Ivan Kolář; Jan Slovák
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ON THE GEOMETRIC FUNCTORS ON MANIFOLDS

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A. Nijenhuis pointed out that the classical bundles of geometric objects can be viewed as certain functors transforming manifolds and their local diffeomorphisms into fibred manifolds and their morphisms, [10]. Some of those functors are defined on the whole category Mf of all manifolds and all smooth maps. However, one can observe that several general aspects of the theory of the bundle functors on the whole category Mf are rather different from the case of the classical bundles of geometric objects. Recently it has been deduced, [1], [5], [7], that the product-preserving bundle functors on Mf coincide with the functors defined by means of the finite-dimensional local algebras by A. Weil, [15]. The main aim of our present paper are some general properties of the non-product-preserving bundle functors on Mf. We deduce that the fibres of the bundle functors with the so-called point property, [6], are diffeomorphic to numerical spaces. We show that the product-preserving functors are fully characterized by the corresponding condition on dimensions. We prove that every bundle functor on Mf satisfies the so-called prolongation axiom by J. Pradines, [13]. Finally we deduce that the bundle functors without the point property can be interpreted as certain parametrized systems of functors with the point property. - We assume all manifolds and maps to be infinitely differentiable and all manifolds to be paracompact.

1. ORDER OF A BUNDLE FUNCTOR

Let Mf be the category of all manifolds and all maps, FM be the category of all fibred manifolds and their morphisms and B: FM $\to$ Mf be the base functor. Given a functor F: Mf $\to$ FM satisfy-

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This paper is in final form and no version of it will be submitted for publication elsewhere.

ing $B\circ F = \mathrm{id}_{\underline{Mf}}$, we denote by $p_M\colon FM \to M$ its value on a manifold M and by $F_xf\colon F_xM \to F_{f(x)}N$ the restriction of its value $Ff\colon FM \to FN$ on $f\colon M \to N$ to the fibres of FM over x and of FN over $f(x)$, $x \in M$.

Definition 1. A bundle functor on $\underline{Mf}$ is a functor $F\colon \underline{Mf} \to \underline{FM}$ satisfying $B\circ F = \mathrm{id}_{\underline{Mf}}$ and the localization condition: if $i\colon U \hookrightarrow M$ is the inclusion of an open subset, then $FU = p_M^{-1}(U)$ and Fi is the inclusion $p_M^{-1}(U) \hookrightarrow FM$.

If we replace the category $\underline{Mf}$ by the category $\underline{Mf}_m$ of all m-dimensional manifolds and their local diffeomorphisms, we obtain the classical concept of a natural bundle in dimension m by Nijenhuis, [10], and Palais-Terng, [11]. Hence the restriction F_m of a bundle functor F on $\underline{Mf}$ to $\underline{Mf}_m$ is a natural bundle in dimension m.

Let M, N, P be manifolds. A parametrized system of smooth maps $f_p\colon M \to N$, $p \in P$ is said to be smoothly parametrized, if the resulting map $f\colon M \times P \to N$ is smooth. The following result was deduced in a more general context in [14], but we find it useful to present a direct proof here.

Proposition 1. Every bundle functor $F\colon \underline{Mf} \to \underline{FM}$ satisfies the regularity condition: if $f\colon M \times P \to N$ is a smoothly parametrized family, then the family $\tilde{F}f\colon FM \times P \to FN$ defined by $(\tilde{F}f)_p = F(f_p)$ is also smoothly parametrized.

Proof. Since smoothness is local property, it suffices to discuss the case $f\colon R^m \times R^k \to N$. Denote by $i\colon R^m \to R^m \times R^k$ the injection $x \mapsto (x,0)$ and by t_p the translation on $R^m \times R^k$ transforming the origin into $(0,p)$, $p \in R^k$. A deep analytical result by Epstein and Thurston reads that Ft_p, $p \in R^k$, is a smoothly parametrized family, [2]. We have $f_p = f\circ t_p\circ i$, so that $Ff_p = Ff\circ Ft_p\circ Fi$ is a smoothly parametrized family as well, QED.

According to Palais-Terng, [11], every natural bundle F_m has a finite order $r(m)$, i.e. $j^{r(m)}f(x) = j^{r(m)}g(x)$ implies $F_xf = F_xg$ for any two local diffeomorphisms of m-dimensional manifolds. We deduce a similar result for arbitrary maps. Write $m = \dim M$, $n = \dim N$ and $r(m,n) = r(\max(m,n))$.

Proposition 2. For any maps $f,g\colon M \to N$, $j^{r(m,n)}f(x) = j^{r(m,n)}g(x)$ implies $F_xf = F_xg$.

Proof. By locality, it suffices to consider two maps $f,g\colon R^m \to R^n$.

We have to discuss three cases.

I. Let any two maps $f,g\colon R^m \longrightarrow R^m$ satisfy $j^rf(x) = j^rg(x)$ with $r = r(m)$. Consider one-parameter families $f_t = f + t\,\mathrm{id}_{R^m}$, $g_t = g + t\,\mathrm{id}_{R^m}$, $t \in R$. Since their Jacobians at x are certain non-zero polynomials in t, f_t and g_t are local diffeomorphisms in a neighbourhood of x except a finite number values of t. Since $j^rf_t(x) = j^rg_t(x)$ for all t, the classical result, [11], implies $F_xf_t = F_xg_t$ except a finite number values of t. Then the regularity condition yields $F_xf_o = F_xg_o$.

II. Let $m = n+k$, $k > 0$, and $f,g\colon R^{n+k} \longrightarrow R^n$ satisfy $j^rf(x) = j^rg(x)$ with $r = r(m)$. Consider $\bar f = (f,pr_2)$, $\bar g = (g,pr_2)\colon R^m \longrightarrow R^m$, where $pr_2\colon R^n \times R^k \longrightarrow R^k$ is the second product projection. Obviously, it holds $j^r\bar f(x) = j^r\bar g(x)$. Since $f = pr_1\circ\bar f$, $g = pr_1\circ\bar g$, functoriality and I. imply $F_xf = F_{\bar f(x)}pr_1\circ F_x\bar f = F_{\bar g(x)}pr_1\circ F_x\bar g = F_xg$.

III. Let $m+k = n$, $k > 0$ and $f,g\colon R^m \longrightarrow R^{m+k}$ satisfy $j^rf(x) = j^rg(x)$ with $r = r(n)$. Consider $\bar f = f\circ pr_1$, $\bar g = g\circ pr_1\colon R^n \longrightarrow R^n$, where $pr_1\colon R^m \times R^k \longrightarrow R^m$ is the first product projection. We have $j^r\bar f(y) = j^r\bar g(y)$ for every y satisfying $pr_1(y) = x$. Since $f = \bar f\circ i$, $g = \bar g\circ i$, functoriality and I. yield $F_xf = F_{i(x)}\bar f\circ F_xi = F_{i(x)}\bar g\circ F_xi = F_xg$, QED.

Remark 1. If F is a product-preserving functor, it is a Weil functor, [1], [5], [7]. If r is the order of the corresponding local algebra, it holds $r(m) = r$ for all dimensions m. On the other hand, Mikulski has constructed a non-product-preserving functor of infinite order, i.e. with an unbounded sequence of $r(m)$, [8]:

2. FUNCTORS WITH THE POINT PROPERTY

Let pt denote a one-point manifold.

Definition 2, [6]. A bundle functor $F\colon \underline{Mf} \longrightarrow \underline{FM}$ is said to have the point property, if $F(pt) = pt$.

Obviously, every product-preserving functor has the point property. An example of a non-product-preserving functor with the point property is the r-th order tangent functor in the sense of

F. W. Pohl, [12], [6].

An interesting feature of the bundle functors with the point property is the existence of natural canonical sections $c_M\colon M \to FM$ defined by $c_M(x) = Fi_x(pt)$, where $i_x\colon pt \to M$ is the injection $i_x(pt) = x$ of pt into $x \in M$. The regularity condition of Proposition 1 implies that c_M are smooth maps. Naturality of those maps means $c_N \circ f = Ff \circ c_M$ for all maps $f\colon M \to N$, which follows directly from the definition.

Proposition 3. If F is a bundle functor with the point property, then every fibre F_xM is diffeomorphic to a numerical space $R^{k(m)}$, $m = \dim M$.

The proof is based on a lemma from differential topology.

Lemma. Let S be a paracompact m-dimensional manifold and $s \in S$ be a point. Let h_t be a smoothly parametrized system of maps, $t \in R$, $h_1 = id_S$, $h_o(S) = \{s\}$ and let h_t be diffeomorphisms for all $t \neq 0$. Then S is diffeomorphic to R^m.

Proof. We first recall a well known fact that if $S = \bigcup_{k=0}^{\infty} S_k$, where S_k are open submanifolds diffeomorphic to R^m and $S_k \subset S_{k+1}$ for all k, then S is diffeomorphic to R^m, see [3], Chapter 1, § 2. We are going to construct a sequence with the latter property. Choose an increasing sequence of relatively compact open submanifolds $K_n \subset K_{n+1} \subset S$, $S = \bigcup_{k=1}^{\infty} K_n$ and take a relatively compact neighbourhood U of the point s diffeomorphic to R^m. Put $S_o = U$. Since S_o is relatively compact, there exist an integer n_1 with $K_{n_1} \supset S_o$ and a $t_1 > 0$ with $h_{t_1}(K_{n_1}) \subset U$. Then we define $S_1 = (h_{t_1})^{-1}(U)$. We have $S_1 \supset K_{n_1} \supset S_o$, S_1 is relatively compact and diffeomorphic to R^m. Iterating this procedure, we construct S_k and n_k satisfying $S_k \supset K_{n_k} \supset S_{k-1}$, $n_k > n_{k-1}$, QED.

Proof of Proposition 3. It suffices to deduce that F_oR^m is diffeomorphic to $R^{k(m)}$. Write $S = F_oR^m$, $s = c_{R^m}(0)$ with $0 \in R^m$ and let $g_t\colon R^m \to R^m$ be the system of homotheties $g_t(x) = tx$, $t \in R$. Since

$g_t(0) = 0$ for all t and g_o coincides with the composition $R^m \to pt \to \{0\}$, the smoothly parametrized system $h_t = Fg_t|S$ satisfies all assumptions of the Lemma, QED.

3. PRODUCTS AND DIMENSIONS

Let F be an arbitrary bundle functor on <u>Mf</u>.

Proposition 4. If $f: M \to N$ is an immersion, then $Ff: FM \to FN$ is an immersion as well.

Proof. It suffices to discuss an immersion in its local canonical form $i: R^m \to R^n = R^{m+k}$, $x \mapsto (x,0)$. Since the canonical projection $p: R^n = R^{m+k} \to R^m$ satisfies $p \circ i = id_{R^m}$, we have $Fp \circ Fi = id_{FR^m}$. Hence $TFp \circ T_yFi = id$ for all $y \in FR^m$, which implies that Fi is an immersion.

Remark 2. It is well known that connected paracompact smooth manifolds are exactly smooth neighbourhood retracts in numerical spaces, see [3], Chapter 1, § 1. A direct consequence is that a smooth map $f: M \to N$ is an embedding if and only if there are an open submanifold $U \subset N$ with $f(M) \subset U$ and a map $g: U \to M$ satisfying $g \circ f = id_M$. This implies that any bundle functor F on <u>Mf</u> transforms embeddings into embeddings. Indeed, if $f: M \to N$ is an embedding and $g: U \to M$ is the above map, then $Fg \circ Ff = id_{FM}$ and FU is an open submanifold in FN, so that Ff is an embedding as well.

For technical reasons, we study the bundle functors with the point property in the rest of this section, but we shall show in the next section that similar results hold for arbitrary bundle functors on <u>Mf</u>. Consider a product of two manifolds $M \xleftarrow{p} M \times N \xrightarrow{q} N$. The induced maps $FM \xleftarrow{Fp} F(M \times N) \xrightarrow{Fq} FN$ determine a canonical map $\pi : F(M \times N) \to FM \times FN$.

Proposition 5. If F has the point property then $\pi : F(M\times N) \to FM \times FN$ is a surjective submersion.

Proof. It suffices to discuss the case $M = R^m$, $N = R^n$. Write $0_1 = c_{R^m}(0) \in FR^m$, $0_2 = c_{R^n}(0) \in FR^n$, $0_3 = c_{R^{m+n}}(0) \in FR^{m+n}$. Let $i: R^m \to R^{m+n}$, $x \mapsto (x,0)$ and $j: R^n \to R^{m+n}$, $y \mapsto (0,y)$ be the canonical injections. In the tangent space $T_{0_3}F(R^{m+n})$ we have two subspaces $V = TFi(T_{0_1}FR^m)$ and $W = TFj(T_{0_2}FR^n)$. We are going to deduce $V \cap W = 0$. Let $A \in V \cap W$, $A = TFi(B) = TFj(C)$, $B \in T_{0_1}FR^m$, $C \in T_{0_2}FR^n$. On one hand, $p \circ i = \mathrm{id}_{R^m}$ implies $TFp(A) = TFp(TFi(B)) = B$. On the other hand, $p \circ j$ is the constant map of R^n into the zero point of R^m. The latter map can be factorized as $R^n \to pt \xrightarrow{i_o} R^m$. Hence $Fp \circ Fj$ is the constant map of FR^n into 0_1. This implies $0 = TFp \circ \circ TFj(C) = TFp(A) = B$. Hence $V \cap W = 0$, so that π is a submersion at $0_3 \in FR^{m+n}$ and consequently on a neighbourhood $U \subset FR^{m+n}$ of 0_3. Since all homotheties g_t on R^m, R^n and R^{m+n} commute with the product projections and π is induced by Fp and Fq, the images Fg_t commute with π as well. The family Fg_t is smoothly parametrized and $Fg_0(FR^{m+n}) = \{0_3\}$, so that every point of FR^{m+n} can be mapped into U by a suitable Fg_t, $t > 0$. Taking into account that Fg_t, $t > 0$, are diffeomorphisms, we see that π is a submersion. Therefore the image $\pi(FR^{m+n})$ is an open neighbourhood of $(0_1, 0_2) \in FR^m \times FR^n$. Similarly as above, every point of $FR^m \times FR^n$ can be mapped into $\pi(FR^{m+n})$ by a suitable Fg_t, $t > 0$. This implies that π is surjective, QED.

Remark 3. It is easy to check that Proposition 5 can be extended to an arbitrary finite product of manifolds,

Proposition 6. If F has the point property and $f: M \to N$ is a submersion, then $Ff: FM \to FN$ is also a submersion.

Proof. It suffices to discuss a submersion in its local canonical form $p: R^n \times R^k \to R^n$. Then $Fp = pr_1 \circ \pi$ is a composition of two submersions $\pi: F(R^n \times R^k) \to FR^n \times FR^k$ and $pr_1: FR^n \times FR^k \to FR^n$, QED.

Proposition 7. If F has the point property, then $k(m+n) \geq k(m)+k(n)$. The equality holds if and only if F preserves products in dimensions m and n.

Proof. By Proposition 5, $\pi: F(R^m \times R^n) \to FR^m \times FR^n$ is a submersion, which implies $k(m+n) \geq k(m)+k(n)$. If the equality holds, π is a local diffeomorphism at each point, so that π is a covering. But

$FR^m = R^m \times R^{k(m)}$ by Proposition 3, so that $FR^m \times FR^n$ is simply connected and π must be a global diffeomorphism. Dealing with arbitrary manifolds M, N, we obtain the result by the localization property of the bundle functors and by a standard diagram chasing, QED.

Corollary 1. A bundle functor $F: \underline{Mf} \to \underline{FM}$ preserves products if and only if $F(pt) = pt$ and $k(m) = mk(1)$ for all integers m.

Remark 4. An interesting consequence of our results is that a bundle functor F on $\underline{Mf}$ with the point property can transform principal fibre bundles into principal fibre bundles if and only if F preserves products. More precisely, for any principal fibre bundle (P,p,M,G) we are looking for a natural principal fibre bundle structure on $Fp: FP \to FM$ with respect to a natural group structure on FG. This can be defined by prolongating all maps in question if F preserves products and this is impossible if F does not preserve products for the dimension reasons of Proposition 7. (We remark that there is a natural group structure on FG even for the non-product-preserving functor of the r-th order tangent vectors, [6].)

4. FUNCTORS WITHOUT THE POINT PROPERTY

Consider an arbitrary bundle functor $F: \underline{Mf} \to \underline{FM}$. Hence $Q = F(pt)$ is a manifold and the unique map $q_M: M \to pt$ induces $Fq_M: FM \to Q$. Similarly to § 2, every point $a \in Q$ determines canonical natural sections $c(a)_M: M \to FM$ defined by $c(a)_M(x) = Fi_x(a)$, where i_x is the injection of pt into $x \in M$. Let $G: \underline{Mf} \to \underline{FM}$ be the bundle functor defined by $GM = M \times Q$ and $Gf = f \times id_Q$ for all manifolds and maps.

Proposition 8. The maps $\sigma_M(x,a) = c(a)_M(x)$, $x \in M$, $a \in Q$, and $\rho_M(z) = (p_M(z), Fq_M(z))$, $z \in FM$, define natural transformations $\sigma: G \to F$ and $\rho: F \to G$ satisfying $\rho \circ \sigma = id$. Moreover, σ_M is an embedding and ρ_M is a surjective submersion for every manifold M.

Proof. The proof of the first sentence is straightforward. By Remark 2, the equality $\rho_M \circ \sigma_M = id_{M \times Q}$ implies that σ_M is an embedding. The latter equality also implies that ρ_M is surjective and has the maximal rank on a neighbourhood U of the image

$\sigma_M(M\times Q)$. It suffices to prove that every ρ_{R^m} is a submersion. Consider the homotheties $g_t(x) = tx$ on R^m, Then Fg_t is a smoothly parametrized family with $Fg_1 = \mathrm{id}$ and $Fg_0(FR^m) = Fi_0 \circ Fq_{R^m}(FR^m) \subset \sigma_M(R^m\times Q)$. Hence every point of FR^m is mapped into U by some Fg_t, $t>0$ and consequently ρ_{R^m} has maximal rank everywhere, QED.

Corollary 2. For every $a\in Q$, the rule $F_aM = (Fq_M)^{-1}(a)$, $F_af = Ff|F_aM$ determines a bundle functor with the point property.

Corollary 3. Every bundle functor F: Mf → FM transforms submersions into submersions.

Proof. By Proposition 8, every induced map Ff: FM → FN is a base-preserving morphism of fibred manifold Fq_M: FM → Q into Fq_N: FN → Q. If f: M → N is a submersion, then every F_af: $F_aM \to F_aN$ is a submersion by Proposition 6. Hence Ff must be also a submersion, QED.

The following corollary was deduced by quite different methods by Mikulski, [9].

Corollary 4. Let F: Mf → FM be a bundle functor with compact fibres. Then F is naturally equivalent to a trivial bundle functor of order 0.

Proof. If the standard fibres of F are compact, then all functors F_a of Corollary 2 coincide with the identity functor on Mf by Proposition 3. Hence the natural transformations σ and ρ of Proposition 8 are natural equivalences, QED.

Remark 5. We remark that Proposition 5 does not hold for general bundle functors. However, using Propositions 7 and 8 and Corollary 2, we can deduce the inequality $k(m+n) \geq k(m)+k(n) - \dim Q$. Another simple consequence of Corollary 2 is that the equality holds if and only if all functors F_a, $a\in Q$, preserve products in dimensions m and n.

Let f: Y → X be a submersion and $FX\oplus Y$ be the pullback of Y with respect to p_X: FX → X. Since Ff: FY → FX and p_Y: FY → Y satisfy $p_X \circ Ff = f\circ p_Y$, we have an induced pullback map μ : $FY \to FX\oplus Y$.

Proposition 9. For every submersion f: Y → X, the pullback map

μ: $FY \to FX \oplus Y$ is also a submersion.

Proof. Taking into account the universal property of pullbacks, the fibration ρ_M: $FM \to M \times Q$ from Proposition 8, the functors F_a, $a \in Q$ defined in Corollary 2 and standard diagram chasings, we may restrict ourselves to the functors with the point property. It suffices to discuss a submersion in its local canonical form f: $R^{m+n} \to R^m$, $f(x,y) = x$. Let s_y: $R^m \to R^{m+n}$ be the sections $x \mapsto (x,y)$, $y \in R^n$. Denoting by $FR^m \oplus R^{m+n}$ the Whitney sum over R^m, we define a map t: $FR^m \oplus R^{m+n} \to FR^{m+n}$ by $t(z,(x,y)) = Fs_y(z)$. Then t is smooth and $t(c_{R^m}(x), (x,y)) = c_{R^{m+n}}(x,y)$, so that we have constructed a smooth section of μ through the values $c_{R^{m+n}}(x,y)$. Therefore μ is of maximal rank on a neighbourhood of $c_{R^{m+n}}(R^{m+n})$ and the proof is completed using the family of all homotheties on R^{m+n} in the same way as in the proof of Proposition 8, QED.

Remark 6. We can reformulate Proposition 9 by saying that every bundle functor on Mf satisfies the so-called prolongation axiom introduced by Pradines, [13], in a more general situation.

The simpliest example of a bundle functor without the point property can be constructed as follows. We take any bundle functor G with the point property and any manifold Q and we define $FM = GM \times Q$, $Ff = Gf \times \mathrm{id}_Q$. We present an example showing that not all bundle functors on Mf are of this type, not even if Q is connected. The basic idea of our example is that some of the individual "fibre components" F_a of our functor F coincide with the functor T_1^2 of 1-dimensional velocities of the second order and the other ones are the Whitney sum $T \oplus T$ of the tangent functor with itself in dependence on the zero values of a smooth function on Q. However, to give an exact proof of it, we shall use a formal procedure by Janyška, [4].

Example. Let L^2 be the category, the objects of which are the non-negative integers, the morphisms $L^2(m,n) = J_0^2(R^m,R^n)_0$ are the second order jets of R^m into R^n with source 0 and target 0 and the composition in L^2 is the composition of jets. Consider any manifold Q and define $S_n = Q \times R^n \times R^n$ for $n = 0,1,\dots$. By [4], every action of L^2 on the system $S = \{S_0, S_1, \dots\}$ determines a second order bundle functor on Mf. Let a_i^p, a_{ij}^p be the canonical coordinates on $L^2(m,n)$ and y^i, z^i be the canonical coordinates on $R^n \times R^n$. Take any smooth function $f\colon Q \to R$ and define

$$(a^p_i, a^p_{ij})(q, y^i, z^i) = (q, a^p_i y^i, f(q) a^p_{ij} y^i y^j + a^p_i z^i)$$

$q \in Q$. One verifies easily that this really is an action of L^2 on S. Let F be the bundle functor defined by this action, so that $F(pt) = Q$. Obviously, if $f(q) = 0$, then the functor F_q in the sense of Corollary 2 is the Whitney sum $T \oplus T$. If $f(q) \neq 0$, then F_q is naturally equivalent to the functor T^2_1 mentioned above. This can be easily deduced by the Janyška's method: the maps $R^{2n} \to R^{2n}$, $y^i \mapsto y^i$, $z^i \mapsto f(q) z^i$ are L^2-covariant and invertible, so that they determine a natural equivalence of T^2_1 into F_q.

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INSTITUT OF MATHEMATICS OF THE ČSAV,
BRANCH BRNO
MENDELOVO NÁM. 1
CS-66282 BRNO