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## Some Systematical Features of Gallagher-Moszkowski Splitting in Odd-Odd Rare Earth Deformed Nuclei

D. NOSEK, F. ŠTĚRBA, P. HOLAN

Department of Nuclear Physics, Charles University, Prague\*)

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Detail analysis of the splitting of the Gallagher-Moszkowski pairs in odd-odd rare earth deformed nuclei is performed in frame of the Nilsson model. The analysis is based on wide compiled experimental material. Systematical behaviour of the splitting energy as a function of configuration of odd nucleons, nuclear mass number, nuclear shape and Nilsson model parameters is examined. Substantial role of the  $\Delta N = 2$  interaction is also shown. In the last part of paper method of separated parameters, recommended recently by Singh and Sood, is systematically proved.

V rámci Nilssonova modelu je provedena analýza rozštěpení Gallagher-Moszkowskiho párů v licho-lichých deformovaných jádrech z oblasti vzácných zemin. Analýza se opírá o rozsáhlou kompilaci experimentálního materiálu. Je proveden systematický rozbor závislosti energie rozštěpení na konfiguraci lichých nukleonů, na hmotnostním čísle a tvaru jádra a na parametrech Nilssonova modelu. Je také ukázán podstatný vliv  $\Delta N = 2$  interakce na rozštěpení. Na závěr je provedena systematická prověrka použitelnosti metody separovaných parametrů, navržené nedávno Singhem a Soodem.

В рамках модели Нильссона проводится анализ расщепления Галлагер-Мошковского пар в нечетно-нечетных ядрах из области редких земель. Анализ проводится на основе широкого компилационного экспериментального материала. Проанализировано систематическое поведение энергии расщепления в зависимости от конфигурации нечетных нуклеонов, массового числа ядра, ядерной формы и параметров модели Нильссона. Показана существенная роль  $\Delta N = 2$  взаиймодейтствия. В последней части проводится систематическая проверка метода разделенных параметров не давно рекоммендированного Сингом и Соодом для анализа нечетно-нечетных деформированных ядер.

### 1. Introduction

The study of the structure of odd-odd deformed nuclei is complicated because the intrinsic states, determined by average potential, are splitted by residual interaction between odd proton and odd neutron. Energy of corresponding states with parallel and antiparallel orientation of the spin ("Gallagher-Moszkowski pair") differs and the separation energy  $\Delta E_{GM}$  is referred to as the Gallagher-Moszkowski

<sup>\*)</sup> V Holešovičkách 2, 180 00 Praha 8, Czechoslovakia.

splitting energy. Exact model calculations of the splitting are difficult and are performed usually in a rather simple approximation (e.g. [1-5]). Recently Singh and Sood [6, 7] suggested a semiempirical method based on the p-n interaction with zero-range radial dependence, which made it possible to calculate the states in odd-odd nuclei from the states known in neighbour odd-A (odd-proton and odd-neutron) ones. The matrix elements of the p-n interaction used in the method can be calculated in a definite nuclear model.

In the present paper we analyse the splitting of the G-M pairs using wide experimental material published elsewhere. The influence of the model parameters including deformation parameters  $\delta$  and  $\alpha_{40}$  on splitting energy  $\Delta E_{GM}$  is examined in details. Rather preliminary informations on the role of the  $\Delta N = 2$  interaction is also shown. However, in the calculations were included only the diagonal matrix elements of the Coriolis interaction which is known to be rather important in odd-A deformed nuclei [8-10].

## 2. The structure of odd-odd deformed nuclei

## 2.1. Model description

Our analysis is performed in frame of the unified model with pairing interaction included [8-10]. The band head states for different rotational bands are considered as the noncollective states resulting from the coupling of the single proton and/or neutron to the deformed even-even core. Introduction of the pairing interaction in the nucleus by transformation from particle to quasiparticle description made it possible to consider intrinsic states in odd-A nuclei as one-quasiparticle while in odd-odd nuclei as two-quasiparticle ones [9, 10].

The states of definite rotational band in nucleus with axial symmetry are characterized by projection K of the total angular momentum  $\vec{I}$  onto the symmetry axis. In odd-odd nucleus two values of  $K, K = K_{\pm} = |\Omega_p \pm \Omega_n|$  are obtained for parallel  $(K_+)$  and antiparallel  $(K_-)$  orientation of the projection  $\Omega$  of the intrinsic proton(p) and neutron (n) angular momentum  $\vec{j}$  onto symmetry axis.

The model hamiltonian of odd-odd nucleus may be written in form [1, 2, 8, 9]

$$(1) H = H_{\rm in} + H_{\rm rot} + V_{\rm pn}$$

where  $H_{in}$  describes intrinsic (nonrotational) nuclear motion,  $V_{pn}$  is potential of residual interaction between odd proton and odd neutron.  $H_{rot}$  represents rotational motion of the nucleus as a whole including influence of rotational motion on the intrinsic degrees of freedom of nucleus (Coriolis interaction – CI).  $H_{rot}$  can be expressed as

$$H_{\rm rot} = H_{\rm R} + H_{\rm CI},$$

(3) 
$$H_{\rm R} = \frac{\hbar^2}{2\mathscr{P}_{00}} (I^2 - I_z^2)$$

(4) 
$$H_{\rm CI} = -\frac{\hbar^2}{2\mathscr{P}_{00}} \left[ \left( I_+ J_- + I_- J_+ \right) + \left( j_+^{\rm p} j_-^{\rm n} + j_-^{\rm p} j_+^{\rm n} \right) \right].$$

Here  $\mathscr{P}_{00}$  is moment of inertia of odd-odd nucleus,  $\vec{J}$  is total intrinsic impulsmoment,  $I_{\pm}, J_{\pm}, j_{\pm}^{p}$  and  $j_{\pm}^{n}$  are usual step operators and  $I_{z}$  is operator of projection of total nuclear angular momentum onto symmetry axis. Using adiabatic wave functions of axially symmetric nucleus [3, 8, 9] and neglecting nondiagonal matrix elements of the Coriolis interaction (4) energy E(I) of the members of rotational bands built on definite intrinsic states with intrinsic energy  $\varepsilon_{in}$  becomes [8, 9, 11]

(5) 
$$E(I) = \varepsilon_{\rm in} + \frac{\hbar^2}{2\mathscr{P}_{00}} \left[ I(I+1) - K^2 \right] + E_{\rm int}^K + (-1)^I (B_{\rm int} - E_{\rm a}) \,\delta_{K0} \,.$$

Here  $E_{int}$  and  $B_{int}$  are determined by the residual p-n interaction between odd proton and odd neutron and  $E_a$  is diagonal matrix element of the Coriolis interaction depending on the decoupling parameters  $a_p$  and  $a_n$  of the one-proton and one-neutron states respectively

(6) 
$$E_{a} = -\frac{\hbar^{2}}{2\mathscr{P}_{00}} a_{p} \cdot a_{n} \cdot \delta_{\Omega_{p}1/2} \cdot \delta_{\Omega_{n}1/2} \cdot \delta_{K0}$$

Energy of corresponding two-quasiparticle band head states may be written in form [6, 7]

(7) 
$$E(A, Z)_{K} = {}^{0}E(A - 1, Z - 1)_{\Omega_{n}} + {}^{0}E(A - 1, Z)_{\Omega_{p}} + E_{rot} + E_{a} + E_{pn}$$

where  ${}^{0}E(A-1, Z-1)$  and  ${}^{0}E(A-1, Z)$  are band head energies of one-quasiparticle states in neighbour odd-A nuclei normalised so that the energy of the ground state equals zero. The term  $E_{rot}$  is connected with different rotational energy in odd-A and odd-odd nuclei and equals

(8) 
$$E_{\rm rot} = \frac{\hbar^2}{2\mathscr{P}_{00}} K - \left(\frac{\hbar^2}{2\mathscr{P}_p} \Omega_p + \frac{\hbar^2}{2\mathscr{P}_n} \Omega_n\right) \sim -2 \frac{\hbar^2}{2\mathscr{P}} \Omega_{<} \cdot \delta_{KK_{-}}.$$

Here  $\mathscr{P}_{00}$ ,  $\mathscr{P}_p$  and  $\mathscr{P}_n$  are the moments of inertia of odd-odd, odd-proton and oddneutron nuclei **r**espectively while  $\mathscr{P}$  is corresponding average value.  $\Omega_{<}$  is smaller of the  $\Omega_p$  and  $\Omega_n$  values. Exactly,  $E_{rot}$  is difference in rotational energy of odd-odd and odd-A nuclei.

Last term in (7), determined fully by the p-n interaction is responsible for the splitting of the  $K_+$  and  $K_-$  rotational bands in odd-odd nuclei. Using for the p-n interaction simple zero-range potential  $V_{pn}(r)$  in form [5]

(9) 
$$V_{pn}(r) = -4\pi g \cdot \delta(r) \left[ (1-\alpha) + \alpha \cdot \vec{\sigma}_{p} \cdot \vec{\sigma}_{n} \right]$$
$$r = |\vec{r}_{p} - \vec{r}_{n}|$$

 $E_{pn}$  can be written in the form [12]

(10) 
$$E_{pn}^{K} = E_{int}^{K} + B_{int} \cdot \delta_{K0}$$

where

(11) 
$$E_{\text{int}}^{K_{\pm}} = W.(1-\alpha).A_0 \pm \alpha.W.A_{\sigma} \quad B_{\text{int}}^{K=0} = W.\alpha.B_{\sigma}.$$

Here  $A_0$  and  $A_\sigma$  are diagonal matrix elements of the spin-independent and spindependent part of the p-n interaction (9) respectively. Matrix element  $B_\sigma$  is responsible for the Newby odd-even shift in the K = 0 rotational bands [2]. Parameter W is directly connected with the strength parameter g of p-n interaction potential (9) and in the Nilsson model becomes

(12) 
$$W = g \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{m\omega_0}{\hbar}\right)^{3/2} = \text{konst. } A^{1/2}$$

what indicates rather weak dependence of parameter W on nuclear mass number A.

All matrix elements in (11) can be calculated in definite model describing intrinsic states of deformed nuclei. It is substantial that the diagonal matrix elements are not affected by pairing interaction and can be therefore calculated directly from the single particle model [2, 12]. Explicite form of matrix elements  $A_0$ ,  $A_{\sigma}$  and  $B_{\sigma}$ , calculated with the Nilsson wave functions is [12]

(13a) 
$$A_{0} = \sum_{(IA)} a_{(IA)} \cdot \langle \mathbf{p} | \langle \mathbf{n} | \delta(\mathbf{r}) | \mathbf{n}' \rangle | \mathbf{p}' \rangle$$

(13b) 
$$A_{\sigma} = \sum_{(lA)} a_{(lA)} \cdot \langle \mathbf{p} | \langle \mathbf{n} | \delta(r) \cdot \vec{\sigma}_{\mathbf{p}} \cdot \vec{\sigma}_{\mathbf{n}} | \mathbf{n}' \rangle | \mathbf{p}' \rangle$$

(13c) 
$$B_{\sigma} = \sum_{(lA)} a_{(lA)} \cdot \langle \mathbf{p} | \langle \tilde{n} | \delta(r) \cdot \vec{\sigma}_{\mathbf{p}} \cdot \vec{\sigma}_{\mathbf{n}} | \mathbf{n}' \rangle | \mathbf{p}' \rangle$$

Here  $|i\rangle = |Nl\Lambda\Sigma\rangle$  are proton (i = p) and neutron (i = n) wave functions of the spherical part of the Nilsson potential and  $a_{(lA)} = a_{l_pA_p} \cdot a_{l_nA_n} \cdot a_{l_p'A_{p'}} \cdot a_{l_{n'A_{n'}}}$  is product of the Nilsson coefficients  $a_{l_iA_i}$ . Addittion in (13) is performed over all permited values of  $l_i\Lambda_i$ .

The splitting energy  $\Delta E_{GM}$  of the band head states of rotational bands with  $K = K_+$  and  $K = K_-$  in odd-odd deformed nuclei ("Gallagher-Moszkowski splitting") is simply given by [3, 6, 7, 12]

(14) 
$$\Delta E_{\rm GM} = E_{\rm int}^{K_-} - E_{\rm int}^{K_+}$$

As follows from Eqs. (11), for the  $K \neq 0$  rotational bands the G-M splitting posses simple form

(15) 
$$\Delta E_{\rm GM} = -2 \cdot \alpha \cdot W \cdot A_{\sigma}$$

depending only on the common parameter  $D = \alpha$ . W and on the spin-dependent matrix element  $A_{\sigma}$ . Expression (15) can be used for calculation of band head energies of one component of the G-M pair if second component and the common parameter D are known. Nevertheless, explicite values of parameters  $\alpha$  and W are necessary

if direct calculation of both components of the G-M pair should be calculated from definite nuclear model (Eqs. (6)-(8), (10) and (11)).

Singh and Sood [7] analysed influence of the p-n interaction on the energy of the ground states of odd-odd deformed nuclei using atomic masses of neighbour odd-odd and odd-A isotopes. Considering rather weak dependence of the parameter W on mass number A (Eq. (12)) they got for parameter  $\alpha$  explicite expression

(16) 
$$\alpha = \left[1 + k \left|\frac{A_{\sigma}(K)}{A_{0}(K)}\right|\right]^{-1}$$

where k was in [7] determined from experimental atomic masses as  $k = 5.5 \pm 0.5$ . Using experimental value of the G-M splitting energy,  $\Delta E_{GM}^{exp}$ , the parameter W becomes

(17) 
$$W = \frac{\Delta E_{\rm GM}^{\rm exp}}{2\alpha A_{\sigma}(K)}$$

Both parameters,  $\alpha$  and W, are model dependent through matrix elements  $A_0$  and  $A_{\sigma}$ . Matrix elements are generally expected to be strongly dependent on the specific particle states of proton and neutron (configuration dependence). For given configuration the variation of  $\alpha$  and W from nucleus to nucleus should be concentrated in dependence of  $A_0$  and  $A_{\sigma}$  on the nuclear shape and on other parameters and are expected to be rather weak [7].

In the present paper we analyse validity of described simple model in more details. The model calculations of the matrix elements  $A_0$  and  $A_{\sigma}$  are applicated to the experimentally observed G-M pairs in rare earth deformed nuclei [2, 4, 11, 13-26]. Rather substantial influence of nuclear shape on the matrix elements (13) and finally on the G-M splitting energy  $\Delta E_{\rm GM}$  is shown. More, preliminary calculations of  $A_0$  and  $A_{\sigma}$  from the Nilsson model with included  $\Delta N = 2$  interaction point out substantial influence of the interaction on the G-M splitting as well. In the next part of the paper the experimentally observed G-M pairs are analysed using Eq. (15) while in the last part some systematical behaviour of parameters  $\alpha$  and W is shown.

#### 2.2. Experimental informations on odd-odd deformed nuclei

Experimental informations about odd-odd deformed nuclei known up to end of 1974 year were collected in 1976 year by Boisson et al. [2] and Elmore and Alford [4]. As present experimental material is much more rich, we collected in Tab. 1 all experimental informations obtained and published up to end of the 1983 year. In first and second columns the proton and neutron Nilsson states are given in asymptotic quantum numbers asignment while the values of K and corresponding band head energies for both members of the G-M pairs are in next four columns.

The sources of experimental informations are shown in last column of the table. If informations on definite nucleus have been collected by other authors, only this reference is included in Tab. 1.

$\Omega_{\mathbf{p}}[Nn_{z}\Lambda]_{\mathbf{p}}$	$\Omega_{n}[Nn_{z}\Lambda]_{n}$	Κ_	$E_{K}$ (keV)	<i>K</i> <sub>+</sub>	$E_{K+}$ (keV)	⊿E <sup>exp</sup> (keV)	∆E <sup>th</sup> (keV)	G <sup>a</sup> )	R <sup>b</sup> )
				<sup>152</sup> Eu					
5/2 [413]	11/2 [505]	0-	0	3—	148	-95	- 121	1	
5/2 [413]	5/2 [642]			5+	108				
5/2 [413]	3/2 [402]			4+	90				
5/2 [413]	3/2 [532]			4—	142				[2]
5/2 [413]	3/2 [521]	1—	65						[16]
5/2 [532]	3/2 [532]	1+	78						
3/2 [411]	3/2 [532]	0	47						
3/2 [411]	3/2 [521]			3—	77				
				<sup>154</sup> Eu					
5/2 [413]	11/2 [505]	3—	0						
5/2 [413]	3/2 [651]	1+	72						
5/2 [413]	3/2 [521]	1—	83						[3]
5/2 [532]	3/2 [521]	1+	135						
5/2 [532]	3/2 [651]	1—	162						
				<sup>156</sup> Eu					
5/2 [413]	5/2 [642]	0+	0						
5/2 [413]	3/2 [521]	1—	88						[23]
5/2 [532]	3/2 [521]	1+	291						
<u> </u>				<sup>156</sup> Tb					
3/2 [411]	3/2 [521]	0-	112	3-	0	122	138	1	[16]
3/2 [411]	3/2 [651]	<b>0</b> +	88						[23]
				<sup>158</sup> Tb					
3/2 [411]	3/2 [521]	0-	111	3—	0	133	138	1	
3/2 [411]	11/2 [505]			7—	340				[3]
3/2 [411]	5/2 [642]			4+	130				[16]
3/2 [411]	3/2 [402]	0+	420	3+	593	-108	150		[23]
3/2 [411]	1/2 [400]	1+	767	2+	641	139	138	3	
3/2 [411]	1/2 [530]			2—	678				
5/2 [402]	3/2 [521]					139	68	1°)	
7/2 [404]	3/2 [521]					- 70	— 52	3°)	
5/2 [532]	3/2 [521]					119	57	3°)	

.

Table 1. States observed in rare earth odd-odd deformed nuclei

Table 1. (Continued 1)

$\Omega_{\mathfrak{p}}[Nn_{z}\Lambda]_{\mathfrak{p}}$	$\Omega_{n}[Nn_{z}\Lambda]_{n}$	K_	<i>E<sub>K</sub></i> _(keV)	K <sub>+</sub>	<i>E<sub>K↓</sub></i> (keV)	⊿E <sup>exp</sup> (keV)	⊿E <sup>th</sup> (keV)	G <sup>a</sup> )	R <sup>b</sup> )
				<sup>160</sup> Tb					
5/2 [413]	5/2 [642]	0+	223						
5/2 [413]	5/2 [523]	0	235						
3/2 [411]	3/2 [651]	0+	479						
3/2 [411]	5/2 [642]	1+	139	4+	64	93	75	1	[3]
3/2 [411]	3/2 [521]	0	79	3—	0	126	138	1	[23]
3/2 [411]	5/2 [523]	1—	64	4—	258	-161	- 75	1	
3/2 [411]	1/2 [521]	1—	381						
				<sup>162</sup> Tb					
3/2 [411]	5/2 [523]	1—	0						[23]
7/2 [523]	5/2 [523]	1+	442						
				<sup>158</sup> Ho					
7/2 [523]	3/2 [521]	1+	72	5+	0	144	33	1	[16]
1/2 [411]	3/2 [521]			4	139				[23]
				<sup>160</sup> Ho					
7/2 [523]	3/2 [521]			5+	0				
7/2 [523]	3/2 [651]	2—	60						
7/2 [523]	11/2 [505]			9+	110				[3]
7/2 [523]	5/2 [642]			6—	169				
				<sup>162</sup> Ho					
7/2 [523]	5/2 [523]	1+	0						
7/2 [523]	5/2 [642]			6	106				[3]
7/2 [523]	5/2 [512]			6+	286				
				<sup>164</sup> Ho					
7/2 [523]	5/2 [523]	1+	0	6+	191	-146	-156	1	
7/2 [523]	5/2 [642]	1—	159	6—	140	53	90	1	
7/2 [523]	1/2 [521]	3+	187						[3]
7/2 [523]	3/2 [521]	2+	486	5+	343	160	33	1	[16]
7/2 [523]	3/2 [402]	2—	620	5—	733	- 88	-111	1	
7/2 [523]	1/2 [400]	3-	925	4	833	101	22	1	
				<sup>166</sup> Ho					
7/2 [523]	7/2 [633]	0—	0	7—	5	83	128	1	
7/2 [523]	1/2 [521]	3+	191	4+	372	-168	- 30	1	
7/2 [523]	5/2 [512]	1+	568	6+	294	310	78	1	
7/2 [523]	5/2 [523]	1+	426						
7/2 [523]	1/2 [510] ,	3+	815	4+	559	266	27	1	
3/2 [411]	7/2 [633]	2+	425	5+	259	194	102	1	[12]
1/2 [411]	5/2 [523]	2—	416	3—	563	-138	- 68	1	
1/2 [411]	7/2 [633]	3+	718	4+	884	-119	- 69	1	

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$\Omega_{\rm p}[Nn_z\Lambda]_{\rm p}$	$\Omega_{n}[Nn_{z}\Lambda]_{n}$	Κ_	$E_{K}$ (keV)	$K_+$	$E_{K_+}$ (keV)	⊿E <sup>exp</sup> (keV)	⊿E <sup>th</sup> (keV)	G <sup>a</sup> )	R <sup>b</sup> )
7/2 [404]	7/2 [633]	0+	801	7+-	915	34	- 86	3	
5/2 [413]	7/2 [633]	1+	1150	6 +	1272	— 70	-109	2	
5/2 [532]	7/2 [633]			6—	1560				
				<sup>160</sup> Tm					
1/2 [411]	3/2 [521]	1—	0						
1/2 [411]	5/2 [642]	2+	140						
7/2 [523]	3/2 [521]	2+	174						[1]
7/2 [523]	5/2 [523]	1+	216						
				<sup>162</sup> Tm					
1/2 [411]	3/2 [521]	1-	0						
7/2 [523]	5/2 [523]	1+	164						
7/2 [523]	3/2 [521]	2+	192						[23]
7/2 [404]	3/2 [521]	2+	67						
				<sup>164</sup> Tm					
7/2 [523]	5/2 [523]	1+	0						
7/2 [404]	5/2 [523]			6	50				[23]
3/2 [411]	5/2 [523]	1+	87						
				<sup>166</sup> Tm					
7/2 [523]	5/2 [523]	1+	82						[23]
				<sup>168</sup> Tm					
1/2 [411]	7/2 [633]	3+-	0	4+	147	-138		3	
1/2 [411]	1/2 [521]	0	167	1—	3	191	133	3	
1/2 [411]	5/2 [512]	2—	246	3—	499	-233	<u> </u>	3	
1/2 [411]	3/2 [521]	1—	611	2—	702	— 72	-112	3	
1/2 [411]	1/2 [510]	0	789	1—	882	-113	-153	3	
1/2 [411]	5/2 [523]	2	904	3—	853	86	68	3	[3]
1/2 [411]	1/2 [400]	0+	1057	1+	1347	-258	-150	2	
1/2 [411]	3/2 [402]	1+	1427	2+	1116	324	73	2	
1/2 [541]	7/2 [633]	3—	200	4	336	- 65	- 27	1	
5/2 [402]	1/2 [510]	2—	732	3—	815				
1/2 [530]	7/2 [633]	3—	1437	4—	1389	49	6	3	
				<sup>170</sup> Tm					
1/2 [411]	1/2 [521]	0-	148	1	0	181	133	1	
1/2 [411]	7/2 [633]	3+	183						
1/2 [411]	5/2 [512]	2—	194	3-	449	-237	<u> </u>	1	
7/2 [404]	1/2 [521]	3—	774	4-	644	142	85	1	[14]
5/2 [402]	1/2 [521]	2-	716	3—	867	-139	- 85	1	
3/2 [411]	1/2 [521]	1	700	2	851	-146	-123	1	
7/2 [523]	1/2 [521]	3+	671	4+	690	- 10	- 30	1	
5/2 [413]	1/2 [521]	2	1382	3—	1213	149	25	2	

Table 1. (Continued 2)

$\Omega_{n}[Nn_{z}\Lambda]_{n}$	Κ_	$E_{K-}$ (keV)	$K_+$	$E_{K+}$ (keV)	⊿E <sup>exp</sup> (keV)	⊿E <sup>th</sup> (keV)	G <sup>a</sup> )	R <sup>b</sup> )
			<sup>172</sup> Tm					
5/2 [512]	2—	0	3	240	-230	- 194	1	[3]
1/2 [521]	0	475	1—	407	94	133	3	[16]
5/2 [523]					144	94	1°)	
			<sup>170</sup> Lu					
7/2 [633]	0+	0						
1/2 [521]	3—	96	4—	93	13	85	3	[3]
5/2 [512]	1—	165						[23]
1/2 [521]	0-	408	1	245	192	133	1	[45]
1/2 [521]	0+	437	1+	349	97	43	3	
			<sup>172</sup> Lu					
1/2 [521]	3-	68	4—	0	76	85	1	
1/2 [521]	0+	237	1+	66	60	43	3	[3]
1/2 [521]					- 84	-85	1°)	[16]
1/2 [521]					-125	-35	3°)	
			<sup>174</sup> Lu			· · · · · · · ·		
5/2 [512]	1—	0	6—	171	-119	- 50	1	
7/2 [633]	0+-	277	7+	431	-138	- 86	1	
1/2 [521]	3_	432	4	365	77	85	1	
3/2 [521]	2	1178	5	1304	- 87	- 53	1	
5/2 [512]	2+	278	3+	414	- 25	-16	1	[3]
5/2 [512]	0-	555	5	455	130	124	3	[-]
5/2 [512]	2+	693	7+	530	168	92	1	
5/2 [512]	2+	1293	3+	1262	30	145	1	
5/2 [512]	- 1	-=/0	4+	1439	20		-	
			<sup>176</sup> Lu					
7/2 [514]	0—	241	7	0	252	137	1	
9/2 [624]	1+	198	8+	404	-118	-119	1	
5/2 [512]	1—	390						
1/2 [510]	3—	662	4—	791	-114	- 48	1	[3]
7/2 [514]	1+	327	8+	486	- 83	-229	1	[13]
7/2 [514]	1-	391	6	565	- 104	- 93	1	
7/2 [514]	3—	840	4	723	127	16	1	
7/2 [514]	0+	1057	7+	1273	7	-125	3	
7/2 [514]			5—	1395				
			<sup>178</sup> Lu					
9/2 [624]	1+	0						[16]
7/2 [514]	0—	80	7—	40	70	137	3	[23]
7/2 [514]	1	201					-	[ ]
	$\begin{split} & \Omega_n[Nn_zA]_n \\ & 5/2 [512] \\ & 1/2 [521] \\ & 5/2 [523] \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\$	$\begin{array}{c} \mathfrak{Q}_{n}[Nn_{z}\mathcal{A}]_{n}  K_{-} \\ \\ \hline \\ 5/2 [512]  2- \\ 1/2 [521]  0- \\ 5/2 [523] \\ \hline \\ \hline \\ 7/2 [633]  0+ \\ 1/2 [521]  3- \\ 5/2 [512]  1- \\ 1/2 [521]  0- \\ 1/2 [521]  0- \\ 1/2 [521]  0+ \\ 1/2 [521]  0+ \\ 1/2 [521]  0+ \\ 1/2 [521]  0+ \\ 1/2 [521]  0+ \\ 1/2 [521]  0- \\ 1/2 [521]  0- \\ 1/2 [521]  0- \\ 1/2 [521]  0- \\ 1/2 [521]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [512]  2- \\ 5/2 [5$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

Table 1. (Continued 3)

		(keV)	м <sub>+</sub>	$L_{K+}$ (keV)	(keV)	$\frac{\Delta E_{GM}}{(\text{keV})}$	0)	R)
			<sup>176</sup> Ta					
5/2 [512]	1—	0						
7/2 [633]	<b>0</b> +	100						[23]
7/2 [514]	1+	195						
			<sup>178</sup> Ta					
7/2 [514]			7—	0				
7/2 [514]			6—	290				[15]
7/2 [514]			8+	220				
9/2 [624]			9—	393				
			<sup>180</sup> Ta					
9/2 [624]	1+	0	8+	177	-104	-119	1	
7/2 [514]	0	382	7—	176	283	137	1	
5/2 [512]	1	412	6—	575	- 99	- 50	1	
1/2 [510]	3-	534	4	659	-116	- 48	1	
1/2 [521]	3—	788	4—	727	70	85	1	[46]
3/2 [512]	2—	1030	5-	974	83	22	1	
9/2 [624]	0-	121	9	82	126	145	1	
9/2 [624]	2+	563	7+	361	255	91	1	
	2		<sup>182</sup> Ta					
1/2 [510]	3—	0	4—	114	- 84	- 48	1	
3/2 [512]	2—	270	5—	173	125	22		1
11/2 [615]	2+	402						
7/2 [503]	0-	584	7—	<b>7</b> 77	-123	-152	1	
9/2 [624]	1+	593						[3]
1/2 [510]	4+	150	5+	16	147	102	1	[16]
3/2 [512]	3+	250	6+	390	- 97	- 66	3	
11/2 [615]	1—	689			285	281	3°)	
3/2 [512]	1—	444						
1/2 [510]	2—	660	3—	547	130	55	3	
11/2 [615]	3+	749						
			<sup>184</sup> Ta					
3/2 [512]			5—	0				[3], [16]
7/2 [503]	0—	272						[23]
			<sup>180</sup> Re					
7/2 [514]	1-	0						[3]
7/2 [514]	1+	60						[23]
	5/2 [512] 7/2 [633] 7/2 [514] 7/2 [514] 7/2 [514] 7/2 [514] 7/2 [514] 9/2 [624] 9/2 [624] 9/2 [624] 1/2 [510] 1/2 [512] 1/2 [510] 3/2 [512] 1/2 [513] 7/2 [514] 7/2 [514]	$\begin{array}{c} 5/2 \ [512] \ 1-\\ 7/2 \ [633] \ 0+\\ 7/2 \ [514] \ 1+\\ \hline\\ \hline\\ 7/2 \ [514] \ 1+\\ \hline\\ 7/2 \ [514] \ 7/2 \ [514] \\ 7/2 \ [514] \ 7/2 \ [514] \\ 7/2 \ [512] \ 1-\\ 1/2 \ [510] \ 3-\\ 1/2 \ [512] \ 2-\\ 9/2 \ [624] \ 0-\\ 9/2 \ [624] \ 2+\\ \hline\\ \hline\\ 7/2 \ [512] \ 2-\\ 9/2 \ [624] \ 0-\\ 9/2 \ [624] \ 2+\\ \hline\\ \hline\\ 7/2 \ [510] \ 3-\\ 3/2 \ [512] \ 2-\\ 9/2 \ [624] \ 1+\\ 1/2 \ [510] \ 3-\\ 3/2 \ [512] \ 2-\\ 11/2 \ [615] \ 2+\\ 7/2 \ [503] \ 0-\\ 9/2 \ [624] \ 1+\\ 1/2 \ [510] \ 3+\\ \hline\\ 3/2 \ [512] \ 1-\\ 1/2 \ [510] \ 2-\\ 11/2 \ [615] \ 3+\\ \hline\\ \hline\\ 3/2 \ [512] \ 1-\\ 1/2 \ [510] \ 2-\\ 11/2 \ [615] \ 3+\\ \hline\\ \hline\\ 3/2 \ [512] \ 3-\\ 1/2 \ [510] \ 2-\\ 11/2 \ [615] \ 3+\\ \hline\\ \hline\\ 7/2 \ [514] \ 1-\\ 7/2 \ [514] \ 1-\\ 7/2 \ [514] \ 1-\\ 7/2 \ [514] \ 1-\\ \hline\end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $

Table 1. (Continued 4)

$\Omega_{\rm p}[Nn_{\rm z}\Lambda]_{\rm p}$	$\Omega_{n}[Nn_{z}\Lambda]_{n}$	K_	E <sub>K</sub> _ (keV)	К+	$E_{K+}$ (keV)	⊿E <sup>exp</sup> (keV)	⊿E <sup>th</sup> (keV)	G <sup>a</sup> )	R <sup>b</sup> )
				<sup>182</sup> Re					
5/2 [402]	1/2 [510]	2—	236						
5/2 [402]	3/2 [512]	1—	263						[3]
5/2 [402]	9/2 [624]	2+	0						[23]
9/2 [514]	7/2 [514]	1+	510						
				<sup>184</sup> Re					
5/2 [402]	1/2 [510]	2—	74	3	0	89	55	1	[3]
5/2 [402]	3/2 [512]	1	311	4	56	-209	-118	1	[16]
5/2 [402]	7/2 [503]	1—	440	6—	347	163	295	3	[21]
5/2 [402]	9/2 [624]			7+	590				[23]
5/2 [402]	11/2 [615]	3+	474	8+	188	266	119	3	
				<sup>186</sup> Re					
5/2 [402]	1/2 [510]	2—	210	3	99	136	55	1	
5/2 [402]	3/2 [512]	1	0	4—	173	-130	-118	1	[3]
5/2 [402]	7/2 [503]	1—	316	6—	186	206	295	1	[16]
5/2 [402]	9/2 [505]	2—	577						[21]
5/2 [402]	11/2 [615]	3+	314	8+	150	231	119	1	[23]
9/2 [514]	3/2 [512]	3+	351	6+	562	-155	- 66	3	[31]
9/2 [514]	1/2 [510]	4+	330	5+	471	160	- 102	3	
9/2 [514]	7/2 [503]	1+	601						
				<sup>188</sup> Re					
5/2 [402]	1/2 [510]	2—	256	3—	169	100	55	1	
5/2 [402]	3/2 [512]	1—	0	4—	182	- 149	118	1	
5/2 [402]	7/2 [503]	1—	<b>2</b> 90	6-	172	209	295	1	
5/2 [402]	9/2 [505]	2—	205						[3]
5/2 [402]	11/2 [615]	3+	440						[7]
5/2 [402]	3/2 [501]	1—	557	4—	284	271	162	2	[23]
9/2 [514]	1/2 [510]			5+	361				[38]
9/2 [514]	3/2 [512]	3+	231						[44]
9/2 [514]	7/2 [503]	1+	482						
9/2 [514]	9/2 [505]	0+	208						
1/2 [411]	3/2 [512]	1—	784	2—	591	167	179	3	

Table 1. (Continued 5)

<sup>a</sup>) Reliability of experimental identification of G-M pairs, "1" being the highest one.

<sup>b</sup>) Source of information.

<sup>c</sup>) The G-M splitting experimental energy  $\Delta E_{GM}^{exp}$  overtaken from Ref. [4].

Experimental values of the splitting energy,  $\Delta E_{GM}^{exp}$ , can be obtained from experimental energies  $E_K^{exp}$  of  $K_+$  and  $K_-$  band head states. As the quasiparticle energy  ${}^{0}E$  of odd proton and odd neutron is equal for both,  $K_+$  and  $K_-$  members of the G-M pair (see Eq. (7)),  $\Delta E_{GM}^{exp}$  can be calculated from experimental energies  $E_K^{exp}$  by sub-

stitution of Eqs. (6)-(8) and (10) into (14)

(18) 
$$\Delta E_{GM}^{exp} = E_{K_{-}}^{exp} - E_{K_{+}}^{exp} + \frac{\hbar^2}{2\mathscr{P}_{K_{+}}} K_{+} - \frac{\hbar^2}{2\mathscr{P}_{K_{-}}} K_{-} + (B^{exp} - E_a) \delta_{K0} .$$

Here the Newby shift can be calculated from energy  $E_{I+1}$  and  $E_I$  of the K = 0 rotational band

(19) 
$$B^{\exp} = \frac{(-1)^{l+1}}{2} E_{I+1}^{\exp} - E_{I}^{\exp} - 2 \frac{\hbar^2}{2\mathscr{P}} (l+1) + \frac{\hbar^2}{2\mathscr{P}} a_{p} a_{n} \delta_{\Omega_{p} 1/2} \delta_{\Omega_{n} 1/2}$$

The values of  $\Delta E_{GM}^{exp}$  for known G-M pairs obtained from (18) are collected in seventh column of Tab. 1. The inertial parameters  $\hbar^2/2\mathcal{P}$  for  $K_+$  and  $K_-$  rotational bands used in calculations were determined from first two states of the bands.

#### 3. Results of analysis

Model description of the odd-odd deformed nuclei described in part 2.1 was investigated using experimental informations collected in Tab. 1. First, the dependence of the G-M splitting on different parameters of the Nilsson model was examined. The results were then used in the analysis of the systematical behaviour of the splitting energy  $\Delta E_{\rm GM}$  calculated from Eq. (15). Finally, method of "separated parameters" was examined in more details and compared with experimental results for observed G-M pairs.

#### 3.1. Dependence of the G-M splitting on model calculations

## 3.1.1. Dependence on the model parameters

The G-M splitting energy,  $\Delta E_{GM}$ , was calculated from Eq. (15). Average value of common parameter  $D = \alpha W = 789$  keV, taken from Ref. [2] was used everywhere. Matrix elements  $A_{\sigma}$  for individual configuration were calculated from Nilsson model wave functions using expanded Nilsson potential [9, 10]. In the calculation were neglected the nondiagonal matrix elements of Coriolis interaction.

To analyse dependence of the G-M splitting on model parameters matrix elements  $A_{\sigma}$  were calculated with different sets of parameters. In the first step parameters  $\varkappa$  and  $\mu$  were changed while deformation parameters  $\delta$  and  $\alpha_{40}$  were kept constant. The  $\Delta N = 2$  interaction was not included in this step. Typical results for the deformation parameters  $\delta = 0.30$  and  $\alpha_{40} = 0.04$  and variable neutron parameters  $\varkappa$ and  $\mu$  are shown in Tab. 2. Generally, very weak dependence of the G-M splitting on parameters  $\varkappa$  and  $\mu$  was found. Therefore in further analysis these parameters were kept constant for all nuclei and their values for different proton and neutron N-shells were taken from Solovjev's monography [9] (see Tab. 3).

		,		
×n	0.0	630	0.0	640
μ <sub>n</sub>	0.41	0.43	0.41	0.43
$\Delta E_{\rm GM}^{\rm th}$ (keV)		-116.0	-116.5	-114.9

Table 2. Influence of neutron parameters  $\varkappa$  and  $\mu$  on the splitting energy  $\Delta E_{GM}^{th}$  of the  $\{5/2 + [402]_p, 3/2 - [512]_n\}$  configuration

N	4	ł	4	5		6
Nucleons	р	n	р	n	р	n
μ	0.60	0.35	0.60	0.42	0.30	0.30
×	0.0637	0.0637	0.0637	0.0637	0.0637	0.0660

Table 3. Model parameters  $\varkappa$  and  $\mu$  used in calculations



Fig. 1. Dependence of the splitting energy  $\Delta E_{GM}$  on the nuclear deformation parameters  $\delta$  and  $\alpha_{40}$ .

In the next step of the analysis deformation parameters  $\delta$  and  $\alpha_{40}$  were changed and calculations for  $\delta = 0.18 - 0.30$  and  $\alpha_{40} = -0.04 - 0.04$  were done. The  $\Delta N = 2$ interaction was again omitted. Typical strong dependence of the G-M splitting energy  $\Delta E_{\rm GM}$  on deformation parameters is shown on Fig. 1 for configuration  $\{5/2 + [402]_{\rm p}, 3/2 - [512]_{\rm n}\}$ . Similar results for other configurations show that calculated values  $\Delta E_{\rm GM}^{\rm th}$  differ in considered region of deformation parameters by more than 25% of average value. Therefore calculation of the G-M splitting in definite odd-odd nucleus has to be performed with proper deformation parameters.

#### 3.1.2. Inclusion of the $\Delta N = 2$ interaction

In the Nilsson model calculations the matrix elements between main N-shels (the  $\Delta N = 2$  interaction) are usually neglected except some rather special cases (see e.g. [9]). Nevertheless, as was shown in [28], the influence of the  $\Delta N = 2$  interaction on some nuclear properties (e.g. transition probabilities, reaction cross sections etc.) is rather general feature of the Nilsson model. Therefore we tried to calculate the G-M splitting energy,  $\Delta E_{\rm GM}$ , in a more accurate model including the  $\Delta N = 2$  matrix elements. Matrix elements  $A_0$  and  $A_{\sigma}$  (Eqs. (13)) were calculated with the wave functions, calculated with computer programme written on the Department of nuclear physics of MFF UK in Prague [29]. Using average value of parameter D = 789 keV the splitting energy  $\Delta E_{\rm GM}^{\rm th}$  can be calculated from Eq. (15) as a function of amplitude of the N-mixing. Because the calculations are rather tedious and memory consuming, no more than three N-shells were included and the calculations performed only for a few states should be considered as a rather pre-liminary. A sample of typical results is shown in Tab. 4 for three different con-

Configuration	N <sub>p</sub>	N <sub>n</sub>	⊿E <sup>th</sup> (keV)
${5/2 + [402]_p, \ 1/2 - [510]_n}$	4	5	29.5
	4	5, 7, 9	15.7
	4, 6, 8	5, 7, 9	16.0
$ \begin{cases} 5/2 + [402]_{p}, \\ 3/2 - [512]_{n} \end{cases} $	4	5	86.9
	4	5, 7, 9	77.9
	4, 6, 8	5, 7, 9	83.8
$ \begin{cases} 5/2 + [402]_{p}, \\ 7/2 - [503]_{n} \end{cases} $	4	5	230.3
	4	5, 7, 9	208.6
	4, 6, 8	5, 7, 9	194.9

Table 4. Influence of the  $\Delta N = 2$  interaction on the calculated splitting energy  $\Delta E_{GM}^{th}$ 

figurations. The principal quantum number N = 4, 6 and 8 for proton and N = 5, 7 and 9 for neutron states was considered in the calculations while the deformation parameters  $\delta = 0.18$ ,  $\alpha_{40} = 0.04$  were used everywhere. As is seen from the table, the changes in the splitting energy  $\Delta E_{GM}^{th}$  are remarkable and seem to be strongly dependent on the configuration. Although performed calculations are rather preliminary, it is obvious that exact analysis of the G-M splitting in odd-odd nuclei in frame of the Nilsson model cannot be performed without the  $\Delta N = 2$  interaction to be included.

#### 3.2. Comparison of calculated and experimental splitting energies

For all states experimentally established in rare earth odd-odd deformed nuclei (see Tab. 1) we have calculated the splitting energy  $\Delta E_{GM}$  using Eq. (15). Average value of common parameter D = 789 keV was used everywhere. Nilsson model parameters  $\varkappa$  and  $\mu$  for considered N-shells and regions of nuclei were taken from Solovjev's monography [9] (see Tab. 3) alike as deformation parameters  $\delta$  and  $\alpha_{40}$  proper for each nucleus. Neither the  $\Delta N = 2$  nor nondiagonal Coriolis interaction matrix elements were considered in the calculations. Obtained values of splitting energy,  $\Delta E_{GM}^{th}$ , are given in eight column of Tab. 1.

Before we will analyse agreement between calculated and experimental splitting energies some notes should be done. First, if should be considered the Coriolis interaction neglected in our analysis, the calculated energies could be shifted with respect to  $\Delta E_{GM}^{th}$  in Tab. 1 by values reaching up to a few hundreds of keV. More, corresponding shift should be rather strongly dependent on the proton and neutron states as well as on the shape of the nucleus. Obviously, if Coriolis interaction is correctly included, the calculated energies should be shifted to the experimental ones.

Second note concerns the  $\Delta N = 2$  interaction discussed in part 3.1.2. Although this interaction is expected to be generally less important than Coriolis one, it can remarkably affect the matrix elements  $A_0$  and  $A_{\sigma}$  what can leed to substantial change of calculated splitting energies  $\Delta E_{GM}^{th}$ .

Agreement between calculated  $(\Delta E_{GM}^{th})$  and experimental  $(\Delta E_{GM}^{exp})$  splitting energies included in Tab. 1 is similar as in Refs. [2] and [4]. It has to be emphasized that the number of the G-M pairs included in our analysis is much higher than in [2] and [4] and, with respect to method of analysis, some substantial differences between  $\Delta E_{GM}^{th}$  and  $\Delta E_{GM}^{exp}$  are sufficiently probable. More, some substantial disagreement may be connected with the only tentative assignment of experimentally observed G-M pairs (reliability "3" in Tab. 1).

With respect to these facts and to precedent notes the agreement between experimental and calculated values of  $\Delta E_{GM}$  for analysed G-M pairs is rather good. Systematical behaviour of splitting energy exhibits some important features: i: The splitting energy  $\Delta E_{GM}$  for different configuration in definite nucleus differs generally very remarkably. ii: For the same configuration the splitting energy changes rather weakly from nucleus to nucleus. iii: The difference between calculated and experimental splitting energies is more remarkable for the pairs with  $\Delta K = K_{+} - K_{-} = 1$  than for other pairs. Performed analysis of experimental informations on the G-M pairs shows that violation of these rules for experimental splitting energies is usually connected with tentative assignment of the observed states.

First two features are in agreement with expected fact that the G-M splitting is mostly determined by the configuration of both odd particles in odd-odd nuclei. The properties of individual nucleus affect then the splitting energy only weakly (e.g. over the nuclear shape, spacing of individual states etc.). Last feature is obviously connected with Coriolis interaction between both states of the G-M pair. It supports the importance of the Coriolis interaction for odd-odd deformed nuclei and shows the shortcoming of the theoretical analysis of odd-odd nuclei, if this interaction is neglected. Nevertheless, the study of the systematics in the G-M splitting should be supported by more wide analysis of dependence of the splitting energy on nuclear mass number A which should give also other valuable informations about quasiparticle states in odd-odd deformed nuclei. Unfortunately, performance of such analysis in present work was impossible because each configuration was usually not observed in more than three nuclei. Therefore further experimental studies of odd-odd nuclei are very desirable.

#### 3.3. Method of separated parameters

## 3.3.1. Model dependence of the parameter $\alpha$

Analysis in parts 3.1 and 3.2 was performed with common parameter  $D = \alpha$ . W. Nevertheless, if "method of separated parameters" is used for determination of parameters  $\alpha$  and W(see Eqs. (16) and (17)), model calculations of band head energies of both G-M components can be done independently. More, if the value of coefficient k in (16) is established by model independent method, as was done by Singh



Fig. 2. Dependence of the splitting parameter  $\alpha$  on the nuclear deformation parameters  $\delta$  and  $\alpha_{AO}$ .

and Sood [6], parameter  $\alpha$  can be examined as a function of nuclear shape and other parameters.

In the present paper we have calculated the parameter  $\alpha$  from Eq. (16) as a function of deformation parameters  $\delta$  and  $\alpha_{40}$ . Parameters  $\varkappa$  and  $\mu$  of the Nilsson potential were kept constant everywhere and were the same as in part 3.1. Typical behaviour of parameter  $\alpha$  is shown in Fig. 2 for the  $\{5/2 + \lfloor 402 \rfloor_p, 3/2 - \lfloor 512 \rfloor_n\}$  configuration. The value of  $\alpha$  for different nuclear shape differs rather substantially, the differences attaining more than 20% of average value of  $\alpha$  for deformation parameters corresponding to rare earth deformed nuclei. Therefore calculation of parameter  $\alpha$ for the G-M pairs in fixed nucleus should be performed with proper nuclear deformation parameters.

Parameters W for considered G-M pairs may be calculated from Eq. (17). Beside the dependence on model parameters through parameter  $\alpha$  and matrix element  $A_{\sigma}$  parameter W depends also on experimental values of splitting energy  $\Delta E_{\rm GM}^{\rm exp}$ . But, as was shown in precedent parts, the values of  $\Delta E_{\rm GM}^{\rm exp}$  reflect all effects not included in the model calculations and therefore the dependence of the parameter W on the model parameters cannot be directly extracted. From the same reason the common parameter  $D = \alpha$ . W calculated as a product of parameters  $\alpha$  and W established independently from Eqs. (16) and (17) should generally differ from the average value established experimentally in [2] and [4] and used in part 3.1 and 3.2 of present paper. On the other hand, the deviation of calculated values of D from average one may be used as a valuable information about importance of neglected effects for definite G-M pair in definite nucleus.

## 3.3.2. Systematical behaviour of the splitting parameters

As the last part of our analysis we have calculated parameters  $\alpha$  and W for all experimentally observed G-M pairs. Matrix elements  $A_0$  and  $A_{\sigma}$  for each nucleus were calculated with proper deformation parameters taken from [9]. Parameter W was calculated from the splitting energy  $\Delta E_{\rm GM}^{\rm exp}$  established in part 3.2. Obtained values of  $\alpha$  and W are collected in Tab. 5 for each proton-neutron configuration observed experimentally in at least two nuclei, the results for each configuration being presented separately. In third and fourth columns are given the deformation parameters  $\delta$  and  $\alpha_{40}$  used for individual nuclei. Corresponding values of common parameter  $D = \alpha$ . W calculated as product of  $\alpha$  and W are in seventh column of Tab. 5.

As is seen from the table, the parameters  $\alpha$  and W for the same p-n configuration in different nuclei are generally rather close while the parameters for different configurations differ remarkably enough even in the same nucleus. More essential deviations from this rule are connected mainly with tentative assignment of experimentally observed states. Very close values of  $\alpha$ , calculated from Eq. (16) for given p-n configuration in different nuclei, reflect slow variation of matrix elements  $A_0$  and  $A_{\sigma}$ as a function of the deformation parameters. It is substantial, that in the model used

Nucleus	δ	α <sub>40</sub>	α	W [MeV]	α. W [MeV]	Note
		{3/2	[411] <sub>p</sub> , 3/2 [5	21] <sub>n</sub> }		
<sup>156</sup> Tb	0.26	0.05	0.230	3.37	0.77	
<sup>158</sup> Tb	0.27	0.04	0.226	3.60	0.82	
<sup>160</sup> Tb	0.27	0.04	0.228	3.42	0.78	
		{7/2	[523] <sub>p</sub> , 3/2 [5	21] <sub>n</sub> }		
<sup>158</sup> Ho	0.27	0.04	0.414	10.05	4.16	
<sup>164</sup> Ho	0.27	0.02	0.434	11.57	5.02	
		{7/2	[523] <sub>p</sub> , 1/2 [5	21] <sub>n</sub> }		
<sup>166</sup> Ho	0.27	0.02	0.450	12.30	5.53	
170 Tm	0.27	-0.01	0.450	0.73	0.32	ь)
		{7/2	$[404]_n, 3/2[52]$	21] <sub>n</sub> }		
<sup>158</sup> Tb	0.27	0.04	0.363	3.61	1.31	
<sup>174</sup> Lu	0.26	-0.03	0.443	5.07	2.25	·
		{7/2	[404] <sub>p</sub> , 7/2 [63	33] <sub>n</sub> }		
<sup>166</sup> Ho	0.27	0.01	0.419	1.45	0.61	<sup>a</sup> )
<sup>174</sup> Lu	0.26	-0.03	0.218	6.04	1.31	
		{7/2	[404] <sub>p</sub> , 1/2 [52	21] <sub>n</sub> }		
<sup>170</sup> Tm	0.27	-0.01	0.199	7.19	1.43	
<sup>170</sup> Lu	0.25	-0.01	0.203	0.68	0.14	
172Lu	0.26	-0.02	0.200	3.91	0.78	
<sup>1</sup> / <sup>4</sup> Lu	0.26	-0.03	0.200	3.96	0.79	
та Та	0.24	-0.05	0.206	3.76	0.77	
		{7/2	[404] <sub>p</sub> , 5/2 [51	2] <sub>n</sub> }		
<sup>174</sup> Lu	0.26	-0.03	0.334	6.78	2.26	
<sup>180</sup> Ta	0.24	-0.05	0.374	5.88	2.20	
		{7/2	[404] <sub>p</sub> , 7/2 [51	4] <sub>n</sub> }		
<sup>176</sup> Lu	0.26	-0.04	0.211	6.35	1.34	
<sup>178</sup> Lu	0.26	-0.04	0.211	3.35	0.71	<sup>a</sup> )
<sup>180</sup> Ta	0.24	-0.05	0.214	7.16	1.53	
		{7/2 [	404] <sub>p</sub> , 9/2 [62	4] <sub>n</sub> }		
<sup>76</sup> Lu	0.26	-0.04	0.193	4.11	0.79	
<sup>i 80</sup> Ta	0.24	-0.05	0.194	3.62	0.70	

Table 5. The splitting parameters  $\alpha$  and W

Nucleus	δ	α <sub>40</sub>	α	<i>W</i> [MeV]	α. W [MeV]	Note
		{7/2	[404] <sub>p</sub> , 1/2 [5	10] <sub>n</sub> }		
<sup>176</sup> Lu	0.26	-0.04	0.315	13.18	4.15	
<sup>180</sup> Ta	0.24	-0.05	0.293	12.31	3.61	
<sup>182</sup> Ta	0.24	-0.05	0.293	8.91	2.61	
		{7/2	[404] <sub>p</sub> , 3/2 [5	12] <sub>n</sub> }		
<sup>180</sup> Ta	0.24	-0.05	0.560	7.41	4.15	
<sup>182</sup> Ta	0.24	-0.05	0.560	11.21	6.27	
		{1/2	[411] <sub>p</sub> , 5/3[5	23] <sub>n</sub> }		
<sup>166</sup> Ho	0.27	0.01	0.331	5.44	1.80	
<sup>168</sup> Tm	0.26	0.00	0.341	3.42	1.16	<b>a</b> )
		{1/2	[411] <sub>p</sub> 7/2 [6	33] <sub>n</sub> }		
<sup>166</sup> Ho	0.27	0.01	0.303	4.81	1.46	
<sup>168</sup> Tm	0.26	0.00	0.307	5.65	1.73	<sup>a</sup> )
		{1/2	[411] <sub>p</sub> , 1/2 [5	21] <sub>n</sub> }		
<sup>168</sup> Tm	0.26	0.00	0.252	4.88	1.22	<b>a</b> )
<sup>170</sup> Tm	0.27	-0.01	0.248	4.61	1.14	,
<sup>170</sup> Lu	0.25	-0.01	0.256	4.81	1.23	
<sup>172</sup> Tm	0.28	-0.02	0.244	2.38	0.58	<b>a</b> )
		{1/2	[411] <sub>p</sub> , 5/2 [5	12] <sub>n</sub> }		
<sup>168</sup> Tm	0.26	0.00	0.208	5.00	1.04	<sup>a</sup> )
<sup>170</sup> Tm	0.27	-0.01	0.205	5.03	1.03	
<sup>172</sup> Tm	0.28	-0.02	0.203	4.83	0.98	
		{1/2	[411] <sub>p</sub> , 1/2 [5	21] <sub>n</sub> }		
<sup>170</sup> Lu	0.25	-0.01	0.567	5.70	3.23	<b>a</b> )
<sup>172</sup> Lu	0.26	-0.02	0.543	3.36	1.82	<sup>9</sup> )
		{5/2	[402] <sub>p</sub> , 1/2 [5	21] <sub>n</sub> }		
<sup>170</sup> Tm	0.27	-0.01	0.215	6.29	1.35	
<sup>172</sup> Lu	0.26	-0.02	0.217	3.84	0.83	
		{5/2	[402] <sub>p</sub> , 1/2 [5	10] <sub>n</sub> }	·····	
<sup>182</sup> Ta	0.24	0.05	0.310	6.80	2.11	<b>a</b> )
<sup>184</sup> Re	0.22	-0.05	0.335	4.78	1.60	,
<sup>186</sup> Re	0.20	-0.05	0.369	7.58	2.03	
188 <sub>D</sub>	0.18	-0.05	0.416	5.01	2 16	

Table 5. (Continued 1)

Nucleus .	δ	$\alpha_{40}$	α	W [MeV]	α. W [MeV]	Note
		{5/2	[402] <sub>p</sub> , 3/2 [5	512] <sub>n</sub> }		
<sup>184</sup> Re	0.22	-0.05	0.229	6.59	1.51	
<sup>186</sup> Re	0.20	-0.05	0.238	4.13	0.98	
<sup>188</sup> Re	0.18	-0.05	0.248	4.78	1.18	
		{5/2 [	402] <sub>p</sub> , 11/2 [	615] <sub>n</sub> }		
<sup>184</sup> Re	0.22	-0.05	0.195	9.51	1.88	<sup>a</sup> )
<sup>186</sup> Re	0.20	-0.05	0.195	8.27	1.61	,
		{5/2	[402] <sub>p</sub> , 7/2 [5	603] <sub>n</sub> }		
<sup>186</sup> Re	0.20	-0.05	0.156	3.52	0.55	
<sup>188</sup> Re	0.18	-0.05	0.157	3.58	0.56	
		{9/2	[514] <sub>p</sub> , 3/2 [5	12] <sub>n</sub> }		
<sup>182</sup> Ta	0.24	-0.05	0.289	4.87	1.41	a)
<sup>186</sup> Re	0.20	-0.05	0.329	8.26	2.72	<sup>a</sup> )
		{9/2	[514] <sub>p</sub> , 1/2 [5	10] <sub>n</sub> }		
<sup>182</sup> Ta	0.24	-0.05	0.615	40.30	24.78	
<sup>186</sup> Re	0.20	-0.05	0.463	30.27	14.01	<b>a</b> )

Table 5. (Continued 2)

<sup>a</sup>) Experimental splitting energy is of reliability "3" in Tab. 1.

<sup>b</sup>) Extremally low value of  $\Delta E_{GM}^{exp}$  indicates probably noncorrect interpretation of the G-M pair.

for present analysis (part 2.1)) parameter  $\alpha$  is calculated with simple single particle Nilsson wave functions. On the other hand, parameter W calculated from Eq. (17) depends on the experimental separation energy  $\Delta E_{GM}^{exp}$  and reflects all effects neglected in the model description used in the analysis. Here  $\Delta N = 2$  and especially Coriolis interactions in nucleus can generally change band heads energies by a value up to few hundreds keV and corresponding shifts should be transferred to the experimental energies  $\Delta E_{GM}^{exp}$ . Therefore greater dispersion of W for definite configuration is well in agreement with model assumptions. More, from values of W, calculated from  $\Delta E_{GM}^{exp}$  it can be in principle extracted some information on the neglected effects. Nevertheless, extraction should be very complicated and, with respect to many uncertainties about structure of odd-odd deformed nuclei it should be rather sophistic. In present work no attempt in this direction was done.

Some note should be done about the comon parameter  $D = \alpha$ . W. As is seen from Tab. 5, the values calculated directly as the product of  $\alpha$  and W exhibit smaller variations than parameter W, nevertheless, the dependence on the p-n configuration

is well pronounced. The average value over all rare-earth region is close to 1600 keV, what is about two times higher than the values used by Boisson et al. [2] and Elmore and al. [4]. In [2] and [4] was in some extent considered the Coriolis interaction what made it possible to consider corresponding values of D as an lower limit of the common parameter for unperturbed states. Nevertheless, for the more realistic approximate calculations higher value of  $\alpha$ . W should be used.

## 4. Conclusions

Analysis, performed in the present work shows that irrespective to high complexity of the structure of odd-odd deformed nuclei many substantial features of excited states are connected with the simple particle (quasiparticle) degrees of freedom. Beside the method, using average common parameter  $D = \alpha \cdot W$ , the "method of separated parameters" appears to be very convenient step in the analysis of experimental material, giving first information about approximate energies of expected states in fixed nucleus. Our analysis, based on wide experimental material, fully proved rough acceptability of the method at least for the rare earth region of nuclei. Usefulness of the method in two directions should be emphasized. First, the possibility to calculate expected energies of the band head states of unknown G-M pairs for given nucleus using proton and neutron one-particle energies in neighbour odd-A nuclei and average values of parameters  $\alpha$  and W established for considered p-n configuration in other odd-odd nuclei. Second, expected energies of the band head states can be taken as a starting information for more accurate analysis including such important effects in deformed nuclei as is Coriolis interaction.

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