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SEVENTY YEARS OF PROFESSOR TIBOR ŠALÁT

Tibor Šalát was born on May 13, 1926 in teacher's family at Vajka on Žitava, in the southern part of Slovakia. He was educated at the grammar school in Zlaté Moravce and continued in Šurany. He studied at the Faculty of Natural Sciences of Charles University in Prague and graduated in 1950. His intention to make his career in mathematics was reinforced by inspirational Czech mathematicians such as E. Čech, V. Jarník and M. Kössler. He spent the subsequent two years at Nové Zámky as a teacher. In 1952, he went at the Faculty of Natural Sciences of Comenius University in Bratislava. In 1962 he was appointed Associate Professor and in 1977 he became Full Professor at the Comenius University, where he works until this time. In 1952 he received RNDr. degree, in 1958 he defended the candidate thesis dealt with the Hausdorff dimension of real sets, and in 1974 he defended his doctoral thesis which concerned expansions of real numbers.

Tibor Šalát is one of the most appreciated Slovak mathematicians from scientific, pedagogical and social point of view. He published more than 120 papers and wrote 11 books and monographs and 10 textbooks and lecture notes, some by himself and some jointly with T. Neubrunn, P. Kostyrko, J. Smítal and others.

Scientific results of Tibor Šalát can be divided into two parts: theory of numbers and theory of real functions. A common feature of his papers is an application of measure and topological methods (probability methods, Lebesgue and Hausdorff measure theory methods, Baire category methods). Papers of Tibor Šalát concern the following areas of number theory: Cantor's expansions, Lüroth's expansions, continued fractions, summation methods, statistical convergence, uniform distribution $\bmod 1$, ratio sets and some parts of elementary theory of numbers. Papers which concern the theory of real functions deal with: generalizations of the notion of continuity (quasi-continuity, cliquishness, somewhat continuity, A-continuity, Denjoy property), functions with closed graphs, symmetric properties of real functions, Darboux property. Some of his results are quoted in monographs [ZB], [V], [Sch], [KN], [G]. We give here a selected description of his work, which is an extension of an account of his contribution to some branches of mathematics, given in [KS] on the occasion of his 60th birthday.

Some papers of Tibor Šalát are devoted to the study of sets W of numbers of the form $\sum\limits_{n=1}^{\infty} \varepsilon_n a_n$, where $\sum\limits_{n=1}^{\infty} a_n$ is a given convergent series with positive terms and $\varepsilon_n \in \{1,-1\}$, $n = 1,2,\dots$. It is known, that Lebesgue measure μ of this sets depends on properties of the series $\sum\limits_{n=1}^{\infty} a_n$. If $a_k > R_k = \sum\limits_{n=1}^{\infty} a_{k+n}$ $(k = 1,2,\dots)$, then $\mu(W) = \lim\limits_{n\to\infty} 2^{n+1}R_n$. The set W was investigated also by other authors (P. Kesava Menon, A. Turowicz and others). Tibor Šalát investigated distribution of factors $+1$ and -1 with respect to Lebesgue and Hausdorff measure. He generalized known results of B. Volkmann, V. Knichal, E. Borel and A. S. Besicovitch concerning dyadic expansions, on the set W. E.g., in [15], it is shown that an analogy of the well known Borel theorem on a distribution of therms in dyadic expansions holds for distributions of factors $+1$ and -1 for a large class of sets W with $\mu(W) = 0$. Obviously, this result is formulated using the notion of Hausdorff dimension. This is an affirmative answer on a problem risen by V. Jarník.

Other papers of Tibor Šalát deal with metric theory of expansions of real numbers. Especially, with continuous fractions, Cantor and Lüroth expansions of real numbers. Foundations of the metric theory of Cantor expansions were formulated by A. Rényi and P. Erdös. Their results are improved, using topological and Hausdorff measure methods in papers [16], [28]. Let $\{q_k\}_{k=1}^{\infty}$ be a sequence of integers with $q_k \ge 2$ for $k = 1,2,\dots$. Any real number $0 < x < 1$ leads to the Cantor series $x = \sum\limits_{k=1}^{\infty} \frac{\varepsilon_k(x)}{q_1q_2\dots q_k}$. Assuming $\sum\limits_{k=1}^{\infty} \frac{q_k}{(q_1q_2\dots q_k)^{\varepsilon}} < \infty$ for every $\varepsilon > 0$, Tibor Šalát, in [16], has given the following explicit formula

$$\dim M_{(a_k)} = \liminf_{n\to\infty} \frac{\log \prod\limits_{k=1}^{n} \min(a_k, q_k)}{\log \prod\limits_{k=1}^{n} q_k}$$

for Hausdorff dimension of the set $M_{(a_k)}$ of all $x \in (0,1)$ satisfying $\varepsilon_k(x) < a_k$, $k = 1,2,\dots$, for given positive integers a_k. This result was extended by H. Wegmann [W]. In [21], certain "metric valuations" of efficiency criteria for irrationality of sums of Cantor's series, which belong to A. Oppenheim (cf. [34]), are given.

Problems on metric theory of continuous fractions and Cantor expansions are investigated in [33],[36],[38],[42]. In [38], he studied a transformation $T\big(\{x_k\}_{k=1}^{\infty}\big) = \sum\limits_{k=1}^{\infty} \frac{[x_kq_k]}{q_1q_2\dots q_k}$ and proved that if $\lim\limits_{n\to\infty} \frac{1}{n} \sum\limits_{k=1}^{n} \frac{1}{q_k} = 0$, then the set $T\big(\{x_k\}_{k=1}^{\infty}\big)$, where $\{x_k\}_{k=1}^{\infty}$ belongs to the family of all uniformly distributed sequences, is a set of full measure in $[0,1]$. The work [G2] of J. Galambos is strongly related to this result.

Foundations of the metric theory of Lüroth's series were established by L. Holzer. A detailed investigation of Lüroth expansions is done in [40] by using probabilistic methods and Lebesgue and Hausdorff measure theory. This paper completes a basic paper of L. Holzer published in 1928. Almost all basic results of papers devoted to Cantor and Lüroth series are quoted in the monograph [G]. A recent quotation of the paper [40] is in [BBDK].

In papers [43] and [52], ratio sets of sets of natural numbers are introduced and investigated. For the set $A \subset \mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers, $R(A)$ denotes the set of all numbers of the form a/b, where $a, b \in A$. A set A is said to be a ratio base for the set $\mathbb{Q}^+$ of all positive rational numbers if $R(A) = \mathbb{Q}^+$ and A is called rationally dense (in the interval $(0,\infty)$) if $R(A)$ is dense in $(0,\infty)$. In [43], it is shown that if the upper asymptotic density of A is 1, then A is rationally dense. In the above mentioned papers, Tibor Šalát used the following method of dyadic numbers of sets $A \subset \mathbb{N}$, which was introduced by German mathematicians (cf. H. H. Ostmann [O; pp. 17, 189–201]): For $A = \{a_1 < a_2 < \dots\} \subset \mathbb{N}$, put $\rho(A) = \sum\limits_{k=1}^{\infty} 2^{-a_k}$. Then ρ is a one-to-one map of the family $\mathcal{U}$ of all infinite subsets A of $\mathbb{N}$ to $(0,1]$. If $\mathcal{S} \subset \mathcal{U}$, then $\rho(\mathcal{S}) = \{\rho(A) : A \in \mathcal{S}\} \subset (0,1]$, and "greatness" of $\rho(\mathcal{S})$ has an influence on a "greatness" of the family $\mathcal{S}$. Tibor Šalát proved, e.g., for the family $\mathcal{S}$ of all ratio bases of $\mathbb{Q}^+$, that the set $\rho(\mathcal{S})$ has full measure and is residual in $(0,1]$ (cf. [43], [52]). The method of dyadic numbers $\rho(A)$ of sets $A \subset \mathbb{N}$ is employed in [60], [66], [68] for an investigation of some additive problems in the theory of numbers.

Some papers of Tibor Šalát are devoted to the study of properties of asymptotic density of sets $A \subset \mathbb{N}$ and to applications of this notion in other areas of mathematics. E.g., the statistical convergence of sequences, which was introduced by H. Fast and I. J. Schoenberg using the notion of asymptotic density is investigated in [71]. Here a distribution of statistically convergent sequences in spaces of sequences is described. In [120] (with A. Schinzel) and [122], Tibor Šalát gave three interesting examples of number-theoretic sequences statistically convergent to zero.

In papers [83], [86] (with R. Tijedeman), and [111] (with M. Paštéka), some new properties of the asymptotic density and measure density are studied. In [77] (with F. Schweiger), from the above point of view, a connection between sets $A \subset \mathbb{N}$ and their g-adic expansions are investigated. (If $A = \{a_k\}_{k=1}^{\infty}$ is an increasing sequence of positive integers, then the real number $\alpha(A) = 0, c_1, c_2, \dots$ arises in such a way that after the symbol 0 the expressions of the numbers $a_1, a_2, \dots$ in g-adic scale successively are written.) If $\mathcal{S}$ denotes the family of all $A \subset \mathbb{N}$ for which $\alpha(A)$ is simply normal, then $\rho(\mathcal{S})$ is a set of the first category in $(0,1]$ of the type $G_{\delta\sigma\delta}$ and its Lebesgue measure is 1.

H. Steinhaus observed that the set of all prime numbers has the following property (S): For every $x \in (0,\infty)$ there exists a sequence $\{q_n\}_{n=1}^{\infty}$ of primes such that $q_n/n \to x$. In paper [89] (with W. Narkiewicz), the family of all sets

$A \subset \mathbb{N}$ with the property (S) is investigated in details and compared with the family of all sets $A \subset \mathbb{N}$ rationally dense in $(0, \infty)$. It is shown, e.g., that every set with the property (S) is rationally dense.

The notion of uniform distribution mod 1 is studied in the papers [69], [97], [98], [102], [106], [107]. In [102] (with Š. Porubský and O. Strauch) a new type of functions, so called uniform distribution preserving functions, is introduced. This paper was a motivation for some other papers, e.g., a generalization of this notion on compact spaces can be found in the paper of R. F. Tichy and R. Winkler [TW].

In the papers which deal with the theory of real functions mostly topological methods and measure theory methods are used. These papers can be divided into several areas.

In the first collection of papers, various classes of functions are investigated ([25], [26], [47], [60], [64], [70]).

Another collection of papers deal with some generalizations of the notion of continuity ([53], [63], [67], [75], [79], [81], [82]). In [53] (with J. S. Lipiński), the set of points of quasicontinuity of real function is characterized: A set E is the set of all quasicontinuity points of a function $f\colon \mathbb{R}^n \to \mathbb{R}$ if and only if $(\operatorname{Int} E^c) - E$ is a set of the first Baire category.

Another papers are devoted to the study of properties of some important functions. E.g., a generalized Banach indicatrix in [55] and [58] is introduced and investigated. Papers [80] and [90] are devoted to the study of the exponent of convergence $\lambda(A)$, which was introduced by G. Pólya and G. Szegö for every unbounded nondecreasing sequence $A = \{a_k\}_{k=1}^{\infty}$ of positive numbers in the following way: $\lambda(A) = \inf\Bigl\{\sigma > 0\,;\ \sum\limits_{k=1}^{\infty} a_k^{-\sigma} < +\infty\Bigr\}$. Let S^+ be the metric space of all sequences of the above mentioned type endowed with the Fréchet metric. The exponent of convergence is a function $\lambda\colon S^+ \to [0, +\infty)$. It is shown that λ is a function of Baire class two, everywhere discontinuous, without the Darboux property.

Some results of Tibor Šalát deal with the structural properties of some functional spaces from the Baire category view-point. E.g., in [91] (with P. Kostyrko), there is shown: Let $b\Delta$ (bA) be the metric space of all bounded derivatives (approximately continuous functions) on the interval $[0, 1]$ furnished with the metric of uniform convergence. Then the set of all functions in $b\Delta$ (bA), the set of discontinuity points of which has full measure, is a residual G_δ-set in $b\Delta$ (bA). In papers [114], [115], [119] (with J. Tóth and L. Zsilinszky), the structure of the space of metrics on a given set is studied.

Tibor Šalát was the editor-in-chief and held a variety of editorial responsibility in *Acta Mathematica Universitatis Comenianae* for many years. Further-more, he is currently on the editorial boards of *Mathematica Slovaca* and *Tatra Mountains Mathematical Publications.*

A great part of Tibor Šalát professional career is connected with his pedagogical activities. He is a born teacher, partly owing to his mastery of the language, but, most of all, he is an excellent teacher because of his extreme clarity of mind and his profound intellectual honesty. During more than forty years he gave almost all courses in mathematical analysis and wrote more than 10 textbooks and lecture notes for students. For many years Tibor Šalát has been leading scientific seminars on the theory of real functions and on the theory of numbers. Further, he was involved in the Ph.D. research of many students and served as the principal supervisor for a number of them. List of his Ph.D. students follows.

For a long time Tibor Šalát was noted as one of the most important persons in the organization of scientific life in Slovakia. At present, his organizing activity includes the directorship of the Slovak Mathematical Society. During his scientific and pedagogical work, Tibor Šalát received many appreciations and awards. Note some of them. In 1980 he was awarded the gold medal of the Comenius University, in 1986 the gold honourable plaquette of Jur Hronec of Slovak Academy of Sciences for his merits in mathematical sciences, and in 1996 he was awarded the Jur Hronec prize of the foundation of *Matica slovenská.*

It is a pleasant fact that Tibor Šalát meets his anniversary in good health and full activity. We thank him for his scientific, pedagogical and organizational work. We all wish Tibor Šalát many more healthy and creative years with a lot of enjoyment from mathematics.

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J. Smítal (1972)
P. Kostyrko (1972)
A. Neubrunnová (1974)
V. László (1979)
O. Strauch (1980)
J. Doboš (1986)
J. Borsík (1987)
M. Paštéka (1992)