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Otakar Borůvka, his life and work

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THIS ISSUE OF ARCHIVUM MATHEMATICUM  
IS DEDICATED TO THE MEMORY OF PROFESSOR OTAKAR BORŮVKA

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OTAKAR BORŮVKA, an outstanding personality in the history of the Czech and Slovak mathematics and a founder of several basic concepts in different parts of mathematics, died on July 22, 1995, in his 96 years.

Otakar Borůvka was born on May 10, 1899 in Uherský Ostroh (Moravia, Czech Republic). After studies at the Masaryk University of Brno, he became a lecturer (1921), a reader (1928), and Professor (1934) at this University till 1970. He studied in Paris (1926 and 1929) with Prof. É. Cartan and in Hamburg (1930) with Prof. W. Blaschke, where he also met Professors E. Artin, E. Borel, J. Douglas, M. Fréchet, E. Goursat, J. Hadamard, H. Lebesgue, E. Picard, B. Segre, E. Vessiot and many others. In 1953 he became a corresponding member and in 1965 an ordinary member (Academician) of the Czechoslovak Academy of Sciences. From 1969 he worked in the Mathematical Institute of the Academy, branch Brno.

O. Borůvka's scientific work has covered extensive fields of Mathematics and reflects the main trends of the development of 20th century Mathematics not only in the Czech and Slovak Republics but throughout the world. His excellent results and new methods concern the theory of graphs, differential geometry, algebra and the theory of differential equations. He substantially contributed to the development of all these areas and established research schools. During his pedagogical and scientific activity he taught numbers of mathematicians; most of the mathematicians in Moravia and Slovakia are his pupils or pupils of his pupils. His incredible enthusiasm for work inspired them in a very large range of problems especially concerning abstract algebra, differential geometry and the theory of differential equations. Borůvka's results in classical analysis belong to the period 1923-1925, having been achieved mainly under the influence of his teacher, Prof. M. Lerch. In the pioneering paper *On a certain minimal problem* from 1926, Borůvka algorithmically solved the problem of a minimal cost of an electric network, a kind of the transport problem belonging to an essential part of the graph theory, at least ten years before the graph theory was established as a mathematical discipline.

In his monumental work on projective differential geometry O. Borůvka was the first who studied analytic correspondences between two projective planes. The results of his extensive paper from 1933 on (two dimensional) spherical surfaces in

2n-dimensional spaces of constant curvatures have found important applications in modern differential geometry. The research school in Bologna has been continuing Borůvka's original study in many respects. For example, S. S. Chern in his paper on minimal submanifolds immersed into spheres calls certain differential equations "Frenet-Borůvka formulae".

O. Borůvka is also one of the founders of some important conceptions of the general algebra. He established the theory of groupoids and collected his original methods and results in the monograph *Foundations of the Theory of Groupoids and Groups*, published in German (1960), English (1974), and several times in Czech.

In 1950 O. Borůvka started his systematic study of differential equations. On the basis of his perfect knowledge of classical analysis, differential geometry and algebra, he developed an original and fruitful theory of global transformations of linear differential equations of the second order. He introduced several new notions and methods, solved many open problems in this field, for example, the problem of global equivalence of such equations. The results of this qualitative theory of a global character, which exhibits a high degree of geometrization and algebraization is collected in his monograph *Lineare Differentialtransformationen 2. Ordnung*, published in German (Berlin 1967) and in English (London 1971). As was the case with differential geometry and algebra, numerous Czech and Slovak as well as foreign mathematicians have exploited Borůvka's methods and results in the theory of differential equations to solve various problems concerning not only equations of the second but also of higher orders. The assistance he was giving to the Komenský University in Bratislava for more than ten years in addition to his duties in Brno is highly appreciated by Slovak mathematicians as a substantial contribution to the development of mathematics in Slovakia.

Academician O. Borůvka has also well deserved of establishing the Institute of Mathematics of the Czechoslovak Academy of Sciences branch Brno and of founding the well-known mathematical journal, *Archivum Mathematicum*, issued by the Masaryk University since 1965.

The great importance of Borůvka's achievements has had wide response in a number of honours awarded to him in Czechoslovakia and abroad, and in numerous invitations to lecture at foreign universities and conferences. His contribution to the world science will never be forgotten.

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## SCIENTIFIC PUBLICATIONS OF OTAKAR BORŮVKA

- [1] *On complex roots of equation  $\Gamma(z) = a$*  (in Czech), Spisy přír. fak. MU, **26**, 1923.
- [2] *To the theory of certain transcendents of integral calculus* (in Czech), Spisy přír. fak. MU, **37**, 1924.
- [3] *On certain types of surfaces that are projectively transformable into themselves* (in Czech), Spisy přír. fak. MU, **43**, 1924.
- [4] *A note to Kummer's formula* (in Czech), Čas. pěst. mat. fys. **54**, 1925, 109–113.
- [5] *Sur les correspondences analytiques entre deux plans projectifs I*, Spisy přír. fak. MU, **72**, 1926.
- [6] *On a certain minimal problem* (in Czech), Práce Moravské přírodovědecké společnosti, **3**, No. 3, 1926, 37–58.
- [7] *Sur les correspondences analytiques entre deux plans projectifs II*, Spisy přír. fak. MU, **85**, 1927.
- [8] *Géométrie projective des correspondences analytiques entre deux plans*, C. R. Acad. Sci. Paris **184**, 1927, 1518.
- [9] *On correspondences with characteristic curves of equation  $dx^3 - dy^3 = 0$*  (in Czech), Čas. pěst. mat. fys. **57**, 1928, 183–185.
- [10] *On a certain type of minimal surfaces in 4-dimensional space of constant curvature* (in Czech), Rozpravy II. České akademie **XXXVII**, 1928, 37.
- [11] *Sur une classe de surfaces minima plongées dans un espace à quatre dimensions à courbure constante*, Bulletin int. Acad. Tchèque Sci., 1928.
- [12] *Sur une classe de surfaces minima plongées dans un espace à quatre dimensions à courbure constante*, C. R. Acad. Sci. Paris **187**, 1928, 334.
- [13] *Sur une classe de surfaces minima plongées dans un espace à cinq dimensions à courbure constante*, C. R. Acad. Sci. Paris **187**, 1928, 1271.
- [14] *Sur une classe de surfaces minima plongées dans un espace à cinq dimensions à courbure constante*, Spisy přír. fak. MU, **106**, 1929.
- [15] *Sur les surfaces projectivement déformables qui admettent un groupe de  $\infty^1$  transformations projectives en elles-mêmes*, C. R. Acad. Sci. Paris **189**, 1929, 964.
- [16] *Sur les surfaces dont le réseau conjugué de déformation projective est formé par les lignes de Segre-Darboux*, Bulletin Sci. math. **53**, 1929.
- [17] *Sur les surfaces par les fonctions sphériques de première espèce*, C. R. Acad. Sci. Paris **190**, 1930, 1336.
- [18] *Sur les surfaces dont les lignes de Segre sont des géodésiques*, Tôhoku Math. Journal **32**, 1930, 292–302.
- [19] *Sur les hypercirconférences et certaines surfaces paraboliques dans l'espace euclidien à quatre dimensions*, Spisy přír. fak. MU, **146**, 1931.
- [20] *Sur les hypercirconférences et certaines surfaces paraboliques dans l'espace euclidien à quatre dimensions*, C. R. Acad. Sci. Paris **193**, 1931, 633.
- [21] *Recherches sur la courbure des surfaces dans des espaces à  $n$  dimensions à courbure constante  $I$* , Spisy přír. fak. MU, **165**, 1932.

- [22] *On a certain parabolic surfaces in  $2n$ -dimensional Euclidean spaces* (in Czech), Čas. pěst. mat. fys. **62**, 1932, 140–153.
- [23] *Sur les surfaces représentées par les fonctions sphériques de premières espèce*, Journal Math. Pures Appl. **12**, 1933, 337–383.
- [24] *Sur une extension des formules de Frenet dans l'espace complexe et leur image réelle*, C. R. Acad. Sci. Paris **197**, 1933, 109.
- [25] *Über die partiellen Differentialgleichungen, denen hermitesche Formen genügen*, Abh. math. Sem. Hamb. Univ. **11**, 1934, 65–72.
- [26] *Recherches sur la courbure des surfaces dans des espaces à  $n$  dimensions à courbure constante II*, Spisy přír. fak. MU, **212**, 1935.
- [27] *Recherches sur la courbure des surfaces dans des espaces à  $n$  dimensions à courbure constante III*, Spisy přír. fak. MU, **214**, 1935.
- [28] *Sur les courbes analytiques dans les espaces hermitiens*, Čas. pěst. mat. fys. **64**, 1935, 187–188.
- [29] *Sur les matrices singulières*, C. R. Acad. Sci. Paris **203**, 1936, 600.
- [30] *Sur les systèmes multiplicatifs*, C. R. Acad. Sci. Paris **204**, 1937, 1779.
- [31] *Studies on multiplicative systems. Part I*, Spisy přír. fak. MU, **245**, 1937.
- [32] *Studies on multiplicative systems. Part II*, Spisy přír. fak. MU, **265**, 1938.
- [33] *Theory of groupoids. Part one* (in Czech), Spisy přír. fak. MU, **275**, 1939.
- [34] *Über Ketten von Faktoroiden*, Math. Ann. **118**, 1941, 41–64.
- [35] *On decompositions of sets* (in Czech), Rozpravy II. České akademie **LIII**, No. 23, 1943.
- [36] *Über Zerlegungen von Mengen*, Mitteilungen der Tschechischen Akad. der Wiss. **LIII**, No. 23, 1943.
- [37] *Introduction to the theory of groups* (in Czech), Královská česká společnost nauk. Praha 1944, 80 pp.
- [38] *Theory of decompositions in set. Part I* (in Czech), Spisy přír. fak. MU, **278**, 1946.
- [39] *Introduction to the theory of groups* (in Czech), Přírodovědecké vydavatelství, Praha 1952.
- [40] *О колеблющихся интегралах дифференциальных линейных уравнений 2-ого порядка*, Czech. Math. J. **3 (78)**, 1953, 199–255.
- [41] *A note to an application of the Weyr theory of matrices to the integration of systems of differential linear equations with constant coefficients* (in Czech), Čas. pěst. mat. fys. **79**, 1954, 151–155.
- [42] *Sur la transformation des intégrales des équations différentielles linéaires ordinaires du second ordre*. Ann. Mat. Pura Appl., S. IV, T. **XLI**, 1956, 325–342.
- [43] *Über eine Verallgemeinerung der Eindeutigkeitsätze für Integrale der Differentialgleichung  $y' = f(x, y)$* , Acta F. R. N. Univ. Comenianae, Mathematica, IV–VI, 1956, 155–167.
- [44] *Works of Matyáš Lerch in the mathematical analysis* (in Czech), Práce Brněnské základny Československé akademie věd, **XXIX**, 1957, 417–540.

- [45] *Théorie analytique et constructive des transformations différentielles linéaires du second ordre*, Bulletin Math. de la Soc. Sci. Math. Phys. de la R. P. R. **1** (49), 1957, 125–130.
- [46] *Mathias Lerch als Fortsetzer der Klassiker in der Theorie der Gammafunktion*, Euler-Festschrift, Berlin 1959, 78–86.
- [47] *Grundlagen der Gruppoid- und Gruppentheorie*, VEB Deutscher Verlag der Wissenschaften Berlin, 1960, xii + 198 str.
- [48] *Sur les transformations différentielles linéaires complètes du second ordre*, Ann. Mat. Pura Appl., XLIX, 1960, 229–252.
- [49] *Transformations des équations différentielles linéaires du deuxième ordre*, Paris, Séminaire Dubreil-Pisot, 3 mai 1961 (Algèbre et Théorie des nombres 14e année, **22**, 1960/61, 1–18).
- [50] *Décompositions dans les ensembles et théorie des groupoïdes*, Paris Séminaire Dubreil-Pisot, 8 mai 1961 (Algèbre et Théorie des nombres 14e année, **22** bis, 1960/61, 19–35).
- [51] *Sur la structure de l'ensemble des transformations différentielles linéaires complètes du second ordre*, Ann. Mat. Pura Appl., LVIII, 1962, 317–334.
- [52] *Foundations of the theory of groupoids and groups* (in Czech), Nakladatelství ČSAV, Praha, 1962, 216 pp.
- [53] *Über einige Ergebnisse aus der Theorie der linearen Differentialtransformationen 2. Ordnung*, Heft 13 der Schriftenreihe der Intistitute für Mathematik, Bericht von der Dirichlet-Tagung-Akademie-Verlag Berlin 1963, 51–57.
- [54] *Sur l'ensemble des équations différentielles linéaires ordinaires du deuxième ordre qui ont la même dispersion fondamentale*, Bul. Inst. Politechn. Iași, Serie noua, IX (XIII) 1963, 11–20.
- [55] *Transformation of Ordinary Second-Order Linear Differential Equations. Differential Equations and their Applications*, Proceedings of the Conference held in Prague in September 1962, Prague 1964, 27–38.
- [56] *Über die algebraische Struktur der Phasenmenge der linearen oszillatorischen Differentialgleichungen 2. Ordnung*, Bericht von der Tagung über geordnete Mengen, Brno, November 1963. Publ. Fac. Sci. Univ. J. E. P., Brno, **457**, 1964, 461–462.
- [57] *Sur quelques applications des dispersions centrales dans la théorie des équations différentielles linéaires du deuxième ordre*, Arch. Math. (Brno), **1**, 1965, 1–20.
- [58] *Über die allgemeinen Dispersionen der linearen Differentialgleichungen 2. Ordnung*, Ann. Sci. Univ. "Al. I. Cuza" Iași XI<sub>B</sub>, 1965, 217–238.
- [59] *Sur une application géométrique des dispersions centrales des équations différentielles linéaires du deuxième ordre*, Ann. Mat. Pura Appl., LXXI, 1966, 165–188.
- [60] *Neuere Ergebnisse in der Transformationstheorie der gewöhnlichen linearen Differentialgleichungen 2. Ordnung*, Vortrage der 3. Tagung über Probleme und Methoden der mathematischen Physik, Heft 1, 13–27. Technische Hochschule Karl-Marx-Stadt, 1966, 13–27.

- [61] *Lineare Differentialtransformationen 2. Ordnung*, VEB Deutscher Verlag der Wissenschaften Berlin, 1967, xiv + 218 pp.
- [62] *L'état actuel de la théorie des transformations des équations différentielles linéaires du deuxième ordre*, Colloque sur la théorie de l'approximation des fonctions. Cluj, 15–20 Septembre, 1967, 1–14.
- [63] *Théorie des transformations des équations différentielles linéaires du deuxième ordre*, Rend. Mat., **26**, 1967, 187–246.
- [64] *Éléments géométriques dans la théorie transformations des équations différentielles linéaires et ordinaires du deuxième ordre*, Atti del Convegno Internazionale di Geometria Differenziale (Bologna, 28–30, IX, 1967).
- [65] *Über eine Charakterisierung der allgemeinen Dispersionen linearer Differentialgleichungen 2. Ordnung*, Math. Nachr. 1968, **38**, No. 5/6, 261–266.
- [66] *Sur les solutions simultanées de deux équations différentielles de Kummer*, IVème Congrès des mathématiciens d'expression latine et Commémoration d'Elie Cartan, Bucaresti-Brasov, 1969, Résumés, 3–4.
- [67] *Algebraic elements in the transformation theory of 2nd order linear oscillatory differential equations*, Acta F. R. N. Univ. Comenianae, Mathematica **XVII**, 1967, 27–36 (Equadiff II, Bratislava).
- [68] *Geometric elements in the theory of transformations of ordinary second-order linear differential equations*, Symposium of Differential Equations and Dynamical Systems. Mathematics Institute, University of Warwick, 1968–69, 19–22.
- [69] *Sur quelques propriétés de structure du groupe des phases des équations différentielles linéaires du deuxième ordre*, Rev. Roumaine de Math. pures et appl., **XV**, (1970), 1345–1356.
- [70] *Linear Differential Transformations of the Second Order*, The English Universities Press, London, 1971, 254 pp.
- [71] *Foundations of the matrix theory* (in Czech), Academia, naklad. ČSAV, Praha, 1971, 177 pp.
- [72] *Sur la périodicité de la distance des zéros des intégrales de l'équation différentielle  $y'' = q(t)y$* , Tensor, N. S. **26**, (1972), 121–128.
- [73] *Foundations of the Theory of Groupoids and Groups*, VEB Deutscher Verlag der Wissenschaften, Berlin, 1974, 215 pp.
- [74] *On central dispersions of the differential equations  $y'' = q(t)y$  with periodic coefficients*, Lecture Notes in Mathematics, **415**. Ordinary and Partial Differential Equations. Proceedings of the Conference held at Dundee, Scotland, 26–29 March, 1974; 47–61.
- [75] *Sur la structure algébrique de la théorie des transformations différentielles linéaires du deuxième ordre*, Acta F. R. N. Univ. Comenianae, Mathematica **XXXI**, 1975, 59–71.
- [76] *Sur quelques compléments à la théorie de Floquet pour les équations différentielles du deuxième ordre*, Ann. Mat. Pura Appl., IV, **CII**, 1975, 71–77.
- [77] *Sur les blocs des équations différentielles  $y'' = g(t)y$  aux coefficients périodiques*, Rend. di Mat., (2), **8**, VI, 1975, 519–532.

- [78] *Über die Differentialgleichungen  $y'' = g(t)y$  mit periodischen Abständen der Nullstellen ihrer Integrale*, Wissenschaftliche Schriftenreihe der Technischen Hochschule Karl-Marx-Stadt, 1975. (5. Tagung über Probleme und Methoden der mathematischen Physik, 1975), 239–255.
- [79] *Теория глобальных свойств обыкновенных линейных дифференциальных уравнений второго порядка, Дифференциальные уравнения, Минск, 12, 1976, 1347–1383. English translation: Theory of the global properties of second order linear ordinary differential equations, Differential equations, 12, No. 8, 1977, 949–975.*
- [80] *Contribution à la théorie algébrique des équations  $Y'' = Q(T)Y$* , Bollettino U.M.I., (5) **13-B**, 1976, 896–915.
- [81] *Algebraic methods in the theory of global properties of the oscillatory equations  $y'' = q(t)y$* , Equadiff IV – Proceedings, Prague 1977, Lecture Notes in Mathematics **703**, Springer, Berlin – New York 1979, 35–45.
- [82] *Sur une classe des groupes continus à un paramètre formés des fonctions réelles d'une variable*, Ann. Polon. Math., **XLII**, 1983, 25–35.
- [83] *Sur les transformations simultanées de deux équations différentielles linéaires du deuxième ordre dans elles-mêmes*, Applicable Analysis **15**, 1983, 187–200.
- [84] *Sur les sous-groupes des groupes des dispersions des équations différentielles linéaires du deuxième ordre*, Proc. Roy. Soc. Edinburgh **97 A**, 1984, 35–41.
- [85] *Sur les blocs des équations différentielles linéaires du deuxième ordre et leurs transformations*, Čas. pěst. mat. fys. **111**, 1986, 78–88.
- [86] *Algebraic spaces with operators and their realization by differential equations*. Text of “Seminar on differential equations”, 1988, 35 pp.

Further publications of Otakar Borůvka, mostly of a bibliographical or historical nature, are listed in a special monograph *Otakar Borůvka*, 240 pages, published by the Masaryk University, Granos Brno 1996, (in Czech with English, French and German summaries), ISBN: 80 - 902004 - 0 - 0.

This book contains also his very interesting comments to the development of mathematics in this century and personal remembrances to meetings with famous mathematicians in the last fifty years.