Commentationes Mathematicae Universitatis Carolinae

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Commentationes Mathematicae Universitatis Carolinae, Vol. 20 (1979), No. 1, 107--110

Persistent URL: http://dml.cz/dmlcz/105906

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20, 1 (1979)

FACTORING UNCONDITIONALLY CONVERGING OPERATORS

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Abstract: It is shown that an unconditionally converging operator factors through a Banach space containing no isomorphs of c_0.

Key words and phrases: Unconditionally converging operator, Banach space.

AMS: 47A05

An operator T mapping a Banach space X into a Banach space Y is unconditionally converging (uc) if it maps weakly unconditionally converging (wuc) series of X into unconditionally converging (uc) series in Y. On page 260 of [2] the usefulness of factoring a uc operator is pointed out. Our aim is to show that such a factorization does occur, that is, if T is a uc operator, then T factors through a Banach space containing no isomorphs of c_0. The proof is similar to that for weakly compact operators in [1]. We use NX to denote the set $\{F \in X''$: there exists a wuc series $\sum x_n$ in X such that $F = \sigma(X'',X')\text{-}\lim_n \sum_{i=1}^{n} Jx_i\}$. Here J is the canonical embedding map of X into X". Well known facts are that wuc series are uc if and only if X does not contain an isomorph of c_0 if and only if JX = NX (see [3]). Let KX be the weak* sequential closure of JX in X". Note that KX and NX are norm closed in X". This is proven in [4] for KX and a similar proof holds for NX. Let W be a convex, symmetric and bounded subset of X. For $n = 1,2,\ldots$ the gauge $\| \|_n$ of the set $U_n = 2^n W + 2^{-n}B_X$ (B_X is the unit ball of X) is a norm equivalent to $\| \|$. Define, for $x \in X$, $|||x||| = (\sum_{n=1}^{\infty} \|x\|_n^2)^{1/2}$ and let $Y = \{x \in X: |||x||| < \infty\}$ and j denote the identity embedding of Y into X.

Lemma 1 ([1]) (i) $W \subseteq B_Y$

(ii) $(Y, |||\cdot|||)$ is a Banach space and j is continuous.

(iii) $j'': Y'' \longrightarrow X''$ is one to one and $(j'')^{-1}(X) = Y$.

Lemma 2 $JY = NY$ if and only if every wuc series is uc in W (as a subset of X).

Proof: We first show that the $\sigma(NX,X')$ closure of B_Y in NX is $j''(B_{NY})$. B_{NY} is norm closed and bounded in Y'', hence $\sigma(Y'',Y')$ - compact; and thus $\sigma(NY,Y')$ - compact. B_Y is $\sigma(Y'',Y')$ dense in $B_{Y''}$ (Goldstine Theorem), so $\sigma(Y'',Y')$ dense in B_{NY}, and hence $\sigma(NY,Y')$ dense in B_{NY}. Since j'' is weak* continuous, $j''(B_{NY})$ is $\sigma(NX,X')$ closed (being $\sigma(NX,X')$ compact) and $j''(B_Y) = B_Y$ is $\sigma(NX,X')$ dense in it.

Now, if every wuc series is a uc series in W ($W \subseteq X$), and $\overline{W}$ denotes W together with all limit points of wuc series in W, then $2^n\overline{W}+2^{-n} B_{NX}$, $n = 1,2,\ldots$ contain B_Y and are $\sigma(NX,X')$ closed, hence they contain $j''(B_{NY})$. Since

$$\bigcap_n (2^n \overline{W}+2^{-n} B_{NX}) \subseteq \bigcap_n (X+2^{-n}B_{X''}) = X$$

it follows $j''(B_{NY}) \subseteq X$, hence by Lemma 1 (iii), $NY \subseteq Y$.

The converse follows by using Lemma 1 (i) and the weak topology for uc series (Orlicz-Pettis Theorem).

Theorem 3 Every uc operator factors through Banach spaces containing no isomorphs of c_0.

Proof: Let $T:Z \longrightarrow X$ be uc and let W of Lemma 1 be $T(B_Z)$. Then the operators $j^{-1} \circ T:Z \longrightarrow Y$ and $j:Y \longrightarrow X$ provide the required factorization.

As in [3] we say $T:X \longrightarrow Y$ is weakly completely continuous (wcc) if T sends weak Cauchy sequences into weakly convergent sequences. As NX is to uc operators, so KX is to wcc operators and similar results can be obtained (see [3]): Note that KX = JX if and only if X is weakly sequentially complete. Since it is a matter of using sequences instead of series, we state without proof the following.

Lemma 4 JY = KY if and only if W is weakly sequentially complete (as a subset of X).

Theorem 5 Every wcc operator factors through weakly sequentially complete spaces.

R e f e r e n c e s

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(Oblatum 26.10. 1978)