ABSTRACTS OF THESES* IN MATHEMATICS

defended recently at Charles University, Prague

THE THEORY AND APPLICATIONS OF SPATIAL STATISTICS AND STOCHASTIC GEOMETRY

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(March 14, 2000; supervisor V. Beneš)

The doctoral thesis presents five years work. The research work started in September 1994 and did not stop until autumn of 1999. The results are two independent collections belonging to the closely related subjects of spatial statistics and stochastic geometry, respectively. The first part of the thesis is the collection of published papers. The papers included are as follows.

Papers

(i) Pavel Krejčíř, *Development of the Kriging method with application*, Applications of Mathematics, submitted. Short version published in Proceedings S4G Conf., V.Benes et al. Eds., JCMF Praha, 1999, 153–158.

(ii) Pavel Krejčíř, *A maximum likelihood estimator of an inhomogenous Poisson point process intensity using beta splines*, Kybernetika, submitted. Short version published in Proceedings S4G Conf., V. Benes et al., Eds., JCMF Praha, 1999, 159–164.

(iii) Pavel Krejčíř, Viktor Beneš, *Orientation analysis in second-order stereology*, Acta Stereologica **15.1** (1996), 59–64.

(iv) Pavel Krejčíř, Viktor Beneš, *Stereological analysis of spatial surface processes*, Journal of Microscopy **186.2** (1997), 185–197.

(v) Viktor Beneš, Jan Rataj, Pavel Krejčíř, Joachim Ohser, *Projection measures and estimation variances of intensities*, Statistics **32** (1999), 369–393.

(vi) Viktor Beneš, Pavel Krejčíř, *Decomposition in stereological unfolding problem*, Kybernetika **33.3** (1997), 245–258.

Thesis: An Estimator of the Radioactivity Inventory in Solway Firth and Ribble Estuary

The second part is the PhD. thesis created from 1995 to 1997 in the United Kingdom. The part of the UK work is an autonomy section. The work has been concerned with the measurement of the radioactivity in coastal areas of north-west England and south-west Scotland. The aim was to establish a statistical

*An equivalent to PhD.

model of radioactivity and estimate the total radioactivity inventory in selected areas.

A complex mathematical approach was applied with strength to spatial statistics. The appropriate data transformation has been applied, the data preprocessing involved change point analysis. Finally, a modification of the popular kriging method has been derived and the total amount of radioactivity estimated.

DETECTION OF CHANGES IN ECONOMETRIC MODELS

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The main aim of our work is to study the stability of econometric models, that means, whether the parameters of build econometric model are constant over the whole sample of available observations. We consider the following two models. The first one is the model of change in location, scale or both at the same time in the sequence of independent random variables:

$$(1) \qquad \begin{aligned} Y_i &= \mu + \sigma\epsilon_i & i &= 1, \ldots, m \\ Y_i &= \mu + \delta_n + (\sigma + h_n)\epsilon_i & i &= m+1, \ldots, n \end{aligned}$$

where $1 \leq m \leq n$ is an unknown change point, μ, σ and $\mu + \delta_n$, $\sigma + h_n$ are unknown parameters before and after change and $(\delta_n, h_n) \neq (0, 0)$, $\sigma > 0$ and $\sigma + h_n > 0$. Further ϵ_i are independent identically distributed random variables with some properties. The model is referred in literature as change point model.

The second one is the model for change in linear regression:

$$(2) \qquad \begin{aligned} Y_i &= x_{i\cdot}\beta + \sigma\epsilon_i & i &= 1, \ldots, m \\ Y_i &= x_{i\cdot}\beta + x_{i\cdot}\delta_n + (\sigma + h_n)\epsilon_i & i &= m+1, \ldots, n \end{aligned}$$

where $X_n = (x_{ij})_{i=1,\ldots,n; j=1,\ldots,p}$ is the known design matrix, p is the known number of regression coefficients, n is the number of observations, $x_{i1} = 1$, for all $i = 1, \ldots, n$, $x_{i\cdot} = (x_{i1}, \ldots, x_{ip})$ denotes i-th row of the design matrix X_n, $1 \leq m \leq n$ is an unknown change point, $\beta = (\beta_1, \ldots, \beta_p)^T$ and $\delta_n = (\delta_{n1}, \ldots, \delta_{np})^T$ are unknown regression coefficients and σ and h_n are unknown parameters of error terms with $\sigma > 0$ and $\sigma + h_n > 0$ and ϵ_i are the same as above. This model is referred in literature as the model of constancy of regression relationship over time.

The main aim of work is to develop and study the L_1-test statistics and change point estimator for both models (1) and (2). We can point out several topics:

- Construction of the L_1-change point estimators for the model (1) in the case of change in location only. Derivation of their asymptotic distribution.

- Construction of the L_1-test statistic and change point estimators for the model (1) in the case of change in location, scale or both at the same time.
- Construction of the L_1-test statistic for the model (2) in the case of change in location, scale or both at the same time.

The second topic inherits the study of asymptotic behavior of L_1-estimator and residuals in the model with present change in location and scale at the same time.

CONTACT REPRESENTATIONS OF GRAPHS

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Informally speaking, an intersection graph of a set family has the sets as vertices and the intersecting pairs of sets as edges. This work concerns mainly *contact representations of graphs* (a special case) and related algorithmic problems. If an underlying set family of an intersection graph is given in geometrical terms, one may define when this is a contact representation. The background meaning is that the geometric objects are not allowed to "cross", but only to "touch" each other.

Contact graphs of *curves* (and of *line-segments* in particular) in the plane, and contact graphs of *balls* in the Euclidean space are defined in this work. We study basic properties of these contact graphs, and mainly, we consider the complexity of the recognition problem for these classes of graphs. We prove that the recognition of curve 3-contact graphs is NP-complete for planar graphs, while the same question for planar triangulations is polynomial. We also prove that the recognition of line-segment 3-contact graphs is NP-complete. Concerning ball contact graphs, we prove that it is NP-hard to recognize d-unit-ball contact graphs in dimensions $d = 3, 4, 8$, and we present some ideas applicable to a general dimension d.

FINITE VOLUME — FINITE ELEMENT SOLUTION OF COMPRESSIBLE FLOW

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Fluid dynamics has extensive applications in a wide range of applied sciences, but at the same time continues the most difficult collection of problems in the whole of mathematical analysis. In this work, we develop numerical methods for a two-dimensional compressible rotational viscous or inviscid fluid flow, which is described by a system of Navier-Stokes or Euler equations, respectively.

We start in Chapter 2 with formulation of the governing equations and boundary conditions describing the mentioned fluid flow. Using the conservation laws and the equation of state, we derive the system written for an unknown vector variable w. This system can be written for the dimensionless quantities.

Chapter 3 is devoted to the introduction of the discrete problem, i.e. to the development of the numerical methods for gaining the steady-state approximate solution. Especially, we will introduce the finite volume method for the Euler equations describing the inviscid flow and the so-called operator splitting method for the Navier-Stokes equations. The operator splitting method is based on the combination of the finite volume and finite element method, especially we combine the triangular finite volumes with conforming triangular finite elements or barycentric finite volumes with nonconforming finite elements.

Chapter 4, the major part of this work, is dealing with the adaptive mesh refinement. This approach is used for making the computation more efficient and for gaining more precise approximate solution. It is based on evaluation of the so-called error indicators. Error indicators are in general cases developed from the a-posteriori error estimates. For elliptic and parabolic problems there exists a large theory of a-posteriori error estimates and their applications in automatic adaptive mesh refinement.
However, the case of solving problems in the area of the flow is much more complicated. Despite the intensive research in this domain, the results have been obtained only for scalar equations, not for systems.
In case of the systems, such methods are usually applied which use as the error indicator an approximation of the gradient of some variable describing the flow (velocity, density, pressure, etc.). However, these methods have no relation to the error estimates and moreover, the numerical experiments proved that they do not in many cases yield satisfactory results. This is why we are interested in development of such adaptive mesh refinement methods, which are based on the rigorous criteria and on the analogues with the efficient methods carried out for numerical solutions of elliptic and parabolic equations.
The goal is to detect automatically with using the *error indicator* such regions of the domain, where the error of the method highly grows. After that, it is possible to refine only such part of the computational mesh, where it is useful for the numerical calculation. In our work we present two error indicators: the superconvergence error indicator for the Euler equations and the $\mathbf{H}^{-1}$-norm residual error indicator for the Navier-Stokes equations. Both of the indicators yield satisfactory results. They are tested on three types of domains: the GAMM channel, the flow past a cascade of profiles and the flow past an isolated profile NACA 0012.

In Chapter 5 we present the derivation of the numerical method for the calculation of the drag and lift coefficients for an isolated profile. These coefficients are very useful mainly in the industrial application. The method derived in Chapter 5 is motivated by the weak formulation of the problem. Results of this method are compared with those obtained with the aid of other numerical approaches. This comparison indicates a good agreement of results.

The last chapter is focused on the theoretical analysis of error estimates for the combined finite volume — finite element method used for the numerical solution of nonstationary nonlinear convection-diffusion problems. The error estimates are obtained in the discrete analogues of $L^2(L^2)$ and $L^2(H^1)$ norms.

BAYESIAN ANALYSIS OF MODELS WITH NON-NEGATIVE RESIDUALS

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This paper stresses the Bayesian estimation of the autoregressive (AR) parameters in the one-dimensional stationar linear AR models. Classical non-Bayesian estimates are mentioned too, to be compared with them. So Chapter 2 deals with non-Bayesian estimates in the AR model of the first order AR(1), Chapter 3 deals with Bayesian estimates in the same model. Similarly, Chapters 4 and 5 discuss non-Bayesian and Bayesian estimates in the model of the second order AR(2). These chapters serve as an overview of previously published results of other authors.

Models with the uniformly distributed residuals were discussed analogically as the own result. A simulation study was performed for the case of AR(2) models, summarized in Chapter 5.4, to illustrate the behaviour of all the estimates. The last two chapters contain original results too. There is a generalization of the estimation procedure for the models of higher orders — the non-Bayesian approach in Chapter 6, the Bayesian approach in Chapter 7. There is shortly presented a new iterative method for finding the stationary density of the AR process in Chapter 8 — this method was published in Anděl, Hrach, *On calculation of stationary density of autoregressive processes*, Kybernetika **36** (2000), 311–319.

M-ESTIMATORS IN THE LINEAR MODEL FOR NONREGULAR DENSITIES

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M-estimation of regression parameters $\boldsymbol{\beta}$ in regression model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{E}$ is studied for nonregular density of errors $f(e)$. The limit distribution of M-estimators is derived under reasonable conditions on $\mathbf{X}$ for various types of objective functions. The estimators are defined as minimizers of $\sum_{i=1}^{n} \varrho(Y_i - \mathbf{x}_i'\boldsymbol{\beta})$ for $\boldsymbol{\beta} \in \mathbb{R}^p$. In the thesis, real-valued convex ϱ are used. The study of estimators for L_1-based objective function is crucial for studying the difference between regular and nonregular cases. The nonregularities analyzed are of several kinds: smooth, with density having the following property: $\exists k \in \mathbb{N}$ such that $f^{(i)}(r) = 0$, $i = 1, \dots, k-2$, $f^{(k-1)}(r) \neq 0$, and non-smooth with singularities of first and second kind, according to [2].

The limit behaviour of M-estimators of location parameters under special nonregular conditions was studied in [1] (Ghosh, Sukhatme-1981), with interesting results for sample quantiles of nonregular distributions. As there is a similarity between estimation of location parameters and estimation of regression parameters (we can compare, for example, sample quantiles and regression quantiles), we would expect analogous results for regression case. Another approach to the problem of M-estimators of regression vector for nonregular densities comes from developing the results for standard cases. The monography [3] gives a clear overview of methods and results for M-estimator of regression vector. These results are used and extended for densities with isolated zero points of density.

Some standard and some special conditions on the design matrix are imposed to get results for the limit behaviour of M-estimators. These conditions are not strict and it would be possible to relax them.

Methods used for deriving the results include those used in [1], [3] and some advanced probabilistic results for the limit properties of processes from [6] and [5].

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EXTREMAL MARTINGALE MEASURES IN FINANCE

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The dissertation studies the characteristics of sets of processes, their distributions, and especially their structure and the possibility of their representation, stability with respect to weak convergency, optimization, convexity and measure convexity. Furthermore, in the thesis, the above mentioned characteristics are successfully applied to sets of weak solutions of stochastic differential equations. The thesis is concluded by a practical application of the results in the area of continuous financial markets, thereby showing a potential use in at optimization problems, or by solving the stochastic differential equations with random initial conditions, describing the development of elements of the continuous financial market.

Continuous financial mathematics is one of the most rapidly developing areas of contemporary mathematics. The description of the development of an element of the financial market through the use of stochastic differential equations becomes the basis for a great number of works dedicated to this theme. The sources listed in the first section were its prime inspiration for writing the thesis. It was mainly the two terms, convexity and extremal points of sets of probability distributions, that form the large part of this thesis.

The thesis begins with the description of pairs of processes (X, Y) such that the process Y is a martingale (local martingale) with respect to the augmented filtration generated by a process X (the sets being denoted as $\mathbb{M}^0$ and $\mathbb{L}^0$). The second part studies characteristics of these sets. A pair of processes (X, Y) can be represented by a different pair — process and distribution (G, m) so that $Y \overset{a.s.}{=} G(X)$ and $\mathcal{L}(X) = m$. We call this representation a *canonical representation* and its characteristics are described in Theorem 2.8.

The initial objective was to prove the closedness of the set $\mathbb{L}^0$ with respect to convergence in distribution. Based on the relation of the process Y to the process X it is possible to divide the set $\mathbb{L}^0$ — a set where the adaptability is preserved $(\mathbb{A}^0)$ and a set where the martingale property is satisfied $(\mathbb{PL})$. Members of the latter set are referred to as the *local premartingales*. Unfortunately, the first of the two sets is not closed with respect to convergence in distribution and, therefore, the maximum result is a relative closedness — presented by Proposition. At the end of Section 3, this theme is discussed.

The proof of the above statements was found by proving the Stopping theorem for the set of bounded premartingales — Lemma 2.3 and by following the time change conditions under which a premartingale becomes a martingale — Lemma 2.2.

In Sections 4 and 5 we study the geometry and topology of sets of solutions to local martingale problems, the inspiration being delivered by M. Yor (1978, 1979) and by D.W. Stroock, S.R.S. Varadhan (1969). The mathematical tools, coming from the Choquet theory for convex sets in measure spaces, are in a general form developed in Section 4. Supported by the results of G. Winkler (1980), G. Winkler (1978), H. von Weizsäcker, G. Winkler (1979), H. von Weizsäcker, G. Winkler (1980) and J. Štěpán (1984), we study measure convex sets $\mathfrak{M}$ of Borel probability measures on a Polish space, especially the sets $\mathfrak{M}$, called here Choquet sets, that are generated as the measure convex hull of their respective extremal boundaries. Theorem 4.6 offers sufficient conditions for a set $\mathfrak{M}$ to be a Choquet set or even a Choquet simplex that are applied in Section 5 to the sets $\mathcal{W}_{\mathcal{G},B}$ of $(\mathcal{G}, B)$ — a local martingale problem by Theorem 5.10.

The second part of Section 4 is dedicated to the characteristics of the direct image $H \circ \mathfrak{M}$ of a Choquet set with a Borel (analytic) extremal boundary, where H is a Borel map. This problem becomes vivid when trying to prove the existence of solutions of stochastic differential equations with boundary condition, which is represented by the map H. Theorems 4.9, 4.10 and Corollary 4.12 are successfully applied in Section 5, to the sets of local martingale problem in Theorems 5.12, 5.13 and Corollary 5.14. Both Sections, 4 and 5, contain examples of the sets being studied and demonstrate their possibility of application of the Krein-Milman Theorem.

The results of the above mentioned sections are applied to a financial market in the last section. In the first example, the Stroock-Varadhan Theorem is applied, and a solution of the stochastic differential equation is the value of portfolio. Utility optimization is a primary request of investors, this being the second example. The last example studies distributions of processes that within a finite time horizon double their values.

The thesis presents original results in the two main areas. The first one is the study of properties of a set $\mathbb{L}^0$. The second one is presented in Sections 4, 5 and 6 and it concerns research of (measure) convexity, closedness and Choquet property of the set of solutions of a local martingale problem and more importantly of a stochastic differential equation.

ROBUST SEQUENTIAL ESTIMATION

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The most popular sequential methods based on the sample mean are nonrobust and can produce misleading results if the real distribution is not Gaussian. We improve the behaviour of the sequential procedures by using more robust estimators.

We focus on the three-stage procedure in order to balance the applicability and ease of calculation. We generalize the sequential procedure based on bootstrapping sample mean suggested by Aerts and Gijbels (1993). We suggest to base the sequential method on the more general and more robust M-estimators. Three approximations of the distribution of the M-estimator are investigated: approach based on asymptotic normality, approach based on standardized bootstrap, and approach based on studentized bootstrap.

Asymptotic properties of the proposed procedures are established in Chapter 4. The main theorems describe the asymptotic behaviour of the bootstrap for M-

estimators and the behaviour of the final sample size of the sequential procedure based on the bootstrap critical points. The proofs are based on the approach of Jurečková and Sen (1981). The asymptotic behaviour of the bootstrap critical points is derived using the results of Lahiri (1992).

Simulation study is carried out in Chapter 5 in order to investigate the behaviour of the robust three-stage procedure for smaller sample sizes. The results suggest that the method based on studentized bootstrap critical points yields the best results.

The work concludes with a discussion on possible further development of the proposed methods.

References

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EQUATIONS IN FREE MONOIDS

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The thesis consists of four chapters. Each chapter can be read independently.

Chapters 1, 2 and 3 deal with the test set of the language

$$L = \{x_1^i \dots x_n^i | i \in \mathbb{N}\}.$$

The first chapter will appear in Theoretical Computer Science. It shows that any

$$T = \{x_1^k \dots x_n^k \mid k = a, a+1, a+2\},$$

with an integer $a > 1$, is a test set of L.

Chapters 2 and 3 result from the attempt to find out whether the set

$$T' = \{x_1^i \dots x_n^i | i = 1, 2, 3\}$$

is a test set of L, a question that remains open.

Chapter 2 was published in *Computational and Geometric Aspects of Modern Algebra*, London Math. Soc. Lect. Notes **275** (2000), 120–127. It presents some useful notions regarding equations in a free monoid and applies them to the open question.

Chapter 3 generalizes a method of finding solutions of equations in free semigroups. The method yields a non-trivial solution of the equation

$$(x_1^2 \dots x_n^2)^3 = (x_1^3 \dots x_n^3)^2.$$

The existence of such a solution is a necessary condition for T' not to be a test set of L. The chapter is a corrected version of a paper published in *Contributions to General Algebra 11*, Verlag Johanes Heyn, Klagenfurt, 1999, pp. 105–111.

Chapter 4 deals with binary equality sets. It claims that a binary equality language cannot be of the form $(\alpha\gamma^*\beta)^*$.

MATHEMATICAL METHODS OF STATE CHANGE ASSESSMENT IN MEDICAL RESEARCH

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The topic studied in the thesis may be characterized as the measurement of improvement (or deterioration) of an individual's health state and, especially, contradictory properties of such measurement systems. The whole work has been inspired by, and is primarily oriented to the psychiatric research. Applications of results in other areas of medical research, in psychology or, conceivably, in other research fields, are possible.

The author, as a statistician involved in the psychiatric research, had to face growing popularity of the so called Rakús index – a "strange" scale designed for measuring the extent of decrease (or increase) of schizophrenia symptoms. The index seemed to be defective: It is possible, for instance, to reach in several steps, each of them being assessed by the index as an improvement, a state, that is clearly worse than the initial one.

Specific criticism of the Rakús index was followed by a more general study of the systems of improvement assessment. The objectives of this research were (i) to formulate a normative criterion whose application eliminates "unreasonable" properties of change measurement scales, and (ii) to characterize those scales that fulfill the criterion.

Problem (i) was resolved by the definition of so called *regularity* criterion, a key concept of the thesis. More precisely, several versions of regularity, corresponding to different data types, were introduced, all of them being covered by a general definition. The solution of problem (ii) can be derived from results of the utility theory of the sixties.

The first objective of the thesis is *to present the results of the above research thoroughly.* Another objective is *to establish theoretical foundations of measuring nonregularity*. Apart from general considerations relevant to violation of all versions of regularity, so called *measures of cardinal nonregularity* are studied in some detail.

Most of the material of the thesis (motivation, definitions and theorems) is independent of any literature known to the author.

MATHEMATICAL MODELS OF HEALTH INSURANCE FOR COMMERCIAL INSURANCE COMPANIES — EMBEDDED VALUE OF ACCIDENT INSURANCE

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Traditional accounting statements, as required by a supervisory authority, contain basic information about an insurance company's financial situation primarily with respect to solvency. Nevertheless, they are insufficient for investors or experts in investment analysis, who should be able to judge an insurance company value regarding a return of an invested capital. Therefore an approach based on a discounting of expected cash flows was developed. Especially life insurance companies started using indicators like an embedded value, embedded profit or appraisal value.

The most important and, from an actuarial point of view, most interesting part of the above mentioned approaches is the present value of expected cash flows from an insurance portfolio. The theory of its estimation is very well developed and used in the practice for life insurance business. We have tried to extend it to accident insurance as well.

Expected future profits are calculated separately for each model point, i.e. a group of insurance contracts with the same parameters, and for particular components like accidental death, permanent injury consequences, daily allowance etc.

Profits from premium may be modeled similarly to life insurance. Since accident insurance is purely risk insurance, no premium reserve is, however, involved but we calculate with a reserve of unearned premium. A fictive premium fund is introduced to show the difference between insurance with regular and single premium. Model for inflation adjustment of premium and sum insured is included.

Claims are estimated proportionate to a risk volume expressed by earned premium. The coefficient of proportionality is there a claims ratio, which enters model from the outside. We have also to take into account the fact that claims are not paid immediately after they arise. A delay in payments appears and therefore we create claims reserves. Run-off triangles with appropriate methods are used to deal with this problem. Operating expenses are based on their real development and make a part of earned premium. Investment income reflects earnings achieved by an insurance company from an investment related to premiums, reserves of unearned premium, claim, claims reserves, etc.

Beside the model for future profits, their decomposition according to sources is done. Furthermore, we dealt with an estimation of a new business and profits expected to be gained out of it. All results are illustrated by a practical numeric example.

BAYESIAN ESTIMATION IN EXPONENT COMPETING RISKS AND RELATED MODELS WITH APPLICATIONS TO INSURANCE

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The thesis deals with the competing risks model with independent exponential distributions of risks, the natural conjugate prior and three its generalizations are considered for parameters of the model.

Bayes estimators under the quadratic loss function are explored. Their convergence and asymptotic normality (including the weak asymptotics of the reliability function estimator considered as a process in C) are given and Bayes risks (and the integrated Bayes risk), or at least their asymptotic expansion, are derived. The latter are used to measure sensitivity of the estimators to changes in the prior density by asymptotic deficiency.

The results are generalized to a multiple-state model with constant transition intensities (a homogeneous Markov process). The Koziol-Green model of random censorship is presented as a special case, models with more general forms of intensities are discussed.

An appendix provides a proposition suitable for asymptotic expansion of functions of random variables by means of conditional moments.

LOCALLY INJECTIVE HOMOMORPHISMS

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The graph covering projection, in other words a graph homomorphism that is locally isomorphic, appeared during the past four decades in various graph-theoretic concepts as well as there were presented its application in computer science, e.g. in distributed computing. In the thesis we exhibit further application of graph covering projections, namely in the graph-theoretic model of the channel assignment problem. Like the application of covers in distributed computing, results in the frequency assignment field have high interest in computer and telecommunication industry.

We started exploring properties of graph covering projections by exposing the degree refinement, i.e., a factorization of the vertex set, that restricts the image of a vertex under a possible covering projection.

Further we have extended the result of Kratochvíl, Proskurowski and Telle, proving that all k-regular graphs H of $k \geq 3$ are *NP*-complete instances for the H-cover problem.

Then we have exploited the class of H-partial covering problems, i.e. decision problems whether there exist locally injective homomorphisms. We concentrated on graphs H, where it is known that the H-covering problem is polynomially solvable, and we investigated whether this is still valid when asking for a partial covering projection. Both *NP*-complete and polynomially solvable instances were found.

In the last chapter, we have described a simple graph theoretic model for the channel assignment problem, and have showed that several cases can be reduced to partial covering projections. The same argument glues together the characterization of the computational complexity of several classes of the $\lambda(p_1,\dots,p_k)$-labeling problem with corresponding classes of the H-partial covering problem, even on both sides, there are problems requiring their own approach.

We hope that we presented several interesting aspects connecting algebraic nature of graph homomorphism with the combinatorial optimization methods that might find an application in the channel assignment industry.

QUALITATIVE PROPERTIES OF SOLUTIONS OF SYSTEMS OF FLUID MECHANICS

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The governing equations for an isothermal process of incompressible materials in a bounded domain $\Omega \subset \mathbb{R}^2$ read

$$\operatorname{div} u = 0$$

(*)
$$\rho\frac{\partial u}{\partial t} + \rho\sum_{k=1}^{2} u_k\frac{\partial u}{\partial x_k} - \operatorname{div}\mathcal{T}(Du) = \nabla\pi + \rho f.$$

Here, ρ is a given positive constant expressing the density of the fluid, $u = (u_1, u_2)$ is the velocity field, π is the pressure and $f = (f_1, f_2)$ stands for the vector of external body forces. Du denotes the symmetrized gradient of u, $\mathcal{T}$ is a given generally nonlinear tensor function mapping the space of symmetric 2×2 matrices to itself.

In my thesis I study the regularity of solutions of both steady and unsteady variant of (*) assuming that the density ρ is normalized to 1 and that $\mathcal{T}$ has growth $p-1$, $p>1$.

In the case of steady problem we consider two types of boundary condition, namely the periodic boundary condition and the non-homogeneous Dirichlet boundary condition.

Considering the periodic boundary condition we prove the existence of a Hölder continuous solution u, π: $u \in C^{1,\alpha}(\Omega)$, $\pi \in C^{0,\alpha}(\Omega)$ for all $p > 1$, while for the Dirichlet boundary problem there occur two different bounds: If $p > 6/5$ we prove the existence of a locally Hölder continuous solution, i.e. $u \in C^{1,\alpha}_{\mathrm{loc}}(\Omega)$, $\pi \in C^{0,\alpha}_{\mathrm{loc}}(\Omega)$, and if $p > 3/2$ we obtain regularity up to the boundary; $u \in C^{1,\alpha}(\Omega)$ and $\pi \in C^{0,\alpha}(\Omega)$.

The crucial step of the proof of these results, the passage from the integrability of second derivatives of solution to $C^{1,\alpha}$ regularity, is solved by using the ideas from articles of Nečas and Stará (1967, 1968, 1971). It is based on L^p theory for systems of Stokes type with coefficients which are only measurable and bounded.

For unsteady flows we present an analogous result in the case of periodic boundary condition. We prove the existence of $C^{1,\alpha}$-solution to (*) if $p > 4/3$.

The first ingredient of the proof is an L^q-estimate of time derivative of velocity field u for the problem of Stokes type inspired by the paper of Nečas and Šverák (1991).

The second ingredient is our analysis of steady flows.

SELFDISTRIBUTIVE RINGS AND NEAR-RINGS

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The (left and right) equations (or identities, laws, etc.) of selfdistributivity for a binary operation (say multiplication) are expressed as $x(yz) = (xy)(xz)$ and $(zy)x = (zx)(yx)$. The first explicit allusion to selfdistributivity seems to appear in C.S. Peirce, *On the algebra of logic*, Amer. J. Math. III (1880), 15–57. On the page 33–34 of this article we can read the following comment:

"These are another cases of the distributive principle. ... These formulae, which have hither to escaped notice, are not without interest."

The present thesis is meant as a systematic treatment on one-sided and two-sided selfdistributive rings.

Besides, some results on selfdistributive semirings and nearrings are also included.