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SELF–TUNING CONTROLLERS BASED ON ORTHONORMAL FUNCTIONS

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Problems of the system identification using orthonormal functions are discussed and algorithms of computing parameters of the discrete time state-space model of the plant based on the generalized orthonormal functions and the Laguerre functions are derived. The adaptive LQ regulator and the predictive controller based on the Laguerre function model are also presented. The stability and the robustness of the closed loop using the predictive controller are investigated.

1. INTRODUCTION

During the past decades several adaptive control systems have been developed. Most of the systems are model based. In particular, in the input-output case, ARMA or FIR models have been widely used. Recently, the search for robust adaptive control systems which require a minimal *a priori* information has led to the development of unstructured adaptive control. Using this approach, the conventional ARMA or FIR model is replaced by an orthonormal series representation of the plant dynamics. The main advantage of this approach is that any stable plant can be modeled without knowledge of the structure, for example without assumption about the true plant order or time delay. The Laguerre functions are usually used to model plant dynamics because of their simplicity and their similarity to transient signals, however, the other orthonormal functions may be used, such as the Laning–Battin functions. Compared with ARMA or FIR model, using of the model based on orthonormal series representation has several advantages: good approximation of a system with a delay; tolerance to unmodeled dynamics and a reduced sensitivity of the estimated parameters; convenient filter network realization; good low frequency match between the estimated model and the true plant model, etc.

One of the main problem when currently identifying plants using orthonormal series representation is to express the model in the discrete time domain. In this paper, memory saving algorithms for computing the discrete state-space orthonormal models are presented.

One of the regulators, the parameters of which can be computed using the linear state-space model, is an LQ regulator. Using receding horizon LQ regulator several iterations of the Riccati difference equation have to be made. In this article, fast and memory saving algorithm of the adaptive LQ regulator, which is based on the orthonormal Laguerre model, is proposed.

Predictive controllers are based on predicting the process behavior, and result from the minimization of a cost function over a future time horizon and under certain process constraints. The cost function is defined in terms of tracking error, i. e. the difference between the predicted output and the desired set-point. In the past decade, several predictive control laws based on the Laguerre model have been proposed, see e. g. [3, 4, 5, 6, 7, 8, 11, 13]. Their major advantage is simplicity of use, intuitive appeal, and easy handling of non-minimum phase behavior. In this paper, we also present a simple predictive controller based on the Laguerre orthonormal discrete state-space model. In addition, the analysis of the stability and robustness of the closed loop using this predictive controller are discussed.

2. PROBLEM FORMULATION FOR MODELING

The Laguerre functions, a complete orthonormal set in $L_2\langle 0,\infty)$, have been often used because of their convenient network realization and their similarity to transient signals. They may be expressed by

$$l_i(t) = \sqrt{2p}\frac{\exp(pt)}{(i-1)!}\frac{d^{i-1}}{dt^{i-1}}\left[t^{i-1}\exp(-2pt)\right], \quad i = 1,\dots,n$$

where i is the function order and p is the time scale. A convenient representation of the Laguerre function in the frequency domain is given by

$$L_i(s) = \sqrt{2p}\frac{(s-p)^{i-1}}{(s+p)^i}, \quad i = 1,\dots,n. \tag{2.1}$$

For a given real continuous function $q(t)$, there exist a real number ϵ and an integer N such that

$$\int_0^\infty |q(t) - S(t)|^2 \, \mathrm{d}t < \epsilon$$

where

$$S(t) = \sum_{i=1}^{N} c_i l_i(t).$$

The coefficients c_i are called the Laguerre spectra gains and for the deterministic signals they can be computed by

$$c_i = \int_0^\infty q(t) l_i(t) \, \mathrm{d}t.$$

In practice the Laguerre spectra gains c_i are estimated using the least-squares parameter estimation.

The Laning–Battin functions can be described in the continuous time domain by

$$w_i(t) = \sum_{j=1}^{i} q_{ij} \exp(-ipt), \quad i = 1,\dots,n$$

where

$$q_{ij} = \sqrt{2pi\frac{2j}{i+j}\prod_{k=1 k\neq j}^{i}\frac{j+k}{j-k}}.$$

The Laplace transform of these functions is

$$W_i(s) = \sqrt{2ip}\frac{1}{s+ip}\prod_{k=1}^{i-1}\frac{s-kp}{s+kp}. \tag{2.2}$$

Comparing Laplace transforms of the Laguerre (2.1) and the Laning–Battin functions (2.2) a new set of generalized orthonormal functions can be found [9]. In the frequency domain they can be given by

$$Z_i(s) = \sqrt{2[1+(i-1)\,m]\,p}\frac{1}{s+[1+(i-1)\,m]\,p}\prod_{k=1}^{i-1}\frac{s-[1+(k-1)\,m]\,p}{s+[1+(k-1)\,m]\,p}$$

where $m \in \langle 0, \infty)$. In the continuous time domain they can be expressed by

$$z_i(t) = \exp(-pt)\sum_{k=1}^{i} q_{ik}\exp(mkpt)$$

where

$$q_{ik} = \sqrt{2[1+(i-1)\,m]\,p}\frac{1}{m^{i-1}}\frac{\sum_{j=0}^{i-2}[2+(k+j)\,m]}{\sum_{j=0,j\neq i}^{i-1}k-j}.$$

To estimate parameters of the discrete state-space model it is necessary to express the generalized orthonormal functions in the discrete time domain. The continuous state-space model based on generalized orthonormal functions can be written as

$$\begin{aligned}\frac{\mathrm{d}\bar{x}(t)}{\mathrm{d}t} &= \bar{A}\bar{x}(t) + \bar{b}u(t)\\ \bar{z}(t) &= \bar{F}\bar{x}(t)\\ y(t) &= \bar{c}\bar{z}(t)\end{aligned}$$

where $\bar{A}$ is a lower triangular $n \times n$ matrix of the form

$$\bar{A} = \begin{bmatrix} -p & 0 & 0 & \dots & 0 \\ -2p & -(1+m)\,p & 0 & \dots & 0 \\ -2p & -2(1+m)\,p & -(1+2m)\,p & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -2p & -2(1+m)\,p & -2(1+2m)\,p & \dots & -(1+(n-1)\,m)\,p \end{bmatrix}$$

$$= \begin{bmatrix} a_1 & 0 & 0 & \dots & 0 \\ 2a_1 & a_2 & 0 & \dots & 0 \\ 2a_1 & 2a_2 & a_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2a_1 & 2a_2 & 2a_3 & \dots & a_n \end{bmatrix}$$

$\bar{b}$ is the n-dimensional vector in the form

$$\bar{b} = [\ 1 \quad 1 \quad 1 \quad \ldots \quad 1\ ]^T \tag{2.3}$$

$\bar{F}$ is the diagonal $n \times n$ matrix in the form

$$\bar{F} = \begin{bmatrix} \sqrt{2p} & 0 & \ldots & 0 \\ 0 & \sqrt{2(1+m)p} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sqrt{2(1+(n-1)m)p} \end{bmatrix} \tag{2.4}$$

and $\bar{c}$ is the estimated n-dimensional row vector. The discrete state-space model based on the generalized orthonormal functions can be expressed by

$$\begin{aligned} \bar{x}(k+1) &= \bar{E}\bar{x}(k) + \bar{g}u(k) \\ \bar{z}(k) &= \bar{F}\bar{x}(k) \\ y(k) &= \bar{c}\bar{z}(k). \end{aligned}$$

Using the matrix $\bar{A}$ and the vector $\bar{b}$, the matrix $\bar{E}$ and the vector $\bar{g}$ can be written as

$$\begin{aligned} \bar{E} &= \exp(\bar{A}T) \\ \bar{g} &= (\bar{E} - \bar{I})\bar{A}^{-1}\bar{b} \end{aligned} \tag{2.5}$$

where T is the sample period, $\bar{I}$ is the $n \times n$ identity matrix and $\exp(\bar{A}T)$ is the exponential matrix function

$$\exp(\bar{A}T) = \sum_{i=0}^{\infty} \frac{(\bar{A}T)^i}{i!}. \tag{2.6}$$

Because of the limited precision of the computer arithmetic only the first k elements of the exponential matrix function (2.6) can be used to compute the matrix $\bar{E}$. It is easy to show that the matrix $\bar{E}$ is a lower triangular matrix in the form

$$\bar{E} = \begin{bmatrix} e_{11} & 0 & 0 & \ldots & 0 \\ e_{21} & e_{22} & 0 & \ldots & 0 \\ e_{31} & e_{32} & e_{33} & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e_{n1} & e_{n2} & e_{n3} & \ldots & e_{nn} \end{bmatrix}.$$

The exponential series can be rewritten as

$$\exp(\bar{A}T) = \sum_{i=0}^{\infty} \frac{\bar{S}^i}{i!} \tag{2.7}$$

where $\bar{S}^i = (AT)^i$ are lower triangular matrices in the form

$$\bar{S}^i = \begin{bmatrix} s_{11}^i & 0 & 0 & \dots & 0 \\ s_{21}^i & s_{22}^i & 0 & \dots & 0 \\ s_{31}^i & s_{32}^i & s_{33}^i & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_{n1}^i & s_{n2}^i & s_{n3}^i & \dots & s_{nn}^i \end{bmatrix}.$$

The elements of the matrices $\bar{S}^i$, which are not equal to zero, can be computed using the recursive formula

$$s_{ij}^{k+1} = -(1+(j-1)m)\,pT\left(s_{ij}^k + 2\sum_{l=j+1}^{i} s_{il}^k\right), \tag{2.8}$$

$$i = n, \dots, 1,\ j = 1, \dots, i,\ k = 1, \dots, \infty,\ \bar{S}^1 = \bar{A}T.$$

To compute the elements of the matrix $\bar{E}$ we introduce auxiliary matrices $\bar{H}^k$, elements of which are given by

$$h_{ij}^k = 1 + \sum_{l=1}^{k} \frac{s_{ij}^l}{l!}, \quad i = 1, \dots, n,\ j = 1, \dots, i. \tag{2.9}$$

Using the matrices $\bar{H}^k$ elements of the matrix $\bar{E}$ can be expressed by

$$e_{ij} = \lim_{k \to \infty} h_{ij}^k.$$

Thus, the recurrent relations have been derived to compute the elements of the matrix $\bar{E}$. As stated above, in practical realization it is necessary to carry out only a finite number of iterations using equations (2.8) and (2.9). The number of iterations depends on the time-scale size. An appropriate condition to terminate the iterative process can be

$$|s_{ij}^k| < \epsilon, \quad i = 1, \dots, n,\ j = 1, \dots, i$$

where ϵ is an appropriately chosen small real number.
Multiplying matrices, it can be proved that the inverse matrix corresponding to the matrix $\bar{A}$ is a lower triangular matrix in the form

$$\bar{A}^{-1} = \begin{bmatrix} -1 & 0 & 0 & \dots & 0 \\ \frac{2}{1+m} & -\frac{1}{1+m} & 0 & \dots & 0 \\ -\frac{2}{1+2m} & \frac{2}{1+2m} & -\frac{1}{1+2m} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{2(-1)^n}{1+(n-1)m} & \frac{2(-1)^{n+1}}{1+(n-1)m} & \frac{2(-1)^{n+2}}{1+(n-1)m} & \dots & -\frac{1}{1+(n-1)m} \end{bmatrix} \frac{1}{p}. \tag{2.10}$$

Using (2.3), (2.5) and (2.10) the following expression can be obtained

$$g_i = \left[(-1)^i \frac{e_{ii} - 1}{(1 + (i-1)\,m)} + \sum_{j=1}^{i-1} (-1)^j \frac{e_{ij}}{(1 + (j-1)\,m)} \right] \frac{1}{p}, \quad i = 1, \ldots, n. \quad (2.11)$$

Thus, all the relations necessary for computing the elements of the matrix $\bar{E}$ and the vector $\bar{g}$ of the discrete state-space model based on generalized orthonormal functions have been derived. The algorithm consists of the equations (2.8), (2.9) and (2.11).

An algorithm, which is similar to that described above, can be derived for computing matrices of the model based on Laguerre functions. The continuous time state-space Laguerre model can be written as

$$\begin{aligned} \frac{\mathrm{d}\bar{l}(t)}{\mathrm{d}t} &= \bar{A}\bar{l}(t) + \bar{b}u(t) \\ y(t) &= \bar{c}\bar{l}(t) \end{aligned}$$

where $\bar{A}$ is a lower triangular matrix in the form

$$\bar{A} = \begin{bmatrix} -p & 0 & 0 & \ldots & 0 \\ -2p & -p & 0 & \ldots & 0 \\ -2p & -2p & -p & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -2p & -2p & -2p & \ldots & -p \end{bmatrix} = \begin{bmatrix} a_n & 0 & 0 & \ldots & 0 \\ a_{n-1} & a_n & 0 & \ldots & 0 \\ a_{n-2} & a_{n-1} & a_n & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_1 & a_2 & a_3 & \ldots & a_n \end{bmatrix}$$

and $\bar{b}$ is an n-dimensional vector expressed by

$$\bar{b} = (\begin{array}{ccccc} 1 & 1 & 1 & \ldots & 1 \end{array})^T \sqrt{2p}, \quad (2.12)$$

$\bar{l}$ is the vector of the Laguerre functions and $\bar{c}$ is the estimated vector of the Laguerre spectra gains.

A discrete time state-space Lagurerre model can be expressed by

$$\bar{l}(k+1) = \bar{E}\bar{l}(k) + \bar{g}u(k) \quad (2.13)$$

$$y(k) = \bar{c}\bar{l}(k) \quad (2.14)$$

where $\bar{g}$ is an n-dimensional vector and $\bar{E}$ is a lower triangular $n \times n$ matrix. It is easy to prove that the matrix $\bar{E}$ has the form

$$\bar{E} = \begin{bmatrix} e_n & 0 & 0 & \ldots & 0 \\ e_{n-1} & e_n & 0 & \ldots & 0 \\ e_{n-2} & e_{n-1} & e_n & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e_1 & e_2 & e_3 & \ldots & e_n \end{bmatrix} = \exp(\bar{A}T) = \sum_{i=0}^{\infty} \frac{(\bar{A}T)^i}{i!} = \sum_{i=0}^{\infty} \frac{\bar{S}^i}{i!} \quad (2.15)$$

where T is the sampling period and the matrices $\bar{S}^i$ are lower triangular matrices in the form

$$\bar{S}^i = \begin{bmatrix} s_n^i & 0 & 0 & \dots & 0 \\ s_{n-1}^i & s_n^i & 0 & \dots & 0 \\ s_{n-2}^i & s_{n-1}^i & s_n^i & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_1^i & s_2^i & s_3^i & \dots & s_n^i \end{bmatrix}.$$

The equations (2.8), (2.9) for computing the elements of the matrices $\bar{S}^i$ and $\bar{H}$, which are not equal to zero, can be rewritten as

$$s_i^{k+1} = -pT\left(s_i^k + 2\sum_{j=i+1}^{n} s_j^k\right), \tag{2.16}$$

$$i = 1,\dots,n,\ k = 1,\dots,\infty,\ s_i^1 = a_i T,$$

$$h_i = 1 + \sum_{l=1}^{k} \frac{s_i^l}{l!}, \tag{2.17}$$

$$i = 1,\dots,n,\ k = 1,\dots,\infty.$$

The inverse matrix $\bar{A}^{-1}$ can be expressed for the Laguerre model by

$$\bar{A}^{-1} = \begin{bmatrix} -1 & 0 & 0 & \dots & 0 \\ 2 & -1 & 0 & \dots & 0 \\ -2 & 2 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-1)^n 2 & (-1)^{n+1} 2 & (-1)^{n+2} 2 & \dots & -1 \end{bmatrix} \frac{1}{p}. \tag{2.18}$$

Using (2.12), (2.15) and (2.18) it can be proved easily that

$$g_i = \left[(-1)^i (e_n - 1) + \sum_{j=n+1-i}^{n-1} (-1)^{i+j-1} e_j\right] \sqrt{\frac{2}{p}},\quad i = 1,\dots,n. \tag{2.19}$$

Thus, we have described all the relations necessary to compute the elements of the matrix $\bar{E}$ and vector $\bar{g}$ of the Laguerre function model. The algorithm consists of equations (2.16), (2.17) and (2.19)

3. RECEDING HORIZON LQ REGULATOR BASED ON LAGUERRE MODEL

As orthonormal models of controlled plant are expressed in the state-space form, the state-space control design techniques may be used. One of the controllers, parameters of which can be computed using the state-space model, is a receding horizon LQ regulator [2]. LQ regulators are studied in many books and they are well understood. In this part of the article, we describe a simple memory saving algorithm of the LQ regulator based on Laguerre orthonormal functions.

Let us assume that the model of the plant to be controlled is given by (2.13), (2.14). The quadratic cost criterion, which is to be minimized by the controller, is expressed by

$$\begin{aligned} J(N,k) &= \bar{l}^T(k+N)\bar{c}^T q_N \bar{c}\bar{l}(k+N) + \sum_{j=0}^{N-1}(\bar{l}^T(k+j)\bar{c}^T q \bar{c}\bar{l}(k+j) + ru^2(k+j)) \\ &= \bar{l}^T(k+N)\bar{Q}_N\bar{l}(k+N) + \sum_{j=0}^{N-1}(\bar{l}^T(k+j)\bar{Q}\bar{l}(k+j) + ru^2(k+j)) \end{aligned}$$

where N is the control horizon and q_N, q and r are weighting coefficients. The receding horizon LQ controller, which minimizes the criteria, is given by

$$u(k) = -\frac{1}{d(0)}\bar{t}(0)\bar{l}(k)$$

where

$$\begin{aligned} \bar{P}(j) &= \bar{E}^T\bar{P}(j+1)\bar{E} + \bar{Q} - \frac{1}{d(j+1)}\bar{t}^T(j+1)\bar{t}(j+1) \\ \bar{t}(j+1) &= \bar{g}^T\bar{P}(j+1)\bar{E} \\ d(j+1) &= \bar{g}^T\bar{P}(j+1)\bar{g} + r \\ \bar{P}_N &= \bar{Q}_N. \end{aligned}$$

To save computer memory and make an evaluation of the control variable faster only the lower triangular part of the symmetric matrices $P(j)$ should be computed and stored in a computer memory. Using matrix multiplication the following relations for computing values of the scalars $d(j)$, and the elements of the matrices $\bar{P}(j)$ and vectors $\bar{t}(j)$ can be proved easily

$$p_{ij}(j) = \sum_{l=i}^{n} e_{n+i-l}\sum_{k=j}^{n} e_{n+j-k}p_{\max(l,k),\min(l,k)}(j+1) + qc_ic_j - \frac{t_i(j+1)t_j(j+1)}{d(j+1)},$$

$$i = 1,\dots,n,\ j = 1,\dots,i$$

$$\begin{aligned} t_j(j+1) &= \sum_{l=1}^{n} g_l \sum_{k=j}^{n} e_{n+j-k}p_{\max(l,k),\min(l,k)}(j+1), \quad j = 1,\dots,n \\ d(j+1) &= r + \sum_{k=1}^{n} g_k \sum_{l=1}^{n} g_l p_{\max(k,l),\min(k,l)}(j+1) \end{aligned}$$

where n is the model order.

4. PREDICTIVE CONTROLLERS BASED ON LAGUERRE MODEL

To develop the control law, let the model of the controlled plant be expressed by

$$\bar{l}(k+1) = \bar{E}\bar{l}(k) + \bar{g}(u(k-1) + \Delta u(k)) \qquad (4.1)$$

$$\hat{y}(k) = \bar{c}\bar{l}(k) \qquad (4.2)$$

where $\bar{E}$ and $\bar{g}$ are the matrices of the Laguerre state-space model, $\bar{l}(k)$ is the Laguerre model state vector, $\bar{c}$ is the estimated vector of the Laguerre spectra gains, $u(k-1)$ is the control action in the previous sample period, $\Delta u(k)$ is the actual increment of the control action and $\hat{y}(k)$ is the estimated output of the system. The predictive control approach is based on the minimization of a cost function, that penalizes the error between the predicted output and the set-point. Let the minimized cost function be given by

$$J(k) = \frac{1}{2}\left[q(y_r - \hat{y}(k+d))^2 + r \bigtriangleup u(k)^2\right]$$

where d is the prediction horizon of output, $\hat{y}(k+d)$ is the d-step ahead output prediction and y_r is the set point. If we assume that

$$u(k) = u(k+1) = \cdots = u(k+d-1)$$

the following expression can be proved easily

$$\bar{l}(k+d) = \bar{E}^d \bar{l}(k) + \bar{\beta}_d u(k) \tag{4.3}$$

where

$$\bar{\beta}_d = (\bar{E}^{d-1} + \bar{E}^{d-2} + \cdots + \bar{E} + \bar{I})\bar{g}. \tag{4.4}$$

Substituting (4.4) and (4.3) into the expression for the d-step ahead output

$$\hat{y}(k+d) = y(k) + \bar{c}\left[\bar{l}(k+d) - \bar{l}(k)\right]$$

we finally obtain

$$\hat{y}(k+d) = \bar{c}\left[(\bar{E}^d - \bar{I})\bar{l}(k) + \bar{\beta}_d(u(k-1) + \Delta u(k))\right] + y(k). \tag{4.5}$$

Using (4.5) the quadratic cost function can be formulated as follows

$$J(k) = \frac{1}{2}\left\{q\left[y_r - y(k) - \bar{c}\left[(\bar{E}^d - \bar{I})\bar{l}(k) + \bar{\beta}_d(u(k-1) + \Delta u(k))\right]\right]^2 + r \bigtriangleup u(k)^2\right\}. \tag{4.6}$$

To minimize this cost function the control law has to satisfy the condition

$$\frac{\mathrm{d}J}{\mathrm{d} \bigtriangleup u(k)} = 0.$$

Then $\Delta u(k)$ is given by

$$\Delta u(k) = \frac{q\bar{c}\bar{\beta}_d\left[y_r - y(k)\right]}{r + q\left[\bar{c}\bar{\beta}_d\right]^2} - \frac{q\bar{c}\bar{\beta}_d\left[\bar{c}(\bar{E}^d - \bar{I})\bar{l}(k) + \bar{c}\bar{\beta}_d u(k-1)\right]}{r + q\left[\bar{c}\bar{\beta}_d\right]^2}. \tag{4.7}$$

It is easy to prove that

$$\frac{\mathrm{d}^2 J}{\Delta u(k)^2} = r + q\left[\bar{c}\bar{\beta}_d\right]^2 \geq 0.$$

Thus, the control law (4.7) always minimizes the cost function (4.6).

Stability

Theorem 1. Let the system described by (2.13), (2.14) be controlled by (4.7), and assume $y(0)$ and $u(0)$ are bounded. Let D be the time delay of the system and T be the sample period. Then, for any $q \in (0, \infty)$ there are $r \in (0, \infty)$ and $d \in N^+, Td > D$, such that

$$\begin{aligned} \lim_{k \to \infty} y(k) &= y_r \\ \lim_{k \to \infty} \Delta u(k) &= 0 \\ \lim_{k \to \infty} u(k) &= u \end{aligned}$$

where $u \in R$ is a constant.

Proof. Using (4.7) and (4.1) the closed-loop system can be expressed by

$$\bar{l}(k+1) = \bar{\Gamma}\bar{l}(k) + \frac{q\bar{c}\bar{\beta}_d}{r + q\left[\bar{c}\bar{\beta}_d\right]^2}\bar{g}(y_r - y(k)) + \frac{r}{r + q\left[\bar{c}\bar{\beta}_d\right]^2}\bar{g}u(k-1)$$

where

$$\bar{\Gamma} = \left[\bar{E} - \frac{q\bar{c}\bar{\beta}_d\bar{g}\bar{c}\bar{E}^d}{r + q\left[\bar{c}\bar{\beta}_d\right]^2} + \frac{q\bar{c}\bar{\beta}_d\bar{g}\bar{c}}{r + q\left[\bar{c}\bar{\beta}_d\right]^2}\right]. \tag{4.8}$$

As long as the eigenvalues of the matrix $\bar{\Gamma}$, for some values of d, q and r, are inside the unit disk in the $\mathcal{Z}$-plane then the closed loop system is stable. The matrix $\bar{E}$ is lower triangular and the eigenvalues of $\bar{E}$ appear along its main diagonal. Then it is straightforward to show that the powers $\bar{E}^d$ approach to zero because all the eigenvalues of $\bar{E}$ are less than one in modulus ($|\lambda(\bar{E})| < 1$). Thus, the eigenvalues of the second term approach to zero. For sufficiently large r and $d \in N^+$, $Td > D$ the eigenvalues of the third term in the expression (4.8) also approach to zero for any $q \in (0, \infty)$. The first term in expression (4.8) is always a stable matrix $\bar{E}$ and the closed-loop system is thus stable. From (4.7) is then obvious that the steady-state is such that $y(k) = y_r$.

Robustness

Because the identification based on the orthonormal functions does not rely on a predefined model structure and does not separate the delay from the dynamics, we expect the controller to be more robust than the schemes based on the transfer function models. The simple analysis that follows seems to indicate that this is the case.

Let the true deterministic plant be represented by

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{b}(u(k-1) + \Delta u(k)) \tag{4.9}$$

$$y(k) = \bar{c}_s\bar{x}(k) \tag{4.10}$$

and be identified by the model based on the Laguerre functions (4.1), (4.2). Let the system be under self-tuning control using predictive control law (4.7). Substituting for (4.7) in (4.1) and (4.9) we obtain the following set of equations

$$\begin{aligned}\bar{x}(k+1) &= \bar{A}\bar{x}(k) - \bar{b}\frac{q\bar{c}\bar{\beta}_d\bar{c}(\bar{E}^d - \bar{I})}{r + q(\bar{c}\bar{\beta}_d)^2}\bar{l}(k) + +\bar{b}\frac{r}{r + q(\bar{c}\bar{\beta}_d)^2}u(k-1) \\ &\quad + \bar{b}\frac{q\bar{c}\bar{\beta}_d}{r + q(\bar{c}\bar{\beta}_d)^2}(y_r - y(k)) \qquad (4.11)\\ \bar{l}(k+1) &= \left[\bar{E} - \bar{g}\frac{q\bar{c}\bar{\beta}_d\bar{c}(\bar{E}^d - \bar{I})}{r + q(\bar{c}\bar{\beta}_d)^2}\right]\bar{l}(k) + \bar{g}\frac{r}{r + q(\bar{c}\bar{\beta}_d)^2}u(k-1) \\ &\quad + \bar{g}\frac{q\bar{c}\bar{\beta}_d}{r + q(\bar{c}\bar{\beta}_d)^2}(y_r - y(k)). \qquad (4.12)\end{aligned}$$

Let further assume that the output model mismatch between the true plant and the identified one can be described by the function $\zeta(k)$, that is always bounded for any $\forall\, k \in R$, i.e.

$$|\zeta(k)| \le Z, \qquad Z \in R^+.$$

Then we can write

$$y(k) = \hat{y}(k) + \zeta(k), \qquad \text{or} \qquad |y(k) - \hat{y}(k)| = |\zeta(k)|\,. \qquad (4.13)$$

Substituting for (4.13) and (4.2) in (4.11) resp. (4.12) we obtain an extended state equation

$$\begin{aligned}\begin{bmatrix}\bar{x}(k+1)\\ \bar{l}(k+1)\end{bmatrix} &= \begin{bmatrix}\bar{A} & \bar{b}\frac{q\bar{c}\bar{\beta}_d\bar{c}(\bar{E}^d - \bar{I})}{r+q(\bar{c}\bar{\beta}_d)^2}\\ \bar{0} & \bar{E} - \bar{g}\frac{q\bar{c}\bar{\beta}_d\bar{c}(\bar{E}^d - \bar{I})}{r+q(\bar{c}\bar{\beta}_d)^2}\end{bmatrix}\begin{bmatrix}\bar{x}(k)\\ \bar{l}(k)\end{bmatrix} \qquad (4.14)\\ &\quad + \begin{bmatrix}\bar{b}\\ \bar{g}\end{bmatrix}\frac{q\bar{c}\bar{\beta}_d}{r + q(\bar{c}\bar{\beta}_d)^2}(y_r - \zeta(k)) + \begin{bmatrix}\bar{b}\\ \bar{g}\end{bmatrix}\frac{r}{r + q(\bar{c}\bar{\beta}_d)^2}u(k-1)\end{aligned}$$

where $\bar{0}$ is an all-zero matrix of the appropriate dimensions.

It is obvious that the robustness of the controller (4.7) against the disturbance $\zeta(k)$ at the output of the controlled plant can be evaluated by the stability of the system (4.15) that is determined by the placement of the eigenvalues of the upper block-triangular matrix

$$\bar{A}_c = \begin{bmatrix}\bar{A} & \bar{b}\frac{q\bar{c}\bar{\beta}_d\bar{c}(\bar{E}^d - \bar{I})}{r+q(\bar{c}\bar{\beta}_d)^2}\\ \bar{0} & \bar{E} - \bar{g}\frac{q\bar{c}\bar{\beta}_d\bar{c}(\bar{E}^d - \bar{I})}{r+q(\bar{c}\bar{\beta}_d)^2}\end{bmatrix} = \begin{bmatrix}\bar{A}_{c11} & \bar{A}_{c12}\\ \bar{A}_{c21} & \bar{A}_{c22}\end{bmatrix}$$

in the $\mathcal{Z}$-plane. Since the true plant under study was assumed to be stable then the eigenvalues of the matrix $\bar{A}$ are inside the unit disk in the $\mathcal{Z}$-plane. Thus, the stability of the system (4.15) is given by the placement of the eigenvalues of the matrix

$$\bar{A}_{c22} = \bar{E} - \bar{g}\frac{q\bar{c}\bar{\beta}_d\bar{c}\bar{E}^d}{r + q(\bar{c}\bar{\beta}_d)^2} + \frac{q\bar{c}\bar{\beta}_d\bar{c}}{r + q(\bar{c}\bar{\beta}_d)^2}$$

Besides the standard arguments from the proof of Theorem 1 indicate that the matrix $\bar{A}_{c22}$ is also stable. As a result the closed-loop is always stable. The facts mentioned above prove the following theorem.

Theorem 2. Let a true deterministic plant be represented by the equations (4.9), (4.10) and be identified by the model (4.1), (4.2) based on Laguerre functions. Let it be under predictive control law (4.7). Let D be a time delay and T is a sample period. Assume that the output model mismatch between the true plant and the identified model can be expressed by any bounded deterministic or stochastic signal $\zeta(k)$ such that $\zeta(k) \leq Z < \infty$. Then, for any $q \in (0, \infty)$, there are $r \in (0, \infty)$ and $d \in N^+$, $Td > D$, such that the closed loop adaptive system remains stable.

5. SIMULATION RESULTS

5.1. LQ controller

Let the true plant model be given by the transfer function

$$S(s) = \frac{3s^2 + s + 1}{s^3 + 3s^2 + 2s + 0.1}$$

and be controlled by the LQ regulator based on the Laguerre orthonormal functions described in Section 3. The simulated true plant model is identified by the 3rd order model based on the Laguerre function (4.1), (4.2), with the time scale $p = 0.1$, the sample period $T = 0.5$, the forgetting factor $\phi = 0.985$ and the initial values of the estimated parameters

$$c_1 = 1.496422, \quad c_2 = -0.504039, \quad c_3 = 0.2212.$$

The LDFIL version of the least-squares parameter estimation is used to estimate the elements of the vector $\bar{c}$ of the model [12].
The values of the regulator parameters are

$$N = 50, \quad q = 10, \quad r = 5.$$

The true plant output time response and the output of the LQ regulator are shown in Figure 5.1.

5.2. Predictive controller

Consider the non-minimum phase plant described by

$$S(s) = \frac{3s^2 + s + 1}{s^3 + 3s^2 + 1.5s + 2} e^{-1.5s}.$$

The simulated plant is identified using the 15th order Laguerre model (4.1), (4.2) set with the time scale $p = 1$, the sample period $T = 0.1$ and the forgetting factor

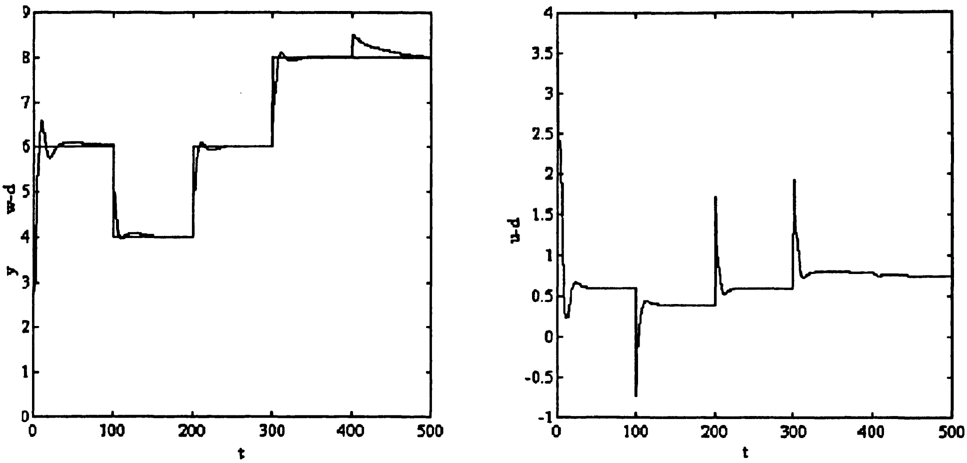

Fig. 5.1. Plant output time response and LQ controller output.

$\phi = 0.985$. The initial values of the estimated parameters are

$$\begin{array}{lllll} c_1 = 0.131, & c_2 = -0.414, & c_3 = -0.173, & c_4 = 0.430, & c_5 = 0.441, \\ c_6 = -0.410, & c_7 = -0.693, & c_8 = -0.268, & c_9 = -0.142, & c_{10} = -0.181, \\ c_{11} = -0.055, & c_{12} = 0.183, & c_{13} = 0.038, & c_{14} = -0.083, & c_{15} = -0.023. \end{array}$$

The plant is controlled by the predictive regulator (4.7) and the values of controller parameters are

$$r = 0.7, q = 0.1, d = 19.$$

Figure 5.2 shows the output of the plant and the setpoint.

6. CONCLUSION

We have presented a novel unstructured adaptive control approach, which uses the orthonormal functions to model the plant dynamics. A new algorithm of computing the elements of the controlled plant model based on the generalized orthonormal functions is proposed. The version of this algorithm, which uses the Laguerre functions to model the plant dynamics, is also described.

The result of the identification based on Laguerre function is the discrete state-space model of the plant, the matrices of which have a special structure. This model has been used to develop a new algorithm of the receding horizon LQ controller, which is memory saving and simple from the numerical point of view.

We have also presented a new predictive controller based on the Laguerre filter model, which is robust, simple to use and require minimal a priori information. Compared to other robust predictive control strategies, the proposed algorithm has some advantages. First, the analysis of the closed-loop stability can be accomplished. Second, the robustness issues of the proposed algorithms can be presented.

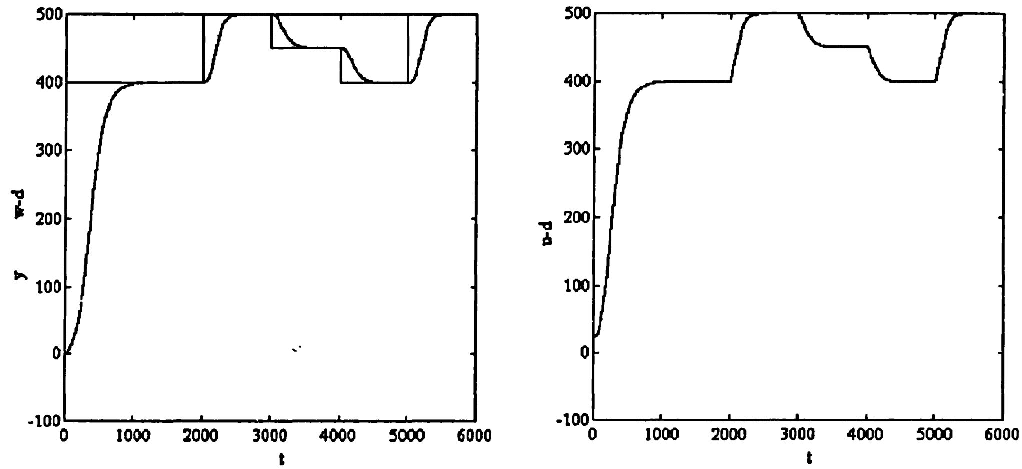

Fig. 5.2. Output response of the plant and controller output.

Finally, compared with the other predictive control laws the suggested controller is considerably simpler from the numerical point of view.

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