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CRITERIA OF CORRECTNESS OF LINEAR BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF GENERALIZED ORDINARY DIFFERENTIAL EQUATIONS

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Statement of the problems and formulation of the main results

Let the matrix-and vector-functions $A_0\colon [a,b] \to \mathbb{R}^{n\times n}$ and $f_0\colon [a,b] \to \mathbb{R}^n$ be of bounded variation, $c_0 \in \mathbb{R}^n$ and $l_0\colon BV_n(a,b) \to \mathbb{R}_n$ be a bounded linear operator such that the boundary value problem

$$\mathrm{d}x(t) = \mathrm{d}A_0(t)\cdot x(t) + \mathrm{d}f_0(t), \tag{1.1$_0$}$$

$$l_0(x) = c_0 \tag{1.2$_0$}$$

has a unique solution x_0.

Consider the sequences of matrix-and vector-functions $A_k\colon [a,b] \to \mathbb{R}^{n\times n}$ $(k = 1,2,\dots)$ and $f_k\colon [a,b] \to \mathbb{R}^n$ $(k = 1,2,\dots)$ of bounded variation, a sequence of constant vectors $C_k \in \mathbb{R}^n$ $(k = 1,2,\dots)$ and a sequence of linear continuous operators $l_k\colon BV_n(a,b) \to \mathbb{R}^n (k = 1,2,\dots)$. In [1] sufficient conditions are given for the problem

$$\mathrm{d}x(t) = \mathrm{d}A_k(t)\cdot x(t) + \mathrm{d}f_k(t), \tag{1.1$_k$}$$

$$l_k(x) = c_k \tag{1.2$_0$}$$

to have a unique solution x_k for sufficiently large k and

$$\lim_{k\to+\infty} x_k(t) = x_0(t) \tag{1.3}$$

uniformly on $[a,b]$.

In this paper the necessary and sufficient conditions are established for a sequence of boundary value problems of form (1.1_k), (1.2_k) to have the above mentioned property.

An analogous question is studied in [2–7] for the boundary value problem for a system of ordinary differential equations.

The theory of generalized ordinary differential equations enables one to investigate ordinary differential and difference equations from the commonly accepted standpoint. Moreover, the convergence conditions for difference schemes corresponding to boundary value problems for systems of ordinary differential equations can be obtained from the results on the correctness of boundary value problems for systems of generalized ordinary differential equations [1, 8–11].

Throughout the paper the following notation and definitions will be used:

$\mathbb{R} =]-\infty, +\infty[$;

$\mathbb{R}^n$ is the space of all real column n-vectors $x = (x_i)_{i=1}^n$ with the norm

$$\|x\| = \sum_{i=1}^{n} |x_i|;$$

$\mathbb{R}^{n\times m}$ is the space of all real $n \times m$-matrices $X = (x_{ij})_{i,j=1}^{n,m}$ with the norm

$$\|X\| = \max_{j=1,\dots,m} \sum_{i=1}^{n} |x_{ij}|.$$

If $X \in \mathbb{R}^{n\times n}$, then X^{-1} and $\det(X)$ are respectively the matrix inverse to X and the determinant of X; I is the identity $n \times n$-matrix;

$\bigvee\limits_a^b(X)$ is the total variation of a matrix-function $X\colon [a,b] \to \mathbb{R}^{n\times m}$, i.e. the sum of total variations of the latter's components;

$X(t-)$ and $X(t+)$ $\big(X(a-) = X(a), X(b+) = X(b)\big)$ are the left and the right limit of the matrix-function $X\colon [a,b] \to \mathbb{R}^{n\times m}$ at the point t;

$d_1X(t) = X(t) - X(t-)$, $d_2X(t) = X(t+) - X(t)$;

$C_n(a,b)$ is the space of all continuous vector-functions $x\colon [a,b] \to \mathbb{R}^n$ with the norm

$$\|x\|_C = \max\big\{\|x(t)\| : t \in [a,b]\big\};$$

$\widetilde{C}_n(a,b)$ and $\widetilde{C}_{n\times n}(a,b)$ are respectively the sets of all absolutely continuous vector- and matrix-functions;

$BV_n(a,b)$ is the space of all vector-functions of bounded variation $x\colon [a,b] \to \mathbb{R}^n$ with the norm

$$\|x\|_{\sup} = \sup\big\{\|x(t)\| : t \in [a,b]\big\};$$

$BV_{n\times n}(a,b)$ is the set of all matrix-functions of bounded variation $X\colon [a,b] \to \mathbb{R}^{n\times n}$, i.e., such that $\bigvee\limits_a^b(X) < +\infty$;

If $g \in BV_1(a,b)$, $x\colon [a,b] \to \mathbb{R}$ and $a \leqslant s < t \leqslant b$, then

$$\int_s^t x(\tau)\,\mathrm{d}g(\tau) = x(t)\,\mathrm{d}_1 g(t) + x(s)\,\mathrm{d}_2 g(s) + \int_{]s,t[} x(\tau)\,\mathrm{d}g(\tau),$$

where $\int_{]s,t[} x(\tau)\,\mathrm{d}g(\tau)$ is Lebesgue-Stieltjes integral over the open interval $]s,t[$ (if $s = t$, then $\int_s^t x(\tau)\,\mathrm{d}g(\tau) = 0$);

If $G = (g_{ij})_{i,j=1}^n \in BV_{n\times n}(a,b)$, $x = (x_i)_{i=1}^n\colon [a,b] \to \mathbb{R}^n$, $X = (x_{ij})_{i,j=1}^n\colon [a,b] \to \mathbb{R}^{n\times n}$ and $a \leqslant s \leqslant t \leqslant b$, then

$$\int_s^t \mathrm{d}G(\tau)\cdot x(\tau) = \left(\sum_{k=1}^n \int_s^t x_k(\tau)\,\mathrm{d}g_{ik}(\tau)\right)_{i=1}^n,$$

$$\int_s^t \mathrm{d}G(\tau)\cdot X(\tau) = \left(\sum_{k=1}^n \int_s^t x_{kj}(\tau)\,\mathrm{d}g_{ik}(\tau)\right)_{i,j=1}^n,$$

$$\mathcal{B}(G,x)(t) = G(t)x(t) - G(a)x(a) - \int_a^t \mathrm{d}G(\tau)\cdot x(\tau),$$

$$\mathcal{B}(G,X)(t) = G(t)X(t) - G(a)X(a) - \int_a^t \mathrm{d}G(\tau)\cdot X(\tau),$$

$$\mathcal{I}(G,X)(t) = \int_a^t \mathrm{d}\big[G(\tau) + \mathcal{B}(G,X)(\tau)\big]\cdot G^{-1}(\tau);$$

$\|l\|$ is the usual norm of the bounded linear continuous operator l.

Let $k \in \{0,1,\ldots\}$ be fixed. A function $x \in BV_n(a,b)$ is called a solution of problem (1.1_k), (1.2_k) if it satisfies condition (1.2_k) and

$$x(t) = x(s) + \int_s^t \mathrm{d}A_k(\tau)\cdot x(\tau) + f_k(t) - f_k(s) \quad \text{for} \quad a \leqslant s \leqslant t \leqslant b. \tag{1.4}$$

Alongside with (1.1_k), we shall consider the corresponding homogeneous system

$$\mathrm{d}x(t) = \mathrm{d}A_k(t)\cdot x(t). \tag{1.5_k}$$

A matrix-function $Y \in BV_{n\times n}(a,b)$ is called a fundamental matrix of the homogeneous system (1.5_k) if

$$Y(t) = Y(s) + \int_s^t \mathrm{d}A_k(\tau)\cdot Y(\tau) \quad \text{for} \quad a \leqslant s \leqslant t \leqslant b$$

and

$$\det\big(Y(t)\big) \neq 0 \quad \text{for} \quad t \in [a,b].$$

Definition 1.1. We say that a sequence (A_k, f_k, l_k) $(k = 1, 2, \ldots)$ belongs to the set $S(A_0, f_0, l_0)$ if for every $c_0 \in \mathbb{R}^n$ and $c_k \in \mathbb{R}^n$ $(k = 1, 2, \ldots)$ satisfying the condition

$$\lim_{k \to +\infty} C_k = C_0 \tag{1.6}$$

the problem (1.1_k), (1.2_k) has a unique solution x_k for any sufficiently large k and (1.3) holds.

Without loss of generality we may assume that $A_k(a) = 0$ $(k = 0, 1, \ldots)$ and $f_k(a) = 0$ $(k = 0, 1, \ldots)$.

Theorem 1.1. *Let conditions* (1.6),

$$\lim_{k \to +\infty} l_k(y) = l_0(y) \quad \text{for} \quad y \in BV_n(a, b), \tag{1.7}$$

$$\lim_{k \to +\infty} \sup \|l_k\| < +\infty \tag{1.8}$$

and

$$\det\bigl(I + (-1)^j \, \mathrm{d}_j A_0(t)\bigr) \neq 0 \quad \text{for} \quad t \in [a, b] \ (j = 1, 2) \tag{1.9}$$

hold. Then

$$\bigl((A_k, f_k, l_k)\bigr)_{k=1}^{+\infty} \in S(A_0, f_0, l_0) \tag{1.10}$$

if and only if there exists a sequence of matrix-functions $H_k \in BV_{n\times n}(a, b)$ $(k = 0, 1, \ldots)$ *such that*

$$\lim_{k \to +\infty} \sup \bigvee_a^b \bigl(H_k + \mathcal{B}(H_k, A_k)\bigr) < +\infty, \tag{1.11}$$

$$\inf\bigl\{|\det\bigl(H_0(t)\bigr)| : t \in [a, b]\bigr\} > 0 \tag{1.12}$$

and the conditions

$$\lim_{k \to +\infty} H_k(t) = H_0(t), \tag{1.13}$$

$$\lim_{k \to +\infty} \mathcal{B}(H_k, A_k)(t) = \mathcal{B}(H_0, A_0)(t), \tag{1.14}$$

$$\lim_{k \to +\infty} \mathcal{B}(H_k, f_k)(t) = \mathcal{B}(H_0, f_0)(t) \tag{1.15}$$

are fulfilled uniformly on $[a, b]$.

Corollary 1.1. *Let $A_k \in \widetilde{C}_{n\times n}(a,b)$ $(k=0,1,\dots)$, $f_k \in \widetilde{C}_n(a,b)$ $(k=0,1,\dots)$ and $l_k: C_n(a,b) \to \mathbb{R}^n (k=0,1,\dots)$ be a sequence of linear continuous operators satisfying equality (1.7) on the set $\widetilde{C}_n(a,b)$. Let, moreover, conditions (1.6) and (1.8) be satisfied. Then (1.10) holds if and only if there exists a sequence of matrix-functions $H_k \in \widetilde{C}_{n\times n}(a,b)$ $(k=0,1,\dots)$ such that*

$$\lim_{k\to+\infty} \sup \int_a^b \|H_k'(s) + H_k(s)P_k(s)\| \,\mathrm{d}s < +\infty$$

and conditions (1.13),

$$\lim_{k\to+\infty} \int_a^t H_k(s)P_k(s)\,\mathrm{d}s = \int_a^t H_0(s)P_0(s)\,\mathrm{d}s$$

and

$$\lim_{k\to+\infty} \int_a^t H_k(s)q_k(s)\,\mathrm{d}s = \int_a^t H_0(s)q_0(s)\,\mathrm{d}s$$

are fulfilled uniformly on $[a,b]$, where

$$P_k(t) = A_k'(t), \quad q_k(t) = f_k'(t).$$

Note that Corollary 1.1 represents a more precise version of Theorem 1 from [7].

Theorem 1.2. *Let $A_0^* \in BV_{n\times n}(a,b)$, $f_0^* \in BV_n(a,b)$, $c_0^* \in \mathbb{R}^n$ and $l_0^*: BV_n(a,b) \to \mathbb{R}^n$ be a linear continuous operator such that*

$$\det\big(I + (-1)^j \,\mathrm{d}_j A_0^*(t)\big) \neq 0 \quad \text{for} \quad t \in [a,b] (j=1,2) \tag{1.16}$$

and the boundary value problem

$$\mathrm{d}x(t) = \mathrm{d}A_0^*(t)\cdot x(t) + \mathrm{d}f_0^*(t), \tag{1.17}$$

$$l_0^*(x) = c_0^* \tag{1.18}$$

has a unique solution x_0^*. Moreover, let there exist sequences of matrix-and vector-functions, $H_k \in BV_{n\times n}(a,b)$ $(k=1,2,\dots)$ and $h_k \in BV_n(a,b)$ $(k=1,2,\dots)$ such that

$$\inf\big\{|\det\big(H_k(t)\big)|: t \in [a,b]\big\} > 0 \tag{1.19}$$

for sufficiently large k and for the sequences

$$l_k^*(y) = l_k(H_k^{-1}y) \ (k=1,2,\dots), \quad A_k^*(t) = \mathcal{I}(H_k, A_k)(t) \ (k=1,2,\dots)$$

and

$$f_k^*(t) = h_k(t) - h_k(a) + \mathcal{B}(H_k, f_k)(t) - \int_a^t \mathrm{d}A_k^*(s)\cdot h_k(s) \quad (k = 1, 2, \dots)$$

the conditions

$$\lim_{k\to+\infty} \big[C_k + l_k^*(h_k)\big] = C_0^*, \tag{1.20}$$

$$\lim_{k\to+\infty} l_k^*(y) = l_0^*(y) \quad \text{for} \quad y \in BV_n(a, b), \tag{1.21}$$

$$\lim_{k\to+\infty} \sup \|l_k^*\| < +\infty, \tag{1.22}$$

$$\lim_{k\to+\infty} \sup \bigvee_a^b (A_k^*) < +\infty \tag{1.23}$$

hold and the following conditions

$$\lim_{k\to+\infty} A_k^*(t) = A_0^*(t), \tag{1.24}$$

$$\lim_{k\to+\infty} f_k^*(t) = f_0^*(t), \tag{1.25}$$

are fulfilled uniformly on $[a, b]$. *Then for any sufficiently large* k *problem* (1.1_k), (1.2_k) *has a unique solution* x_k *and*

$$\lim_{k\to+\infty} \|H_k x_k + h_k - x_0^*\|_{\sup} = 0. \tag{1.26}$$

Corollary 1.2. *Let* (1.7)–(1.9), (1.11), (1.12),

$$\lim_{k\to+\infty} \big[c_k - l_k(\varphi_k)\big] = c_0 \tag{1.27}$$

hold and conditions (1.13), (1.14) *and*

$$\lim_{k\to+\infty} \Big[\mathcal{B}(H_k, f_k - \varphi_k)(t) + \int_a^t \mathrm{d}\mathcal{B}(H_k, A_k)(s)\cdot\varphi_k(s)\Big] = \mathcal{B}(H_0, f_0)(t) \tag{1.28}$$

be fulfilled uniformly on $[a, b]$, *where* $H_k \in BV_{n\times n}(a, b)$, $\varphi_k \in BV_n(a, b)$. *Then for any sufficiently large* k *problem* (1.1_k), (1.2_k) *has a unique solution* x_k *and*

$$\lim_{k\to+\infty} \|x_k - \varphi_k - x_0\|_{\sup} = 0. \tag{1.29}$$

Corollary 1.3. *Let* (1.6)–(1.9), (1.11), (1.12) *hold and conditions* (1.13),

$$\lim_{k\to+\infty}\int_a^t H_k(s)\,\mathrm{d}A_k(s)=\int_a^t H_0(s)\,\mathrm{d}A_0(s), \tag{1.30}$$

$$\lim_{k\to+\infty}\int_a^t H_k(s)\,\mathrm{d}f_k(s)=\int_a^t H_0(s)\,\mathrm{d}f_0(s), \tag{1.31}$$

$$\lim_{k\to+\infty}\mathrm{d}_jA_k(t)=\mathrm{d}_jA_0(t)\quad(j=1,2) \tag{1.32}$$

and

$$\lim_{k\to+\infty}\mathrm{d}_jf_k(t)=\mathrm{d}_jf_0(t)\quad(j=1,2) \tag{1.33}$$

be fulfilled uniformly on $[a,b]$, *where* $H_k\in BV_{n\times n}(a,b)$. *Let, moreover, either*

$$\lim_{k\to+\infty}\sup\sum_{a\leqslant t\leqslant b}\big(\|\mathrm{d}_jA_k(t)\|+\|\mathrm{d}_jf_k(t)\|\big)<+\infty\quad(j=1,2) \tag{1.34}$$

or

$$\lim_{k\to+\infty}\sup\sum_{a\leqslant t\leqslant b}\|\mathrm{d}_jH_k(t)\|<+\infty\quad(j=1,2) \tag{1.35}$$

Then (1.10) *holds.*

Corollary 1.4. *Let* (1.6)–(1.9), (1.11), (1.12) *hold and conditions* (1.13),

$$\lim_{k\to+\infty}A_k(t)=A_0(t), \tag{1.36}$$

$$\lim_{k\to+\infty}f_k(t)=f_0(t), \tag{1.37}$$

$$\lim_{k\to+\infty}\int_a^t \mathrm{d}\big[H_0^{-1}(s)H_k(s)\big]\cdot A_k(s)=A_*(t) \tag{1.38}$$

and

$$\lim_{k\to+\infty}\int_a^t \mathrm{d}\big[H_0^{-1}(s)H_k(s)\big]\cdot f_k(s)=f_*(t) \tag{1.39}$$

be fulfilled uniformly on $[a,b]$, *where* $A_*,\ H_k\in BV_{n\times n}(a,b)$; $f_*, f_k\in BV_n(a,b)$. *Moreover, let the system*

$$\mathrm{d}x(t)=\mathrm{d}A_{0*}(t)\cdot x(t)+\mathrm{d}f_{0*}(t),$$

where $A_{0*}(t) = A_0(t) - A_*(t)$, $f_{0*}(t) = f_0(t) - f_*(t)$ *has the unique solution satisfying condition* (1.2_0). *Then*

$$\big((A_k, f_k, l_k)\big)_{k=1}^{+\infty} \in S(A_{0*}, f_{0*}, l_0).$$

Corollary 1.5. *Let* (1.6)–(1.9) *hold and there exist a natural number* m *and matrix-and vector-functions* $A_{0j} \in BV_{n\times n}(a,b)$ $(j = 1,\dots,m)$, $A_{0m}(t) = A_0(t)$ *and* $f_{0j} \in BV_n(a,b)$ $(j = 1,\dots,m)$, $f_{0m}(t) = f_0(t)$ *such that*

$$\lim_{k\to+\infty} \sup \bigvee_a^b (A_{km}) < +\infty$$

and for every $j \in \{1,\dots,m\}$ *the conditions*

$$\lim_{k\to+\infty} A_{kj}(t) = A_{0j}(t),$$
$$\lim_{k\to+\infty} f_{kj}(t) = f_{0j}(t),$$

be fulfilled uniformly on $[a,b]$, *where*

$$A_{k1}(t) = A_k(t), \quad A_{kj+1}(t) = H_{kj}(t) + \mathcal{B}(H_{kj}, A_k)(t),$$
$$f_{k1}(t) = f_k(t), \quad f_{kj+1}(t) = \mathcal{B}(H_{kj}, f_k)(t),$$
$$H_{kj}(t) = \big[I - A_{kj}(t) + A_{0j}(t)\big] \times \dots \times \big[I - A_{k1}(t) + A_{01}(t)\big].$$

Then (1.10) *holds.*

Remark. Identity (2.5) from Lemma 2.1 below shows that in Theorem 1.1 and Corollaries 1.1–1.4 we can assume without loss of generality that $H_0(t) = I$ $(t \in [a,b])$.

Let $f, g, h \in BV_1(a,b)$ and $[c,d] \subset [a,b]$. We shall make use of the following formulas:

$$\int_c^d f(s)\,\mathrm{d}\Big[\int_c^s g(\tau)\,\mathrm{d}h(\tau)\Big] = \int_c^d f(s)g(s)\,\mathrm{d}h(s), \tag{2.1}$$

$$\begin{aligned}\int_c^d f(s)\,\mathrm{d}g(s) + \int_c^d g(s)\,\mathrm{d}f(s) &= f(d)g(d) - f(c)g(c)\\ &+ \sum_{c<s\leqslant d} \mathrm{d}_1 f(s)\cdot \mathrm{d}_1 g(s) - \sum_{c\leqslant s<d} \mathrm{d}_2 f(s)\cdot \mathrm{d}_2 g(s),\end{aligned} \tag{2.2}$$

$$\int_c^d f(s)\,\mathrm{d}\Big[\sum_{c<\tau\leqslant s} \mathrm{d}_1 g(\tau)\Big] = \sum_{c<s\leqslant d} f(s)\,\mathrm{d}_1 g(s) \tag{2.3}$$

and

$$\int_c^d f(s)\,\mathrm{d}\Big[\sum_{c\leqslant\tau<s} \mathrm{d}_2 g(\tau)\Big] = \sum_{c\leqslant s<d} f(s)\,\mathrm{d}_2 g(s) \tag{2.4}$$

(see [9], Theorem I.4.25, I.4.33 and Lemma I.4.23).

Lemma 2.1. *Let G, H, $A \in BV_{n\times n}(a,b)$ and $\varphi \in BV_n(a,b)$; then for any $t \in [a,b]$ we have*

$$\mathcal{B}\big(G, \mathcal{B}(H,A)\big)(t) = \mathcal{B}(GH, A)(t), \tag{2.5}$$

$$\mathcal{B}\big(G, \mathcal{B}(H,\varphi)\big)(t) = \mathcal{B}(GH, \varphi)(t), \tag{2.6}$$

$$\mathcal{B}\Big(G, \int_a \mathrm{d}H(s)\cdot\varphi(s)\Big)(t) = \int_a^t \mathrm{d}\mathcal{B}(G,H)(s)\cdot\varphi(s) \tag{2.7}$$

and

$$\mathcal{I}\big(G, \mathcal{I}(H,A)\big)(t) = \mathcal{I}(GH, A)(t).$$

Proof. Let us show that (2.5) is valid. According to equalities (2.1)–(2.4) and

$$\mathrm{d}_j\Big[\int_a^t \mathrm{d}H(s)\cdot A(s)\Big] = \mathrm{d}_j H(t)\cdot A(t) \quad (j=1,2)$$

we have

$$
\begin{aligned}
\mathcal{B}\big(G,\mathcal{B}(H,A)\big)(t) &= G(t)\mathcal{B}(H,A)(t) - \int_a^t \mathrm{d}G(s)\cdot\mathcal{B}(H,A)(s) \\
&= G(t)\cdot\Big[H(t)A(t) - H(a)A(a) - \int_a^t \mathrm{d}H(s)\cdot A(s)\Big] \\
&\quad - \int_a^t \mathrm{d}G(s)\cdot\Big[H(s)A(s) - H(a)A(a) - \int_a^s \mathrm{d}H(\tau)\cdot A(\tau)\Big] \\
&= G(t)H(t)A(t) - G(a)H(a)A(a) - G(t)\int_a^t \mathrm{d}H(s)\cdot A(s) \\
&\quad - \int_a^t \mathrm{d}G(s)\cdot H(s)A(s) + \int_a^t \mathrm{d}G(s)\cdot\int_a^s \mathrm{d}H(\tau)\cdot A(\tau) \\
&= G(t)H(t)A(t) - G(a)H(a)A(a) - \int_a^t \mathrm{d}G(s)\cdot H(s)A(s) \\
&\quad - \int_a^t G(s)\,\mathrm{d}H(s)\cdot A(s) + \sum_{a<s\leqslant t} \mathrm{d}_1G(s)\cdot\mathrm{d}_1H(s)\cdot A(s) \\
&\quad - \sum_{a\leqslant s<t} \mathrm{d}_2G(s)\cdot\mathrm{d}_2H(s)\cdot A(s) = G(t)H(t)A(t) - G(a)H(a)A(a) \\
&\quad - \int_a^t \mathrm{d}\Big[\int_a^s \mathrm{d}G(\tau)\cdot H(\tau) + \int_a^s G(\tau)\,\mathrm{d}H(\tau) \\
&\quad - \sum_{a\leqslant\tau<s} \mathrm{d}_1G(\tau)\cdot\mathrm{d}_1H(\tau) + \sum_{a<\tau\leqslant s} \mathrm{d}_2G(\tau)\cdot\mathrm{d}_2H(\tau)\Big]\cdot A(s) \\
&= G(t)H(t)A(t) - G(a)H(a)A(a) - \int_a^t \mathrm{d}\big[G(s)H(s)\big]\cdot A(s) \\
&= \mathcal{B}(GH,A)(t).
\end{aligned}
$$

The proof of equality (2.6) is analogous.

Let us verify (2.7). By (2.1) and (2.6) it can be easily shown that

$$
\begin{aligned}
\mathcal{B}\Big(G,\int_a \mathrm{d}H(s)\cdot\varphi(s)\Big)(t) &= \mathcal{B}\big(G, H\varphi - \mathcal{B}(H,\varphi)\big)(t) \\
&= \mathcal{B}(G,H\varphi)(t) - \mathcal{B}\big(G,\mathcal{B}(H,\varphi)\big)(t) \\
&= \mathcal{B}(G,H\varphi)(t) - \mathcal{B}(GH,\varphi)(t) \\
&= \int_a^t \mathrm{d}\big[G(s)H(s)\big]\cdot\varphi(s) - \int_a^t \mathrm{d}G(s)\cdot H(s)\varphi(s) \\
&= \int_a^t \mathrm{d}\mathcal{B}(G,H)(s)\cdot\varphi(s).
\end{aligned}
$$

Finally, using (2.1), (2.5) and (2.7) we have

$$\begin{aligned}
&\mathcal{I}\big(G,\mathcal{I}(H,A)\big)(t)\\
&\quad=\int_a^t \mathrm{d}\big[G(\tau)+\mathcal{B}\big(G,\mathcal{I}(H,A)\big)(\tau)\big]\cdot G^{-1}(\tau)\\
&\quad=\int_a^t \mathrm{d}\Big[G(\tau)+\mathcal{B}\Big(G,\int_a \mathrm{d}\big[H(s)+\mathcal{B}(H,A)(s)\big]\cdot H^{-1}(s)\Big)(\tau)\Big]\cdot G^{-1}(\tau)\\
&\quad=\int_a^t \mathrm{d}\Big[G(\tau)+\int_a^\tau \mathrm{d}\mathcal{B}\big(G,H+\mathcal{B}(H,A)\big)(s)\cdot H^{-1}(s)\Big]\cdot G^{-1}(\tau)\\
&\quad=\int_a^t \mathrm{d}\Big[G(\tau)+\int_a^\tau \mathrm{d}\mathcal{B}(G,H)(s)\cdot H^{-1}(s)\\
&\qquad+\int_a^\tau \mathrm{d}\mathcal{B}\big(G,\mathcal{B}(H,A)\big)(s)\cdot H^{-1}(s)\Big]\cdot G^{-1}(\tau)\\
&\quad=\int_a^t \mathrm{d}\Big[G(\tau)+\int_a^\tau \mathrm{d}\big[G(s)H(s)-\int_a^s \mathrm{d}G(\sigma)\cdot H(\sigma)\big]\cdot H^{-1}(s)\\
&\qquad+\int_a^\tau \mathrm{d}\mathcal{B}(GH,A)(s)\cdot H^{-1}(s)\Big]\cdot G^{-1}(\tau)\\
&\quad=\int_a^t \mathrm{d}\Big[\int_a^\tau \mathrm{d}\big[G(s)H(s)\big]\cdot H^{-1}(s)+\int_a^\tau \mathrm{d}\mathcal{B}(GH,A)(s)\cdot H^{-1}(s)\Big]\cdot G^{-1}(\tau)\\
&\quad=\int_a^t \mathrm{d}\big[G(\tau)H(\tau)+\mathcal{B}(GH,A)(\tau)\big]\cdot H^{-1}(\tau)G^{-1}(\tau)=\mathcal{I}(GH,A)(t).
\end{aligned}$$

Therefore (2.8) is proved. □

Lemma 2.2. *Let $h\in BV_n(a,b)$ and $H\in BV_{n\times n}(a,b)$ be a nonsingular matrix-function on $[a,b]$. Then the mapping*

$$x\to y=Hx+h$$

establishes a one-to-one correspondence between the solutions x and y of the systems

$$\mathrm{d}x(t)=\mathrm{d}A(t)\cdot x(t)+\mathrm{d}f(t) \tag{2.9}$$

and

$$\mathrm{d}y(t)=\mathrm{d}A^*(t)\cdot y(t)+\mathrm{d}f^*(t), \tag{2.10}$$

respectively, where

$$\begin{aligned}
A^*(t)&=\mathcal{I}(H,A)(t),\\
f^*(t)&=h(t)-h(a)+\mathcal{B}(H,f)(t)-\int_a^t \mathrm{d}A^*(\tau)\cdot h(\tau)
\end{aligned}$$

Besides, for every $j \in \{1, 2\}$ and $t \in [a, b]$

$$I + (-1)^j \,\mathrm{d}_j A^*(t) = \big[H(t) + (-1)^j \,\mathrm{d}_j H(t)\big]\big[I + (-1)^j \,\mathrm{d}_j A(t)\big] H^{-1}(t). \tag{2.11}$$

P r o o f. Let x be a solution of the system (2.9) and let $y(t) = H(t)x(t) + h(t)$ for $t \in [a, b]$. In view of (2.7) and the definition of a solution we have

$$\int_a^t \mathrm{d}\mathcal{B}(H, A)(s) \cdot x(s) = \mathcal{B}(H, x - f)(t) \quad \text{for} \quad t \in [a, b].$$

By this and (2.1) we obtain

$$\begin{aligned}
&\int_a^t \mathrm{d}A^*(s) \cdot y(s) + f^*(t) - f^*(a) \\
&\quad = \int_a^t \mathrm{d}A^*(s) \cdot \big[y(s) - h(s)\big] + \mathcal{B}(H, f)(t) + h(t) - h(a) \\
&\quad = \int_a^t \mathrm{d}\bigg[\int_a^s \mathrm{d}\big[H(\tau) + \mathcal{B}(H, A)(\tau)\big] \cdot H^{-1}(\tau)\bigg] \cdot H(s)x(s) \\
&\qquad + \mathcal{B}(H, f)(t) + h(t) - h(a) \\
&\quad = \int_a^t \mathrm{d}\big[H(s) + \mathcal{B}(H, A)(s)\big] \cdot x(s) + \mathcal{B}(H, f)(t) + h(t) - h(a) \\
&\quad = \int_a^t \mathrm{d}H(s) \cdot x(s) + \mathcal{B}(H, x - f)(t) + \mathcal{B}(H, f)(t) + h(t) - h(a) \\
&\quad = \int_a^t \mathrm{d}H(s) \cdot x(s) + \mathcal{B}(H, x)(t) + h(t) - h(a) \\
&\quad = H(t)x(t) - H(a)x(a) + h(t) - h(a) \\
&\quad = y(t) - y(a) \quad \text{for} \quad t \in [a, b],
\end{aligned}$$

i.e. y is a solution of the system (2.10).

Let us prove the converse assertion. It is sufficient to show that for every $t \in [a, b]$

$$\mathcal{I}(H^{-1}, A^*)(t) = A(t) - A(a) \tag{2.12}$$

and

$$\begin{aligned}
&-H^{-1}(t)h(t) + H^{-1}(a)h(a) + \mathcal{B}(H^{-1}, f^*)(t) \\
&\qquad + \int_a^t \mathrm{d}\mathcal{I}(H^{-1}, A^*)(\tau) \cdot H^{-1}(\tau)h(\tau) = f(t) - f(a).
\end{aligned} \tag{2.13}$$

By (2.8) we have

$$\begin{aligned}\mathcal{I}(H^{-1}, A^*)(t) &= \mathcal{I}\big(H^{-1}, \mathcal{I}(H, A)\big)(t) = \mathcal{I}(I, A)(t) \\ &= \int_a^t \mathrm{d}\big[I + \mathcal{B}(I, A)(s)\big] = \mathcal{B}(I, A)(t) = A(t) - A(a).\end{aligned}$$

Therefore equality (2.12) is proved.

Let us show that (2.13) is valid. Let $\mathcal{R}(t)$ be the left hand side od equality (2.13). In view of (2.5) and (2.7) it is easy to verify that

$$\mathcal{B}\left(H^{-1}, \int_a \mathrm{d}\mathcal{B}(H, A)(s) \cdot H^{-1}(s)h(s)\right)(t) = \int_a^t \mathrm{d}A(s) \cdot H^{-1}(s)h(s)$$

and

$$\mathcal{B}\left(H^{-1}, \int_a \mathrm{d}H(s) \cdot H^{-1}(s)h(s)\right)(t) = -\int_a^t \mathrm{d}H^{-1}(s) \cdot h(s).$$

Taking these equalities, (2.1), (2.5), (2.7) and (2.12) into account, we obtain

$$\begin{aligned}\mathcal{R}(t) &= -H^{-1}(t)h(t) + H^{-1}(a)h(a) + \mathcal{B}(H^{-1}, h)(t) \\ &\quad + \mathcal{B}\big(H^{-1}, \mathcal{B}(H, f)\big)(t) - \mathcal{B}\left(H^{-1}, \int_a \mathrm{d}A^*(s) \cdot h(s)\right)(t) \\ &\quad + \int_a^t \mathrm{d}A(s) \cdot H^{-1}(s)h(s) = \mathcal{B}(I, f)(t) - \int_a^t \mathrm{d}H^{-1}(s) \cdot h(s) \\ &\quad - \mathcal{B}\left(H^{-1}, \int_a \mathrm{d}\mathcal{I}(H, A)(s) \cdot h(s)\right)(t) + \int_a^t \mathrm{d}A(s) \cdot H^{-1}(s)h(s) \\ &= f(t) - f(a) - \int_a^t \mathrm{d}H^{-1}(s) \cdot h(s) - \mathcal{B}\left(H^{-1}, \int_a \mathrm{d}H(s) \cdot H^{-1}(s)h(s)\right)(t) \\ &\quad - \mathcal{B}\left(H^{-1}, \int_a \mathrm{d}\mathcal{B}(H, A)(s) \cdot H^{-1}(s)h(s)\right)(t) + \int_a^t \mathrm{d}A(s) \cdot H^{-1}(s)h(s) \\ &= f(t) - f(a).\end{aligned}$$

Hence (2.13) is valid.

Equality (2.11) follows from the equalities

$$\mathrm{d}_j A^*(t) = \mathrm{d}_j\big[H(t) + \mathcal{B}(H, A)(t)\big] \cdot H^{-1}(t) \quad (j = 1, 2)$$

and

$$\mathrm{d}_j\mathcal{B}(H, A)(t) = \mathrm{d}_j\big[H(t)A(t)\big] \cdot \mathrm{d}_j H(t) \cdot A(t) \quad (j = 1, 2)$$

The lemma is proved. □

Lemma 2.3. *Let $\alpha_k, \beta_k \in BV_1(a,b)$ $(k = 0, 1, \dots)$.*

$$\lim_{k\to+\infty} \|\beta_k - \beta_0\|_{\sup} = 0,$$

$$\lim_{k\to+\infty} \sup \bigvee_a^b(\alpha_k) < +\infty$$

and the condition

$$\lim_{k\to+\infty} \big[\alpha_k(t) - \alpha_k(a)\big] = \alpha_0(t) - \alpha_0(a)$$

be fulfilled uniformly on $[a,b]$. Then

$$\lim_{k\to+\infty} \int_a^t \beta_k(\tau)\,\mathrm{d}\alpha_k(\tau) = \int_a^t \beta_0(\tau)\,\mathrm{d}\alpha_0(\tau)$$

uniformly on $[a,b]$.

Lemma 2.4. *Let condition* (1.9) *hold and let*

$$\lim_{k\to+\infty} Y_k(t) = Y_0(t) \tag{2.14}$$

uniformly on $[a,b]$, where Y_k is a fundamental matrix of the system (1.5_k) for any $k \in \{0, 1, \dots\}$. Then

$$\begin{gathered}\inf\big\{\big|\det\big(Y_0(t)\big)\big| : t \in [a,b]\big\} > 0,\\ \inf\big\{\big|\det\big(Y_0^{-1}(t)\big)\big| : t \in [a,b]\big\} > 0\end{gathered} \tag{2.15}$$

and

$$\lim_{k\to+\infty} Y_k^{-1}(t) = Y_0^{-1}(t) \tag{2.16}$$

uniformly on $[a,b]$.

The proofs of Lemmas 2.3 and 2.4 are given in [1].

3. Proof of the main results

Proof of Theorem 1.2. In view of (1.19) $l_k^*\colon BV_n(a,b) \to \mathbb{R}^n$ is a bounded linear operator for every sufficiently large k. Moreover, it is not difficult to see that by the mapping

$$x \to y = H_k x + h_k$$

a one-to-one correspondence between solutions of problem (1.1_k), (1.2_k) and solutions of the problem

$$(3.1_k) \qquad \mathrm{d}y(t) = \mathrm{d}A_k^*(t) \cdot y(t) + \mathrm{d}f_k^*(t),$$

$$(3.2_k) \qquad l_k^*(y) = c_k^*,$$

is given, where $c_k^* = c_k + l_k^*(h_k)$. In fact, according to Lemma 2.2 it is sufficient to show that equality (1.2_k) implies equality (3.2_k) and conversely. But this is obvious, since

$$l_k^*(y) = l_k(H_k^{-1}y) = l_k(x) + l_k^*(h_k).$$

It follows from (1.20)–(1.25) that the conditions of Theorem 1 from [1] are fulfilled for the sequence of problems (3.1_k), (3.2_k) $(k = 1, 2, \ldots)$ and problem (1.17), (1.18). Hence by the same theorem problem (3.1_k), (3.2_k) has the unique solution y_k for any sufficiently large k and

$$\lim_{k\to+\infty} \|y_k - x_0^*\|_{\sup} = \lim_{k\to+\infty} \|H_k x_k + h_k - x_0^*\|_{\sup} = 0,$$

where $x_k(t) = H_k^{-1}(t)\big[y_k(t) - h_k(t)\big]$ is the unique solution of problem (1.1_k), (1.2_k). $\square$

Proof of Corollary 1.2. Verifying the conditions of Theorem 1.2. (1.12) and (1.13) we obtain (1.19) and

$$(3.3) \qquad \lim_{k\to+\infty} \|H_k^{-1} - H_0^{-1}\|_{\sup} = 0.$$

Put

$$h_k(t) = -H_k(t)\varphi_k(t) \quad \text{for} \quad t \in [a,b] \ (k = 1, 2, \ldots)$$

Then by (1.7), (1.8), (1.27) and (3.3) conditions (1.20)–(1.22), where $c_0^* = c_0$ and $l_0^*(y) = l_0(H_0^{-1}y)$, are satisfied.

Applying Lemma 2.3, from (1.11), (1.13), (1.14) and (3.3) we find that (1.23) holds and (1.24) is fulfilled uniformly on $[a,b]$, where

$$A_0^*(t) = \mathcal{I}(H_0, A_0)(t).$$

On the other hand,

$$f_k^*(t) = \mathcal{B}(H_k, f_k - \varphi_k)(t) + \int_a^t \mathrm{d}\mathcal{B}(H_k, A_k)(s) \cdot \varphi_k(s) \quad \text{for} \quad t \in [a, b]$$

for any natural k. Therefore, (1.28) implies that (1.25), where

$$f_0^*(t) = \mathcal{B}(H_0, f_0)(t),$$

is fulfilled uniformly on $[a, b]$.

Taking into account Lemma 2.2 and the equalities

$$l_0^*(H_0 x_0) = l_0(x_0) = c_0$$

it is not difficult to see that problem (1.17), (1.18) has a unique solution

$$x_0^*(t) = H_0(t) x_0(t).$$

Moreover, it can be easily shown that inequality (1.12) is equivalent to the condition

$$\det\big(H_0(t+) \cdot H_0(t-)\big) \neq 0 \quad \text{for} \quad t \in [a, b].$$

Thus in virtue of (1.9) and (2.11) condition (1.16) is fulfilled.

According to theorem 1.2 condition (1.26) holds. Hence (1.29) follows from (1.26) and (3.3). □

P r o o f o f C o r o l l a r y 1.3. By (1.32), (1.33) and (1.34) (or (1.35)) we have

$$\lim_{k\to+\infty} \sum_{a<s\leqslant t} \big[\mathrm{d}_1 H_k(s) \cdot \mathrm{d}_1 A_k(s) - \mathrm{d}_1 H_0(s) \cdot \mathrm{d}_1 A_0(s)\big] = 0$$

$$\lim_{k\to+\infty} \sum_{a<s\leqslant t} \big[\mathrm{d}_1 H_k(s) \cdot \mathrm{d}_1 f_k(s) - \mathrm{d}_1 H_0(s) \cdot \mathrm{d}_1 f_0(s)\big] = 0$$

$$\lim_{k\to+\infty} \sum_{a\leqslant s<t} \big[\mathrm{d}_2 H_k(s) \cdot \mathrm{d}_2 A_k(s) - \mathrm{d}_2 H_0(s) \cdot \mathrm{d}_2 A_0(s)\big] = 0$$

and

$$\lim_{k\to+\infty} \sum_{a\leqslant s<t} \big[\mathrm{d}_2 H_k(s) \cdot \mathrm{d}_2 f_k(s) - \mathrm{d}_2 H_0(s) \cdot \mathrm{d}_2 f_0(s)\big] = 0$$

uniformly on $[a, b]$. From this, the integration-by-parts formula (2.2), (1.30) and (1.31) we obtain that the conditions (1.14) and (1.28), where $\varphi_k(t) \equiv 0$, are fulfilled uniformly on $[a, b]$.

Therefore, Corollary 1.3 follows from Corollary 1.2. □

Proof of Corollary 1.4. Using (1.13), (1.36) and (1.38) we have

$$\mathrm{d}_j A_*(t) = 0 \quad \text{for} \quad t \in [a,b] \ (j = 1,2).$$

Hence, in view of (1.9)

$$\det\bigl(I + (-1)^j \,\mathrm{d}_j A_{0*}(t)\bigr) \neq 0 \quad \text{for} \quad t \in [a,b] \ (j = 1,2).$$

On the other hand, (1.13), (1.36)–(1.39) yield

$$\lim_{k\to+\infty} \mathcal{B}(H_0^{-1}H_k, A_k)(t) = \mathcal{B}(I, A_{0*})(t)$$

and

$$\lim_{k\to+\infty} \mathcal{B}(H_0^{-1}H_k, f_k)(t) = \mathcal{B}(I, f_{0*})(t)$$

uniformly on $[a,b]$. Thus, Corollary 1.4 is a direct consequence of Corollary 1.2. □

Proof of Corollary 1.5. It is suffices to assume in Corollary 1.2 that $\varphi_k(t) = 0$ and $H_k(t) = H_{km}(t)$ and to notice that by (2.5), (2.6) and (2.8) for every $j \in \{1,\dots,m\}$ and $t \in [a,b]$

$$\mathcal{B}\bigl(B_{kj}, \mathcal{B}(B_{kj-1},\dots,\mathcal{B}(B_{k1},A_k)\dots)\bigr)(t) = \mathcal{B}(H_{kj},A_k)(t),$$
$$\mathcal{B}\bigl(B_{kj}, \mathcal{B}(B_{kj-1},\dots,\mathcal{B}(B_{k1},f_k)\dots)\bigr)(t) = \mathcal{B}(H_{kj},f_k)(t),$$

and

$$\mathcal{I}\bigl(B_{kj}, \mathcal{I}(B_{kj-1},\dots,\mathcal{I}(B_{k1},A_k)\dots)\bigr)(t) = \mathcal{I}(H_{kj},A_k)(t),$$

where $B_{kj}(t) = I - A_{kj}(t) + A_{0j}(t)$. □

Proof of Theorem 1.1. Sufficiency follows from Corollary 1.2.

Let us show necessity. Let $c_k \in \mathbb{R}^n$ $(k = 0,1,\dots)$ be an arbitrary sequence satisfying (1.6) and let $e_j = (\delta_{ij})_{i=1}^n$ $(j = 1,\dots,n)$, where $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$.

In view of (1.10) we may assume without loss of generality that problem (1.1_k), (1.2_k) has a unique solution x_k for every natural k.

For any $k \in \{0,1,\dots\}$ and $j \in \{1,\dots,n\}$ let us denote

$$y_{kj}(t) = x_k(t) - x_{kj}(t) \quad \text{for} \quad t \in [a,b],$$

where x_{kj} is the unique solution of (1.1_k) satisfying

$$l_k(x) = c_k - e_j.$$

Moreover, by $Y_k(t)$ let us denote the matrix-function whose columns are $y_{k1}(t), \dots, y_{kn}(t)$.

It can be easily shown that y_{kj} is a solution of (1.5_k) and

$$l_k(y_{kj}) = e_j \quad (j = 1, \dots, n; \; k = 0, 1, \dots). \tag{3.4}$$

If for some k and $\alpha_j \in \mathbb{R}$ $(j = 1, \dots, n)$

$$\sum_{j=1}^{n} \alpha_j y_{kj}(t) = 0 \quad \text{for} \quad t \in [a, b],$$

then using (3.4)

$$\sum_{j=1}^{n} \alpha_j e_j = 0$$

and therefore

$$\alpha_1 = \dots = \alpha_n = 0.$$

i.e. Y_k is the fundamental matrix of system (1.5_k) for every $k \in \{0, 1, \dots\}$. Hence by (1.6), (1.10) and Lemma 2.4 conditions (2.14)–(2.16) hold. We may assume without loss of generality that

$$Y_k(a) = I \quad (k = 0, 1, \dots).$$

For every $k \in \{0, 1, \dots\}$ and $t \in [a, b]$ assume that

$$H_k(t) = Y_k^{-1}(t)$$

and verify (1.11)–(1.15).

Condition (1.12) coincides with (2.11).

According to Proposition III.2.15 from [9] for every $k \in \{0, 1, \dots\}$ and $t \in [a, b]$ we have

$$Y_k^{-1}(t) = I - \mathcal{B}(Y_k^{-1}, A_k)(t). \tag{3.5}$$

Thus (1.11) is evident. On the other hand, by (2.7) and (3.5), from (1.4) we have

$$
\begin{aligned}
\mathcal{B}(H_k, f_k)(t) &= \mathcal{B}\Big(Y_k^{-1}, x_k - \int_a \mathrm{d}A_k(s)\cdot x_k(s)\Big)(t)\\
&= \mathcal{B}\Big(Y_k^{-1}, x_k)(t) - \mathcal{B}(Y_k^{-1}, \int_a \mathrm{d}A_k(s)\cdot x_k(s)\Big)(t)\\
&= \mathcal{B}(Y_k^{-1}, x_k)(t) - \int_a^t \mathrm{d}\mathcal{B}(Y_k^{-1}, A_k)(s)\cdot x_k(s)\\
&= Y_k^{-1}(t)x_k(t) - x_k(a) - \int_a^t \mathrm{d}Y_k^{-1}(s)\cdot x_k(s)\\
&\quad - \int_a^t \mathrm{d}\big[I - Y_k^{-1}(s)\big]\cdot x_k(s)\\
&= Y_k^{-1}(t)x_k(t) - x_k(a).
\end{aligned}
$$

This, (1.3), (2.16) and (3.5) imply that conditions (1.13)–(1.15) are fulfilled uniformly on $[a, b]$. The theorem is proved. □

Corollary 1.1 follows from Theorem 1.1, since for any linear continuous operator $l\colon C_n(a,b) \to \mathbb{R}^n$ there exists a linear continuous operator $\widetilde{l}\colon BV_n(a,b) \to \mathbb{R}^n$ such that $\|l\| = \|\widetilde{l}\|$ and $l(y) = \widetilde{l}(y)$ for $y \in \widetilde{C}_n(a,b)$.

References

[1] *M. Ashordia*: On the correctness of linear boundary value problems for systems of generalized ordinary differential equations. Proceedings of the Georgian Academy of Sciences. Mathematics *1* (1993), No. 4, 385–394.

[2] *И.Т. Кигурадзе*: Краевые задачи для систем обыкновенных дифференциальных уравнений, Современные проблеы математики. Новейшие достижения (Итоги науки и техники. ВИНИТИ Акад. Наук СССР), Москва 1987, т. 30, 3–103.

[3] *Д.Г. Бицадзе, И.Т. Кигурадзе*: Об устойчивости множестба решений нелинейных краевых задач. Дифференциальные уравнения *20* (1984), No. 9, 1495–1501.

[4] *Д.Г. Бицадзе*: К вопросу о зависимости от параметра решений многоточечных нелилейных краевыч задач. Труды Ин-та прикл. мат. Тбилисс. ун-та *22* (1987), 42–55.

[5] *М.А. Красносельский, С.Г. Крейн*: О принципе усреднения в нелинейноймеханике. Успехи мат. наук *10* (1955), No. 3, 147–152.

[6] *J. Kurzweil, J. Jarník*: Interated Lie brackets in limit processes in ordinary differential equations, Results in Mathematics. Birkhäuser Verlag. Basel *14* (1988), 125–137.

[7] *M. Ashordia*: On the stability of solutions of linear boundary value problems for the system of ordinary differential equations. Proceedings of the Georgian Academy of Sciences. Mathematic *1* (1993), No. 2, 129–141.

[8] *J. Kurzweil*: Generalized ordinary differential equations and continuous dependence on a parameter. Czechoslovak Math. J. *7(82)* (1957), 418–449.

[9] *Š. Schwabik, M. Tvrdý, O. Vejvoda*: Differential and Integral Equations. Boundary value Problems and Adjoints, Praha, 1979.

[10] *P.C. Das, R.R. Sharma*: Existence and stability of measure differential equations. Czechoslovak Math. J. *22(97)* (1972), 145–158.
[11] *М.Т. Ашордия*: О непрерывной зависимости решения задачи Коши для системы обобщенных обыкновенных дифференциальных уравнений от параметра. Труды Ин-та. прикл. мат. Тбилисс. ун-та *31* (1988), 5–22.

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