t^e(\alpha)$. The left hand side decreases in M_t^e: Increasing the mass of entrants decreases future operating profits $\pi_{ij,s\geq t}(z,h,k)$. The right hand side constitutes the start-up investment firms have to conduct to turn active. Its value is constant. Hence, restricting $F > 0$ to lie in between $\lim_{M_t^e \to \infty} E_{m_t^e}[\Pi_{i\emptyset}] = 0$ and $\lim_{M_t^e \to 0} E_{m_t^e}[\Pi_{i\emptyset}] > 0$, condition (III.12) closes the model and ensures positive and finite mass of entrants $0 < M_t^e < \infty$ and in turn a positive and finite mass of incumbents $0 < M_t^i < \infty$.[5]

Proposition 13. *Given identical initial conditions $m_{t=0}^e(\alpha)$, $m_{t=0}^i(\alpha)$ in all countries $j \leq n$, there exists a symmetric general equilibrium such that firm profit is given by (III.9), cut-off values are defined by (III.10) and (III.11), and firm entry solves (III.12) for every $t \geq 0$.*

Proof: Appendix.

The focus is on existence. Uniqueness is neglected. The reason is our sole interest in exploring individual firm life cycles. I am not interested in analyzing or comparing aggregate outcomes or dynamics or discussing aspects of firm distributions. Therefore I do not distinguish between two different general equilibria that give rise to identical firm profit (III.9). Both yield identical firm life cycles. Hence, they belong to the same equivalence class with respect to their implications, i.e. all outcomes are independent of their possible difference.

III.4 Firm Life Cycles

Having introduced the model structure and the general equilibrium above, the next step is to explore individual firm decisions within this framework. I do not consider any macro shocks, and let the system to evolve according to the balanced growth path specified in Proposition (13).

Current Productivity Dynamics

Consider a firm with true productivity potential α. Then its productivity samples are randomly generated with respect to the distribution $g_\alpha(z)$. Whenever a new

[5] Here I implicitly assume that the costs f and K are sufficiently small for $\lim_{M_t^e \to 0} E_{m_t^e}[\Pi_{i\emptyset}]$ to be strictly positive.

productivity sample z_t dominates its current productivity z it switches to this new productivity $z = z_t$. As the true productivity potential α is fixed and does not change over the firm's life cycle, the probability of a productivity draw z_t that increases a firm's current productivity $z = \max\{z_{t-a}, \ldots, z_t\}$ decreases in firm age a. Let $\triangle_t z$ denote a firm's current productivity change in period t, i.e. $\triangle_t z = \max\{z_{t-a}, \ldots, z_t\} - \max\{z_{t-a}, \ldots, z_{t-1}\}$ and consider two firms (z, h, k) that solely differ in age $a > a'$. Then, the young firm is more likely to experience a positive productivity shock, $Prob_{a'}[\Delta_t z > 0] > Prob_a[\Delta_t z > 0]$. Moreover, given a positive productivity shock, the shock of the young firm is higher in expectation. This means, $E_{g_\alpha}[\Delta_t z | \Delta_t z > 0]$ decreases in firm age.

Corollary 5. *Productivity shocks are either positive or zero. The likelihood and the expected size of positive productivity shocks decrease in firm age.*

Perceived Productivity Dynamics

A firm with perceived productivity potential $h(\alpha)$ expects its productivity samples z_t to be distributed according to $E_h[g_\alpha(z)]$. If a productivity sample z_t dominates the expected value of this distribution, i.e. $z_t > E_h[E_{g_\alpha}[z]]$, than it is better than expected and prompts the firm to become more optimistic about its perceived productivity potential $h(\alpha)$. If z_t is smaller than $E_h[E_{g_\alpha}[z]]$, the firm turns less optimistic about $h(\alpha)$. As $g_\alpha(z)$ first order stochastically dominates $g_{\alpha'}(z)$ whenever $\alpha \geq \alpha'$, turning more or less optimistic about $h(\alpha)$ corresponds to shifting weight to higher or lower values of α respectively. To formalize the change in optimism, consider total firm profit $\Pi_{i\lambda_{t-1}}(z, h, k)$. For given values of z, k and λ_{t-1}, it constitutes an ordering on the set of Lebesgue integrable functions on $[0, 1]$ via $\Pi_{i\lambda_{t-1}}(z, \cdot, k) : L^1[0, 1] \to \mathbb{R}, h \mapsto \Pi_{i\lambda_{t-1}}(z, h, k)$. Suppose the firm's perceived productivity potential equals h_{t-1} in period $t-1$ and h_t in period t. Then I define the size of belief revision in period t via $\Delta_t h = \Pi_{i\lambda_{t-1}}(z, h_t, k) - \Pi_{i\lambda_{t-1}}(z, h_{t-1}, k)$. Thus $\Delta_t h$ captures the relative impact of h_t and h_{t-1} on total firm profit all else equal. Positive values of $\triangle_t h$ correspond to increasing optimism, negative values to decreasing optimism.

Corollary 6. *Firms can get more or less optimistic about their productivity potential. Belief revisions $\Delta_t h$ depend on productivity samples $(z_s)_{t-a\leq s<t}$:*

$$\begin{aligned} z_t > E_h[E_{g_\alpha}[z]] &\Rightarrow \triangle_t h > 0, \\ z_t = E_h[E_{g_\alpha}[z]] &\Rightarrow \triangle_t h = 0, \\ z_t < E_h[E_{g_\alpha}[z]] &\Rightarrow \triangle_t h < 0. \end{aligned}$$

Moreover, the expected size of belief revisions decreases in firm age.

The last point follows from decreasing impact of new productivity samples z_t in firm age: Suppose there is a firm that survives for an infinite number of periods. Then, its productivity sample z_∞ has zero significance relative to its productivity sample history $(z_s)_{s\leq\infty}$.

Perceived Demand Dynamics

Consider a firm with observable type $(z, h(\alpha), (k_i(\delta))_{i\leq n})_t$ and information set $((z_s)_{t-a\leq s<t}, (\zeta_i)_{i\leq n}, (\chi_{i,s})_{i\leq n, t-a\leq s<t})$. There are three events that trigger changes in its perceived demand risk $k_j(\delta)$ for country j: New demand signals ζ_l, survival information $\chi_{l,t} = 0$ and demand shocks $\chi_{l,t} = 2$. Whenever a demand signal ζ_l is above its expected value $\zeta_l > E_{k_l}[\delta]$, it causes an upward correction of the corresponding belief. This means $k_l(\delta)$ shifts weight towards high values of δ. If markets l and j are correlated, this upward correction of $k_l(\delta)$ yields an upward correction of $k_j(\delta)$ and the firm turns less optimistic about its demand stability in market j. Analogously $\chi_{l,t} = 0$, $\chi_{l,t} = 2$ triggers a downward respectively an upward adjustment of $k_j(\delta)$. The stronger the demand correlation between market j and l, the greater the impact of these belief revisions. Similarly to $\triangle_t h$, I define the size of such a belief revision $\triangle_t k_j$ with respect to the relative impact of $k_{j,t-1}$ and $k_{j,t}$ on total firm profit. Let $k_{-j} = (k_i(\delta))_{i\leq n, i\neq j}$ denote all perceived demand risks except $k_j(\delta)$. Then, for given values of z, h, k_{-j} and λ_{t-1}, total firm profits constitute an ordering on the space of Lebesgue integrable functions on $[0,1]$ via $\Pi_{i\lambda_{t-1}}(z, h, k_{-j}, \cdot) : L^1(0,1) \to \mathbb{R}, k_j \mapsto \Pi_{i\lambda_{t-1}}(z, h, k_{-j}, k_j)$. Defining $\Delta_t k_j = \Pi_{i\lambda_{t-1}}(z, h, k_{-j}, k_{j,t}) - \Pi_{i\lambda_{t-1}}(z, h, k_{-j}, k_{j,t-1})$, positive values of $\Delta_t k_j$ correspond to increasing optimism, and negative values to decreasing optimism.

Corollary 7. *Firms can get more or less optimistic about their demand stability in country j. Belief revisions $\Delta_t k_j$ depend on market entry signals $(\zeta_l)_{l\leq n}$ and survival* *histories* $(\chi_{l,s})_{l\leq n,t-a\leq s<t}$ *on correlated markets* $l \leq n$*:*

$$\begin{aligned}
\zeta_l > E_{k_l}[\delta] \quad &\Rightarrow \quad \triangle k_j < 0,\\
\zeta_l < E_{k_l}[\delta] \quad &\Rightarrow \quad \triangle k_j > 0,\\
\chi_{l,t} = 0 \quad &\Rightarrow \quad \triangle k_j > 0,\\
\chi_{l,t} = 2 \quad &\Rightarrow \quad \triangle k_j < 0.
\end{aligned}$$

The stronger the correlation between market j *and* l*, the higher the expected size of belief revisions triggered by market* l*.*

Entry-/Exit-Dynamics

Firm decisions are based on a comparison of costs and benefits. Market entry costs arise from F, expected costs from staying active arise from f. Benefits from market entry correspond to additional operating profits (III.6) and a decreasing exogenous firm death probability. The comparison of these costs and benefits is implicitly carried out by comparing the value of firms current productivities z with cut-off values for entry $z^*_{ij,t}(\lambda_{t-1}, h, k)$ and exit $z^{**}_{ij,t}(\lambda_{t-1}, h, k)$. As f constitutes the sole source of fixed costs from staying active, it follows $z^{**}_{ij,t}(\lambda_{t-1}, h, k) \to 0$ for $f \to 0$. And as the sole cost difference of entry and of staying active is given by F, it follows $z^*_{ij,t}(\lambda_{t-1}, h, k) > z^{**}_{ij,t}(\lambda_{t-1}, h, k)$ whenever $F > 0$, and $z^*_{ij,t}(\lambda_{t-1}, h, k) \to z^{**}_{ij,t}(\lambda_{t-1}, h, k)$ for $F \to 0$.

Corollary 8 (Cut off values)**.** *There is no endogenous market exit if flow fixed costs vanish:*

$$z^{**}_{ij,t}(\lambda_{t-1}, h, k) \to 0 \textit{ for } f \to 0.$$

Cut off productivities for entry and exit converge if market entry costs vanish:

$$z^*_{ij,t}(\lambda_{t-1}, h, k) \to z^{**}_{ij,t}(\lambda_{t-1}, h, k) \textit{ for } F \to 0.$$

Whenever entry is costly, it only occurs at strictly higher productivity levels than exit:

$$z^*_{ij,t}(\lambda_{t-1}, h, k) > z^{**}_{ij,t}(\lambda_{t-1}, h, k) \textit{ if } F > 0.$$

Firms entry- and exit-dynamics are completely determined by the movement of the firm's current productivity and its cut off values. Consider a firm (z, h, k) that serves markets $\lambda_{t-1} \in \Lambda_{-j}$ in period $t-1$, i.e. it is not active in country j. If the size of its current productivity or the size of the entry cut-off changes, and the new value of z dominates the new cut-off value $z > z^*_{ij,t}(\lambda_{t-1}, h, k)$, the firm enters market j. It does not matter which value moves in which direction. Only their relative size in period t matters. Analogously, the firm exits market j if $\lambda_{t-1} \in \Lambda_j$ and z drops below $z^{**}_{ij,t}(\lambda_{t-1}, h, k)$. The evolution of z is specified in Corollary 5. Holding λ_{t-1} and t fixed, the evolution of cut-off values is completely determined by the movement of h and k. These are specified in Corollary 6 and Corollary 7. If the firm becomes more optimistic, i.e. $\Delta_t h > 0$ or $\Delta_t k_j > 0$, expected total profits $\Pi_{ij,t}\big(z, h, k\big) = \sum_{s \geq t} E_{k_j, \psi_s}\big[(1-\delta)^{s-t} \pi_{ij,s}(z, h, k)\big] - F$ increase and cut-off values $z^*_{ij,t}(\lambda_{t-1}, h, k)$, $z^{**}_{ij,t}(\lambda_{t-1}, h, k)$ drop. Finally, for given values of h, k both cut-off productivities $z^*_{ij,t}(\lambda \in \Lambda_{-j}, h, k)$, $z^{**}_{ij,t}(\lambda \in \Lambda_j, h, k)$ increase in t. This effect is driven by technology diffusion $\vartheta > 0$: The average productivity of competitors $(\bar{z}_s)_{0 \leq s \leq t}$ increases monotonically, slowly crowding out firms whose current productivity evolution $(z_s)_{t-a \leq s \leq t}$ is not keeping pace. If $\vartheta = 0$, then cut off values are constant with respect to t.

Corollary 9 (Entry)**.** *Opening up to new markets $j \leq n$ is always preceded by positive productivity shocks $\triangle_t z > 0$, or positive belief revisions $\triangle_t h > 0$, $\triangle_t k_j > 0$. Thus, entry only occurs after sufficiently good productivity samples $z_t > \min(z, E_h[E_{g_\alpha}[z]])$ or affirmative demand signals $\zeta_l > E_{k_l}[\delta]$, $\chi_{l,t} = 0$ on positively correlated markets $l \leq n$.*

While entry only occurs after positive signals, positive signals do not always imply entry. Positive signals can even be followed by market exit: Given positive technology diffusion cut off values feature a constant upward drift in t. If a firm receives a positive signal, but this signal is not sufficiently good to outbalance the upward drift of the cut off value for a certain market, the firm might still find it optimal to exit this market. Another characteristic of market exit is its increased appearance directly after entry: Every period a firm is active in a certain market it generates some survival information $\chi_{j,t}$. Only in its first period of activity it additionally generates the market entry signal ζ_j. Thus, expected belief revisions with respect to demand signals triggered by this market are most volatile during the first period of activity.

Corollary 10 (Exit)**.** *There is exogenous and endogenous market exit. Exogenous exit occurs if a firm is hit by a demand shock. Endogenous exit occurs either because the firm observed negative demand signals on correlated markets, i.e. $\Delta_t k_j < 0$, or because it got more pessimistic about its productivity potential, i.e. $\Delta_t h < 0$. If* *there is positive technology diffusion $\vartheta > 0$, endogenous exit can also be caused by the monotonically increasing average productivity of competitors. Endogenous exit is most likely directly after entry.*

Firm Expansion and Shrinkage

Consider a firm located in country i. From the firm's perspective, markets j and l solely differ in variable trade costs τ_{ij}, τ_{il} and perceived demands $k_j(\delta)$, $k_l(\delta)$. Their relationship completely determines the firm's entry decision: The lower the variable trade costs and the lower the perceived demand risk, the higher ex-ante expected total profits from entering the market. Suppose the firm has just been created and received a start-up demand signal ζ_i that is better than average. Then, it also expects better-than-average demand characteristics in positively correlated countries. The higher the correlation the stronger the positive effect. In case the firm starts exporting and there were no variable trade costs, it would enter the market with the highest demand correlation first. Possibly, this market is not a neighboring country. This solely depends on the correlation $cor_v(\delta_i, \delta_j)$. So possibly a firm might find it profitable to export to the most distant market first. In case of positive variable trade costs the firm might find it more profitable to not deviate that far from its home country. Suppose now the firm starts with a demand stability signal ζ_i below average, then it would seek markets with low correlation for exporting. The same holds true for all other firms analogously: Any firm prefers to enter markets that are geographically close to its home country in order to minimize variable trade costs and that are strongly correlated to markets that exhibit a robust demand for the firm's product. Hence, the pattern of a firm's expansion path is determined by the trade-off between perceived demand characteristics $k_j(\delta)$ and variable trade costs τ_{ij}. As variable trade costs are identical for all firms, expansion path differences are driven by idiosyncratic demand signal histories. Depending on the variable trade cost function $\tau_{ij} = \tau(d(i, j))$ and the correlation $cor_v(\delta_i, \delta_j)$ any expansion path is possible.

Corollary 11 (Expansion path)**.** *Expanding firms prefer new destinations with low variable trade costs, that are strongly correlated with existing markets exhibiting robust demand for the firm's product. Idiosyncratic expansion paths of different firms are driven by their idiosyncratic demand signal histories.*

Excluding degenerate distributions $g_\alpha(z)$, that assign their entire density to one outcome z', expected productivity growth is strictly positive for young firms (see

Corollary 5). Choosing a sufficiently low technology diffusion $\vartheta > 0$, expected current productivity growth rates of young firms are higher than expected growth rates of cut off productivities.[6] As long as current productivity growth rates dominate cut off productivity growth rates, firms will expand into more and more countries. However, as firms turn older, expected current productivity growth rates converge towards zero. Hence, from some age onwards, expected growth of cut off productivities dominates expected growth of current productivities and firms start exiting countries. Assuming a sufficiently small technology diffusion $\vartheta > 0$, it follows:

Corollary 12 (Firm life cycle). *If a firm is not hit by an exogenous death shock its life cycle consists of firm birth, firm expansion (while firm's expected current productivity growth dominates expected growth of cut off productivities), firm shrinkage (when firm's expected current productivity growth drops below expected growth of cut off productivities), and finally firm death when the firm endogenously exits the last market.*

A firm is dead if it is not active on any market. There is endogenous and exogenous firm death. If a firm decides to exit its last market because its expected total profit turns negative, it dies endogenously. If a firm expects to generate positive total profits and is hit by a demand shock on all its markets simultaneously, it dies exogenously. Hence, a firm's exogenous death probability is given by the product of all demand shock risks $\Pi_{j \in \lambda} \delta_j$ of all the markets it serves $j \in \lambda$. Consider two newly created firms that solely differ in their productivity potential $\alpha > \alpha'$. Then the firm with the higher productivity potential exhibits higher expected current productivities, $E_{\psi_{t,\alpha}}[z] > E_{\psi_{t,\alpha'}}[z]$ for all t, and chooses a later period for endogenous exit in expectation. This introduces a negative correlation between productivity potential and endogenous firm death. Moreover, the firm with the higher productivity potential enters more markets in expectation. As all demand shock risks attain values in between zero and one, the more countries the firm is active in the lower its exogenous death probability $\Pi_{j \in \lambda} \delta_j$. Hence, productivity potential and exogenous death probability are also correlated negatively.

Corollary 13 (Firm Death). *Endogenous and exogenous death probabilities are negatively correlated with firm productivity.*

[6] Choosing $\vartheta = 0$ growth rates of cut off productivities are zero. By continuity of the model, those growth rates can be decreased arbitrarily by choosing a sufficiently small ϑ.

III.5 Conclusion

This chapter replicates gradual firm growth and decay in open economies by combining three approaches that have hitherto been studied separately. First, there is supply uncertainty leading to gradual improvement of production techniques on firm level. Second, there is demand uncertainty resulting in firm specific expansion paths. Third, there are knowledge spillovers which constantly intensify competition and lead to crowding out of old firms.

Each firm is assigned an unobservable productivity distribution upon its birth. This distribution generates a new productivity sample every period. Whenever a new productivity sample dominates the firm's current productivity it switches technology and produces according to this new productivity. This generates firm growth and firm learning (from observing lengthening sample histories). Firm learning has no effect on firm productivities, but it makes firms more or less optimistic about their future productivity evolution, and via this channel influences market entry/exit decisions. Furthermore, each firm is assigned an unobservable per period demand shock probability for every country. Demand is either positive or - when hit by a shock - completely vanishes. Firms can learn about these market specific default probabilities by various means. As demand characteristics are positively correlated across countries, firm learning for a specific country also comprises observing demand signals in other countries. Firm specific demand signals entail firm specific learning and result in firm specific expansion paths. However, generally, more productive firms enter more markets. This generates a positive correlation of firm productivity and firm life expectancy. Finally, there are knowledge spillovers from incumbent firms to entrants. Start-ups draw their productivity type from a distribution that depends on the productivity type distribution of existing firms. This generates monotonically increasing average productivities and results in crowding out of firms that are not able to keep pace with the industry average.

Appendix A

A Simple Theory of Trade, Finance, and Firm Dynamics

A.1 Baseline Model

Existence and Uniqueness of Cut-Off Values

Starting with (I.9) and applying (I.7), we get:

$$\begin{aligned}
K/G(\delta_d^*) &= E_g(\pi^n(\delta)/\delta|\delta \le \delta_d^*) \\
&= E_g(\pi_d^n(\delta)/\delta|\delta \le \delta_d^*) + (G(\delta_x^*)/G(\delta_d^*))E_g(n\pi_x^n(\delta)/\delta|\delta \le \delta_x^*) \\
&= E_g((\delta_d^* - \delta)f_d/\delta|\delta \le \delta_d^*) + (G(\delta_x^*)/G(\delta_d^*))E_g(n(\delta_x^* - \delta)f_x/\delta|\delta \le \delta_x^*) \\
&= f_d(\delta_d^* E_g(1/\delta|\delta \le \delta_d^*) - 1) + nf_x(G(\delta_x^*)/G(\delta_d^*))(\delta_x^* E_g(1/\delta|\delta \le \delta_x^*) - 1),
\end{aligned}$$

yielding:

$$f_d G(\delta_d^*)(\delta_d^* E_g(1/\delta|\delta \le \delta_d^*) - 1) + nf_x G(\delta_x^*)(\delta_x^* E_g(1/\delta|\delta \le \delta_x^*) - 1) = K. \quad \text{(A.1)}$$

Replacing δ_x^* by δ_d^* via (I.8), the left hand side of (A.1) is a continuous function of δ_d^* that equals 0 for $\delta_d^* = 0$ and is strictly positive for $\delta_d^* = 1$ as $E(1/\delta) > 1$. Thus, it is always possible to choose $K > 0$ sufficiently small in order to ensure the existence of a solution of (A.1). Uniqueness follows from proof by contradiction: Assume there are at least two different domestic cut-off values $\delta_d^\natural < \delta_d^\flat$ solving (A.1). Then net per period profits of firms $\delta \in (\delta_d^\natural, \delta_d^\flat)$ have to be less or equal to net per period profits of firm $\delta^\flat$, which yields a contradiction as net per period profits strictly decrease in δ.

Proposition 1

Equation (I.8) exhibits a direct effect $\tau \downarrow \Rightarrow \delta_x^* \uparrow$. As $\delta_x^* \uparrow$ yields $\delta_x^* \downarrow$ via (A.1) and as there is no direct effect of τ on (A.1). The statements contained in the Proposition follow.

Derivation of (I.11)

Labor market clearing $L = Mq_d + nM_x\tau q_x$ yields $wL = M(r_d - \pi_d) + nM_x(r_x - \pi_x) = M((r_d - \pi_d) + nH(\delta_x^*)(r_x - \pi_x))$. Transforming r_d and r_x according to (I.4) and (I.6) and replacing π_d and π_x via (I.6) we get:

$$\begin{aligned} wL &= M((r_d - \pi_d) + nH(\delta_x^*)(r_x - \pi_x)) = M(\sigma\pi_d - \pi_d + nH(\delta_x^*)(\sigma\pi_x - \pi_x)) \\ &= M((\sigma - 1)\delta_d^* f_d + nH(\delta_x^*)(\sigma - 1)\delta_x^* f_x) = M(\sigma - 1)(\delta_d^* f_d + nH(\delta_x^*)\delta_x^* f_x) \end{aligned}$$

yielding (I.11).

Derivation of (I.12)

From $1 = P = (Mp_d^{1-\sigma} + nM_x p_x^{1-\sigma})^{\frac{1}{1-\sigma}}$ we get:

$$\begin{aligned} w &= w/P = w(Mp_d^{1-\sigma} + nM_x p_x^{1-\sigma})^{\frac{1}{\sigma-1}} = w(Mp_d^{1-\sigma} + nH(\delta_x^*)M(\tau p_d)^{1-\sigma})^{\frac{1}{\sigma-1}} \\ &= (w/p_d)(1 + nH(\delta_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}} = \rho(1 + nH(\delta_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}}. \end{aligned}$$

Together with $C = Lw/P$, this implies (I.12).

A.2 Uncertain Firm Types (Symmetric Countries)

Existence and Uniqueness of Cut-Off Values

Starting with (I.17) and applying (I.15) in the fourth and (I.14) in the fifth step of the calculation, we get:

$$\begin{aligned} K/G(\hat{\delta}_d^*) &= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \pi^n(\hat{\delta}_t))|\hat{\delta}_0 \leq \hat{\delta}_d^*) \\ &= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \pi_d^n(\hat{\delta}_t) + \Sigma_{t=t(\hat{\delta}_0)}^{\infty}(1-\delta)^t n\pi_x^n(\hat{\delta}_t))|\hat{\delta}_0 \leq \hat{\delta}_d^*) \\ &= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \pi_d^n(\hat{\delta}_t)) \\ &\quad + E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}\Sigma_{t=0}^{\infty}(1-\delta)^t n\pi_x^n(\hat{\delta}_{t(\hat{\delta}_0)+t}))|\hat{\delta}_0 \leq \hat{\delta}_d^*) \end{aligned}$$

$$
\begin{aligned}
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t(\pi_d - \hat{\delta}_t f_d) \\
&\quad + E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}\Sigma_{t=0}^{\infty}(1-\delta)^t n(\pi_x - \hat{\delta}_{t(\hat{\delta}_0)+t} f_x))|\hat{\delta}_0 \leq \hat{\delta}_d^*) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t((\bar{\delta}(\hat{\delta}_d^*) - \hat{\delta}_t) f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))n f_x) \\
&\quad + E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}\Sigma_{t=0}^{\infty}(1-\delta)^t n(\hat{\delta}_x^* - \hat{\delta}_{t(\hat{\delta}_0)+t}) f_x)|\hat{\delta}_0 \leq \hat{\delta}_d^*) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t((\bar{\delta}(\hat{\delta}_d^*) - \bar{\delta}(\hat{\delta}_0)) f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))n f_x) \\
&\quad + E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}\Sigma_{t=0}^{\infty}(1-\delta)^t(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))n f_x)|\hat{\delta}_0 \leq \hat{\delta}_d^*) \\
&= E_g(((\bar{\delta}(\hat{\delta}_d^*) - \bar{\delta}(\hat{\delta}_0)) f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))n f_x) E_{b_{\hat{\delta}_0}}(1/\delta) \\
&\quad + (\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))n f_x E_{b_{\hat{\delta}_0}}((1-\delta)^{t(\hat{\delta}_0)}/\delta)|\hat{\delta}_0 \leq \hat{\delta}_d^*) \\
&= E_g(((\bar{\delta}(\hat{\delta}_d^*) - \bar{\delta}(\hat{\delta}_0)) f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))n f_x \\
&\quad + \psi(\hat{\delta}_0)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))n f_x) E_{b_{\hat{\delta}_0}}(1/\delta)|\hat{\delta}_0 \leq \hat{\delta}_d^*)
\end{aligned}
$$

yielding:

$$
\begin{aligned}
K = G(\hat{\delta}_d^*) E_g[[(\bar{\delta}(\hat{\delta}_d^*) - \bar{\delta}(\hat{\delta}_0)) f_d - \psi(\hat{\delta}_d^*)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_d^*)}))n f_x \\
+ \psi(\hat{\delta}_0)(\hat{\delta}_x^* - \bar{\delta}(\hat{\delta}_{t(\hat{\delta}_0)}))n f_x] E_{b_{\hat{\delta}_0}}(1/\delta)|\hat{\delta}_0 \leq \hat{\delta}_d^*],
\end{aligned} \tag{A.2}
$$

with $t(\hat{\delta}_0)$ denoting the period in which a firm of start-up perceived type $\hat{\delta}_0$ would enter the export market in case of survival. Existence and uniqueness of the solution $\hat{\delta}_d^*$ follows from analog arguments as in the known firm type case.

Effects of trade liberalization

From (I.16) we get $\partial\hat{\delta}_x^*/\partial\tau < 0$, yielding $\tau \downarrow \Rightarrow \hat{\delta}_x^* \uparrow$. As $\psi(\hat{\delta}_d^*) \leq \psi(\hat{\delta}_0)$ yields the correspondence $\hat{\delta}_x^* \uparrow \Rightarrow \hat{\delta}_x^* \downarrow$ via (A.2), and as there is no direct effect of τ on (A.2), we obtain the claims made in the text.

Derivation of (I.20)

Labor market clearing $L = Mq_d + nM_x\tau q_x$ yields $wL = M(r_d - \pi_d) + nM_x(r_x - \pi_x) = M(r_d - \pi_d + n\hat{H}(\hat{\delta}_x^*)(r_x - \pi_x))$. Transforming r_d and r_x via (I.4) and (**??**) and replacing π_d and π_x according to (I.14), yields:

$$
\begin{aligned}
wL &= M(r_d - \pi_d + n\hat{H}(\hat{\delta}_x^*)(r_x - \pi_x)) \\
&= M(\sigma\pi_d - \pi_d + n\hat{H}(\hat{\delta}_x^*)(\sigma\pi_x - \pi_x)) \\
&= M((\sigma-1)(\bar{\delta}(\hat{\delta}_d^*) f_d - \psi(\hat{\delta}_d^*)\hat{\delta}_x^* n f_x) + n\hat{H}(\hat{\delta}_x^*)((\sigma-1)\hat{\delta}_x^* f_x) \\
&= M(\sigma-1)(f_d\bar{\delta}(\hat{\delta}_d^*) + n f_x(\hat{H}(\hat{\delta}_x^*) - \psi(\hat{\delta}_d^*))\hat{\delta}_x^*),
\end{aligned}
$$

yielding (I.20).

Derivation of (I.21)

From $1 = P = (Mp_d^{1-\sigma} + nM_x p_x^{1-\sigma})^{\frac{1}{1-\sigma}}$ we get:

$$\begin{aligned} w &= w/P \\ &= w(Mp_d^{1-\sigma} + nM_x p_x^{1-\sigma})^{\frac{1}{\sigma-1}} \\ &= w(Mp_d^{1-\sigma} + n\hat{H}(\hat{\delta}_x^*)M(\tau p_d)^{1-\sigma})^{\frac{1}{\sigma-1}} \\ &= (w/p_d)(1 + n\hat{H}(\hat{\delta}_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}} \\ &= \rho(1 + n\hat{H}(\hat{\delta}_x^*)\tau^{1-\sigma})^{\frac{1}{\sigma-1}} M^{\frac{1}{\sigma-1}}. \end{aligned}$$

Together with $C = Lw/P$, this implies (I.21).

Proposition 8

If a firm starts exporting by a misjudgment of its true type, expected profits from exporting $\Sigma_{t=0}^{\infty}(1-\delta)^t \pi_x$ are dominated by costs f_x. In this case the firm uses up more units of final good of a country than it produces, yielding a negative welfare effect. By constructing a specific ex-ante distribution g' of true and perceived firm types, we can increase the fraction of firms that enter by misjudgment of their type almost to 1. Let $(\hat{\delta}_x^*)_{-1}$ denote the value of the start-up perceived type that coincides with the exporting cut-off value after one period of updating and let $R = \int_{\hat{\delta}_x^*}^{(\hat{\delta}_x^*)_{-1}} g(\hat{\delta}_0) d\hat{\delta}_0$ denote the fraction of start-up firms $\hat{\delta}_0$ within $(\hat{\delta}_x^*, (\hat{\delta}_x^*)_{-1})$. Then, those start-up firms will enter foreign markets in their second period of operation, yielding a negative aggregate welfare effect, as start-up perceived firm types are correct in expectation. By shifting probability density towards a value $\hat{\delta}_0'$ within the open interval $(\hat{\delta}_x^*, (\hat{\delta}_x^*)_{-1})$, we can push R arbitrarily close towards 1.[1] For some value of R' (close enough to 1) the negative welfare effect from export entry of firms belonging to this fraction will outweigh the possibly positive welfare effect from export entry of the residual $1 - R'$. Under such an ex-ante distribution g' of true and perceived firm types a change to prohibitive variable trade costs $\tau \to \infty$ or to $n \to 0$ accessible foreign markets increases welfare. Hence, by the mean value theorem of differential calculus, there exists a τ' and a n' at which liberalizing trade yields negative welfare effects.

[1] As this shifting of probability density draws $\hat{\delta}_x^*$ and $(\hat{\delta}_x^*)_{-1}$ closer together, $\hat{\delta}_0'$ has to belong to the interval subsequent the shifting of probability density. As $\hat{\delta}_x^* < (\hat{\delta}_x^*)_{-1}$ for all non-degenerate distributions g, such a $\hat{\delta}_0'$ always exists.

A.3 Uncertain Firm Types (Asymmetric Countries)

Existence and Uniqueness of Cut-Off Values

Using the zero cut-off profit conditions $\pi_\iota = \hat{\delta}^*_\iota f_\iota$ and $\hat{\delta}^*_\iota = (f_\kappa/f_\iota)\hat{\delta}^*_\kappa$ we can transform the free entry condition into an equation with only one unknown $\hat{\delta}^*_0$:

$$\begin{aligned}
K &= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)} (\pi_\kappa - \hat{\delta}_t f_\kappa) d\kappa)) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)} ((\hat{\delta}^*_\kappa - \hat{\delta}_t) f_\kappa) d\kappa)) \\
&= E_g(E_{b_{\hat{\delta}_0}}(\Sigma_{t=0}^{\infty}(1-\delta)^t \int_0^{\kappa(\hat{\delta}_t)} ((f_0/f_\kappa)\hat{\delta}^*_0 - \hat{\delta}_t) f_\kappa d\kappa)) \qquad (A.3)
\end{aligned}$$

The right hand side of (A.3) is a continuous monotonically increasing function of $\hat{\delta}^*_0$. It equals zero for $\hat{\delta}^*_0 = 0$, as in this case all cut-off values vanish $\hat{\delta}^*_\kappa = (f_0/f_\kappa)\hat{\delta}^*_0 = 0$ and thus no firm will enter into exporting. If $\hat{\delta}^*_0 = 1$ all firms will export to country $\iota = 0$ and to countries with similarly low market entry costs $\iota = 0 + \epsilon$.[2] Hence $\int_0^{\kappa(\hat{\delta}_0)} ((f_0/f_\kappa)\hat{\delta}^*_0 - \hat{\delta}_0) f_\kappa d\kappa > 0$ for all $\hat{\delta}_0$, yielding a strictly positive right hand side of (A.3). Thus, for all sufficiently small $K > 0$, there exists a unique solution $\hat{\delta}^*_0$ of (A.3).

Derivation of (I.25)

From the labor market clearing condition $L = \int_0^1 M_\kappa \tau q_\kappa d\kappa$, we get:

$$\begin{aligned}
wL &= \int_0^1 M_\kappa w \tau q_\kappa d\kappa = \int_0^1 M_\kappa (p_\kappa q_\kappa - (p_\kappa - w\tau) q_\kappa) d\kappa \\
&= \int_0^1 M_\kappa (r_\kappa - \pi_\kappa) d\kappa = \int_0^1 M_\kappa (\sigma - 1) \pi_\kappa d\kappa \\
&= \int_0^1 M_\kappa (\sigma - 1) \hat{\delta}^*_\kappa f_\kappa d\kappa = \int_0^1 \hat{H}(\hat{\delta}^*_\kappa) M (\sigma - 1) \hat{\delta}^*_\kappa f_\kappa d\kappa \\
&= M(\sigma - 1) \int_0^1 \hat{H}(\hat{\delta}^*_\kappa) \hat{\delta}^*_\kappa f_\kappa d\kappa,
\end{aligned}$$

yielding (I.25).

[2] We assume that market entry costs f_ι increase continuously in ι.

Derivation of (I.26)

Determining the country index

$$\begin{aligned} P_\iota &= \Big(\int_0^1 \Big(\int_{\Omega_{\kappa,\iota}} p(\omega_{\kappa,\iota})^{1-\sigma} d\omega_{\kappa,\iota}\Big) d\kappa\Big)^{1/(1-\sigma)} \\ &= \Big(\int_0^1 \Big(\int_{\Omega_{\kappa,\iota}} p_\iota^{1-\sigma} d\omega_{\kappa,\iota}\Big) d\kappa\Big)^{1/(1-\sigma)} = \Big(\int_0^1 p_\iota^{1-\sigma} M_\iota d\kappa\Big)^{1/(1-\sigma)} \\ &= p_\iota M_\iota^{1/(1-\sigma)} = w(\tau/\rho)(\hat{H}(\hat{\delta}_\iota^*)M)^{1/(1-\sigma)} \end{aligned}$$

and plugging it into $P = \Big(\int_0^1 P_\iota^{1-\sigma} d\iota\Big)^{1/(1-\sigma)} = w(\tau/\rho)\Big(M\int_0^1 \hat{H}(\hat{\delta}_\iota^*)d\iota\Big)^{1/(1-\sigma)}$ we receive $C = Lw/P = L(\rho/\tau)\Big(M\int_0^1 \hat{H}(\hat{\delta}_\iota^*)d\iota\Big)^{1/(\sigma-1)}$.

Appendix B

Labor Market Dynamics and Trade

B.1 Equivalence of Simplified and Competitive Search Equilibria

Proof of Proposition 11. I show that if J_t solves the recursive equation (II.5), then $S_t = J_t + A_t$ solves the recursive equation (II.8). Suppose A_t denotes a constant that does not interfere with the firms maximization problem. Then, whenever a firm maximizes J_t according to the competitive search setting, it maximizes $S_t = J_t + A_t$ according to the simplified search setting and the resultant equilibrium allocations are identical. This proves that every competitive search equilibrium allocation constitutes a simplified search equilibrium allocation. The reverse direction follows from performing the Main Calculation for $J_t = S_t - A_t$.

Main Calculation

Defining:

$$A_t(z, a) = \sum_{\hat{t}=t-a}^{t-1} \frac{\ell_{\hat{t},t}}{1 - \eta_{\hat{t},t}} \Big(u_t(B_{\hat{t}}) - u_t(0) \Big) \tag{B.1}$$

the expression is independent of the firms maximization problem in period t as it only depends on contracts the firm signed in previous periods $\hat{t} < t$ and does not depend on current firm behavior. This allows us to perform step (B.6) below. Applying (II.6) and (II.7) in step (B.2), (B.11) in step (B.3), (B.1) in step (B.4), (B.10) in step (B.5), the Bellman Equation for the unemployed $u_t(0) = b + \rho_t + \beta u_{t+1}(0)$ and

(II.6) in step (B.7), and (II.6) in step (B.8) and (B.9) we get:

$$
\begin{aligned}
S_t(z,a) &= J_t(z,a) + A_t(z,a) \\
&= \max_{\iota,V,B_t}(1-\delta)\Big\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - W + \beta J_{t+1}(z,\ell_t,(B_{\hat{t}})_{\hat{t}=t-a}^t)\Big\} \\
&\quad + A_t(z,a) \\
&= \max_{\iota,V,B_t}(1-\delta)\Big\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - \sum_{\hat{t}=t-a}^{t-1} w_{\hat{t},t}\ell_{\hat{t},t} - w_{t,t}m(\lambda)V \qquad \text{(B.2)} \\
&\quad + \beta J_{t+1}(z,\ell_t,(B_{\hat{t}})_{\hat{t}=t-a}^t)\Big\} + A_t(z,a) \\
&= \max_{\iota,V,B_t}(1-\delta)\Big\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - \sum_{\hat{t}=t-a}^{t-1} w_{\hat{t},t}\ell_{\hat{t},t} - \Big(b + \beta u_{t+1}(0) \qquad \text{(B.3)} \\
&\quad + \frac{\lambda}{m(\lambda)}\rho_t\Big)m(\lambda)V + \beta\Big\{J_{t+1}(z,\ell_t,(B_{\hat{t}})_{\hat{t}=t-a}^t) + u_{t+1}(B_t)m(\lambda)V\Big\}\Big\} + A_t(z,a) \\
&= \max_{\iota,V,B_t}(1-\delta)\Big\{\dots\Big\} + \sum_{\hat{t}=t-a}^{t-1}\frac{\ell_{\hat{t},t}}{1-\eta_{\hat{t},t}}\Big(u_t(B_{\hat{t}}) - u_t(0)\Big) \qquad \text{(B.4)} \\
&= \max_{\iota,V,B_t}(1-\delta)\Big\{\dots\Big\} + \sum_{\hat{t}=t-a}^{t-1}\frac{\ell_{\hat{t},t}}{1-\eta_{\hat{t},t}}\Big([\delta + (1-\delta)\eta_{\hat{t},t}]u_t(0) \qquad \text{(B.5)} \\
&\quad + [1-(\delta+(1-\delta)\eta_{\hat{t},t})](w_{\hat{t},t} + \beta u_{t+1}(B_{\hat{t}})) - u_t(0)\Big) \\
&= \max_{\iota,V,B_t}(1-\delta)\Big\{\dots\Big\} + \sum_{\hat{t}=t-a}^{t-1}\frac{\ell_{\hat{t},t}}{1-\eta_{\hat{t},t}}\Big([1-(\delta+(1-\delta)\eta_{\hat{t},t})](w_{\hat{t},t} - u_t(0) + \beta u_{t+1}(B_{\hat{t}}))\Big) \\
&= \max_{\iota,V,B_t}(1-\delta)\Big\{\dots\Big\} + (1-\delta)\sum_{\hat{t}=t-a}^{t-1}\ell_{\hat{t},t}\Big(w_{\hat{t},t} - u_t(0) + \beta u_{t+1}(B_{\hat{t}})\Big) \\
&= \max_{\iota,V,B_t}(1-\delta)\Big\{\dots + \sum_{\hat{t}=t-a}^{t-1}\ell_{\hat{t},t}\Big(w_{\hat{t},t} - u_t(0) + \beta u_{t+1}(B_{\hat{t}})\Big)\Big\} \qquad \text{(B.6)} \\
&= \max_{\iota,V,B_t}(1-\delta)\Bigg\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - \sum_{\hat{t}=t-a}^{t-1} w_{\hat{t},t}\ell_{\hat{t},t} - \Big(b + \beta u_{t+1}(0) \\
&\quad + \frac{\lambda}{m(\lambda)}\rho_t\Big)m(\lambda)V + \beta\Big\{J_{t+1}(z,\ell_t,(B_{\hat{t}})_{\hat{t}=t-a}^t) + u_{t+1}(B_t)m(\lambda)V\Big\} \\
&\quad + \sum_{\hat{t}=t-a}^{t-1}\ell_{\hat{t},t}\Big(w_{\hat{t},t} - u_t(0) + \beta u_{t+1}(B_{\hat{t}})\Big)\Bigg\} \\
&= \max_{\iota,V,B_t}(1-\delta)\Bigg\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - \Big(b + \beta u_{t+1}(0) + \frac{\lambda}{m(\lambda)}\rho_t\Big)m(\lambda)V \\
&\quad + \beta\Big\{J_{t+1}(z,\ell_t,(B_{\hat{t}})_{\hat{t}=t-a}^t) + u_{t+1}(B_t)m(\lambda)V\Big\} + \sum_{\hat{t}=t-a}^{t-1}\ell_{\hat{t},t}\Big(-u_t(0) + \beta u_{t+1}(B_{\hat{t}})\Big)\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&= \max_{\iota,V,B_t}(1-\delta)\Bigg\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - \rho_t\lambda V - (b+\beta u_{t+1}(0))m(\lambda)V \\
&\quad - \sum_{\hat{t}=t-a}^{t-1}\ell_{\hat{t},t}u_t(0) + \beta\Big\{J_{t+1}(z,\ell_t,(B_{\hat{t}})_{\hat{t}=t-a}^{t}) + \sum_{\hat{t}=t-a}^{t-1}\ell_{\hat{t},t}u_{t+1}(B_{\hat{t}}) + u_{t+1}(B_t)m(\lambda)V\Big\}\Bigg\} \\
&= \max_{\iota,V,B_t}(1-\delta)\Bigg\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - \rho_t\lambda V - (b+\beta u_{t+1}(0))m(\lambda)V \\
&\quad - \sum_{\hat{t}=t-a}^{t-1}\ell_{\hat{t},t}u_t(0) + \beta\Big\{J_{t+1}(z,\ell_t,(B_{\hat{t}})_{\hat{t}=t-a}^{t}) + \sum_{\hat{t}=t-a}^{t}\ell_{\hat{t},t}u_{t+1}(B_{\hat{t}})\Big\}\Bigg\} \\
&= \max_{\iota,V,B_t}(1-\delta)\Bigg\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - \rho_t\lambda V - \ell_{t,t}(b+\beta u_{t+1}(0)) \qquad \text{(B.7)} \\
&\quad - \sum_{\hat{t}=t-a}^{t-1}\ell_{\hat{t},t}(b+\rho_t+\beta u_{t+1}(0)) + \beta\Big\{J_{t+1}(z,\ell_t,(B_{\hat{t}})_{\hat{t}=t-a}^{t}) + \sum_{\hat{t}=t-a}^{t}\ell_{\hat{t},t}u_{t+1}(B_{\hat{t}})\Big\}\Bigg\} \\
&= \max_{\iota,V,B_t}(1-\delta)\Bigg\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - \rho_t\lambda V - \ell_{t,t}b - \sum_{\hat{t}=t-a}^{t-1}\ell_{\hat{t},t}(b+\rho_t) \\
&\quad +\beta\Big\{J_{t+1}(z,\ell_t,(B_{\hat{t}})_{\hat{t}=t-a}^{t}) + \sum_{\hat{t}=t-a}^{t}\ell_{\hat{t},t}(u_{t+1}(B_{\hat{t}})-u_{t+1}(0))\Big\}\Bigg\} \\
&= \max_{\iota,V,B_t}(1-\delta)\Bigg\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - \rho_t(\ell_{t-1}+\lambda V) - b(\ell_{t-1}+\ell_{t,t}) \qquad \text{(B.8)} \\
&\quad +\beta\Big\{J_{t+1}(z,\ell_t,(B_{\hat{t}})_{\hat{t}=t-a}^{t}) + \sum_{\hat{t}=t-a}^{t}\ell_{\hat{t},t}(u_{t+1}(B_{\hat{t}})-u_{t+1}(0))\Big\}\Bigg\} \\
&= \max_{\iota,V,B_t}(1-\delta)\Bigg\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - \rho_t(\ell_{t-1}+\lambda V) - b(\ell_{t-1}+\ell_{t,t}) \qquad \text{(B.9)} \\
&\quad +\beta\Big\{J_{t+1}(z,\ell_t,(B_{\hat{t}})_{\hat{t}=t-a}^{t}) + \sum_{\hat{t}=t-a}^{t}\frac{\ell_{\hat{t},t+1}}{1-\eta_{\hat{t},t}}\Big(u_{t+1}(B_{\hat{t}})-u_{t+1}(0)\Big)\Big\}\Bigg\} \\
&= \max_{\iota,V,B_t}(1-\delta)\Bigg\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - C(V) - \rho_t(\ell_{t-1}+\lambda V) - b(\ell_{t-1}+\ell_{t,t}) \\
&\quad +\beta\Big\{J_{t+1}(z,a) + A_{t+1}(z,a)\Big\}\Bigg\} \\
&= \max_{\iota,V,\lambda,\eta_t}(1-\delta)\Big\{r_\iota(z,\ell_{t-1}+\ell_{t,t}) - b(\ell_{t-1}+\ell_{t,t}) - \mu_t(\ell_{t-1}+\lambda V) - C(V) \\
&\quad +\beta S_{t+1}(z,\ell_t)\Big\}.
\end{aligned}
$$

Thus, for $\rho_t = \mu_t$, the function $S_t(z,a) = J_t(z,a) + A_t(z,a)$ solves the simplified control problem (II.8).

Auxiliary Calculation

A contract $B_{\hat{t}}$ signed in period $\hat{t} < t$ satisfies the recursive equation:

$$u_t(B_{\hat{t}}) = [\delta + (1-\delta)\eta_{\hat{t},t}]u_t(0) + [1 - (\delta + (1-\delta)\eta_{\hat{t},t})](w_{\hat{t},t} + \beta u_{t+1}(B_{\hat{t}})). \quad \text{(B.10)}$$

If the firm defaults or the firm-worker match breaks (which happens with probability $\delta + (1-\delta)\eta_{\hat{t},t}$) the worker switches to the utility of an unemployed worker $u_t(0)$. If the workers stays employed he receives his salary $w_{\hat{t},t}$ and enters the next period with same contract $B_{\hat{t}}$ yielding a present value in period t of $\beta u_{t+1}(B_{\hat{t}})$. As firm death and firm-worker match default is impossible directly after signing the contract (period t, stage $s = 3$) for the first period of the contract, equation (B.10) transforms into:

$$u_t(B_t) = w_{t,t} + \beta u_{t+1}(B_t)$$

and hence:

$$w_{t,t} = b + \beta u_{t+1}(0) + \frac{\lambda}{m(\lambda)}\rho_t - \beta u_{t+1}(B_t), \quad \text{(B.11)}$$

where we replaced $u_t(B_t)$ according to (II.7). □

B.2 Firm Policy Functions

Proof of Proposition 12. Surplus of a firm $(z, \ell = \ell_{t-1})$ is given by:

$$\begin{aligned} S(z,\ell) &= \max_{\iota,V,\lambda}(1-\delta)\Big\{r_\iota(z,\ell+\ell_{t,t}) - b(\ell+\ell_{t,t}) - \mu_t(\ell+\lambda V) \\ &\quad -C(V) + \beta S_{t+1}(z,\ell_t)\Big\}. \end{aligned} \quad \text{(B.12)}$$

Employment growth fulfills $\ell_{t,t} = m(\lambda)V$ and $\ell_t = (1-\eta)(\ell+\ell_{t,t})$. Firm revenue reads $r_\iota(z,\ell) = zA(\ell) - f_d$ for $\iota = d$ and $r_\iota(z,\ell) = pzA(\ell)/\tau - f_x$ for $\iota = x$, with $A(\ell)$ denoting a strictly increasing and concave function with $A(0) = 0$.

Recruiting Firms

If $V \geq 0$, vacancy costs $C(V)$ are positive, strictly increasing and convex and the matching function $m(\lambda)$ is positive, strictly increasing and concave in the worker job ratio λ. The first part of statement 1 in proposition 12 follows directly from comparing domestic and exporting revenues. The second part of statement 1 affords more work. Without loss of generality we execute following calculation for domestic firm revenue $r_\iota(z,\ell) = r_d(z,\ell)$. Differentiating (B.12) with respect to λ and V and applying the envelope theorem we get:

$$\begin{aligned} S_\lambda(z,\ell) &= (1-\delta)\{zA'(\hat{\ell})m'(\lambda)V - bm'(\lambda)V - \mu V\} = 0 \\ &\Rightarrow zA'(\hat{\ell}) = b + \mu/m'(\lambda), \qquad \text{(B.13)} \\ S_V(z,\ell) &= (1-\delta)\{zA'(\hat{\ell})m(\lambda) - bm(\lambda) - \mu\lambda - C'(V)\} = 0 \\ &\Rightarrow C'(V) = zA'(\hat{\ell})m(\lambda) - bm(\lambda) - \mu\lambda, \qquad \text{(B.14)} \end{aligned}$$

with $\hat{\ell} = \ell + \ell_{t,t}$ Plugging (B.13) in (B.14) yields:

$$C'(V) = \mu\Big(\frac{m(\lambda)}{m'(\lambda)} - \lambda\Big). \qquad \text{(B.15)}$$

Consider V as a function of λ and differentiate both sides with respect to λ:

$$C''(V)V_\lambda = \mu\Big(\frac{m'(\lambda)^2 - m(\lambda)m''(\lambda)}{m'(\lambda)^2} - 1\Big) = -\mu\Big(\frac{m(\lambda)m''(\lambda)}{m'(\lambda)^2}\Big).$$

From the convexity of $C(V)$ and the concavity of $m(\lambda)$ it follows $V_\lambda > 0$, i.e. the higher the number of vacancies V a firm posts the higher the unemployment vacancy ratio λ of the sub market it targets. Applying the implicit function theorem on (B.13) (in the form $K(\lambda,z,\ell) = zA'(\hat{\ell}) - \mu/m'(\lambda) - b = 0$) we can determine the effect of z and ℓ on λ:

$$\begin{aligned} K_z(\lambda,z,\ell) &= A'(\hat{\ell}) > 0, \\ K_\ell(\lambda,z,\ell) &= zA''(\hat{\ell}) < 0, \\ K_\lambda(\lambda,z,\ell) &= zA''(\hat{\ell})\Big(m'(\lambda)V + m(\lambda)V_\lambda\Big) + \mu\frac{m''(\lambda)}{m'(\lambda)^2} < 0 \end{aligned}$$

and hence

$$\lambda_z(z,\ell) = \frac{K_z(\lambda,z,\ell)}{K_\lambda(\lambda,z,\ell)} > 0, \quad \lambda_\ell(z,\ell) = \frac{K_\ell(\lambda,z,\ell)}{K_\lambda(\lambda,z,\ell)} < 0.$$

From (B.15) and $V_\lambda > 0$, it follows:

$$V_z(z,\ell) > 0, \quad V_\ell(z,\ell) < 0.$$

Thus, the higher the firms productivity z the more vacancies V it posts and the higher the worker job ratio λ it targets. The reverse is true for firm size ℓ.

Shrinking Firms

Again the first part of the statement follows from comparing domestic and export revenue. Before we prove the second part of the statement we want to show that it is indeed equivalent for firms to separate from workers by choosing a separation probability $\eta_{t-1} > \eta$, or to separate from workers by choosing negative vacancies $V < 0$, in which case we impose $\lambda = 1$, $m(\lambda) == 1$ and $C(V) == 0$. In order to prove equivalence we show that a firm can generate identical surplus $r_\iota(z, \ell_{t-1} + \ell_{t,t}) - b(\ell_{t-1} + \ell_{t,t}) - \mu_t(\ell_{t-1} + \lambda V) - C(V)$ in period t (see (B.12)) by either strategy. From $\eta_{t-1} > \eta$ and $V = 0$ follows $\ell_{t-1} = (1 - \eta_{t-1})\ell_{t-2}$ and $\ell_{t,t} = 0$, and surplus equals $r_\iota(z, (1 - \eta_{t-1})\ell_{t-2}) - (b + \mu)((1 - \eta_{t-1})\ell_{t-2})$. If $\eta_{t-1} = \eta$, $V < 0$, $C(V) == 0$, $\lambda = 1$ and $m(\lambda) == 1$ we get $\ell_{t-1} = (1 - \eta)\ell_{t-2}$ and $\ell_{t,t} = V < 0$ yielding surplus $r_\iota(z, (1 - \eta)\ell_{t-2} + V) - (b + \mu)((1 - \eta)\ell_{t-2} + V)$. Hence, for $V = (\eta - \eta_{t-1})\ell_{t-2} < 0$ both strategies coincide. We proceed to prove the second part of the statement. If $C(V) == 0$, $\lambda = 1$ and $m(\lambda) == 1$, the first order condition of (B.12) with respect to V reads:

$$S_V(z, \ell) = zA'(\ell + V) - b - \mu = 0.$$

It follows $V = (A')^{-1}((b + \mu)/z) - \ell$ which completes the proof of the second statement.

Exiting Firms

If a firm is not able to generate positive surplus it chooses $V = -\ell$ and turns inactive endogenously. □

Appendix C

Firm Life Cycles in a Global Economy

C.1 Balanced Growth Path

Proof of Proposition 13. The proof proceeds in three steps. First, I calculate market supply probabilities ρ_{λ_s}, which are needed to determine total profits $\Pi_{i\lambda_t}(z,h,k) = \sum_{s\geq t}\sum_{\lambda_s\in\Lambda} E_{k,\psi_s}[\rho_{\lambda_s}(z,h,k)\pi_{i\lambda_s}(z,h,k) - |\lambda_s\backslash\lambda_{s-1}|F]$ defined in (III.9). Second, I verify existence and uniqueness of the cut-off values for entry $z^*_{ij,t}(\lambda_{t-1},h,k)$ and exit $z^{**}_{ij,t}(\lambda_{t-1},h,k)$ defined via (III.10) and (III.11). Third, I show how the free entry condition (III.12) pins down the general equilibrium.

Market Supply Probabilities

Consider a firm (z,h,k) in period t. For a specific $\lambda_s \in \Lambda$ the expression $\rho_{\lambda_s}(z,h,k)$ denotes the probability that this firm will serve markets $\lambda_s \in \Lambda$ in $t+s$ periods. This probability of activity in λ_s depends on the firms destinations λ_{t+s-1} in the previous period. Hence, ρ_{λ_s} depends on $\rho_{\lambda_{s-1}}$ whenever $s > t$. Only ρ_{λ_t} can be determined directly. It equals one if λ_t represents the firms actual destinations in period t, otherwise it equals zero. Let $\rho_{\lambda_{s-1}\to\lambda_s}$ denote the firms probability to switch from λ_{s-1} to λ_s, then we have:

$$\rho_{\lambda_s} = \sum_{\lambda_{s-1}\in\Lambda} \rho_{\lambda_{s-1}}\rho_{\lambda_{s-1}\to\lambda_s}. \tag{C.1}$$

The sum collects all possible previous states $\lambda_{s-1} \in \Lambda$ and multiplies their probability $\rho_{\lambda_{s-1}}$ with the corresponding switching-probability $\rho_{\lambda_{s-1}\to\lambda_s}$. This adds up the probabilities of all possible paths leading to λ_s. The switching-probability can be factorized into three components:

$$\rho_{\lambda_{s-1}\to\lambda_s} = \rho^{co}_{\lambda_{s-1}\to\lambda_s}\rho^{en}_{\lambda_{s-1}\to\lambda_s}\rho^{ex}_{\lambda_{s-1}\to\lambda_s}. \tag{C.2}$$

The probability $\rho^{co}_{\lambda_{s-1}\to\lambda_s}$ that the firm continues serving markets $\lambda_{s-1}\cap\lambda_s$, the probability $\rho^{en}_{\lambda_{s-1}\to\lambda_s}$ that it enters markets $\lambda_s\backslash(\lambda_{s-1}\cap\lambda_s)$, and the probability $\rho^{ex}_{\lambda_{s-1}\to\lambda_s}$ that it exits markets $\lambda_{s-1}\backslash(\lambda_{s-1}\cap\lambda_s)$. If and only if all conditions that are expressed by those probabilities are fulfilled, the firm switches from λ_{s-1} to λ_s. Let $z^*_{ij,s}$ and $z^{**}_{ij,s}$ denote the cut-off productivities for endogenous entry into, respectively endogenous exit from market j. If $\lambda_{s-1}\cap\lambda_s=\emptyset$ there is no market the firm needs to continue serving in order to switch from λ_{s-1} to λ_s. In this case $\rho^{co}_{\lambda_{s-1}\to\lambda_s}=1$. If $\lambda_{s-1}\cap\lambda_s\neq\emptyset$ then:

$$\rho^{co}_{\lambda_{s-1}\to\lambda_s} = \sum_{\iota\in P(\lambda_{s-1}\cap\lambda_s)} E_{k,\psi_s}[\Pi_{l\in\iota}\delta_l(1-\Psi_s(z^*_{il,s}))\Pi_{m\in(\lambda_{s-1}\cap\lambda_s)\backslash\iota}(1-\delta_m)(1-\Psi_s(z^{**}_{im,s}))] \tag{C.3}$$

with $\Psi_s(\hat{z})=\int_0^{\hat{z}}\psi_s(z'|z,h)dz'$ denoting the cdf of ψ_s. The term $\Pi_{l\in\iota}\delta_l\Pi_{m\in(\lambda_{s-1}\cap\lambda_s)\backslash\iota}(1-\delta_m)$ of $\rho^{co}_{\lambda_{s-1}\to\lambda_s}$ expresses the probability that a firm suffers demand shocks in all markets $\iota\subset\lambda_{s-1}\cap\lambda_s$ and does not suffer shocks in the remaining countries of $\lambda_{s-1}\cap\lambda_s$. In order to stay active in $\lambda_{s-1}\cap\lambda_s$ the firm has to re-enter markets ι and not leave any of the remaining markets endogenously. The re-entry probability is given by $\Pi_{l\in\iota}(1-\Psi_s(z^*_{il,s}))$, and the non-endogenous exit probability by $\Pi_{m\in\iota}(1-\Psi_s(z^{**}_{im,s}))$. Hence, $E_k[\Pi_{l\in\iota}\delta_l(1-\Psi_s(z^*_{il,s}))\Pi_{m\in(\lambda_{s-1}\cap\lambda_s)\backslash\iota}(1-\delta_m)(1-\Psi_s(z^{**}_{im,s}))]$ denotes the probability that the firm defaults at $\iota\subset\lambda_{s-1}\cap\lambda_s$ but still ends up being active at $\lambda_{s-1}\cap\lambda_s$ in the next period. Adding up this term for all ι in the power set $P(\lambda_{s-1}\cap\lambda_s)$ we add up the probabilities of all possible transitions from λ_{s-1} to λ_s that keep the firm active in $\lambda_{s-1}\cap\lambda_s$, i.e. we get $\rho^{co}_{\lambda_{s-1}\to\lambda_s}$. Accordingly, we get $\rho^{en}_{\lambda_{s-1}\to\lambda_s}=1$ if $\lambda_s\backslash(\lambda_{s-1}\cap\lambda_s)=\emptyset$ and

$$\rho^{en}_{\lambda_{s-1}\to\lambda_s} = E_{k,\psi_s}[\Pi_{l\in\lambda_s\backslash(\lambda_{s-1}\cap\lambda_s)}(1-\Psi_s(z^*_{il,s}))\Pi_{m\in\{1,\dots,n\}\backslash(\lambda_s\cup\lambda_{s-1})}\Psi_s(z^*_{im,s})] \tag{C.4}$$

else. The only possibility that the firm decides to enter all markets $\lambda_s\backslash(\lambda_{s-1}\cap\lambda_s)$ is that its current productivity z_s overshoots all entry cut-off values of those markets $z^*_{il,s}$, $l\in\lambda_s\backslash(\lambda_{s-1}\cap\lambda_s)$. This is expressed by the first product. At the same time it must hold that the firm does not enter any markets outside λ_s. This is partly reflected by the second product. We restrict the second product to $\{1,\dots,n\}\backslash(\lambda_s\cup\lambda_{s-1})\subset\{1,\dots,n\}\backslash\lambda_s$ as the no-entry condition for markets $\lambda_{s-1}\backslash(\lambda_{s-1}\cap\lambda_s)$ is already implemented by $\rho^{ex}_{\lambda_{s-1}\to\lambda_s}$. The last factor of $\rho_{\lambda_{s-1}\to\lambda_s}$ is given by $\rho^{ex}_{\lambda_{s-1}\to\lambda_s} = 1$ if $\lambda_s \backslash (\lambda_{s-1} \cap \lambda_s) = \emptyset$ or else by:

$$\rho^{ex}_{\lambda_{s-1}\to\lambda_s} = \sum_{\iota \in P(\lambda_s \backslash (\lambda_{s-1}\cap\lambda_s))} E_{k,\psi_s}[\Pi_{l\in\iota}\delta_l \Psi_t(z^*_{il,s})\Pi_{m\in(\lambda_s\backslash(\lambda_{s-1}\cap\lambda_s))\backslash\iota}(1-\delta_m)\Psi_s(z^{**}_{im,s})]. \tag{C.5}$$

Either the firm exits markets $\iota \in \lambda_s \backslash (\lambda_{s-1} \cap \lambda_s)$ by exogenous shocks and does not find re-entry profitable (product over l), or it decides to exit markets $\iota \in \lambda_s \backslash (\lambda_{s-1} \cap \lambda_t s)$ endogenously (product over m). Applying (C.1)-(C.5) we can trace ρ_{λ_s} back to ρ_{λ_t}:

$$\begin{aligned}
\rho_{\lambda_s} &= \sum_{\lambda_{s-1}\in\Lambda} \rho_{\lambda_{s-1}}\rho_{\lambda_{s-1}\to\lambda_s} = \sum_{\lambda_{s-1}\in\Lambda}\Big(\sum_{\lambda_{s-2}\in\Lambda}\rho_{\lambda_{s-2}}\rho_{\lambda_{s-2}\to\lambda_{s-1}}\Big)\rho_{\lambda_{s-1}\to\lambda_s} \\
&= \sum_{\lambda_{s-1}\in\Lambda}\Big(\sum_{\lambda_{s-2}\in\Lambda}\Big(\cdots\sum_{\lambda_{t+1}\in\Lambda}\Big(\sum_{\lambda_t\in\Lambda}\rho_{\lambda_t}\rho_{\lambda_t\to\lambda_{t+1}}\Big)\rho_{\lambda_{t+1}\to\lambda_{t+2}}\cdots\Big)\rho_{\lambda_{s-2}\to\lambda_{s-1}}\Big)\rho_{\lambda_{s-1}\to\lambda_s},
\end{aligned}$$

with $\rho_{\lambda_t} = 1$ in case λ_t constitutes the firms current set of destinations, or else $\rho_{\lambda_t} = 0$.

Cut-Off Values

Lemma 14 ensures existence and uniqueness of cut-off values. Existence of cut-off values follows from statement (i) and (ii): If a firm exits all markets for $z \to 0$ and enters all markets for $z \to \infty$ it must hold that there are values of z that trigger entry $z^*_{ij,t}$ or exit $z^{**}_{ij,t}$ for every market $j \in \{1, \dots, n\}$. Uniqueness of cut-off values follows from monotonicity of firm expansion expressed in statement (iii): If a firm enters country j at a certain current productivity z, then it also enters at all productivities $z' \geq z$. Hence, there is a unique smallest value triggering entry $z^*_{ij,t}$, respectively a unique largest value $z^{**}_{ij,t}$ triggering exit for every market $j \in \{1, \dots, n\}$.

Lemma 14. *Suppose fix costs $F > 0$ and $f > 0$ are sufficiently small. Consider a firm in stage three of period t, i.e. it just received a new productivity draw z_t and has to decide which markets to leave or to enter. Then, fixing the firms demand risks $k_i(\delta)$ and its productivity outlook $h(\alpha)$, there is following correspondence of its current productivity z and its expansion: (i) $z \to \infty$ yields $\lambda_t \to \{1, \dots, n\}$, (ii) if the firm is not too optimistic about its perceived type (h, k), then $z \to 0$ yields $\lambda_t \to \emptyset$ and (iii) $z < z'$ implies $\lambda_t(z) \subset \lambda_t(z')$.*

Statement (i) claims that the firm will enter all markets $\lambda_t = \{1, \dots, n\}$ if z is sufficiently big. Consider an arbitrary market $j \in \{1, \dots, n\}$. The firms per period operating profit $\pi_{ij,t}(z,h,k) = \max_{q\in\mathbb{R}_+}\{r_t(q) - w_t\tau_{ij}\frac{q}{z} - E_{k_j}[\delta]f\}$ from serving this market consists of three components $r_t(q)$, $-w_t\tau_{ij}q/z$ and $-E_{k_j}[\delta]f$. Firm revenue $r_t(q)$ is strictly positive. The first cost component $-w_t\tau_{ij}q/z$ vanishes as z tends towards infinity. Thus, $r_t(q)-w_t\tau_{ij}q/z$ is strictly positive and it is possible to choose $f > 0$ such that also $\pi_{ij,t}$ is strictly positive. As the firm only switches to a new productivity level if it dominates its current productivity, we have $\psi_{s+1}(\pi_{ij,s+1}) \geq \psi_s(\pi_{ij,s})$ for all $s \geq t$. Hence, also the continuation operating profit from serving market j is strictly positive, $\sum_{s\geq t} E_{k_j,\psi_s}[(1-\delta)^s\pi_{ij,s}] > 0$. Again choosing $F > 0$ sufficiently small, we end up with strictly positive total profits from entering market j, $\Pi_{ij,t} > 0$ (see formula (III.8)). As entering a new market does not only change firm profits, but decreases the firms default probability at the same time, $\Pi_{ij,t} > 0$ is a sufficient property for entry. Hence, $j \in \lambda_t$. As this holds for arbitrary $j \in \{1,\ldots,n\}$ it follows $\lambda_t = \{1,\ldots,n\}$. Statement (*ii*) claims that the firm will exit all markets $\lambda_t = \emptyset$ if z is sufficiently small. This statement is restricted to sufficiently pessimistic perceived types (h,k). If the firm is very optimistic about its future productivity evolution and demand stability, potential future gains could outweigh losses in present periods and the firm would stay active, i.e. $\lambda_t \neq \emptyset$. Hence, there possibly exists a region of perceived types (h,k) without endogenous exit for some markets $j \in \{1,\ldots,n\}$. However, there exist endogenous exit cut-off values for all markets if the firm is not too optimistic about (h,k). If the firm faces very high demand risks $E_{k_j}[\delta] \to 1$ within all countries $j \in \{1,\ldots,n\}$, then its default probability $\rho_{\lambda_{t-1}\emptyset}$ also tends towards one. From $f > 0$ it follows that $z \to 0$ yields negative per period operating profits $\pi_{ij,t} < 0$ on all markets $j \in \{1,\ldots,n\}$. Hence, the firm will prefer to exit all markets and turn inactive endogenously in stage three, rather than realizing negative per period operating profits $\pi_{ij} < 0$ in stage four and then default due to exogenous demand shocks in stage five. This is the extreme case, where endogenous exit occurs independent from the future productivity evolution h. However, endogenous exit can also occur if the default probability is not as drastic. Consider a firm that is not very optimistic about its productivity evolution h. If it expects to default before its current productivity increased sufficiently in order to balance the initial negative per period operating profits, it will maximize its total profits by turning inactive immediately. Hence there exists a region of values of (h,k) that guarantees the existence of exit cut-off values $z^{**}_{ij,t}$ for all markets $j \in \{1,\ldots,n\}$. Statement (*iii*) ensures monotonicity, i.e. if a firm finds it profitable to enter market j at current productivity level z, then it would also enter market j at all dominating current productivity levels $z' > z$. Let $\iota \subset \{1,\ldots,n\}$ denote the subset of markets that the firm does not serve and let $\sigma : \iota \rightarrow \iota$ denote the permutation that orders the markets $l \in \iota$ according to their entry cut-off values, i.e. $\sigma(l) < \sigma(m)$ yields $z^{*}_{i\sigma(l),t} < z^{*}_{i\sigma(m),t}$. In terms of σ, it is straight forward to prove monotonicity of market entry. Monotonicity holds if increasing z does not alter σ. This is true as varying z while holding (h, k) fix affects $\Pi_{i\lambda_a,t}$ only via changing per period operating profits $\pi_{il,t}$ directly. Those profits are strictly increasing in z. Hence, leapfrogging of cut-off values is not possible. Monotonicity of market exit follows from the same arguments by replacing $\iota \subset \{1, \ldots, n\}$ with the markets that the firm serves and $\sigma : \iota \rightarrow \iota$ with the permutation that orders those markets such that $\sigma(l) < \sigma(m)$ yields $z^{**}_{i\sigma(l),t} < z^{**}_{i\sigma(m),t}$.

Free Entry Condition

The free entry condition closes the model by pinning down the mass of entrants M^e_t in every period $t \geq 1$ along the balanced growth path. Existence and uniqueness of a general equilibrium with a positive mass of entrants $M^e_t > 0$ follows from Lemma 15 via choosing a value $K > 0$ in between $\lim_{M^e_t \rightarrow \infty} E_{m^e_t}[\Pi_{i\emptyset,t}] = 0$ and $\lim_{M^e_t \rightarrow 0} E_{m^e_t}[\Pi_{i\emptyset,t}] > 0$. Similar to Lemma 14, existence of M^e_t follows from (i) and (ii) and uniqueness from (iii).

Lemma 15. *Given the restrictions on f and F from Lemma 14, the relationship of mass M^e_t and expected total profit $E_{m^e_t}[\Pi_{i\emptyset,t}]$ in stage one (i.e. before observing the first productivity draw z_0) fulfills: (i) $M^e_t \rightarrow \infty$ yields $E_{m^e_t}[\Pi_{i\emptyset,t}] \rightarrow 0$, (ii) $M^e_t = 0$ yields $E_{m^e_t}[\Pi_{i\emptyset,t}] > 0$, and (iii) $\tilde{M}^e_t < M^e_t$ yields $E_{m^e_t}[\tilde{\Pi}_{i\emptyset,t}] > E_{m^e_t}[\Pi_{i\emptyset,t}]$.*

Statement (i) claims that as the mass of entrants M^e_t approaches infinity, their expected total profit $E_{m^e_t}[\Pi_{i\emptyset,t}]$ approaches zero. As there is only a finite mass of workers $L > 0$ in every country $i \in \{1, \ldots, n\}$, increasing the mass of entrants M^e_t without bound pushes the expected labor share $l = q/z$ of each firm towards zero. Either a subset of firm types of positive measure decides to turn active, or a Lebesgue zero set of firm types turns active. The first case implies an infinite mass of new incumbents, hence, the labor share of each individual firm equals zero. The second case also incorporates finite masses of new incumbents and yields positive actual labor shares in this case. However, as the ex-ante probability to turn active is zero, ex-ante expected labor shares equal zero as well. Hence, expected production quantities q converge towards zero in either case and expected firm revenue $r_t(q)$ vanishes. This yields negative per period operating profits $\pi_{ij,t} = \max_{q \in \mathbb{R}_+} \{r_t(q) - w_t \tau_{ij} q/z - E_{k_j}[\delta] f\}$. Thus, new born firms will not find it optimal to invest in market entry and decide to turn inactive generating zero total profits. Statement (*ii*) claims that if the mass of entrants M_t^e converges towards zero, their expected total profit $E_{m_t^e}[\Pi_{i\emptyset,t}]$ becomes strictly positive. By choice of f and F (see proof of Lemma 14) there exist some values of z_0 that yield strictly positive firm profits. Possibly there exist some values of z_0 that yield zero profits, but negative profits are not possible as the firm can always decide to turn inactive. Hence, expected total profits are strictly positive. Statement (*iii*) ensures monotonicity, i.e. the larger the mass of entrants M_t^e the smaller the expected total profit $E_{m_t^e}[\Pi_{i\emptyset,t}]$. This result is driven via two channels. Consider a firm (z, h, k). As M_t^e rises, the mass of competitors increases. Hence, the workforce $l = q/z$ of firm (z, h, k) shrinks. This reduces q, and consequently operating profits $\pi_{ij,t}(z, h, k)$ decline. Operating profits affect total profits $\Pi_{i\lambda_t}(z, h, k) = \sum_{s\geq t} \sum_{\lambda_s \in \Lambda} E_{k,\psi_s}[\rho_{\lambda_s}(z, h, k)\pi_{i\lambda_s}(z, h, k) - |\lambda_s \backslash \lambda_{s-1}|F]$ directly via $\pi_{i\lambda_s}(z, h, k)$ and indirectly via $\rho_{\lambda_s}(z, h, k)$. The probabilities $\rho_{\lambda_s}(z, h, k)$ give rise to the probability of firm expansion $\sum_{\lambda_t \supset \lambda_{t-a}} \rho_{\lambda_t}(z, h, k)$ or firm shrinkage $\sum_{\lambda_t \subset \lambda_{t-a}} \rho_{\lambda_t}(z, h, k)$. Both, $\pi_{i\lambda_t}(z, h, k)$ and $\sum_{\lambda_t \supset \lambda_{t-a}} \rho_{\lambda_t}(z, h, k)$ are downsized by a decline in operating profits $\pi_{ij,s}(z, h, k)$. Hence, increasing the mass of entrants M_t^e reduces total profits $\Pi_{i\lambda_t}(z, h, k)$ for any firm (z, h, k) and thus ex-ante expected total profits $E_{m_t^e}[\Pi_{i\emptyset,t}]$ shrink. □

List of Figures

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