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A GENERALIZATION OF NORMAL SPACES

V. Renukadevi and D. Sivaraj

Abstract. A new class of spaces which contains the class of all normal spaces is defined and its characterization and properties are discussed.

1. Introduction and preliminaries

Ideals in topological spaces have been considered since 1930. This topic has won its importance by the paper of Vaidyanathaswamy [8]. An *ideal* $\mathcal{I}$ on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies (i) $A\in\mathcal{I}$ and $B\subset A$ implies $B\in\mathcal{I}$ and (ii) $A\in\mathcal{I}$ and $B\in\mathcal{I}$ implies $A\cup B\in\mathcal{I}$. Given a topological space (X,τ) with an ideal $\mathcal{I}$ on X and if $\wp(X)$ is the set of all subsets of X, a set operator $(\cdot)^\star\colon\wp(X)\to\wp(X)$, called the *local function* [4] of A with respect to τ and $\mathcal{I}$, is defined as follows: for $A\subset X$, $A^\star(\mathcal{I},\tau)=\big\{x\in X\mid U\cap A\notin\mathcal{I}$ for every $U\in\tau(x)\big\}$ where $\tau(x)=\big\{U\in\tau\mid x\in U\big\}$. A Kuratowski closure operator $\mathrm{cl}^\star(\cdot)$ for a topology $\tau^\star(\mathcal{I},\tau)$, called the $\star$-*topology*, finer than τ is defined by $\mathrm{cl}^\star(A)=A\cup A^\star(\mathcal{I},\tau)$ [9]. When there is no chance for confusion, we will simply write $A^\star$ for $A^\star(\mathcal{I},\tau)$. We will make use of the properties of the local function established in Theorem 2.3 of [3] without mentioning it explicitly. The aim of this paper is to introduce a new class of spaces called $\mathcal{I}$-*normal* spaces which contains the class of all normal spaces and discuss some of its properties.

By a space, we always mean a topological space (X,τ) with no separation properties assumed. If $A\subset X$, $\mathrm{cl}(A)$ and $int(A)$ will, respectively, denote the closure and interior of A in (X,τ). An ideal $\mathcal{I}$ is said to be *codense or a boundary* ideal [5] if $\tau\cap\mathcal{I}=\{\emptyset\}$. The following lemmas will be useful in the sequel.

Lemma 1.1 ([6, Theorem 5])**.** *Let $(X,\tau,\mathcal{I})$ be an ideal space and A be a subset of X. If $A\subset A^\star$, then $A^\star=\mathrm{cl}(A^\star)=\mathrm{cl}(A)=\mathrm{cl}^\star(A)$.*

Lemma 1.2 ([3, Theorem 6.1])**.** *If $(X,\tau,\mathcal{I})$ is an ideal space, then $\mathcal{I}$ is codense if and only if $G\subset G^\star$ for every open set G in X.*

2. $\mathcal{I}$-normal spaces

An ideal space $(X,\tau,\mathcal{I})$ is said to be $\mathcal{I}$-*normal* if for every pair of disjoint closed sets A and B of X, there exist disjoint open sets U and V such that $A-U\in\mathcal{I}$ and

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$B - V \in \mathcal{I}$. Clearly, if $\mathcal{I} = \{\emptyset\}$, then normality and $\mathcal{I}-$normality coincide. Also, if $\mathcal{I}$ and $\mathcal{J}$ are ideals on X such that $\mathcal{I} \subset \mathcal{J}$ and $(X, \tau, \mathcal{I})$ is $\mathcal{I}$-normal, then $(X, \tau, \mathcal{J})$ is $\mathcal{J}$-normal. Since $\emptyset \in \mathcal{I}$, it is clear that every normal space is an $\mathcal{I}$-normal space for every ideal $\mathcal{I}$ but not the converse, as shown by the following Example 2.1.

Example 2.1. *Consider the Modified Fort space* [7, Example 27] *in which* $X = \mathbb{N} \cup \{x_1\} \cup \{x_2\}$, *where* $\mathbb{N}$ *is the set of all natural numbers, with the topology* τ *defined as follows: Any subset of* $\mathbb{N}$ *is open and any set containing* x_1 *or* x_2 *is open if and only if it contains all but a finite number of points of* $\mathbb{N}$. *This space is not normal. Consider* $\mathcal{I}_f$, *the ideal of all finite subsets of* X. *We prove that* $(X, \tau, \mathcal{I}_f)$ *is* $\mathcal{I}$*-normal. Let* A *and* B *be two disjoint closed sets in* X.

Case (i). If A and B are subsets of $\mathbb{N}$, then A and B are open. If $G = A$ and $H = B$, since $\emptyset \in \mathcal{I}$, $A - G \in \mathcal{I}_f$ and $B - H \in \mathcal{I}_f$.

Case (ii). Suppose $x_1 \in A$ and $x_2 \notin A$. Let $G = A - \{x_1\}$ and $H = (X - A) - \{x_2\}$. Then G and H are disjoint. Since $G \subset \mathbb{N}$, G is open and $A - G = \{x_1\} \in \mathcal{I}_f$. Since $H \subset \mathbb{N}$, H is open and $B - H \subset B \cap A \subset A$. Since $x_2 \notin A$, A is finite and so $A \in \mathcal{I}_f$ which implies that $B - H \in \mathcal{I}_f$. Thus, there exist disjoint open sets G and H such that $A - G \in \mathcal{I}_f$ and $B - H \in \mathcal{I}_f$.

Case (iii). Suppose $x_1, x_2 \in A$. Let $G = A - \{x_1, x_2\}$ and $H = B$. Then G and H are disjoint. Since $G \subset \mathbb{N}$, G is open and $A - G = \{x_1, x_2\} \in \mathcal{I}_f$. $x_1, x_2 \notin B$ implies that $B \subset \mathbb{N}$ and so B is open. Thus there exist disjoint open sets G and H such that $A - G \in \mathcal{I}_f$ and $B - H \in \mathcal{I}_f$.

Thus, in all the three cases, there exist disjoint open sets G and H such that $A - G \in \mathcal{I}_f$ and $B - H \in \mathcal{I}_f$. Hence $(X, \tau, \mathcal{I}_f)$ is $\mathcal{I}$-normal.

The following Theorem 2.2 characterizes $\mathcal{I}$-normal spaces.

Theorem 2.2. *Let* $(X, \tau, \mathcal{I})$ *be an ideal space. Then the following are equivalent.*

(a) $(X, \tau, \mathcal{I})$ *is* $\mathcal{I}$*-normal.*

(b) *For every closed set* F *and open set* G *containing* F, *there exists an open set* V *such that* $F - V \in \mathcal{I}$ *and* $\mathrm{cl}(V) - G \in \mathcal{I}$.

(c) *For each pair of disjoint closed sets* A *and* B, *there exists an open set* U *such that* $A - U \in \mathcal{I}$ *and* $\mathrm{cl}(U) \cap B \in \mathcal{I}$.

Proof. (a)⇒(b). Let F be closed and G be open such that $F \subset G$. Then $X - G$ is a closed set such that $(X - G) \cap F = \emptyset$. By hypothesis, there exist disjoint open sets U and V such that $(X - G) - U \in \mathcal{I}$ and $F - V \in \mathcal{I}$. Now $U \cap V = \emptyset$ implies that $\mathrm{cl}(V) \subset X - U$ and so $(X - G) \cap \mathrm{cl}(V) \subset (X - G) \cap (X - U)$ which in turn implies that $\mathrm{cl}(V) - G \subset (X - G) - U \in \mathcal{I}$. Therefore, $\mathrm{cl}(V) - G \in \mathcal{I}$.
(b)⇒(c). Let A and B be disjoint closed subsets of X. Then there exists an open set U such that $A - U \in \mathcal{I}$ and $\mathrm{cl}(U) - (X - B) \in \mathcal{I}$ which implies that $A - U \in \mathcal{I}$ and $\mathrm{cl}(U) \cap B \in \mathcal{I}$.
(c)⇒(a). Let A and B be disjoint closed subsets in X. Then there exists an open set U such that $A - U \in \mathcal{I}$ and $\mathrm{cl}(U) \cap B \in \mathcal{I}$. Now $\mathrm{cl}(U) \cap B \in \mathcal{I}$ implies that $B - (X - cl(U)) \in \mathcal{I}$. If $V = X - \mathrm{cl}(U)$, then V is an open set such that $B - V \in \mathcal{I}$ and $U \cap V = U \cap (X - \mathrm{cl}(U)) = \emptyset$. Hence $(X, \tau, \mathcal{I})$ is $\mathcal{I}$-normal. □

The following Corollary 2.3 follows from Theorem 2.2 and Lemmas 1.1 and 1.2.

Corollary 2.3. *Let $(X, \tau, \mathcal{I})$ be an ideal space where $\mathcal{I}$ be codense. Then the following are equivalent.*

(a) *$(X, \tau, \mathcal{I})$ is $\mathcal{I}$-normal.*

(b) *For every closed set F and open set G containing F, there exists an open set V such that $F - V \in \mathcal{I}$ and $V^\star - G \in \mathcal{I}$.*

(c) *For each pair of disjoint closed sets A and B, there exists an open set U such that $A - U \in \mathcal{I}$ and $U^\star \cap B \in \mathcal{I}$.*

If $\mathcal{I}$ is an ideal of subsets of X and Y is a subset of X, then $\mathcal{I}_Y = \{Y \cap I \mid I \in \mathcal{I}\} = \{I \in \mathcal{I} \mid I \subset Y\}$ is an ideal of subsets of Y [5]. The following Theorem 2.4 shows that $\mathcal{I}$-normality is closed hereditary. Since every space (X, τ) is the ideal space $(X, \tau, \mathcal{I})$ where $\mathcal{I} = \{\emptyset\}$, it follows that the condition *closed* on the subset cannot be dropped.

Theorem 2.4. *If $(X, \tau, \mathcal{I})$ is an $\mathcal{I}$-normal ideal space and $Y \subset X$ is closed, then $(Y, \tau_Y, \mathcal{I}_Y)$ is $\mathcal{I}_Y$-normal.*

Proof. Let A and B be disjoint τ_Y closed subsets of Y. Since Y is closed, A and B are disjoint closed subsets of X. By hypothesis, there exist disjoint open sets U and V such that $A - U \in \mathcal{I}$ and $B - V \in \mathcal{I}$. If $A - U = I \in \mathcal{I}$ and $B - V = J \in \mathcal{I}$, then $A \subset U \cup I$ and $B \subset V \cup J$. Since $A \subset Y$, $A \subset Y \cap (U \cup I)$ and so $A \subset (Y \cap U) \cup (Y \cap I)$. Therefore, $A - (Y \cap U) \subset (Y \cap I) \in \mathcal{I}_Y$. Similarly, $B - (Y \cap V) \subset (Y \cap J) \in \mathcal{I}_Y$. If $U_1 = Y \cap U$ and $V_1 = Y \cap V$, then U_1 and V_1 are disjoint τ_Y open sets such that $A - U_1 \in \mathcal{I}_Y$ and $B - V_1 \in \mathcal{I}_Y$. Hence $(Y, \tau_Y, \mathcal{I}_Y)$ is $\mathcal{I}_Y$-normal. $\square$

If $(X, \tau, \mathcal{I})$ is an ideal space, (Y, σ) is a topological space and $f\colon (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is a function, then $f(\mathcal{I}) = \{f(I) \mid I \in \mathcal{I}\}$ is an ideal on Y [5]. The following Theorem 2.5 shows that $\mathcal{I}$-normality is a topological property.

Theorem 2.5. *If $(X, \tau, \mathcal{I})$ is an $\mathcal{I}$-normal space and $f\colon (X, \tau, \mathcal{I}) \to \big(Y, \sigma, f(\mathcal{I})\big)$ is a homeomorphism, then $\big(Y, \sigma, f(\mathcal{I})\big)$ is a $f(\mathcal{I})$-normal space.*

Proof. Let A and B be disjoint σ-closed subsets of Y. Since f is continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint closed subsets of X. Since $(X, \tau, \mathcal{I})$ is $\mathcal{I}$-normal, there exist disjoint open sets U and V in X such that $f^{-1}(A) - U \in \mathcal{I}$ and $f^{-1}(B) - V \in \mathcal{I}$, $f^{-1}(A) - U \in \mathcal{I} \Rightarrow f(f^{-1}(A) - U) \in f(\mathcal{I}) \Rightarrow A - f(U) \in f(\mathcal{I})$. Similarly, $B - f(V) \in f(\mathcal{I})$. Since $f(U)$ and $f(V)$ are disjoint σ-open sets in Y, it follows that $\big(Y, \sigma, f(\mathcal{I})\big)$ is $f(\mathcal{I})$-normal. $\square$

An ideal space $(X, \tau, \mathcal{I})$ is said to be *paracompact modulo $\mathcal{I}$* or *$\mathcal{I}$-paracompact* [10] if for every open cover $\mathcal{U}$ of X, there exists a locally finite refinement $\mathcal{V}$ such that $X - \cup\{V \mid V \in \mathcal{V}\} \in \mathcal{I}$. A space $(X, \tau, \mathcal{I})$ is said to be *$\mathcal{I}$-regular* [2], if for each closed set F and a point $p \notin F$, there exist disjoint open sets U and V such that $p \in U$ and $F - V \in \mathcal{I}$. Clearly, for the ideal $\mathcal{I} = \{\emptyset\}$, regularity and $\mathcal{I}$-regularity coincide. Also, it is clear that $\mathcal{I}$-regularity and $\mathcal{I}$-normality are independent concepts and for T_1spaces, $\mathcal{I}$-normality implies $\mathcal{I}$-regularity. In [2, Theorem 2.1], it was established that every $\mathcal{I}$-paracompact, Hausdorff space is $\mathcal{I}$-regular. The following Theorem 2.6 shows that it is even $\mathcal{I}$-normal.

Theorem 2.6. *If $(X,\tau,\mathcal{I})$ is an $\mathcal{I}$-paracompact, Hausdorff space, then $(X,\tau,\mathcal{I})$ is $\mathcal{I}$-normal.*

Proof. Let A and B be disjoint closed subsets of X. Since $(X,\tau,\mathcal{I})$ is $\mathcal{I}$-regular, for each $x\in A$, there exist disjoint open sets U_x and V_x such that $x\in U_x$ and $B-V_x\in\mathcal{I}$. The collection $\mathcal{U}=\{U_x\mid x\in A\}\cup(X-A)$ is an open cover of X. Since $(X,\tau,\mathcal{I})$ is $\mathcal{I}$-paracompact, there exists a precise locally finite open refinement $\mathcal{V}=\{W_x\mid x\in A\}\cup G$ such that $W_x\subset U_x$ for every $x\in A$, $G\subset X-A$ and $X-\cup\{H\mid H\in\mathcal{V}\}\in\mathcal{I}$. Let $V=\cup\{W_x\mid x\in A\}$. Then V is open. Now $\big(X-\cup\{H\mid H\in\mathcal{V}\}\big)\cap A=\big(X-(\cup\{W_x\mid x\in A\}\cup G)\big)\cap A=\big(X-\cup\{W_x\mid x\in A\}\big)\cap A=A-\cup\{W_x\mid x\in A\}=A-V$. Since $\big(X-\cup\{H\mid H\in\mathcal{V}\}\big)\cap A\subset\big(X-\cup\{H\mid H\in\mathcal{V}\}\big)\in\mathcal{I}$, $A-V\in\mathcal{I}$. For each $x\in X$, $U_x\cap V_x=\emptyset$ implies that $\mathrm{cl}(U_x)\subset X-V_x$ and so $\mathrm{cl}(W_x)\subset X-V_x$. Now $\cup\{\mathrm{cl}(W_x)\mid x\in X\}\subset\cup\{X-V_x\mid x\in X\}$ which implies that $B\cap\mathrm{cl}(V)=B\cap\big(\cup\{\mathrm{cl}(W_x)\mid x\in X\}\big)\subset B\cap\big(\cup\{X-V_x\mid x\in X\}\big)=\cup\{B-V_x\mid x\in X\}\in\mathcal{I}$. Hence by Theorem 2.2(c), $(X,\tau,\mathcal{I})$ is $\mathcal{I}$-normal. □

A subset A of an ideal space $(X,\tau,\mathcal{I})$ is said to be $\mathcal{I}$-*compact* [5], if for every open cover $\{U_\alpha\mid\alpha\in\triangle\}$ of A that $A-\cup\{U_\alpha\mid\alpha\in\triangle_0\}\in\mathcal{I}$. $(X,\tau,\mathcal{I})$ is said to be $\mathcal{I}$-compact if X is $\mathcal{I}$-compact as a subset. In [2, Theorem 2.9], it was established that every $\mathcal{I}$-compact, Hausdorff space $(X,\tau,\mathcal{I})$ is $\mathcal{I}$-regular. The following Corollary 2.7 shows that every $\mathcal{I}$-compact, Hausdorff space is $\mathcal{I}$-normal, which follows from the fact that every $\mathcal{I}$-compact space is $\mathcal{I}$-paracompact.

Corollary 2.7. *If $(X,\tau,\mathcal{I})$ is $\mathcal{I}$-compact and Hausdorff, then $(X,\tau,\mathcal{I})$ is $\mathcal{I}$-normal.*

The following Lemma 2.8 gives characterizations of $\mathcal{I}-$regular spaces, which is necessary to prove Theorem 2.9.

Lemma 2.8. *Let $(X,\tau,\mathcal{I})$ be an ideal space. Then the following are equivalent.*

(a) *X is $\mathcal{I}$-regular.*

(b) *For each $x\in X$ and open set U containing x, there is an open set V containing x such that $\mathrm{cl}(V)-U\in\mathcal{I}$.*

(c) *For each $x\in X$ and closed set A not containing x, there is an open set V containing x such that $\mathrm{cl}(V)\cap A\in\mathcal{I}$.*

Proof. (a)⇒(b). Let $x\in X$ and U be an open set containing x. Then, there exist disjoint open sets V and W such that $x\in V$ and $(X-U)-W\in\mathcal{I}$. If $(X-U)-W=I\in\mathcal{I}$, then $(X-U)\subset W\cup I$. Now $V\cap W=\emptyset$ implies that $V\subset X-W$ and so $\mathrm{cl}(V)\subset X-W$. Now $\mathrm{cl}(V)-U\subset(X-W)\cap(W\cup I)=(X-W)\cap I\subset I\in\mathcal{I}$.
(b)⇒(c). Let A be closed in X such that $x\notin A$. Then, there exists an open set V containing x such that $\mathrm{cl}(V)-(X-A)\in\mathcal{I}$ which implies that $\mathrm{cl}(V)\cap A\in\mathcal{I}$.
(c)⇒(a). Let A be closed in X such that $x\notin A$. Then, there is an open set V containing x such that $\mathrm{cl}(V)\cap A\in\mathcal{I}$. If $\mathrm{cl}(V)\cap A=I\in\mathcal{I}$, then $A-\big(X-\mathrm{cl}(V)\big)=I\in\mathcal{I}$. V and $\big(X-\mathrm{cl}(V)\big)$ are the required disjoint open sets such that $x\in V$ and $A-\big(X-\mathrm{cl}(V)\big)\in\mathcal{I}$. Hence X is $\mathcal{I}$-regular. □

Theorem 2.9. *If $(X, \tau, \mathcal{I})$ is a Lindelof, $\mathcal{I}$-regular space, then $(X, \tau, \mathcal{I})$ is $\mathcal{I}$-normal.*

Proof. Let A and B be two disjoint closed subsets of X. Since $(X, \tau, \mathcal{I})$ is $\mathcal{I}-$regular, by Lemma 2.8(b), for each $a \in A$, there is an open set U_a such that $a \in U_a$ and $cl(U_a) \cap B \in \mathcal{I}$. Since the collection $\{U_a \cap A \mid a \in A\}$ is a cover of A by open subsets of A and A is a Lindelof subspace of X, $A = \cup\{U_i \cap A \mid i \in \mathbb{N}\}$ where $\mathbb{N}$ is the set of all natural numbers, which implies that $A \subset \cup\{U_i \mid i \in \mathbb{N}\}$. Also $\mathrm{cl}(U_i) \cap B \in \mathcal{I}$ for every $i \in \mathbb{N}$. Similarly, we can find a countable collection $\{V_i \mid i \in \mathbb{N}\}$ of open sets such that $B \subset \cup\{V_i \mid i \in \mathbb{N}\}$ and $\mathrm{cl}(V_i) \cap A = I_i \in \mathcal{I}$ for every $i \in \mathbb{N}$. For each $n \in \mathbb{N}$, let $G_n = U_n - \cup\{\mathrm{cl}(V_i) \mid i = 1, 2, \ldots, n\}$ and $H_n = V_n - \cup\{\mathrm{cl}(U_i) \mid i = 1, 2, \ldots, n\}$. Let $G = \cup\{G_n \mid n \in \mathbb{N}\}$ and $H = \cup\{H_n \mid n \in \mathbb{N}\}$. Since G_n and H_n are open for each $n \in \mathbb{N}$, G and H are open subsets of X. Clearly, $G \cap H = \emptyset$. Now we prove that $A - G \in \mathcal{I}$. Let $x \in A$. Then $x \in U_m$ for some m. Also, $\mathrm{cl}(V_n) \cap A = I_n \in \mathcal{I}$ for every n implies that $A \subset I_n \cup (X - \mathrm{cl}(V_n))$ for every n. Therefore, $x \in A$ implies that $x \in I_n \cup \big(X - \mathrm{cl}(V_n)\big)$ for every n and so $x \in I_n$ or $x \notin \mathrm{cl}(V_n)$ for every n. Hence $x \in U_m - \cup\big\{ \mathrm{cl}(V_j) \mid j = 1, 2, \ldots, m\big\}$ or $x \in \cap\{I_j \mid j \in \mathbb{N}\} = I \in \mathcal{I}$. Since $x \in G_m$, $x \in G$ and so $x \in G \cup I$. Hence $A \subset G \cup I$ which implies that $A - G \subset I \in \mathcal{I}$. Similarly, we can prove that $B - H \in \mathcal{I}$. Hence $(X, \tau, \mathcal{I})$ is $\mathcal{I}$-normal. □

The following Corollary 2.10 follows from Theorem 1.3 of [1], which says that if $(X, \tau, \mathcal{I}_c)$ is $\mathcal{I}_c$-compact, then the space X is Lindelof, where $\mathcal{I}_c$ is the ideal of all countable subsets of X.

Corollary 2.10. *If $\mathcal{I} = \mathcal{I}_c$, $(X, \tau, \mathcal{I})$ is $\mathcal{I}$-compact and $\mathcal{I}$-regular, then $(X, \tau, \mathcal{I})$ is $\mathcal{I}$-normal.*

References

[1] Hamlett, T. R., Jancović, D., *Compactness with respect to an ideal*, Boll. Un. Mat. Ital. B (7) **4** (1990), 849–861.

[2] Hamlett, T. R., Jancović, D., *On weaker forms of paracompactness, countable compactness and Lindelofness*, Ann. New York Acad. Sci. **728** (1994), 41–49.

[3] Jancović, D., Hamlett, T. R., *New Topologies from old via ideals*, Amer. Math. Monthly **97** (4) (1990), 295 – 310.

[4] Kuratowski, K., *Topology, Vol. I*, Academic Press, New York, 1966.

[5] Newcomb, R. L., *Topologies which are compact modulo an ideal*, Ph.D. thesis, University of Cal. at Santa Barbara, 1967.

[6] Renukadevi, V., Sivaraj, D., Tamizh Chelvam, T., *Codense and Completely codense ideals*, Acta Math. Hungar. **108** (3) (2005), 197–205.

[7] Steen, L. A., Seebach, J. A., *Counterexamples in Topology*, Springer-Verlag, New York, 1978.

[8] Vaidyanathaswamy, R., *The localization theory in set topology*, Proc. Indian Acad. Sci. Math. Sci. **20** (1945), 51–61.

[9] Vaidyanathaswamy, R., *Set Topology*, Chelsea Publishing Company, 1946.

[10] Zahid, M. I., *Para-H-closed spaces, locally para-H-closed spaces and their minimal topologies*, Ph.D. thesis, Univ. of Pittsburgh, 1981.

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