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A GEOMETRIC SOLUTION TO THE DYNAMIC DISTURBANCE DECOUPLING FOR DISCRETE–TIME NONLINEAR SYSTEMS

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The notion of controlled invariance under quasi-static state feedback for discrete-time nonlinear systems has been recently introduced and shown to provide a geometric solution to the dynamic disturbance decoupling problem (DDDP). However, the proof relies heavily on the inversion (structure) algorithm. This paper presents an intrinsic, algorithm-independent, proof of the solvability conditions to the DDDP.

Keywords: controlled invariance, dynamic state feedback, disturbance decoupling, differential forms

AMS Subject Classification: 93C10, 93C55, 93B25, 58A10

1. INTRODUCTION

The key concept in the geometric solution of the regular static disturbance decoupling problem (DDP), both in continuous-time and discrete-time, is the so-called largest controlled invariant distribution $\mathcal{D}^*$, contained in the kernel of the output map [16]. In the discrete-time case, invariant distributions were first studied in [8, 15]. In the definition of $\mathcal{D}^*$, invariance is considered under regular static state feedback.

Recently, for continuous-time systems, a generalized notion of controlled invariance has been introduced under the enlarged class of quasi-static state feedback transformations [10, 11], and has shown to be useful to derive a geometric solution to the dynamic disturbance decoupling problem (DDDP). The proposed geometric solution to the DDDP is completely parallel to the solution of the static disturbance decoupling problem: the only difference in the solvability conditions is that the classical controlled invariant codistribution is replaced by the generalized controlled invariant subspace.

In [3] a discrete-time analogue of the notion of controlled invariance under quasi-static state feedback was given and shown to provide a geometric solution of the DDDP. However, the proof presented in [3] relies heavily on the inversion algorithm. Though this algorithm-based proof provides an explicit way to compute the smallest controlled invariant subspace, our goal in this paper is to obtain an intrinsic algorithm-independent proof. The proof follows the same line of reasoning as in [11].

For other approaches to the DDDP in discrete-time, see [1] (a linear algebraic solution), [13] (a structural solution), [6, 12, 14] (an inversion algorithm based solution).

The rest of the paper is organized as follows. The problem statement is given in Section 2. Section 3 recalls the notion of generalized controlled invariance with respect to quasi-static state feedback. An intrinsic algorithm-independent proof of the solvability conditions of the DDDP is given in Section 4. In Section 5 we present some examples, discuss differences between the continuous-time and discrete-time solutions and between linear and nonlinear solutions. Moreover, we discuss some properties of the quasi-static state feedback for systems with disturbances. Finally, the restrictiveness of the submersivity assumption of the control system is discussed here, and the means how this assumption can be relaxed, are suggested.

2. PRELIMINARIES

Consider a discrete-time nonlinear system Σ, described by equations of the form

$$\Sigma : \left\{ \begin{array}{rcl} x(t+1) & = & f(x(t), u(t), w(t)), \ x(0) = x_0, \\ y(t) & = & h(x(t)), \end{array} \right. \tag{1}$$

where the state $x(t) \in \mathbb{R}^n$, the control $u(t) \in \mathbb{R}^m$, the disturbance $w(t) \in \mathbb{R}^r$ and the output $y(t) \in \mathbb{R}^p$. The mappings $f(\cdot)$ and $h(\cdot)$ are supposed to be real analytic. Throughout the paper it is also assumed that the mapping $f(\cdot)$ generically defines a submersion, i. e. that generically

$$\operatorname{rank} \frac{\partial f(x,u,w)}{\partial(x,u)} = n.$$

The algebraic framework, that we describe below, was formulated by Grizzle [9] for discrete-time nonlinear systems. This framework is related with Fliess' difference-algebraic approach [7] and has been modified in [2] to end up with an inversive difference field.

Let $\mathcal{K}$ be the field of meromorphic functions of a finite number of the variables of the following (infinite) set $\{x(0), u(t), w(t), t \geq 0\}$. Denote by $\mathcal{E}$ the formal vector space spanned by the differentials of the elements of $\mathcal{K}$; that is, $\mathcal{E} := \operatorname{span}_{\mathcal{K}}\{\mathrm{d}\varphi \mid \varphi \in \mathcal{K}\}$. The elements of $\mathcal{E}$ are called differential forms of order one, or simply one-forms. The space $\mathcal{E}$ can be decomposed into the direct sum of three subspaces, $\mathcal{E} = \mathcal{X} \oplus \mathcal{U} \oplus \mathcal{W}$, where $\mathcal{X} := \operatorname{span}_{\mathcal{K}}\{\mathrm{d}x(0)\}$, $\mathcal{U} := \operatorname{span}_{\mathcal{K}}\{\mathrm{d}u(k),\ k \geq 0\}$, $\mathcal{W} := \operatorname{span}_{\mathcal{K}}\{\mathrm{d}w(k),\ k \geq 0\}$. Define the difference output space $\mathcal{Y} := \operatorname{span}_{\mathcal{K}}\{\mathrm{d}y(k),\ k \geq 0\}$. By (1), $\mathcal{Y} \subset \mathcal{E} = \mathcal{X} \oplus \mathcal{U} \oplus \mathcal{W}$.

The forward-shift operator $\delta : \mathcal{K} \to \mathcal{K}$ is defined by

$$\delta\varphi[x(0), u(j), w(k)] = \varphi[f(x(0), u(0), w(0)), u(j+1), w(k+1)].$$

The operator δ induces a forward-shift operator $\Delta : \mathcal{E} \to \mathcal{E}$ by

$$\sum_i a_i \mathrm{d}\varphi_i \mapsto \sum_i (\delta a_i)\mathrm{d}(\delta\varphi_i), \quad a_i, \varphi_i \in \mathcal{K}.$$

Given a function $\varphi \in \mathcal{K}$ and a one-form $\omega \in \mathcal{E}$, sometimes we use the abridged notations $\varphi^+ = \delta\varphi$ and $\omega^+ = \Delta\,\omega$.

The compensator used to control the system Σ is a dynamic state feedback C described by equations of the form

$$C \; : \; \left\{ \begin{array}{rcl} z(t+1) & = & \psi(z(t), x(t), \nu(t)), \;\; z(0) = z_0, \\ u(t) & = & \varphi(z(t), x(t), \nu(t)), \end{array} \right. \tag{2}$$

with state $z(t) \in \mathbb{R}^q$, with new control $\nu(t) \in \mathbb{R}^m$ and with real analytic $\psi(\cdot)$ and $\varphi(\cdot)$.

We call the compensator C described by equations (2) regular if the nonlinear control system

$$\begin{array}{rcl} x(t+1) & = & f(x(t), \varphi(z(t), x(t), \nu(t)), w(t)) \\ z(t+1) & = & \psi(z(t), x(t), \nu(t)) \\ u(t) & = & \varphi(z(t), x(t), \nu(t)), \end{array} \tag{3}$$

with inputs $\nu(t)$ and outputs $u(t)$ is invertible; see [7, 9] for details about the notion of invertibility.

The closed-loop system (1) – (2), initialized at (x_0, z_0) is denoted by $\Sigma \circ C$. For the study of the closed-loop system $\Sigma \circ C$, we need to consider the field of meromorphic functions of a finite number of the variables $\{x(0), z(0), \nu(t), w(t), t \geq 0\}$. By abuse of notation, we use the same symbol $\mathcal{K}$ to denote this new field. Notice that the invertibility of the compensator C implies that there exists a (x, z, w)-dependent bijection between the variables $u(t)$ and $\nu(t)$. Therefore, $\nu(t)$ and $u(t)$ can be used indistinctly in the definition of $\mathcal{K}$. For the closed-loop system $\Sigma \circ C$, define $\mathcal{Z} = \mathrm{span}_{\mathcal{K}}\{\mathrm{d}z(0)\}$, $\mathcal{V} = \mathrm{span}_{\mathcal{K}}\{\mathrm{d}\nu(k),\; k \geq 0\}$, and $\mathcal{Y}^* = \mathrm{span}_{\mathcal{K}}\{\mathrm{d}y(k),\; k \geq 0\}$.

Definition 1. (Dynamic disturbance decoupling problem DDDP.) Find, if possible, a regular dynamic state feedback of the form (2) such that the difference output space $\mathcal{Y}^*$ of the closed-loop system $\Sigma \circ C$ satisfies $\mathcal{Y}^* \subset \mathcal{Z} \oplus \mathcal{X} \oplus \mathcal{V}$.

3. GENERALIZED CONTROLLED INVARIANCE

Consider the discrete-time nonlinear system Σ_0 which is obtained from Σ by setting $w(t) \equiv 0$, for $t \geq 0$, that is:

$$\Sigma_0 \; : \; \left\{ \begin{array}{rcl} x(t+1) & = & f(x(t), u(t)), \;\; x(0) = x_0, \\ y(t) & = & h(x(t)). \end{array} \right. \tag{4}$$

A feedback of the form

$$u(t) = \varphi[x(t), v(t), v(t+1), \ldots, v(t+\beta)], \tag{5}$$

where β is a finite nonnegative integer and dim $u =$ dim v is said to be a quasi-static state feedback for (4) if there exist an integer $\gamma \geq 0$, and a map ξ such that, locally,

$$v(t) = \xi[x(t), u(t), u(t+1), \ldots, u(t+\gamma)]. \tag{6}$$

When $\beta = \gamma = 0$, this notion reduces to a regular static state feedback.

Remark 1. The class of quasi-static state feedbacks may be considered as a mathematical tool used to describe intrinsic properties of the system under dynamic feedback, computed on the basis of the inversion algorithm, rather than a new class of compensators which are used in practical applications [11]. As a matter of fact, a quasi-static state feedback of the form (5) may be put in the form (2) defining $z_i(t) = v(t+i-1)$, $1 \leq i \leq \beta$, $v(t+\beta) = \nu(t)$. Conversely, a dynamic state feedback computed using the inversion algorithm has a structure which reduces to (5).

For continuous-time systems the class of quasi-static state feedback was applied first in [17] and formalized in [4, 5]. In the discrete-time case, quasi-static state feedback was first considered in [1] for systems with disturbances.

Denote by Δ_f the forward-shift operator induced by the dynamics of system (4) and by $\Delta_{\tilde{f}}$ the forward-shift operator that corresponds to the dynamics

$$\tilde{f}[x(t), v(t), \ldots, v(t+\beta)] = f[x(t), \varphi(x(t), v(t), \ldots, v(t+\beta))]$$

of the closed-loop system (4) – (5). Given a subspace $\Omega \subset \mathcal{X}$, we define the subspace $\Delta_f \Omega$ by $\Delta_f \Omega = \mathrm{span}_{\mathcal{K}}\{\Delta_f \omega \mid \omega \in \Omega\}$. Subspace $\Delta_{\tilde{f}} \Omega$ is defined in a similar manner.

The following definitions are the discrete-time analogues of Definitions 2.1 and 3.7 in [10]:

Definition 2. A subspace $\Omega \subset \mathcal{X}$ is said to be an invariant subspace of (4) if $\Delta_f \Omega \subset \Omega + \mathrm{span}_{\mathcal{K}}\{\mathrm{d}u(0)\}$.

Definition 3. A subspace $\Omega \subset \mathcal{X}$ is said to be a controlled invariant subspace of (4) with respect to quasi-static state feedback if there exists a quasi-static state feedback (5), such that for (4) – (5) one has $\Delta_{\tilde{f}} \Omega \subset \Omega + \mathcal{V}$.

4. GEOMETRIC SOLUTION OF THE DDDP

In this section we give an intrinsic, algorithm-independent, proof for Theorem 4 in [3]. In order to state our main Theorem, we define $\mathcal{Y}_0 = \mathrm{span}_{\mathcal{K}}\{\mathrm{d}y(0)\}$.

Theorem 1. The DDDP is solvable if and only if there exists a controlled invariant subspace $\Omega \subset \mathcal{X}$ such that

$$\mathcal{Y}_0 \subset \Omega_{f(x,u,w)} \subset \mathrm{span}_{\mathcal{K}} \left\{ \frac{\partial f}{\partial w} \right\}^{\perp}_{(x,u,w)} \tag{7}$$

Proof. (*Sufficiency*) Since DDP is solvable by dynamic state feedback if and only if it is solvable by quasi-static state feedback [3], to prove the theorem, it suffices to show that the DDP is solvable by quasi-static state feedback.

Controlled invariance of Ω implies that there exists a quasi-static state feedback of the form (5) such that for the compensated dynamics

$$\Delta_{\tilde{f}}\Omega \subset \Omega + \mathcal{V}. \tag{8}$$

By (7) and (8) one has that for the compensated system for $\forall t \geq 0$

$$\mathrm{d}y(t) \subset \Omega + \mathcal{V}.$$

Thus in the closed loop system the output y is decoupled from the disturbance.

(*Necessity*) Suppose that the quasi-static state feedback of the form (5) solves the DDP. Then for the closed-loop system on has for $\forall t \geq 0$

$$\mathrm{d}y(t) \subset \mathrm{span}_{\mathcal{K}}\{\mathrm{d}x(0), \mathrm{d}v(0), \ldots, \mathrm{d}v(\beta + t - 1)\}.$$

Define the sequence Ω_μ as

$$\Omega_0 = \left\{\frac{\partial f}{\partial w}\right\}^{\perp}$$

$$\Omega_{\mu+1} = \mathrm{span}_{\mathcal{K}}\{\omega \in \Omega_\mu \mid \Delta_{\tilde{f}}\,\omega \in \Omega_\mu + \mathcal{V}\}, \quad \mu \geq 1$$

and

$$\Omega = \lim_{\mu \to \infty} \Omega_\mu.$$

Notice that, by definition, $\Omega_{\mu+1} \subset \Omega_\mu$. Since Ω_0 is finite dimensional, the sequence $\{\Omega_\mu\}$ necessarily stabilizes. Therefore, Ω is well-defined.

Obviously $\Delta_{\tilde{f}}\Omega \subset \Omega + \mathcal{V}$. Thus, Ω is a controlled invariant subspace. By construction, Ω is the subspace decoupled from the perturbation $\mathrm{d}w$. On the other hand, by the assumption the one-forms $\mathrm{d}y(t)$ of the closed-loop system are independent of the perturbation $\mathrm{d}w$, for all $t \geq 0$. Then, necessarily, $\mathrm{span}_{\mathcal{K}}\{\mathrm{d}y(0)\} \subset \Omega$ and $\Omega \subset \left\{\frac{\partial f}{\partial w}\right\}^{\perp}$, and so (7) also holds. □

Condition (7) in Theorem 1 is not constructive. The corresponding constructive condition is obtained when considering the smallest controlled invariant subspace Ω^* containing $\mathcal{Y}_0 = \mathrm{span}_{\mathcal{K}}\{\mathrm{d}y(0)\}$. In [3] it has been proved, that $\Omega^* = \mathcal{X} \cap \mathcal{Y}$. An immediate consequence of Theorem 1 is then the following theorem.

Theorem 2. The DDDP is solvable if and only if

$$\Omega^*_{f(x,u,w)} \subset \mathrm{span}_{\mathcal{K}}\left\{\frac{\partial f}{\partial w}\right\}^{\perp}_{(x,u,w)}. \tag{9}$$

5. DISCUSSION AND EXAMPLES

5.1. Differences with respect to the continuous-time case

The geometric solvability condition of the DDDP (9) differs from its continuous-time analogue [10, 11], as well as from the linear case, in the following manner: the coefficients of the one-forms which define the subspace Ω^* must not be evaluated at time instant t, but at time instant $t+1$.

The difference between the continuous-time and the discrete-time cases can be explained because the time-differentiation operator $\frac{\mathrm{d}}{\mathrm{d}t}$ obeys Leibnitz rule, while the forward-shift operator Δ is a homomorphism. Therefore, both operators act differently on one-forms.

For instance, consider the one-form $\omega = \sum_i a_i \mathrm{d}x_i = a\mathrm{d}x$. The time-derivative of the form ω along the trajectories of the continuous-time system $\dot{x} = f(x,u,w)$ and the forward-shift of the form ω along the trajectories of the discrete-time system $x^+ = f(x,u,w)$ are given, respectively, by

$$\dot{\omega} = \dot{a}\,\mathrm{d}x + a\left[\frac{\partial f}{\partial x}\mathrm{d}x + \frac{\partial f}{\partial u}\mathrm{d}u + \frac{\partial f}{\partial w}\mathrm{d}w\right],$$

and

$$\omega^+ = a^+\left[\frac{\partial f}{\partial x}\mathrm{d}x + \frac{\partial f}{\partial u}\mathrm{d}u + \frac{\partial f}{\partial w}\mathrm{d}w\right].$$

It can be seen that the one-forms $\dot{\omega}$ and ω^+ are independent of the disturbance $\mathrm{d}w$ if and only if the row vector a, respectively a^+, annihilate the matrix $\frac{\partial f}{\partial w}$, whence the difference arises.

The difference from the linear case stems from the fact that in the case of linear systems, the notion of a one-form reduces to a constant row vector. In the example above, if the coefficients a_i are constant, then $a^+ = a$.

This is also the reason why the necessity of considering Ω^* at x^+ is not apparent for all nonlinear examples. The next example demonstrates, that in general, one has to be careful at which point Ω^* should be evaluated.

Example 1. Consider the system

$$\begin{aligned}
x_1(t+1) &= u_1(t)\\
x_2(t+1) &= x_3(t)u_1(t) + x_2(t)x_4(t) - x_1(t)x_5(t)\\
x_3(t+1) &= u_2(t)\\
x_4(t+1) &= x_1(t)w(t)\\
x_5(t+1) &= x_2(t)w(t)\\
y_1(t) &= x_1(t)\\
y_2(t) &= x_2(t).
\end{aligned}$$

For this system, $\Omega^* = \mathrm{span}_{\mathcal{K}}\{\mathrm{d}x_1, \mathrm{d}x_2, u_1\mathrm{d}x_3 + x_2\mathrm{d}x_4 - x_1\mathrm{d}x_5\}$ and

$$\frac{\partial f}{\partial w} = (0,0,0,x_1,x_2)^T.$$

We easily find that for $\omega_3 = u_1 \mathrm{d}x_3 + x_2 \mathrm{d}x_4 - x_1 \mathrm{d}x_5$

$$\omega_3(x)\frac{\partial f}{\partial w} = x_1x_2 - x_2x_1 = 0;$$

however, the DDDP is not solvable. The reason is that

$$\omega_3(x^+)\frac{\partial f}{\partial w} = -u_1x_2 + x_1x_3u_1 + x_1x_2x_4 - x_1^2x_5 \neq 0.$$

5.2. The role of the submersitivity assumption

Throughout the paper we have assumed that the discrete-time control system under study is submersive. The forward-shift operator δ might not be well defined for nonsubmersive systems, as it is illustrated by the following example

Example 2. Consider the discrete-time nonlinear system

$$\begin{array}{rcl} x_1(t+1) & = & x_2(t) \\ x_2(t+1) & = & -x_1(t) \\ x_3(t+1) & = & x_1(t)x_2(t). \end{array} \tag{10}$$

System (10) is not submersive, because rank of the Jacobian matrix

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ x_2 & x_1 & 0 \end{bmatrix}$$

is equal to 2.

Define the function $\mu = \frac{1}{x_3 + x_1x_2}$. Straightforward computations show that μ^+ is not defined. Moreover, if we define $\eta = x_3 + x_1x_2$, then $\delta\eta = 0$ does not necessarily imply $\eta = 0$, and therefore, the forward-shift operator δ is not one-to-one.

The effects are, however, shortlived and can be eliminated from the analysis by ignoring the first n (at maximum) time instances which means that submersitivity assumption can be relaxed. However, we have chosen to keep this technical assumption in order to keep the proofs shorter and more transparent (see the proofs for nonsubmersive systems, related to invertibility problem in [9]).

Example 3. Consider the nonsubmersive system

$$\begin{array}{rcl} x_1(t+1) & = & u_1(t) + x_4(t)w(t) \\ x_2(t+1) & = & x_3(t)u_1(t) \\ x_3(t+1) & = & u_2(t) + x_7(t) \\ x_4(t+1) & = & x_5(t) - x_6(t) \\ x_5(t+1) & = & x_6(t) \\ x_6(t+1) & = & x_6(t) \\ x_7(t+1) & = & x_7(t)w(t) \\ y_1(t) & = & x_1(t) \\ y_2(t) & = & x_2(t) \end{array}$$

The solvability conditions of Theorem 1 (alternatively Theorem 2) are *not satisfied* since

$$y_1(t+1) = u_1(t) + x_4(t)w(t)$$

and

$$y_2(t+1) = x_3(t)[y_1(t+1) - x_4(t)w(t)]$$

depend on disturbance $w(t)$. However, the effect of the nilpotent part of the system on the output sequence is shortlived and can be eliminated from the analysis by ignoring the first n time instances. The compensator

$$\begin{aligned} z(t+1) &= v_1(t) \\ u_1(t) &= z(t) \\ u_2(t) &= v_2(t)/v_1(t) - x_7(t) \end{aligned}$$

will solve the DDDP.

5.3. Properties of quasi-static state feedbacks

A minor mistake in a previous publication is clarified in this section. Indeed, equation (13) in [1], which reflects the properties of the quasi-static state feedback, contains a mistake: despite of the fact that $u(t)$ does not depend on $w(t)$, $v(t)$ may still depend on $w(t)$; see Example 4 below. Equations (5) and (6) do not depend on $w(t)$ because they are defined for system Σ_0, which is obtained by setting $w(t) = 0$.

Example 4. Consider the system

$$\begin{aligned} x_1(t+1) &= x_2(t) + u_1(t) \\ x_2(t+1) &= x_3(t) + w(t) \\ x_3(t+1) &= x_4(t) + u_1(t) \\ x_4(t+1) &= x_1(t)x_2(t) + u_2(t) + w(t) \\ y_1(t) &= x_1(t) \\ y_2(t) &= x_3(t). \end{aligned}$$

This example illustrates the fact that though the feedback of the form (5) that solves the disturbance decoupling problem does not depend on the disturbances $w(t)$,

$$\begin{aligned} u_1(t) &= v_1(t) - x_2(t) \\ u_2(t) &= v_2(t) - v_1(t+1) - x_1(t)x_2(t) + x_3(t), \end{aligned}$$

the equation (6), reflecting the properties of the feedback, may still depend on $w(t)$

$$\begin{aligned} v_1(t) &= u_1(t) + x_2(t) \\ v_2(t) &= u_2(t) + u_1(t+1) + x_1(t)x_2(t) + w(t). \end{aligned}$$

6. CONCLUSIONS

Recently the notion of controlled invariance under quasi-static state feedback was given and shown to provide a geometric solution of the dynamic disturbance decoupling problem. However, the proof relies heavily on the inversion (structure) algorithm. This paper presents an intrinsic algorithm-independent proof for the geometric solution. Moreover, several worked examples have been presented to clarify the differences of the discrete-time case with respect to the continuous-time and the linear cases. Also, the role of the submersivity assumption is put forward, and some insights about its relaxation are suggested.

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REFERENCES

[1] E. Aranda-Bricaire and Ü. Kotta: Dynamic disturbance decoupling for discrete-time nonlinear systems: a linear algebraic solution. In: Proc. IFAC Conference on System Structure and Control, Nantes, France, July 1995, pp. 155–160.

[2] E. Aranda-Bricaire, Ü. Kotta, and C. Moog: Linearization of discrete-time systems. SIAM J. Control Optim. *34* (1996), 1999–2023.

[3] E. Aranda-Bricaire and Ü. Kotta: Generalized controlled invariance for discrete-time nonlinear systems with an application to the dynamic disturbance decoupling problem. IEEE Trans. Automat. Control *46* (2001), 165–171.

[4] E. Delaleau and M. Fliess: Algorithme de structure, filtrations et découplage. C. R. Acad. Sci. Paris *315* (1992), Serie I, 101–106.

[5] E. Delaleau and M. Fliess: An algebraic interpretation of the structure algorithm with an application to feedback decoupling. In: Nonlinear Control Systems Design - Selected Papers from the 2nd IFAC Symposium (M. Fliess, ed.), Pergamon Press, Oxford 1993, pp. 489–494.

[6] T. Fliegner and H. Nijmeijer: Dynamic disturbance decoupling for nonlinear discrete-time systems. In: Proc. 33rd IEEE Conference on Decision and Control, Buena Vista, Florida 1994, Volume 2, pp. 1790–1791.

[7] M. Fliess: Automatique en temps discret et algèbre aux différences. Forum Mathematicum *2* (1990), 213–232.

[8] J. W. Grizzle: Controlled invariance for discrete-time nonlinear systems with an application to the disturbance decoupling problem. IEEE Trans. Automat. Control *30* (1985), 868–873.

[9] J. W. Grizzle: A linear algebraic framework for the analysis of discrete-time nonlinear systems. SIAM J. Control Optim. *31* (1993), 1026–1044.

[10] H. J. C. Huijberts and C. H. Moog: Controlled invariance of nonlinear systems: nonexact forms speak louder than exact forms. In: Systems and Networks: Mathematical Theory and Application, Volume II (U. Helmke, R. Mennicken, and J. Saurer, eds.), Akademie Verlag, Berlin 1994, pp. 245–248.

[11] H. J. C. Huijberts, C. H. Moog, and R. Andiarti: Generalized controlled invariance for nonlinear systems. SIAM J. Control Optim. *35* (1997), 953–979.
[12] Ü. Kotta: Dynamic disturbance decoupling for discrete-time nonlinear systems: the nonsquare and noninvertible case. Proc. Estonian Academy of Sciences. Phys. Math. *41* (1992), 14–22.
[13] Ü. Kotta: Dynamic disturbance decoupling for discrete-time nonlinear systems: a solution in terms of system invariants. Proc. Estonian Academy of Sciences Phys. Math. *43* (1994), 147–159.
[14] Ü. Kotta and H. Nijmeijer: Dynamic disturbance decoupling for nonlinear discrete-time systems (in Russian). Proc. Academy of Sciences of USSR,. Technical Cybernetics, 1991, pp. 52–59.
[15] S. Monaco and D. Normand-Cyrot: Invariant distributions for discrete-time nonlinear systems. Systems Control Lett. *5* (1984), 191–196.
[16] H. Nijmeijer and A. van der Schaft: Nonlinear Dynamical Control Systems. Springer-Verlag, Berlin 1990.
[17] A. M. Perdon, G. Conte, and C. H. Moog: Some canonical properties of nonlinear systems. In: Realization and Modeling in System Theory (M. A. Kaashoek, J. H. van Schuppen, and A. C. M. Ran, eds.), Birkhäuser, Boston 1990, pp. 89–96.

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