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A POLYNOMIAL SOLUTION TO REGULATION AND TRACKING

Part I. Deterministic Problem

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Recent results on polynomial techniques in solving the discrete-time linear-quadratic regulation and/or tracking problems are presented. Both deterministic and stochastic problems are considered in order to let appear their formal similarity and to contrast the inherent differences. The analysis is based on external polynomial models and the construction of the optimal controller or control sequence is reduced to the solution of linear polynomial equations, combined with spectral factorization. The existence of admissible controls that yield a finite performance index is studied and all such controls are specified in a parametric form. The optimal control then corresponds to the zero parameter and is shown to be recurrent, i.e. realizable by a linear finite dimensional system.

The paper is divided into two parts. Part I is concerned with the deterministic problem, i.e. with the existence of open-loop control strategies and their realization by various feedback schemes. Part II investigates the stochastic problem, i.e. the existence of closed-loop control strategies including the constraints of causality and stability.

1. INTRODUCTION

1.1. General

Polynomial techniques have been successfully applied to solve various problems of linear control theory. Basic ideas and numerous results can be found in the books by Volgin [13], Åström [1], Rosenbrock [10], Wolovich [14], Kučera [4], [5] and Kailath [2].

The aim of this paper is to present, in a compact and unified way, the recent results concerning the polynomial solution to the *discrete-time linear-quadratic regulation and/or tracking* problems. Such problems were considered by Volgin [13], Åström [1], Peterka [8], [9], Kučera [4], [5], [6], [7], Šebek [10], [11] and Šebek and Kučera [12]. Different techniques were used depending on the author and on the particular problem at hand. The unifying idea, however, was to make use of input-output *polynomial models* and reduce the synthesis of the optimal control to the solution of linear *polynomial equations*, possibly in conjunction with the spectral factorization.

The results presented here are much deeper, however. A detailed analysis of the problem is given for *single-input single-output* linear systems and *infinite control horizon*. The analysis results in a necessary and sufficient condition for the existence of admissible controls that make the given performance criterion finite, and all such controls are specified in a parametric form. The optimal control is then obtained by setting the parameter to zero. The requirements of stability and optimality are treated separately where appropriate. This provides further insight as to the best attainable performance and to realizability of the optimal control via state feedback. Finally the effect of initial conditions is considered in order to let appear the inherent differences between the *deterministic* and the *stochastic* problems.

1.2. Sequences and Polynomials

Discrete-time signals are represented by (two-sided) real *sequences* $s = \{s_t\}$, where t ranges over integers; they are denoted by lower case letters throughout the paper. If $s_t = 0$ for $t < T$, where T is an integer (either negative, or positive, or zero), then we speak of a *one-sided sequence* s. The set of all one-sided sequences forms a field under the usual elementwise addition and convolutory multiplication. A sequence s is said to be *causal* if $s_t = 0$ for $t < 0$ and *bi-causal* if it is causal together with its inverse $1/s$. Furthermore, s is an *l_2-sequence* if $\sum_{t=-\infty}^{\infty} s_t^2 < \infty$; it is *stable* if $\lim s_t = 0$; and it is *Hurwitz* if there exist a real α and integers $p \geqq 0$, T_1 such that $|s_t| < \alpha t^p$ for all $t < T_1$.

Two-sided sequences can be added in the usual way; multiplication of sequences $w = uv$ is defined by the convolution formula $w_t = \sum_{i+j=t} u_i v_j$ whenever the sum converges absolutely (it always does for l_2-sequences). The *conjugate sequence* s_* of s is defined by $s_{*t} = s_{-t}$. The symbol $\langle s \rangle$ is used to denote s_0, the *zero-position element* of s. For l_2-sequences u, v the sum $\sum_{t=-\infty}^{\infty} u_t v_t$ is finite and can be written in terms of the *inner product*

$$\sum_{t=-\infty}^{\infty} u_t v_t = \langle u_* v \rangle .$$

In particular,

$$\sum_{t=-\infty}^{\infty} u_t^2 = \langle u_* u \rangle .$$

The *delay operator* $d : s_t \to s_{t-1}$ is introduced for any sequence s. By means of it, and of the inverse operator d^{-1}, every sequence can be thought of as a *formal* *power series* $s = \sum_{t=-\infty}^{\infty} s_t d^t$. A one-sided sequence s is called *recurrent* if there exist integers $n \geqq 0$, T_2 and reals $\alpha_0, \alpha_1, \ldots, \alpha_n$ such that $\sum_{i=0}^{n} \alpha_i s_{t+i} = 0$ for all $t < T_2$. A recurrent sequence is an l_2-sequence if and only if it is stable. Finite causal sequences (i.e. *polynomials* in d) are of special importance; they are denoted by upper case letters. Every recurrent sequence can be expressed as a ratio of two polynomials. A polynomial $P(d)$ is said to be *causal*, *Hurwitz*, and *stable* if the recurrent sequence $1/P$ (obtained by long division into ascending powers of d) is respectively causal, Hurwitz, and stable.

2. DETERMINISTIC REGULATION AND TRACKING

2.1. Formulation

Consider the *plant*

(2.1a)
$$\begin{aligned} x_{t+1} &= \boldsymbol{A}x_t + \boldsymbol{B}u_t \\ y_t &= \boldsymbol{C}x_t + \boldsymbol{D}u_t \end{aligned}$$

and the *reference generator*

(2.2a)
$$\begin{aligned} z_{t+1} &= \boldsymbol{F}z_t \\ y_{Rt} &= \boldsymbol{H}z_t \end{aligned}$$

for the discrete times $t = 0, 1, \ldots$. Here $u_t \in \mathbb{R}$ is the control input, $x_t \in \mathbb{R}^n$ is the plant state, $y_t \in \mathbb{R}$ is the output and $z_t \in \mathbb{R}^m$ is the generator state, $y_{Rt} \in \mathbb{R}$ is the reference. Denote u, x, y and z, y_R the causal sequences formed respectively from u_t, x_t, y_t and z_t, y_{Rt} fot $t \geqq 0$.

Given the initial states x_0 and z_0 at $t = 0$, the problem is to find a *causal* control sequence u such that the cost

(2.3a)
$$J = \sum_{t=0}^{\infty} \lambda u_t^2 + \mu(y_{Rt} - y_t)^2$$

is finite and attains its minimum. Here $\lambda \geqq 0$ and $\mu \geqq 0$ are real constants, not both zero.

This is the standard formulation of the infinite-horizon linear-quadratic *tracking* problem and $y_R - y$ is the tracking error. The special case when $y_R = 0$ is called the *regulation* problem. The interpretation of J depends on actual values of λ and μ: If $\lambda = 0$ the output y is to follow the reference y_R as closely as possible; if $\mu = 0$ the control effort is to be minimized; and if $\lambda\mu > 0$ a compromise of the two is to be found with λ and μ weighting the relative importance of both requirements.

In addition to the internal models (2.1a) and (2.2a) it is convenient to introduce the external model of the plant

(2.1b)
$$Ay = Bu + C$$

where A, B, and C are relatively prime polynomials in d defined by

$$\frac{B(d)}{A(d)} = \boldsymbol{D} + \boldsymbol{C}(\boldsymbol{I}_n - \boldsymbol{A}d)^{-1} \boldsymbol{B}d$$

$$\frac{C(d)}{A(d)} = \boldsymbol{C}(\boldsymbol{I}_n - \boldsymbol{A}d)^{-1} x_0$$

and the external model of the reference generator

$$F y_R = G \tag{2.2b}$$

where F and G are relatively prime polynomials in d defined by

$$\frac{G(d)}{F(d)} = \boldsymbol{H}(\boldsymbol{I}_m - \boldsymbol{F}d)^{-1} z_0 .$$

Note that both A and F are causal polynomials. To avoid trivia, it is assumed that $B \neq 0$.

The cost J is finite if and only if λu and $\mu(y_R - y)$ are both l_2-sequences. Then (2.3a) can be written as

$$J = \langle u_* \lambda u + (y_R - y)_* \, \mu(y_R - y) \rangle . \tag{2.3b}$$

2.2. Solution

For further reference we define relatively prime polynomials A_0 and F_0 such that

$$\frac{A_0}{F_0} = \frac{A}{F} \tag{2.4}$$

and denote

$$E = A_0 G - F_0 C .$$

Write D for the greatest common divisor of A and B, i.e.

$$A = A'D , \quad B = B'D$$

where A' and B' are relatively prime. Let $\bar{B}$ be the greatest *causal factor* of B', i.e.

$$B' = d^k \bar{B} \tag{2.5}$$

for some $k \geqq 0$ and let $\bar{A}$ be the greatest causal factor of A', i.e.

$$A' = \bar{A} \tag{2.6}$$

as A' itself is causal. Also let $\bar{H}$ be a causal Hurwitz polynomial such that

$$A'_* \lambda A' + B'_* \mu B' = \bar{H}_* \bar{H} . \tag{2.7}$$

Such an $\bar{H}$ is called the *spectral factor*; it exists and is unique up to the sign.

Theorem 1. Define

$$H = \begin{cases} \sqrt{(\mu)}\,\bar{B} & \text{if} \quad \lambda = 0 \\ \sqrt{(\lambda)}\,\bar{A} & \text{if} \quad \mu = 0 \\ \bar{H} & \text{if} \quad \lambda\mu > 0\,. \end{cases} \tag{2.8}$$

Let P, Q, and T be the polynomial solution of the equations

$$H_*P - T_*BF_0 = A'_*\lambda E \tag{2.9a}$$

$$H_*Q + T_*AF_0 = B'_*\mu E$$

that satisfies $\langle T \rangle = 0$.

Then

a) there exists a causal sequence u which makes J finite if and only if $\lambda\mu E/DF_0$ is a stable sequence;

b) the set of all causal u's that yield finite J is generated by the formula

$$u = \frac{Q - AF_0 w}{DHF_0} \tag{2.10}$$

where w is any causal l_2-sequence;

c) the causal $\hat{u}$ which minimizes J is unique and given by

$$\hat{u} = \frac{Q}{DHF_0} \tag{2.11}$$

i.e. it corresponds to $w = 0$ and is recurrent. The associated error is

$$y_R - \hat{y} = \frac{P}{DHF_0} \tag{2.12}$$

and the associated minimal cost

$$\hat{J} = \left\langle \frac{\lambda\mu}{H_*H} \frac{E_*E}{F_{0*}D_*DF_0} \right\rangle + \left\langle \frac{T_*T}{H_*H} \right\rangle. \tag{2.13}$$

Proof. We shall manipulate the cost so as to make our claims evident. Substitute

$$y_R - y = \frac{E}{AF_0} - \frac{B}{A}u \tag{2.14}$$

in (2.3b). Make use of (2.5)–(2.8) and complete the squares to obtain

$$J = J_1 + \langle w_{1*}w_1 \rangle \tag{2.15}$$

where

$$J_1 = \left\langle \frac{\lambda\mu}{H_*H} \frac{E_*E}{F_{0*}D_*DF_0} \right\rangle$$

and

$$w_1 = \frac{B'_* \mu E}{H_* A F_0} - \frac{H}{A'} u .$$

Use equation (2.9b) to decompose the first term of w_1 as follows

$$\frac{B'_* \mu E}{H_* A F_0} = \frac{T_*}{H_*} + \frac{Q}{A F_0} . \tag{2.16}$$

Then

$$\langle w_{1*} w_1 \rangle = J_2 - 2 \left\langle \frac{T}{H} w \right\rangle + \langle w_* w \rangle$$

where

$$J_2 = \left\langle \frac{T_* T}{H_* H} \right\rangle$$

and

$$w = \frac{Q}{A F_0} - \frac{H}{A'} u . \tag{2.17}$$

For a causal sequence u the sequence w is also causal. Then $\langle T \rangle = 0$ entails

$$\left\langle \frac{T}{H} w \right\rangle = 0$$

and we finally get

$$J = J_1 + J_2 + \langle w_* w \rangle . \tag{2.18}$$

Claim a) is evident for $\lambda\mu = 0$. To prove it for $\lambda\mu > 0$ suppose that J is finite for some causal u. Then both u and $y_R - y$ are l_2-sequences. According to (2.14) these sequences are coupled by the equation

$$A'(y_R - y) + B'u = \frac{E}{D F_0} . \tag{2.19}$$

Hence E/DF_0 is an l_2-sequence and as it is recurrent, it is stable. Conversely let E/DF_0 be stable. Then the greatest common divisor of H_* and DF_0 is contained in E. As a result, equations (2.9) are solvable. Define $u = Q/DHF_0$. This u is causal and yields $w = 0$, see (2.17). Hence $J = J_1 + J_2$ by (2.18). As E/DF_0 is an l_2-sequence, J_1 is finite. As $\lambda A'$ and $\mu B'$ are relatively prime, $\bar{H} = H$ is not merely Hurwitz but stable and J_2 is finite, too. Therefore J is finite.

Claim b) follows immediately from (2.17) on taking into account that w is to be causal (so that u may be causal) and l_2 (so that J may be finite).

Claim c) is proved simply by noting that J_1 and J_2 in (2.18) are independent of the control sequence u. The best one can do to minimize J is put to $w = 0$ whence

(2.11) results. Then insert (2.11) into (2.14) to get

$$y_R - y = \frac{HE - B'Q}{A'DHF_0}.$$

On multiplying equation (2.9a) by A', equation (2.9b) by B' and adding them up, one verifies that

$$A'P + B'Q = HE$$

whence (2.12) follows. As $w = 0$, the associated cost is $\hat{J} = J_1 + J_2$. □

The idea underlying the proof is simple: to separate the cost into two parts of which only one depends on the control. This part is then set to zero in order to obtain the optimal control; the remaining part identifies the minimal cost. This is accomplished by completing the squares (by means of H) in several stages. The first completion results in (2.15). It is temping to minimize J by setting $w_1 = 0$ but it would yield a non-causal u. Therefore we isolate the non-causal part of w_1; it is done through the decomposition (2.16). The requirement $\langle T \rangle = 0$ is crucial in obtaining the final complete square (2.18). Here $w = 0$ already yields a causal u. Thus J can be reduced to J_1 by non-causal controls only, the minimum atteinable by causal controls is $J_1 + J_2$.

Theorem 1 covers the "regular" case of $\lambda\mu > 0$ as well as the "singular" cases characterized by $\lambda = 0$ and $\mu = 0$. This is made possible through the way H is defined. In the regular case we just take H to be $\bar{H}$ of (2.7). When $\lambda = 0$ the synthesis of optimal control simplifies. In view of (2.5) and (2.8) equations (2.9) reduce to the single equation

$$d^k Q' + T'AF_0 = E$$

where deg $T' < k$, and

$$P' = \bar{B}DF_0T'.$$

The relationships

$$P = \sqrt{(\mu)}\, P', \quad Q = \sqrt{(\mu)}\, Q', \quad T_* = \mu\, B'_* T'$$

then yield

$$\hat{u} = \frac{Q'}{\bar{B}DF_0}, \quad y_R - \hat{y} = T', \quad \hat{J} = \langle T'_* \mu T' \rangle.$$

When $\mu = 0$ the problem becomes trivial. Now (2.6) and (2.8) result in

$$P = \sqrt{(\lambda)}\, E, \quad Q = 0, \quad T_* = 0$$

whence

$$\hat{u} = 0, \quad y_R - \hat{y} = \frac{E}{AF_0}, \quad \hat{J} = 0.$$

It is important to note that the singular cases are *not* obtained as limit for $\lambda \to 0$ or $\mu \to 0$ of the regular case. The difference stems from the fact that u need not be an l_2-sequence when $\lambda = 0$ and similarly for $y_R - y$ when $\mu = 0$. For positive λ and μ, no matter how small, both u and $y_R - y$ must be l_2-sequences. This discontinuity is embodied in the definition (2.8) of H. The limit cases would correspond to taking $H = \bar{H}$ for any λ and μ.

In any case, however, it is seen that the optimal control sequence $\hat{u}$ is *recurrent* while the family of controls that yield finite cost is much broader. As a consequence, $\hat{u}$ can be generated by a linear finite-dimensional system.

2.3. Feedback Realization

It is worthwhile to note that the solution of the deterministic problem is an open-loop one. The optimal control strategy is obtained in the form of a sequence that depends on the given data including the initial states x_0 and z_0. There is no need for feedback control when x_0 and z_0 are known.

The optimal control sequence $\hat{u}$ can nevertheless be realized via *state* feedback of the form

$$\hat{u}_t = \boldsymbol{L}_1 x_t + \boldsymbol{L}_2 z_t \,. \tag{2.20}$$

The major advantage of this realization is that the matrices L_1 and L_2 are independent of x_0 and z_0, hence they generate the optimal control sequence for *every* x_0 and z_0. On the other hand, the resulting system

$$\begin{bmatrix} x_{t+1} \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} + \boldsymbol{B}\boldsymbol{L}_1 & \boldsymbol{B}\boldsymbol{L}_2 \\ 0 & \boldsymbol{F} \end{bmatrix} \begin{bmatrix} x_t \\ z_t \end{bmatrix}$$

is not practicable unless it is stable in some sense. The reference generator is fixed and not stable in most applications; we can do nothing about $\boldsymbol{F}$. But $\boldsymbol{A} + \boldsymbol{B}\boldsymbol{L}_1$, the matrix governing the closed-loop part of the system, should be as stable as possible.

It is therefore of interest to identify the spectrum of $\boldsymbol{A} + \boldsymbol{B}\boldsymbol{L}_1$. First of all, it contains the unreachable eigenvalues of the plant. In particular, the unreachable but observable eigenvalues are associated with the roots of D. In the regular case of $\lambda\mu > 0$ it is the spectral factor $\bar{H}$ that determines the nonzero reachable eigenvalues of the closed-loop system, see Kučera [7]. For $\lambda = 0$ we obtain in fact the deadbeat strategy studied by Kučera [3]. The closed-loop eigenvalues are those of the causal inverse of the plant. When $\mu = 0$, no control is applied and the closed-loop eigenvalues are simply those of the plant. To summarize, we have

Theorem 2. The spectrum of $\boldsymbol{A} + \boldsymbol{B}\boldsymbol{L}_1$ is the union of

1) the unreachable eigenvalues of $(\boldsymbol{A}, \boldsymbol{B})$
2) the roots of H
3) the zeros to complete the spectrum to n items.

Let us illustrate the results by a simple example. Consider plant (2.1) with

$$\boldsymbol{A} = [1], \quad \boldsymbol{B} = [0]$$
$$\boldsymbol{C} = [1], \quad \boldsymbol{D} = [1]$$

and reference generator (2.2) with

$$\boldsymbol{F} = [1]$$
$$\boldsymbol{H} = [1].$$

Let the cost (2.3) be specified by $\lambda = 0$ and $\mu = 1$.

The plant and the reference generator give rise to the polynomials

$$A = 1 - d, \quad B = 1 - d, \quad C = x_0$$

and

$$F = 1 - d, \quad G = z_0$$

where x_0 and z_0 are the initial states, arbitrary but fixed.

We calculate

$$A_0 = 1, \quad F_0 = 1$$
$$D = 1 - d, \quad B' = 1, \quad H = 1$$
$$E = z_0 - x_0$$

and solve equations (2.9) to get

$$P = 0, \quad Q = z_0 - x_0, \quad T = 0.$$

Then all causal control sequences that yield finite cost are given by (2.10) as

$$u = \frac{z_0 - x_0}{1 - d} + w$$

where w is any causal l_2-sequence. The associated tracking error is

$$y_R - y = w$$

and hence $J = \langle w_* w \rangle$. The optimal control sequence results on setting $w = 0$, i.e.

$$\hat{u} = \frac{z_0 - x_0}{1 - d} \tag{2.21}$$

and $\hat{J} = 0$.

This control sequence can be realized by the state feedback (2.20) where

$$\boldsymbol{L}_1 = [-1], \quad \boldsymbol{L}_2 = [1].$$

The resulting system is governed by the equation

$$\begin{bmatrix} x_{t+1} \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ z_t \end{bmatrix}$$

and its closed-loop part fails to be stable. Its spectrum is given by $D = 1 - d$, i.e. by the unstable common factor of A and B. For fixed initial states x_0 and z_0, however, the control sequence (2.21) is easy to realize.

2.4. Observer-Based Controller

The complete information on the system, namely on its initial state, makes an open loop strategy possible. The situation is drastically different if the initial state x_0 or z_0 is not available. Then we have to resort to output feedback. However, the optimal feedback control law does depend on x_0 and z_0 and hence it cannot be found. It is only the state feedback law in which the x_0 and z_0 enter in a non-parametric way.

This impass is usually obviated by *state reconstruction*. The state sequences x and z are reconstructed by means of (Luenberger) observers and these approximations are used in place of the true states in (2.20). This observer-based control law is by no means optimal but it is a reasonable solution frequently used in practice.

The whole problem is best illustrated by an example. Consider plant (2.1) given by

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \boldsymbol{D} = \begin{bmatrix} 0 \end{bmatrix}$$

and solve the regulation problem (i.e. zero reference) for $\lambda = 1$ and $\mu = 2$ in (2.3).

Let us first suppose that the initial state

$$x_0 = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix}$$

is *known* and find the open-loop optimal control strategy. The polynomial description of the plant is

$$A = 1\,, \quad B = d + d^2\,, \quad C = x_{10} + x_{20}d$$

and we calculate

$$E = -x_{10} - x_{20}d$$
$$H = 2 + d\,.$$

Equations (2.9) are satisfied by

$$P = -2x_{10} - (x_{10} + x_{20})\,d$$
$$Q = -\ x_{20}$$
$$R = -(2x_{10} + x_{20})\,d - 2x_{10}d^2\,.$$

The optimal control sequence (2.11) is therefore given by

$$\hat{u} = \frac{x_{20}}{2 + d}\,. \qquad (2.22)$$

The associated cost is

$$\hat{J} = x_{10}^2 + x_{20}^2 . \tag{2.23}$$

The optimal control sequence (2.22) can be realized by the state feedback $u_t = L_1 x_t$ where

$$L_1 = [0 \ -0{\cdot}5] .$$

The closed loop system is then described by the equation

$$\begin{bmatrix} x_{1t+1} \\ x_{2t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0{\cdot}5 \\ 0 & -0{\cdot}5 \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} .$$

Note that the feedback matrix L_1 is independent of x_{10} and x_{20}, hence it is able to generate the optimal control sequence for every initial state.

Suppose now that the initial state x_{20} is *not* available. Then the optimal control sequence (2.22) cannot be realized by a controller that processes the available information, namely y. We therefore set up the Luenberger observer for $L_1 x$ with arbitrary dynamics, described by

$$\begin{aligned} w_{t+1} &= \bar{A} w_t + \bar{B}_1 y_t + \bar{B}_2 u_t \\ v_t &= \bar{C} w_t + \bar{D} y_t \end{aligned}$$

where, see e.g. Kučera [7],

$$\bar{A} = [\alpha] , \quad \bar{B}_1 = [-\alpha^2] , \quad \bar{B}_2 = [1 + \alpha] \quad \bar{C} = [-0{\cdot}5] ,$$
$$\bar{D} = [0{\cdot}5\alpha]$$

and α is a real number such that $-1 < \alpha < 1$. The observer output v_t then approximates $u_t = L_1 x_t$ with the reconstruction error

$$e_t = w_t - \alpha x_{1t} - x_{2t} .$$

When v_t is used to replace u_t, the overall system obeys the equation

$$\begin{bmatrix} x_{1t+1} \\ x_{2t+1} \\ e_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0{\cdot}5 & -0{\cdot}5 \\ 0 & -0{\cdot}5 & -0{\cdot}5 \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \\ e_t \end{bmatrix}$$

The resulting control sequence is given by

$$u_t = [0 \ -0{\cdot}5 \ 0] \begin{bmatrix} x_{1t} \\ x_{2t} \\ e_t \end{bmatrix}$$

namely

$$u = -\tfrac{1}{2}x_{20} + \left(\tfrac{1}{4}x_{20} + \tfrac{1}{4}e_0\right) d - \left(\tfrac{1}{8}x_{20} + \tfrac{1}{8}e_0 - \tfrac{1}{4}\alpha e_0\right) d^2 +$$
$$+ \left(\tfrac{1}{16}x_{20} + \tfrac{1}{16}e_0 - \tfrac{1}{8}\alpha e_0 + \tfrac{1}{4}\alpha^2 e_0\right) d^3 - \ldots .$$

The associated cost equals

$$J = \hat{J} + J_\alpha$$

where $\hat{J}$ is given by (2.23) and J_α depends on x_{20}, e_0 and α. The observer-based controller is seen to be optimal only for $e_0 = 0$, an unrealistic situation when x_{20} is not known. Moreover, the minimum of J_α with respect to α depends on x_{20} and e_0; hence there is no observer which would minimize J.

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