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UNIFYING APPROACH TO OBSERVER–FILTER DESIGN

Václav Černý

The paper examines similarities between observer design as introduced in Automatic Control Theory and filter design as established in Signal Processing. It is shown in the paper that there are obvious connections between them in spite of different aims for their design. Therefore, it is prospective to make them be compatible from the structural point of view. Introduced error invariance and error convergence properties of both of them are unifying tools for their design. Lyapunov's stability theory, signal power, system energy and a power balance relation are other basic terms used in the paper.

Keywords: observer, invariance, convergence, filter, signal power, system energy
AMS Subject Classification: 93C10, 93C05

1. INTRODUCTION

Observer design is one of large fields investigated in Automatic Control Theory and a lot of articles have already been dedicated to it in technical literature. One of possible approaches to non-linear observer design is to transform a given system representation into a proper canonical form [3, 12, 20, 22, 23, 29]. Then it is feasible in new coordinates to design an observer with homogeneous error dynamics which can be made asymptotically stable (e. g. by pole assignment). The error dynamics of such observers is mostly *linear*. The approach presented in the paper is a straightforward extension of those methods mentioned above in such a way that the error dynamics of an observer is a *priori* selected as *non-linear*.

In the area of Signal Processing [16, 26, 32], main functions of a filter are to remove unwanted parts of a signal such as a random noise and other measurement disturbances or to extract useful parts of the signal such as its certain components lying within a specific frequency range. From this point of view, it seems to be familiar to start with general theory of stochastic processes and stochastic estimation theory [4, 17, 19] in case a quantitative description of the uncertainty is provided.

On the other hand, frequency domain characteristics of both the useful and unwanted parts of the signal seem to be more acceptable in many practical situations and therefore frequency domain methods based on a concept of an *ideal frequency filter* are preferred. The ideal frequency filter would have a *rectangular* magnitude frequency response. Unfortunately, it is non-causal on principle. However, there are several filter design techniques that approximate the ideal frequency filter characteristics (Butterworth, Chebyshev, Bessel, etc.). Each of the major types optimizes a different aspect of the approximation.

The main contribution of the paper consists in demonstrating that it is possible to make both *observer* and *filter* design approaches be unified from the structural point of view and in spite of different aims for their design. Introduced *error invariance* and *error convergence* properties are the tools for the unification. The concrete new result presented shows that *optimal* as well as *frequency domain* filters are derived as special cases of a developed *asymptotic* observer-filter design approach.

2. DISSIPATION NORMAL FORM

Definition 1. Consider the representation $R_D(S)$ of a system S in the form:

$$R_D(S): \frac{\mathrm{d}x(t)}{\mathrm{d}t} = f[x(t)] \tag{1}$$
$$y(t) = h[x(t)] \tag{2}$$

where $x(t) \in X \subset \mathbb{R}^n$ is a state, X is a smooth manifold defined on $\mathbb{R}^n$, $n \in \mathbb{N} \setminus \{0\}$, $y(t) \in \mathbb{R}^1$ is an output, $f : X \to \mathbb{R}^n$ is a smooth vector field and $h : X \to \mathbb{R}^1$ is a smooth scalar function. Let x_e be an equilibrium state of the representation $R_D(S)$. Assume that there exists a function $W : Y \to \mathbb{R}^1$ defined on a neighborhood $Y \subset \mathbb{R}^n$ of the equilibrium state x_e. The representation $R_D(S)$ will be called the *dissipation normal form* if the function W fulfills the following conditions:

$$W[x(t)] = \|x(t)\|^2 \tag{3}$$
$$L_f\{W[x(t)]\} = \beta[y(t)] \leq 0. \tag{4}$$

Remark 1. There is an obvious connection between the function $W[x(t)]$ and the Lyapunov function. The function $W[x(t)]$ is also related to the available storage [33] and a non-linear function $\beta[y(t)]$ corresponds to the Rayleigh function [30].

Theorem 1. Let $k_2, \dots, k_n \in \mathbb{R}$; $k_2, \dots, k_n \neq 0$ and $\alpha, \varphi_1 : \mathbb{R}^1 \to \mathbb{R}^1$ are continuous functions satisfying the following conditions: $\forall x(t) \in X : \frac{\mathrm{d}\alpha[x_1(t)]}{\mathrm{d}x_1(t)} \neq 0$; $\forall x(t) \in Z,\ Z \subset Y : \varphi_1[x_1(t)] < 0 \Leftrightarrow x_1(t) \neq 0$. If the representation $R_D(S)$ has the following structure [14]:

$$R_D(S): \frac{\mathrm{d}x(t)}{\mathrm{d}t} = \begin{bmatrix} \varphi_1[x_1(t)] & k_2 & 0 & \cdots & 0 \\ -k_2 & 0 & k_3 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -k_{n-1} & 0 & k_n \\ 0 & \cdots & 0 & -k_n & 0 \end{bmatrix} x(t) \tag{5}$$
$$y(t) = \alpha[x_1(t)] \tag{6}$$

then it is observable as defined in [13] and the equilibrium state $x_e = 0$, $x_e \in Z$ is asymptotically stable in Z. Furthermore, the function $W[x(t)]$ fulfills the conditions

(3), (4) for any $\alpha[x_1(t)]$, $\varphi_1[x_1(t)]$ and $k_2, \ldots, k_n$ on Z satisfying the premises given at the beginning of the theorem.

Proof. At first, observability of the representation $R_D(S)$ will be proved and subsequently a proof of asymptotical stability of its equilibrium state will follow using the second (direct) Lyapunov stability method.

1. It holds that:

$$\det H_0[x(t)] = \det \frac{\partial}{\partial x(t)} \begin{bmatrix} \alpha[x_1(t)] \\ L_f\{\alpha[x_1(t)]\} \\ \vdots \\ L_f^{n-1}\{\alpha[x_1(t)]\} \end{bmatrix} = k_2^{n-1} k_3^{n-2} \cdots k_n \left\{ \frac{\mathrm{d}\alpha[x_1(t)]}{\mathrm{d}x_1(t)} \right\}^n,$$
$$\det H_0[x(t)] \neq 0 \quad \forall x(t). \tag{7}$$

   It follows from the relation (7) that the representation $R_D(S)$ is observable under the assumptions stated at the beginning of the theorem.

2. Assume that the representation $R_D(S)$ has the form (5), (6) and consider the function $W[x(t)] = \|x(t)\|^2$ defined on $\mathbb{R}^n$.

   - The relation (5) implies that:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = 0 \iff x(t) = x_e = 0. \tag{8}$$

     Hence, $x_e = 0$, $x_e \in Z$ is the equilibrium state of the representation $R_D(S)$.
   - It holds that:

$$\begin{aligned} W[x(t)] &> 0 \quad \text{for} \quad x(t) \neq 0 && (9) \\ W[x(t)] &= 0 \quad \text{for} \quad x(t) = 0 && (10) \\ L_f\{W[x(t)]\} &= 2x_1^2(t)\varphi_1[x_1(t)] = 2\{\alpha^{-1}[y(t)]\}^2\varphi_1\{\alpha^{-1}[y(t)]\} \\ &= \beta[y(t)] < 0 \quad \text{for} \quad x(t) \notin M \subset Z && (11) \\ L_f\{W[x(t)]\} &= 2x_1^2(t)\varphi_1[x_1(t)] = 2\{\alpha^{-1}[y(t)]\}^2\varphi_1\{\alpha^{-1}[y(t)]\} \\ &= \beta[y(t)] = 0 \quad \text{for} \quad x(t) \in M && (12) \end{aligned}$$

     where $M = \{x(t) \in Z, L_f\{W[x(t)]\} = 0\} = \{x(t) \in Z, x_1(t) = 0\}$. However, if $x(t) \in M$ then $\frac{\mathrm{d}x(t)}{\mathrm{d}t} \neq 0$ except for $x(t) = x_e = 0$. This means that the set M does not contain any equilibrium trajectory (for example any limit cycle) except for the equilibrium state $x(t) = x_e = 0$. Consequently, the relations (9), (10), (11), (12) and the Krasovskij–LaSalle principle [11] imply that the function $W[x(t)]$ is a Lyapunov function on Z. Thus, the equilibrium state $x_e = 0$ is asymptotically stable in Z. It is also obvious that the function $W[x(t)]$ fulfills the conditions (3), (4) for any $\alpha[x_1(t)]$, $\varphi_1[x_1(t)]$ and $k_2, \ldots, k_n$ on Z under the presumptions presented at the beginning of the theorem. □

Remark 2. The dissipation normal form is the generalization of the Schwarz matrix [31] reached by including the two non-linear functions.

Remark 3. In linear case, if the coefficients of the form are as follows:

$$R_D(S) : \frac{\mathrm{d}x(t)}{\mathrm{d}t} = \omega_0 \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -1 & 0 & 1 \\ 0 & \cdots & 0 & -1 & 0 \end{bmatrix} x(t) \tag{13}$$

$$y(t) = x_1(t) \tag{14}$$

where $\omega_0 \in \mathbb{R}$, $\omega_0 > 0$ then it is optimal with respect to the following optimality criterion established as output signal energy [14, 27]:

$$J = \int_{t_0}^{\infty} \|y(t)\|^2 \, \mathrm{d}t. \tag{15}$$

3. NON–LINEAR OBSERVER DESIGN

3.1. Problem formulation

Consider the representation $R(S)$ of a system S in the form:

$$R(S) : \frac{\mathrm{d}x(t)}{\mathrm{d}t} = f[x(t), u(t)] \tag{16}$$

$$y(t) = h[x(t)] \tag{17}$$

where $x(t) \in X \subset \mathbb{R}^n$ is a state, $u(t) \in U \subset \mathbb{R}^p$ is an input, $n, p \in \mathbb{N} \setminus \{0\}$, $y(t) \in \mathbb{R}^1$ is an output, $f \in C^n : X \times U \to \mathbb{R}^n$ is a vector function and $h \in C^n : X \to \mathbb{R}^1$ is a scalar function. The input and output signals $u(t)$ and $y(t)$ are supposed to be measured. The state $x(t)$ and its initial value x_0 are assumed to be unknown. The representation $R(S)$ is supposed to be observable for any $u(t)$ [9].

The aim is to design an observer $\hat{S}$:

$$R(\hat{S}) : \frac{\mathrm{d}\hat{x}(t)}{\mathrm{d}t} = \hat{f}[\hat{x}(t), u(t), y(t)] \tag{18}$$

that will generate an estimate $\hat{x}(t)$ of the state $x(t)$ in such a way that the following two properties will be held:

- An *observer structure* should have a *state error invariance property*:

$$R(\tilde{S}) : \frac{\mathrm{d}\tilde{x}(t)}{\mathrm{d}t} = \tilde{f}[\tilde{x}(t), x(t), \hat{x}(t), u(t), y(t)] = \tilde{f}[\tilde{x}(t)] \tag{19}$$

  where $\tilde{S}$ is a state error system and $\tilde{x}(t)$ is a state error defined as:

$$\tilde{x}(t) = x(t) - \hat{x}(t). \tag{20}$$

- *Observer parametrization* should have a *state error convergence property*. This means that the state error will be uniformly convergent to zero and it will be possible to choose proper rate and/or mode of the convergence.

3.2. Problem solution

The method consists in a *priori* choice of a state error system representation specified in order to fulfill the two properties mentioned above in advance. The representation of the state error system is chosen in the *dissipation normal form*:

$$R_D^*(\tilde{S}) : \frac{\mathrm{d}\tilde{x}^*(t)}{\mathrm{d}t} = \omega_0 \begin{bmatrix} \varepsilon_1^*[\tilde{x}_1^*(t)] & \varepsilon_2^* & 0 & \cdots & 0 \\ -\varepsilon_2^* & 0 & \varepsilon_3^* & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -\varepsilon_{n-1}^* & 0 & \varepsilon_n^* \\ 0 & \cdots & 0 & -\varepsilon_n^* & 0 \end{bmatrix} \tilde{x}^*(t)$$

where $\varepsilon_1^*[\tilde{x}_1^*(t)]$, $\varepsilon_2^*, \ldots, \varepsilon_n^*$ and $\omega_0 > 0$ are design (selectable) parameters.
It holds that:

$$L_{\tilde{f}^*}\{\tilde{W}^*[\tilde{x}^*(t)]\} = 2\omega_0 \tilde{x}_1^{*^2}(t)\varepsilon_1^*[\tilde{x}_1^*(t)]. \tag{21}$$

The relation (21) implies that the *state error invariance property* is held. Subsequently, it results from the relation (21) that the *state error convergence property* is also retained when the design parameters are properly chosen (see Theorem 1.). The constant ω_0 represents a time scale transformation and therefore it affects convergence rate. A non-linear function $\varepsilon_1^*[\tilde{x}_1^*(t)]$ designates in proportion as system energy dissipates and accordingly it specifies convergence mode. It is obvious from the relation (21) that the constants $\varepsilon_2^*, \ldots, \varepsilon_n^* \neq 0$ do not have any effect on the rate and/or mode of the convergence. From this point of view, they can in principle be chosen in an arbitrary way. It is even possible them to be non-linear functions in general. This means that the state error invariance property does not have to be satisfied. However, the general case is redundant to be considered with respect to the objective of the paper.

Remark 4. If $\omega_0 \to \infty$ then an appropriate observer corresponds to the high-gain observer [1, 2, 8] in the sense of possible setting error convergence to zero properly fast enough so that a closed loop system is asymptotically stable.

Further, assume that the original representation $R(S)$ of a system S (16), (17) can be transformed into the following canonical form induced by the structure of the state error system representation [5]:

$$R^*(S) : \frac{\mathrm{d}x^*(t)}{\mathrm{d}t} = A^* x^*(t) + \psi^*[x_1^*(t), u(t), u_d(t)] \tag{22}$$

$$y(t) = h^*[x_1^*(t)] \tag{23}$$

where $A^* = \begin{bmatrix} 0 & a_2^* & 0 & \cdots & 0 \\ -a_2^* & 0 & a_3^* & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -a_{n-1}^* & 0 & a_n^* \\ 0 & \cdots & 0 & -a_n^* & 0 \end{bmatrix}$; $a_2^*, \ldots, a_n^* \in \mathbb{R}$; $a_2^*, \ldots, a_n^* \neq 0$; $u_d(t) =$

$\frac{\mathrm{d}u(t)}{\mathrm{d}t}, \dots, \frac{\mathrm{d}^{n-1}u(t)}{\mathrm{d}t^{n-1}}$ and $\psi^*[x_1^*(t), u(t), u_d(t)] = \begin{bmatrix} \psi_1^*[x_1^*(t), u(t)] \\ \psi_2^*[x_1^*(t), u(t), \frac{\mathrm{d}u(t)}{\mathrm{d}t}] \\ \vdots \\ \psi_n^*[x_1^*(t), u(t), u_d(t)] \end{bmatrix}.$

Then substituting to the derivated (with respect to t) relation (20) from (21), (22) and after its slight modifications we get the *structure* of the observer:

$$R^*(\hat{S}) : \frac{\mathrm{d}\hat{x}^*(t)}{\mathrm{d}t} = A^*\hat{x}^*(t) + \psi^*\{c[y(t)], u(t), u_d(t)\} - \omega_0 \begin{bmatrix} \varepsilon_1^*\{c[y(t)] - \hat{x}_1^*(t)\} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \{c[y(t)] - \hat{x}_1^*(t)\} \tag{24}$$

where the inverse:

$$x_1^*(t) = h^{*^{-1}}[y(t)] = c[y(t)] \tag{25}$$

is supposed to exist.

Parametrization of the observer (i. e., determination of the unknown terms in (24) and in (22), (23) indeed) is performed through the generalized observability canonical form [34] and consists in solving a system of differential equations [5]. Finally, the proposed observer is transformed into original coordinates.

4. LINEAR TIME–VARYING OBSERVER DESIGN

4.1. Problem formulation

Consider the representation $R(S)$ of a system S in the form:

$$R(S) : \frac{\mathrm{d}x(t)}{\mathrm{d}t} = A(t)x(t) + B(t)u(t) \tag{26}$$

$$y(t) = C(t)x(t) \tag{27}$$

where $x(t) \in \mathbb{R}^n$ is a state, $u(t) \in \mathbb{R}^p$ is an input, $y(t) \in \mathbb{R}^q$ is an output, $n, p, q \in \mathbb{N} \setminus \{0\}$, and $A(t)$, $B(t)$, $C(t)$ are matrices with appropriate dimensions. The input and output signals $u(t)$ and $y(t)$ are supposed to be measured. The state $x(t)$ and its initial value x_0 are assumed to be unknown. The representation $R(S)$ is supposed to be observable.

The aim is again to design an observer $\hat{S}$:

$$R(\hat{S}) : \frac{\mathrm{d}\hat{x}(t)}{\mathrm{d}t} = \hat{F}(t)\hat{x}(t) + \hat{G}(t)u(t) + \hat{K}(t)y(t) \tag{28}$$

$$\hat{y}(t) = \hat{H}(t)\hat{x}(t) \tag{29}$$

that will generate an estimate $\hat{x}(t)$ of the state $x(t)$ in such a way that the following – already familiar two properties will be held:

- The *state-output error invariance property* expressed by the formulas:

$$R(\tilde{S}):\ \frac{\mathrm{d}\tilde{x}(t)}{\mathrm{d}t} = \tilde{M}(t)\tilde{x}(t) \quad (30)$$

$$\tilde{y}(t) = \tilde{N}(t)\tilde{x}(t) \quad (31)$$

  where $\tilde{S}$ is a state-output error system and $\tilde{y}(t) = y(t) - \hat{y}(t)$ is an output error signal.

- The *state-output error convergence property* that will be specified in detail thereinafter.

4.2. Problem solution

When the *state-output error invariance property* is employed within the derivated (with respect to t) relation (20) by substituting to it from (26), (28), (30) and followed by its obvious modifications, we get the *structure* of the observer again:

$$R(\hat{S}):\ \frac{\mathrm{d}\hat{x}(t)}{\mathrm{d}t} = A(t)\hat{x}(t) + B(t)u(t) + \hat{K}(t)[y(t) - C(t)\hat{x}(t)] \quad (32)$$

$$\hat{y}(t) = C(t)\hat{x}(t). \quad (33)$$

Consequently, $\tilde{M}(t) = \hat{F}(t) = A(t) - \hat{K}(t)C(t)$, $\tilde{N}(t) = \hat{H}(t) = C(t)$, $\hat{G}(t) = B(t)$.

Remark 5. This is consistent with the structure of the Kalman–Bucy filter [4], [19]. In case of the time-invariant restriction: $A(t) = A$, $B(t) = B$, $C(t) = C$, $\hat{K}(t) = \hat{K}$, then it is also identical with the structure of the n-order Luenberger observer [24].

Let *output error signal power* $\tilde{P}(t)$ [16, 26, 32] and an *energy function* $\tilde{E}(t)$ of the state-output error system be introduced as follows in the specific coordinates (see the Section 2):

$$\tilde{P}(t) = \|\tilde{y}(t)\|^2 \quad (34)$$

$$\tilde{E}(t) = \delta\|\tilde{x}(t)\|^2 \quad (35)$$

where $\delta > 0$ is a scaling parameter.

The *state-output error convergence property* is equivalent to the *power balance relation* [15, 18]:

$$\frac{\mathrm{d}\tilde{E}(t)}{\mathrm{d}t} = -[\rho^{-1}(t)\tilde{P}(t) + \sigma(t)\|\tilde{v}(t)\|^2] \quad (36)$$

where $\tilde{v}(t)$ is a dual output error signal and $\rho(t) > 0$, $\sigma(t) > 0$ are design parameters. The design parameters specify required rate and/or mode of convergence.

Let us combine the power balance relations for the state-output error system $\tilde{S}$ (36) and for the original system S:

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = \rho^{-1}(t)P(t) - \sigma(t)\|v(t)\|^2 \quad (37)$$

where $v(t)$ is a dual output signal, $P(t) = \|y(t)\|^2$ and $E(t) = \delta\|x(t)\|^2$. We get the *parametrization* of the observer [6]:

$$\hat{K}(t) = S(t)C^T(t)\rho^{-1}(t) \quad (38)$$

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = A(t)S(t) + S(t)A^T(t) + B(t)\frac{\sigma(t)}{\delta}B^T(t) - S(t)C^T(t)\frac{\rho^{-1}(t)}{\delta}C(t)S(t). \quad (39)$$

Consequently, the Lyapunov function for the given system S has the form:

$$V[x(t)] = x^T(t)S^{-1}(t)x(t). \quad (40)$$

Remark 6. The dual signals are introduced with respect to generalizing Tellegen's theorem [7, 28].

Relation to optimal filtering: Let us invoke the design parameters as follows: $\delta = 1$, $\sigma(t) \cdot I = Q(t)$ and $\rho(t) \cdot I = R(t)$. Then the relations (38), (39) become to:

$$\hat{K}(t) = S(t)C^T(t)R^{-1}(t) \quad (41)$$

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = A(t)S(t) + S(t)A^T(t) + B(t)Q(t)B^T(t) - S(t)C^T(t)R^{-1}(t)C(t)S(t). \quad (42)$$

It is obvious that the formulas are equivalent to the relation for the *Kalman gain matrix* and *Riccati differential equation* [4, 19] in case of input and output signals disturbances being mutually independent white noises with zero mean and covariance matrices $Q(t)$ and $R(t)$. This yields that the variance functions $\sigma(t)$ and $\rho(t)$ in the matrices (placed on their diagonals) determine the convergence characteristics (36).

Consider for now a time-invariant restriction in order to get more explicit results. The appropriate representation of the state-output error system has the form:

$$R(\tilde{S}) : \frac{\mathrm{d}\tilde{x}(t)}{\mathrm{d}t} = \tilde{M}\tilde{x}(t) = (A - \hat{K}C)\tilde{x}(t) \quad (43)$$

$$\tilde{y}(t) = \tilde{N}\tilde{x}(t) = C\tilde{x}(t). \quad (44)$$

When the output error signal energy (15) is chosen as an *optimality criterion*:

$$\tilde{J} = \int_{t_0}^{\infty} \tilde{P}(t)\,\mathrm{d}t = \int_{t_0}^{\infty} \|\tilde{y}(t)\|^2\,\mathrm{d}t \quad (45)$$

then the *optimal* solution is specified as the following matrices (see (13), (14)) having the dissipation normal form structure:

$$\tilde{M} = A - \hat{K}C = \omega_0 \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -1 & 0 & 1 \\ 0 & \cdots & 0 & -1 & 0 \end{bmatrix}, \ \omega_0 > 0 \quad (46)$$

$$\tilde{N} = C = [1 \ \ 0 \ \ldots \ 0] \quad (47)$$

or as the recursively given *optimal* transfer function (for $B = [0\ \dots\ 0\ \omega_0]^T$):

$$F_n(s) = \frac{\omega_0^n}{P_n(s)} \quad (48)$$

$$P_0(s) = 1 \quad (49)$$

$$P_1(s) = s + \omega_0 \quad (50)$$

$$P_k(s) = sP_{k-1}(s) + \omega_0^2 P_{k-2}(s) \text{ for } k \in \{2, \dots, n\}. \quad (51)$$

Relation to frequency domain filtering: Let us have the following optimal transfer functions for various orders:

- $n = 1$, $\omega_0 = 1$: $F_1(s) = \frac{1}{P_1(s)}$
- $n = 2$, $\omega_0 = 1$: $F_2(s) = \frac{1}{P_2(s)}$
- $n = 7$, $\omega_0 = 1$: $F_7(s) = \frac{1}{P_7(s)}$
- $n = 14$, $\omega_0 = 1$: $F_{14}(s) = \frac{1}{P_{14}(s)}$
- $n = 21$, $\omega_0 = 1$: $F_{21}(s) = \frac{1}{P_{21}(s)}$

Corresponding frequency responses of the transfer functions are depicted in Figure 1. It is shown in the figure that the responses converge to the response of the *ideal low pass frequency filter* with increasing order. The cutoff frequency is given by ω_0.

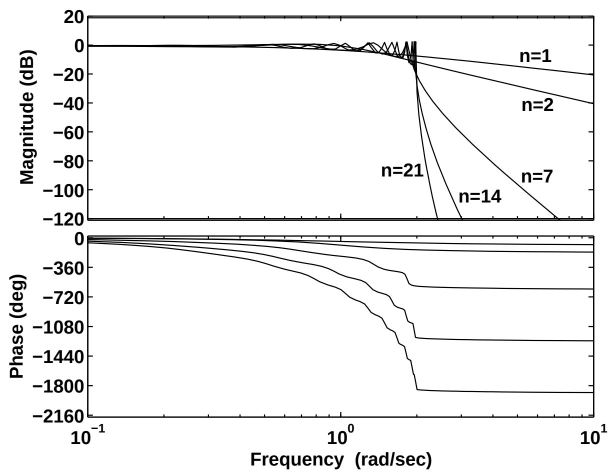

Fig. 1. Frequency responses for $\omega_0 = 1$ and the orders $n = 1, 2, 7, 14, 21$.

The matrix $\tilde{M}$ implies the *lattice* structure of the filters shown in Figure 2. Its *leap-frog* realization [10] is robust with respect to system parameters uncertainties.

The discrete-time modification depicted in Figure 3 gives the *lattice-ladder* realization of IIR digital filters [21]. On condition that the parameter values $\delta_i = 1$ then it is reduced to a FIR digital filter. Moreover, the realization is robust with respect to rounding errors. Another interesting thing is the fact that the optimal filter described by (46), (47) eliminates a constant systematic error from a disturbed signal [6].

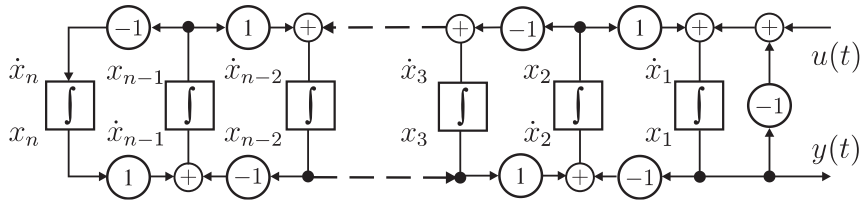

Fig. 2. Lattice structure of the filters.

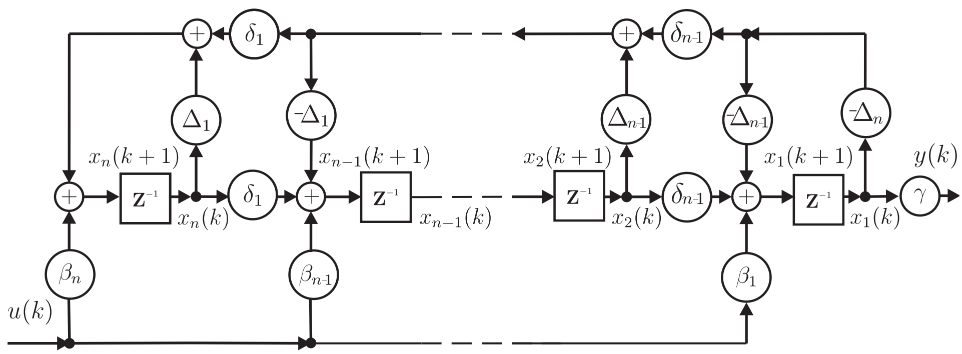

Fig. 3. Lattice-leader realization of IIR digital filters.

5. CONCLUSIONS

It is shown in the paper that it is possible to make be compatible both the *observer* and the *filter* design approaches from the structural point of view and in spite of the different aims for their design. The *error invariance* and the *error convergence properties* are introduced to be unifying tools in it. While the error invariance property specifies their *structure*, the error convergence property provides their *parametrization*, respectively. Moreover, it is demonstrated that the *asymptotic* observer corresponds to the *optimal* Kalman–Bucy filter under introducing the power balance relations as well as to the *low pass frequency* filters of various orders under minimizing the output error signal energy as an optimality criterion. These facts are very interesting and constitute concrete new findings of the paper. At first, no optimization is required and yet the optimal Kalman–Bucy filter appears. On the other hand, the frequency filters and their realizations arise when the optimality criterion is used. Finally, the filters are obtained as special cases of the developed asymptotic observer-filter design approach.

In addition, an optimal low pass frequency filter of the third order given by the formulas (48) – (51) has already been implemented in practice into a vibrodiagnostic device for pre-processing of signals from relative turbine rotor vibration sensors. The signals are then used in an estimation procedure that performs turbine rotor analyzes under low RPM [25] (i. e. it evaluates turbine rotor shaft eccentricity and ovality). On one hand, the filter provides very good filtration results with respect to possible measurement noise elimination and decay of higher harmonic component amplitudes contained in the signals (e. g. grid frequency and its higher harmonics). On the other hand, it preserves amplitude and phase values of useful harmonic components of the signals. It is much effective than for example averaging commonly used in industrial practice within this kind of application (the estimation procedure is more accurate).

Finally, it should be stated that the class of filters investigated in the paper does not include such important filters as for example particle or fractional filters.

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