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ADAPTIVE CONTROL OF UNCERTAIN NONHOLONOMIC SYSTEMS IN FINITE TIME

Jiankui Wang, Guoshan Zhang and Hongyi Li

In this paper, the finite-time stabilization problem of chained form systems with parametric uncertainties is investigated. A novel switching control strategy is proposed for adaptive finite-time control design with the help of Lyapunov-based method and time-rescaling technique. With the proposed control law, the uncertain closed-loop system under consideration is finite-time stable within a given settling time. An illustrative example is also given to show the effectiveness of the proposed controller.

Keywords: finite-time convergence, parameter uncertainty, adaptive control, nonholonomic systems

AMS Subject Classification: 93D15, 93D21

1. INTRODUCTION

The stabilization and adaptive control of nonholonomic systems have drawn much research attention in the nonlinear control community over the last few decades and many results have been obtained ([4, 10, 11, 16]). For example, in [4], adaptive state feedback and output feedback control strategies were proposed for a class of uncertain nonholonomic systems in chained form using backstepping techniques. In [11], a constructive adaptive control scheme was reported for a new class of linearly parametrized nonlinear systems by virtue of backstepping and time-varying control techniques. On the other hand, non-smooth finite-time control, which makes the controlled system reach the target in a finite time, provides fast response and high tracking precision, and moreover, shows disturbance-rejection properties. In recent years, some explicitly-constructed continuous (but non-smooth) finite-time controllers for nonlinear systems have been proposed [2, 7, 6, 8, 13, 14, 15].

In [6], Y. Hong gave a class of non-smooth finite time controllers for a class of high order nonlinear systems by homogeneous method. Based on the design method of [6], Y. Hong et al. [9] proposed a novel switching finite time control strategy to nonholonomic systems in chained form with help of time-rescaling, and Lyapunov-based method. The controllers of [9] can only make the chained form systems with uncertain parameters and perturbed terms finite time stable locally because the design methods of controllers rely on the homogeneous method. The purpose of

this paper is to obtain adaptive finite-time stabilization for a class of chained form systems with uncertain parameters and perturbed terms. Inspired by the design method of adaptive control of [8], in this paper we extend the finite-time stabilizing control for nonholonomic systems in [9] to adaptive finite-time stabilization for the chained form systems with uncertain parameters and perturbed terms.

The remainder of this paper is organized as follows. In Section 2, the problem formation and preliminary knowledge are given. In Section 3 adaptive control laws are constructed, using backstepping-like method and time-rescaling technique, to make the closed-loop system finite-time convergent within any given settling time. Moreover, Lyapunov stability of the considered closed-loop system is also discussed. In Section 4, concluding remarks are given.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we consider a class of uncertain chained form system in the following form:

$$\begin{cases} \dot{x}_0 = u_0 + x_0\phi_0(t, x_0, \theta_0), \\ \dot{x}_1 = x_2 u_0 + \phi_1(t, x_0, x_1, \theta), \\ \vdots \\ \dot{x}_{n-1} = x_n u_0 + \phi_{n-1}(t, x_0, x_1, \ldots, x_{n-1}, \theta), \\ \dot{x}_n = u + \phi_n(t, x_0, x, \theta) \end{cases} \tag{1}$$

where $x = (x_1, \cdots, x_n)^T \in \mathbb{R}^n$ denotes the state vector for the n state variables; $\phi_i(t, 0, 0, \ldots, 0) = 0$ for $i = 0, 1, \ldots, n$; $\theta_0 \in \mathbb{R}^p$ and $\theta \in \mathbb{R}^m$ are bounded uncertain parameter vector as assumed by [4]; u_0, u are control inputs. Here we assume $n \geq 2$.

Define

$$\nu = \frac{p_0}{q_0} - 1 < 0, \quad r_i = 1 + (i-1)\nu > 0, \quad i = 1, \ldots, n \tag{2}$$

where $p_0 < q_0$ are two positive odd integers, and define

$$\beta_0 = r_2, \quad (\beta_i + 1)r_{i+1} = (\beta_{i-1} + 1)r_i > 0, \quad i = 1, \ldots, n-1. \tag{3}$$

These notations will be used in the construction of adaptive controllers.

Two assumptions are given for ϕ_i, $(i = 0, \ldots, n)$:

Assumption 2.1. There is a known nonnegative function $b_0(x_0)$ such that

$$|\phi_0(t, x_0, \theta_0)| \leq b_0(x_0).$$

Assumption 2.2. For $1 \leq l \leq i$, $1 \leq i \leq n$,

$$|\phi_i(t, x_0, x_1, \ldots, x_i, \theta)| \leq \sum_{l=1}^{i} |x_l| \bar{\phi}_i(\theta) \leq \sum_{l=1}^{i} |x_l| \sigma$$

with $\sigma \triangleq \max\{\bar{\phi}_i(\theta)^{\frac{4(1+\nu)}{-\nu(1+n\nu)}}, 1\}$, where $\bar{\phi}_i(\theta)$ is nonnegative smooth function.

At first, the concepts about finite-time stability are introduced.

Definition 2.3. Consider a system

$$\dot{x} = f(x,t), \qquad f(0,t) = 0, \quad x \in U_0 \subset \mathbb{R}^n, \tag{4}$$

where $f : U_0 \times \mathbb{R}^+ \to \mathbb{R}^n$ is continuous with respect to x on an open neighborhood U_0 of the origin $x = 0$. The equilibrium $x = 0$ of the system is finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq U_0$ of the origin. By 'finite-time convergence', we mean that, if, for any initial condition $x(t_0) = x_0 \in U$ at any given initial time t_0, there is a settling time $T > 0$, such that every solution $x(t; t_0, x_0)$ of system (4) is defined with $x(t; t_0, x_0) \in U/\{0\}$ for $t \in [t_0, T)$ and

$$\lim_{t \to T} x(t; t_0, x_0) = 0, \quad x(t; t_0, x_0) = 0, \ \forall\, t > T.$$

Next lemma is quite straightforward [2].

Lemma 2.4. Suppose that, for system (4), there is a C^1 function $V(x,t)$ ($V(x,t) = 0$ if and only if $x = 0$), defined on $\hat{U} \times \mathbb{R}$, where $\hat{U} \subset U_0 \subset \mathbb{R}^n$ is a neighborhood of the origin, real numbers $c > 0$ and $0 < \alpha < 1$, such that $V(x,t)$ is positive definite on $\hat{U}$ for any t and $-c_2 V^\alpha(x,t) \le \dot{V}(x,t) \le -c_1 V^\alpha(x,t)$ (along the trajectory) on $\hat{U}$. Then $V(x,t)$ is locally finite-time convergent, or equivalently, becomes 0 locally in finite time, with its settling time $\frac{V(x_0,t_0)^{1-\alpha}}{c_2(1-\alpha)} \le T \le \frac{V(x_0,t_0)^{1-\alpha}}{c_1(1-\alpha)}$ for a given initial condition x_0 in a neighborhood of the origin in $\hat{U}$ at initial time t_0.

The following inequalities are well-known [1].

Lemma 2.5. (Jensen's inequality) For $x_i \ge 0, \ i = 1, \ldots, n$ and $0 < c_1 < c_2$,

$$\left(\sum_{i=1}^{n} x_i^{c_2} \right)^{1/c_2} \le \left(\sum_{i=1}^{n} x_i^{c_1} \right)^{1/c_1}.$$

Lemma 2.6. (Young's inequality)

$$ab \le \frac{a^{1+c}}{1+c} + \frac{c\, b^{1+\frac{1}{c}}}{1+c},$$

for any $a \ge 0, \ b \ge 0, \ c > 0$.

The objective of this paper is that for any given initial condition $(x_0(0), x(0), \hat{\sigma}(0))$, we find two controllers

$$\begin{cases} u_0 = u_0\,(x_0) \\ u = u(x_0, x, \hat{\sigma}) \end{cases} \tag{5}$$

along with update laws

$$\dot{\hat{\sigma}} = \mu_2(x_0, x, \hat{\sigma}) \tag{6}$$

such that the equilibrium $(0, 0, \sigma)$ of the closed-loop system

$$\begin{cases} \dot{x}_0 = u_0 + x_0\phi_0(t, x_0, \theta_0), \\ \dot{x}_1 = x_2 u_0 + \phi_1(t, x_0, x_1, \theta), \\ \vdots \\ \dot{x}_{n-1} = x_n u_0 + \phi_{n-1}(t, x_0, x_1, \ldots, x_{n-1}, \theta), \\ \dot{x}_n = u + \phi_n(t, x_0, x, \theta), \\ \dot{\hat{\sigma}} = \mu_2(x_0, x, \hat{\sigma}). \end{cases} \tag{7}$$

with u_0 and u defined in (5) is Lyapunov stable and $x_0(t) = 0, x(t) = 0,\ \forall\, t \geq T$, where T is any given settling time.

Remark 2.7. When $\phi_i = 0,\ i = 0, 1, \ldots, n-1$ and θ is known, system (1) is the special case of the system discussed in [9]. The controller of [9] can locally finite time stabilize the system (1) because high order perturbed terms $\phi_i = 0,\ (i = 1, 2, \ldots, n-1)$ with the same dilation coefficients as x-subsystem in [9] lead to the homogeneousness of the system only in the local sense, not in the global sense.

3. ADAPTIVE FINITE-TIME STABILIZATION

In this section, we give a constructive procedure for the adaptive finite-time stabilizing control of system (1) within any given settling time T. As usual, we first discuss the problem in a special case when $x_0(0) \neq 0$, and then we extend our result to the case when $x_0(0) = 0$.

3.1. Control for $x_0(0) \neq 0$

For x_0-subsystem, we can take a finite-time control law in the form of

$$u_0 = -k_0 x_0^{\alpha} - |x_0| b_0, \tag{8}$$

where k_0 is a positive design parameter, $0 \leq \alpha = \frac{a_1}{a_2} < 1$ with $a_i,\ i = 1, 2$ being positive odd integers, $u_0 = -k_0 \operatorname{sign}(x_0) - |x_0| b_0$ when $\alpha = 0$.

Take a Lyapunov function $V_0 \triangleq \frac{1}{2} x_0^2$ for system

$$\dot{x}_0 = u_0 + x_0\phi_0(t, x_0, \theta_0). \tag{9}$$

Then

$$-k_0 x_0^{1+\alpha} - 2x_0^2 b_0 \leq \dot{V}_0|_{(9)} = -k_0 x_0^{1+\alpha} - x_0^2 b_0 + x_0\phi_0(t, x_0, \theta_0) \leq -k_0 x_0^{1+\alpha} \leq 0, \tag{10}$$

which implies $|x_0(t)| \leq |x_0(0)|$. Set $\bar{k}_0 = \max_{|x_0(t)| \leq |x_0(0)|} b_0(x_0)$.

From (10), we have

$$-(k_0 + 2|x_0(0)|^{1-\alpha}\bar{k}_0) x_0^{1+\alpha} \leq \dot{V}_0|_{(9)} \leq -k_0 x_0^{1+\alpha}.$$

If we define $\underline{K} \triangleq (k_0 + 2|x_0(0)|^{1-\alpha}\bar{k}_0)$, then we have:

$$-\underline{K} V_0^{\frac{1+\alpha}{2}} \leq \dot{V}_0 \leq -k_0 V_0^{\frac{1+\alpha}{2}}. \tag{11}$$

Thus by Lemma 2.4, x_0 tends to 0 within a settling time denoted by T_0. Moreover,

$$\frac{2V_0^{\frac{1-\alpha}{2}}}{\underline{K}(1-\alpha)} \leq T_0 \leq \frac{2V_0^{\frac{1-\alpha}{2}}}{k_0(1-\alpha)}. \tag{12}$$

To secure finite-time convergence within T for any $x_0(0) \neq 0$, we need to keep $T_0 \leq \frac{2V_0^{\frac{1-\alpha}{2}}}{k_0(1-\alpha)} < T$ by taking $k_0 > \frac{2V_0^{\frac{1-\alpha}{2}}}{T(1-\alpha)}$.

Remark 3.1. For the one-dimensional system $\dot{x}_0 = u_0 + x_0\phi_0$, when $x_0(t_0) \neq 0$, the trajectory $x_0(t; x_0(t_0), t_0)$ (or $x_0(t)$ for short) of the system satisfies: $x_0(t_0) \cdot x_0(t) > 0,\ t < T_0$. Namely, the state $x_0(t)$ cannot becomes 0 when $t < T_0$. Take $T_* < \frac{2V_0^{\frac{1-\alpha}{2}}}{\underline{K}(1-\alpha)} \leq T_0 < T$, and then

$$x_0(0) \cdot x_0(t) > 0, \quad t \in [0, T_*]. \tag{13}$$

On the one hand, from(11),

$$-\frac{1-\alpha}{2}\underline{K}\,\mathrm{d}t \leq \mathrm{d}V_0^{\frac{1-\alpha}{2}}.$$

Integrating the above inequality from 0 to T_*, we have

$$0 < \left(\frac{{x_*}^2}{2}\right)^{\frac{1-\alpha}{2}} \triangleq V_0(0)^{\frac{1-\alpha}{2}} - \frac{1-\alpha}{2}\underline{K}T_* \leq V_0(T_*)^{\frac{1-\alpha}{2}},$$

which yields $0 < x_* \leq |x_0(T_*)|$.

On the other hand, according to $\dot{V}_0 \leq -k_0 V_0^{\frac{1+\alpha}{2}} \leq 0$,

$$|x_0(T_*)| \leq |x_0(t)| \leq |x_0(0)|,\ t \in [0, T_*].$$

Therefore, it is not difficult to get

$$x_* \leq |x_0(t)| \leq |x_0(0)|,\ t \in [0, T_*].$$

Note that controller (8) guarantees that $u_0(t) \neq 0$ when $0 \leq t \leq T_*$, then we have

$$0 < \underline{u}_0 \triangleq k_0 x_*^{\alpha} + x_* \min_{x_* \leq |x_0(t)| \leq |x_0(0)|} b_0 \leq |u_0| \leq k_0 |x_0(0)|^{\alpha} + |x_0(0)|\bar{k}_0 \triangleq \overline{u}_0.$$

Remark 3.2. For x_0-subsystem of system (1), the case with $\phi_0 = 0$ is the special case of x_0-subsystem discussed in [9]. In [9] the main work lies on how to estimate the uncertain coefficient term of u_0, however in this paper we mainly work on the estimation of perturbed term ϕ_i.

Then the task is completed if we can adaptively stabilize the time-varying x-subsystem within the given settling time T_*:

$$\begin{cases} \dot{x}_1 = x_2 u_0 + \phi_1(t, x_0, x_1, \theta), \\ \vdots \\ \dot{x}_{n-1} = x_n u_0 + \phi_{n-1}(t, x_0, x_1, \ldots, x_{n-1}, \theta), \\ \dot{x}_n = u + \phi_n(t, x_0, x, \theta) \end{cases} \tag{14}$$

If u_0 is a known constant, then the system (14) has been solved in [8]. Here u_0 is unknown but it is bounded and will never be zero (when $t \in [0, T_*]$). Therefore, with almost the same idea, we can extend the design procedure given in [8] to this uncertain time-varying system (14). In the following, a procedure (almost the same as in [8]) to construct an adaptive finite-time control for system (14) is only briefly introduced (the detailed construction can be found in [8]).

The control law can be given as $u = v_n$ in a recursive form as follows

$$v_0 = 0, \quad v_j = -\frac{\text{sign}(u_0)}{\underline{u}_0} w_j^{\frac{r_j+\nu}{r_j\beta_{j-1}}} \Phi_j, \quad 1 \le j \le n-1, \quad v_n = -w_n^{\frac{r_n+\nu}{r_n\beta_{n-1}}} \Phi_n \tag{15}$$

with

$$w_1 = x_1^{1+\nu}, \; w_j \triangleq x_j^{\beta_{j-1}} - v_{j-1}(x_1, \ldots, x_{j-1}, \hat{\sigma})^{\beta_{j-1}}, \quad 2 \le j \le n, \tag{16}$$

where $\hat{\sigma}$ is the estimate of σ, $\Phi_j(x, \hat{\sigma})$ is a C^1 positive function ($1 \le j \le n$), which will be determined later. For convenience, we also define

$$Q_j = (|w_1|^{\frac{2(1+\nu)}{r_2}} + \ldots + |w_j|^{\frac{(2(1+\nu))}{r_j\beta_{j-1}}})^{\frac{1}{2(1+\nu)}}. \tag{17}$$

In what follows, we consider the adaptive finite-time control design for system (14), consistent with the procedures given in [8].

Step 1: Consider system

$$\dot{x}_1 = x_2 u_0 + \phi_1(t, x_0, x_1, \theta), \tag{18}$$

where $|\phi_1| \le |x_1|\sigma$. Take $v_1 = -\frac{\text{sign}(u_0)}{\underline{u}_0} w_1 \Phi_1 = -\frac{\text{sign}(u_0)}{\underline{u}_0} x_1^{1+\nu} \Phi_1$, where $\Phi_1 = \frac{2+\nu}{2} - \frac{\nu}{2}\hat{\sigma}^{\frac{2}{-\nu}} x_1^2 + l_1$ is C^1 according to $-\frac{2}{k} > 2n$ (because $r_n + \nu = 1 + n\nu > 0$). Note that $\frac{2+\nu}{2} - \frac{\nu}{2}\hat{\sigma}^{\frac{2}{-\nu}} x_1^2 \ge x_1^{-\nu}\hat{\sigma}$ by Young's inequality and $\beta_0 = 1 + \nu$ from (3), then we have

$$x_1^{r_2} u_0 v_1 \le -x_1^{2(1+\nu)} \Phi_1 \le -l_1 x_1^{2(1+\nu)} - \hat{\sigma} x_1^{2+\nu}. \tag{19}$$

Take $L_1 = l_1$, $V_1^* = W_1 \triangleq \frac{x_1^{2+\nu}}{2+\nu}$(noting that $2 + \nu = \frac{p_0}{q_0} + 1$ according to (2), V_1^* is nonnegative), and $V_1 = V_1^* + \frac{\tilde{\sigma}^2}{2}$, where $\tilde{\sigma} = \sigma - \hat{\sigma}$. Obviously, $\dot{\tilde{\sigma}} = -\dot{\hat{\sigma}}$ because σ is a constant. Then by (19) we have

$$\dot{V}_1 \le \overline{u}_0 |x_1^{r_2}||x_2 - v_1| - L_1 x_1^{2(1+\nu)} + |x_1^{2+\nu}|\tilde{\sigma} - \dot{\hat{\sigma}}\tilde{\sigma}$$

$$\le -L_1 x_1^{2(1+\nu)} + \overline{u}_0 |w_1||x_2 - v_1| + (\tilde{\sigma} + \eta_1)[\dot{\tilde{\phi}}_1 - \dot{\hat{\sigma}}], \tag{20}$$

where $\eta_1 = 0$ and $\tilde{\phi}_1 = |x_1|^{2+\nu}$ is C^1.

After Step $j-1$ ($2 \le j \le n$): For system

$$\begin{cases} \dot{x}_1 = x_2 u_0 + \phi_1, \\ \vdots \\ \dot{x}_{j-1} = x_j u_0 + \phi_{j-1}, \end{cases} \tag{21}$$

we assume that we already have

(i) $v_i^{\beta_i}$ with v_i defined in (15), is C^1. Moreover, $\Phi_i(x,\hat{\sigma})$ is positive and C^1, for $1 \le i \le j-1$.

(ii) There is a C^1 function $\rho_{i,l} \ge 0$, for any given $1 \le i \le l \le j-1$, such that $|\frac{\partial v_l^{\beta_l}}{\partial x_i}| \le Q_l^{(r_l+\nu)\beta_l - r_i}(x)\rho_{i,l}(x,\hat{\sigma})$; Meanwhile, there is a C^1 nonnegative function $\hat{v}_i$ for $1 \le i \le j-1$ such that $|\frac{\partial v_i^{\beta_i}}{\partial \hat{\sigma}}| \le \hat{v}_i(x,\hat{\sigma})$.

(iii) For the function $V_{j-1} = V_{j-1}^* + \tilde{\sigma}^2/2$, we have

$$\dot{V}_{j-1} \le -L_{j-1}Q_{j-1}^{2(1+\nu)} + \overline{u}_0|w_{j-1}||x_j - v_{j-1}| + [\tilde{\sigma} + \eta_{j-1}][\tilde{\phi}_{j-1} - \dot{\hat{\sigma}}]. \tag{22}$$

In Step 1, the above three conditions can be verified easily. All of them will also be checked later in Step j.

Step j: It is time to consider system

$$\begin{cases} \dot{x}_1 = x_2u_0 + \phi_1, \\ \vdots \\ \dot{x}_{j-1} = x_ju_0 + \phi_{j-1}, \\ \dot{x}_j = x_{j+1}u_0 + \phi_j. \end{cases} \tag{23}$$

Set

$$W_j(x) \triangleq \int_{v_{j-1}}^{x_j} w_j(x_1,\ldots,x_{j-1},s)\,\mathrm{d}s = \int_{v_{j-1}}^{x_j} [s^{\beta_{j-1}} - v_{j-1}^{\beta_{j-1}}]\,\mathrm{d}s, \quad 2 \le j \le n \tag{24}$$

is nonnegative and even positive when $x_j \ne v_{j-1}(x_1,\ldots,x_{j-1},\hat{\sigma})$, In fact, we can divide it into two cases: when $x_j \ge v_{j-1}$, based on $[s^{\beta_{j-1}} - v_{j-1}^{\beta_{j-1}}] \ge 0$, we have $W_j \ge 0$; when $x_j \le v_{j-1}$, $W_j = \int_{v_{j-1}}^{x_j}[s^{\beta_{j-1}} - v_{j-1}^{\beta_{j-1}}]\,\mathrm{d}s = \int_{x_j}^{v_{j-1}}[v_{j-1}^{\beta_{j-1}} - s^{\beta_{j-1}}]\,\mathrm{d}s \ge 0$. Moreover, W_j is C^1 owing to Assumption (i) in Step $j-1$. Then we can construct C^1 and positive-definite function (with respect to $x_1,\ldots,x_j$ and $\tilde{\sigma}$):

$$V_j = V_j^* + \frac{1}{2}\tilde{\sigma}^2 = V_{j-1}^* + W_j(x,\hat{\sigma}) + \frac{1}{2}\tilde{\sigma}^2. \tag{25}$$

Let us consider the derivative of V_j:

$$\dot{V}_j \le -L_{j-1}Q_{j-1}^{2(1+\nu)} + \overline{u}_0|w_{j-1}||x_j - v_{j-1}| + \sum_{i=1}^{j}\frac{\partial W_j}{\partial x_i}(x_{i+1}u_0 + \phi_i)$$

$$+(\tilde{\sigma} + \eta_{j-1})\tilde{\phi}_{j-1} - \left(\eta_{j-1} - \frac{\partial W_j}{\partial \hat{\sigma}}\right)\dot{\hat{\sigma}} - \tilde{\sigma}\dot{\hat{\sigma}}.$$

Then we analyze each term on the right hand side of the above inequality.

i) By virtue of Young's inequality, we have

$$\overline{u}_0|w_{j-1}||x_j - v_{j-1}| \le 2\overline{u}_0|w_{j-1}||w_j|^{\frac{1}{\beta_{j-1}}} \le \frac{L_{j-1}}{3n}Q_{j-1}^{2(1+\nu)} + l_j^0|w_j|^{\frac{2(1+\nu)}{r_j\beta_{j-1}}}, \tag{26}$$

where l_j^0 is a positive constant depending on k, L_{j-1} and $\overline{u}_0$.

ii) There exist C^1 nonnegative functions $\tilde{\psi}_j$ and $\tilde{\gamma}_j$ such that

$$\sum_{i=1}^{j}\frac{\partial W_j}{\partial x_i}(x_{i+1}u_0+\phi_i) \leq \frac{L_{j-1}}{3n}Q_j^{2(1+\nu)}+|w_j|^{\frac{2(1+\nu)}{r_j\beta_{j-1}}}[\tilde{\psi}_j+\sigma_2\tilde{\gamma}_j] \tag{27}$$

$$+ \overline{u}_0|w_j||x_{j+1}-v_j|+w_ju_0v_j, \tag{28}$$

where $\tilde{\gamma}_j(0,\hat{\sigma}_2)=0$.

iii) Similarly, we can obtain that, for $\forall\, j\geq 1$, there exists a C^1 positive function $\bar{\phi}_j$ satisfying

$$\tilde{\phi}_{j-1}(\tilde{\sigma}+\eta_{j-1})-(\eta_{j-1}-\frac{\partial W_j}{\partial\hat{\sigma}})\dot{\hat{\sigma}}\leq\frac{L_{j-1}}{3n}Q_{j-1}^{2(1+\nu)}$$

$$+|w_j|^{\frac{2(1+\nu)}{r_j\beta_{j-1}}}\bar{\phi}_j-|w_j|^{\frac{2(1+\nu)}{r_j\beta_{j-1}}}\tilde{\gamma}_j\tilde{\sigma}+(\tilde{\sigma}+\eta_j)\tilde{\phi}_j-\eta_j\dot{\hat{\sigma}}, \tag{29}$$

where $\eta_j=\eta_{j-1}-\frac{\partial W_j}{\partial\hat{\sigma}}$, $\tilde{\phi}_j=\tilde{\phi}_{j-1}+|w_j|^{\frac{2(1+\nu)}{r_j\beta_{j-1}}}\tilde{\gamma}_j$.

Then we can construct

$$v_j=-\frac{\mathrm{sign}(u_0)}{\underline{u}_0}w_j^{\frac{r_j+\nu}{r_j\beta_{j-1}}}\Phi_j, \tag{30}$$

where

$$\Phi_j(x,\hat{\sigma})=l_j+\frac{L_{j-1}}{3n}+l_j^0+\tilde{\psi}_j(x,\hat{\sigma})+\bar{\phi}_j(x,\hat{\sigma})+(1+\hat{\sigma}^2)^{\frac{1}{2}}\tilde{\gamma}_j(x,\hat{\sigma}),\quad j\geq 2$$

is positive. Obviously,

$$u_0w_jv_j+|w_j|^{\frac{2(1+\nu)}{r_j\beta_{j-1}}}\left(\frac{L_{j-1}}{3n}+l_j^0+\tilde{\psi}_j+\bar{\phi}_j+\hat{\sigma}\tilde{\gamma}_j\right)\leq -l_j|w_j|^{\frac{2(1+\nu)}{r_j\beta_{j-1}}}.$$

Therefore, putting (26), (27) and (29) together leads to

$$\dot{V}_j|_{(23)}\leq -L_jQ_j^{2(1+\nu)}+\overline{u}_0|w_j||x_{j+1}-v_j|+(\tilde{\sigma}+\eta_j)(\dot{\tilde{\phi}}_j-\dot{\hat{\sigma}}), \tag{31}$$

where $L_j=\min\{\frac{n-1}{n}L_{j-1},l_j\}$, which is consistent with condition (iii).

Before the end of Step j, we need to verify the three assumptions listed in Step $j-1$ for Step j. Φ_j is C^1 because $\tilde{\psi}_j$, $\bar{\phi}_j$, $\tilde{\gamma}_j$ are so. Therefore, $v_j^{\beta_j}$ is C^1 because w_j is C^1 and $(r_j+\nu)\beta_j\geq r_j\beta_{j-1}$. Therefore, condition (i) given in Step $j-1$ are still valid in Step j.

Moreover, by induction, it is not hard to confirm condition (ii) for Step j, namely, there are C^1 nonnegative functions $\rho_{i,j}$ and $\hat{v}_j$ for any given $1\leq i\leq j$ such that

$$\left|\frac{\partial v_j^{\beta_j}}{\partial x_i}\right|\leq Q_j^{(r_j+\nu)\beta_j-r_i}\rho_{i,j}(x,\hat{\sigma}),\quad \left|\frac{\partial v_j^{\beta_j}}{\partial\hat{\sigma}}\right|\leq\hat{v}_j. \tag{32}$$

Up to Step n: Take

$$V_n = V_n^* + \frac{1}{2}\tilde{\sigma}^2 = \sum_{i=1}^{n} W_i + \frac{1}{2}\tilde{\sigma}^2 \tag{33}$$

which is positive definite with respect to $x_1, \ldots, x_n, \tilde{\sigma}$, and the adaptive control law

$$\begin{cases} u = v_n = -\frac{\operatorname{sign}(u_0)}{\underline{u}_0} w_n^{\frac{r_n+\nu}{r_n \beta_{n-1}}} \Phi_n, \\ \dot{\hat{\sigma}} = \tilde{\phi}_n(x, \hat{\sigma}). \end{cases} \tag{34}$$

Then, with (31), we have

$$\dot{V}_n|_{(14)} \le -L_n Q_n(w_1, \ldots, w_n)^{2(1+\nu)}, \tag{35}$$

where Q_n is positive definite with respect to $w_1, \ldots, w_n$ (and therefore $x_1, \ldots, x_n$). Hence, the equilibrium $(0, \sigma)$ of the closed-loop x-subsystem with the adaptive control law (34) is Lyapunov stable. Moreover, according to (33) and (35), for any given $(x(0), \hat{\sigma}(0))$ with $\hat{\sigma}(0) \ge 0$, we also have

$$|\tilde{\sigma}(t)| \le \sqrt{2V_n^*(x(0), \hat{\sigma}(0)) + (\sigma - \hat{\sigma}(0))^2} \triangleq \tilde{C}, \tag{36}$$

and therefore,

$$0 \le \hat{\sigma}(t) \le C \triangleq \tilde{C} + \sigma, \tag{37}$$

where C only depends on the initial condition $(x(0), \hat{\sigma}(0))$ and σ (or θ).

In the following, we will prove the adaptive finite-time stabilization of the x-subsystem within a given settling time T. To do this, we give some lemmas. The next lemma was shown in [8].

Lemma 3.3. For every continuous function $\tilde{V}(x, \hat{\sigma})$ satisfying $\tilde{V}(0, \hat{\sigma}) = 0$ and for every $C > 0$, there is a constant $\rho > 0$ depending on C such that $\tilde{V}(x, \hat{\sigma}) \le 1$ for all $(x, \hat{\sigma})$ satisfying $\bar{V}(x, \hat{\sigma}) \le \rho$ and $\hat{\sigma} \in [0, C]$, where $\bar{V}(x, \hat{\sigma})$ is positive definite with respect to x and satisfying $\bar{V}(0, \hat{\sigma}) = 0$.

Then we can obtain:

Lemma 3.4. There is a positive constant ρ depending on C such that for any $(x(0), \hat{\sigma}(0)) \in \Omega \triangleq \{(x, \hat{\sigma}_2) : V_n^*(x, \hat{\sigma}_2) \le \rho\}$, the settling time T_x for the closed-loop x-subsystem satisfies

$$T_x \le \frac{4(2+\nu)V_n^*(x(0), \hat{\sigma}(0))^{\frac{-\nu}{2+\nu}}}{-\nu L_n} \tag{38}$$

Proof. According to $V_n^* = \sum_{j=1}^{n} \int_{v_{j-1}}^{x_j} [s^{\beta_{j-1}} - v_{j-1}^{\beta_{j-1}}]\, \mathrm{d}s \le \sum_{j=1}^{n} 2|w_j|^{\frac{2+\nu}{r_j \beta_{j-1}}}$ and Lemma 2.5, we have

$$(V_n^*)^{\frac{2(1+\nu)}{2+\nu}} \le 2Q_n^{2(1+\nu)}. \tag{39}$$

Noting that $\tilde{\phi}_1(x) = Q_1^{2(1+\nu)} x_1^{-\nu}$ is independent of $\hat{\sigma}$, $|w_j|^{\frac{2(1+\nu)}{r_j \beta_{j-1}}} \tilde{\gamma}_j \le Q_j^{2(1+\nu)} \tilde{\gamma}_j$, and recalling the definition of $\tilde{\phi}_n$, we have

$$\tilde{\phi}_n = \tilde{\phi}_1(x) + \sum_{j=2}^{n} |w_j|^{\frac{2(1+\nu)}{r_j \beta_{j-1}}} \tilde{\gamma}_j \le Q_n^{2(1+\nu)} \phi_0, \tag{40}$$

where $\phi_0 = x_1^{-\nu} + \sum_{j=2}^{n} \tilde{\gamma}_j$. By (35), (39) and (40), $\dot{V}_n^* \le -L_n Q_n^{2(1+\nu)} + \tilde{\sigma}\tilde{\phi}_n \le -\frac{L_n}{4}(V_n^*)^{\frac{2(1+\nu)}{2+\nu}} - \frac{L_n}{2} Q_n^{2(1+\nu)} (1 - \frac{2\tilde{C}}{L_n}\phi_0)$. Then, based on Lemma 3.3, we can get a constant ρ such that for any $(x, \hat{\sigma}) \in \Omega \triangleq \{(x, \hat{\sigma}) : V_n^*(x, \hat{\sigma}(t)) \le \rho\}$, we have $\frac{2\tilde{C}}{L_n}\phi_0 < 1$, and therefore, $\dot{V}_n^* \le -\frac{L_n}{4}(V_n^*)^{\frac{2(1+\nu)}{2+\nu}}$. By Lemma 2.4, the proof is completed. □

The following is one of our main results.

Theorem 3.5. If $x_0(0) \neq 0$, system (7) is adaptively finite-time stable within any given settling time T under the controllers in the form of

$$u_0 = -k_0 x_0^{\alpha} - |x_0| b_0, \tag{41}$$

and

$$\begin{cases} u = u_* = K v_n(\frac{x_1}{K}, \frac{x_2}{K}, \ldots, \frac{x_n}{K}), \\ \dot{\hat{\sigma}} = \tilde{\phi}_n(\frac{x_1}{K}, \frac{x_2}{K}, \ldots, \frac{x_n}{K}), \end{cases} \tag{42}$$

where v_n and $\tilde{\phi}_n$ are defined in (34) with suitable $K \ge 1$.

Proof. As discussed, we have selected a suitable k_0 such that the state x_0 converges to zero within $T_0 \le T$ for the system (9). Now we should construct an adaptive controller for x-subsystem (14) to make its settling time $T_x \le T_* < \frac{2V_0^{*\frac{1-\alpha}{2}}}{K(1-\alpha)}$ as mentioned in Remark 3.1.

If $(x(0), \hat{\sigma}(0)) \in \Omega$ and obtained T_x satisfies $T_x \le T_*$, then we can take adaptive control laws in the form of (42) with $K = 1$.

If $(x(0), \hat{\sigma}(0)) \in \Omega$ and $T_x > T_*$, we will employ a *time-rescaling* technique to re-construct an adaptive finite-time controller to make the closed-loop system with a "modified" settling time, $T_x^K \le T_*$.

Take $\bar{x}_i = \frac{x_i}{K}$ with $K \ge 1$, then we get equations from system (14):

$$\begin{cases} \frac{\mathrm{d}\bar{x}_1}{\mathrm{d}t} = \bar{x}_2 u_0 + \frac{\phi_1}{K}, \\ \vdots \\ \frac{\mathrm{d}\bar{x}_{n-1}}{\mathrm{d}t} = \bar{x}_n u_0 + \frac{\phi_{n-1}}{K}, \\ \frac{\mathrm{d}\bar{x}_n}{\mathrm{d}t} = \frac{u}{K} + \frac{\phi_n}{K} \triangleq \bar{u} + \bar{\phi}_n(\bar{x}) \end{cases} \tag{43}$$

where $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_n)^T$. $|\frac{\phi_i}{K}| \le \sum_{l=1}^{i} |\bar{x}_l| \sigma$, $1 \le i \le n$ makes Assumption 2 still valid for system (43). Therefore, control $\bar{u} = v_n(\bar{x}, \hat{\bar{\sigma}})$ and update law $\dot{\hat{\bar{\sigma}}} = \tilde{\phi}_n(\bar{x}, \hat{\bar{\sigma}})$

given in the form of (34) with the same $l_i, i = 1, \ldots, n$ can adaptively finite-time stabilize system (43), where $\hat{\bar{\sigma}}$ is the estimator of σ . Here, we only require that (13) should hold when $t \in [0, T_x^K]$, where $T_x^K(\bar{x})$ is the settling time of the system (43). On the one hand, similar to (36), $|\tilde{\bar{\sigma}}(t)| \leq \sqrt{2V_n^*(\bar{x}(0), \hat{\bar{\sigma}}(0)) + (\sigma - \hat{\bar{\sigma}}(0))^2} \triangleq \tilde{\bar{C}}$, and therefore, $0 \leq \hat{\bar{\sigma}}(t) \leq \bar{C} \triangleq \tilde{\bar{C}} + \sigma$. Noting that $\lim_{K\to\infty} V_n^*(\bar{x}(0), \hat{\bar{\sigma}}(0)) = 0$, we can find $K_1 \geq 1$ such that, when $K \geq K_1$, $\tilde{\bar{C}} \leq \tilde{C}$, and then $\bar{C} \leq C$. Meanwhile, $(\bar{x}(0), \hat{\bar{\sigma}}(0)) \in \Omega$ when $K \geq K_1$. Applying Lemma 3.4 to system (43) gives that

$$T_x^K \leq \frac{4(2+\nu)(V_n^*(\bar{x}(0), \hat{\bar{\sigma}}(0)))^{\frac{-\nu}{2+\nu}}}{-\nu L_n}, \tag{44}$$

when $K \geq K_1$. Since $V_n^*(0, \hat{\bar{\sigma}}(0)) = 0$, there exists a positive number $K_2 \geq K_1$ such that $T_x^K \leq T_*$ for any $K \geq K_2$.

In the case when $(x(0), \hat{\sigma}(0)) \notin \Omega$, we can first find enough big K_0 such that $(\bar{x}(0), \hat{\bar{\sigma}}(0)) \in \Omega$ when $K > K_0$. Then the analysis can be completed in a way similar to the case when $(x(0), \hat{\sigma}(0)) \in \Omega$ and $T_x > T_*$.

□

3.2. Control for $x_0(0) = 0$

We have discussed the case when $x_0(0) \neq 0$. Now we show how to propose adaptive finite time control laws for system (1) within any given finite settling time T when $x_0(0) = 0$. Obviously, if $x(0)$ is also 0, we can certainly take

$$\begin{cases} u_0 = u = 0; \\ \dot{\hat{\sigma}} = 0. \end{cases}$$

Therefore, we will only study the case when $(x_0(0), x(0)) \in \Gamma = \{(0, x) : \|x\| \neq 0\}$. At first, we give a lemma.

Lemma 3.6. Consider the the one-dimensional system

$$\dot{x}_0 = u_0 + x_0\phi_0(t, x_0, \theta_0), \quad x_0(0) = 0, \tag{45}$$

the closed-loop system with $u_0 = \beta - x_0 b_0(x_0)$ enjoys the following properties:

$$|x_0(t)| \leq \beta t, \quad x_0(t) > 0, \quad t > 0,$$

where $\beta > 0$.

P r o o f. Based on $u_0(0) = \beta > 0$, we can get $\dot{x}_0(0) > 0$. According to Remark 3.1, it is not difficult to know that $x_0(t) \geq 0$, $\forall\, t > 0$. On the one hand, according to $\dot{x}_0 = u_0 + x_0\phi_0(t, x_0, \theta_0) \leq \beta$ we have $|x_0(t)| = x_0(t) \leq \beta t$, $\forall\, t > 0$. On the other hand, based on $\dot{x}_0 = u_0 + x_0\phi_0(t, x_0, \theta_0) \geq \beta - 2x_0 b_0$, we have

$$x_0(t) \geq \beta e^{-\mu(t)} \int_0^t e^{\mu(\tau)}\, \mathrm{d}\tau \geq \beta e^{-\mu(t)} t > 0, \quad \forall\, t > 0,$$

where $\mu(t) = 2\int_0^t b_0(x_0(s))\, \mathrm{d}s \geq 0$. □

Theorem 3.7. Consider the system (1) satisfying Assumptions 1 and 2 in the case when $(x_0(0), x(0)) \in \Gamma$. Let T be its settling time. Take real numbers $\beta > 0$ and $\epsilon > 0$ and take α as in (8). Select $t_s(\|x(0)\|) = \min\left\{\frac{1}{\max\limits_{\tau\in[0,\ \|x(0)\|]} 4\{\epsilon+b_0(x_0(\tau))\}}, \frac{T}{4}, \|x(0)\|\right\}$, and k_0 such that $k_0 \geq \frac{8V_0^*(x_0(t_s))^{\frac{1-\alpha}{2}}}{3T(1-\alpha)}$.

Then the controller in the following form:

$$u_0 = \begin{cases} \beta - x_0 b_0(x_0), & \text{if} \quad t < t_s\left(\|x(0)\|\right) \\ -k_0 x_0^{\alpha} - x_0 b_0, & \text{otherwise} \end{cases} \tag{46}$$

$$\begin{cases} u(t) = u_*(x, u_0), \\ \dot{\tilde{\sigma}} = \tilde{\phi}_n(x, u_0), \end{cases} \tag{47}$$

finite-time stabilize the closed-loop system (7) within settling time T, where (47) can be taken in the form of (42) (with $\underline{u}_0 = \frac{\beta}{2}$ when $t < t_s$). Moreover, the equilibrium $(0, 0, \sigma)$ of the closed-loop system (7) is Lyapunov stable.

P r o o f. To keep the controllability of the x-subsystem when $t \in [0, t_s]$, we need to prove

$$u_0(t) \geq \frac{\beta}{2}, \quad t < t_s(\|x(0)\|).$$

In fact,

$$u_0(t) \geq \beta - 2x_0 b_0 \geq \beta - 2\beta t_s b_0$$
$$\geq \beta - \frac{2\beta b_0}{\max\limits_{\tau\in[0,\ \|x(0)\|]} 4\{\epsilon + b_0(x_0(\tau))\}} \geq \frac{\beta}{2}.$$

According to the analysis given for the case when $x_0(0) \neq 0$, it is not hard to see that adaptive control law (47) taken in the form of (42) with suitable K stabilizes x-subsystem within $T_* < t_s + \frac{2V_0^*(x_0(t_s))^{\frac{1-\alpha}{2}}}{\underline{K}(1-\alpha)} \leq T$. Then, with the selection of k_0, x_0-subsystem can reach $x_0 = 0$ within T, which implies that the whole system is finite-time convergent within settling time T.

Next, we investigate the Lyapunov stability of the closed-loop system (7). When $t \in [0, t_s]$, based on (46) and Lemma 3.6, we have

$$|x_0(t)| \leq \beta t \leq \beta t_s \leq \beta\|x(0)\|.$$

In fact, for any $\varepsilon > 0$, when $t \in [0, t_s]$, if $\beta\|x(0)\| \leq \frac{\varepsilon}{2}$, then $|x_0(t)| \leq \frac{\varepsilon}{2}$; when $t \geq t_s$, according to the Lyapunov stability of the closed-loop x_0-subsystem, we have for any $\varepsilon > 0$, there is δ_1 such that if $|x_0(t_s)| \leq \beta\|x(0)\| \leq \delta_1$, then $|x_0(t)| \leq \frac{\varepsilon}{2}$. According to the Lyapunov stability of the closed-loop x-subsystem (14) with update law $\dot{\tilde{\sigma}} = \tilde{\phi}_n$ based on the analysis of the preceding subsection, there exists δ_2 such that if $\|x(0)\| \leq \delta_2$ and $|\tilde{\sigma}(0)| \leq \delta_2$, then $\|x(t), \tilde{\sigma}(t)\| \leq \frac{\varepsilon}{2}$.

Thus, when $\|x(0)\| \leq \min\{\frac{\varepsilon}{2\beta}, \frac{\delta_1}{\beta}, \delta_2\}$ and $|\tilde{\sigma}(0)| \leq \delta_2$, we have

$$\|x_0(t), x(t), \tilde{\sigma}(t)\| \leq |x_0(t)| + \|x(t), \tilde{\sigma}(t)\| \leq \varepsilon.$$

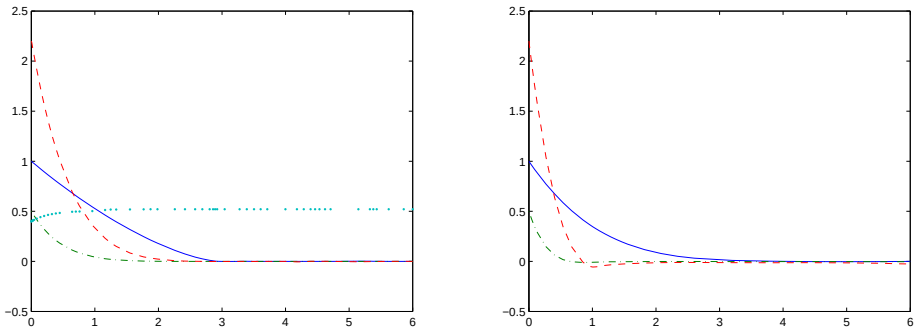

Fig. 1. Trajectories of x_0 (solid line), x_1 (dashdot line), x_2 (dashed line) and $\hat{\sigma}$ (point line) in Case 1.

Hence the closed-loop system (7) is Lyapunov stable. Thus, the proof is completed. □

Remark 3.8. Note that the system considered in [9] is a special case of system (1) when $\phi_0 = \theta = 0$. To handle the uncertain θ, adaptive controller is employed here.

Example 3.9. Consider the following system in the form of (1):

$$\begin{cases} \dot{x}_0 = u_0 + \theta_0 x_0^2/5, \\ \dot{x}_1 = u_0 x_2 + \theta x_1, \\ \dot{x}_2 = u_1 - u_0 x_1, \end{cases} \tag{48}$$

where $\theta_0 = \theta = 1/3$ are uncertain parameters, and $\phi_0 = \theta_0 x_0/5, b_0 = \frac{|x_0|}{10}, \phi_1 = x_1$, where $\theta_0 x_0^2/5$ and θx_1 represent un-modeled dynamics or the perturbations. When $\theta_0 = \theta = 0$, (48) becomes a kinematic model of tricycle-type mobile robot where u_0 and u represent the wheel's angular velocities [10].

Case 1. When $(x_0(0), x_1(0), x_2(0), \hat{\sigma}(0)) = (1,\ 1/2,\ 2.2,\ 0.4)$ and the settling time $T = 6$. For $\alpha = 1/3$, according to $k_0 \geq \frac{2V_0^{\frac{1-\alpha}{2}}}{T(1-\alpha)} = (1/2)^{4/3}$, we can take $k_0 = 1/2$. Thus the adaptive control laws can be given as the follows:

$$u_0 = -\frac{1}{2}x_0^{1/3} - \frac{x_0^2}{10}.$$

$$\begin{cases} u_1 = -(x_2^{9/7} - v_1^{9/7})^{5/9}\{2.6 + 16.5(2^{2/9} + 2(17 + \hat{\sigma}^9 x_1^2))^2 \rho_{1,1}^2/9 \\ \qquad\quad +[(1+\hat{\sigma}^2)^{1/2} + 3|x_2 - v_1|\hat{v}_1 x_1^{\frac{16}{9}}](2x_1)^{4/9}\rho_{1,1}^2\} \\ \dot{\hat{\sigma}} = \tilde{\phi}_2(x, \hat{\sigma}) = x_1^{16/9} + 2w_2^{14/9} x_1^{4/9} \rho_{1,1}^2 \end{cases}$$

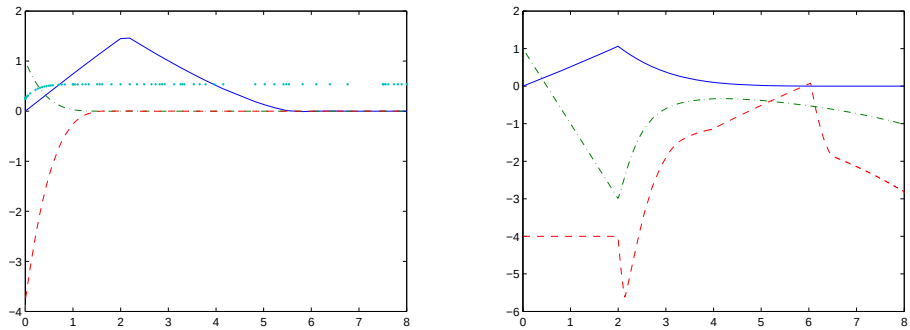

Fig. 2. Trajectories of x_0 (solid line), x_1 (point line), x_2 (dashdot line) and $\hat{\sigma}$ (dashed line) in Case 2.

where

$$v_1 = 2x_1^{7/9}(2 + \hat{\sigma}^9 x_1^2/9), \quad \hat{v}_1 = 9(2 + \hat{\sigma}^9 x_1^2/9)^{2/7}\hat{\sigma}^8|x_2|^3/7,$$

and

$$\rho_{1,1} = (4 + 2\hat{\sigma}^9 x_1^2/9)^{9/7} + 2x_1^2(2 + \hat{\sigma}^9 x_1^2/9)^{2/7}\hat{\sigma}^9/7.$$

Case 2. When $(x_0(0), x_1(0), x_2(0), \hat{\sigma}(0)) = (0,\ 1,\ -4,\ 1/4)$ and the settling time $T = 8$, the adaptive controllers can be given as follows:

$$u_0 = \begin{cases} 3/4 - x_0^2/10, & \text{if} \quad t < 2, \\ -\frac{1}{2}x_0^{1/3} - \frac{x_0^2}{10}, & \text{if} \quad t \geq 2 \end{cases}$$

where u_1 is the same as given in Case 1.

Remark 3.10. If $\theta_0 = \theta = 0$, with the design method of [9] we can get:
When $(x_0(0), x_1(0), x_2(0)) = (0,\ 1,\ -4)$,

$$\begin{cases} u_0(t) = 1/2 \\ u(t) = 0 \end{cases} \qquad \text{if} \quad t < 2, \tag{49}$$

$$\begin{cases} u_0(t) = -x_0^{\frac{9}{11}} \\ u(t) = -4(x_2^{\frac{11}{9}} - 3.5x_1)^{7/11} - 4|x_1|\,\mathrm{sgn}(x_2^{\frac{11}{9}} - 3.5x_1) \end{cases} \qquad \text{if } t \geq 2 \tag{50}$$

When $(x_0(0), x_1(0), x_2(0)) = (1,\ 1/2,\ 2.2)$, controller can be taken as the form of (50).

In Figure 1 and Figure 2, the left figures stands for the simulations for the algorithms in this paper and the right figures represents the simulations for the algorithms (49) and (50) from the reference[9]. From these simulations, it is not difficult for us to get: In Case 1, both algorithms from this paper and reference[9] can work well; however, in Case2 only the algorithms from this paper can work well.

4. CONCLUSIONS

The controllers of [9] can only make the chained form systems with uncertain parameters and perturbed terms finite time stable locally. Inspired by the design method of adaptive control of [8], we extended the finite-time stabilizing control for nonholonomic systems in [9] to adaptive finite-time stabilization for the chained form systems with uncertain parameters and perturbed terms. The existing adaptive finite-time control laws were mainly constructed for the systems with single control input. Here, the nonholonomic system under consideration consists of two coupled control sub-systems. To solve the control problem, we first finite-time stabilized one of the two subsystems within a given settling time, and then we used time-rescaling technique to make the second subsystem convergent faster than the first one. Both rigorous mathematical proofs and a numerical simulation were given.

We note that although controllers in the form of (41) and (42), or, (46) and (47) works well for system (1), it may be interesting to consider how to design controllers such that nonholonomic systems in chained form with dynamic uncertainty are globally or semi-globally finite time stable.

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