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*Mathematica Slovaca*, Vol. 57 (2007), No. 5, [483]--494

Persistent URL: http://dml.cz/dmlcz/136972

VERSITA

Mathematica Slovaca

DOI: 10.2478/s12175-007-0040-5
Math. Slovaca 57 (2007), No. 5, 483–494

# SLANT AND SEMI-SLANT SUBMANIFOLDS OF A KENMOTSU MANIFOLD

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(*Communicated by Július Korbaš*)

ABSTRACT. In the present note we have obtained some basic results pertaining to the geometry of slant and semi-slant submanifolds of a Kenmotsu manifold.



## 1. Introduction

S. Tanno [13] classified connected almost contact metric manifold whose automorphism group has maximum dimension. He classified them into the following three classes.

(1) Homogenous normal contact Riemannian manifolds with constant $\phi$ holomorphic sectional curvature if the sectional curvature of the plane section containing $\xi$, say $K(X,\xi) > 0$.

(2) Global Riemannian product of a line (or a circle) and a Kaehlerian manifold with constant holomorphic sectional curvature, if $K(X,\xi) = 0$.

(3) A warped product space $R \times_f C^n$, if $K(X,\xi) < 0$.

It is known that the manifolds of class (1) are characterized by some tensor equations, it has a Sasakian structure. The manifolds of class (2) are characterized by a tensorial relation admitting a cosymplectic structure. Kenmotsu [9] obtained some tensorial equations to characterize manifolds of class (3). He

2000 Mathematics Subject Classification: Primary 53C40, 53B25.
Keywords: slant, semi-slant, totally umbilical, totally contact umbilical.

obtained geometric properties of these manifolds and paved way for further investigations of these manifolds. From then onwards, these manifolds are termed as Kenmotsu manifolds. In general a Kenmotsu manifold is not Sasakian.

Submanifolds of the manifolds of class (1) and (2) have been explored with different geometric point of view and therefore as a step forward it is natural to explore the submanifolds of the warped product spaces in general and Kenmotsu manifolds in particular. As warped product spaces are geometrically interesting spaces, in the present note we investigate semi-slant submanifolds of Kenmotsu manifolds.

The study of semi-slant submanifolds was initiated by N. Papaghiuc [11]. These submanifolds are a generalized version of CR-submanifolds. J. L. Cabrerizo et al. [7] extended the study of semi-slant submanifolds of Kaehler manifold to the setting of Sasakian manifolds. In view of Tanno's classification, the setting of semi-slant submanifold of Kenmotsu manifold is entirely different from the setting of semi-slant submanifold of Sasakian manifold and therefore worth studying. We have obtained some basic results of this setting with differential geometric point of view.

## 2. Preliminaries

Let $\bar{M}$ be an almost contact metric manifold with structure $(\phi, \xi, \eta, g)$ where $\phi$ is a tensor field of type $(1,1)$, $\xi$ a vector field, $\eta$ is a one form and $g$ is the Riemannian metric on $\bar{M}$. Then they satisfy

$$\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y). \tag{2.1}$$

These conditions also imply that

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \quad \eta(X) = g(X, \xi) \tag{2.2}$$

and

$$g(\phi X, Y) + g(X, \phi Y) = 0 \tag{2.3}$$

for all vector fields $X$, $Y$ on $\bar{M}$. If in addition to the above relations,

$$(\bar{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X \tag{2.4}$$

holds, then $\bar{M}$ is said to be a Kenmotsu manifold. Where $\nabla$ is the Levi-Civita connection of $g$. We also have on Kenmotsu manifold $\bar{M}$,

$$\bar{\nabla}_X \xi = X - \eta(X)\xi. \tag{2.5}$$

Throughout, we denote by $\bar{M}$ a Kenmotsu manifold, $M$ a submanifold of $M$ with structure vector field $\xi$ tangent to $M$. $h$ and $A$ denote the second fundamental form and the shape operator of the immersion of $M$ into $\bar{M}$ respectively. If $\nabla$ is the induced connection on $M$, the Gauss and Weingarten formulae of $M$ into $M$ are then given respectively by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y) \tag{2.6}$$

$$\bar{\nabla}_X V = -A_V X + \nabla^{\perp}_X V \tag{2.7}$$

for all vector fields $X$, $Y$ on $M$ and normal vector fields $V$ on $M$, $\nabla^{\perp}$ denotes the connection on the normal bundle $T^{\perp}M$ of $M$. $h$ and $A$ are related by

$$g(A_V X, Y) = g(h(X, Y), V). \tag{2.8}$$

Where the induced Riemannian metric on $M$ is denoted by the same symbol $g$. Now, for any $x \in M$, $X \in T_xM$ and $V \in T_x^{\perp}M$, we put

$$\phi X = TX + NX \tag{2.9}$$

$$\phi V = tV + nV \tag{2.10}$$

where $TX$ (resp. $NX$) is tangential (resp. normal) part of $\phi X$ and $tV$ (resp.$nV$) is the tangential (resp. normal) part of $\phi V$.

The relation (2.9) gives rise to an endomorphism. $T\colon T_xM \to T_xM$ whose square ($T^2$) will be denoted by $Q$. The tensor fields on $M$ of type $(1,1)$ determined by these endomorphisms will be denoted by the same letters $T$ and $Q$ respectively. From (2.3) and (2.9),

$$g(TX, Y) + g(X, TY) = 0 \tag{2.11}$$

for each $X, Y \in TM$. The covariant derivatives $\nabla T$, $\nabla Q$, and $\nabla N$, are defined by

$$(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y \tag{2.12}$$

$$(\nabla_X Q)Y = \nabla_X QY - Q\nabla_X Y \tag{2.13}$$

$$(\nabla_X N)Y = \nabla^{\perp}_X NY - N\nabla_X Y. \tag{2.14}$$

Now, on a submanifold of a Kenmotsu manifold by equation (2.5) and (2.6), we get

$$\nabla_X \xi = X - \eta(X)\xi \tag{2.15}$$

and

$$h(X, \xi) = 0 \tag{2.16}$$

for each $X \in TM$. Further, from equation (2.16)

$$A_V \xi = 0, \quad \eta(A_V X) = 0 \tag{2.17}$$

for each $V \in T^{\perp}M$ and by using equations (2.4), (2.6), (2.7), (2.9), (2.10), (2.12) and (2.14), we obtain

$$(\nabla_X T)Y - A_{NY}X + th(X,Y) = g(X,TY)\xi - \eta(Y)TX \tag{2.18}$$

$$(\nabla_X N)Y = -h(X,TY) + nh(X,Y) - \eta(Y)NX. \tag{2.19}$$

## 3. Slant submanifolds of a Kenmotsu manifold

For any $x \in M$ and $X \in T_xM$ if the vectors $X$ and $\xi$ are linearly independent, the angle $\theta(X) \in \left[0, \frac{\pi}{2}\right]$ between $\phi X$ and $T_xM$ is well defined. If $\theta(X)$ does not depend on the choice of $x \in M$ and $X \in T_xM$, we say that $M$ is slant in $M$. The constant angle $\theta$ is then called the slant angle of $M$ in $M$. The anti-invariant submanifolds of an almost contact metric manifold are slant submanifolds with slant angle $\frac{\pi}{2}$ and invariant submanifolds are slant submanifolds with slant angle 0. If the slant angle $\theta \neq 0, \pi/2$, then the slant submanifold is called a proper slant submanifold. If $M$ is a slant submanifold of an almost contact manifold then the tangent bundle $TM$ of $M$ is decomposed as

$$TM = D \oplus \langle\xi\rangle$$

where $\langle\xi\rangle$ denotes the distribution spanned by the structure vector field $\xi$ and $D$ is the complementary distribution of $\langle\xi\rangle$ in $TM$, known as the slant distribution. For a proper slant submanifold $M$ of an almost contact manifold $M$ with a slant angle $\theta$, L o t t a [2] proved that

$$QX = -\cos^2\theta(X - \eta(X)\xi)$$

for any $X \in TM$. Recently, C a b r e r i z o et al. [8] extended the above result into a characterization for a slant submanifold in a contact metric manifold. In fact, they obtained the following crucial theorems.

**THEOREM 3.1.** ([8]) *Let $M$ be a submanifold of an almost contact metric manifold $M$ such that $\xi \in TM$. Then, $M$ is slant if and only if there exists a constant $\lambda \in [0,1]$ such that*

$$Q = -\lambda(I - \eta \otimes \xi). \tag{3.1}$$

*Furthermore, in such case, if $\theta$ is the slant angle of $M$, then it verifies that $\lambda = \cos^2\theta$.*

**THEOREM 3.2.** ([8]) *Let $M$ be a slant submanifold of an almost contact metric manifold $\bar{M}$. Then at each point $x \in M$, $Q|_D$ has only one eigenvalue $\lambda$, where $\lambda = \cos^2\theta$, $\theta$ being the slant angle of $M$.*

We first make use of equation (3.1) to see the impact of parallelism of canonical endomorphism $Q$ on a slant submanifold of a Kenmotsu manifold.

**Theorem 3.3.** *Let $M$ be a slant submanifold of a Kenmotsu manifold $\bar{M}$. Then $Q$ is parallel if and only if $M$ is anti-invariant.*

Proof. Let $M$ be a slant submanifold of a Kenmotsu manifold $\bar{M}$. Then for any $X$, $Y$ in $TM$, by equation (3.1)

$$Q\nabla_X Y = \cos^2\theta(-\nabla_X Y + \eta(\nabla_X Y)\xi) \tag{3.2}$$

and

$$QY = \cos^2\theta(-Y + \eta(Y)\xi).$$

Differentiating the last equation covariantly with respect to $X$ and making use of formula (2.13) and (3.2) we obtain

$$(\nabla_X Q)Y = \cos^2\theta[g(X,Y)\xi - 2\eta(X)\eta(Y)\xi + \eta(Y)X]. \tag{3.3}$$

Hence $(\nabla_X Q) = 0$ if and only if either $\theta = \pi/2$ or $TM = \langle\xi\rangle$ and the assertion is proved. □

It is interesting to notice that the formula (3.3), in fact provides a characterization for the existence of a slant submanifold in Kenmotsu manifold. To be more precise, we have

**Theorem 3.4.** *Let $M$ be a submanifold of a Kenmotsu manifold $\bar{M}$ with structure vector field tangential to $M$. Then $M$ is slant if and only if:*

(i) *The endomorphism $Q|_D$ has only one eigenvalue at each point of $M$.*

(ii) *There exists a function*

$$\lambda\colon M' \to [0,1]$$

*such that*

$$(\nabla_X Q)Y = \lambda[g(X,Y)\xi - 2\eta(X)\eta(Y)\xi + \eta(Y)X]$$

*for each $X, Y \in TM$. If $\theta$ is the slant angle of $M$, then $\lambda = \cos^2\theta$.*

Proof. If $M$ is slant, then the statements (i) and (ii) follow directly from Theorem 3.2 and equation (3.3) respectively.

Conversely, let $D = \langle\xi\rangle^{\perp}$ and assume that statement (i) and (ii) hold. Further, let $\lambda_1$ be the eigenvalue of $Q|_D$, then $QY = \lambda_1 Y$ for each $Y \in D$. Now

$$\nabla_X QY = Q\nabla_X Y + \lambda g(X,Y)\xi$$

i.e.,

$$(X\lambda_1)Y + \lambda_1\nabla_X Y = Q\nabla_X Y + \lambda g(X,Y)\xi$$

for any $X$ in $TM$ and $Y$ in $D$. Since $\nabla_X Y$ and $Q\nabla_X Y$ are perpendicular to $Y$, $\lambda_1$ is constant in view of the above equation. Now, for any $X \in TM$, we can write

$$X = \bar{X} + \eta(X)\xi$$

where $\bar{X} \in D$. Thus $\bar{X} = X - \eta(X)\xi$. Since $QX = Q\bar{X}$ by (i)

$$QX = \lambda_1\bar{X} = \lambda_1(X - \eta(X)\xi).$$

Taking $\mu = -\lambda_1$, the above equation is written as

$$QX = -\mu(X - \eta(X)\xi).$$

As $\lambda_1(= -\mu)$ is constant, by Theorem 3.1, $M$ is slant in $\bar{M}$ and $\mu = \cos^2\theta$. □

**Note.** Theorems 3.3 and 3.4 have also been proved by R. S. Gupta et al. [12].

## 4. Semi-slant submanifolds of a Kenmotsu manifold

Semi-slant submanifolds of almost Hermitian manifolds were introduced as a generalized version of slant and CR-submanifolds by N. Papaghiuc [11]. Cabrerizo et al. [7] studied semi-slant submanifolds in the setting of almost contact metric manifold and Sasakian manifolds.

The purpose, in the present section, is to study the semi-slant submanifolds of a Kenmotsu manifold.

A semi-slant submanifold $M$ of an almost contact metric manifold $\bar{M}$ is a submanifold which admits two orthogonal complementary distributions $D_1$ and $D_2$, such that $D_1$ is invariant under $\phi$ and $D_2$ is slant with slant angle $\theta \neq 0$ i.e., $\phi D_1 = D_1$ and $\phi Z$ makes a constant angle $\theta$ with $TM$ for each $Z \in D_2$. In particular, if $\theta = \frac{\pi}{2}$, then a semi-slant submanifold reduces to a semi-invariant submanifold. For a semi-slant submanifold $M$ of an almost contact metric manifold, we have

$$TM = D_1 \oplus D_2 \oplus \langle\xi\rangle.$$

The orthogonal complement of $ND_2$ in the normal bundle $T^\perp M$, is an invariant subbundle of $T^\perp M$ and is denoted by $\mu$. Thus, we have

$$T^\perp M = ND_2 \oplus \mu.$$

Let $M$ be a semi-slant submanifold and $X \in TM$. Then as $TM = D_1 \oplus D_2 \oplus \langle \xi \rangle$, we write

$$X = P_1X + P_2X + \eta(X)\xi \tag{4.1}$$

where $P_1X \in D_1$ and $P_2X \in D_2$. Now by equations (2.9) and (4.1)

$$\phi X = \phi P_1 X + TP_2X + NP_2X. \tag{4.2}$$

It is easy to see that

$$\phi P_1 X = TP_1X, \quad NP_1X = 0, \qquad TP_2X \in D_2. \tag{4.3}$$

Thus

$$TX = \phi P_1X + TP_2X \tag{4.4}$$

and

$$NX = NP_2X. \tag{4.5}$$

Now, for $Y \in D_1 \oplus D_2$, by equation (2.4), we have

$$\bar{\nabla}_\xi \phi Y = \phi \bar{\nabla}_\xi Y. \tag{4.6}$$

In particular, for $Y \in D_1$, the above equation yields

$$\nabla_\xi \phi Y = \phi \nabla_\xi Y.$$

That means $\nabla_\xi Y \in D_1$ for any $Y$ in $D_1$. This observation leads to the following proposition.

**Proposition 4.1.** *On a semi-slant submanifold $M$ of a Kenmotsu manifold $\bar{M}$,*

$$[X, \xi] \in D_1 \qquad \text{and} \qquad [Z, \xi] \in D_2 \tag{4.7}$$

*for any $X \in D_1$ and $Z \in D_2$.*

The result follows an making use of equations (2.5) and (4.6). Furthermore, on taking account of Proposition 4.1 and equation (2.5), we obtain:

**Theorem 4.1.** *The distributions $D_1 \oplus D_2$ on a semi-slant submanifold of a Kenmotsu manifold is integrable.*

For the integrability of the invariant distribution $D_1$ on $M$, we prove:

**Theorem 4.2.** *Let $M$ be a semi-slant submanifold of a Kenmotsu manifold $M$. Then the invariant distribution $D_1$ is integrable if and only if*

$$h(X, \phi Y) = h(\phi X, Y)$$

*for all $X$, $Y$ in $D_1$.*

Proof. For any $V \in T^{\perp}M$,

$$g(\nabla_X \phi Y - \bar{\nabla}_Y \phi X, V) = g(h(X, \phi Y) - h(\phi X, Y), V).$$

By using equations (2.4) and (4.2), the above equation takes the form

$$g(NP_2[X, Y], V) = g(h(X, \phi Y) - h(\phi X, Y), V),$$

from which the assertion follows immediately. □

By applying Proposition 4.1, Theorem 4.1 yields:

**COROLLARY 4.1.** *The distribution $D_1 \oplus \langle \xi \rangle$ on a semi-slant submanifold of a Kenmotsu manifold is integrable if and only if*

$$h(X, \phi Y) = h(\phi X, Y)$$

*for all $X, Y \in D_1$.*

**THEOREM 4.3.** *Let $M$ be a semi-slant submanifold of a Kenmotsu manifold $\bar{M}$. Then the distribution $D_2$ is integrable if and only if*

$$\nabla_Z TW - \nabla_W TZ + A_{NZ}W - A_{NW}Z$$

*lies in $D_2$ for each $Z$, $W$ in $D_2$.*

Proof. By using equations (2.4), (2.6), (2.7) and (2.9), we get

$$g(T[Z, W], X) = g(\nabla_Z TW - \nabla_W TZ + A_{NZ}W - A_{NW}Z, X)$$

for each $X$ in $D_1$ and $Z, W \in D_2$. The assertion follows by virtue of the above equality and formula (2.5). □

In view of Proposition 4.1, above theorem gives:

**COROLLARY 4.2.** *The distribution $D_2 \oplus \langle \xi \rangle$ on a semi-slant submanifold of a Kenmotsu manifold is integrable if and only if*

$$P_1(\nabla_Z TW - \nabla_W TZ + A_{NZ}W - A_{NW}Z) = 0$$

*for any $Z, W \in D_2$.*

The Nijenhuis tensor field $S$ of the tensor $T$ is given by

$$S(X, Y) = [TX, TY] + T^2[X, Y] - T[TX, Y] - T[X, TY]$$

for $X$, $Y$ in $TM$. In particular, for $X \in D_1$ and $Z \in D_2$, the above equation on simplification takes the form

$$S(X, Z) = (\nabla_{TX}T)Z - (\nabla_{TZ}T)X + T(\nabla_Z T)X - T(\nabla_X T)Z,$$

which by applying equation (2.18) becomes

$$S(X,Z) = A_{NZ}TX + th(TX,Z) - th(TZ,X) + th(X,Z) - T(A_{NZ}X + th(X,Z))$$

or,

$$S(X,Z) = A_{NZ}TX + th(TX,Z) - th(TZ,X) - TA_{NZ}X. \tag{4.8}$$

**Theorem 4.4.** *If the invariant distribution $D_1$ on a semi-slant submanifold $M$ of a Kenmotsu manifold $\bar{M}$ is integrable and its leaves are totally geodesic in $M$, then*

(i) $h(D_1, D_1) \in \mu$,

(ii) $S(D_1, D_2) \in D_2$.

Proof. By hypothesis, for any $X, Y$ in $D_1$ and $Z$ in $D_2$

$$g(\nabla_X Y, Z) = 0$$

and therefore by Gauss formula

$$g(\bar{\nabla}_X Y, Z) = 0.$$

The above equation on making use of equations (2.4), (2.6) and (2.9). yields,

$$g(h(X,\phi Y), NZ) = 0$$

from which the first assertion follows. To prove the second, consider $g(S(X,Z),Y)$ for $X, Y \in D_1$ and $Z \in D_2$, which by equation (4.8), gives

$$g(S(X,Z),Y) = g(A_{NZ}TX + th(TX,Z) - th(TZ,X) - TA_{NZ}X, Y).$$

Which is equal to zero by part (i) and thus (ii) is established. □

For the slant distribution, we have.

**Theorem 4.5.** *If the slant distribution $D_2$ on a semi-slant submanifold $M$ of a Kenmotsu manifold $\bar{M}$ is integrable and its leaves are totally geodesic in $M$, then*

(i) $h(D_1, D_2) \in \mu$,

(ii) $S(D_1, D_2) \in D_1$.

Proof. By hypothesis,

$$g(\nabla_Z W, \phi X) = 0$$

for each $Z$, $W$ in $D_2$ and $X$ in $D_1$ and thus equations (2.4), (2.6) and (2.9) give

$$g(h(X,Z), NW) = 0.$$

That proves (i). Now by equation (4.8),

$$g(S(X,Z),W) = g(A_{NZ}TX + th(TX,Z) - th(TZ,X) - TA_{NZ}X, W)$$

for $X \in D_1$ and $Z$, $W$ in $D_2$. The right hand side of the above equation is zero by part (i). This proves part (ii). □

On combining Theorems 4.4 and 4.5, we may state:

**THEOREM 4.6.** *If a semi-slant submanifold $M$ of a Kenmotsu manifold $\bar{M}$ is locally a Riemannian product of the leaves of the distributions $D_1$ and $D_2$ then*

$$A_{ND_2}D_1 = 0.$$

**Note.** The above condition may be viewed as an extension of the necessary and sufficient condition for a CR-submanifold to be a CR-product in a Kaehler manifold to the setting of semi-slant submanifold of a Kenmotsu manifold (cf. [5]).

## 5. Totally umbilical and totally contact umbilical submanifolds of Kenmotsu manifold

To investigate totally umbilical semi-slant submanifolds of a Kenmotsu manifold, we first, prove:

**PROPOSITION 5.1.** *Let $M$ be a semi-slant submanifold of a Kenmotsu manifold $\bar{M}$ with $h(X,TX) = 0$ for each $X \in D_1 \oplus \langle\xi\rangle$. If $D_1 \oplus \langle\xi\rangle$ is integrable then each of its leaves is totally geodesic in $M$ as well as in $\bar{M}$.*

Proof. For $X \in D_1 \oplus \langle\xi\rangle$, by equation (2.19),

$$(\nabla_X N)X = -h(X,TX) + nh(X,X)$$

which by virtue of the hypothesis, formula (2.14) and the fact that $NX = 0$ for each $X \in D_1$, yields

$$N\nabla_X X = nh(X,X). \tag{5.1}$$

Now, making use of Corollary 4.1 and the assumption that $h(X,TX) = 0$ we obtain that $h(X,TY) = 0$ or equivalently $h(X,Y) = 0$ for each $X, Y \in D_1 \oplus \langle\xi\rangle$. This proves that the leaves of $D_1 \oplus \langle\xi\rangle$ are totally geodesic in $\bar{M}$. Making use of this fact in equation (5.1), we obtain that $\nabla_X Y \in D_1 \oplus \langle\xi\rangle$ i.e., the leaves of $D_1 \oplus \langle\xi\rangle$ are totally geodesic in $M$. This proves the proposition completely. □

As an immediate consequence of the above Proposition, we obtain the following geometrically significant result.

**Corollary 5.1.** *Let $M$ be a totally umbilical semi-slant submanifold of a Kenmotsu manifold $\bar{M}$. If the invariant distribution $D_1 \oplus \langle\xi\rangle$ is integrable, then its leaves are totally geodesic in $M$ as well as in $\bar{M}$.*

The proof follows immediately because on a totally umbilical submanifold, the second fundamental form satisfies $h(X,Y) = g(X,Y)H$ for all $X$, $Y$ in $TM$, where $H$ is the mean curvature vector.

**Definition 1.** ([10]) A submanifold $M$ of an almost contact metric manifold is said to be *totally contact umbilical submanifold* if

$$h(X,Y) = g(\phi X, \phi Y)K + \eta(Y)h(X,\xi) + \eta(X)h(Y,\xi)$$

for all $X$, $Y$ in $TM$. Where $K$ is a normal vector field on $M$. If $K = 0$ then $M$ is said to be a totally contact geodesic submanifold. For a submanifold of a Kenmotsu manifold, the condition for totally contact umbilicalness reduces to

$$h(X,Y) = g(\phi X, \phi Y)K.$$

**Theorem 5.1.** *Let $M$ be a totally contact umbilical semi-slant submanifold of a Kenmotsu manifold $\bar{M}$, with $\dim(D_1) \neq 0$. Then the mean curvature vector is a global section of $ND_2$.*

Proof. Take $X \in D_1$, a unit vector and $N \in \mu$. Then by definition, □

$$\begin{aligned} g(H,N) &= g(h(X,X),N) \\ &= g(\bar{\nabla}_X \phi X, \phi N) \\ &= g(h(X,\phi X), \phi N) \\ &= 0. \end{aligned}$$

$\implies H \in ND_2$.

Now, we have the following theorem.

**Theorem 5.2.** *A totally contact umbilical semi-slant submanifold of a Kenmotsu manifold is totally contact geodesic if the invariant distribution $D_1$ is integrable.*

Proof. The proof follows immediately by applying Theorem 4.2. □

REFERENCES


[1] BEJANCU, A.—PAPAGHIUC, N.: *Semi-invariant submanifold of a Sasakian manifold*, An. Ştiinţ. Univ. AI. I. Cuza. Iaşi. Mat. (N.S.) **27** (1981), 163–170.

[2] LOTTA, A.: *Slant submanifolds in contact geometry*, Bull. Math. Soc. Romanie **39** (1996), 183–198.

[3] CHEN, B. Y.: *Geometry of Slant Submanifolds*, Katholieke Universiteit Leuven, Leuven, 1990.

[4] CHEN, B. Y.: *Differential geometry of real submanifold in Kaehler manifold*, Monatsh. Math. **91** (1981), 257–274.

[5] CHEN, B. Y.: *CR-submanifolds of Kähler manifold I*, J. Differential Geom. **16** (1981), 305–323.

[6] BLAIR, D. E.: *Contact Manifolds in Riemannian Geometry.* Lecture Notes in Math. 509, Springer-Verlag, New York, 1976.

[7] CABRERIZO, J. L.—CARRIAZO, A.—FERNANDEZ, L. M.—FERNANDEZ, M.: *Semi-slant submanifolds of a Sasakian manifold*, Geom. Dedicata **78** (1999), 183–199.

[8] CABRERIZO, J. L.—CARRIAZO, A.—FERNANDEZ, L. M.—FERNANDEZ, M.: Slant submanifolds in Sasakian manifolds, Glasg. Math. J. **42** (2000).

[9] KENMOTSU, K.: *A class of almost contact Riemannian manifolds*, Tohoku Math. J. **24** (1972), 93–103.

[10] KON, M.: *Remarks on anti-invariant submanifolds of a Sasakian manifold*, Tensor (N.S.) **30** (1976), 239–245.

[11] PAPAGHIUC, N.: *Semi-slant submanifolds of Kahlerian manifold*, An. Ştiinţ. Univ. AI. I. Cuza. Iaşi. Inform. (N.S.) **9** (1994), 55–61.

[12] GUPTA, R. S.—KHURSHEED HAIDER, S. M.—SHAHID, M. H.: *Slant submanifolds of a Kenmotsu manifold*, Rad. Mat. **12** (2004), 205–214.

[13] TANNO, S.: *The automorphism groups of almost contact Riemannian manifolds*, Tohoku. Math. J. **21** (1969), 21–38.


Received 8. 8. 2005


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