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Aggregation and disaggregation in Markov chains

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ANNOUNCEMENTS OF NEW RESULTS
(of authors having an address in Czechoslovakia)

ON THE CONVERGENCE OF NEUMANN SERIES FOR NONCOMPACT OPERATORS

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We consider solving of the equation

$$(I - T)y = x \tag{1}$$

by means of the Neumann series. We can show that for an element x of the Banach space X and for a bounded linear operator T on X, such that the distance of T from the subspace of all compact linear operators acting on X is less than 1, the series $\sum T^n x$ converges to a solution y of the equation (1) if and only if $T^n x$ converges weakly to zero as $n \to \infty$. This extends the earlier result of N. Suzuki (cf. [1]) dealing with a compact T.

REFERENCES

[1] Suzuki N., *On the convergence of Neumann series in Banach space*, Math. Ann. **220** (1976), 143-146.

AGGREGATION AND DISAGGREGATION IN MARKOV CHAINS

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The aggregation — disaggregation method helps to manage difficulties following from very complicated phenomena so that it enables to investigate the phenomena on two or even more hierarchical nicety levels. This method can also be used to *compute the stationary distribution of the finite homogeneous Markov chain* (with transition probability matrix $\mathbf{P}$).

Let us consider the following algorithm:

0. Choose an *initial distribution* $\mathbf{p}^0 > O$, set $k = 0$.

1.1. *Aggregation:* compute the aggregated transition probability matrix $\bar{\mathbf{P}}(\mathbf{p}^k)$.

1.2. *Solving the aggregated problem:* compute the aggregated stationary distribution $\bar{\mathbf{p}}(\mathbf{p}^k)$ concerning the transition probability matrix $\bar{\mathbf{P}}(\mathbf{p}^k)$.

1.3. *Disaggregation:* compute the corrected distribution $\tilde{\mathbf{p}}^k$ by disaggregating the aggregated stationary distribution.

2. Perform 1 iteration step of *the successive approximations method:* $\mathbf{p}^{k+1} = \tilde{\mathbf{p}}^k\mathbf{P}$.

3. *Convergence test:* if the norm $\|\mathbf{p}^{k+1} - \mathbf{p}^k\|$ is sufficiently small, the computing finishes, otherwise increase k by 1 and repeat the computing from the step 1.1 ($\|x\|_1$ can be used as the norm).

A theorem is valid for this algorithm:

Theorem. *Let* $\mathbf{p}$ *be a stationary distribution of a finite homogenous irreducible Markov chain with the transition probability matrix* $\mathbf{P}$. *Let* $\mathbf{P}$ *have at least one column with all elements positive. Then the iterative aggregation — disaggregation method locally converges to* $\mathbf{p}$, *i.e., an open neighborhood* U *of* $\mathbf{p}$ *exists such that for all* $\mathbf{p}^0 \in U$ *the sequence of vectors generated by the described iteration method converges to* $\mathbf{p}$.

It seems that local convergence can also be proved for a modified algorithm for computing the eigenvectors of other kinds of matrices.

Tiskařské závody, s. p., provoz 66, Příbram